# COLLEGE on SOIL PHYSICS

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## Saturated Flow in Porous Media

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**SATURATED FLOW IN POROUS MEDIA**
LECTURE NOTES
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Catena Verlag, pp. 87-101)

## 5.1   BASIC CONCEPTS

The flow of water in soil can be described microscopically and macroscopically. On the microscopic scale, the flow in each individual pore is considered and for each defined continuous pore, the Navier-Stokes equations apply. For their solution we lack detailed knowledge of the geometrical characteristics of individual pores to obtain a solution for the REV. Even with this knowledge, a tremendous effort would be required necessitating voluminous calculations for even a relatively small soil domain. Nevertheless, this type of procedure is often applied in some theoretical investigations where the basic laws of fluid mechanics are invoked. In such studies the real porous system is usually defined by a model assuming great simplification of reality.

The macroscopic or phenomenological approach of water transport relates to the entire cross-section of the soil with the condition of an REV being satisfied. The rate of water transport through the cross section of the REV (the representative elementary area REA) is the flux. In order to emphasize the fact that water does not flow through the entire macroscopic areal cross section, the term flux density (or flux ratio, macroscopic flow rate et al.) is used to describe the flow realized through only that portion of the area not occupied by the solid phase and, by the air phase eventually when we deal later on with unsaturated soil. Moreover, we use the term flux density understanding that we actually mean the volumetric water flux density having the dimensions of velocity $[LT^{-1}]$.

Inasmuch as the principal equation derived for this macroscopic approach is Darcy's equation, the scale for which this approach is valid is often denoted as the Darcian scale. For soils, the area of this scale is usually in the range of $cm^2$ to $m^2$. Beyond this scale in either direction, larger or smaller, Darcian scale equations may not be realistic. Unless we state otherwise, equations will be derived and solved mainly for the Darcian scale related to a particular REV.

On the Darcian scale, water flow in soils is comparable to other transport processes such as heat flow, molecular diffusion etc. when the appropriate driving force is defined. For example, when the distant ends of a metal rod are kept at different temperatures, heat flow exists. Similarly, molecular diffusion depends upon a difference of concentration in two mutually interconnected pools. Soil water flow is conditioned by the existence of a driving force stemming from a difference of total potentials between two points in the soil. Laymen mistakenly suppose that the driving force of water flow in an unsaturated soil is related to

differences in soil water content. This supposition, valid only for a few specified conditions, generally leads to erroneous conclusions.

Here, we first formulate basic flow equations for the simplest case of flow in a saturated, inert rigid soil. Afterwards, we deal with water flow in a soil not fully saturated with water. This latter type of flow is commonly called unsaturated flow while the former is called saturated flow. To be more precise, we should distinguish the former from the latter flows as those occurring at positive and negative soil water pressures, respectively. If the flow of both air and water in the soil system is simultaneously considered, we speak of two phase flow. Initially, we assume that the concentration of the soil solution does not affect the soil water flow. Subsequently, our discussion is extended to swelling and shrinking soils. Finally, we examine linked or coupled flows together with some specific phenomena of transport at temperatures below 0°C.

All equations that we derive are supposed to be applicable to not only analytical and approximate mathematical solutions of the components of the soil hydrological system but to all deterministic models of soil hydrology.

## 5.2 SATURATED FLOW

We assume that water is flowing in all pores of the soil under a positive pressure head $h$. In field situations the soil rarely reaches complete water saturation. Usually it is quasi-saturated with the soil water content $\theta_W = mP$ where $m$ has values of 0.85 to 0.95 at $h \geq 0$, and $P$ is the porosity. Entrapped air occupies the volume $P(1 - m)$. And for this discussion of saturated flow, the impact of entrapped air is not considered.
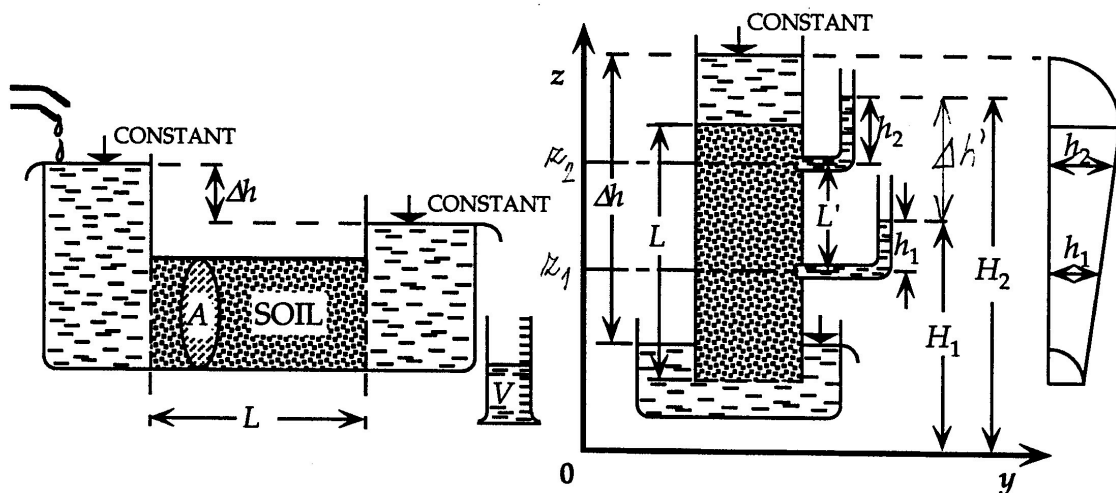
### 5.2.1 Darcy's Equation



*Fig. 5.1. Steady flow experiments on saturated soil columns. On the right, the soil is placed in a cylinder provided with piezometers for measuring pressure heads $h_1$ and $h_2$ at depth $z_1$ and $z_2$, respectively. The total potential head is $H = h + z$.*

