



The Abdus Salam
International Centre for Theoretical Physics



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AUTUMN COLLEGE ON PLASMA PHYSICS

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On some questions in the theory of flowing plasmas

I. Non-Hermitian dynamics of waves in shear flows

Z. Yoshida
University of Tokyo

In collaborations with

S.M. Mahajan, T. Tatsuno, **M. Furukawa**, F. Volponi, M. Iqbal,
A. Ito, S. Ohsaki, R. Numata, **M. Hirota**, D. Hori, **J. Shiraishi**,
V. Berezhiani, N. Shatashvili, H. Saleem, R. Dewar, V. Krishan

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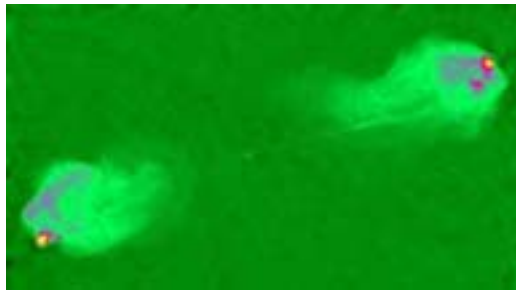
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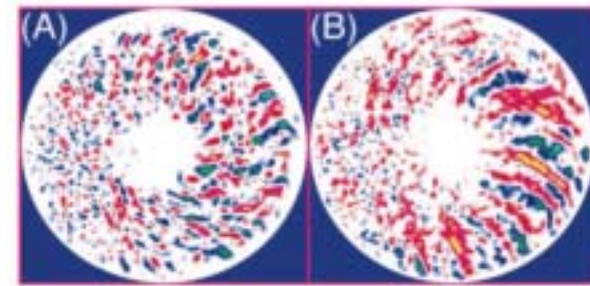
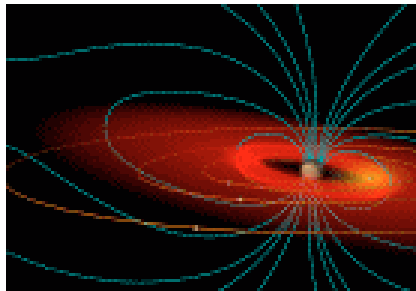
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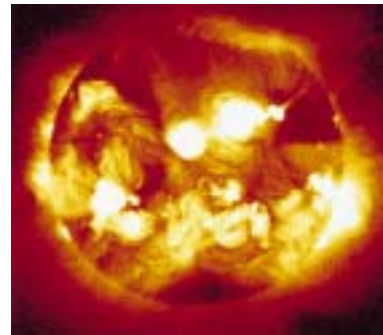
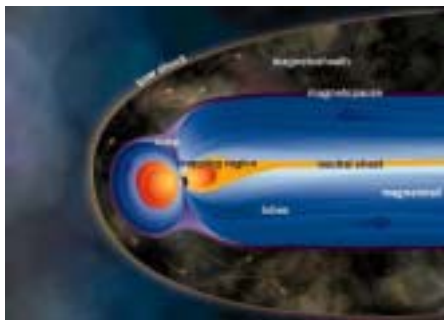
Diverse structures in flowing plasmas



