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Quantum Mechanics and Nonadiabaticity in Charged Particle Dynamics

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QUANTUM MECHANICS and NON-ADIABATICITY in CHARGED PARTICLE DYNAMICS

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Marshall Rosenbluth
August 1965

ADIABATICITY IN CLASSICAL MECHANICS

Adiabatic Invariance of action:

If Hamiltonian for a classical system (one-dimensional, for simplicity) ,depends on a small parameter, ε ,slow time variation of λ

$$H = H(p, q ; \lambda)$$

Then well known that the action, $I = \oint p\, dq$, an Invariant

$$\boxed{d I / dt = 0, \qquad I = \text{const.}}$$

Termed ' Adiabatic' because invariance follows only under the condition

$$\varepsilon = T\, d\lambda/dt \ll 1, \qquad T = \text{period of the motion}$$

NONADIABATICITY: ⟶ Effect of the violation of the Adiabaticity condition $T d\lambda/dt \ll 1/\varepsilon$

CLASSICAL- CHARGED PARTICLE DYNAMICS

Gyro-action as an Adiabatic Invariant:

$\mu = E_\perp / \Omega$, $E_\perp$ = perpendicular energy,

Ω = eB /mc, gyro-frequency

Under the 'adiabaticity condition:

$\varepsilon_t = (1 / \Omega^2)(d\Omega / dt) << 1$, for time variation,

$\varepsilon_x = (v_\perp / \Omega B)(dB / dx) << 1$, for spatial variation

Adiabatic invariant to all orders!!! ?

Change in μ, if these conditions NOT fulfilled ⟶ Non-adiabatic change

$\Delta \mu \sim \exp[-1/\varepsilon]$

It is non-expandible in the small 'adiabaticity parameter'. Belongs to the regime of
'singular' perturbation, and 'asymptotic phenomena

Non-adiabatic effects--- analogous to quantum effects!!!

Kruskal expansion vs WKB. expansion. Both asymptotic series.

The adiabaticity parameter ε has a role analogous to that of $\hbar$ in Q.M
as quantum effect like the tunneling also goes as $\sim \exp[-|S| / \hbar]$

Non-adiabaticity to 'adiabatic motion' as quantum effects to classical dynamics

ADIABATIC INVARIANTS IN OLD QUANTUM THEORY

"Adiabatic Invariants" quantized in old quantum theory:

$$\oint p\, dq = n\hbar \qquad n = \text{action in units of } \hbar$$

Action $\oint p\, dq$ remains conserved as long as the particle stayed in a given orbit

Transition between levels ⟶ alters 'action' by a discrete ammout

⟶ Nonadibaticity-quantum mechanically

The concept of Nonadiabaticity is inherent in the formulation of Old Quantum Theory !!

Adiabaticity condition, quantum mechanically:

The condition that the particle stays in the same orbit n against the perturbing potential V is

$$\tau /(E_n - E_{n+1})[dV/dt] << 1$$

E_n, E_{n+1} energies of neighbouring levels; τ, the period of the motion

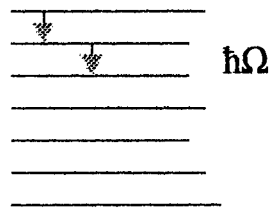
This implies that the 'change of the perturbing potential in a period of the motion Much less than the difference of energy between neighbouring levels

Charged Particle Dynamics in Quantum mechanics

Quantum mechanically charged particle in a magnetic field described by a wave function:

$$\Psi_{Nk}(x, z) = Ħ_N(x)\, e^{ikz}$$

$Ħ_N(x)$ = Harmonic oscillator wave function for the Landau level N
e^{ikz} = plane wave along the magnetic field with $k = mv/\hbar$



Landau levels N

Eigenvalues: $E_N = (N+1/2)\,\hbar\,\Omega$, for the Landau levels
$E_z = (\hbar k)^2 / 2m$, for the plane wave along z

Quantum mechanically, the gyro-action identified as

$$\mu = N\hbar$$

Adiabatic Invariance of μ ⟷ Adiabatic Invariance of N

Quantum mechanically, the adiabaticity condition' gives here

$$[\tau /(E_N - E_{N+1})\, dV/dt << 1$$

τ = period of motion = $2\pi/\Omega$, for 'gyro-motion, $E_N = N\hbar\Omega$, $E_{N+1} = (N+1)\hbar\Omega$, neighbouring levels, $V = \hbar\,\Omega$, transition inducing potential in the present case

With this the above condition reduces to the classical case:

$$(2\pi/\Omega^2)\, d\Omega/dt << 1$$

Object and Focus of the Presentation

To show that:

(i) Nonadiabatic effects in charged particle dynamics are not just 'Quantum-like" but are indeed real quantum effects which manifest in Macroscopic dimensions in the correspondence limit.

(ii) These non-adiabatic effects are found to have unusual and novel manifestations in the form of one-dimensional 'matter wave interference' effects, with matter wave length typically ~ 5cm. This may sound quite astonishing. But I shall show experimental evidence to that effect

(iii) Even more astonishing, motivated by our theoretical prediction, we have detected the presence of a curl-free vector potential, in macroscopic dimensions, in the spirit of the Aharonov-Bohm effect.