For the derivation of Darcy's equation we shall discuss a simple experiment demonstrated in Fig. 5.1. The soil is placed in a horizontal cylinder connected on both sides with vessels containing water maintained at a constant level in each vessel by an overflow valve. If the water level on the left side is higher than that on the right side, water flows to the right. The rate of discharge $Q = V/t$ is simply measured by the volumetric overflow $V$ in time $t$. The flux density $q$ [LT$^{-1}$] (macroscopic flow rate) is

$$q = \frac{V}{A\,t}$$

(5.1)

where $A$ is the cross-sectional area of the soil column perpendicular to the direction of flow. Sometimes, the term $q$ is also called the Darcian flow rate. The mean water flow rate (velocity) in the soil pores $v_p$ is

$$v_p = q/P$$

(5.2)

In 1856, Darcy experimentally demonstrated for columns of sand a linear relationship between the flux density $q$ and the hydraulic gradient $I_h$. In our experiment shown on the right side of Fig. 5.1

$$q = K_s \frac{\Delta h}{L} = K_s \frac{\Delta h'}{L'} = K_s I_h$$

(5.3)

where $\Delta h/L$ or $\Delta h'/L'$ is the hydraulic gradient $I_h$, $\Delta h$ the difference between water levels on both ends of the soil column of length $L$ and $\Delta h'$ the difference between water levels in the piezometers separated by the distance $L'$ in the direction of flow. Both $\Delta h$ and $\Delta h'$ are considered the hydraulic head drop along the soil. Inasmuch as $\Delta h/L$ is dimensionless, $K_S$ has the dimension of $q$ [LT$^{-1}$]. When we read piezometer levels $h_1$ and $h_2$ at elevations $z_1$ and $z_2$, respectively, we have in terms of the total potential $H$

$$q = -K_s \left( \frac{H_2 - H_1}{z_2 - z_1} \right)$$

(5.4)

where the total potential head $H$ ($= h + z$) is related to a unit weight of water. In a more general way (5.4) becomes

$$q = -K_s\, grad H$$

(5.5)

Equation (5.5) states that the flux density is proportional to the driving force of the water flow which is the gradient of the potential. Inasmuch as $K_S$ is a constant for a given soil, we write $\phi^* = K_S H$, and hence,

$$q = -grad\phi^*$$

(5.6)

where $\phi^*$ is $K_S H$. The negative sign in the above equations means that water flows in the direction of decreasing potential or against the positive direction of $z$ in Fig. 5.1. The value of $K_S$ depends upon the nature of the soil and is numerically equal to the flow rate when the hydraulic gradient is unity. Values of $K_S$ commonly range from less than 0.1 cm·day$^{-1}$ ($10^{-8}$ m·s$^{-1}$) to more than $10^2$ cm·day$^{-1}$ ($10^{-5}$ m·s$^{-1}$).

In layered soils we have to specify the direction of the flow relative to that of the layering. When the flow is parallel to the layers, the total flux is the sum of the fluxes for each of the individual layers, see Fig. 5.2. Hence, $Q = Q_1 + Q_2 + Q_3$ and

$$q(b_1 + b_2 + b_3)d = (q_1 b_1 + q_2 b_2 + q_3 b_3)d.$$

(5.7)

For a column of width $d = 1$, length $L$ and thickness $b$ composed of three layers each of thickness $b_i$, the total flux density for a hydraulic head drop $\Delta h$ is

$$q = \left( \frac{K_1 b_1 + K_2 b_2 + K_3 b_3}{b_1 + b_2 + b_3} \right) \frac{\Delta h}{L}$$

or

$$q = K_S' \frac{\Delta h}{L} .$$
(5.8)

Here, the apparent hydraulic conductivity $K_S'$ is the arithmetic mean of the individual values for each layer.

When water flows perpendicular to the layering, we introduce, analogous to an electrical resistance, the hydraulic resistance of each layer $R_i = L_i/K_{Si}$ having units of time. In Fig. 5.2 the flow combined from the three layers is

$$q = \frac{\Delta h}{R_1 + R_2 + R_3} .$$
(5.9)

With the total resistance of the system $R = \Sigma R_i$ we obtain the harmonic mean or the apparent hydraulic conductivity $K_S'' = L/R$.

When the flow is at an angle $< 90°$ to the layers, the difference of $K_S$ in each of the layers causes a change of the direction of streamlines (Zaslavsky and Sinai, 1981, and Miyazaki, 1990), see Fig. 5.3.
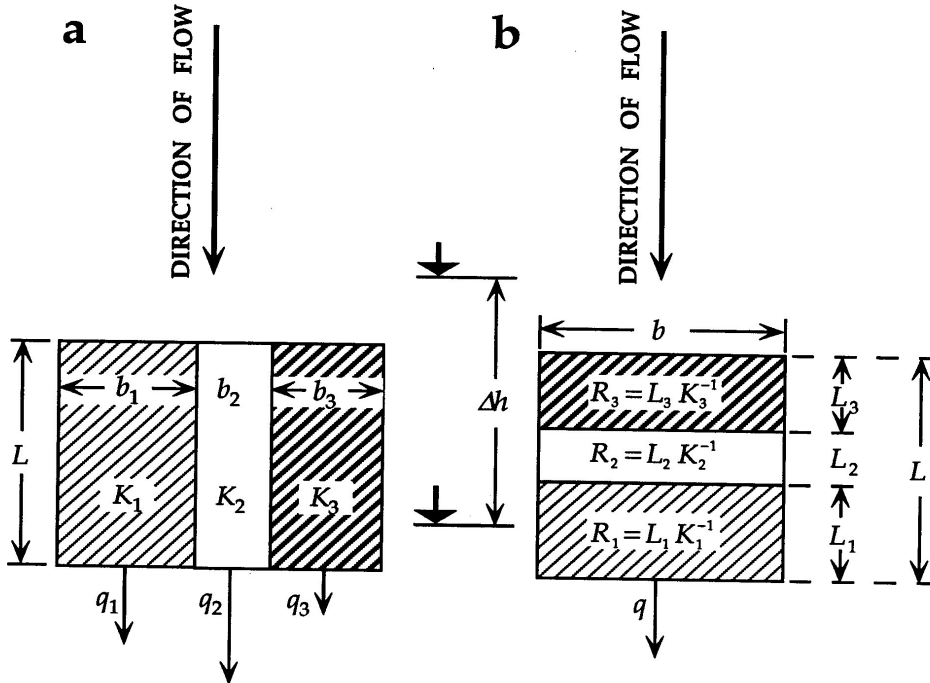