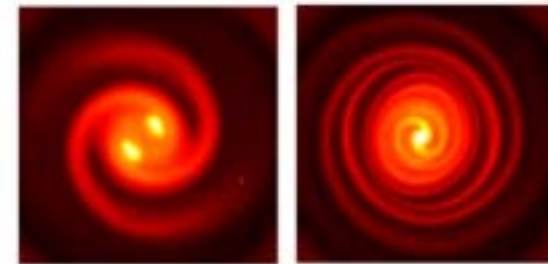
NRAO



Z. Lin, et al., Science (1998).



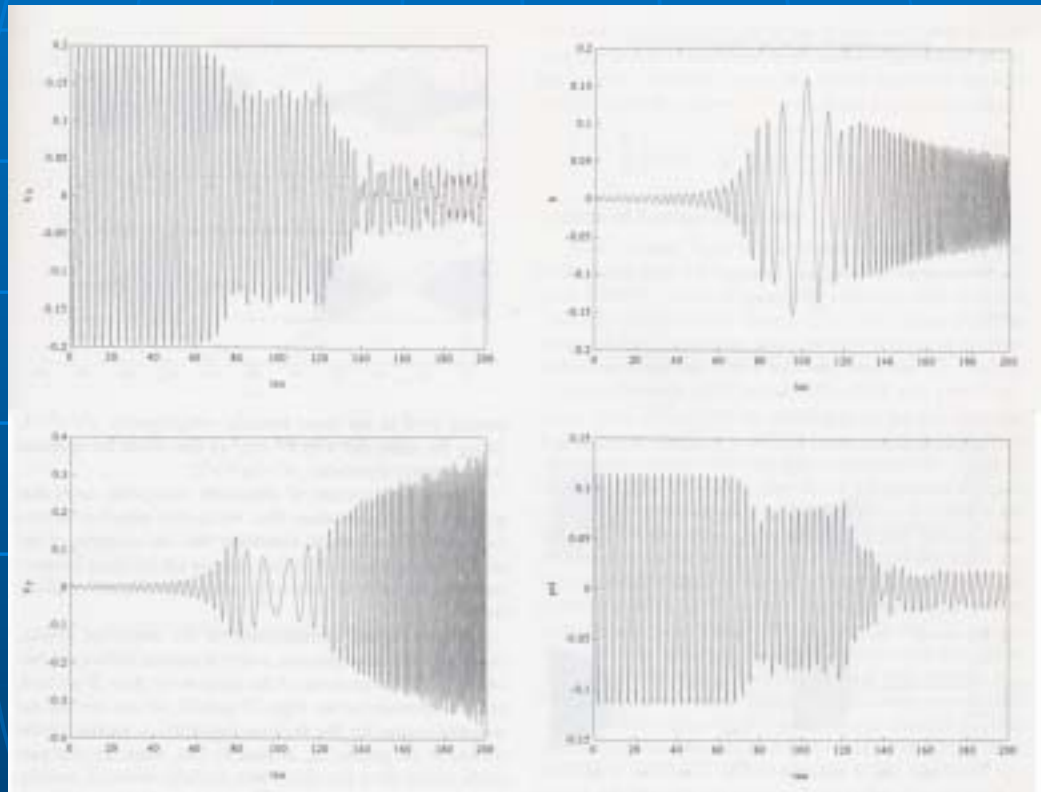
Yohkoh



Y. Kiwamoto

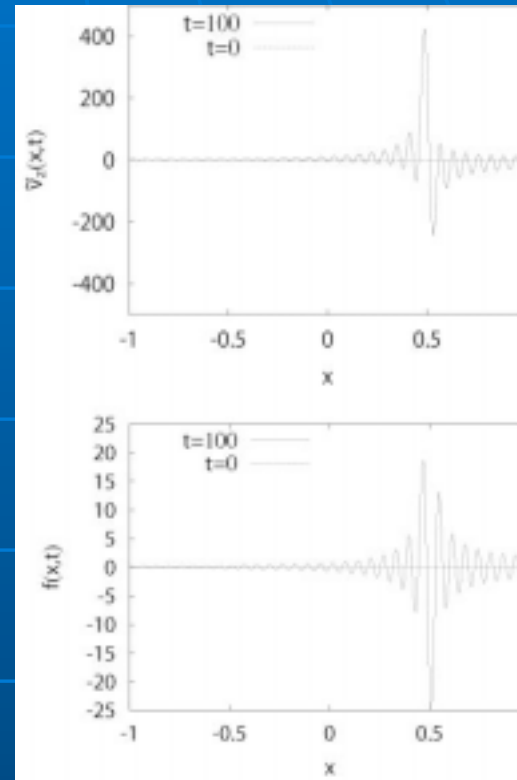
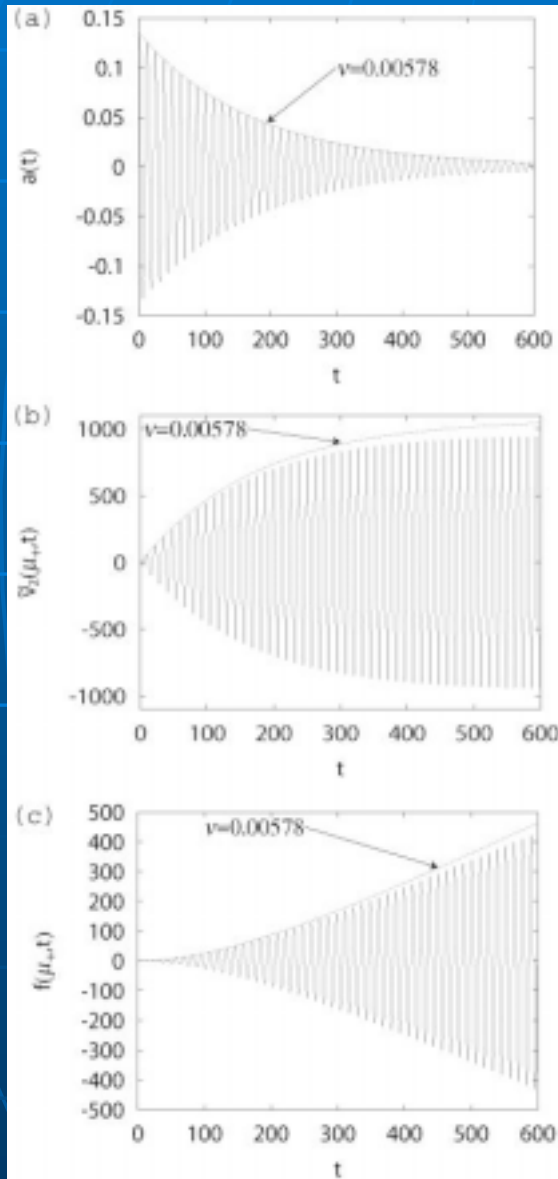
Need new methods to describe and analyze these phenomena

