The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1673/32

AUTUMN COLLEGE ON PLASMA PHYSICS

5 - 30 September 2005

Nonlinear Effects in Fundamental Physics

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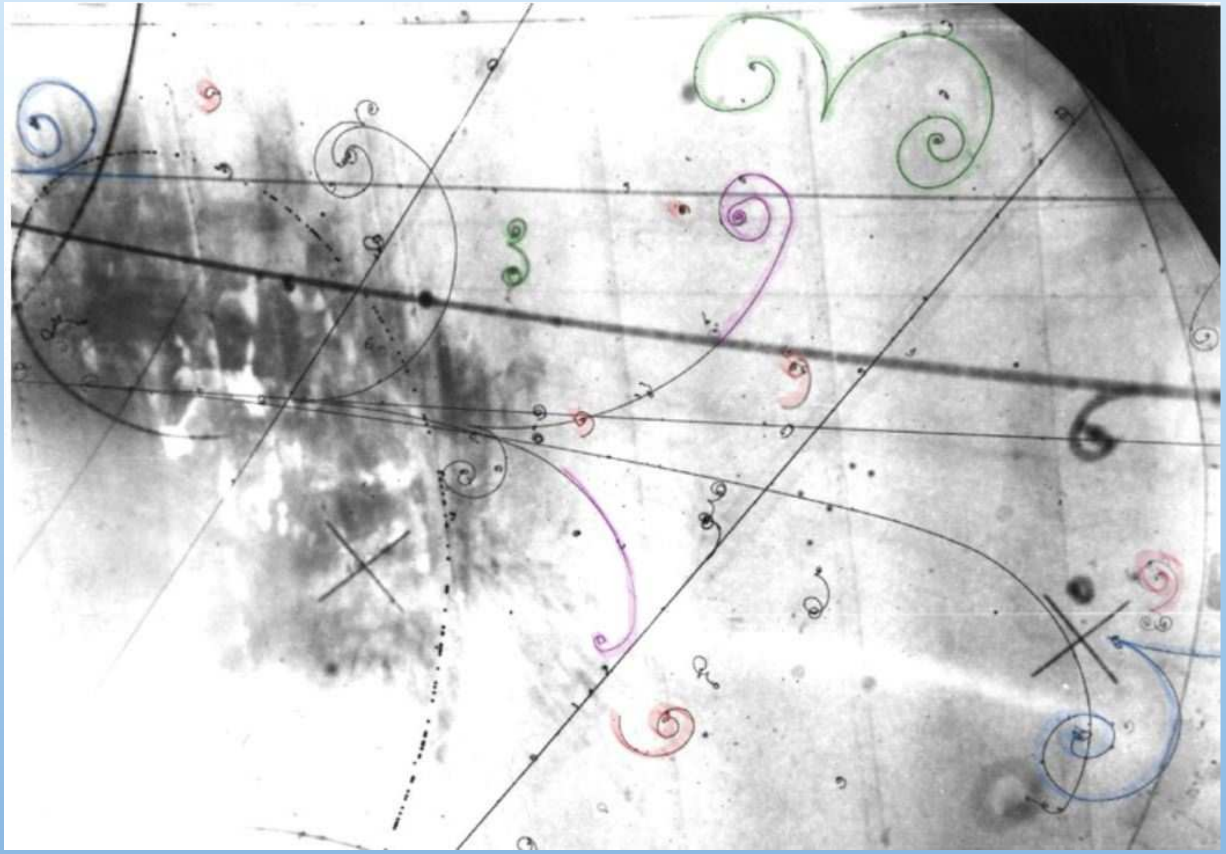
Nonlinear Effects in Fundamental Physics

MATTIAS MARKLUND



Department of Physics, Umeå University, Sweden

Overview

- Nonlinear phenomena
- Fundamental physics, e.g.
    - General relativity
    - Quantum electrodynamics
    - Modified standard model
- Applications to quantum field theory:
    - Axion and dark matter search
    - The Casimir effect
    - The Hawking/Unruh effect
    - Photon–photon scattering vs. pair creation
- Conclusions

Nonlinear phenomena?

Dynamics often nonlinear:

- Atmosphere

Trombes.

Nonlinear phenomena?

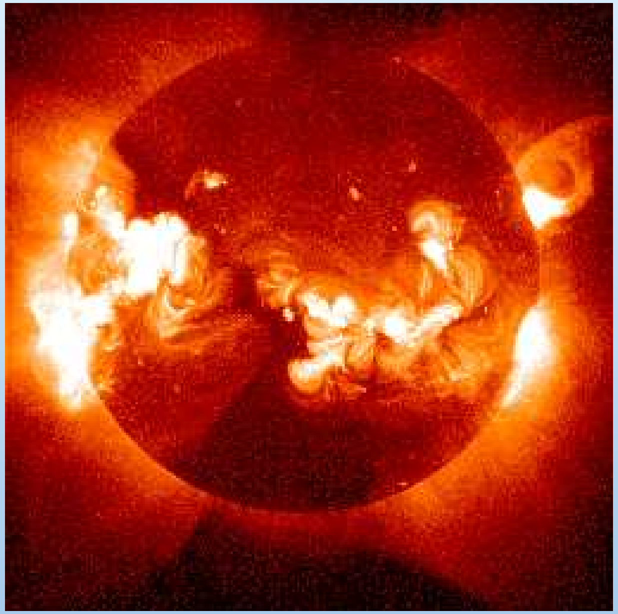
Dynamics often nonlinear:

- Atmosphere
- Ocean

Nonlinear phenomena?

Dynamics often nonlinear:

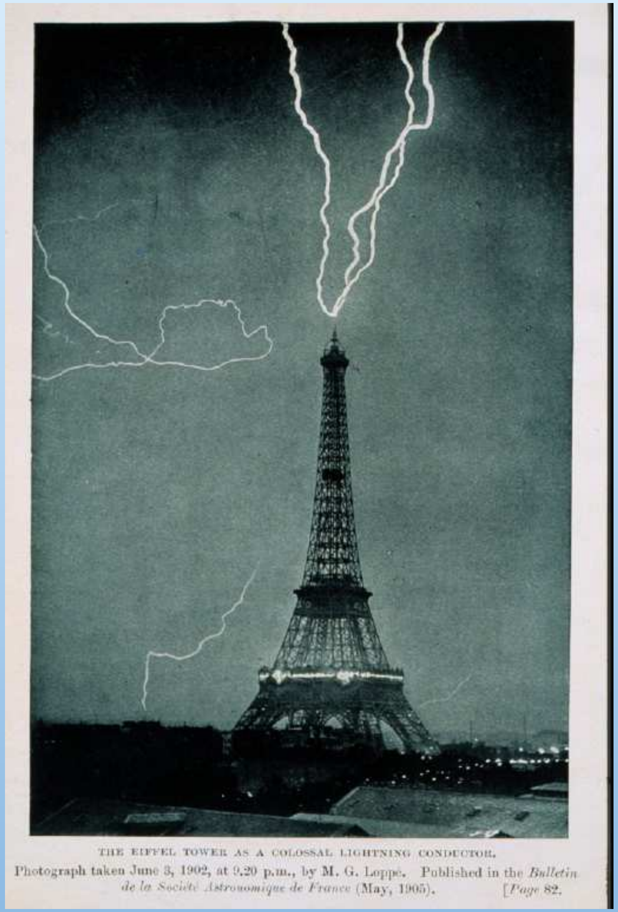
- Atmosphere
- Ocean
- The Sun



Nonlinear phenomena?

Dynamics often nonlinear:

- Atmosphere
- Ocean
- The Sun
- Electromagnetic waves in media



THE EIFFEL TOWER AS A COLOSSAL LIGHTNING CONDUCTOR.

Photograph taken June 3, 1902, at 9.20 p.m., by M. G. Loppé. Published in the *Bulletin de la Société Astronomique de France* (May, 1905). [*Page* 82.

Nonlinear phenomena?

Dynamics often nonlinear:

- Atmosphere
- Ocean
- The Sun
- Electromagnetic waves in media
- Turbulence

Examples

- Dynamics in terms of governing equations:
  - Pendulum: $\theta'' + k\theta' + (g/l)\underline{\sin\theta} = 0$;

    $\Longrightarrow$ *complicated dynamics, depending on damping/forcing* k.
  - Fluids: $\partial_t v + \underline{v\partial_x v} = 0$;

    $\Longrightarrow$ *wave steepening and wave breaking*.
  - Electromagnetic waves $\rightarrow$ nonlinear Schrödinger equation: $i\partial_t E + \nabla^2 E + \underline{|E|^2 E} = 0$;

    $\Longrightarrow$ *soliton formation*.
  - $\lambda\phi^4$-theories: $(\partial_t^2 - c^2\nabla^2)\phi = \underline{\lambda|\phi|^2\phi}$;

    $\Longrightarrow$ *soliton formation*.

Some ‘hot’ topics in physics...

Quantum field theory and intense EM fields

- Fundamental issues regarding the theory of interaction between, e.g. light and matter.
- The structure of the nonlinear quantum vacuum.
- Symmetry breaking and Lorentz invariance.
- Production of anti-matter.
- Necessary component in next generation laser–plasma systems.
- Laboratory astrophysics.

Quantum electrodynamics

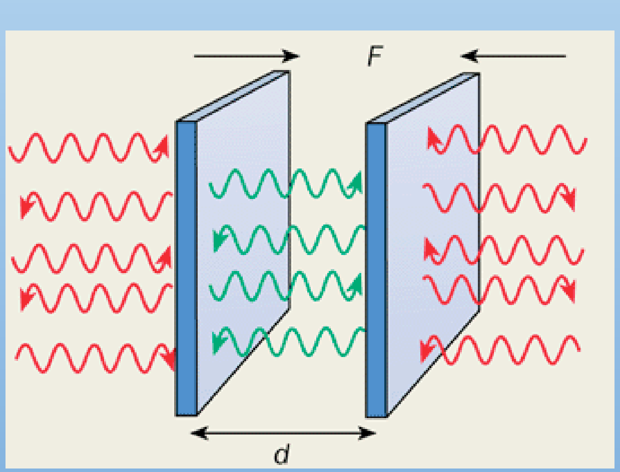
- Classical electromagnetism: Electromagnetic waves indifferent to each other, unless interacting through medium (plasma, optical fibre, ...)
- Quantisation of electromagnetic interactions (Dyson, Feynman, Schwinger, Tomonaga) $\longrightarrow$ Notion of photon (Planck).
- Classical electromagnetic interactions seen as particle scattering (electrons and photons): Electrons exchange virtual photons.
- One of our most well-confirmed theories, in terms of accuracy.

Effects of QED

- The concept of anti-matter.
- Pair creation at high energies: Vacuum fluctuations takes real form when supplied high enough external energy.
  Ex: Electron–positron creation outside neutron stars, due to extreme field strengths ($10^{10} - 10^{13}$ G),
  $\gamma + B \rightarrow e^{+} + e^{-} + B$.

Effects of QED

- The concept of anti-matter.
- Pair creation at high energies: Vacuum fluctuations takes real form when supplied high enough external energy.
  $\gamma + B \rightarrow e^+ + e^- + B$.
- Casimir effect: Attraction of conducting plates in vacuum.



Force $F \sim A/d^4$. Predicted 1948, confirmed 1997.

Effects of QED

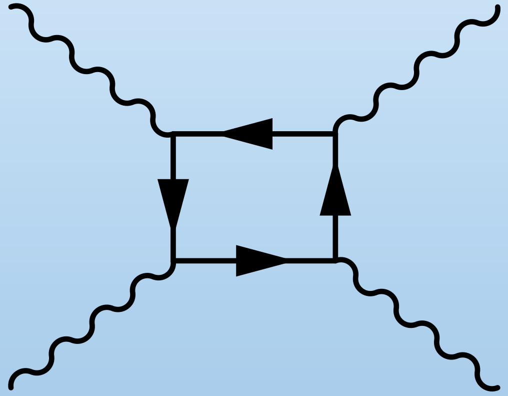
- The concept of anti-matter.
- Pair creation at high energies: Vacuum fluctuations takes real form when supplied high enough external energy.
  $\gamma + B \rightarrow e^+ + e^- + B$.
- Casimir effect: Attraction of conducting plates in vacuum.
- Vacuum polarisation: Radiation travelling through vacuum will experience it differently depending on EM polarisation.
- Vacuum polarisation due to photon–photon scattering.

The Casimir vacuum

- Vacuum can be 'excited' by external electromagnetic fields (Dittrich & Gies, 2000).
- The dispersion relation for a test photon in excited vacuum: $\omega = kc(1-\delta)$. δ constant depending on energy density of exciting field.
- Examples:
  - $\delta \propto (E^2 + c^2B^2) > 0$ for EM field.
  - $\delta \propto T^4 > 0$ for thermal background.
- δ may be negative in curved spacetime (effective photon size $\leftrightarrow$ tidal forces may act).
- For a Casimir vacuum, $\delta \propto -d^{-4}\cos\theta < 0$.
- Thus, vacuum phase and group velocities $> c$ (Scharnhorst, PLB, 1990).
- Boundary conditions → *not* Poincaré invariant.