*Fig. 5.2. Flux density q in layered soils: a. Direction of flow is parallel to layering. B. Direction of flow is perpendicular to layering.*
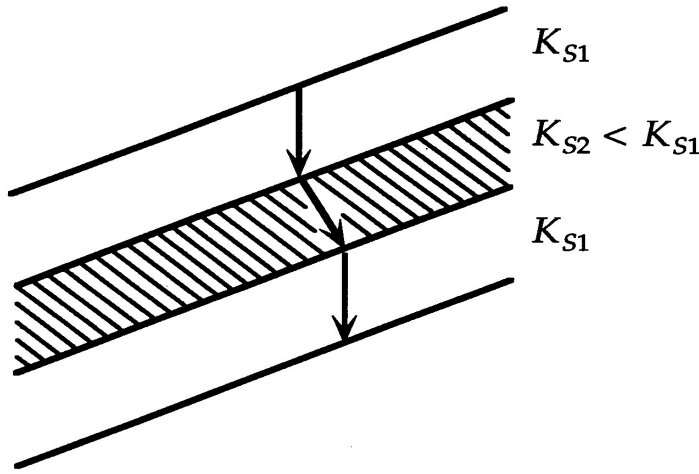
*Fig. 5.3. Change of the direction of flow in a soil with inclined direction of layers.*

## 5.2.2 Saturated Hydraulic Conductivity

Inasmuch as the soil water potential $H$ can be expressed in three modes, the dimension of the hydraulic conductivity is not necessarily $[LT^{-1}]$. From (5.5) we obtain for the three dimensions of $H$ three different dimensions of $K_S$. Although expressing $K_S$ in units of velocity is usually more convenient, any one of the following sets of units is occasionally preferred.

| $H$ | $grad\,H$ | $K_S$ |
|---|---|---|
| J·kg$^{-1}$ [L$^2$T$^{-2}$] | J·kg$^{-1}$·m$^{-1}$ [LT$^{-2}$] | s [T] |
| Pa [ML$^{-1}$T$^{-2}$] | Pa·m$^{-1}$ [ML$^{-2}$T$^{-2}$] | m·s$^{-1}$·$\rho_W^{-1}$·g$^{-1}$ [M$^{-1}$L$^3$T] |
| m [L] | dimensionless | m·s$^{-1}$ [LT$^{-1}$] |

The empirical, intuitive derivation of Darcy's equation (5.5) can be theoretically justified from Navier-Stokes equations applied to an REV of a model of a porous medium and scaled with a characteristic length. In order to obtain (5.5), inertial effects were neglected and the density and viscosity of water were assumed invariant (Bear, 1972; Whitaker, 1986). Scheidegger (1957) showed that $K_S$ should be considered a scalar quantity for isotropic soils, and a tensor of rank 2 for anisotropic soils with the value of $K_S$ dependent upon the direction of flow. When the tensor $K_S$ is assumed to be symmetric, its principal axes, defined by six values, are identical to those of an ellipsoid of conductivity. If the gradient of the potential is not in the direction of a principal axis, the direction of flow is different from that of the gradient.
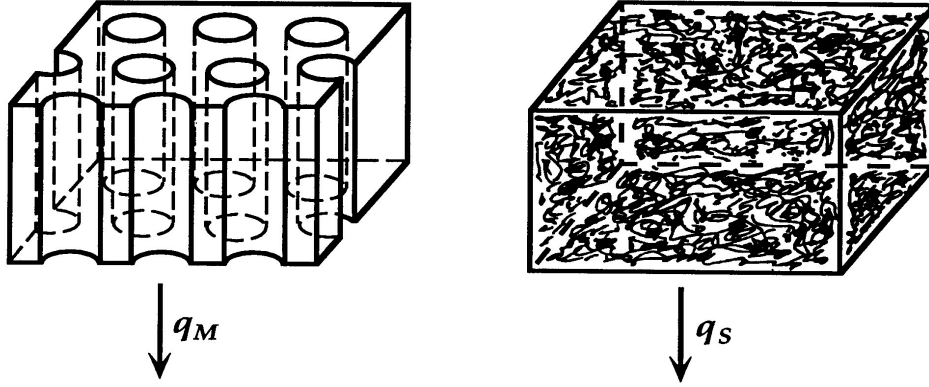
*Fig. 5.4. In Kozeny's model, the complicated soil porous system (right) is represented by a bundle of parallel capillary tubes of uniform radius (left). The flux density q, saturated hydraulic conductivity $K_S$, porosity P and surface area of pores $A_m$ are the same in both, the model and the soil.*

From a theoretical treatment we can obtain a physical interpretation of the hydraulic conductivity. We develop here a modified and simplified model of Kozeny (cf. Scheidegger, 1957) consisting of a bundle of parallel capillary tubes of uniform radius. We assume that the soil and the model are identical with respect to porosity $P$, specific surface $A_m$ [L$^{-1}$] and water flux density $q$ [LT$^{-1}$], see Fig. 5.4. The mean flow rate $v_p$ in a capillary of radius $r$ is described by Hagen-Poiseuille's equation

$$v_p = \frac{\rho_W \, g r^2}{8\mu} \, I_h$$

(5.10)

where $g$ is the acceleration of gravity [LT$^{-2}$], $\rho_W$ the density of water [ML$^{-3}$], $\mu$ the dynamic viscosity [ML$^{-1}$T$^{-1}$] and $I_h$ the hydraulic gradient [dimensionless]. With $n$ being the number of capillaries of unit length $x$, the porosity of the model is

$$P = n\pi r^2 x / V_u$$

(5.11)

where $V_u$ is the unit volume and the specific surface is

$$A_m = 2n\pi r x / V_u.$$

(5.12)

From (5.11) and (5.12) we obtain

$$r = \frac{2P}{A_m}.$$

(5.13)

And, from (5.2) and (5.10) we obtain

$$q = \frac{1}{2} \frac{\rho_W \, g}{\mu} \frac{P^3}{A_m^2} \, I_h$$

(5.14)

Because soil pores are irregularly shaped and mutually interconnected, a shape factor $c$ replaces 1/2 in (5.14). Letting

$$K_p = \frac{cP^3}{A_m^2}$$

(5.15)

we obtain

$$q = K_p \frac{\rho_w\, g}{\mu} I_h$$
(5.16)

which is identical to (5.3). Because the term $K_p$ relates to the flow of any fluid through a soil, it is called the permeability $[L^2]$. The unusual dimension of $K_p$ represents the cross-sectional area of an equivalent pore. Although now almost obsolete, the historical unit of 1 Darcy = 1 $\mu m^2$ was used for describing permeability.