Finally (iv) nonadiabatic loss of charged particles from 'Adiabatic Traps' and multiplicity of 'residence times' observed 20 years back have been Identified as of 'quantum origin' in the correspondence limit

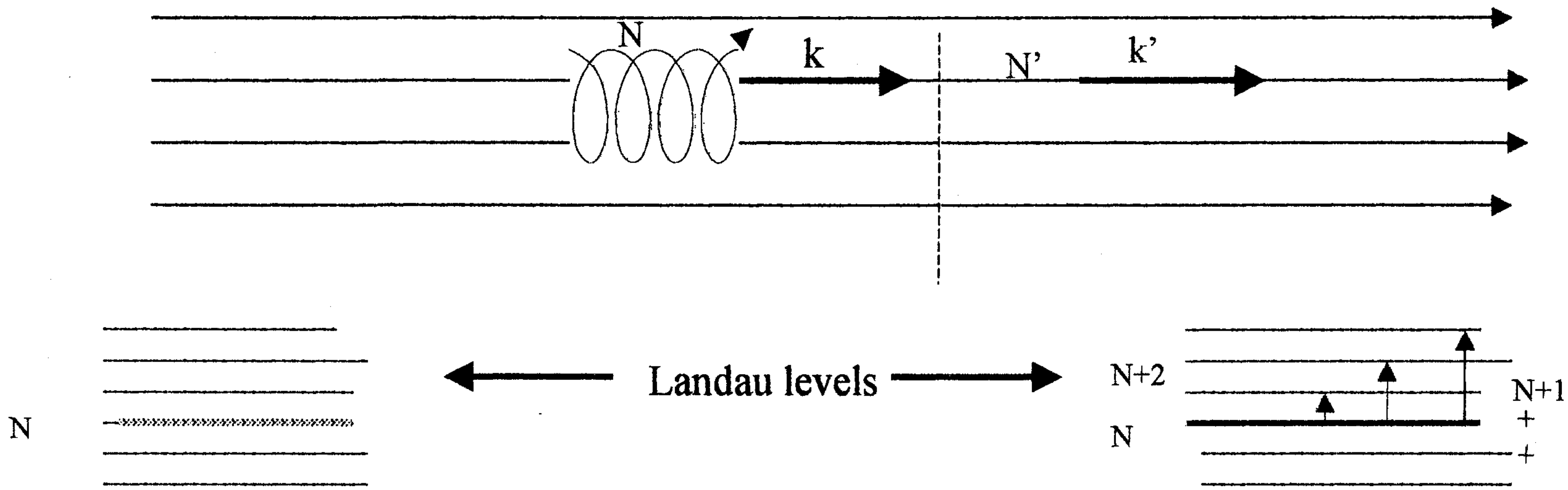
‘Transition Amplitude’ Wave Generated due to Scattering by a Center- a Simple Derivation

Charged Particle Moving in a uniform magnetic field:

Recall, the complete wave function of a charged particle in a Uniform magnetic field, including the ‘plane wave’ along magnetic field given by

$$\Psi_{Nk}(x, z) = Ħ_N(x)\, e^{ikz} \qquad \Psi_{N'k'}(x, z) = Ħ_{N'}(x)\, e^{ik'z}$$

Before scattering After scattering



NONADIBATICITY IN CLASSICAL AND QUANTUM DESCRIPTIONS

Classically, nonadiabaticity at the end of a perturbation operation, is described by one number $\Delta\mu = \Delta N \hbar$,

(i) $\Delta\mu$ – exponentially small if magnetic field B an analytic function of time or space. $\Delta\mu \sim \exp[-1/\varepsilon]$

(ii) $\Delta\mu$, finite, if B or its derivatives undergoes a step function change.

Quantum mechanically, transitions across levels, whereby the (Landau) quantum number changes by ΔN, signifies nonadiabaticity.
Described by a set of " Transition Amplitudes":

$\varphi(N', N) = \varphi(N, n)$, from $N \longrightarrow N' = N + n$, $n = 1, 2, 3, \ldots$

I would like to introduce the concept of "transition amplitudes" specifying non-adiabaticity Quantum Mechanically

As a consequence of a perturbation, which may be

(i) Magnetic field inhomogeneity or time dependence, which is NONADIBATIC

(ii) Any external perturbation V(x) causing the transition $N \rightarrow N' = N + n$

Then

$$\varphi(N', N) = \int dx\, \bar{H}_{N'}(x)\, V(x)\, \bar{H}_N(x) = \varphi(N, n),\ n = 1, 2, 3\ldots$$

These " Transition Amplitudes" specify QUANTUM NONADIBATICITY

Now the transition amplitudes $\Phi(N, N', k)$ for 'transition' from

$$(N, k) \longrightarrow (N', k')$$

$$\Phi(N, N', k) = \int dx\, \Psi^*_{N'k'}(x, z)\, V(x)\, \Psi_{Nk}(x, z)$$

$$= \exp[-i(k'-k)z] \int dx\, H_{N'}(x)\, V(x)\, H_N(x)$$

$$= \exp[-i(k'-k)z]\, \varphi(N', N)$$

Note that this expression represents a plane wave with the 'wave number' (k'-k) along the z-direction--- along the magnetic field.

This is designated as the " Transition Amplitude" wave

To evaluate (k'-k) use the total energy conservation:

$$(\hbar k')^2/2m + N' \hbar\Omega = (\hbar k)^2/2m + N\hbar\Omega$$

This yields: $(k' - k) \simeq (N - N')\,\Omega / (\hbar k/m) = -(n\,\Omega / v)$

$(k' - k) << k$ assumed so that $(k' + k) \simeq 2k$; $\hbar k/m = v$, velocity along z

The "transition amplitude wave" then has the form :

$$\Phi(N, n) = e^{i (n \Omega / v) z} \varphi(N, n)$$

This represents a plane wave along z, the direction of the magnetic field, with wave length

Needless to emphasize that this "wave" has a quantum origin.. but has no ħ in it.!!

$$\lambda_n = 2\pi \ v / n\,\Omega, \quad \text{n, denotes the harmonic number}$$

Recall that n is the Landau level interval subtending the transition.

