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Gravitational Interactions in Plasmas

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Gravitational Interactions in Plasmas

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Overview

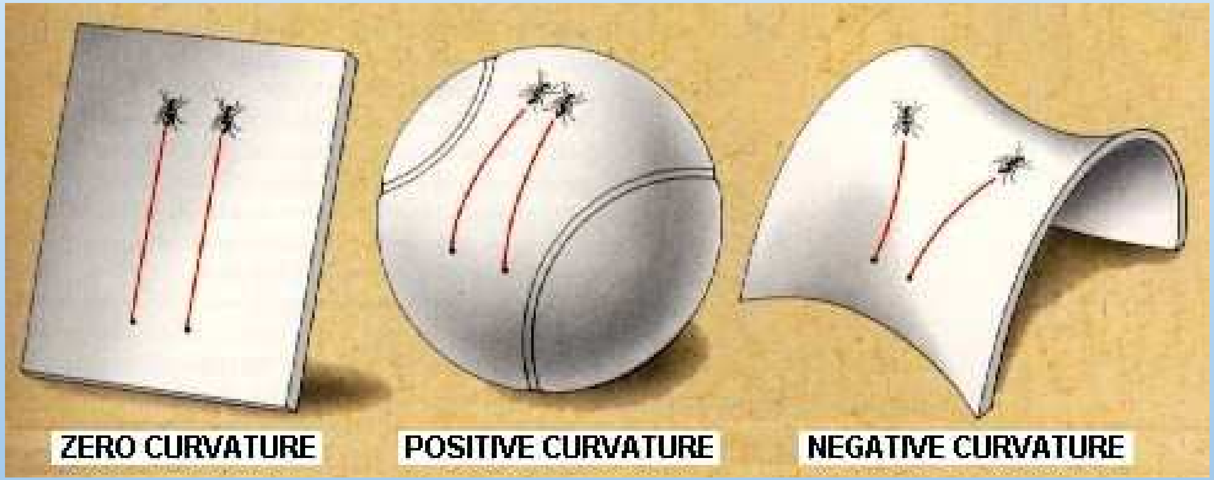
- Nonlinear phenomena
- General relativity
- Applications to general relativity:
  - Magnetic field generation
  - The dynamo
  - Gravitational waves
- Conclusions

Interesting problems in gravitational physics

1. *Gravitational wave propagation and detection*:
   - Weak field limit of general relativity.
   - Analogous to EM plane waves.
   - Carrier of invaluable astrophysical information.
2. *Magnetic field generation*:
   - All scales of the Universe.
   - Seed field generation.
   - Amplification.
   - New physics?

General relativity

- Geometric description of gravity.
- Gravitational force $\longrightarrow$ curved spacetime.



General relativity

- Geometric description of gravity.
- Gravitational force $\to$ curved spacetime.
- Spacetime is dynamic: $R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}$, spacetime curvature (LHS) couples to matter content (RHS). Gives
  - Spacetime collapse $\to$ black holes
  - Expanding Universe
  - Gravitational waves
- Energy-momentum tensors:
  - Perfect fluid: $T_{ab} = (\mu + p)u_a u_b + p g_{ab}$
  - EM field: $T_{ab} = (E^2 + B^2)u_a u_b + \frac{1}{2}(E^2 + B^2)g_{ab} + 2u_{(a}\epsilon_{b)cd}E^c B^d - (E_{(a}E_{b)} + B_{(a}B_{b)})$

Maxwell's equation

- Electromagnetic fields affected by gravity.
- Gravity couples back through Maxwell's equations[1]

$$\dot{\mathbf{E}} = -\tfrac{2}{3}\Theta\mathbf{E} + \bar{\bar{\sigma}}\cdot\mathbf{E} + \boldsymbol{\omega}\times\mathbf{E} + \mathbf{a}\times\mathbf{B} + \nabla\times\mathbf{B} - \mu_0\mathbf{j}\,,$$

$$\dot{\mathbf{B}} = -\tfrac{2}{3}\Theta\mathbf{B} + \bar{\bar{\sigma}}\cdot\mathbf{B} + \boldsymbol{\omega}\times\mathbf{B} - \mathbf{a}\times\mathbf{E} - \nabla\times\mathbf{E}\,,$$

$$\nabla\cdot\mathbf{E} = \rho/\epsilon_0 + 2\boldsymbol{\omega}\cdot\mathbf{B}\,,$$

$$\nabla\cdot\mathbf{B} = -2\boldsymbol{\omega}\cdot\mathbf{E}\,,$$

- Add the fluid equations of motion $\nabla_b T^{ab} = F^{ab} j_b \to$ closed system!

