The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1675 - 5

Workshop on
Noise and Instabilities in Quantum Mechanics

3 - 7 October 2005

Entanglement, decoherence and the correspondence principle

Philipe JACQUOD
Department of Theoretical Physics
University of Geneva
24, Quai Ernest Ansermet
CH-1211 Geneva 4
SWITZERLAND

Entanglement, Decoherence and the Correspondence Principle

Philippe Jacquod - ICTP '05

Colls.:

- I. Adagideli (UBC)
- C. Beenakker (Leiden)
- C. Petitjean (Geneva)

Decoherence - Classical out of Quantum Physics ?

Copenhagen Interpretation

"A quantum system is always measured by a classical apparatus."

But where is the border ?

Decoherence - Classical out of Quantum Physics ?

Copenhagen Interpretation

"A quantum system is always measured by a classical apparatus."

But where is the border ?

Modern view

"There is no fully isolated quantum system. When we measure one, we implicitely integrate out its environment."
("environment"=everything that is not measured)

$$\rho_r(t) = \mathrm{Tr}_\Phi\{\exp[-i\mathcal{H}t]\rho_0 \exp[i\mathcal{H}t]\}$$

- Zurek, RMP '03; Joos et al., Springer '03.

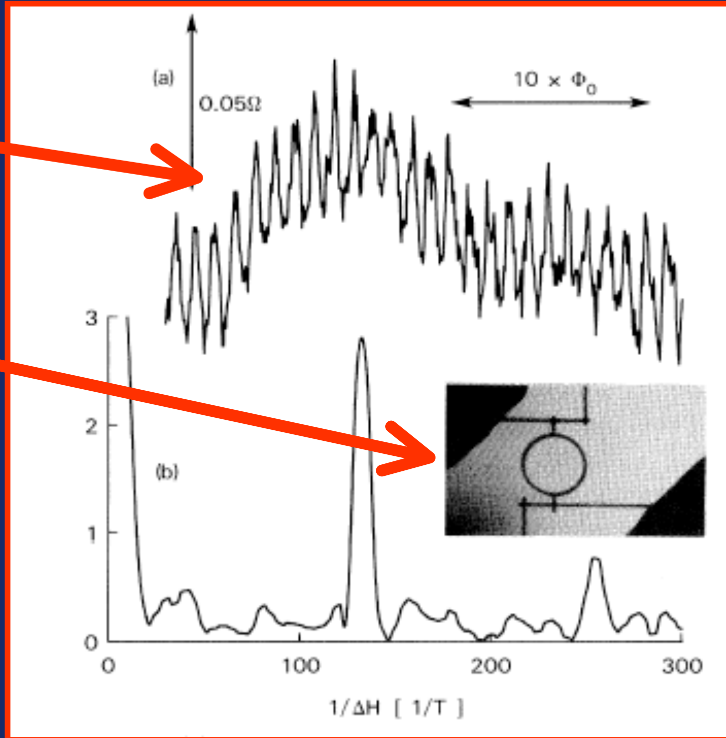
VOLUME 54, NUMBER 25 PHYSICAL REVIEW LETTERS 24 JUNE 1985

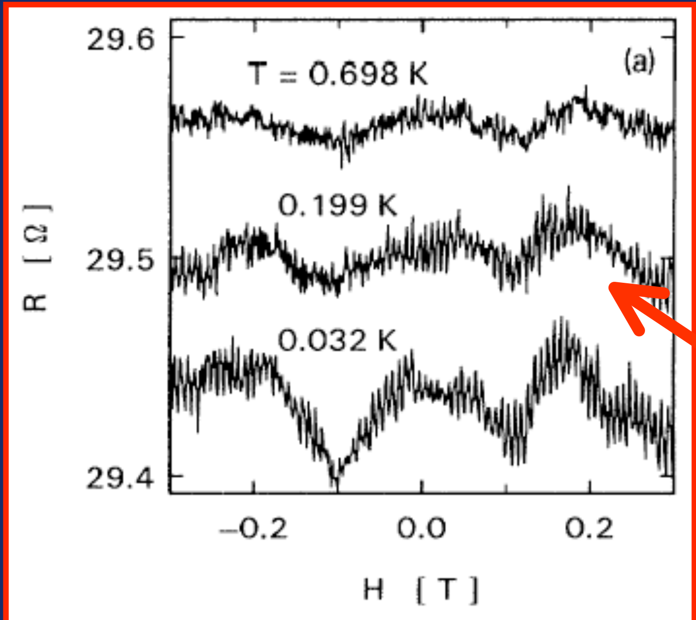
Observation of *h/e* Aharonov-Bohm Oscillations in Normal-Metal Rings

