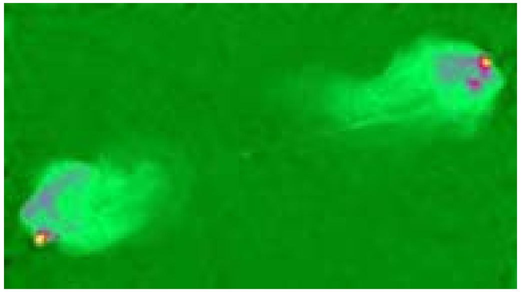
On some questions in the theory of flowing plasmas

I. Non-Hermitian dynamics of waves in shear flows

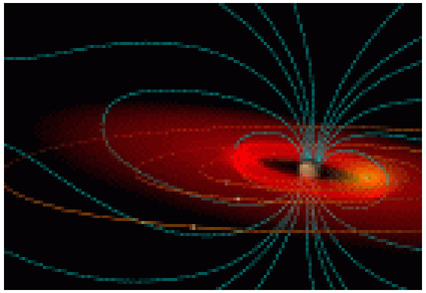
Z. Yoshida
University of Tokyo

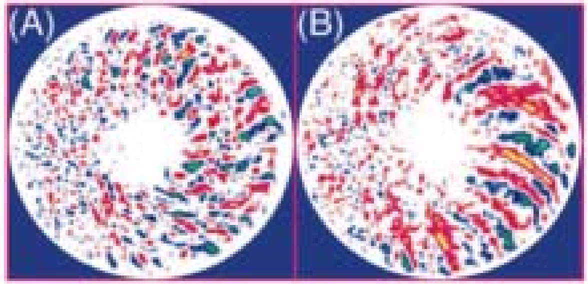
In collaborations with
S.M. Mahajan, T. Tatsuno, **M. Furukawa**, F. Volponi, M. Iqbal, A. Ito, S. Ohsaki, R. Numata, **M. Hirota**, D. Hori, **J. Shiraishi**, V. Berezhiani, N. Shatashvili, H. Saleem, R. Dewar, V. Krishan