Photon–photon scattering

- Photons can collide due to virtual electron–positron pairs



Effective field theory, no parallel plane wave interactions. Yet to be detected!

Photon–photon scattering

- Photons can collide due to virtual electron–positron pairs
- Effective field theory $\longrightarrow$ Heisenberg–Euler Lagrangian (J. Schwinger, PR, 1950).

$$\mathscr{L} = -\tfrac{1}{4}\epsilon_0 F_{ab}F^{ab} + \kappa\epsilon_0^2 \left[4(\tfrac{1}{4}F_{ab}F^{ab})^2 + 7(\tfrac{1}{4}F_{ab}\widehat{F}^{ab})^2\right].$$

Nonlinear corrections to Maxwell's equations.

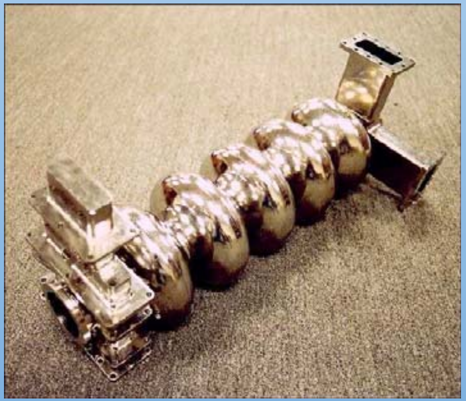
- Modified propagation of photons through 'vacuum'.

Photon–photon scattering

- Photons can collide due to virtual electron–positron pairs
- Effective field theory ⟶ Heisenberg–Euler Lagrangian (J. Schwinger, PR, 1950). Nonlinear corrections to Maxwell's equations.
- Modified propagation of photons through 'vacuum'.
- Single particle effects:
  - ○ Vacuum birefringence: depending on polarisation, photons move with different velocities in strong magnetic fields.
  - ○ Similarly, lensing in strong magnetic fields.
  - ○ Photon splitting.
- Collective effects: Self-interaction of electromagnetic field, cubic corrections to Maxwell's equations.

Where?

- Ultra-intense lasers and plasma accelerators, ‘brute force’ (Soljačić and Segev, PRA, 2000; B. Shen, M.Y. Yu and X. Wang, PoP, 2003). Field strength today 10^{13} $\mathrm{V/m}$ in short pulses (cf. critical value $E_{\text{crit}} \approx 10^{18}$ $\mathrm{V/m}$).
- Super-conducting, ultra-high field strength ($10 - 100$ $\mathrm{MV/m}$) cavities and waveguides, resonant interaction between modes (Brodin, Marklund, and Stenflo, PRL, 2001).



Niobium cavity ~ 0.5 m long (Jefferson Lab).

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- Magnetars, field strength $10^{14}-10^{15}$ G. (C. Kouveliotou *et al.*, Nature, 1998; A.K. Harding, Science, 1991)

Applications

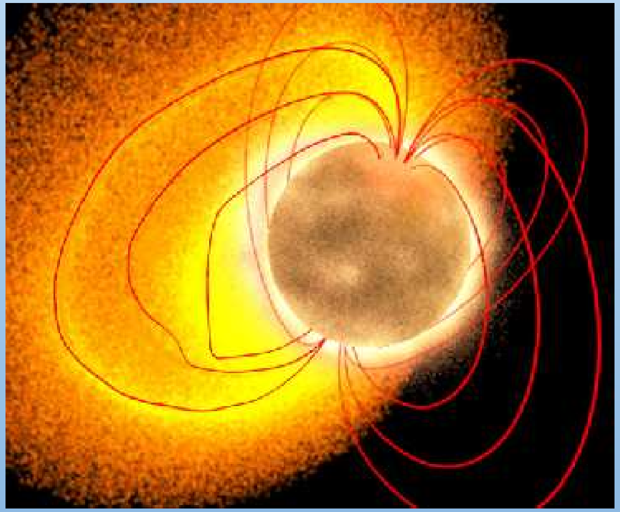
Outside magnetars, only single particle effects have previously been investigated:

1. Lensing of light rays due to ultra-strong magnetic fields (S.L. Adler, Ann. Phys.-NY, 1971; N.J. Shaviv, J.S. Heyl and Y. Lithwick, MNRAS, 999).
2. Vacuum birefringence in strong dipole fields (J.S. Heyl and L. Hernquist, J. Phys. A: Math. Gen.,1997).
3. Photon splitting $\Longrightarrow$ radio quiet magnetars (removal of electron–positron plasma) (M.G. Baring and A.K. Harding, ApJ, 1998).

Applications

Single particle effects, focus on *static* magnetic fields. Other situations predominantly reactive:

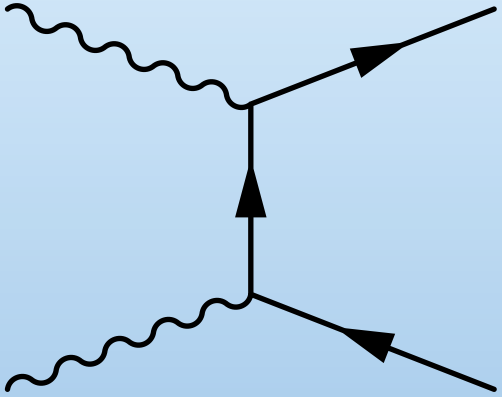
- Early Universe ⟶ interacting random hot photons.
- Magnetars ⟶ quakes from magnetic tension ⟶ stochastic, incoherent strong fields.



- Possible laboratory application: Wave kinetics in ultra-intense lasers, strong fields in plasma accelerators.

Direct creation of pair plasma from light

- Pair creation: $\gamma + \gamma \rightarrow e^+e^-$.