Inasmuch as flow channels in the soil are curved compared with those of a capillary model, a tortuosity factor $\tau$ introduced in (5.15) yields the Kozeny equation

$$K_p = \frac{cP^3}{\tau A_m^2}.$$
(5.17)

The tortuosity $\tau$ is the ratio between the real flow path length $L_e$ and the straight distance $L$ between the two points of the soil. Because $L_e > L$, $\tau > 1$. In a monodispersed sand manifesting a value of $\tau \approx 2$, the flow path forms approximately a sinusoidal curve (Corey, 1977).

Equations identical or of similar type to (5.17) have been derived by many authors. If a model of parallel plates is used instead of capillary tubes and the slits are oriented in the direction of the laminar flow, we obtain the mean flow rate

$$v_p = \frac{\rho_w\, gd^2}{3\mu} I_h$$
(5.18)

where $2d$ is the distance between the plates. When $B$ is the width of the plates, $P = 2ndBx/V_u$ and $A_m = 2nx(2d + B)/V_u$. Taking $x = 1$ and $B = 1$, we obtain $d = P(A_m - 2P)$ and hence,

$$K_p = \frac{cP^3}{\tau (A_m - 2P)^2}.$$
(5.19)

Looking at (5.17) and (5.19), we are reminded of the Kozeny-Carman equation

$$K_p = \frac{P^3}{5A_m^2 (1-P)^2}.$$
(5.20)

Its derivation was shown in detail by Scheidegger (1957).

From (5.3) and (5.16) the relationship between $K_S$ and any formulation of $K_p$ is

$$K_S = K_p \frac{\rho_w\, g}{\mu}.$$
(5.21)


Kozeny's equation shows that $K_S$ is sensitive to porosity. However, in his model the pore radii are considered uniform while those in real soils have broad distributions, see the Fig. 5.4a.

For real soils, we subdivide the pores according to their radii into $j$ categories each having an equivalent radius $r_j$. For $r_j > r_{j+1}$ the flux in each category $q_j(r_j^4, n_j)$ where $n_j$ is the percentage of the $j$-th category in the whole soil, $q_j \gg q_{j+1}$. Total flux $q = \Sigma\, q_j$ as shown for parallel layering. Thus, let us assume for $j = 1$, the percentage of the category of largest pores is eliminated by compaction. Although the porosity may be only marginally reduced, the value of $K_S$ may be reduced by orders of magnitude.
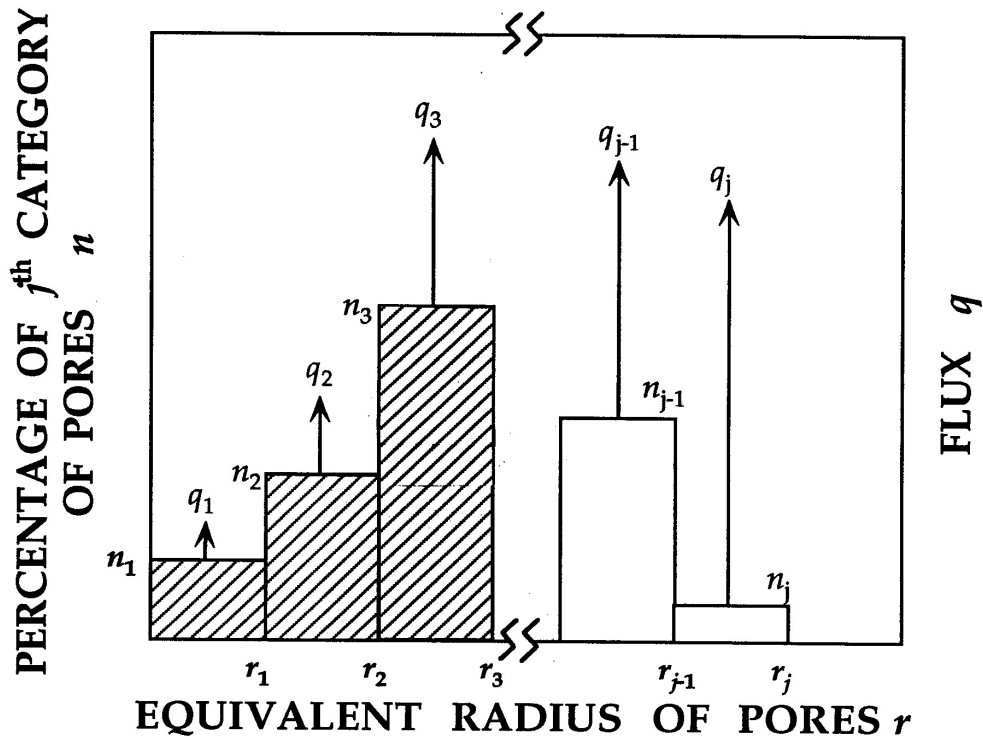
*Fig. 5.4.a Contribution of soil pore categories with $r_1 < r_2 < ...r_j$ and their fluxes $q_1$, $q_2$,..$q_j$. tp the total flux $q = q_1 + q_2 + ...q_j$. Elimination of coarse pores results in substantial reduction of q and for the given grad H reduction of $K_S$. Coarse pores are either eliminated, or their volume is strongly reduced due disaggregation or soil compaction.*

It is logical, therefore, that aggregation of a soil may increase $K_S$ by orders of magnitude, yet the porosity may remain nearly the same. And, vice versa, soil dispersion or disaggregation substantially decreases $K_S$. For example, in a loess soil, the saturated hydraulic conductivity of its surface after a heavy rain decreases 3 to 4 orders of magnitude compared with its original value owing mainly to two processes – disaggregation and the blockage of pores by the released clay particles (McIntyre, 1958). Compaction of soil in the A-horizon and in the bottom of the plowed sub horizon causes a much greater decrease of $K_S$ than that predicted from a decrease of porosity in the simple Kozeny equation because compaction reduces primarily the content of large soil pores associated with values of pressure head $h = 0$ to -100 cm.