1. Marklund et al., Class. Quant. Grav. (2003)

Nonlinear gravitational-EM interaction

1. Magnetic field generation in cosmology:
   - Fluid vorticity $\nabla \times \mathbf{v}$ generates magnetic field.
   - Important issue regarding observed $\mu\mathrm{G}$-fields.[2]
   - Dynamo mechanism can enhance weak pre-existing seed field.[3]
   - Primordial magnetic field diluted by cosmological expansion.
   - How is nonlinear magnetic field generation affected by cosmological expansion?

2. Grasso & Rubinstein, Phys. Rept. (2001); Widrow, Rev. Mod. Phys. (2003)
3. Kronberg, Rept. Prog. Phys. (1994)

Magnetic field generation

- Start with cold plasma Friedmann–Robertson–Walker universe (homogeneous and isotropic):
  - Cosmological expansion Θ,
  - Equilibrium configuration $\longrightarrow$ charge neutrality and zero currents.
- Velocity (i.e., current) perturbations in cold charged two-species plasma model:
  - Nonzero initial fluid curl $\mathscr{K}_{\rm i} \propto |\vec{\nabla} \times \vec{v}|_{\rm i}$.[4]
  - Driven general relativistic wave equation for the magnetic field.[4]

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4. Betschart, Dunsby, & Marklund, Class. Quant. Grav. (2004); Battefeld & Brandenberger, Phys. Rev. D (2004)