R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz

Magnetoresistance
~Aharonov-Bohm oscillations

Measurement sample
diam ~1μm, width ~0.04μm





Amplitude of oscillations decreases
with increasing temperature
~decoherence

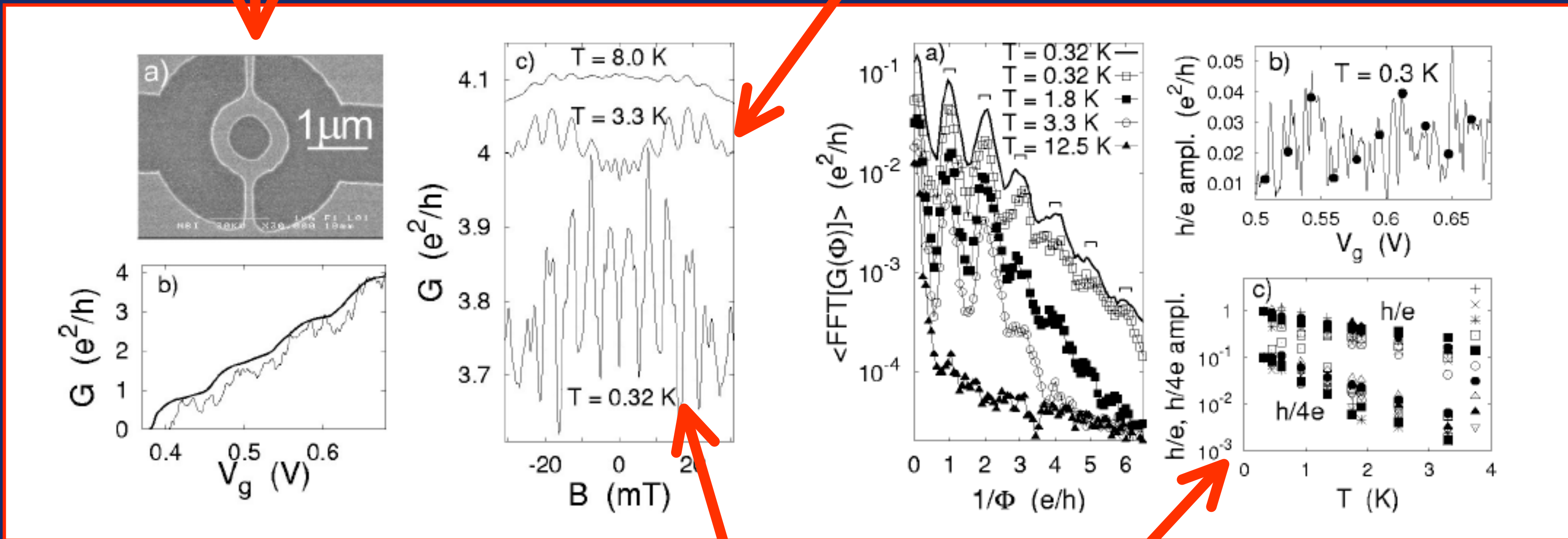
PHYSICAL REVIEW B, VOLUME 64, 045327

Mesoscopic decoherence in Aharonov-Bohm rings

A. E. Hansen,* A. Kristensen,† S. Pedersen,‡ C. B. Sørensen, and P. E. Lindelof
The Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark