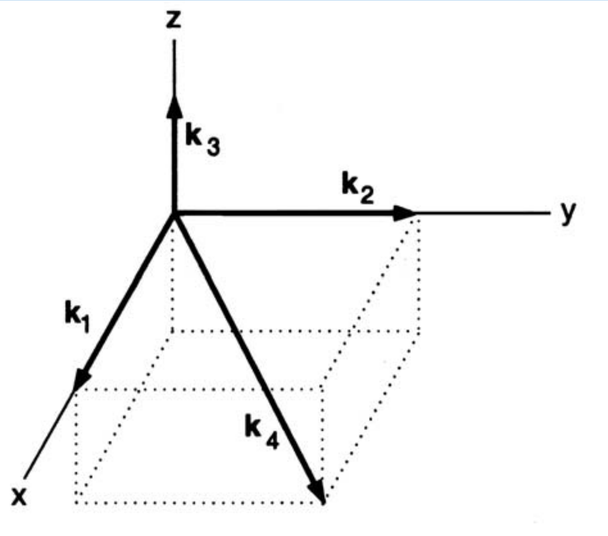


Direct creation of pair plasma from light

- Pair creation: $\gamma + \gamma \to e^+e^-$.
- Creation rate per unit volume $w = \frac{m_e^4 c^5}{(2\pi)^3\hbar^4}\left(\frac{|\mathbf{E}|}{E_{\text{crit}}}\right)^2 \sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n\pi\frac{E_{\text{crit}}}{|\mathbf{E}|}\right)$; Schwinger field $E_{\text{crit}} \approx 10^{18}\,\mathrm{V/m} \leftrightarrow 3\times 10^{29}\,\mathrm{W/cm^2}$.
- *Inelastic* photon–photon scattering.
- Nonlinear Compton (NLC) scattering: $e + n\omega \to e' + \gamma$; single high energy photon created from n photons.
- Laser can induce NLC scattering in high energy electron beam.
- NLC backscattered γ interacts with laser beam photons: Breit-Wheeler process $\gamma + n\omega \to e^+e^-$.
- Achieved by Rochester group (Burke et al., PRL, 1997; Bamber et al., PRD, 1999)!

Detection of elastic γ-γ scattering

- High repetition rate, ultra-intense lasers ideal tools.
- Nonlinearity → four-wave mixing yields new mode; Three interacting waves (1, 2, 3) yields fourth (4) by nonlinearity. Resonance conditions
  $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4$
  $\omega_1 + \omega_2 = \omega_3 + \omega_4$
- Choice:



Detection of elastic γ-γ scattering

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  $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4$
  $\omega_1 + \omega_2 = \omega_3 + \omega_4$
- Number of generated photons through nonlinear quantum vacuum effect:

$$N \approx 1.31\eta^2 G^2 \left(\frac{1\,\mu\mathrm{m}}{\lambda_4}\right)^3 \left(\frac{L}{1\,\mu\mathrm{m}}\right) \left(\frac{P_1 P_2 P_3}{1\,\mathrm{PW}^3}\right),$$

L pulse length, λ_4 generated photon wavelength, P_j incoming pulse power. $\eta^2 \sim 0.025$ and $G^2 \sim 0.77$ factors depending on experimental geometry and setup.

Detection

Power [PW]	N_a	N_b
0.5	0.066	0.27
2.5	8.3	33
5	66	266
25	8.3×10^3	3.3×10^4
50	6.6×10^4	2.7×10^5

Upgraded Astra Gemini Laser (RAL): two 0.5 PW laser beams, $\lambda = 800\,\mathrm{nm}$, pulse energy $\sim 15\,\mathrm{J}$, focal intensity $> 10^{22}\,\mathrm{Wcm}^{-2}$, repetition rate one shot per minute. Here power of incoming beams: $P_1 = P_2 = 0.1\,\mathrm{PW}$, $P_3 = 0.5\,\mathrm{PW}$.

Detection

- Noise:
  - Optical component scattering — solved by experimental geometry.
  - Compton scattering — effect two orders of magnitude below QED effect.
  - Harmonic generation — plasma blow-out creates high vacuum, nonlinear plasma effects non-existent.
- Thus: With several runs, statistics would tell if detection has been achieved.

