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# Recent Advances in Wave Kinetics

presented by

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# Recent advances in wave kinetics

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Collaborators:

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# Outline

- **Kinetic equations for the photon gas;**
- **Wigner representation and Wigner-Moyal equations;**
- **Modulational instabilities of quasi-particle beams;**
- **Photon acceleration in a laser wakefield;**
- **Plasmon driven ion acoustic instability;**
- **Drifton excitation of zonal flows;**
- **Resonant interaction between short and large scale perturbations;**
- **Towards a new view of plasma turbulence.**

# Wigner approach

Schroedinger eq.

$$\left[\frac{\hbar^2}{2m}\nabla^2 - i\hbar\frac{\partial}{\partial t}\right]\psi = -V\psi$$

$$\left(\frac{\hbar^2}{2m}\mathbf{k}\cdot\nabla - i\hbar\frac{\partial}{\partial t}\right)W = -2V(\sin\Lambda)\,W,$$

Wigner-Moyal eq.

$$W(\mathbf{r},\mathbf{k},t) = \int \psi\left(\mathbf{r}+\frac{\mathbf{s}}{2},t\right)\psi^*\left(\mathbf{r}-\frac{\mathbf{s}}{2},t\right)\times \times\exp(-i\mathbf{k}\cdot\mathbf{s})\,d\mathbf{s}.$$

$$\Lambda = \leftarrow\left(\frac{\partial}{\partial\mathbf{r}}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\rightarrow$$

# Quasi-classical approximation
# (sin Λ ~ Λ, h —> 0)

$$\mathbf{F} = -\nabla V$$

$$\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{k}} \right) W = 0,$$

$$\mathbf{v} = \hbar \mathbf{k} / m$$

Conservation of the quasi-probability
(one-particle Liouville equation)

$$\frac{d}{dt} W(\mathbf{r}, \mathbf{k}, t) = 0$$

Of little use in Quantum Physics (W can be directly determined from Schroedinger eq.)

# Wigner-Moyal equation for the electromagnetic field

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\chi\vec{E})$$

Field equation (Maxwell)

$$F_k(\vec{r},t) = \int \vec{E}(\vec{r}+\vec{s}/2,t)\cdot\vec{E}^*(\vec{r}-\vec{s}/2,t)e^{-i\vec{k}.\vec{s}}d\vec{s}$$

$$\left(\frac{\partial}{\partial t} + \vec{v}_k\cdot\nabla\right)F_k + \frac{\partial \ln\epsilon}{\partial t}F_k = -\frac{\omega_k}{\epsilon}[\epsilon \sin\Lambda_k F_k].$$

Kinetic equation

$$\Lambda_k = \frac{1}{2}\frac{\overleftarrow{\partial}}{\partial\vec{r}}\cdot\frac{\overrightarrow{\partial}}{\partial\vec{k}}$$

[Mendonca+Tsintsadze, PRE (2001)]

# Photon number density

$$N_k(\vec{r},t) = \frac{\epsilon_0}{8\hbar}\left(\frac{\partial R}{\partial \omega}\right)_{\omega_k} F_k(\vec{r},t)$$

(R=0 is the dispersion relation in the medium)

**For the simple case of plane waves:**

$$N_k(\vec{r},t) = \frac{\epsilon_0}{8\hbar}\frac{\partial R}{\partial \omega}|E_0|^2\delta(\vec{k}-\vec{k}_0)$$

**Slowly varying medium**

$$\frac{dN_k}{dt} \equiv \left(\frac{\partial}{\partial t} + \vec{v}_k \cdot \nabla + \frac{d\vec{k}}{dt}\cdot\frac{\partial}{\partial \vec{k}}\right) N_k = 0$$

**(photon number conservation)**

# Dispersion relation of electron plasma waves in a photon background

$$1 + \chi_e(\omega, \mathbf{k}) + \chi_{ph}(\omega, \mathbf{k}) = 0$$

Electron susceptibility

Photon susceptibility

$$\chi_{ph}(\omega, \mathrm{k}) = -\frac{\omega_{ph0}^2}{2} \frac{k^2 \omega_{pe0}^2}{\gamma_0^2 m_e \omega^2} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\hbar \mathbf{k} \cdot \frac{\partial \hat{N}_0}{\partial \mathbf{k}}}{\omega_{\mathbf{k}}^2 \left(\omega - \mathbf{k} \cdot \mathbf{v}_g(\mathbf{k})\right)}$$

**Resonant wave-photon interaction,**

**Landau damping is possible**

[Bingham+Mendonca+Dawson, PRL (1997)]