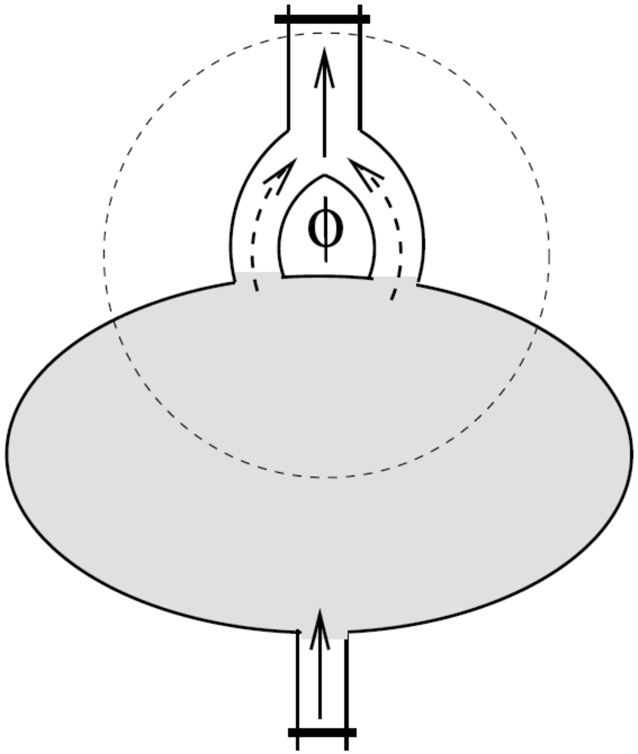
Measurement sample

Magnetoresistance
~Aharonov-Bohm oscillations



Amplitude of oscillations decreases with temperature ~ decoherence

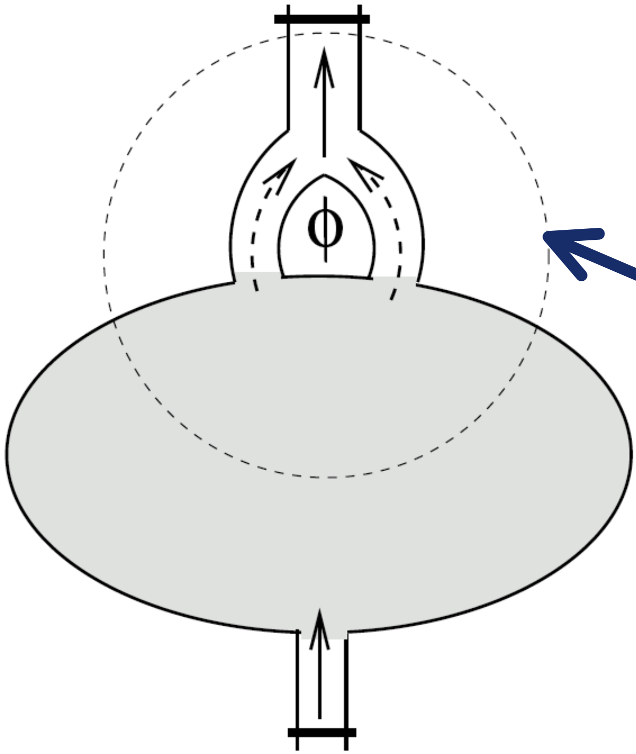
Aharonov-Bohm Transport and Decoherence



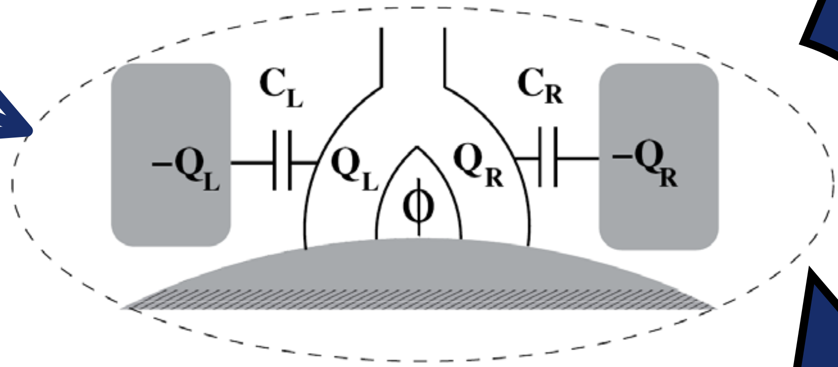
$$\langle \mathbf{g}^2(\phi) \rangle = \frac{\mathbf{N}_\mathrm{B}\,\mathbf{N}_\mathrm{T}}{(\mathbf{N}_\mathrm{B} + \mathbf{N}_\mathrm{T})^2} \cos^2(2\pi\phi/\phi_0)$$

Seelig and Buettiker, PRB '01; PJ, PRE '05.

Aharonov-Bohm Transport and Decoherence



$$\langle g^2(\phi)\rangle = \frac{N_B\, N_T}{(N_B + N_T)^2}\cos^2(2\pi\phi/\phi_0)$$



$$\langle g^2(\phi)\rangle = \frac{N_B\, N_T}{(N_B + N_T)^2}\cos^2(2\pi\phi/\phi_0)e^{-\tau_L/\tau_\varphi}$$

Damping of AB oscillation due to relative phase accumulation !

$$\exp[-\int_0^{\tau_L} dt_1 \int_0^{\tau_L} dt_2 \langle\varphi_s(t_1)\varphi_s(t_2)\rangle_s/2]$$

Seelig and Buettiker, PRB '01; PJ, PRE '05.

VOLUME 69, NUMBER 14 PHYSICAL REVIEW LETTERS 5 OCTOBER 1992

Polarization Echoes in NMR

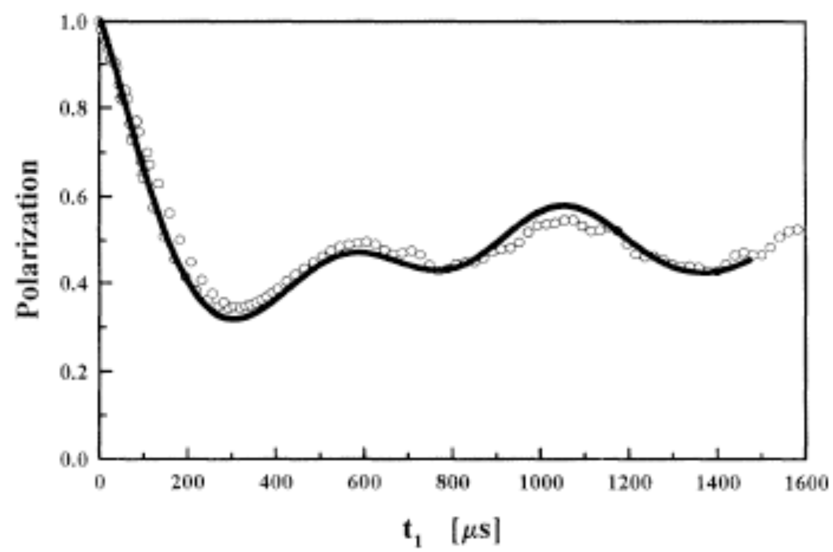
Shanmin Zhang, B. H. Meier, and R. R. Ernst

VOLUME 75, NUMBER 23 PHYSICAL REVIEW LETTERS 4 DECEMBER 1995

Quantum Dynamical Echoes in the Spin Diffusion in Mesoscopic Systems

Horacio M. Pastawski, Patricia R. Levstein, and Gonzalo Usaj

$$M_{\mathrm{PE}}(t) = \langle \Psi_i | \exp[i\mathcal{H}t] \exp[-i\mathcal{H}_0 t] \left[\hat{S}_0^z + \frac{1}{2} \right] \exp[i\mathcal{H}_0 t] \exp[-i\mathcal{H}t] | \Psi_i \rangle$$



- Forward-backward t-evolution of spin polarization
- Imperfect t-reversal ("environment" is not t-reversed)

