The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1675 - 9

Workshop on
Noise and Instabilities in Quantum Mechanics

3 - 7 October 2005

Quantum noise and instabilities in cavity QED

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Quantum Noise and Instabilities in Cavity QED

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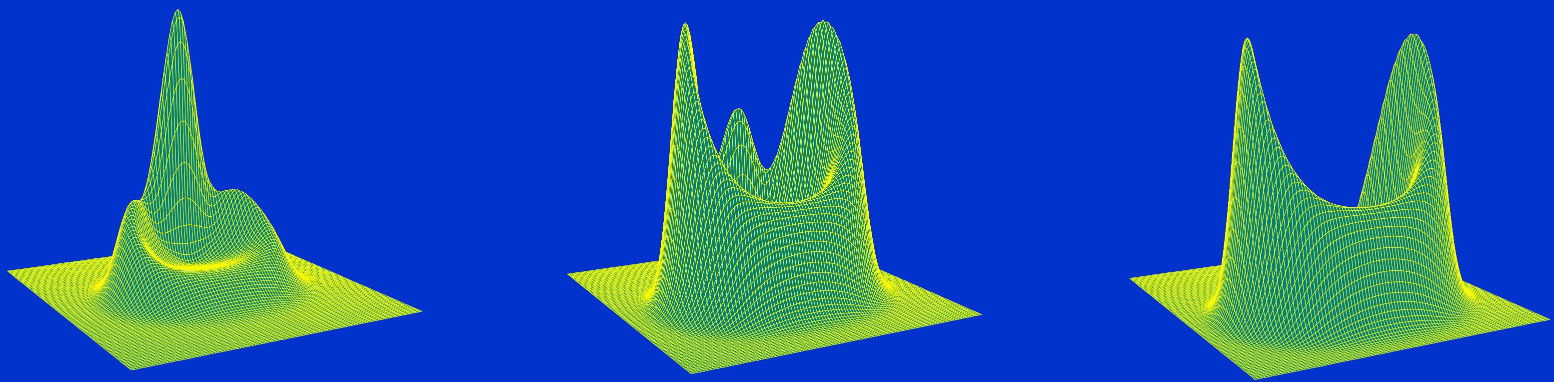
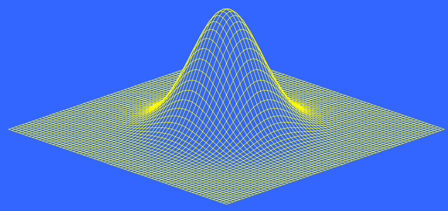
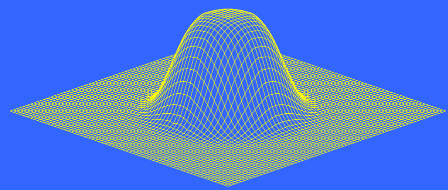
University of Auckland



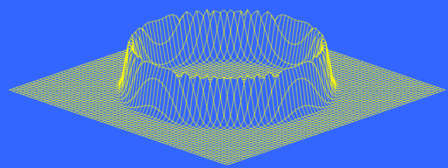
Support by the Marsden Fund of RSNZ

- Background:
  - quantum noise in the laser
  - classical and non-classical fields
  - squeezed light
  - strong coupling/system size





- Cavity QED:
  - optical bistability
  - non-classical intensity noise
  - squeezing/amplitude noise
  - spontaneous dressed-state polarization