# Physical meaning of the Landau resonance

Non-linear three wave interactions

$$\omega - \vec{k}.\vec{v}_g = 0$$

$$\omega + \omega' = \omega''$$
$$\vec{k} + \vec{k}' = \vec{k}''$$

Energy and momentum conservation relations

Limit of low frequency and long wavelength

$$\omega = \omega'' - \omega' = \Delta\omega', with\ |\Delta\omega'| << \omega', \omega''$$
$$\vec{k} = \vec{k}'' - \vec{k}' = \Delta\vec{k}', with\ |\Delta\vec{k}'| << |\vec{k}'|, |\vec{k}''|$$

$$\frac{\omega}{\vec{k}} = \frac{\Delta\omega'}{\Delta\vec{k}'} \Rightarrow \frac{d\omega'}{d\vec{k}'} = \vec{v}_g$$

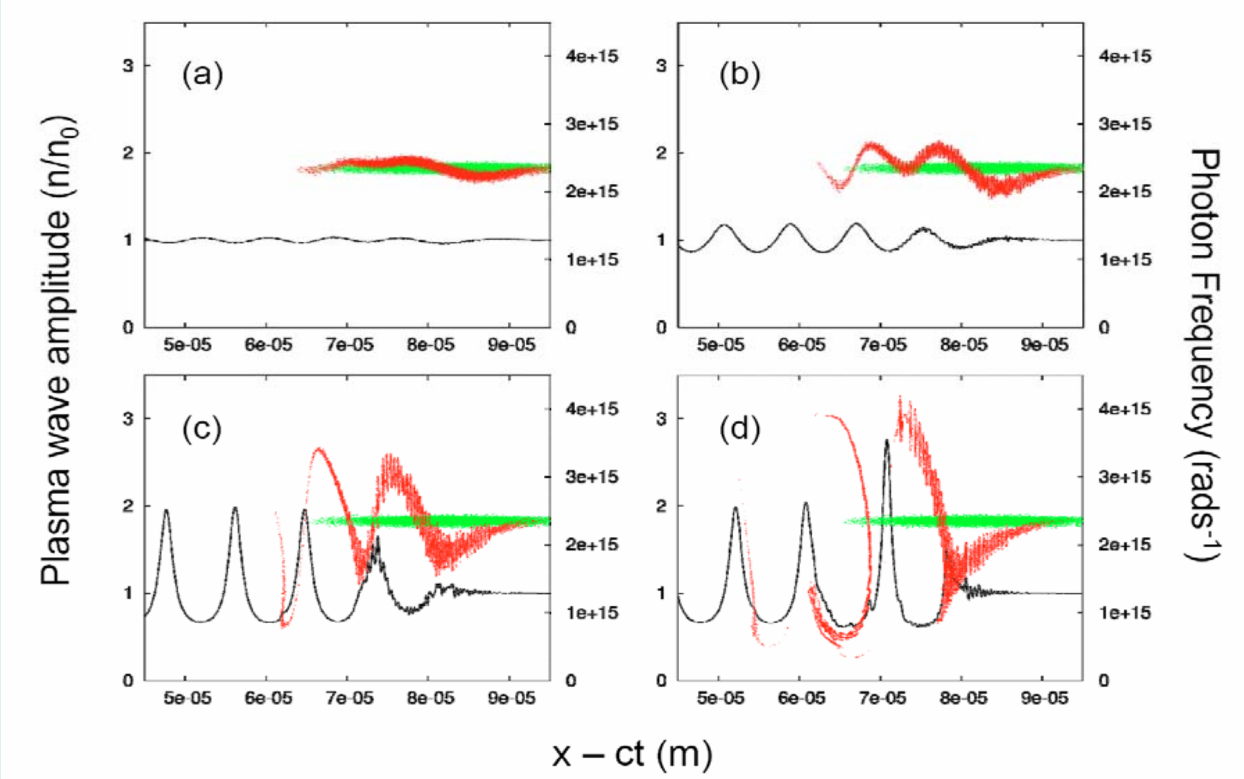
(hints for a quantum description of adiabatic processes)

# Photon dynamics in a laser wakefield

Simulations: R. Trines

Experiments: C. Murphy

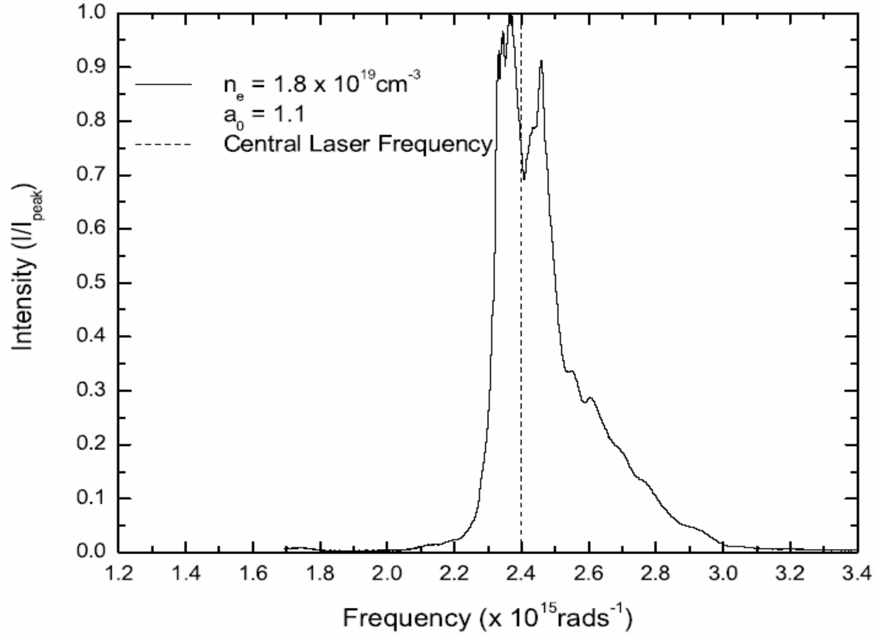
(work performed at RAL)