Diverse structures in flowing plasmas

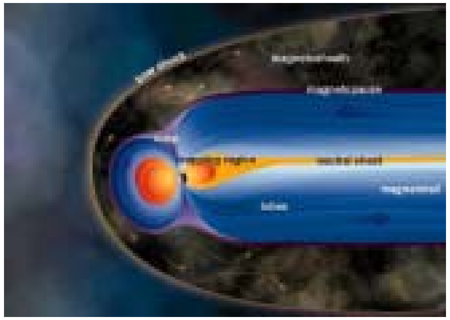


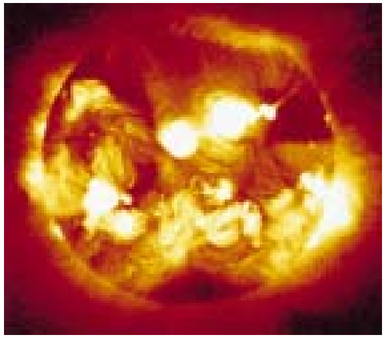
NRAO



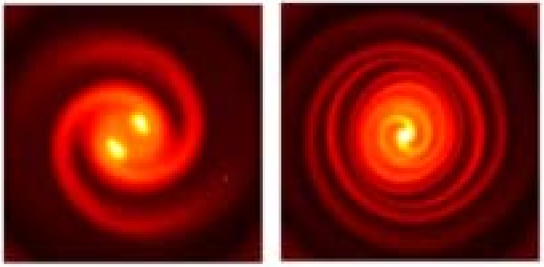


Z. Lin, et al., Science (1998).





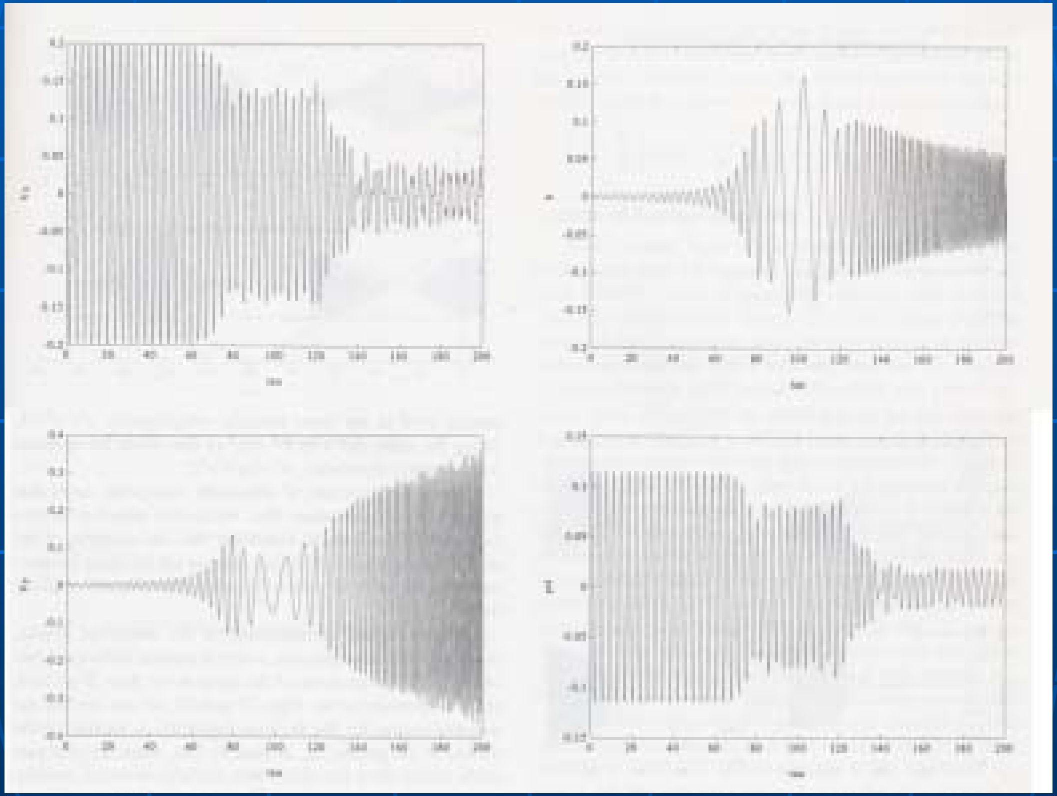
Yohkoh



Y. Kiwamoto

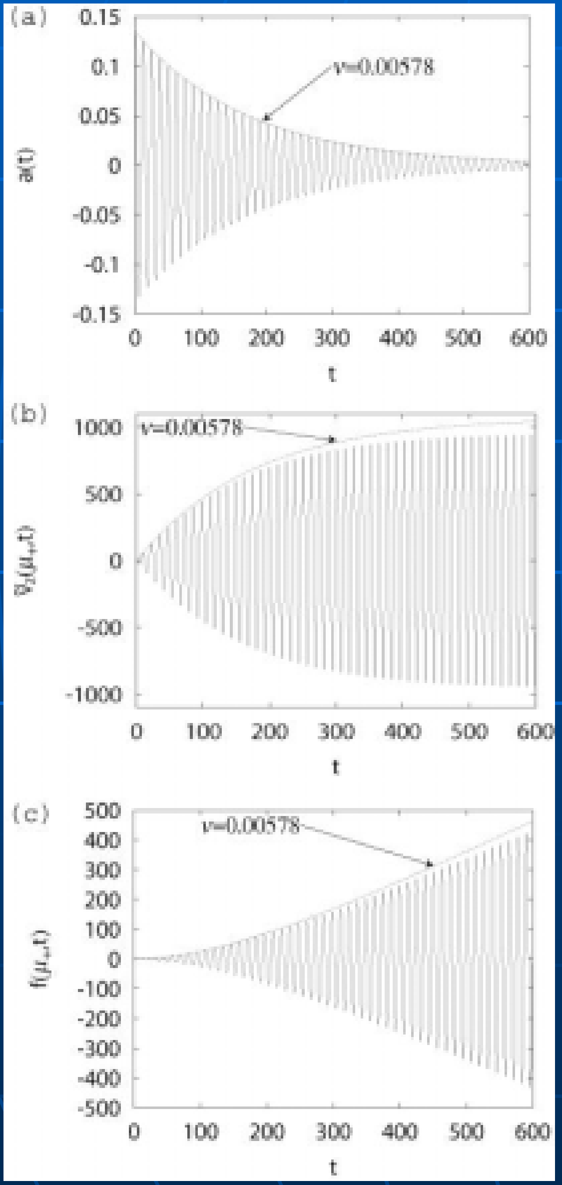
Need new methods to describe and analyze these phenomena