For typical laboratory conditions:

Energy E = 1 keV, Magnetic field B= 100 g

$\lambda_1 \simeq 5$ cm !!!, -- the fundamental

$\lambda_2 \simeq 2.5$ cm ---the first harmonic

It needs to be emphasized that the "transition amplitude" is a new secondary state of the particle whereas the Schrödinger wave function represents its primary state

Such long wave length "Matter Waves" !!! ???

Would they really manifest in Nature ???

Does "Transition Amplitude Wave" have a physically observable consequence in the above form??

WE SHALL SEE

The following state equation governs the evolution of this secondary 'transition amplitude.

A GENERAL FORMALISM for TRANSITION AMPLITUDE:

Equation governing the evolution of 'transition amplitude'; the following Schrödinger-like equation:

$$\frac{i\mu}{n}\frac{\partial \Psi(n)}{\partial t} = \frac{1}{2m}\left[\frac{\mu}{in}\frac{\partial}{\partial z} - \frac{e A_z}{c}\right]^2 \Psi(n) + (\mu\Omega)\,\Psi(n)$$

$\Psi(n)$ = 'transition amplitude' across the Landau quantum number internal $\Delta N = n$, and $\mu = N\hbar$, the initial gyro-action of the quantum state N

For typical laboratory conditions (E ~ 1 keV, B ~ 100 g), $N \sim 10^8$

A_z = z-component of the curl-free vector potential A, which can induce transition across n=1, 2, 3….

This equation offers the possibility of detecting the presence of A_z, through one-dimensional interference effects in macroscopic dimensions

The macro-domain of its operation should be apparent from the magnitude of $\mu/\hbar \sim 10^8 = L/\lambda_{AB}$.

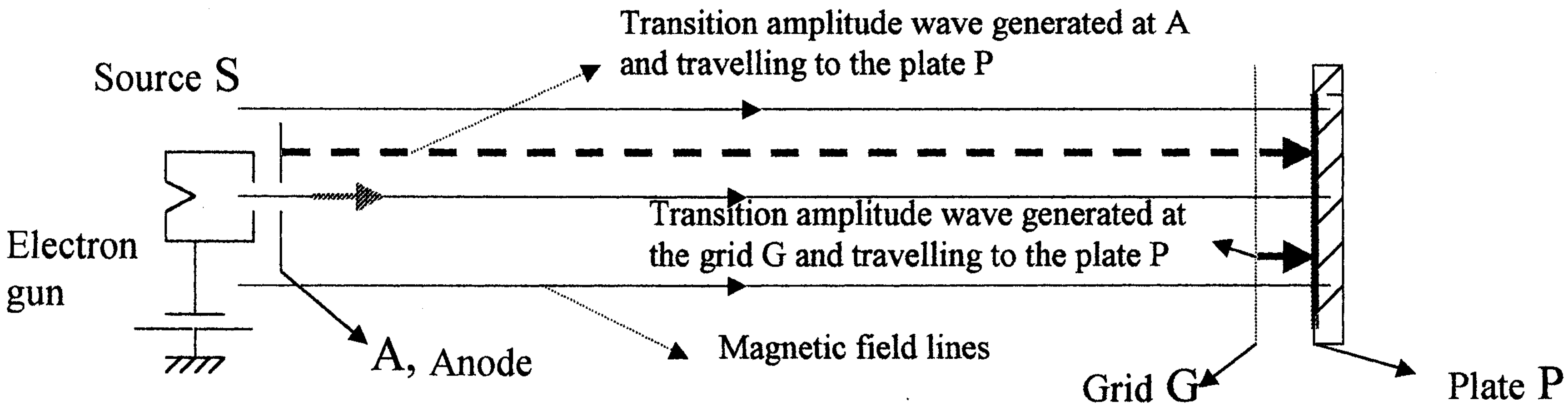
This equation has an evolutionary history.

- It was first obtained in 1977 through an intuitive-heuristic derivation (without A_z in it.)
- Then (again without A_z) it was obtained from the classical Liouville eqn as its Hilbert space representation in 1985.
- In the above form (with A_z) it was obtained from QM in path-integral representation.

References with these developments:

1. R.K.Varma, Phys. Rev. E 64, 036608 (2001), Erratum E 65 019904
   (This reference contains the derivation of this equation from 'quantum mechanics)
2. R.K.Varma, Phys. Reports 378, 301-434 (2003)
   (This reference reviews the work on nonadiabaticity, and the above development including all the associated experimental work.)

Consider a schematic representation of an experiment as below:



Electrons propagate from the source S, the 'electron gun' travelling along the magnetic field B

(i) Primary de Broglie electron wave gets scattered near the anode A by the $E_\perp$ in the gun-region.

$E_\perp$ 'kicks' the electron up the higher Landau levels.

Generates "Transition Amplitude Wave" originating at A, the anode

(ii) The unscattered primary wave amplitude reaching the grid G, is scattered by the grid wires, generating another " Transition Amplitude Wave" originating at G

Important: Transition-amplitude-waves do not exist a priori, but are generated at the scattering centers, e.g. the anode A, grid G, and plate surface P

"Transition-amplitude-wave" amplitude ψ_{AP} originating at the Anode A and reaching the plate P:

$$\psi_{AP} = A \exp[\, i K (z_P - z_A)] \ , \qquad K = \Omega / v$$

"Transition-amplitude-wave" amplitude ψ_{GP} originating at the grid G And reaching the plate P:

$$\psi_{GP} = A' \exp[\, i K (z_P - z_G)] \ , \qquad K = \Omega / v$$

Total amplitude of the " Transition-amplitude-wave" at the plate P

$$\Psi_P = \psi_{AP} + \psi_{GP} = A \exp[i K (z_P - z_A)] + A \exp[i K (z_P - z_G)]$$