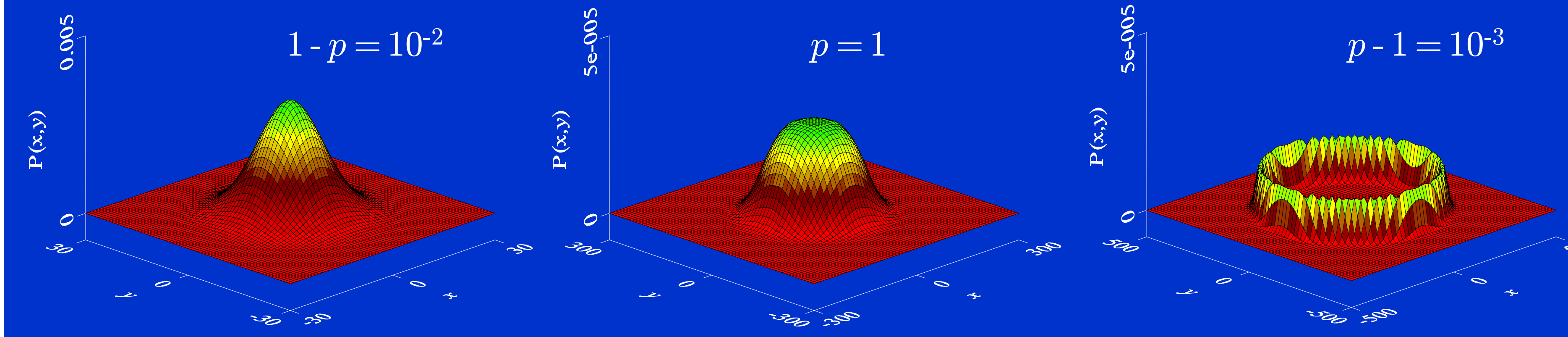


single-mode laser

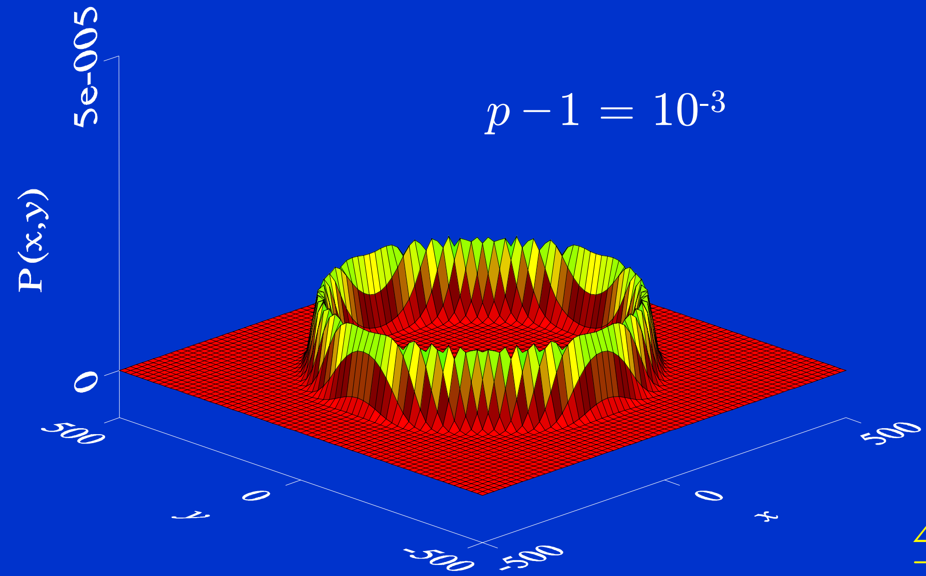
pump parameter

$$d\bar{\alpha} = -\bar{\alpha}(1 - p + p|\bar{\alpha}|^2)dt + \sqrt{\frac{n_{\mathrm{spon}}}{n_{\mathrm{sat}}}}\,(dW_1 + i\,dW_2)$$