Complex transient phenomena -- out of the scope of exponential laws



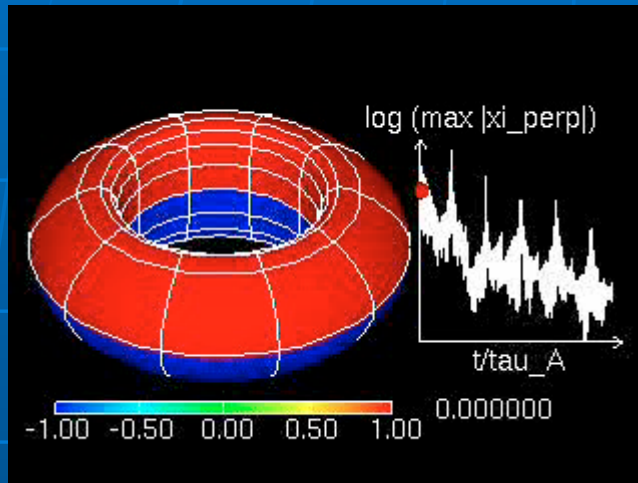
G.D. Chagelishvili, A.D. Rogava, D. Tsiklauri,
Phys. Plasmas 4 (1997), 1182-1195.

Secular amplification in coupled continuous spectra (KH-type modes)

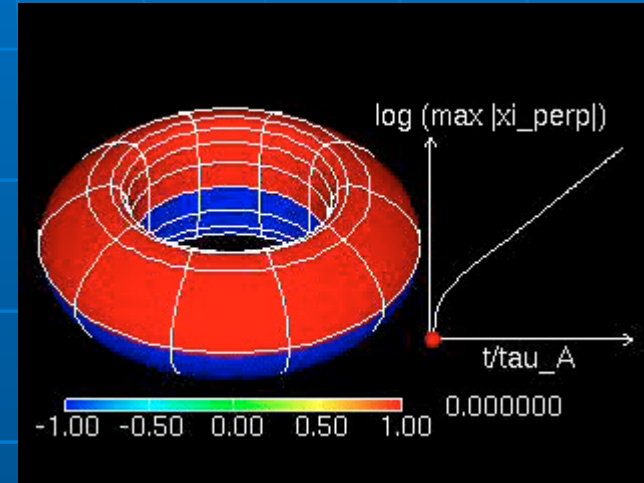


M. Hirota, T. Tatsuno, and Z. Yoshida,
"Degenerate continuous spectra producing
localized secular instability -- An example
in a non-neutral plasma"
J. Plasma Phys. 69, 397 (2003).

Ballooning modes in a flowing tokamak



With flow



Without flow

M. Furukawa and S. Tokuda, Phys. Rev. Lett. 94, 175001 (2005).

What do we know about Hermitian systems?

- We may assume

$$u(x,t) = f(t)\phi(x) = \exp(-it\omega)\phi(x)$$

to derive the **dispersion relation** (= eigenvalue problem).

- The eigenfunctions (including singular eigenfunctions belonging to continuous spectra) are **complete** (von Neumann's theorem).
- The solution to an initial-value problem is given by a group $\{\exp(-itH)\}$ of unitary transforms (propagators):

$$u(t) = \exp(-itH)u(0).$$

- The energy is a constant of motion:

$$\langle u(t), H u(t) \rangle = \text{const.}$$

What can happen in non-Hermitian systems?