Assuming A = A' for simplicity

Intensity at the plate:

$$|\Psi_P|^2 = |\psi_{GP} + \psi_{AP}|^2 = 2 |A|^2 [\, 1 + \cos K L_{AG}]$$

Note that the sweep of $K = \Omega / v$, would cause the Oscillation of plate current At the "frequency" L_{AG}

Interference maxima at:

$$\boxed{K L_{AG} = 2\pi j \ , \quad j = 1, 2, 3 \ldots}$$

$K = \Omega / v$

L_{AG} = Anode grid distance $(z_A - z_G)$

L_{AG} =anode is the Anode-Grid distance. If grid G is close to the plate P, so that $L_{GP} << L_{AG}$, and $L_{AP} \cong L_{AG}$

Then using $K = \Omega / v$, this gives:

$$\Omega L_{AP} = 2 \pi j v, \qquad j=1, 2, 3.....$$

As the condition for interference maxima involving " macroscopic Matter waves" arising from the transition amplitudes

This constitutes the PREDICTION of the above formalism on the existence of one-dimensional 'macroscopic matter waves' with the wave length

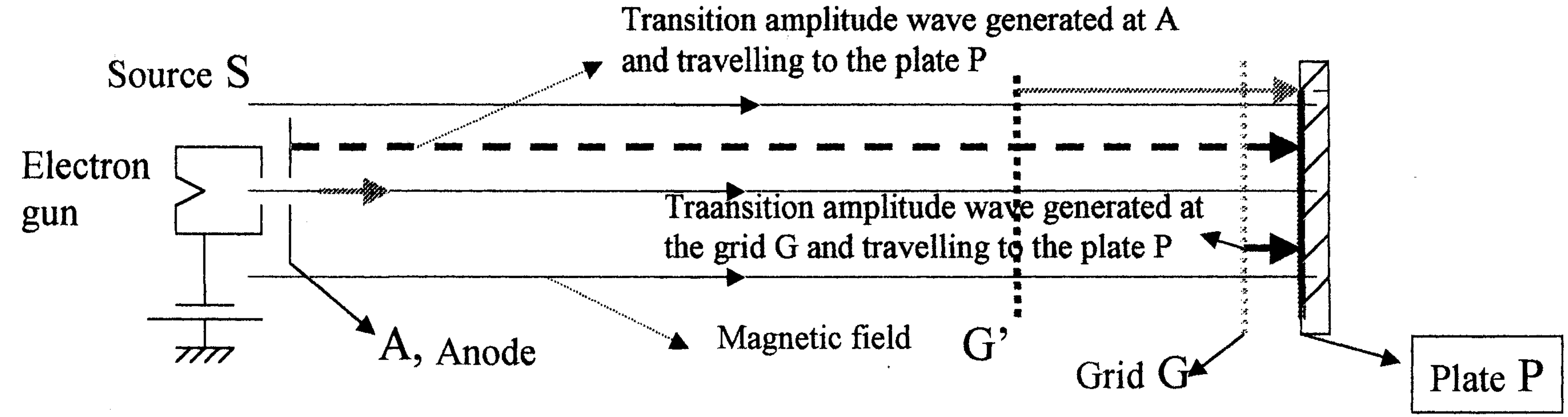
$$\lambda = 2\pi v / \Omega, \qquad \text{for the 'fundamental'}$$
$$\lambda_n = 2\pi v / n \Omega, \qquad \text{for the } n^{th} \text{ harmonic}$$

Above condition implies that $(L_{AP}/ \lambda) = j$,- an integral number of wave lengths in the length L_{AP}

Recall $\lambda_1 \cong 5$ cm, for E = 1 keV, B= 100 g

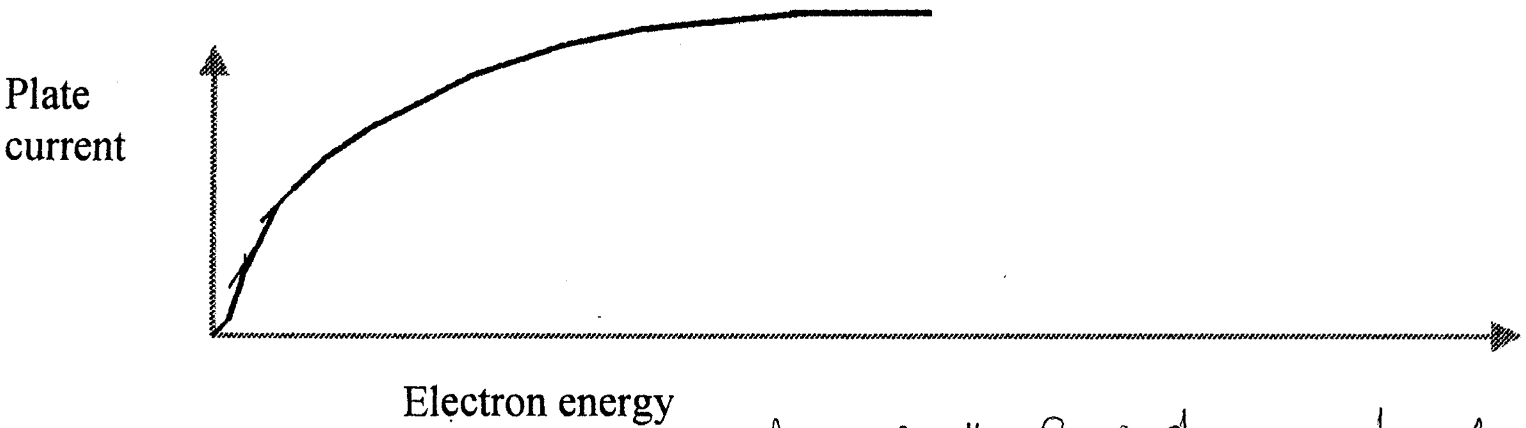
Experimental check ??