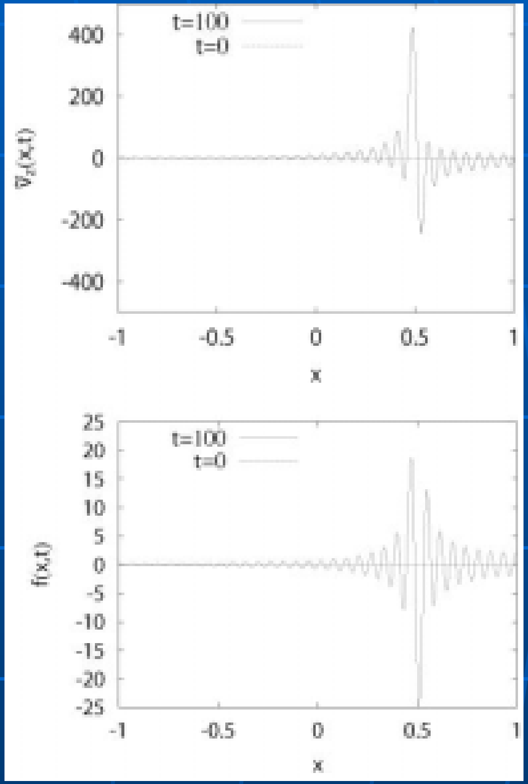
Complex transient phenomena
-- out of the scope of exponential laws



G.D. Chagelishvili, A.D. Rogava, D. Tsiklauri,
Phys. Plasmas **4** (1997), 1182-1195.

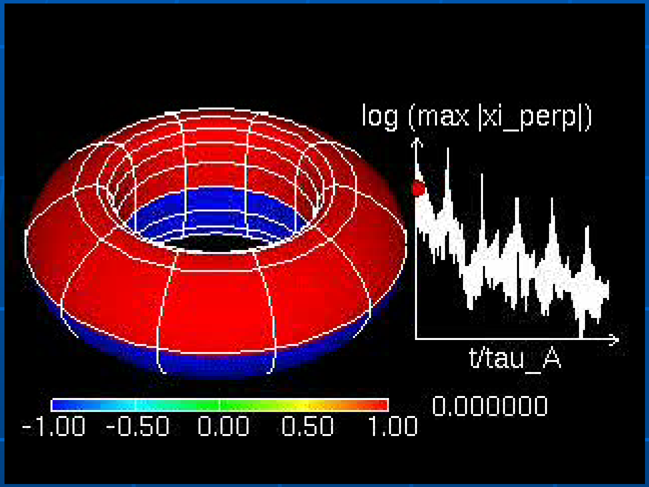
Secular amplification in coupled continuous spectra (KH-type modes)

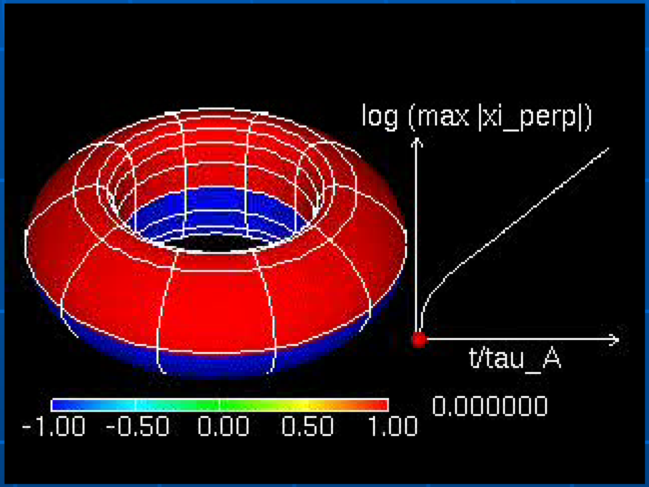




M. Hirota, T. Tatsuno, and Z. Yoshida, "Degenerate continuous spectra producing localized secular instability -- An example in a non-neutral plasma" J. Plasma Phys. 69, 397 (2003).

Ballooning modes in a flowing tokamak





With flow

Without flow

M. Furukawa and S. Tokuda, Phys. Rev. Lett. 94, 175001 (2005).

What do we know about Hermitian systems?

- We may assume

$$u(x,t) = f(t)\phi(x) = \exp(-it\omega)\phi(x)$$

to derive the **dispersion relation** (= eigenvalue problem).

- The eigenfunctions (including singular eigenfunctions belonging to continuous spectra) are **complete**

(von Neumann's theorem).