- We may NOT assume

$$u(x,t) = f(t)\phi(x) = \exp(-it\omega)\phi(x)$$

The dispersion relation falls short to capture a variety of transient phenomena.

- may be unstable even when all ω are real.
- may be stable even when the potential energy can be negative.

- The eigenfunctions may NOT be complete.

Possible nilpotent (higher-order singularity).

- The solution operators $\{\exp(-itH)\}$ may NOT be unitary transforms. \rightarrow wave number is not conserved.

- The energy may NOT be a constant of motion.

Different classes of non-Hermitian generators

- Most pathological generators
 - not a “closed” system.
- Energy-conserving systems
 - generalized Hamiltonian systems.

Some methods of analysis

- Lyapunov stability
 - sufficient for stability (necessary for instability)
 - Z. Yoshida, S. Osaka, A. Ito, S.M. Mahayana, J. Math. Phys. **44**, 2168 (2003).
- Generalized “modes” (Kelvin’s method)
 - integrable if the flow shear is linear.
 - non-conservative property → effective-mass(t)
 - Z. Yoshida, Phys. Plasmas **12**, 024503 (2005).
 - non-integrable dynamics → energy transfer
 - M. Furukawa, Z. Yoshida and S. Tokuda, Phys. Plasmas **12**, 072517 (2005).

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Expansion of ballooning modes in rotating plasmas by “stretching” eigenfunctions

Wave equation for ballooning modes in rotating tokamak plasmas

$$\bar{\rho} \left(\frac{\partial^2 \xi_{\perp}}{\partial t^2} - U \frac{\partial \xi_{\perp}}{\partial t} \right) = \mathcal{L} \xi_{\perp}$$

$$\mathcal{L} \left(\frac{\partial}{\partial \vartheta}, \vartheta, t \right) := \frac{\partial}{\partial \vartheta} \left(f \frac{\partial}{\partial \vartheta} \right) - g$$

$$\begin{aligned} \{f, g, \bar{\rho}, U\}(\vartheta, t) \\ = \{f, g, \bar{\rho}, U\}(\vartheta + 2\pi, t + 2\pi / (d\Omega/dq)) \end{aligned}$$

Consider an eigenvalue problem

$$\mathcal{L} \xi = -\lambda \bar{\rho} h \xi$$

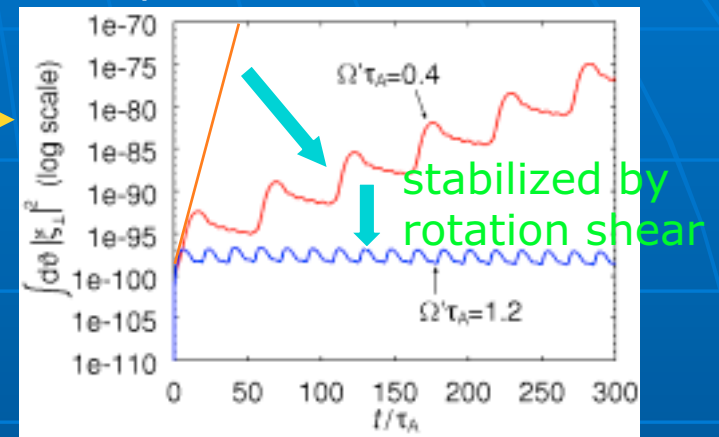
The window function h is chosen so that this equation becomes Sturm-Liouville type

We obtain eigenvalues λ_j and eigenfunctions ξ_j

Then we can expand ξ_{\perp} by the eigenfunctions as

$$\xi_{\perp}(\vartheta, t) = \sum_{j=1}^{\infty} a_j(t) \xi_j(\vartheta, t) \quad a_j(t) = \int_{-\infty}^{\infty} d\vartheta \bar{\rho} h \xi_j \xi_{\perp}$$