To check the above PREDICTIONS, the following experiment carried out:



The electrons from the source shot along magnetic field, and collected at the plate P, as the CATHODE VOLATGE (that is electron energy) IS SWEPT

What kind of plate current response one would expect ??



In our experimental checks we shall be continually called upon to compare the classically expected "results" of experiments with the actually observed results in the same macro-domain – the domain of operation of classical dynamics as generally believed

I would therefore invite your special attention to the comparison between the two.

Observed plate current as a function of the Cathode Voltage:

(a)
(b)
(c)

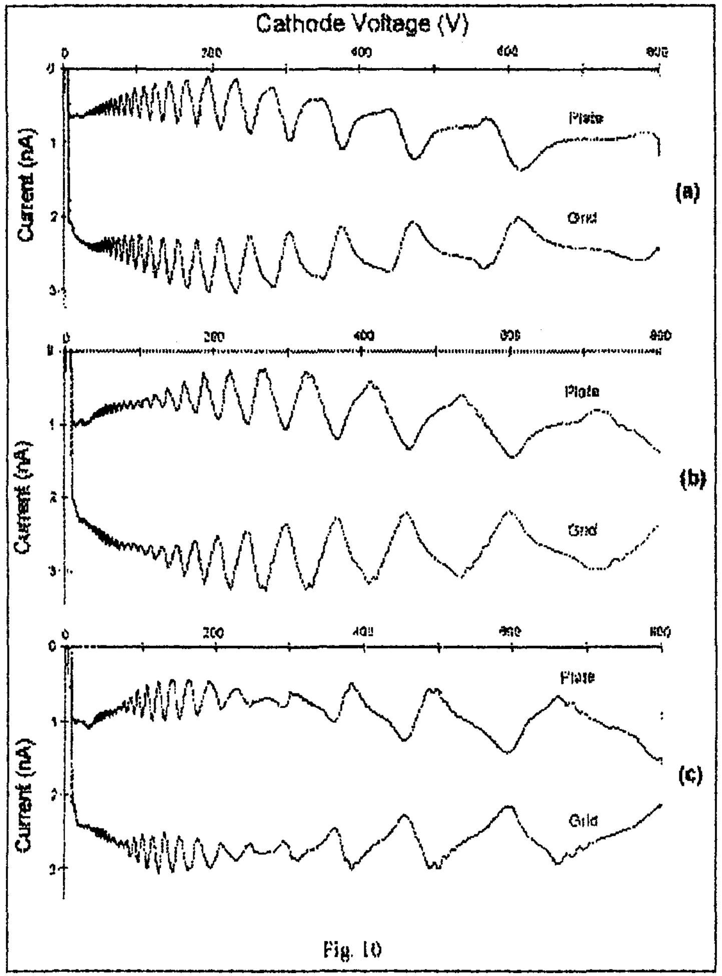


Plate current in nano-amperes

All figures (a), (b), (c) Magnetic field B= 69 gauss. L_{AP}= 51 cm

Grid-plate separation L_{GP} :

(a) 2 cm, (b) 4 cm, (c) 6 cm

NOTICE, the ‘totally unexpected’ nature of the oscillatory plate current response.

There are sharply defined maxima and minima.

Note the progressive formation of “beat-like” structures from (a)-(c)

If they represent interference ‘maxima and minima, they ought to fit the relation

$$\Omega L_{AP} = 2 \pi j v, \qquad j=1, 2, 3.....$$

Recall that the oscillation “frequency” would Correspond to the distance L_{AP}

Table 7

Table showing the fit for Fig (a)

Energy peak positions $\mathcal{E}_j$ "quantum number" identified, L the plate-gun, L_{ded}, deduced from the relation $\Omega L_{ded} = 2\pi j v$, corresponding to the curve of Fig. 10(a). B = ambient magnetic field, $\Omega = eB/mc$, the gyrofrequency, and v the electron beam velocity

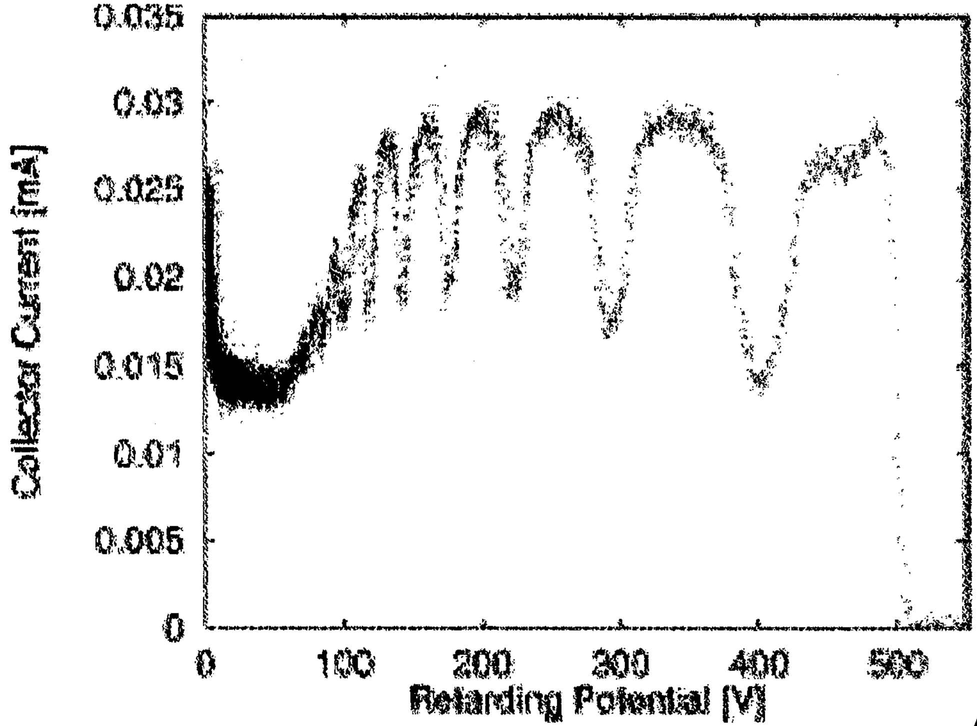
Peak No.	$\mathcal{E}_j$ (eV)	$k = \Omega/2\pi v$ (cm^{-1})	j	$L_{ded} = j(\frac{2\pi v}{\Omega})$ (cm)	Peak No.	$\mathcal{E}_j$ (eV)	$k = \Omega/2\pi v$ (cm^{-1})	j	$L_{ded} = j(\frac{2\pi v}{\Omega})$ (cm)
1	246.7	0.1975	10	50.6	6	110	0.2954	15	50.8
2	206.7	0.2158	11	51.0	7	96.7	0.3153	16	50.7
3	173.3	0.2356	12	50.9	8	85.6	0.3348	17	50.8
4	146.7	0.2561	13	50.8	9	76.6	0.3534	18	50.9
5	126.7	0.2756	14	50.8	10	69.0	0.372	19	51.1