Although the textural class of a soil may have a large influence on the value of $K_S$, any attempt to establish a correlation between the two attributes usually fails. Only for those soils and soil horizons of the same genetic development occurring in the same region and being similarly managed will a correlation between texture and $K_S$ be manifested. On the other hand, a few generalities may exist. For example, the smallest values of $K_S$ in each of the main textural classes can be approximated. In sandy soils, the minimum value of $K_S$ is about 100 cm·day$^{-1}$, in silty loams about 10 cm·day$^{-1}$ and in clays about 0.1 cm·day$^{-1}$. In peats, $K_S$ decreases with an increasing degree of decomposition of the original organic substances. When the degree of decomposition of a peat is about 40 to 50%, the value of $K_S$ diminishes to values of $K_S$ typical of unconsolidated clays. Extreme drainage and concomitant drying of

peat soils causing compaction and an increase of soil bulk density also reduce the magnitude of $K_S$. Moreover, because this drying increases hydrophobism, entrapment of air during wetting is enhanced and contributes even further to the decrease of $K_S$.

In loams and clays, the nature of the prevalent exchangeable cation plays an important role relative to the value of $K_S$, see Fig. 5.5. In vertisols, an increase of the percentage of exchangeable sodium (ESP) is accompanied by a decrease in $K_S$ when the ESP reaches 15 to 20%, provided that the soluble salt content of the soil water is small. For example, with the electrical conductivity of the soil paste EC being 1 mS·cm$^{-1}$ or less, the value of $K_S$ can decrease two or three orders of magnitude. On the other hand, even for the same soil having a
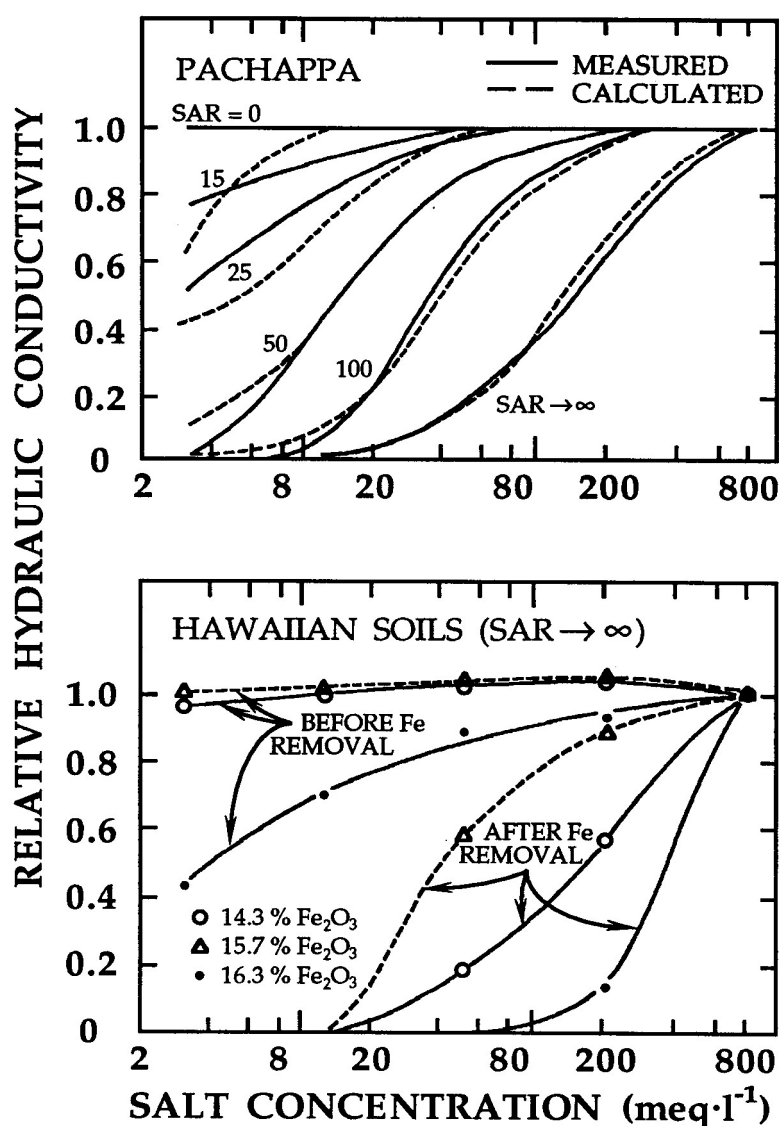


Fig. 5.5. The influence of sodium adsorption ratio SAR and salt concentration upon the value of the saturated hydraulic conductivity is very strong in Aridisols, Ustolls and Vertisols (see top), while in Oxisols with a large accumulation of free iron oxides the influence of SAR and salt concentration is weak (see bottom) (Mc Neal et al., 1968).

