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Relativistic Plasma Physics in the vicinity of black holes

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The Brief History of Black Holes

1783: John Michell surmised that there might be an object massive enough to have an escape velocity greater than the speed of light (called them "dark stars").

1796: Simon Pierre LaPlace "...[It] is therefore possible that the largest luminous bodies in the universe may… be invisible." -- Le Système du Monde.

1916: Karl Schwarzschild gave analytic solution for a nonrotating black hole.

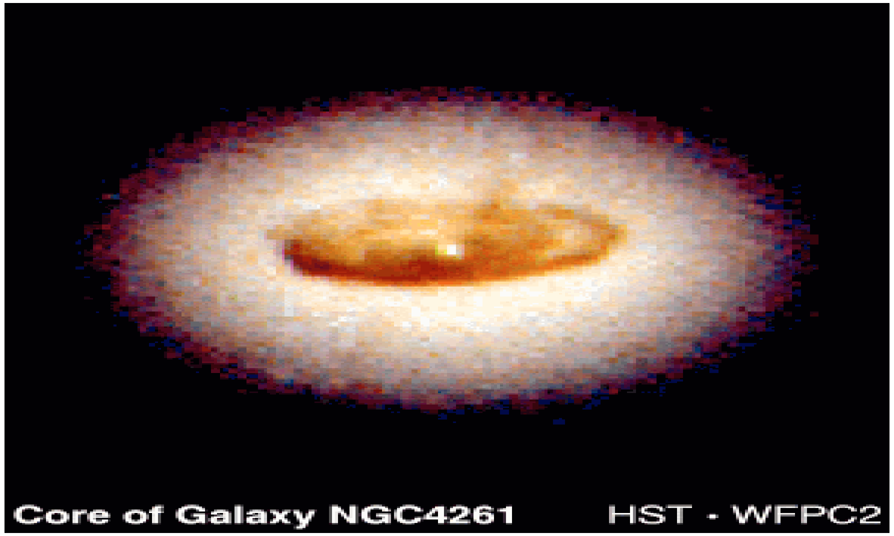
1963: Roy Kerr gave analytic solution for a spinning black hole.

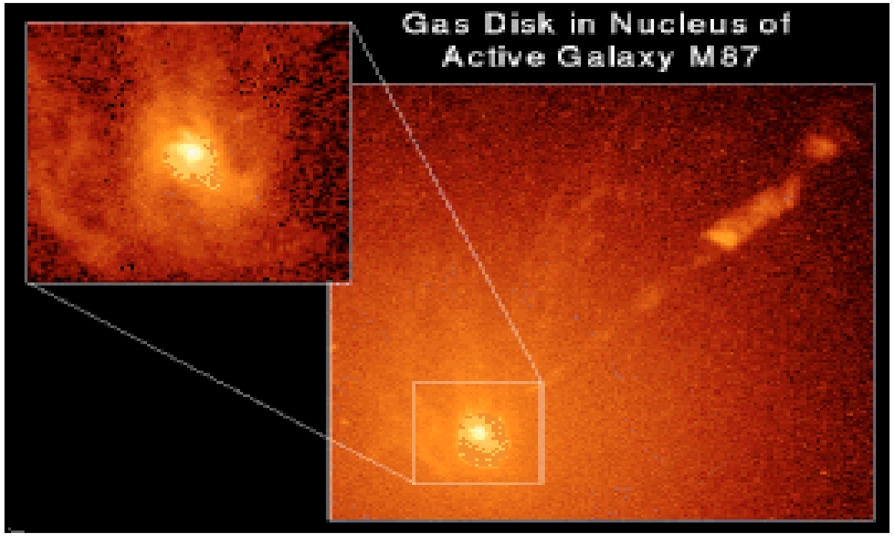
1964: John Wheeler introduced the term, "Black Hole."

1970: Cygnus X – 1 - the first black hole candidate that astronomers found.

2005: We know a number of quite reliable black hole candidates in the Galaxy and a few very reliable supermassive black holes in other galaxies.