Magnetic field $B = 69$ g, $L_p = 51$, and average $\bar{L}_{ded} = 50.8$ cm.

The value of L p = 50.8 cm deduced from observed data using the relation Ω L p = 2 π j v, matches closely with the value L p = 51 cm actually used in the experiment signifies agreement with the theory

Furthermore, various interference peaks have been identified as j = 10, 11,… 16, 17, 18, 19

This establishes the existence of Macroscopic matter waves as also a manifestation of Quantum non-adiabaticity



A. Ito and Z. Yoshida
Phys. Rev. E 63, 026502, (2001)

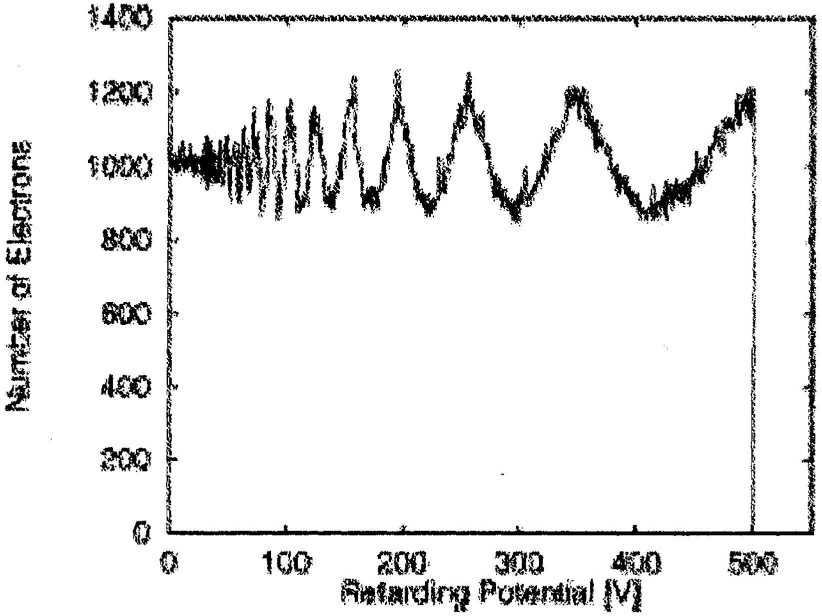
Experimental curve with
B= 99.0 g, and L = 24 cm.

Notice the large amplitude of thedips in the plate current profile~ 60 %

Numerical simulation based on "resonance Production" of secondary electrons

Authors' quotes:

"~~ii! the simulation ~Fig. 7!,with a reasonable secondary electron yield, gives only small and narrow peaks, while in the experiment more drastic changes were observed."



In our experiment where the cathode voltage is swept, the 'resonance Production' mechanism would not be possible.

Existence of Matter Wave "Beats"--- as a further confirmatory evidence

When experiment carried out with grid separated from the plate by a finite distance, the cathode-voltage (electron energy) sweep yields ' beats' in the plate current:

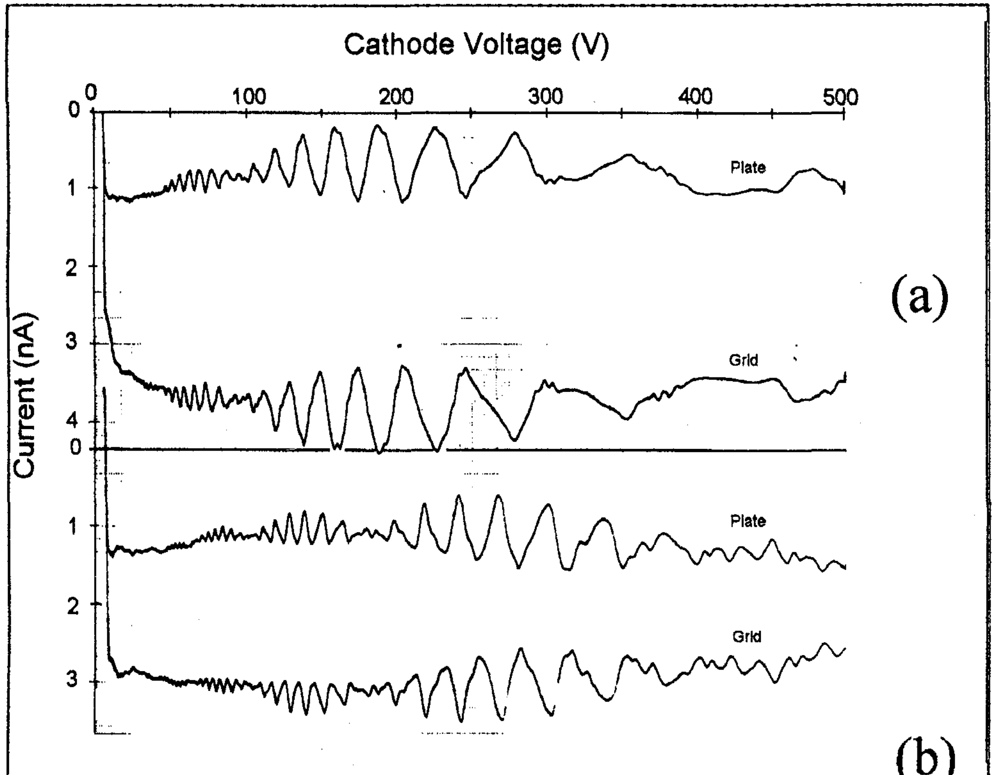


Fig. 11

Note the Matter wave (macroscopic) beats observed :

Grid-plate distance D = 10 cm
Anode-plate distance L_{AP} = 51 cm

Magnetic field B : (a) 69 g (b) 135 g

Electron energy sweep E: 5 eV→ 500 eV implies sweep of $k = \Omega / (2\pi v) = 0.042\ B / V^{1/2}$ from:

(a) 1.3→ 1.3×10^{-2} for B = 69 g

(b) 3.3→ 1.3×10^{-2} for B = 135g

R.K.Varma, A.M. Punithavelu, and S.B. Banerjee Phys. Rev. E 65, 026503 (2002)

Fig. 2

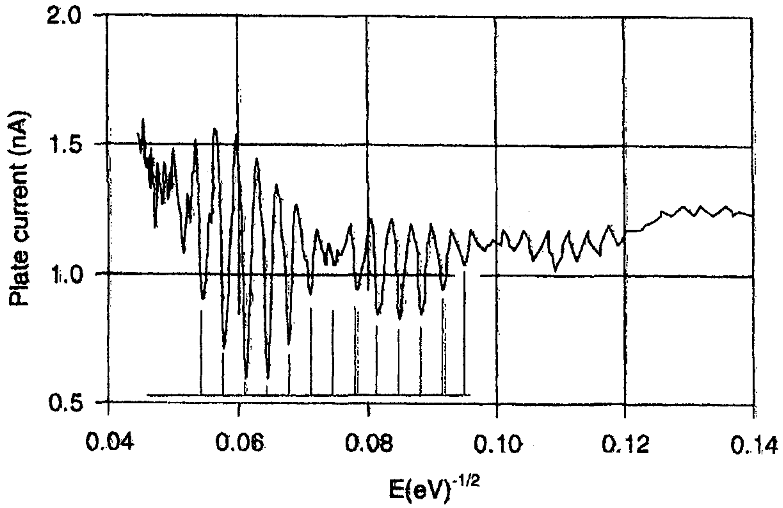


Figure (b) replotted as a function of $[E(eV)]^{-(1/2)} \sim k = (2\pi\ \Omega)[2E/m]^{-(1/2)}$

Sweep of k in the cosine term cos k L, would cause the variation of the plate current with a "frequency" L

Figure shows two frequencies:

(i) A high frequency "carrier", and (ii) a low frequency "beat- envelops"

The two "frequencies" in the experiment which produce the "beats" are
(a) Anode-plate distance L_{AP} (b) Anode-grid distance L_{AG}

If observed beats are "wave-beats", then beat frequency must be $L_{AP} - L_{AG} = L_{GP}$ and "beat-maxima" ought to fit the relation:

$$\Omega L_{GP} = 2\pi j v$$

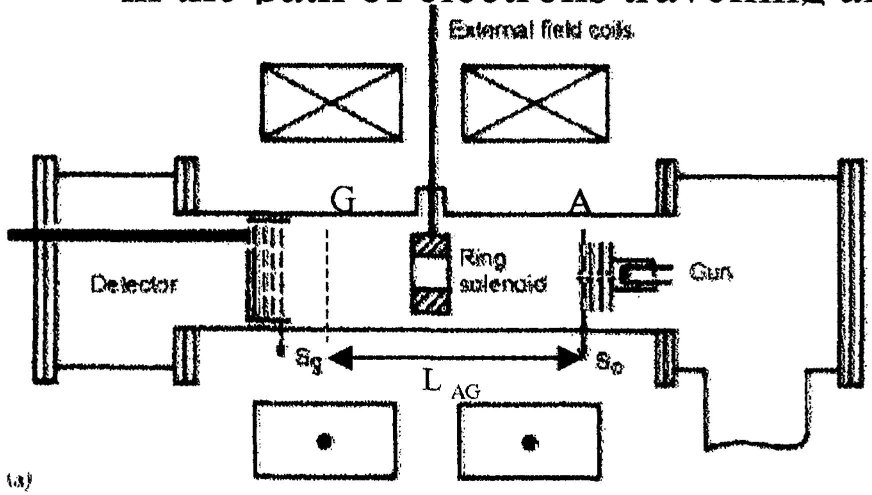
Beat No.	$\mathcal{E}_l$ (eV)	$k = \Omega/2\pi v$ (cm^{-1})	j	$L_{GP} = D = l\frac{2\pi v}{\Omega}$ (cm)
1	55.0	0.820	5	6.1
2	83.3	0.6682	4	6.0
3	141.7	0.5103	3	5.83
4	283.0	0.358	2	5.58

Magnetic field $B = 135$ g, $D = 6$ cm and average $\bar{L} = 5.9$ cm.