- The solution to an initial-value problem is given by a group $\{\exp(-itH)\}$ of unitary transforms (propagators):

$$u(t) = \exp(-itH)u(0).$$

- The energy is a constant of motion:

$$< u(t), H\, u(t) > = \text{const.}$$

What can happen in non-Hermitian systems?

- We may NOT assume

$$u(x,t) = f(t)\phi(x) = \exp(-it\omega)\phi(x)$$

The dispersion relation falls short to capture a variety of transient phenomena.

-- may be unstable even when all ω are real.

-- may be stable even when the potential energy can be negantive.

- The eigenfunctions may NOT be complete.

Possible nilpotent (*higher-order singularity*).

- The solution operators {exp(-*it*H)} may NOT be unitary transforms. → wave number is not conserved.

- The energy may NOT be a constant of motion.

Different classes of non-Hermitian generators

- Most pathological generators

  -- not a “closed” system.

- Energy-conserving systems

  – generalized Hamiltonian systems.

Some methods of analysis

- Lyapunov stability

  -- sufficient for stability (necessary for instability)

  Z. Yoshida, S. Ohsaki, A. Ito, S.M. Mahajan, J. Math. Phys. **44**, 2168 (2003).

- Generalized "modes" (Kelvin's method)

  -- integrable if the flow shear is linear.

  -- non-conservative property → effective-mass(t)

  Z. Yoshida, Phys. Plasmas **12**, 024503 (2005).

  -- non-integrable dynamics → energy transfer

  M. Furukawa, Z. Yoshida and S. Tokuda, Phys. Plasmas **12**, 072517 (2005).

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Expansion of ballooning modes in rotating plasmas by "stretching" eigenfunctions

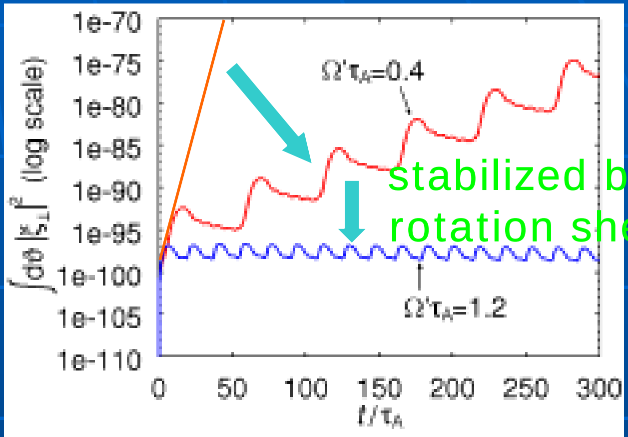
Wave equation for ballooning modes in rotating tokamak plasmas

$$\bar{\rho}\left(\frac{\partial^2 \xi_\perp}{\partial t^2} - U\frac{\partial \xi_\perp}{\partial t}\right) = \mathcal{L}\xi_\perp$$

$$\mathcal{L}\left(\frac{\partial}{\partial\vartheta}, \vartheta, t\right) := \frac{\partial}{\partial\vartheta}\left(f\frac{\partial}{\partial\vartheta}\right) - g$$

$$\{f, g, \bar{\rho}, U\}(\vartheta, t) = \{f, g, \bar{\rho}, U\}(\vartheta + 2\pi, t + 2\pi/(\mathrm{d}\Omega/\mathrm{d}q))$$

Example of numerical solution



Consider an eigenvalue problem

$$\mathcal{L}\xi = -\lambda\bar{\rho}h\xi$$

The window function h is chosen so that this equation becomes Sturm-Liouville type

We obtain eigenvalues λ_j and eigenfunctions ξ_j

$$\mathcal{L}\xi = -\lambda\bar{\rho}\xi$$

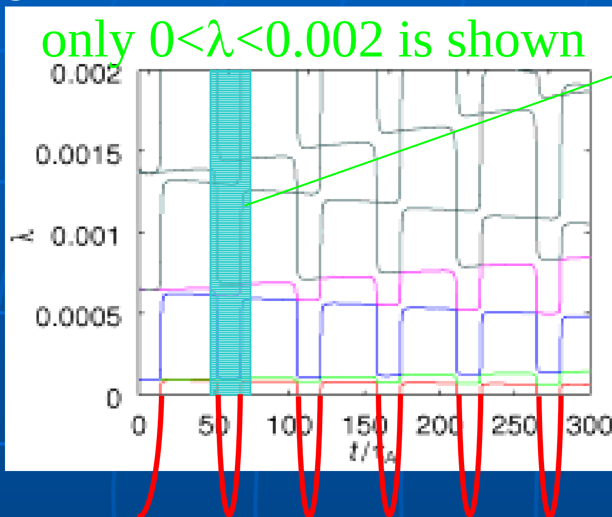
Well-known ballooning equation in static plasmas, which yields the continuous spectrum and the corresponding singular eigenfunctions at the stable side