$\Rightarrow$ Decay of polarization

Time-evolution of the Reduced Density Matrix

$$\rho_r(t) = \mathrm{Tr}_\Phi\{\exp[-i\mathcal{H}t]\rho_0 \exp[i\mathcal{H}t]\}$$

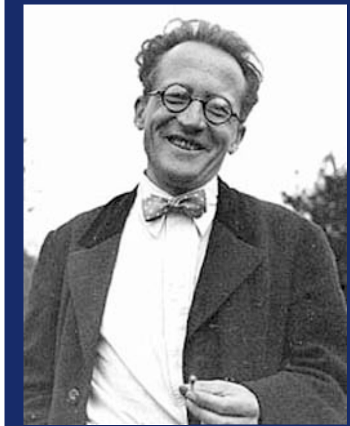
- RDM does not satisfy Neumann's equation !
- Master equations can be derived in specific situations instead, either for RDM or Wigner distribution
  - Separation of time scales (Markov)
  - Environment = uncoupled oscillators
  - High temperatures
  - No recoil, no back-action
  - …

Feynman and Vernon; Caldeira and Leggett; Joos and Zeh; Zurek, Paz and friends; Lutz and Weidenmueller…

This Talk - Semiclassical Approach to Decoherence

- Entanglement generation in bipartite systems
- From two identical particles to one particle + “environment”
- Phase-space Quantum Mechanics : Correspondence Principle
- One particle coupled to (N-1) ***coupled*** particles
- Where are we going from there ?

Entanglement : Purely Quantum Physics



"When two systems (…) enter into temporary interaction, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own."

$$\rho_1(t) = \mathrm{Tr}_2\{\exp[-i\mathcal{H}t]\rho_0 \exp[i\mathcal{H}t]\}$$

$$\mathcal{P}(t) \equiv \mathrm{Tr}[\rho_1(t)^2] \leq 1$$

What determines the rate of entanglement generation ?

Miller and Sarkar '98; Furuya, Nemes and Pellegrino '98;
Lakshminarayan and friends; Tanaka and friends; Prosen and friends…

PHYSICAL REVIEW E VOLUME 60, NUMBER 2 AUGUST 1999

Signatures of chaos in the entanglement of two coupled quantum kicked tops

Paul A. Miller and Sarben Sarkar
Department of Physics, King's College London, Strand, London WC2R 2LS, United Kingdom

"Faster entanglement generation with stronger chaos"

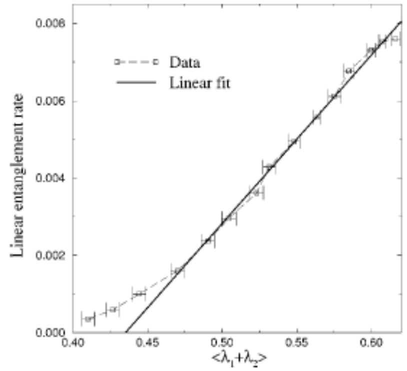


FIG. 7. Linear entanglement rate plotted against the averaged sum of the positive Lyapunov exponents for analagous initial classical distributions. Also plotted is a linear fit to the data away from the first three points.

PHYSICAL REVIEW E **66**, 045201(R) (2002)

Saturation of the production of quantum entanglement between weakly coupled mapping systems in a strongly chaotic region

Atushi Tanaka
Department of Physics, Tokyo Metropolitan University, Minami-Osawa, Hachioji, Tokyo 192-0397, Japan

Hiroshi Fujisaki
Department of Theoretical Studies, Institute for Molecular Science, Myodaiji, Okazaki 444-8585, Japan

Takayuki Miyadera
Department of Information Sciences, Tokyo University of Science, Noda City, Chiba 278-8510, Japan

"Increasing Lyapunov does not increase rate of entanglement in strong chaos region"

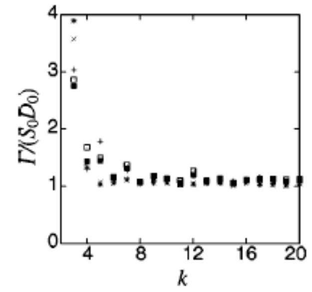


FIG. 4. Dependences of the entanglement production rates Γ, which is measured by linear entropy, on the nonlinear parameter $k = k_1 = k_2$. In order to show typical k dependences, we choose several initial conditions (depicted by different marks) that occur in the chaotic sea. Although the entanglement production heavily depends on the initial condition in the weakly chaotic region, the disappearance of tori weakens the initial condition dependence in the strongly chaotic region. Other parameters are the same as in Fig. 2.

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. 36 (2003) 2463–2481 PII: S0305-4470(03)54131-2

Fidelity and purity decay in weakly coupled composite systems

Marko Žnidarič and Tomaž Prosen

"Faster entanglement generation in chaotic than in regular systems"

Generation of entanglement in bipartite systems

- Two-body Hamiltonian

$$\mathcal{H} = H_1 \otimes I_2 + I_1 \otimes H_2 + \hbar\, \mathcal{U}$$

- Initial product two-particle state / localized Gaussian wavepackets

$$|\psi_1\rangle \otimes |\psi_2\rangle \equiv |\psi_1, \psi_2\rangle$$

- Calculate the reduced density matrix and its purity

$$\rho_1(t) = \mathrm{Tr}_2\{\exp[-i\mathcal{H}t]\rho_0 \exp[i\mathcal{H}t]\} \qquad \mathcal{P}(t) \equiv \mathrm{Tr}[\rho_1(t)^2] \leq 1$$

- Using semiclassics, with two-particle propagator

$$\langle \mathbf{x}, \mathbf{r}| \exp[-i\mathcal{H}t]|\mathbf{y}_1, \mathbf{y}_2\rangle = (-i)^d \sum_{s,s'} C_{s,s'}^{1/2} \exp[i\{S_s + S_{s'} + \mathcal{S}_{s,s'} - \frac{\pi}{2}(\mu_s + \mu_{s'})\}]$$

Classical paths are not affected by the interaction !