large ESP, if the concentration of soluble salts is increased substantially to an EC value of

about 8 mS·cm$^{-1}$ or more, the $K_S$ value is not significantly affected. The value of ESP is closely related to the sodium adsorption ratio SAR of percolating water [SAR = Na/(Ca + Mg)$^{1/2}$]. If Ca is completely absent (i.e. only Mg appears in the denominator), the value of $K_S$ is more greatly reduced than when Mg is absent (McNeal et al., 1968). When monovalent cations are considered in addition to Na, we find that potassium leads to a decrease of $K_S$ but its influence is not as strong as that of exchangeable Na. It has been shown that large organic cations such as pyridinium cause the value of $K_S$ to increase by several orders of magnitude in montmorillonitic clay while their impact on the value of $K_S$ of kaolinitic clays is less significant (Kutílek and Salingerová, 1966). These variations are closely related to the degree of flocculation or peptization of the soil colloidal particles that can be quantified with the value of the $\zeta$-potential derived from double layer theory. Applying this theory, the decrease in $K_S$ owing to the action of rain water (very small EC) is easily predicted for soils having large SAR values. These predictions are not necessarily successful for soils that differ pedologically. For example, a solution of high SAR value percolating through an Oxisol does not decrease the value of $K_S$ even after reducing the solute concentration because the abundant free iron oxides prevent peptization and disaggregation of the soil particles. The value of $K_S$ also depends upon the composition of the clay fraction. It decreases in the order kaolinite, illite and montmorillonite. And, soil organic matter has a profound impact upon the magnitude of $K_S$, owing to its cementing action that promotes aggregate stability. Bacteria and algae may reduce the value of hydraulic conductivity in long term laboratory tests and in some field conditions when sewage treatment effluents are either used for irrigation or disposed on special lands. These same effects can also exist in an irrigated soil well supplied with plant nutrients and sunshine. The process is not necessarily restricted to only anaerobic conditions inasmuch as some aerobic bacteria may cause a reduction in the value of $K_S$. The reduction is partly attributed to cells of bacteria and algae mechanically clogging the soil pores and to slimy, less pervious products of microbial activity being deposited on the walls and necks of the soil pores. For long term laboratory experiments, bactericides are commonly used to prevent the value of $K_S$ from decreasing. In general, there are many factors influencing the value of $K_S$ that are usually not considered in simplified models.

Soils classified according to their values of $K_S$ are

| | |
|---|---|
| *very low permeability* | $K_S < 10^{-7}$ m·s$^{-1}$ |
| *low permeability* | $10^{-7} < K_S < 10^{-6}$ m·s$^{-1}$ |
| *medium permeability* | $10^{-6} < K_S < 10^{-5}$ m·s$^{-1}$ |
| *high permeability* | $10^{-5} < K_S < 10^{-4}$ m·s$^{-1}$ |
| *excessive permeability* | $K_S > 10^{-4}$ m·s$^{-1}$ |

Geological materials are similarly classified as

| | |
|---|---|
| *compacted clays* | $10^{-11} < K_S < 10^{-9}$ m·s$^{-1}$ |
| *gravel* | $10^{-1} < K_S < 10^{1}$ m·s$^{-1}$ |

All such classification schemes above are problematical. For soils in a certain region, a more appropriate classification would be based upon the frequency distribution of $K_S$. Based upon that frequency distribution, we can identify sub regions where a particular range of $K_S$ is expected.

When values of $K_S$ are considered relative to their position within a soil profile, soils are grouped into these seven classes.

1. $K_S$ does not change substantially in the profile.

2. $K_S$ of the A-horizon is substantially greater than that of the remaining soil profile and no horizon of extremely low $K_S$ exists.

3. $K_S$ gradually decreases with soil depth without distinct minima or maxima.

4. $K_S$ manifests a distinct minimum value in the illuvial horizon or in the compacted layer just below the plow layer.

5. Soil of high permeability with its development belonging to one of the first four classes covering the underlying soil of very low permeability.

6. Soil of very low permeability with its development belonging to one of the first four classes covering the underlying soil of very high permeability.

7. $K_S$ changes erratically within the profile owing to extreme heterogeneity in the soil substrata.

The influence of the temperature upon the value of $K_S$ can be examined with (5.21). Inasmuch as $\rho_W$ is negligibly influenced by temperature, changes of $K_S(T)$ depend primarily upon the viscosity $\mu(T)$.


## 5.2.3 Darcian and Non-Darcian Flow

We have already mentioned that Darcy's equation is valid only for small rates when the inertial terms of the Navier-Stokes equations are negligible. For engineering purposes the upper limit of the validity of Darcy's equation given by (5.3) through (5.6) is indicated by the critical value of Reynolds' number for porous media

$$Re = \frac{qd\rho}{\mu} \tag{5.22}$$

where $d$ denotes length. In sands, $d$ is the effective diameter of the particles, or with some corrections, the effective pore diameter. Sometimes $d$ is related to the permeability of the sand, e.g. $d = K_p^{1/2}$. However, in all soils other than sands, $d$ is not at all definable and hence, (5.22) is not applicable. The difficulty in defining $d$ is manifested by controversy in the literature regarding the assignment of critical values of $Re$. Most frequently, critical values of $Re$ have been reported to range from 1 to 100. In this post linear region, the flow is often described by the Forchheimer equation (Bear, 1972)

$$\frac{dH}{dx} = aq + bq^2 \tag{5.23}$$

where $a$ is the material constant analogous to $K_S$ and $b$ is functionally dependent upon the water flux density. This non linearity is caused primarily by inertia and by turbulence starting only at very large values of flux density, see Fig. 5.6. A more detailed theoretical discussion is given by Cvetkovič (1986) .

Deviations from Darcy's equation have also been observed in laboratory experiments for very small flux density values. We define, therefore, the pre linear region of flow where $q$ increases more than proportionally with $I_h$, see Fig. 5.6. This deviation from Darcy's equation, most often observed within pure clay having very large specific surfaces (e.g. $10^2$ m$^2 \cdot$g$^{-1}$), has been explained by the action of three factors: a) Clay particles shift and the clay paste consolidates owing to the imposed hydraulic gradient and the flow of water, b) It is theoretically assumed that the viscosity of water close to the clay surfaces is different than that of bulk water or that in the center of the larger soil pores. According to Eyring's molecular model where the viscosity depends upon the activated Gibbs' free energy, the first