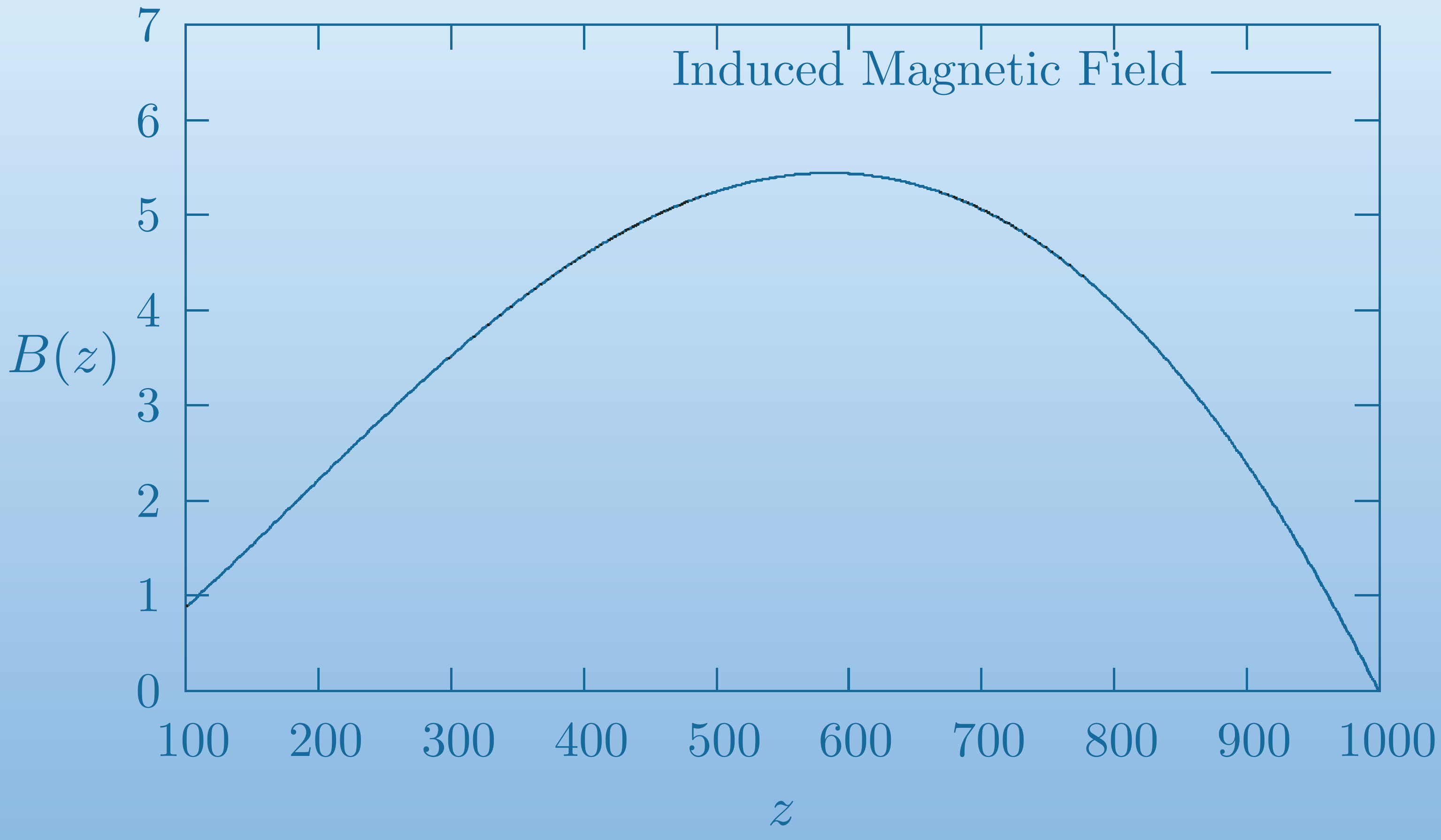
Magnetic field generation

- The magnetic field as a function of the redshift $z = (\lambda_{\text{obs}} - \lambda_{\text{emit}})/\lambda_{\text{emit}}$:

$$|B| \approx \mathscr{K}_{\rm i} h \left(\frac{1+z}{1+z_{\rm i}}\right)^{1/4} (1+z)^{3/2} \times 10^{-24}\,\mathrm{G}.$$

  - Dimensionless Hubble parameter $h \sim 0.7$.
  - CMB measurements give $\mathscr{K}_{\rm i} \lesssim 10^{-5}$ at matter–radiation decoupling ($z \sim 1000$).
- At $z \sim 100 - 10$: $|B| \sim 10^{-26} - 10^{-28}\,\mathrm{G}$
- Cosmic dynamo mechanism requires $|B| \sim 10^{-30}\,\mathrm{G}$. Thus we are well within reach of this!
- Thus, nonlinear effects may play major role.

Magnetic field generation



The dynamo

2. The dynamo equation: MHD approximation with finite conductivity $\sigma \rightarrow$ Flat spacetime

$$\dot{\mathbf{B}} - \nabla \times (\mathbf{v} \times \mathbf{B}) - \lambda \nabla^2 \mathbf{B} = 0,$$

$\lambda \propto 1/\sigma$ *diffusivity*.

The dynamo

2. The dynamo equation: MHD approximation with finite conductivity $\sigma \rightarrow$ Curved spacetime[5]

$$
\begin{aligned}
&\dot{\mathbf{B}} - \nabla \times (\mathbf{v} \times \mathbf{B}) - \lambda \nabla^2 \mathbf{B} \\
&\quad = -\tfrac{2}{3}\lambda\Theta\dot{\mathbf{B}} + 2\lambda\boldsymbol{\omega} \times \dot{\mathbf{B}} + 2\lambda\nabla(\boldsymbol{\omega}\cdot\mathbf{E}) - \lambda\bar{\bar{\mathsf{R}}}\cdot\mathbf{B} \\
&\quad - \left(1 + \tfrac{2}{3}\lambda\Theta\right)\left(\tfrac{2}{3}\Theta\mathbf{B} - \bar{\bar{\sigma}}\cdot\mathbf{B} + \boldsymbol{\omega}\times\mathbf{B} - \mathbf{a}\times\mathbf{E}\right) \\
&\quad - \tfrac{2}{3}\mathbf{E}\times\nabla\Theta - \lambda\nabla\times(\bar{\bar{\sigma}}\cdot\mathbf{E}) \\
&\quad - \lambda\nabla\times(\boldsymbol{\omega}\times\mathbf{E}) - \lambda\nabla\times(\mathbf{a}\times\mathbf{B}) + \lambda\boldsymbol{\Xi},
\end{aligned}
$$

$\lambda \propto 1/\sigma$ *diffusivity*.