PJ, PRL '04; C. Petitjean and PJ, in preparation.

Semiclassical reduced density matrix*

$$\rho_1(\mathbf{x},\mathbf{y};t) = (-4\pi\sigma^2)^d \sum_{s,l} \exp[i\{S_s(\mathbf{r}_1,\mathbf{x};t) - S_l(\mathbf{r}_1,\mathbf{y};t)\}] \times \int d\mathbf{r} \sum_{s'} \mathcal{M}_{s,s'}\mathcal{M}^{\dagger}_{l,s'} \exp[i\{\mathcal{S}_{s,s'}(\mathbf{r}_1,\mathbf{x};\mathbf{r}_2,\mathbf{r};t) - \mathcal{S}_{l,s'}(\mathbf{r}_1,\mathbf{y};\mathbf{r}_2,\mathbf{r};t)\}]$$

- With 2-particle amplitudes

$$\mathcal{M}_{s,s'} = C^{1/2}_{s,s'} \exp[-i\frac{\pi}{2}(\mu_s - \mu_{s'})] \exp[-\frac{\sigma^2}{2}\{(\mathbf{p}_s - \mathbf{p}_0)^2 + (\mathbf{p}_{s'} - \mathbf{p}_0)^2\}]$$

- With 1- and 2-particle action phases

$$\mathcal{S}_{s,s'} = \int_0^t dt_1 \mathcal{U}(\mathbf{q}_s(t_1), \mathbf{q}_{s'}(t_1)) \ll S_s \ , \ S_{s'}$$

➢ Weak interaction regime - no recoil/back-action

➢ Separation of time scales (fast=1-part / slow=2-part)

---

* $\mathrm{Tr}[\rho_1(t)] = 1$ and $\rho_1(\mathbf{x},\mathbf{y};t) = [\rho_1(\mathbf{y},\mathbf{x};t)]^*$

Generation of entanglement in bipartite systems

Purity as sum over four classical trajectories

$$\langle \mathcal{P}(t) \rangle = (4\pi\sigma^2)^{2d} \int \mathrm{dxdy} \int \mathrm{drdr'} \sum_{s,s'} \sum_{l,m} \mathcal{M}_{s,s'} \, \mathcal{M}_{l,m} \, \mathcal{M}^{\dagger}_{l,s'} \, \mathcal{M}^{\dagger}_{s,m} \langle \mathcal{F} \rangle$$

$$\mathcal{F} = \exp[\mathbf{i}\{\mathcal{S}_{\mathbf{s},\mathbf{s}'} - \mathcal{S}_{\mathbf{l},\mathbf{s}'}\}] \; \exp[\mathbf{i}\{\mathcal{S}_{\mathbf{l},\mathbf{m}} - \mathcal{S}_{\mathbf{s},\mathbf{m}}\}]$$

Similarity with Loschmidt echo

⇒diagonal contribution; s'=m; decays with amplitudes

⇒nondiagonal contribution; decay with <F>, i.e. phase

(Jalabert and Pastaski, '01; PJ, Silvestrov and Beenakker, '01)

$$\Rightarrow \quad \langle \mathcal{P}(t) \rangle = \Sigma_0^2(t) + \langle \mathcal{F}(\mathbf{x}, \mathbf{y}; t) \rangle$$

Semiclassical Purity

$$\mathcal{P}(\mathbf{t}) = \exp[-\lambda_1 \mathbf{t}] + \exp[-\lambda_2 \mathbf{t}] + \exp[-\mathbf{2}\Gamma \mathbf{t}]$$

Diagonal contribution of system # 1 Classical Term

Diagonal contribution of system # 2 Classical Term

Non diagonal contribution (Interaction dependent) Quantum Term

$\Gamma \propto \langle \mathcal{U}_{ss'} \mathcal{U}_{ss'} \rangle$, Golden rule width - does not depend on Planck !

Note: (*) regular systems, Lyapunov ⟶ power-law

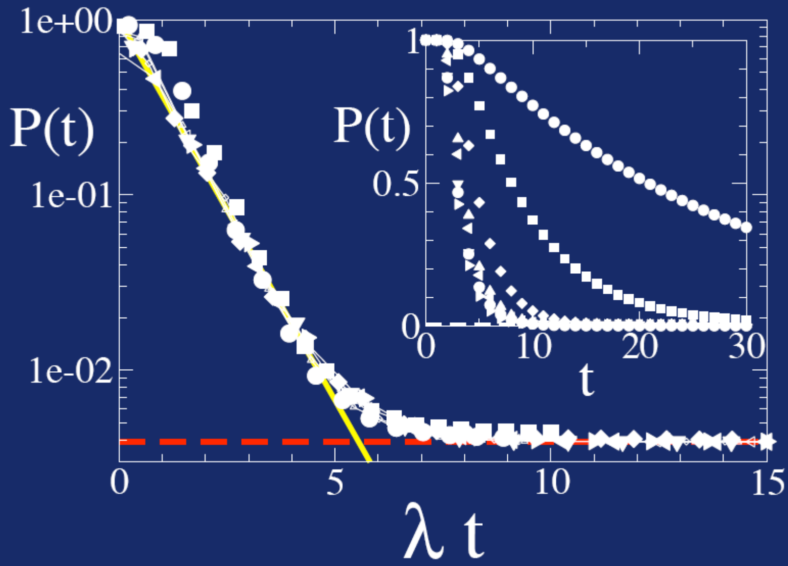
(see also Znidaric and Prosen, PRA '05)