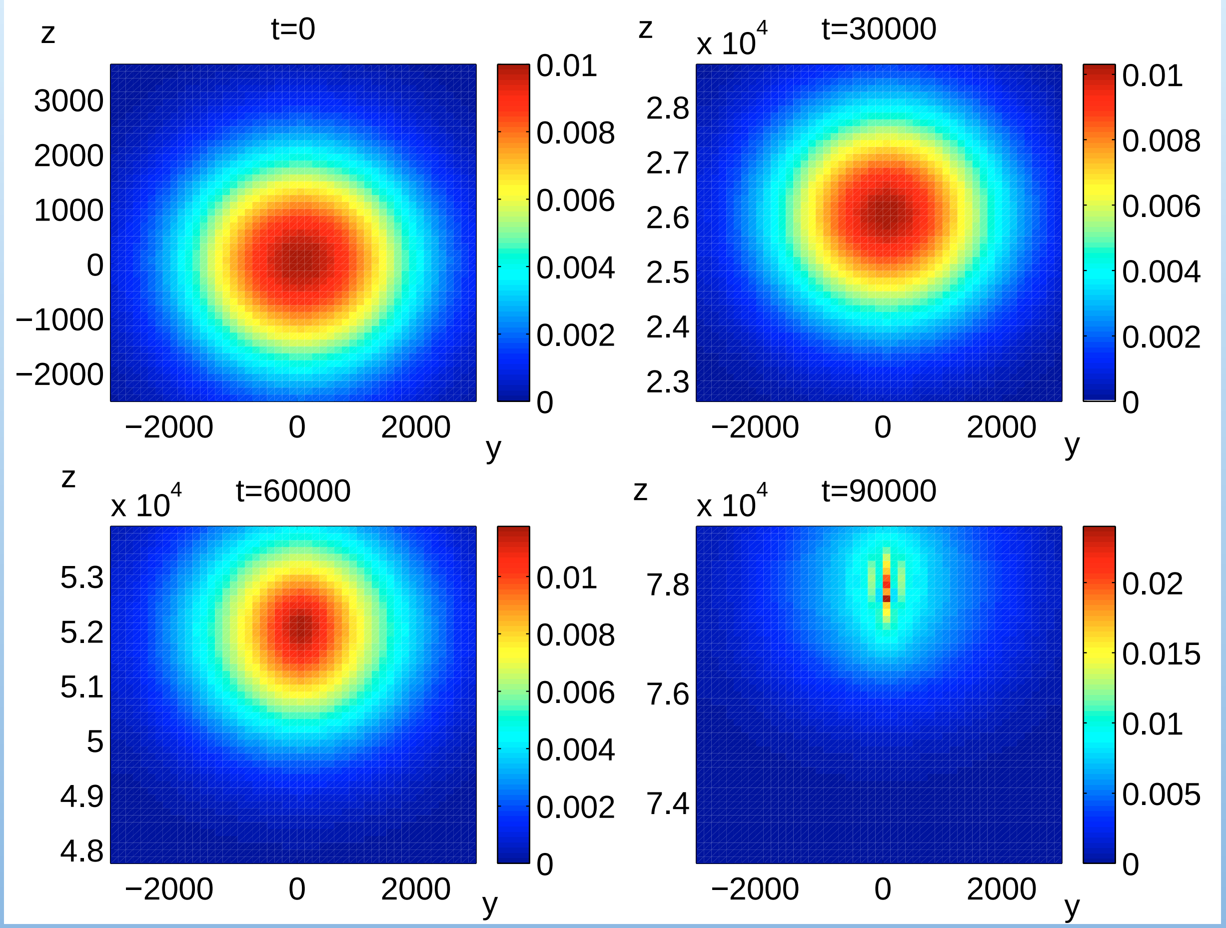
Collective effects

- Background field backscatters $\longrightarrow$ modified background and modified photon propagation. Effectively leads to *self-interaction*. Coupling $\propto \alpha^2$.
- Coherent fields: Crossing lasers, waveguides and cavities. Typically NLSE

$$iE_t + \beta E_{xx} + \kappa |E|^2 E = 0$$

Self-focusing, in 2-D collapse (G. Brodin *et al.*, PLA, 2003).

Collective effects



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$$iE_t + \beta E_{xx} + \kappa |E|^2 E = 0$$

Self-focusing, in 2-D collapse (G. Brodin *et al.*, PLA, 2003).

- Incoherent fields and kinetic effects: Magnetar quakes, early Universe, CMB, gamma-ray bursts, laser pulses. Typically Zakharov–Karpman like system (M. Marklund, G. Brodin, L. Stenflo, PRL, 2003)

$$iE_t + \beta \nabla_\perp^2 E + \kappa \mathscr{E} E = 0,$$

$$\mathscr{E}_{tt} - \tfrac{1}{3} c^2 \nabla^2 \mathscr{E} = -\kappa (|E|^2_{tt} + \nabla^2 |E|^2).$$

Wave collapse and high intensity pulses

- Collapse universal feature, e.g., plasmas, fluids, nonlinear optical media, Bose–Einstein condensates.
- After discovery of laser, indications of possible optical wave collapse for $P_{\text{laser}} > P_{\text{crit}}$.
- Self-action of pulse, due to interaction between matter and radiation $\longrightarrow$ 2-D self-focusing/defocusing in non-dispersive media.
- Halted collapse: Nonlinear absorption, material breakdown etc.
- Dispersive effects $\longrightarrow$ pulse splitting (normal dispersion), 3-D focusing (anomalous dispersion).

Collapse of EM pulses on radiation backgrounds

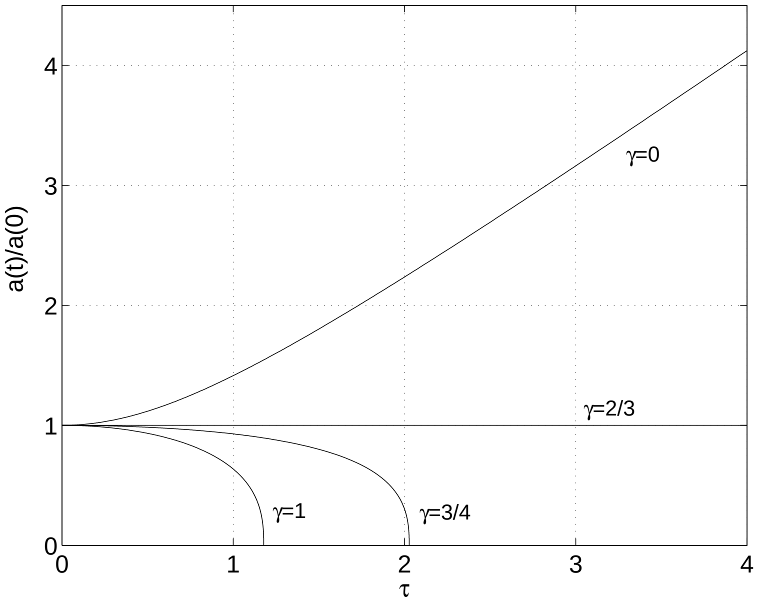
- EM pulse propagating on radiation background: Dynamics governed by coupled NLSE and acoustic wave equation

$$iE_t + \beta\nabla_\perp^2 E + \kappa\mathscr{E}E = 0,$$

$$\mathscr{E}_{tt} - \frac{c^2}{3}\nabla^2\mathscr{E} = -\kappa(|E|_{tt}^2 + \nabla^2|E|^2).$$

Collapse of EM pulses on radiation backgrounds

- EM pulse propagating on radiation background: Dynamics governed by coupled NLSE and acoustic wave equation
- Slow background response $\longrightarrow$ single 2-D NLSE $\Longrightarrow$ 2-D EM pulse collapse. Could be important in pulse propagation in stellar atmospheres, and small angular scales of CMB.



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- Slow background response $\longrightarrow$ single 2-D NLSE $\Longrightarrow$ 2-D EM pulse collapse. Could be important in pulse propagation in stellar atmospheres, and small angular scales of CMB.
- Can show that the system is modulationally and filamentationally unstable, small perturbations grow and collapse (P.K. Shukla, B. Eliasson, PRL, 2003). Inhomogeneities bound to grow.
- This spatial self-focusing lead to ultra-high intensity pulses.

3-D evolution

- Derivative corrections will be appreciable for short wavelengths. In terms of effective Lagrangian (S.G. Mamaev, V.M. Mostepanenko, M.I. Eĭdes, Sov. J. Nucl. Phys. 1981):

$$\mathscr{L}_D = \sigma\epsilon_0\left[(\partial_a F^{ab})(\partial_c F^{c}{}_{b}) - F_{ab}\Box F^{ab}\right]$$

Gives normal dispersive corrections to evolution equations (M. Marklund, B. Eliasson, P.K. Shukla, JETP Lett. 2004; P.K. Shukla, M. Marklund, D.D. Tskhakaya, B. Eliasson, PoP 2004):

$$i\,(\partial_t + \boldsymbol{v}_g\cdot\nabla)\,E + \beta(\nabla_\perp^2 - \beta_\parallel\nabla_\parallel^2)E + \kappa\mathscr{E}\,E = 0,$$