Example of numerical solution

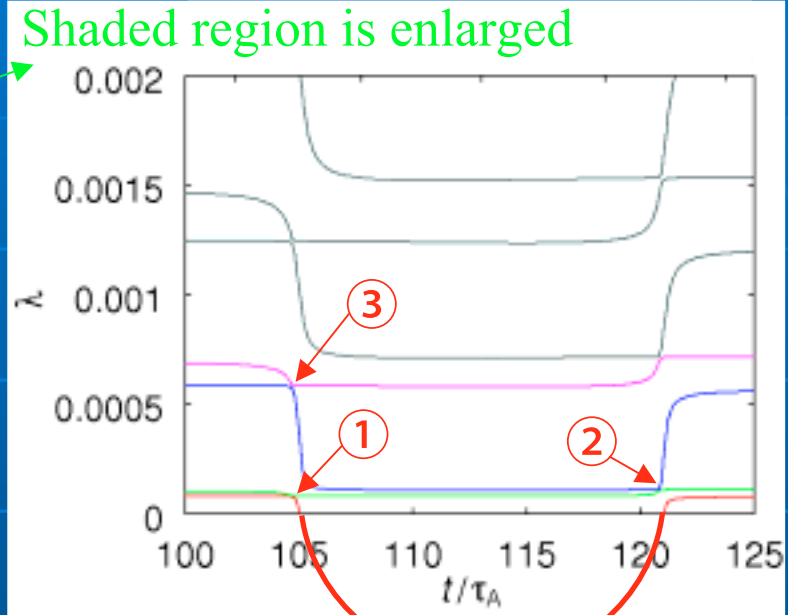
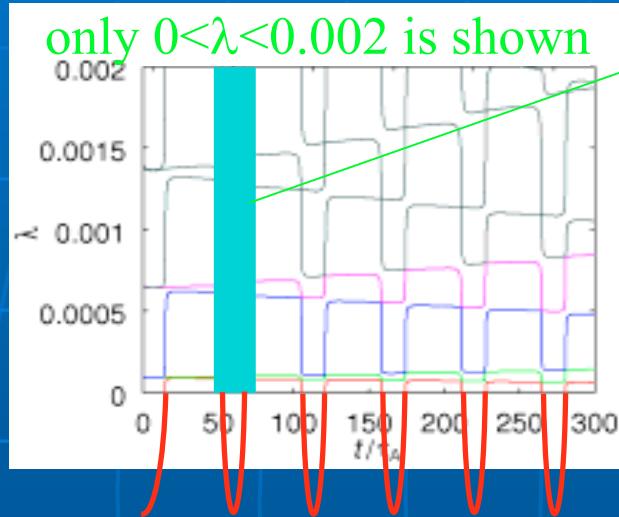


$$\mathcal{L} \xi = -\lambda \bar{\rho} \xi$$

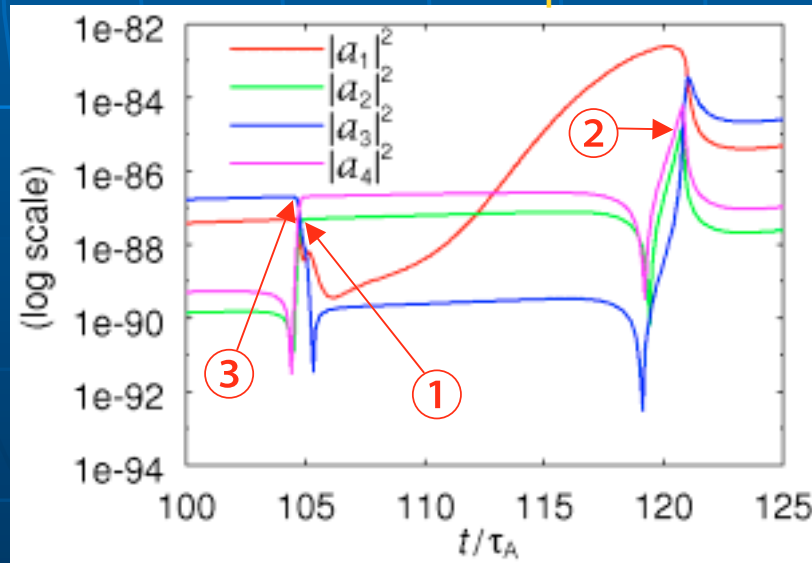
Well-known ballooning equation in static plasmas, which yields the continuous spectrum and the corresponding singular eigenfunctions at the stable side

Energy transfer from an unstable mode to stable modes

Eigenvalues as a function of t



Time evolution of the expansion coefficients



$|a_1|^2$ grows during $\lambda_1 < 0$

However, the energy is transferred to stable modes successively when the eigenvalues cross

Therefore, the unstable mode cannot grow in the time average; i.e., the ballooning mode is stabilized

SUMMARY

- Elementary processes of waves and instabilities in flowing plasmas.
- The theory must go far beyond quantum mechanics (Hermitian operator theory).
- non-canonical, non-conservative, non-exponential properties.