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5. Marklund & Clarkson, Mon. Not. Roy. Astron. Soc. (2005)

The dynamo

Low frequency contribution from displacement current given by

$$
\begin{aligned}
\boldsymbol{\Xi} \equiv & -\left(\tfrac{2}{3}\dot{\Theta}\mathbf{B} - \dot{\bar{\bar{\sigma}}}\cdot\mathbf{B} + \dot{\boldsymbol{\omega}}\times\mathbf{B}\right) - \left(\Theta\dot{\mathbf{B}} - \bar{\bar{\sigma}}\cdot\dot{\mathbf{B}} + \boldsymbol{\omega}\times\dot{\mathbf{B}}\right) \\
& -\tfrac{1}{3}\Theta\left(\tfrac{2}{3}\Theta\mathbf{B} - \bar{\bar{\sigma}}\cdot\mathbf{B} + \boldsymbol{\omega}\times\mathbf{B}\right) \\
& -\mathbf{a}\times\left[\nabla\times\mathbf{B} - \mu_0\mathbf{j} + \mathbf{a}\times\mathbf{B} - \left(\tfrac{1}{3}\Theta\mathbf{E} - \bar{\bar{\sigma}}\cdot\mathbf{E} + \boldsymbol{\omega}\times\mathbf{E}\right)\right] \\
& +\tfrac{1}{2}\mathbf{E}\times\mathbf{q} - \bar{\bar{H}}\cdot\mathbf{E} \\
& \quad -\dot{\mathbf{a}}\times\mathbf{E} + (\bar{\bar{\sigma}}\cdot\nabla)\times\mathbf{E} - (\boldsymbol{\omega}\times\nabla)\times\mathbf{E}
\end{aligned}
$$

General dynamo equation in an arbitrary spacetime

Why look at dynamo?

- Astrophysical and cosmological plasmas satisfies MHD criteria to good approximation:
  - Charge neutrality.
  - No observed stationary electric fields.
  - Low frequency phenomena.
- Magnetic fields observed on almost all scales.
- The dynamo mechanism proposed amplification mechanism.
- Seed fields?
- Effects due to gravitational waves, indirect detection.