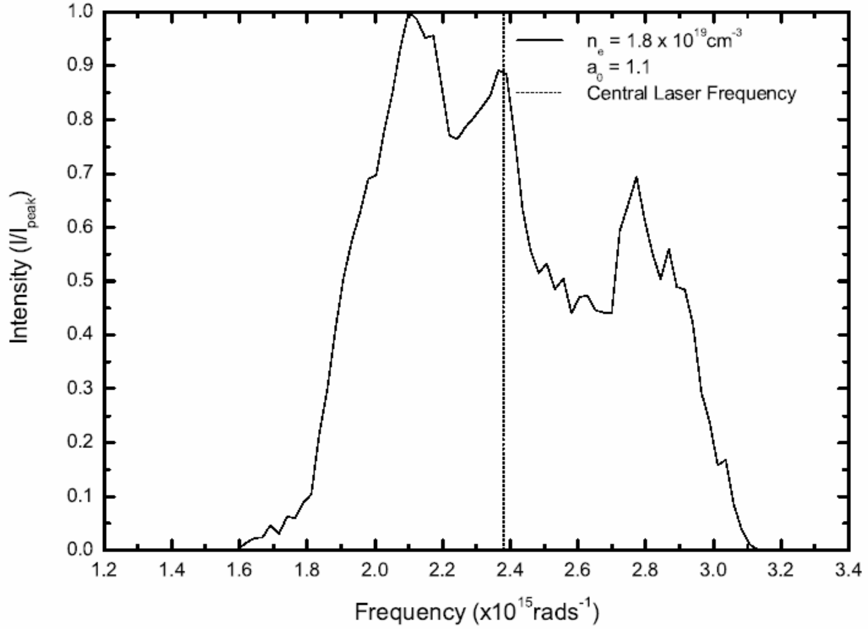

**Spectral features: (a) split peak, (b) bigger split, (c) peak and shoulder, (d) re-split peak**

# Split peak

**Experimental**



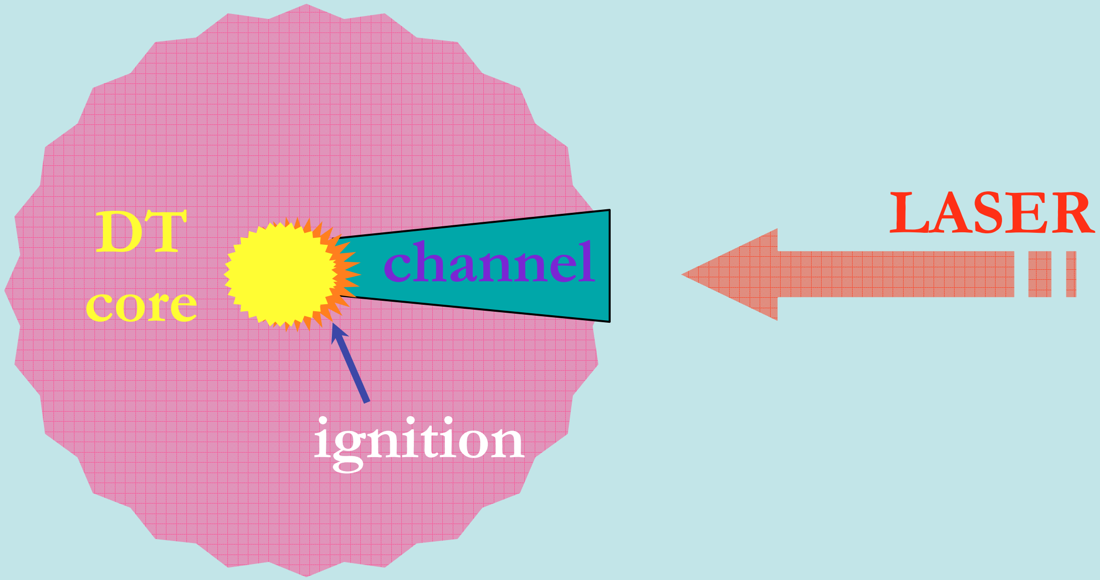

**Numerical 1D**

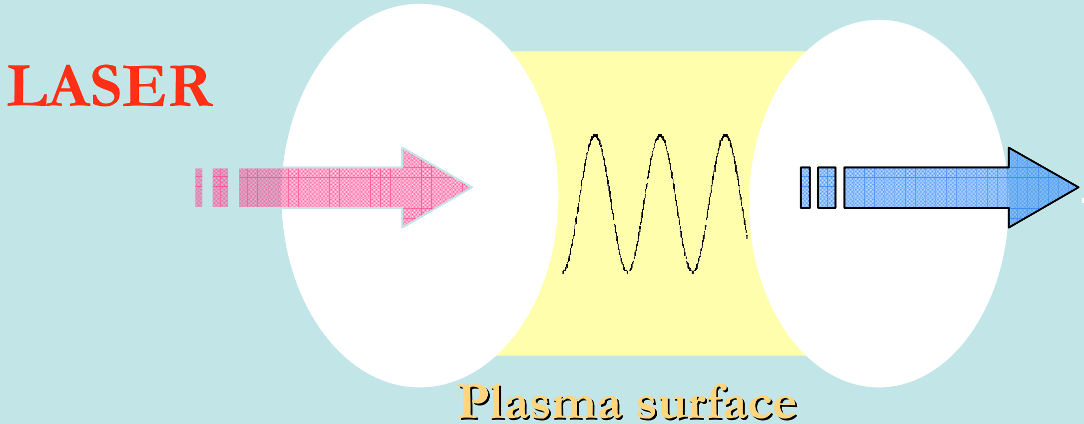



**Also appears in classical particle-in-cell simulations**

**Can be used to estimate wakefield amplitude**

# Anomalous resistivity for Fast Ignition




Fast electron beam
Electron plasma waves
Transverse magnetic fields