two to five molecular layers have a distinct increased viscosity. Owing to the great value of the specific surface in clays, the contribution of the first molecular layers to the alteration of averaged viscosity may not be negligible. c) The coupling of the transfer of water, heat, solutes et al. may also contribute to the existence of the pre linear region (Swartzendruber, 1962; Kutílek, 1964 and 1972; Nerpin and Chudnovskij, 1967).
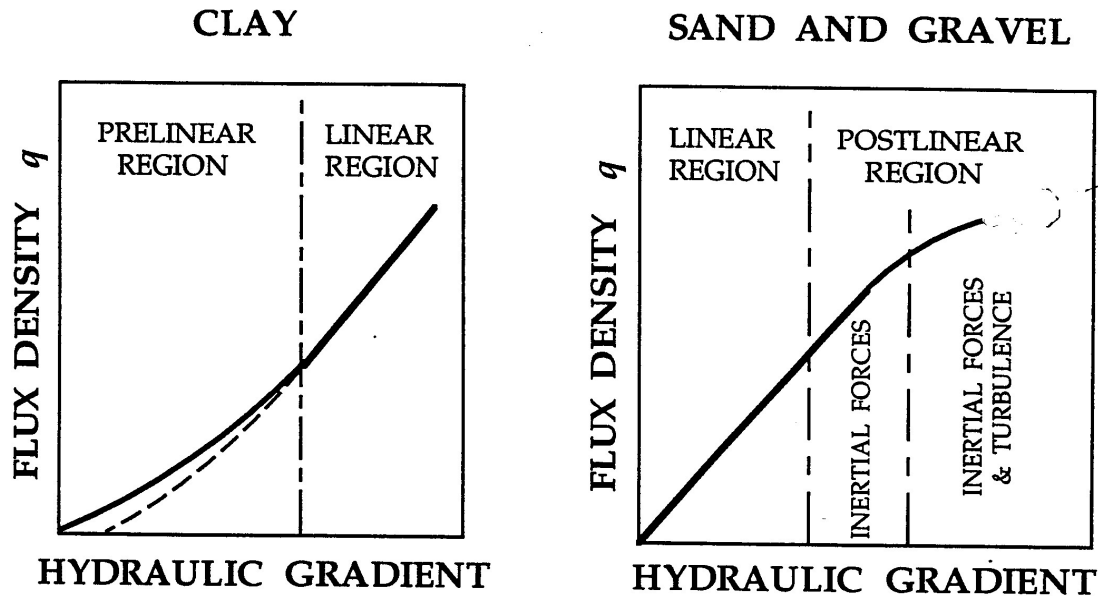


*Fig. 5.6. Deviations from the _inearity of Darcy's equation*

Deviations from Darcian flow are not frequently described or observed, and the post linear region is only rarely reached in sands and gravelly sands. There is not yet any field experimental evidence of the existence of a pre linear region. Darcy's equation is, therefore, either exact or at least a very good approximation entirely adequate for soil hydrology.

## 5.2.4 Measuring $K_S$

Saturated hydraulic conductivity is one of the principal soil characteristics and for its determination, only direct measurement is appropriate. Indirect methods, derived from soil textural characteristics which are sometimes combined with aggregate analyses, generally do not lead to reliable values. Considering soil texture as an example, soil water flow is totally independent from the laboratory procedure of dispersing, separating and measuring the percentage of "individual" soil particles which do not even exist "individually" in natural field soils. It has been shown in section 5.2.2 that the value of $K_S$ is closely related physically to the porous system within a soil. Inasmuch as a quantitative description of this porous system is much more difficult than the measurement of $K_S$, direct measurement of $K_S$ is preferred. When $K_S$ is ascertained by water flux density and potential gradient measurements, we will speak about the determination of $K_S$. In order to avoid misunderstanding, when additional assumptions are used to evaluate these two quantities somewhat less directly, we will speak about the estimation of $K_S$.

Measuring is realized either in the laboratory on soil core samples previously taken from the field, or directly in the field without removing a soil sample. Field methods are preferred. They provide data that better represent the reality of water flow in natural conditions. Their main disadvantage is the lack of rigorous quantitative procedures for measuring soil attributes in the majority of field tests. For laboratory measurements, the size of the REV should be theoretically estimated in order that an appropriate soil core sampler be selected. In practice, because the REV is rarely determined, a standard core or cylinder size is used for most soils. As a result, larger numbers of samples are taken with appropriate statistical evaluation of the data in order to partly reduce the error associated with samples having smaller volumes than the REV. In soils without cracks and frequently occurring macropores a soil sample volume of 200 to 500 $cm^3$ is assumed satisfactory. In subsoils, a 100 $cm^3$ sample will sometimes suffice. Methods relying on "undisturbed" core samples are generally not applicable in stony soils, in forests and in soils that crack excessively upon drying.
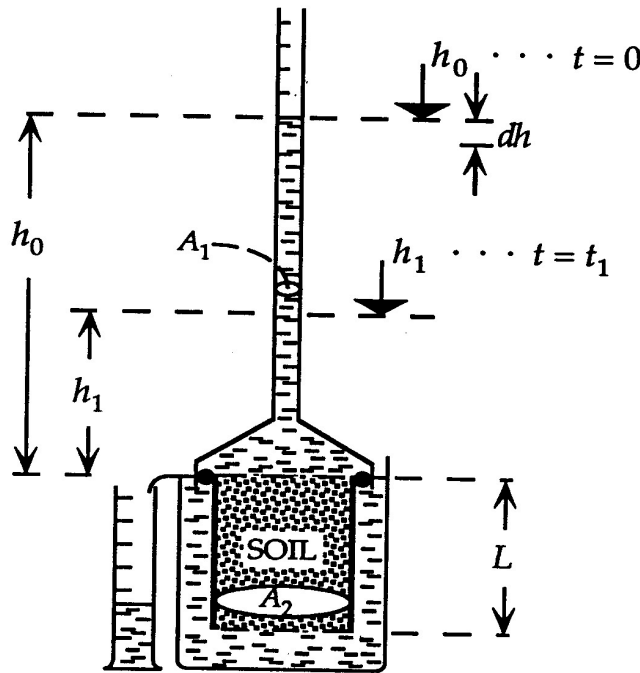