quantum noise

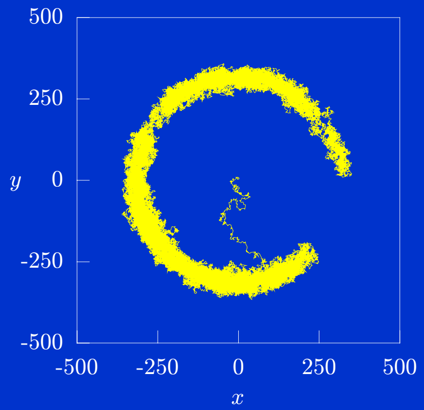


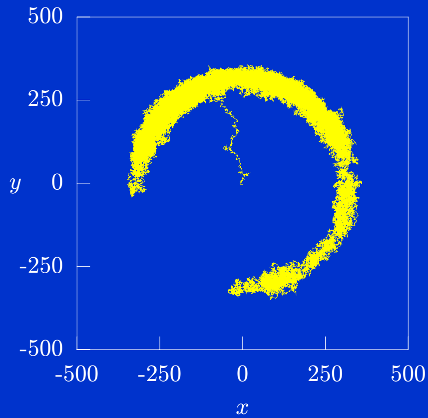
first-order coherence: laser linewidth

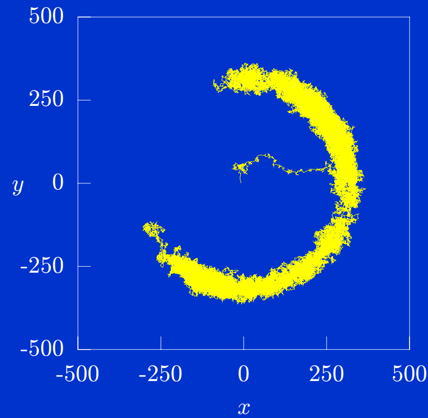


time $= 10^{6}\kappa^{-1}$

$$\frac{\Delta\omega}{\kappa} = \frac{n_{\mathrm{spon}}}{2n_{\mathrm{sat}}(p-1)}$$