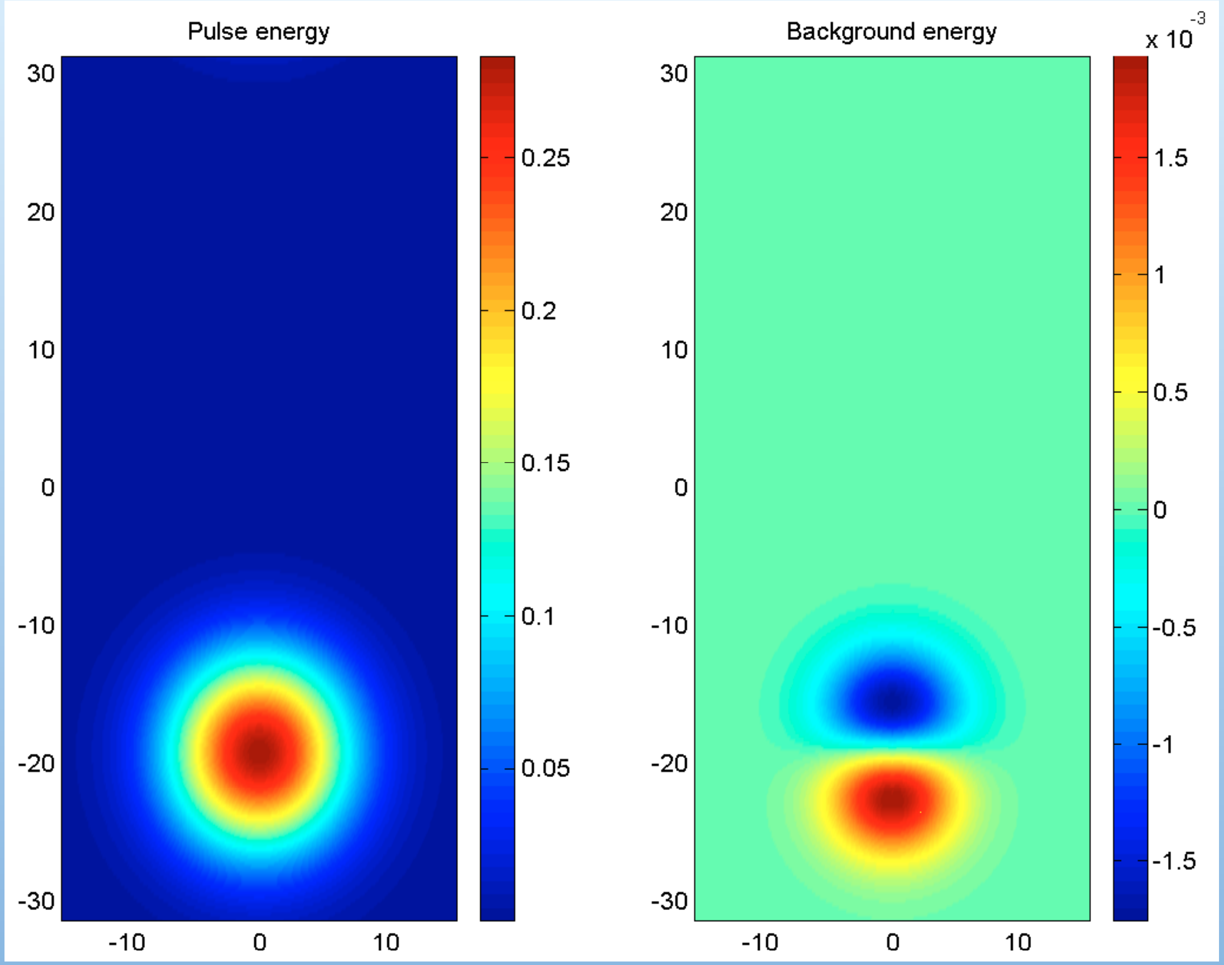
$$\mathscr{E}_{tt} - \tfrac{1}{3}c^2\nabla^2\mathscr{E} = -\kappa(|E|^2_{tt} + \nabla^2|E|^2),$$

where $\beta_\parallel \propto \sigma$.

3-D evolution

- Normal dispersive corrections to NLSE gives rise to pulse splitting and filamentation, ⟶ reduction of pulse intensity, while anomalous dispersion gives 3-D collapse (P. Chernev, V. Petrov, Opt. Lett. 1992; J.E. Rothenberg, Opt. Lett. 1992; N.A. Zharova, A.G. Litvak, V.A. Mironov, JETP 2003; A.L. Gaeta, Science 2003; Y.S. Kivshar, G.P. Agrawal, 2003).
- In QED, time of occurence of filamentation depends on initial parameters ⟶ possible to arrange for super-intense pulses before pulse split (?).

3-D evolution



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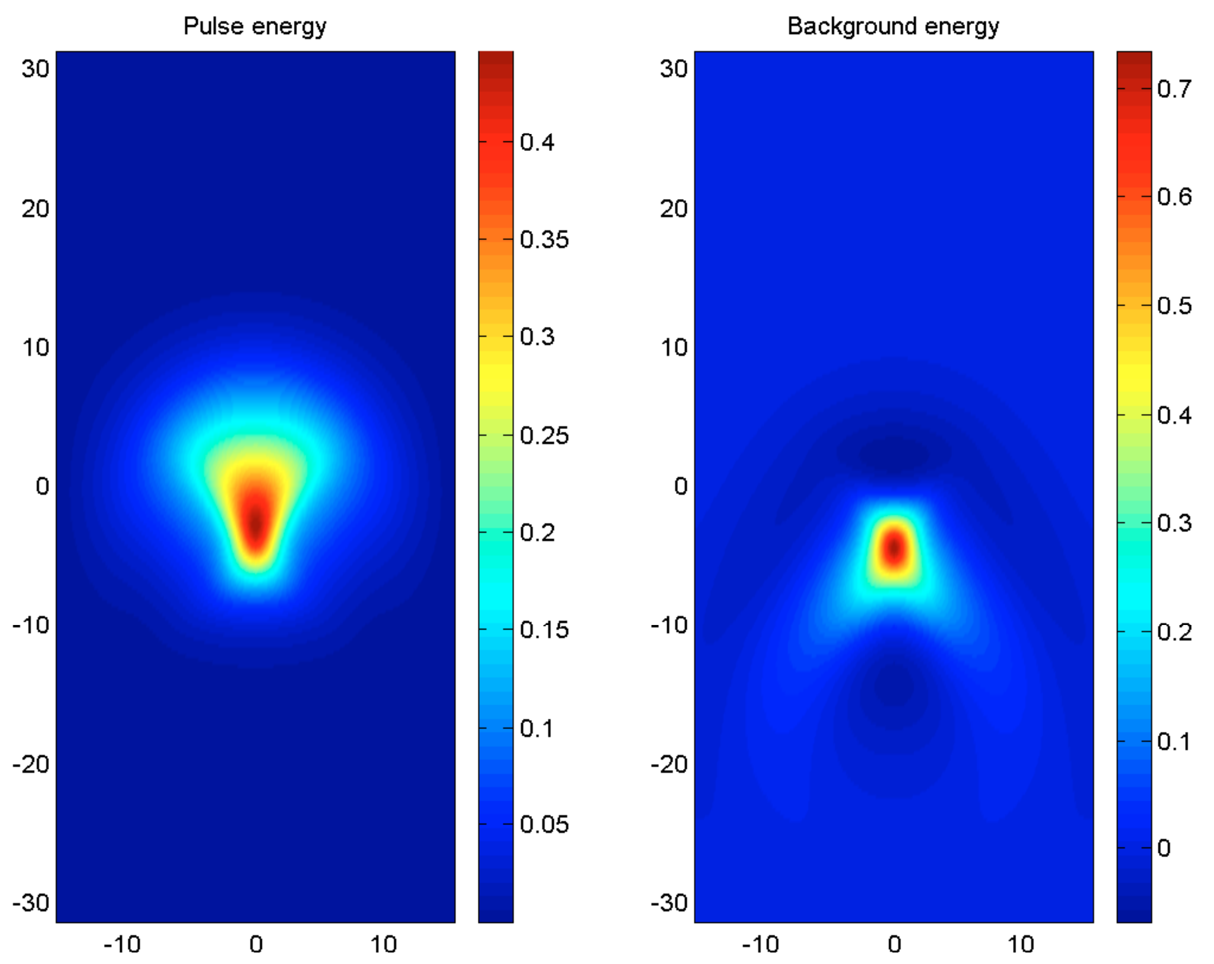