(*) Quantum term also from RMT

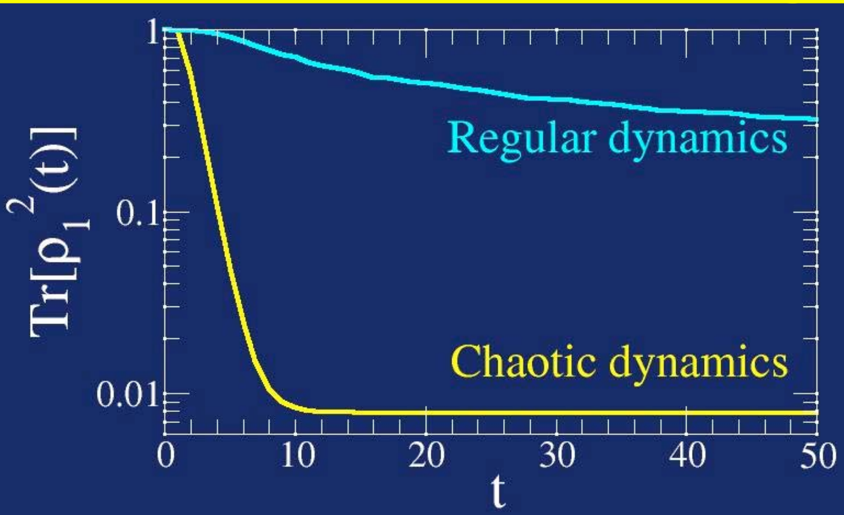
PJ, PRL '04; C. Petitjean and PJ, in preparation.

Numerical confirmation of the Lyapunov decay

- Saturation of Purity decay vs. interaction for $2\Gamma \gtrsim \lambda$
- Scaling of Purity with Lyapunov exponent for



Confirmation of the regular vs. chaotic behavior!



- Much faster decay rate (generation of entanglement) for chaotic dynamics

$\Longrightarrow$ Exponential for chaotic

$\Longrightarrow$ Power-law for regular

From Bipartite Entanglement to Decoherence

Do 2-particle entanglement generation, but the second system…

- has much bigger Hilbert space (it's an environment!)
- has much shorter time scales - faster Hamiltonian flow
- should be more chaotic - much larger Lyapunov exponent

…

- *cannot be initially prepared as Gaussian wavepacket !*
  First approximation : superposition of nonoverlapping Gaussians

From Bipartite Entanglement to Decoherence

Do 2-particle entanglement generation, but the second system…

- has much bigger Hilbert space (it's an environment!)
- has much shorter time scales - faster Hamiltonian flow
- should be more chaotic - much larger Lyapunov exponent

…

- *cannot be initially prepared as Gaussian wavepacket !*

  First approximation : superposition of nonoverlapping Gaussians

$$\mathcal{P}(\mathbf{t}) = \exp[-\lambda_1 \mathbf{t}] + \mathbf{\Omega_2^{-1}} \exp[-\lambda_2 \mathbf{t}] + \exp[-\mathbf{2\Gamma t}]$$

Diagonal contribution of system # 1 Classical Term

Diagonal contribution of environment Classical Term

Non diagonal contribution (Interaction dependent) Quantum Term

Phase-Space QM and the Correspondence Principle

Use Wigner function ~Fourier trsf of density matrix

$$\partial_t W = \left\{ H, W \right\} + \sum_{n \geq 1} \frac{(i\hbar)^{2n}}{2^{2n}(2n+1)!} \partial_q^{2n+1} V \partial_p^{2n+1} W$$

Classical dynamics
Poisson bracket

Purely quantum contributions;
They start to dominate at the
Ehrenfest time

Can one kill the QM contributions soon enough without affecting the Cl dynamics ?

Ehrenfest time : $t_E = \lambda^{-1} \mathrm{Ln}(I/\hbar)$

Phase-Space QM and the Correspondence Principle

Yes ! Example: quantum Brownian motion model;
Ohmic bath of harm. oscillators;
High temperature.