**Ultimate goal: ion heating**

# Theoretical model:

## Wave kinetic description of electron plasma turbulence

- Electron plasma waves described as a plasmon gas;
- Resonant excitation of ion acoustic waves

**Dispersion relation of electrostatic waves**

$$1 + \sum_{\alpha=e,i,f} \chi_\alpha(\omega, \vec{k}) + \chi_{pls}(\omega, \vec{k}) = 0$$

# Electron two stream instability

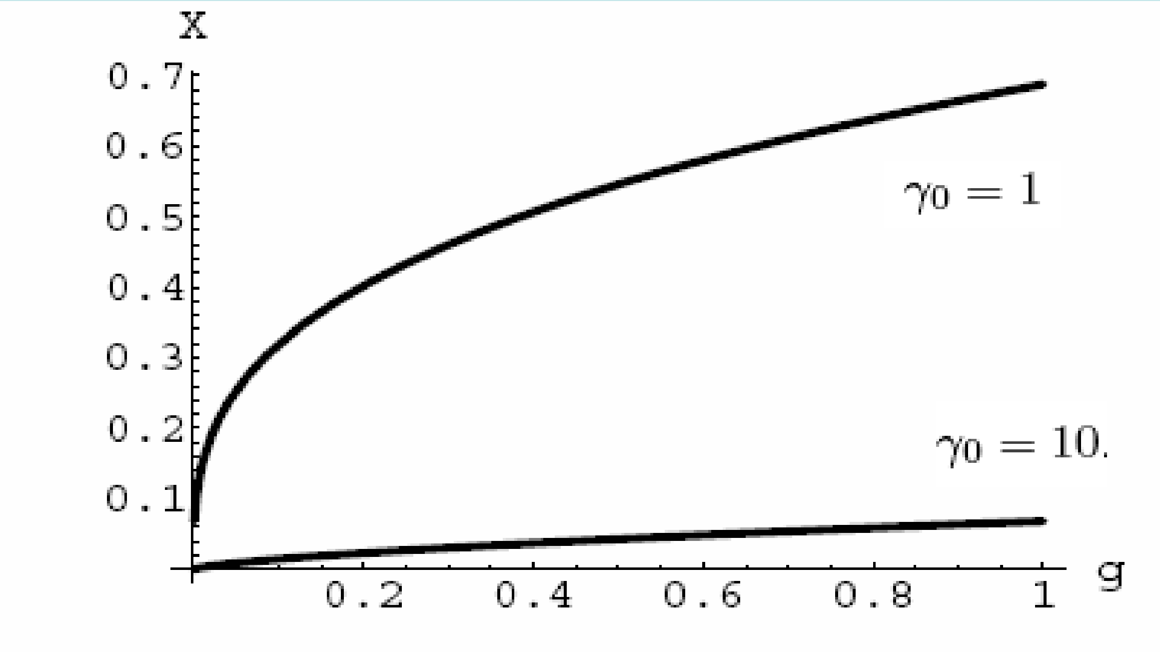
Total plasma current

$$\vec{J} = -e(n_{0e}\vec{u}_e + n_{0f}\vec{u}_f) \simeq 0.$$

$$1 - \frac{\omega_{pe}^2}{(\omega + g\vec{k}\cdot\vec{u}_0)^2} - \frac{\omega_{pf}^2}{\gamma_0^3(\omega - \vec{k}\cdot\vec{u}_0)^2} = 0$$