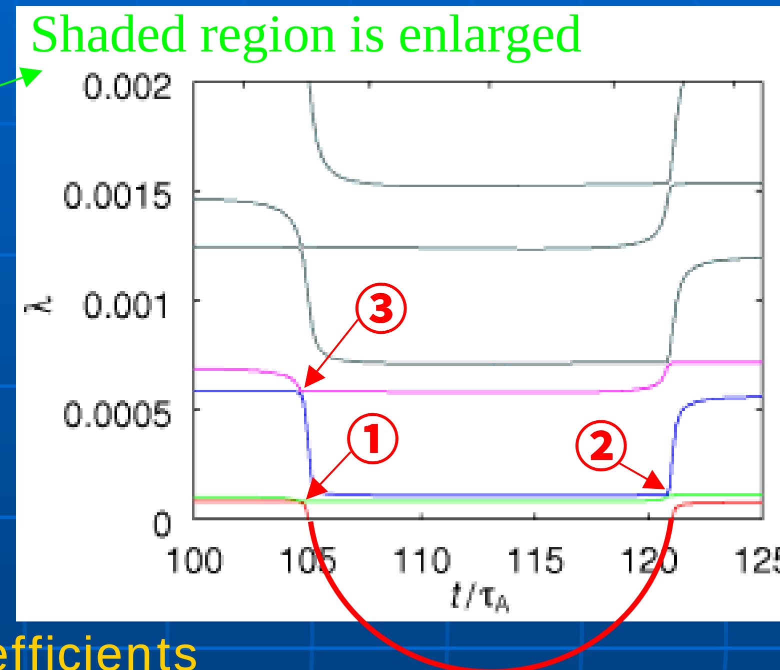
Then we can expand $\xi_\perp$ by the eigenfunctions as

$$\xi_\perp(\vartheta, t) = \sum_{j=1}^{\infty} a_j(t)\xi_j(\vartheta, t) \qquad a_j(t) = \int_{-\infty}^{\infty} \mathrm{d}\vartheta\,\bar{\rho}h\xi_j\xi_\perp$$

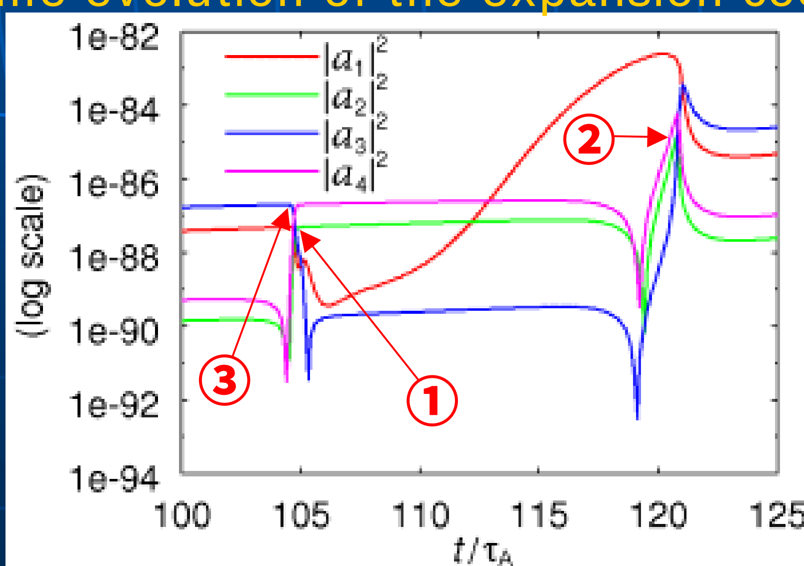
Energy transfer from an unstable mode to stable modes

Eigenvalues as a function of t





Time evolution of the expansion coefficients



$|a_1|^2$ grows during $\lambda_1 < 0$

However, the energy is transferred to stable modes successively when the eigenvalues cross

Therefore, the unstable mode cannot grow in the time average;
i.e., the ballooning mode is stabilized