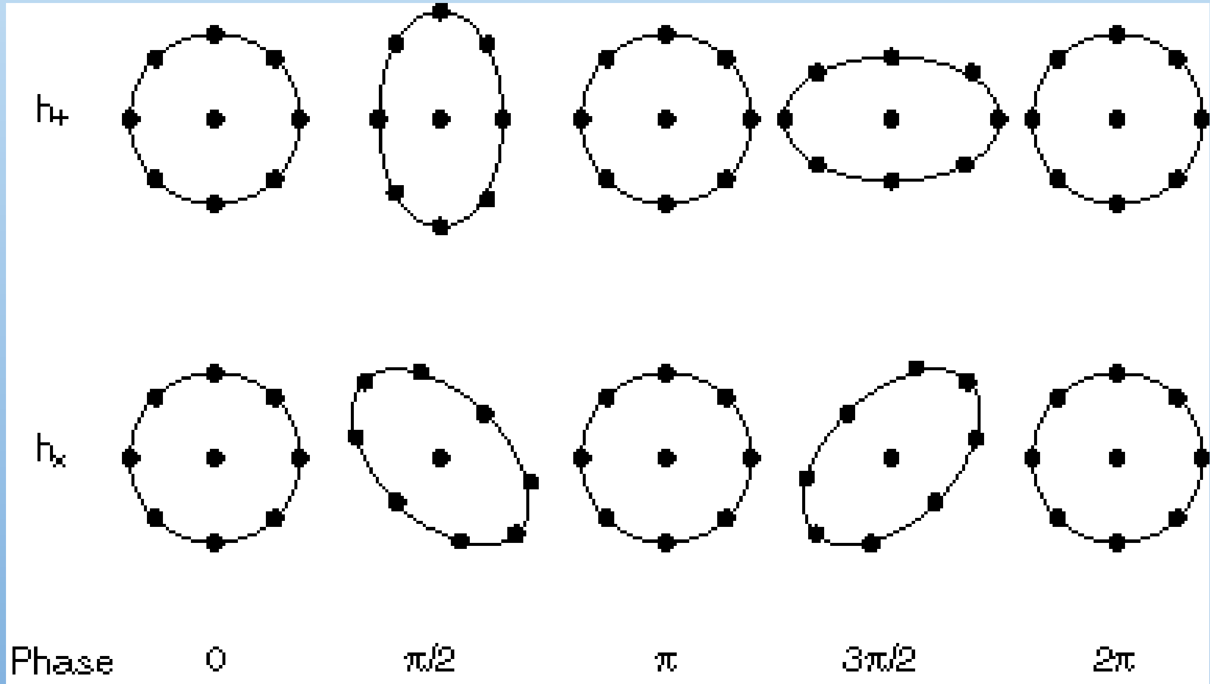
Gravitational waves

3. Gravitational waves: Ripples in spacetime

Gravitational waves

3. Gravitational waves: Ripples in spacetime

- Phase velocity $= c$.
- Wave equation: $\ddot{\bar{\bar{\sigma}}} - c^2\nabla^2\bar{\bar{\sigma}} = 0$
- Tensor field



Gravitational waves

3. Gravitational waves: Ripples in spacetime

- Phase velocity $= c$.
- Wave equation: $\ddot{\bar{\bar{\sigma}}} - c^2\nabla^2\bar{\bar{\sigma}} = 0$
- Tensor field
- Interacts with electromagnetic fields and plasmas.
- Could excite electromagnetic and plasma modes when propagating through space.
- Indeed the case!

Effects due to gravitational waves

- Example: Gravitational wave propagation in magnetized plasma. Then dynamo equation becomes

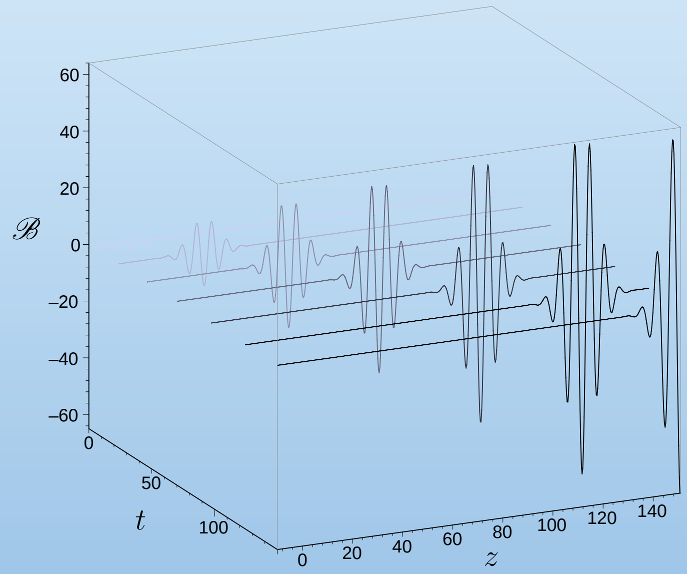
$$\ddot{\mathbf{B}} - \lambda^2\nabla^2\dot{\mathbf{B}} + \frac{1}{\mu_0\mathscr{E}}\left[-\tfrac{1}{2}\mathbf{B}\nabla^2(B^2) + \tfrac{1}{2}(\mathbf{B}\cdot\nabla)\nabla(B^2) - (\mathbf{B}\cdot\nabla)^2\mathbf{B}\right]$$