Dispersion relation

$$g = n_{0f}/n_{0e}$$




Maximum growth rate

$$\Gamma = \frac{\sqrt{3}}{2^{4/3}}\frac{g^{1/3}}{\gamma_0}\omega_{pe}$$

$$X = \Gamma/\omega_{pe}$$

# Kinetic equation for plasmons

$$\left(\frac{\partial}{\partial t} + \vec{v}_{k'} \cdot \frac{\partial}{\partial \vec{r}} + \vec{F}_{k'} \cdot \frac{\partial}{\partial \vec{k}'}\right) N_{k'} = 2\Gamma_{k'} N_{k'}$$

Plasmon occupation number

$$N_{k'} = W_{k'} / \hbar\omega_{k'}$$

$$\omega_{k'} = (\omega_{pe}^2 + S_e^2 k'^2)^{1/2} \simeq \omega_{pe}$$

Plasmon velocity

$$\vec{v}_{k'} = S_e^2 \vec{k}' / \omega_{k'}$$

Force acting on the plasmons

$$F_{k'} = -\frac{e^2}{2\epsilon_0 m_e \omega_{k'}} \nabla n_e$$

# Ion acoutic wave resonantly excited by the plasmon beam

$$\Omega_e^2 \Omega_i^2 = \omega_{pe}^2 \omega_{pi}^2 \left\{ 1 + \frac{\Omega^2}{[(\omega - k u_0) - 2i\Gamma_0]^2} \right\}$$

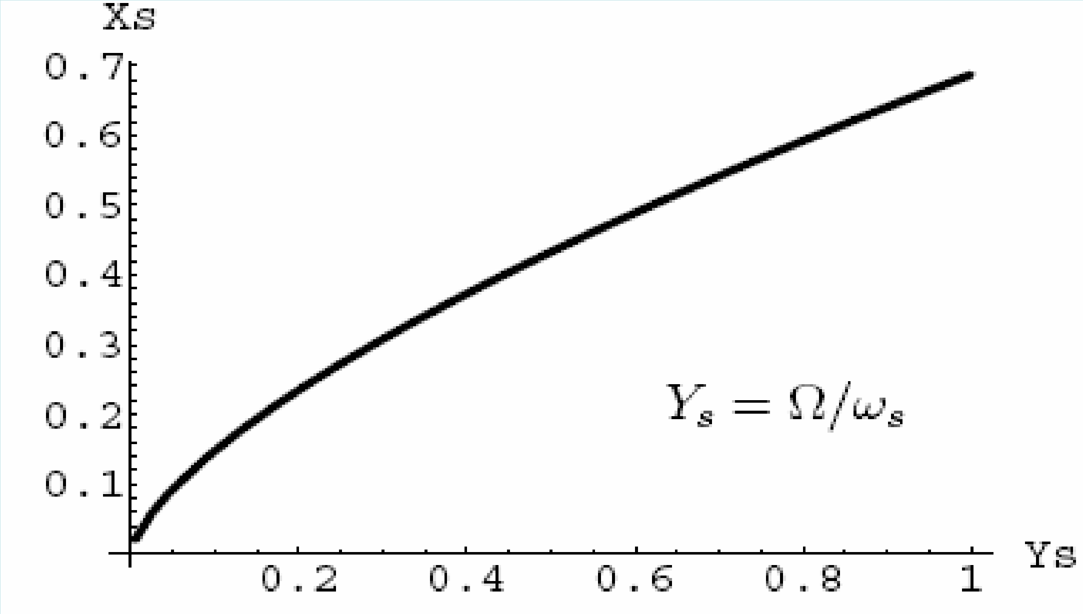
**Effective plasmon frequency**

$$\Omega_\alpha^2 = -(\omega - \vec{k} \cdot \vec{u}_\alpha)[(\omega - \vec{k} \cdot \vec{u}_\alpha) + i\nu_\alpha] + \omega_{p\alpha}^2 \frac{1 + k^2 \lambda_\alpha^2}{\gamma_{0\alpha}^3}$$