$$\partial_t W = \left\{ H, W \right\} + \sum_{n\geq 1} \frac{(i\hbar)^{2n}}{2^{2n}(2n+1)!} \partial_q^{2n+1} V \partial_p^{2n+1} W$$

$$+2\gamma\partial_p(pW) + D\partial_{pp}^2 W$$

Friction Diffusion

- Diffusion kills QM contributions
- Friction disappears

....for $\gamma \to 0$

$$D = 2m\gamma k_B T = \mathrm{Cst}$$

Phase-Space QM and the Correspondence Principle

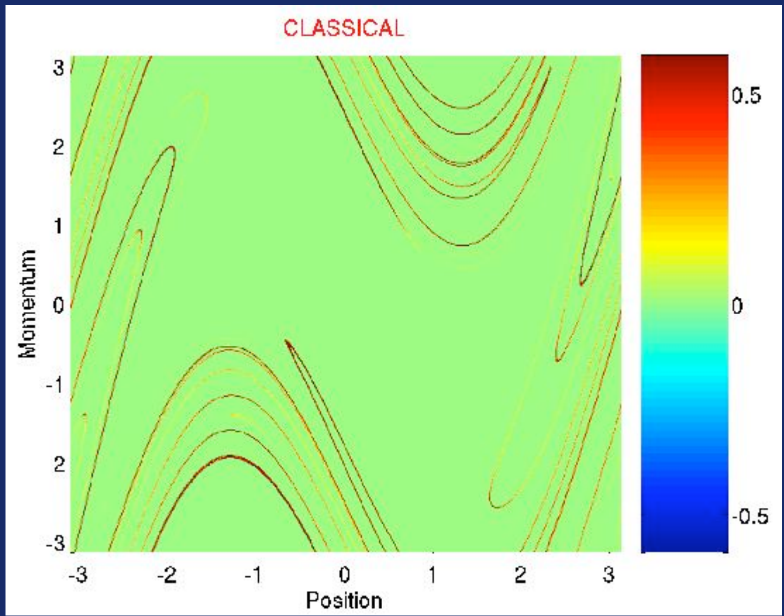
Our semiclassical calculation shows that

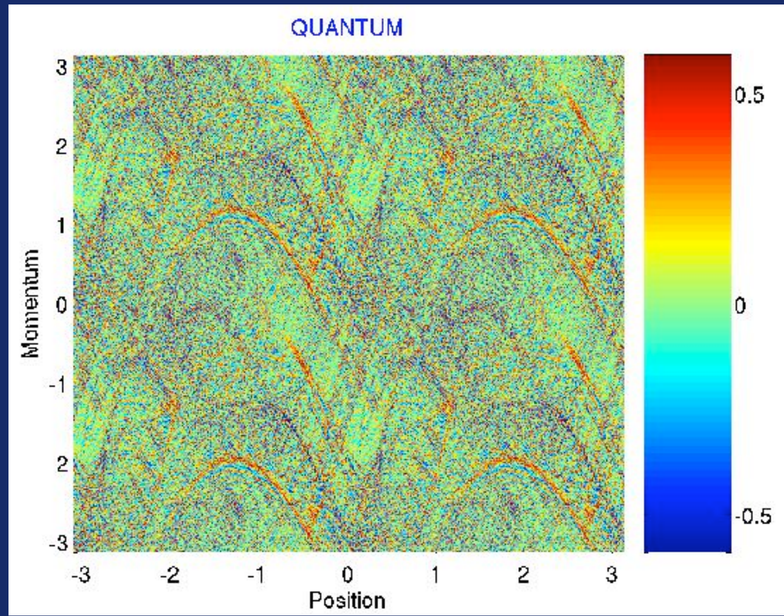
$$\mathcal{P}(\mathbf{t}) = \exp[-\lambda_{\mathbf{1}}\mathbf{t}] + \exp[-\lambda_{\mathbf{2}}\mathbf{t}] + \exp[-\mathbf{2\Gamma t}]$$

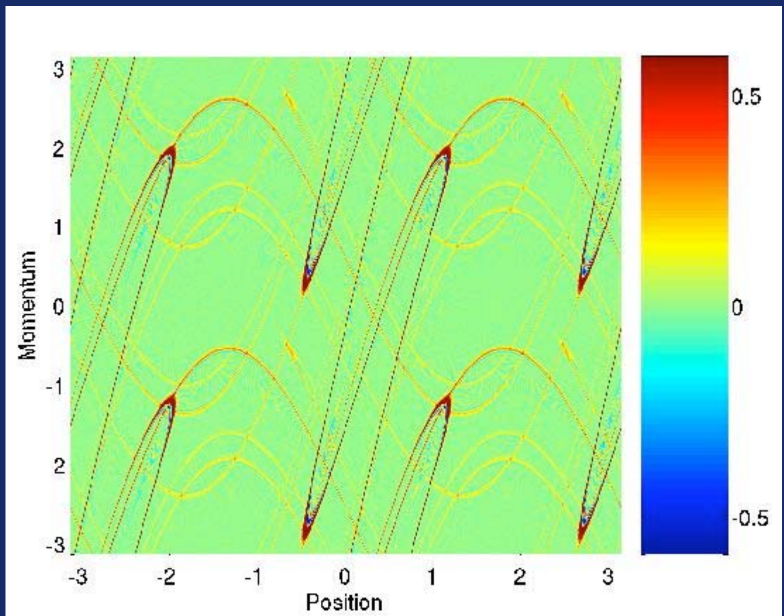
- ✓I.e. for $2\Gamma \gtrsim \lambda$ the purity goes to its minimal value at the Ehrenfest time.
- ✓I.e. off-diagonal matrix elements of the RDM are suppressed during that time.
- ✓I.e. QM contributions are killed before they appear !