Observational Evidence

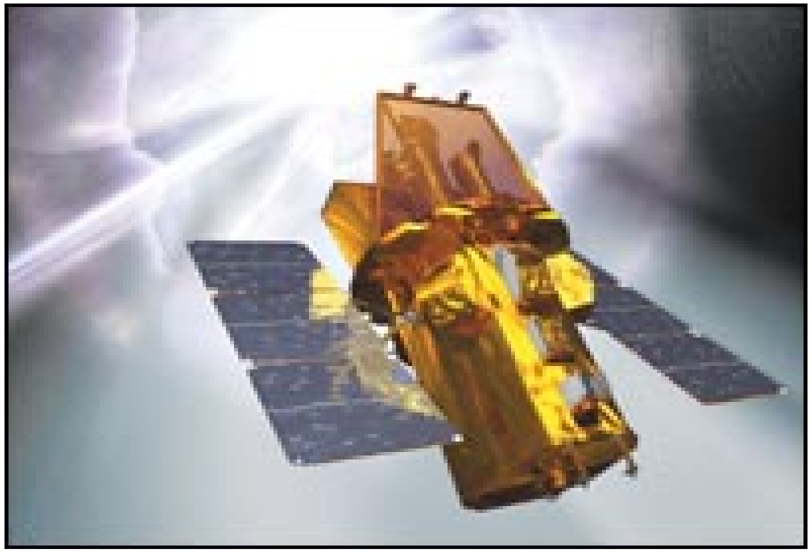




1. galaxy NGC4261. A brown spiral-shaped disk. It weighs on hundred thousand times as much as our sun. Because it is rotating we can measure the radii and speed of its constituents, and weigh the object at its centre. This object is about as large as our solar system, but weighs billion times as much as our sun! **Almost certainly this object is a black hole.**

2. M87: Near its core there is a disc of hot gas. Superposing spectra from opposite sides the speed of rotation of the disk, its size and weight of the invisible object are determined. The object is no bigger than our solar system but it weighs three billion times as much as the sun! It must be a **bona fide black hole.**

Black holes start with many bangs!?



- *"The birth of a black hole is marked by not just one massive explosion but by a series of energetic blasts, according to data from the Swift space telescope." (BBC News, 31.08.2005).*

The general metric

The rotation of the central object (for example, a rapidly rotating neutron star or black hole) introduces off-diagonal terms $g_{t\phi}$ in the metric. It is assumed to be stationary and axisymmetric [$g_{\phi t} = g_{t\phi}$]:

$$\|g\| = \begin{pmatrix} g_{tt} & 0 & 0 & g_{t\phi} \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 \\ g_{\phi t} & 0 & 0 & g_{\phi\phi} \end{pmatrix}$$

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2$$

with the metric coefficients independent of t and ϕ.

Problem 1.: Find all nonzero components of the contravariant tensor $g^{\alpha\beta}$.

“3+1” presentation:

$$dt^2 = -\alpha^2 dt^2 + \gamma_{ik}(dx^i + \beta^i dt)(dx^k + \beta^k dt)$$

Lapse function:

$$\alpha^2 \equiv R^2/g_{\phi\phi} = (g_{t\phi}^2 - g_{tt}g_{\phi\phi})/g_{\phi\phi}$$

γ_{ik} – three-dimensional “absolute” space (diagonal) metric tensor:

$$\|\gamma\| = \begin{pmatrix} \gamma_{rr} & 0 & 0 \\ 0 & \gamma_{\theta\theta} & 0 \\ 0 & 0 & \gamma_{\phi\phi} \end{pmatrix} = \begin{pmatrix} g_{rr} & 0 & 0 \\ 0 & g_{\theta\theta} & 0 \\ 0 & 0 & g_{\phi\phi} \end{pmatrix}$$

$$\beta^i \equiv [0, 0, g_{t\phi}/g_{\phi\phi}], \quad \beta_i = \gamma_{ik}\beta^k$$

Different kinds of black holes:

Kerr-Newman black hole - an exact solution of the Einstein field equations possessing mass M, angular momentum a, and (in principal but not in astrophysical cases) charge Q:

$$ds^2 \equiv -d\tau^2 = -\left[1 - \frac{2Mr}{\Sigma}\right] dt^2 - \left[\frac{4Marsin^2\theta}{\Sigma}\right] dtd\phi$$

$$+\frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left[r^2 + a^2 + \frac{2Ma^2rsin^2\theta}{\Sigma}\right] sin^2\theta d\phi^2$$

with $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$, $\Sigma \equiv r^2 + a^2cos^2\theta$.

When $a = Q = 0$ we have the **Schwarzschild** solution, when $a = 0, Q \neq 0$ we have **Reissner-Nordström** solution, and when $a \neq 0, Q = 0$ we have **Kerr** solution.

Kerr metric through problems:

Problem 2.: Show that if three-velocities of particles, as measured in absolute γ-space $\mathbf{v}$ are defined as: $v^i \equiv (U^i/U^t + \beta^i)/\alpha$ then the following expressions are held:

$$U^t = \Gamma/\alpha; \qquad U_t = \Gamma(-\alpha + \mathbf{v}\cdot\beta)$$

where $\Gamma \equiv (1 - \mathbf{v}\cdot\mathbf{v})^{-1/2}$.