$$\Omega^2 = \frac{Z W_0}{n_0 m_e c^2} \frac{k^4 S_e^2 c^2}{\omega_0'^2}$$

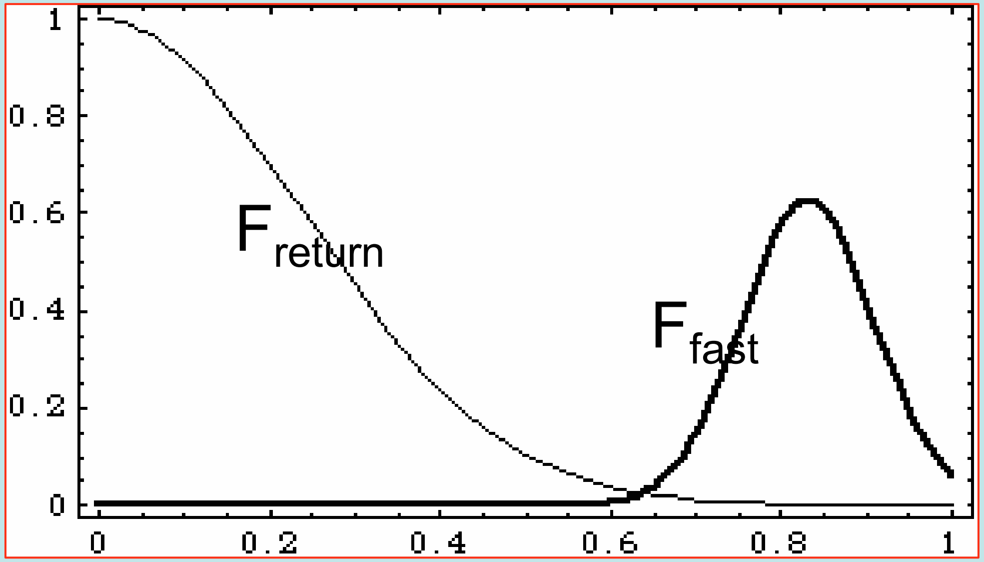
**Maximum growth rate**

$$\Gamma = \sqrt{3} (\omega_s \Omega^2 / 2^4)^{1/3}$$



# Two-stream instability

(interaction between the fast beam and the return current)

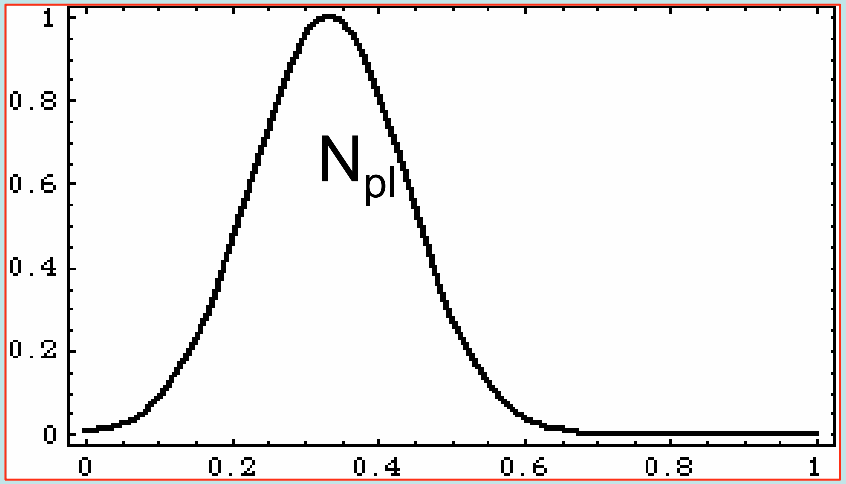



Electron distribution functions

Unstable region:

Plasmon phase velocity $v_{ph} \sim c$

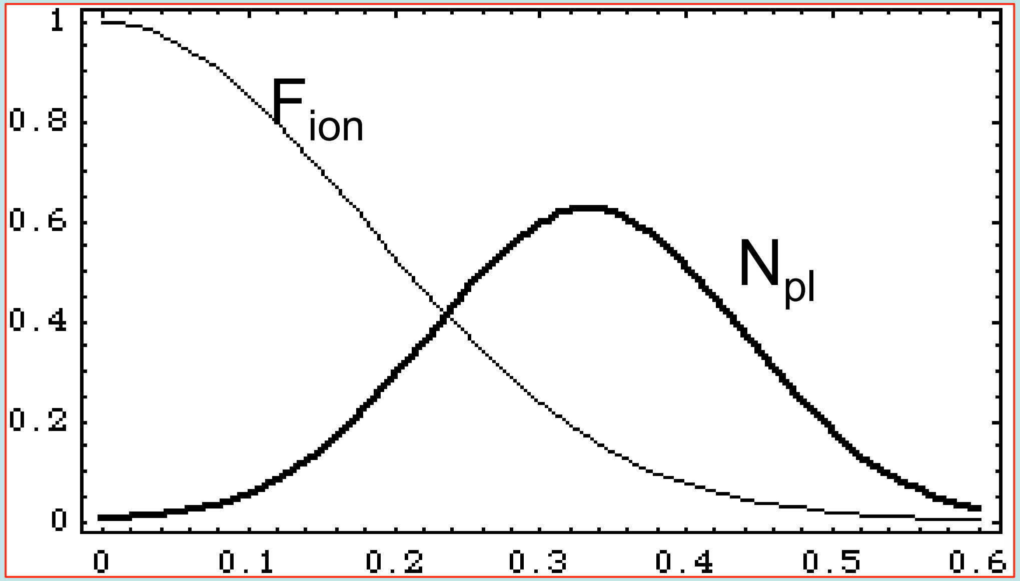
# Low group velocity plasmons: $v_{ph} . v_g = v_{the}^2$