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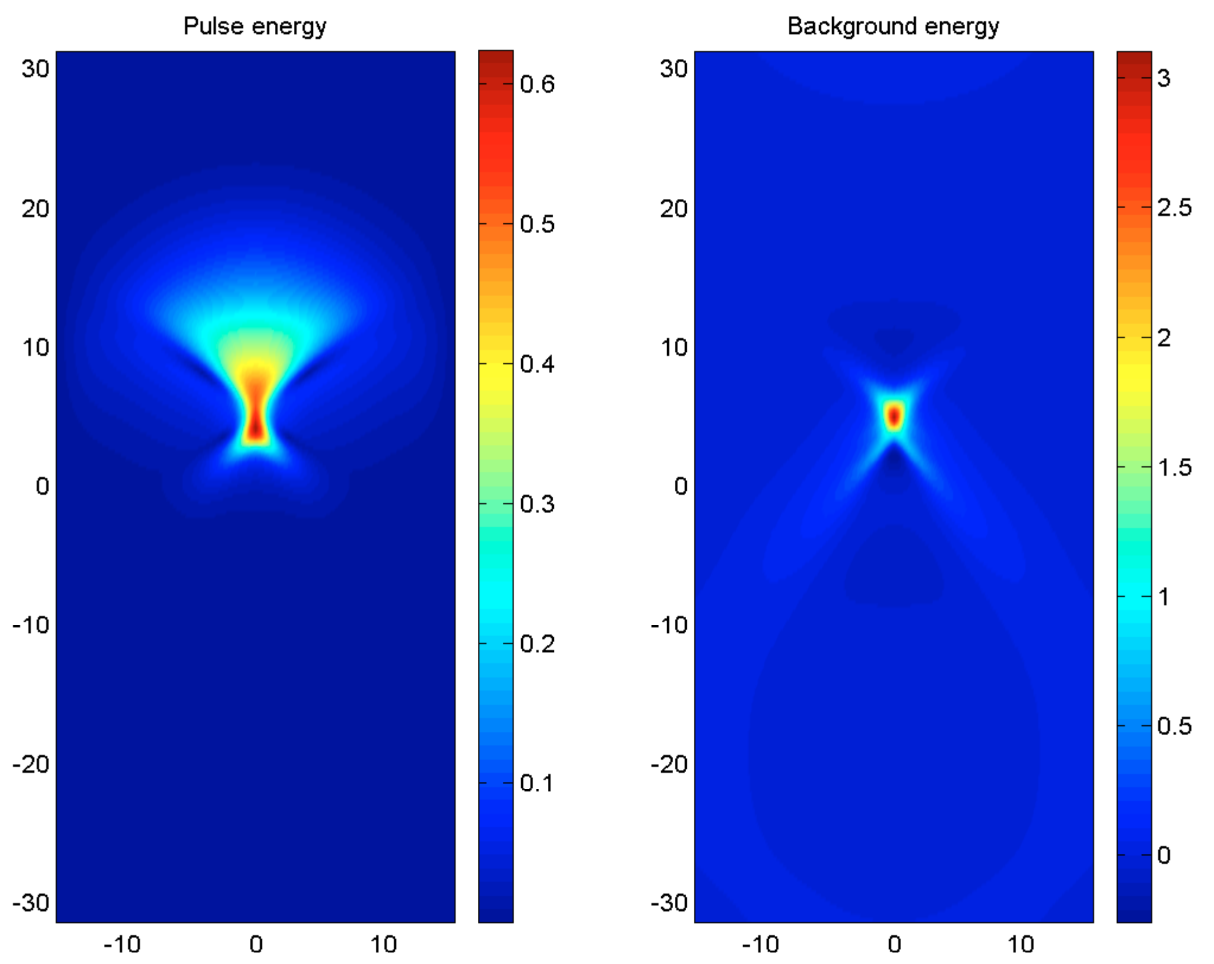