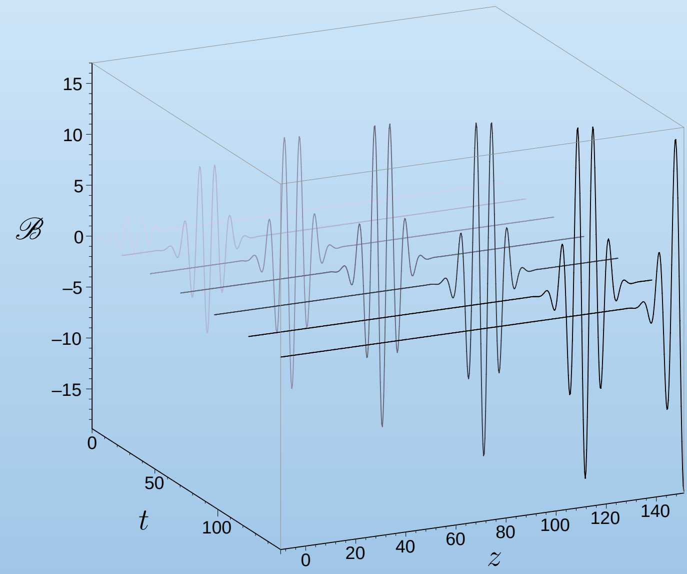
$$= 2\lambda\ddot{\bar{\bar{\sigma}}}\cdot\mathbf{B} - \dot{\bar{\bar{\sigma}}}\cdot\mathbf{B}$$

- Solve with GW as driving term (e.g. binary system).

Effects due to gravitational waves: Typical behaviour

Fast magnetosonic waves (Alfvén speed $C_A \sim 1$):

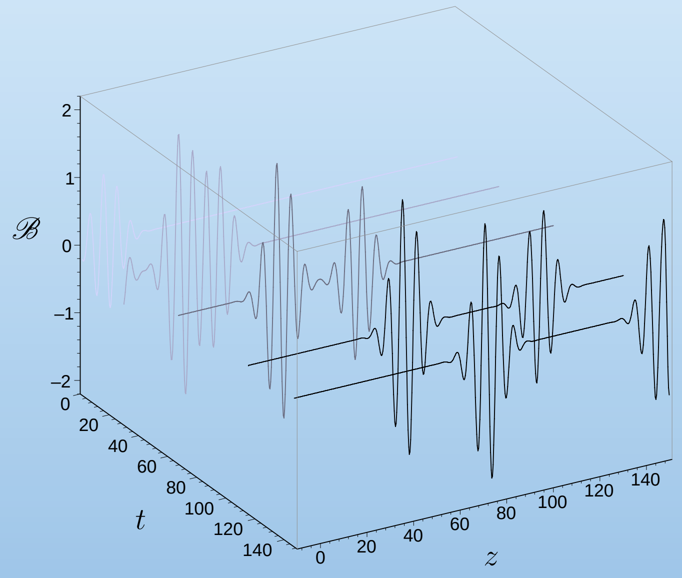


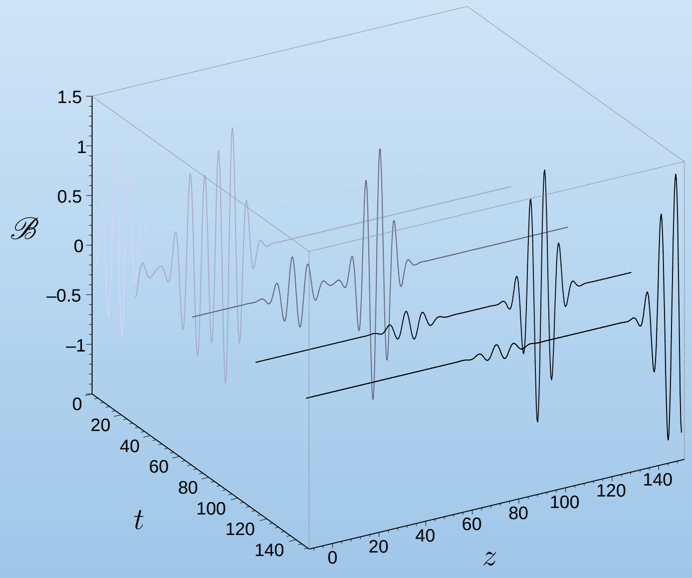


Left fig. shows infinite conductivity, right finite conductivity.

Effects due to gravitational waves: Typical behaviour

Slow magnetosonic waves ($C_A \ll 1$):





Left fig. shows infinite conductivity, right finite conductivity.

CONCLUSIONS

- Gravity may affect plasmas on several levels.
- Modified fluid/kinetic equations.
- Modified Maxwell's equations.
- Interaction with gravitational wave may yield new modes.
- Indirect observations.
- Fundamental tests of general relativity.
- Magnetic field generation.
- Future: Fully nonlinear studies, two-temperature plasmas, dusty plasmas etc.