Plasmon distribution

$V_g \sim v_{the}^2 / c$

Ion distribution (ion acoustic waves are destabilized by the plasmon beam)




$V_{ph/ionac} \sim v_g$

[Mendonça et al., PRL (2005)]

# Preferential ion heating regime

## Laser intensity threshold

$$I > \frac{2^4}{3^{3/2}} \frac{n_{0e} m_e c^3}{f_{abs}} \gamma_0^4 \left( \frac{\nu_e}{\omega_{pe}} \right)^3$$

$\gamma_0$ , $u_{0e} \sim I^{1/2}$

Varies as $I^{-5/4}$

(laser absorption factor) $n_{0f} K_f u_f = f_{abs} I$

**For typical laser target experiments, $n_{0e} \sim 10^{23}$ cm$^{-3}$:**

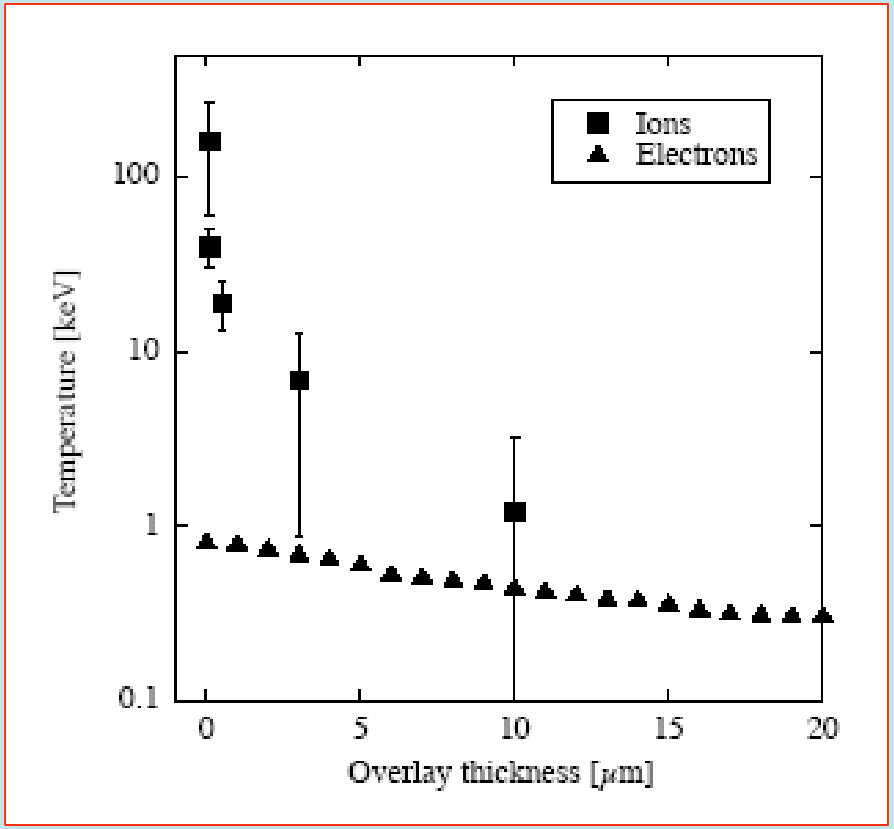
**$I > 10^{20}$ W cm$^{-2}$**

# Experimental evidence

Plastic targets with deuterated layers using Vulcan (RAL)

I = $3 \times 10^{20}$ Watt $cm^{-2}$

Not observed at lower intensities (good agreement with theoretical model)




[P. Norreys et al, PPCF (2005)]

# Coupling of drift waves with zonal flows

**We adapt the 1-D photon code to drift waves:**

- **Two spatial dimensions, cylindrical geometry,**
- **Homogeneous, broadband drifton distribution,**
- **A Gaussian plasma density distribution around the origin.**

**We obtain:**

- **Modulational instability of drift modes,**
- **Excitation of a  zonal flow,**
- **Solitary wave structures drifting outwards.**

# Quasi-particle description of drift waves

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**Fluid model for the plasma (electrostatic potential Φ(r)):**

$$\frac{\partial \Phi}{\partial t} = \int \frac{k_r k_\vartheta}{\left(1 + k_r^2 + k_\vartheta^2\right)^2} N_k d^2 k$$

**Particle model for the "driftons":**

- Drifton number conservation;
- Hamiltonian:

$$\omega_i = k_\vartheta \frac{\partial \Phi}{\partial r} + \frac{k_\vartheta V_*}{\left(1 + k_r^2 + k_\vartheta^2\right)},$$