➡ ***! Classical behavior out of Quantum Mechanics !***

Phase-Space Dynamics





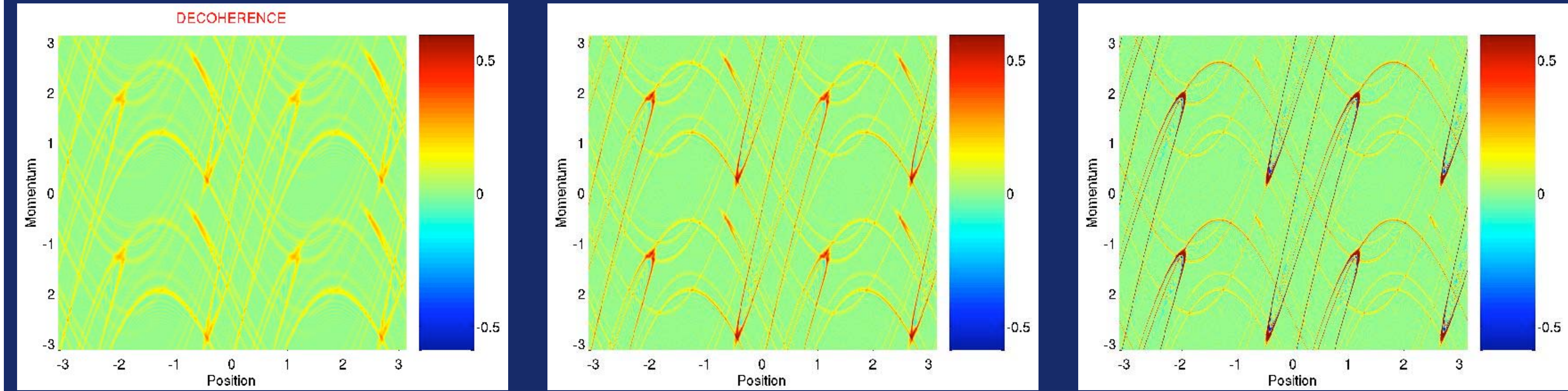


✓Classical mechanics out of QM

✓$\exp[-\lambda_1 t] \sim$

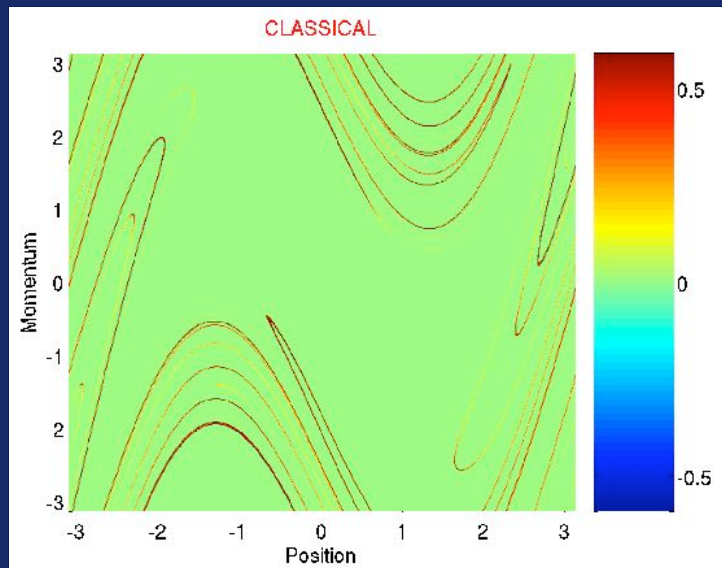
Liouvillian evolution of ρ_1

$$W = \{W, \mathcal{H}\}_{PB} + \cancel{\mathcal{L}_q}$$



DECOHERENCE
Momentum
Position
0.5
0
-0.5

$\hbar \to 0$



CLASSICAL
Momentum
Position
0.5
0
-0.5

Multipartite entanglement

❖N-particle concurrence*

$$\mathcal{C}_N(\Psi) = 2^{1-N/2}\sqrt{(2^N-2)\langle\Psi|\Psi\rangle^2 - \sum_a \mathrm{Tr}[\rho_a^2]}$$

Sum over all nontrivial Subsets of N particles

❖Semiclassical calculation for N=3

$$\mathcal{C}_3^2 = 3 - 3\prod_{i=1}^{3}\exp[-\lambda_i t] - \exp[-2(\Gamma_{13}+\Gamma_{23})t] - \exp[-2(\Gamma_{12}+\Gamma_{23})t] - \exp[-2(\Gamma_{13}+\Gamma_{12})t]$$

➢Cl. and QM stationary phase condition do not mix

➢N-particle concurrence of the form

$$\mathcal{C}_N^2 = A_N - B_N\prod_{i=1}^{N}\exp[-\lambda_i t] - C_N\sum_{i=1}^{N}\exp[-2\sum_{j\neq i}\Gamma_{ij}t]$$

➢*Decoherence* for bath of coupled chaotic systems

$$\mathcal{P}_1(t) = \exp[-\lambda_1 t] + \sum_{i=2}^{N}\exp[-2\Gamma_{1i}t]$$

*Wootters, PRL '98; Rungta et al., PRA '01; Carvalho et al., PRL '04.

Conclusions

- Semiclassical theory for 2-part. Entanglement
  - Classical, dynamically-dependent contribution
  - QM, coupling-dependent contribution
  - Classical out of Quantum mechanics from microscopic model
- Towards semiclassical theory for decoherence
  - Bath of coupled dynamical systems
- TO-DO LIST :
  - Derive Master Equation - new "Lyapunov" term ?