Observation of the Curl-free Vector Potential in the Macro-domain:

[Varma, Punithavelu, and Banerjee Phys. Lett. A 303, 114 (2002)]

Insert a Rowland-ring (producing a curl-free vector potential)
in the path of electrons travelling along an external magnetic field



Transition amplitude wave generated at A and G. The path difference is thus L_{AG}

From the Schrodinger-like equation
The condition for interference maxima
In the presence of the vector potential:

$$\int_A^G dz\,[\,m\,v + (e/c)\,A_z] = 2\pi j\,\mu$$

$j = 1, 2, 3\ldots.$

This leads to:

$$m\,v\,L_{AG} + (e/c)\,\Phi\,\sin\theta o = 2\pi j\,\mu$$

Φ= flux enclosed in the ring solenoid

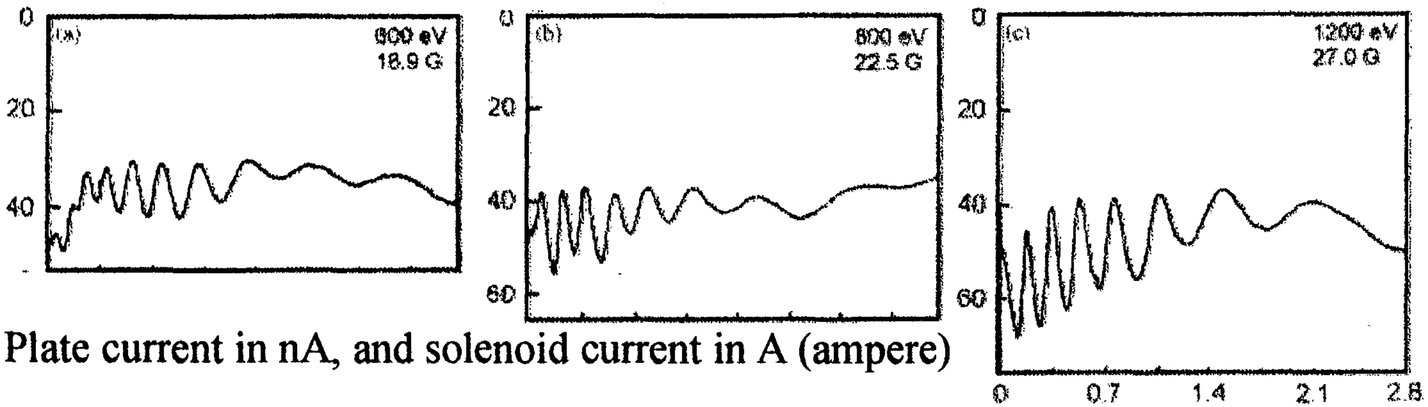


Plate current in nA, and solenoid current in A (ampere)

Plots show the variation of plate current with the variation of current in the ring solenoid, for various electron energies

Two peculiarities of this effect vis-à-vis, the quantum Aharonov-Bohm effect

(i) It is found to exist in the macro-domain in the deci-meter range as against the 'micro-domain' of the Aharonov-Bohm effect.

(ii) It exists in one-dimension as against the minimum of two dimensions required for the A-B effect.

Both these features may cause some consternation :

Because (i) will appear to be in apparent contradiction with the classical Lorentz equation, and
because (ii) is against the well held belief that the A-B effect is of topological Origin, and must occur in no less than two dimensions.

The resolution of (i) should come through the realization that the effect is, in fact, a quantum effect pushed into the macro-domain, the domain also of the 'transition amplitude' in the correspondence limit.

The resolution of (ii) should come from the realization that the "transition amplitudes" which play the seminal role in the phenomena are generated only at the position of scattering, and path differences are reckoned from those points.

The fact that this effect has indeed been observed means in view of (i) and (ii) above that it is indeed an unusual effect--- in fact, an unusual quantum effect.

What is the significance of these observed effects ??

One can view them from at least two angles:

1. From the Nonadiabaticity angle, one can say that:
   These effects are a " novel manifestations of Quantum Nonadiabaticity in the macrodomain"

2. From the Quantum Mechanical angle :
   These are totally unexpected and hitherto unfamiliar revelations of Quantum Dynamics manifesting in the macrodomain, and ought to entail a revision of our water-tight classification that particle related quantum effects belong only to the micro-domain.

Finally, these results bring to focus, the so-far unrecognized importance of the Quantum mechanical object: the "transition amplitude" , which propagates from the point of its generation at a scattering center, and does not exist, a priori. It is this which describes the exotic phenomena discussed above.

The dynamics described by the transition amplitude involved here has been termed as the " Macro-quantum Dynamics" and has been reviewed from both the above angles in

R. K. Varma Phys. Reports 378, 301- 434, (2003)

Marshall in 2002
with Richard Morse

MARSHALL ROSENBLUTH– A PERSONAL TRIBUTE

I would like to submit that the work that I am going to report has been guided and inspired by the spirit of the iconic physicist, Marshall Rosenbluth in whose memory we are holding this symposium today.

It is not just plasma physics that I learnt from him as a student, but a whole manner of thinking, choosing a problem, approaching a problem, and doing science in general.

He was a charming person, with a sense of subtle and sharp humour, not to mention, the sharp scientific mind that he possessed. He thought through a problem fast, keeping his coworkers always on their toes to work out a problem before he would walk into their room with the solution. Those who worked with him could not have remained untouched by the scientific fragrance that he exuded

He was an excellent teacher. The material that he gave us in the 1963 plasma physics course, I have preserved , for I still find some interesting things in it.

My bond with him was very personal, for I was one of his first students, and I remember him with a great deal of affection, respect and fond memories of my association with him. In my tribute to him and to his memory I dedicate this work to him which I have carried out over the past three decades.