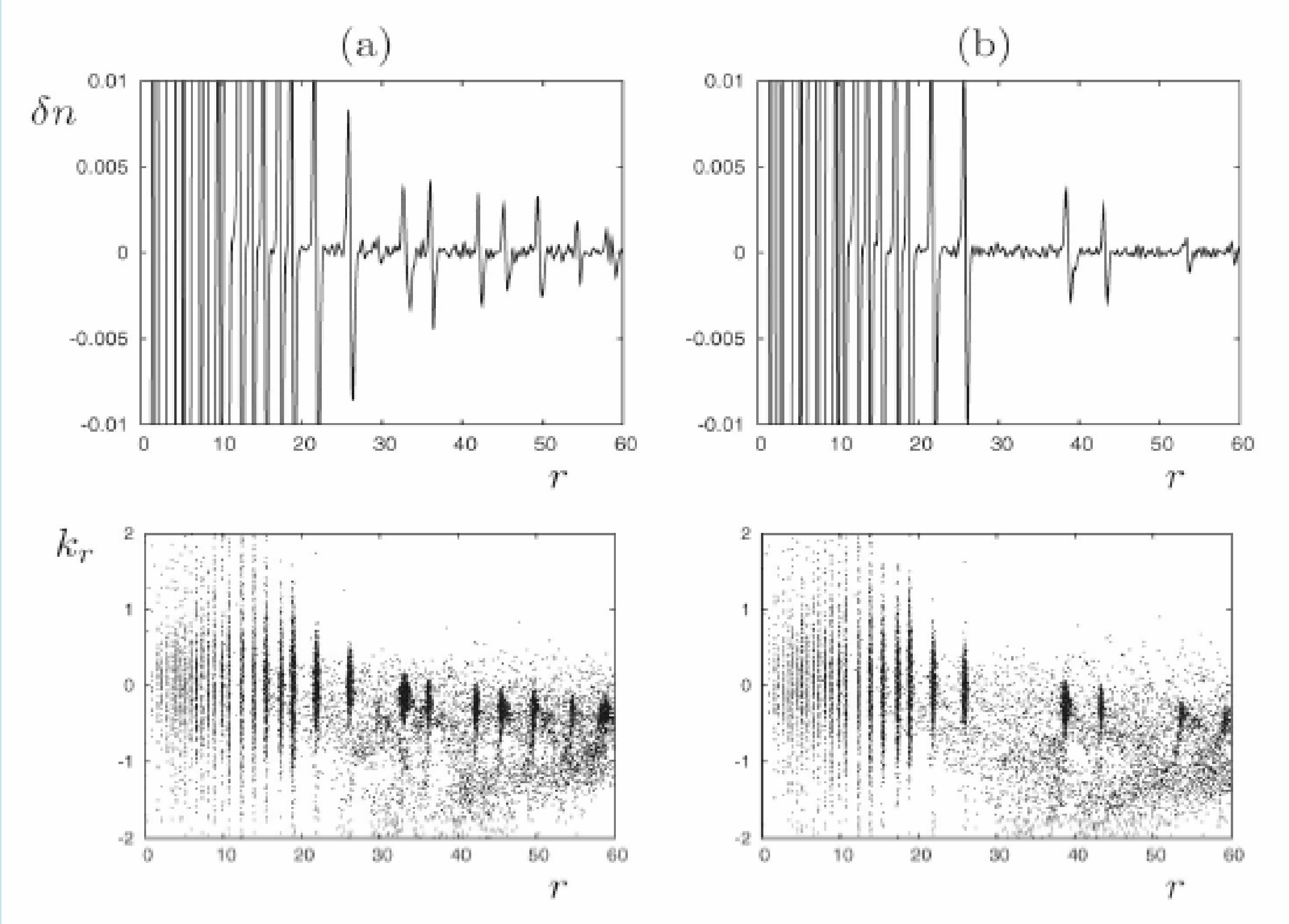
- Equations of motion: from the Hamiltonian

$$V_* = -\frac{1}{n_0}\frac{\partial n_0}{\partial r}$$

[R. Trines et al, PRL (2005)]

# Simulations

**Excitation of a zonal flow for small r, i.e. small background density gradients; Propagation of "zonal" solitons towards larger r.**



# Plasma Physics processes described by wave kinetics

| **Short scales** | **Large scales** | **Physical relevance** |
|---|---|---|
| Photons | Ionization fronts | Photon acceleration |
| Photons | Electron plasma waves | Beam instabilities; photon Landau damping |
| Plasmons | Ion acoustic waves | Anomalous heating |
| Driftons (drift waves) | Zonal flows | Anomalous transport |

# Other Physical Processes

| Short scales | Large scales | Physical relevance |
|---|---|---|
| Photons | laser pulse envelope | Self-phase modulation<br>Cross-phase modulation |
| Photons | polaritons | Tera-Hertz radiation in polar crystals |
| Photons | Gravitational waves | Gamma-ray bursts |
| neutrinos | Electron plasma waves | Supernova explosions |

# Conclusions

• Photon kinetic equations can be derived using the Wigner approach;
• The wave kinetic approach is useful in the quasi-classical limit;
• A simple view of the turbulent plasma processes can be established;
• Resonant interaction from small to large scale fluctuations;
• Successful applications to laser accelerators (wakefield diagnostics); inertial fusion (ion heating) and magnetic fusion (turbulent transport).