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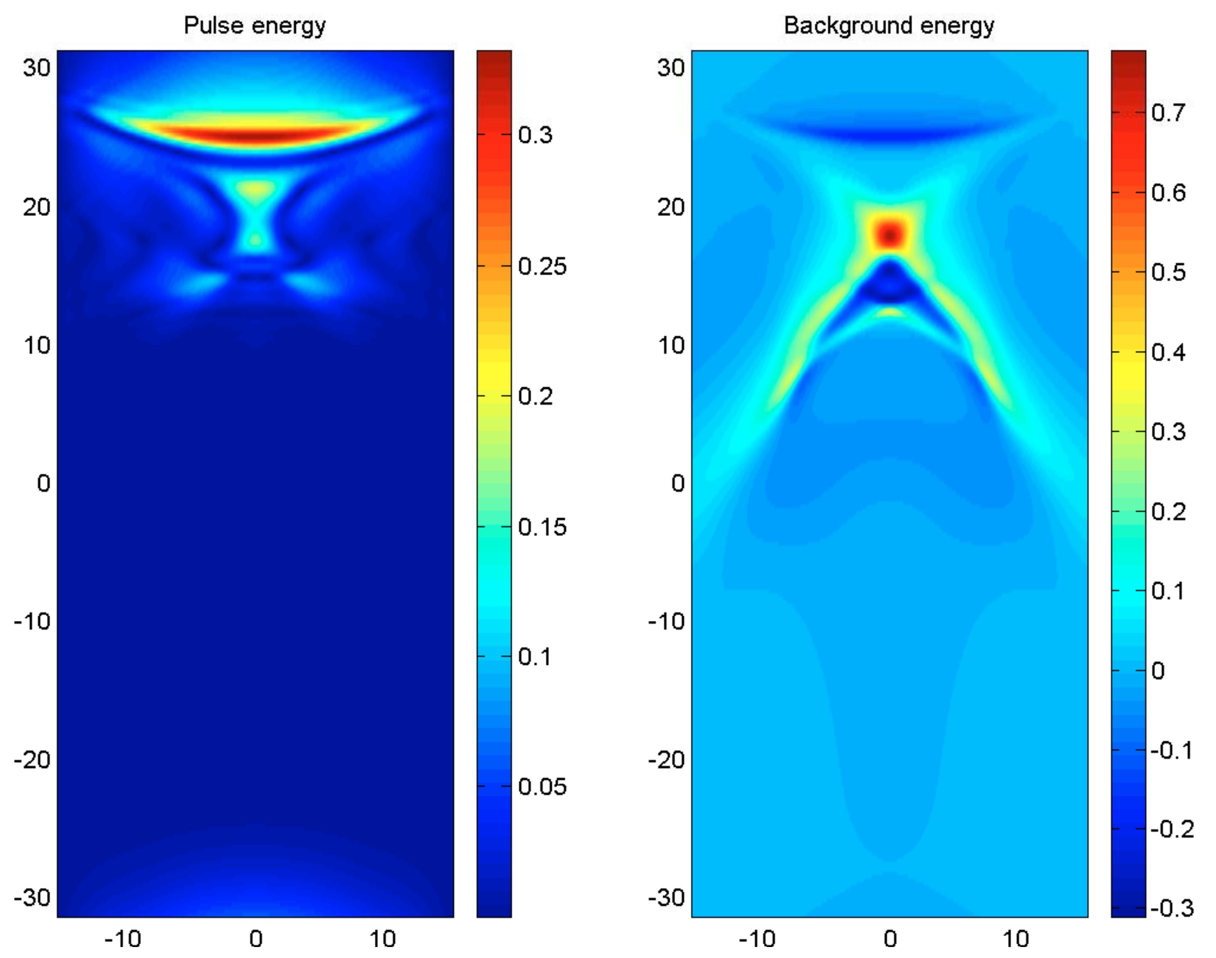
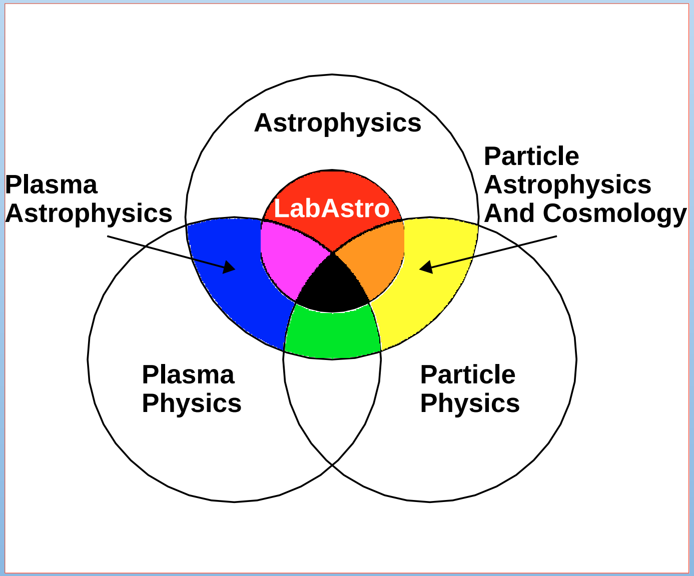


Dark matter and axion detection

- Axions: Light scalar particles.
- Introduced to explain the absence of certain symmetry breaks in standard QCD (e.g. Weinberg, PRL, 1978; Wilczek, PRL, 1978).
- Strong dark matter candidate.
- Virtual light scalars may interact with photons.
- Gives rise to vacuum birefringence (anisotropic refractive index), like γ-γ scattering, and dichroism (rotation of polarization) in magnetic field.
- Dichroism, due to photon absorption via axions, gives rise to unique effect.
- Recent claims of measurement (Zavattini et al., hep-ex/0507107, submitted to PRL).
- Dark matter detection in the lab !?

Laboratory Astrophysics

- Laser development → New energy regimes. Currently petawatt lasers, focal intensities $> 10^{24}\,\mathrm{W/cm^2}$; next step zetawatt lasers, x-ray free electron lasers.
- Astrophysical conditions in the laboratory (Remington, PPCF, 2005).



Laboratory Astrophysics

- Laser development → New energy regimes. Currently petawatt lasers, focal intensities $> 10^{24}\,\mathrm{W/cm^2}$; next step zetawatt lasers, x-ray free electron lasers.
- Astrophysical conditions in the laboratory (Remington, PPCF, 2005).
- Examples: Planetary equation of state, supernova shock, astrophysical turbulence etc.
- Pulsar magnetospheres.
- Hawking radiation.

CONCLUSIONS

- Nonlinear quantum vacuum effect important, both conceptually and as tool.
- Dark matter searches.
- Photon–photon scattering believed to be important in astrophysical environments, such as magnetars.
- Attempts to measure photon–photon scattering in lab unsuccessful, but...
- ...new possibilities for experimental verification.
- Fundamental tests of physical theories.
- Important in next generation laser systems.
- Formation of 2- and 3-D photon bullets.
- Creation of pulse intensities above laser limit.
- Approaching Schwinger limit...?

THANKS!

Centre for Fundamental Physics, Rutherford Appleton Laboratory (UK); G. Brodin, L. Stenflo, P. Shukla, R. Bingham, J. Collier, P. Norreys, D. Tskhakaya, T. Mendonça, E. Lundström, J. Lundin.