*Fig. 5.7. Falling head permeameter*

In the laboratory, the test is usually performed in a way similar to that demonstrated in Fig. 5.1. When both elevations of the water level are kept constant, i.e. in the vessel before water passes the soil ("upper level") and in the vessel after water has passed through the soil ("lower level"), (5.1) and (5.3) are applied. Apparatuses so constructed are usually called constant head permeameters. More frequently, the upper water level is allowed to fall, see Fig. 5.7. The fall of the water level $dh$ in the measuring tube above the soil sample relates to the flux of water through the soil in time $dt$. Equating the two volumes, that moving in the measuring tube and that moving through the soil, we have

$$- A_1 dh = A_2 q\, dt \qquad (5.24)$$

where $A_1$ is the cross-sectional area of the tube above the soil and $A_2$ is the cross-sectional area of the soil. Substituting $q$ from (5.3) into (5.24) and recognizing that $\Delta h$ in (5.3) is the total potential head difference $h$ in Fig. 5.7, we have

$$\frac{A_1 L}{A_2 K_S} \int_{h_0}^{h_1} \frac{dh}{h} = -\int_0^{t_1} dt .$$
(5.25)

After integrating and rearranging,

$$K_S = \frac{A_1 L}{A_2 t_1} \ln\left(\frac{h_0}{h_1}\right) .$$
(5.26)

The method is sometimes modified by having $A_1 = A_2$ and by keeping the bottom vessel without an overflow. Construction details of these apparatuses, usually referred to as falling head permeameters, are given in Klute and Dirksen (1986).

In the field, we generally deal with two cases relative to the determination of $K_S$. In the first case, a water saturated zone is formed by an unconfined aquifer close to the surface with the ground water level being not deeper than about 1.5 m. In the second case, the soil is not fully saturated.

There exists a number of field methods for determining $K_S$ in the saturated zone. Among them, the auger-hole method and the piezometer method are the most commonly practiced.

In the auger-hole method, a hole is drilled to a depth well below the ground water level. After the water from the hole is rapidly pumped out, the rate of rise of the water level in the hole is registered. A general equation for the computation of $K_S$ is simply

$$K_S = C \frac{dz}{dt}$$
(5.27)

where $z$ is the depth of the water level in the hole measured from the hydrostatic ground water level in the soil and $C$ is the shape coefficient dependent upon the geometry of the test. In practice, the derivative in (5.27) is replaced by the differences $\Delta z/\Delta t$ and the coefficient $C$ is evaluated, e.g. according to the potential theory by Ernst (1950) quoted in Maasland and Haskew (1957). Except at the extreme bottom of the hole, it is assumed that the flow paths of water in the soil are perpendicular to the walls of the hole. In order that this assumption be fulfilled, pumping should not be repeated in short sequences of time. Estimation of $C$ for less restrictive geometrical conditions has been described by Boast and Langbartel (1984).

The piezometric method is similar to the auger-hole method except that a tube driven into the augered soil serves as a lining for the hole. The rate of the rise of the water level in the lined hole is measured after pumping a portion of the water from the hole. The lining prevents inflow except through the bottom of the hole. For isotropic soils, $K_S$ is computed from

$$K_S = \frac{\pi r^2}{C \Delta t} \ln\left(\frac{z_1}{z_2}\right)$$
(5.28)

where $r$ is the radius of the tube inserted into the hole, $z_1$ and $z_2$ are the depths of water levels in the hole measured from the hydrostatic ground water level during the time interval $\Delta t$. The shape factor $C \approx 5.6r$ for a flat bottom and $C \approx 9$ for a hemispherical bottom. Note the similarity between (5.28) and (5.26). The falling head permeameter method modified here with a shape factor $C$ applies to the piezometer method. When the lining does not reach the bottom of the hole, water penetrates additionally from the sides, and the value of the shape factor $C$ increases in relation to $L/r$ where $L$ is the height of the unlined portion of the hole (Smiles and Youngs, 1965).

The piezometric method is ideally suited to measure $K_S$ of anisotropic soils. Two measurements are required. For the first measurement, the lining reaches to the bottom of the hole and we compute with (5.28) the product of the vertical and horizontal hydraulic conductivities, $K_V$ and $K_H$. In (5.28), $K_S$ is replaced by $(K_V K_H)^{1/2}$ and we assume that the main axes of the hydraulic conductivity tensor are vertical and horizontal. For the second measurement the hole, deepened by the value $L$ without pushing the lining tube deeper, allows inflow into the hole through both the unlined wall of height $L$ and through the bottom. With a new shape factor $C_2$ in (5.28), the values of $K_V$ and $K_H$, respectively, are simply calculated with data from the second experiment. A detailed description of methods of measurement of $K_S$ in the field is given by Amoozegar and Warrick (1986). If the saturated soil or aquifer is of great thickness and the locality allows installation of several observation wells, pumping tests can be performed in order to obtain $K_S$. This method which deals with ground water hydraulics is frequently described in detail in the literature. Data obtained from these pumping tests are not generally compatible with data determined by the auger-hole method representing a weighted average $K_S$ value in the domain of the cone of depression compared with the "point" data of the auger hole method.

The task of measuring $K_S$ in an unsaturated zone is more difficult. Infiltration tests are most frequently used to measure $K_S$ of the topsoil or other horizons near the soil surface. The value of $K_S$ is estimated from readings of cumulative infiltration as a function of time. Alternatively, after a quasi-steady state infiltration rate is reached, its value together with that of the hydraulic gradient estimated from tensiometers installed at two or more depths are used in (5.5) to calculate the value of $K_S$. The details will be discussed in Chapter 6.

The auger-hole method is commonly modified to determine $K_S$ below the soil surface of an unsaturated soil. Because the zone is not saturated, the modified procedure is opposite to that dealing with a saturated zone. Instead of water being removed from the saturated soil, water is poured into the hole from a marriotte flask. After a relatively short time of 15 to 30 minutes, quasi-steady flow is reached. The steady water flux density $q$ is measured together with the constant height $h$ of the water level above the bottom of the hole. Amoozegar and Warrick (1986) recommend that $K_S$ be calculated according to Glover's solution

$$K_S = \frac{q\left[\sinh^{-1}(h/r) - \left(r^2/h^2 - 1\right)^{1/2} + r/h\right]}{2\pi h^2}$$

(5.29)

where $r$ is the radius of the hole. This procedure is called the constant head well permeameter method. A similar device, the so-called Guelph permeameter (Reynolds and Elrick, 1985, and Elrick and Reynolds, 1990), is broadly used today owing to the efficiency of measurement.