Problem 3.: A toroidal flow field $U^\alpha = (U^t, 0, 0, U^\phi)$ is completely specified by either angular velocity $\Omega = U^\phi/U^t$ or specific angular momentum $\ell = -U_\phi/U_t$. Derive the following relations:

$$U^t = (g_{tt} + 2\Omega g_{t\phi} + \Omega^2 g_{\phi\phi})^{-1/2}$$

$$U_t = R(-\ell^2 g_{tt} - 2\ell g_{t\phi} - g_{\phi\phi})^{-1/2}$$

$$U^t U_t = (1 - \Omega\ell)^{-1}$$

where $R^2 \equiv g_{t\phi}^2 - g_{tt}g_{\phi\phi}$

Kerr metric through more problems:

Problem 4.: Show that angular velocity Ω and specific angular momentum ℓ of the flow from the previous problem can be expressed through each other in the following way :

$$\Omega = -(\ell g_{tt} + g_{t\phi})/(\ell g_{t\phi} + g_{\phi\phi})$$

$$\ell = -(g_{t\phi} + \Omega g_{\phi\phi})/(g_{tt} + \Omega g_{t\phi})$$

Problem 5.: Show that Kepler's law

$$\Omega^2 = M/r^3$$

holds exactly for circular orbits around a Schwarzschild black hole, if r is the curvature coordinate radius, and Ω is the angular frequency as measured from infinity. Derive an analogous law for equatorial orbits around a Kerr black hole of specific angular momentum a.

Locally Nonrotating Frames:

The set of local observers who, in some sense, *"rotate with the geometry"* [Bardeen, ApJ, 171, 52 (1970)]. Each observer carries an orthonormal ($e^{\alpha}_{[a]} e^{\beta}_{[b]} g_{\alpha\beta} = \eta_{[a][b]}$) tetrad of 4-vectors, his locally Minkowskian basis vectors.

$$e^{[t]}_{t} = \alpha, \quad e^{[\phi]}_{t} = \beta_{\phi}/\sqrt{g_{\phi\phi}}, \quad e^{[i]}_{i} = \sqrt{\gamma_{ii}}$$

$$e^{t}_{[t]} = 1/\alpha, \quad e^{\phi}_{[t]} = -\beta^{\phi}/\alpha, \quad e^{i}_{[i]} = 1/\sqrt{\gamma_{ii}}$$

The observers' world lines are: $r = const, \quad \theta = const, \quad \phi = \omega t + const$. They are called **LNRF** or **ZAMO's** (Zero Angular Momentum Observers) or simply **Bardeen's tetrads**.

General co-moving frames (GCMF's):

The set of local observers, **"atached to (co-moving with) the plasma partricles"** [Rogava, Gen. Rel. & Grav. **24** (1992)]. Each observer carries an orthonormal ($e^{\alpha}_{(a]}e^{\beta}_{(b)}g_{\alpha\beta} = \eta_{[(a)(b)}$) tetrad of 4-vectors, his locally Minkowskian basis vectors.

$$e^{t}_{(t)} = \Gamma/\alpha$$

$$e^{t}_{(i)} = \Gamma v^{[i]}/\alpha$$

$$e^{i}_{(t)} = \Gamma(v^{[i]} - \beta^{[i]}/\alpha)(g_{ii})^{-1/2}$$

$$e^{i}_{[k]} = \left[\eta^{[i][k]} + (\Gamma - 1)\frac{v^{[i]}v^{[k]}}{\mathbf{v}^2} - \frac{\Gamma}{\alpha}\beta^{[i]}v^{[k]}\right](g_{ii})^{-1/2}$$

In the case of pure rotational motion they are called **"orbiting systems"** (Novikov & Thorne, 1973).

Problem 6:

Show that :

(a) LNRF really are ZAMO's: tghey have angular momentum:

$$\ell = 0;$$

(b) Lorentz transformation between the LNRF and GCMF tetrads, in the same point of space-time, is described by the tensor:

$$e^{(t)}_{[t]} = \Gamma$$

$$e^{(t)}_{[i]} = e^{(i)}_{[t]} = -\Gamma v^{[i]}$$

$$e^{(i)}_{[k]} = e^{(k)}_{[i]} = \delta^i_k + (\Gamma - 1)\frac{v^{[i]} v^{[k]}}{v^2}$$

[For details see: Rogava, Gen. Rel. & Grav. **24**, 617 (1992)]

“3+1” electrodynamics:

The electromagnetic tensor decomposition:

$$F^{\alpha\beta} = e^{\alpha}_{[t]} E^{\beta} - e^{\beta}_{[t]} E^{\alpha} + \varepsilon^{\alpha\beta\gamma\delta} e_{\gamma[t]} B_{\delta}$$

with $E^0 = E_0 = B^0 = B_0 = 0$ and $e^{\beta}_{[t]} = \{1/\alpha; -\vec{\beta}/\alpha\}$ being LNRF tetrad commponents.

Maxwell equations:

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0$$

can be written in the following “3+1” form:

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times (\alpha \mathbf{E}) = -(\partial_t - \pounds_{\beta})\mathbf{B}$$

with

$$\pounds_{\beta}\mathbf{A} \equiv (\beta \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\beta$$

What we'll do on Friday?

- Write down kinetic equations for the relativistic distribution function;
- Obtain the system of transport equations for the macrosco-pic parameters of plasma: number density, pressure, thermal energy, heat flux density, etc. These quantities will be defined in the GCMF reference frames;
- Derive the closed set of ``3+1'' general-relativistic MHD equations describing collisionless plasma with anisotropic pressure tensor.
- Consider (at least) one astrophysical problem, where this theory has been applied.