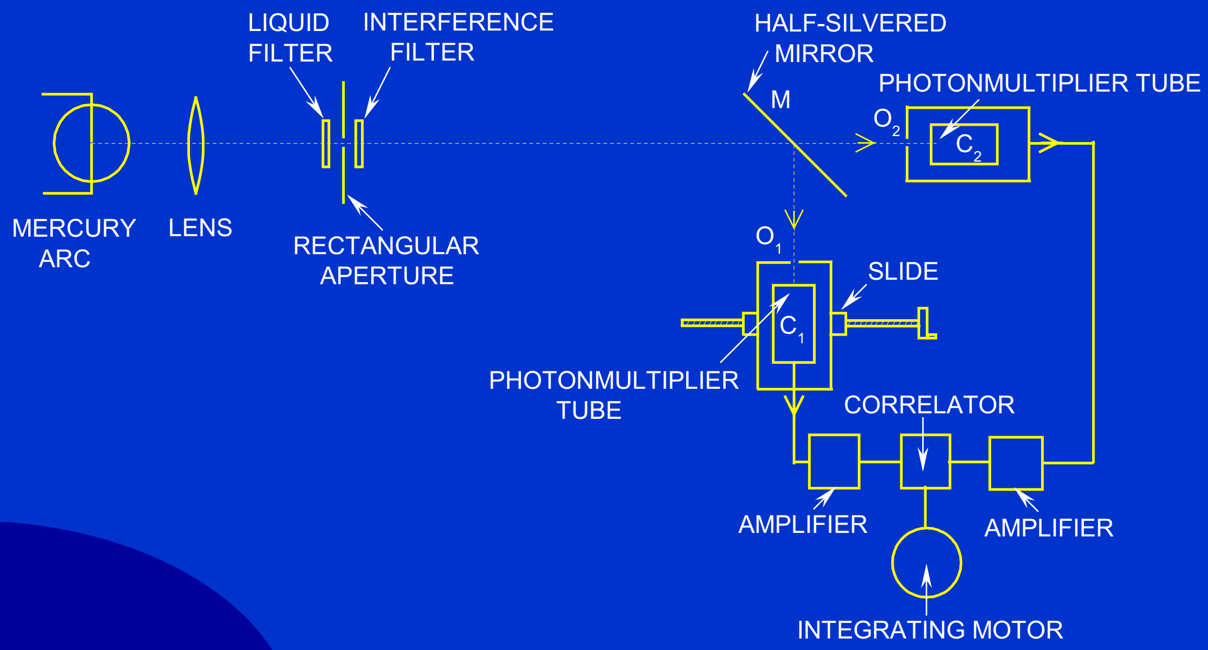


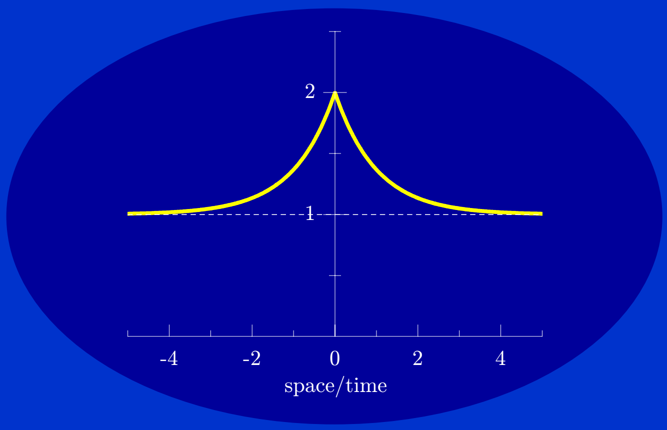




Hanbury Brown and Twiss light intensity interferometer

R. Hanbury Brown and R.Q. Twiss, Nature 177, 27 (1956)
R.Q. Twiss, A.G. Little, and R. Hanbury Brown, Nature 180, 324 (1957)

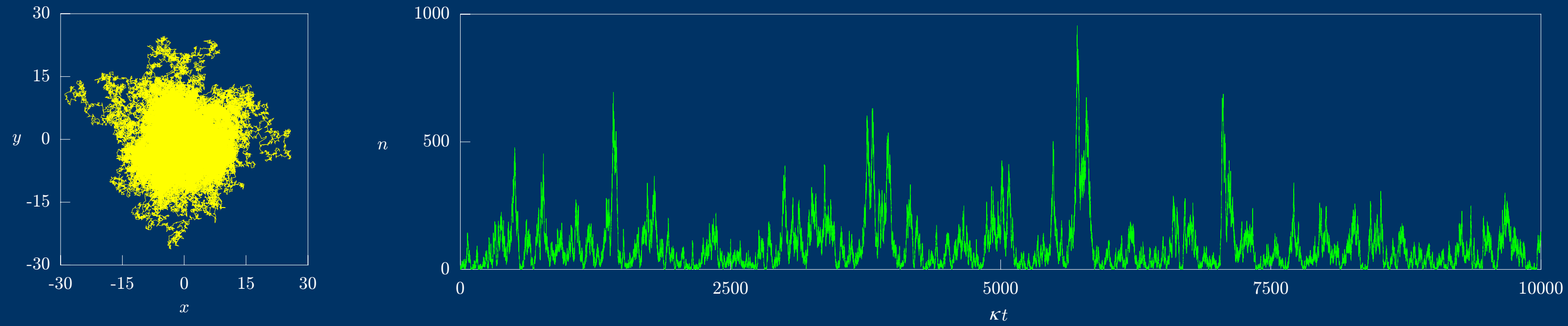




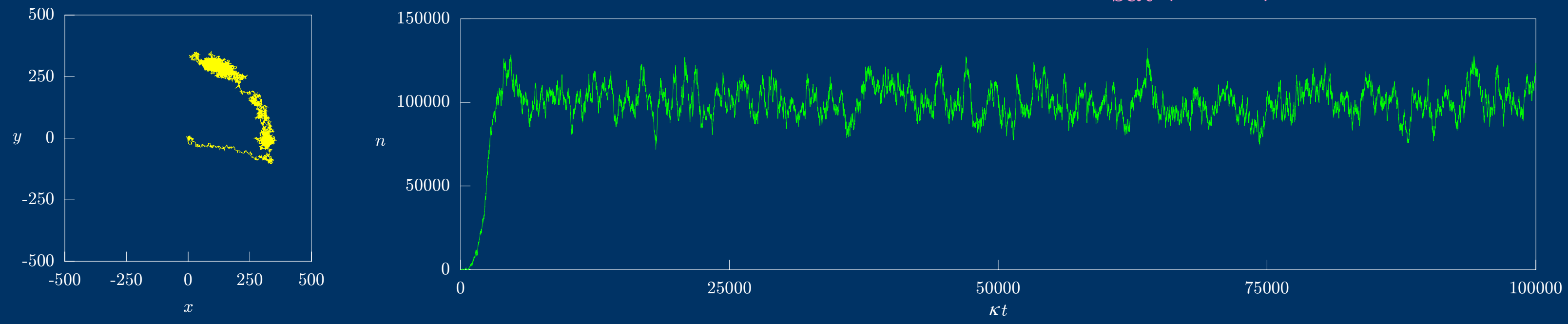
photon bunching

second-order coherence

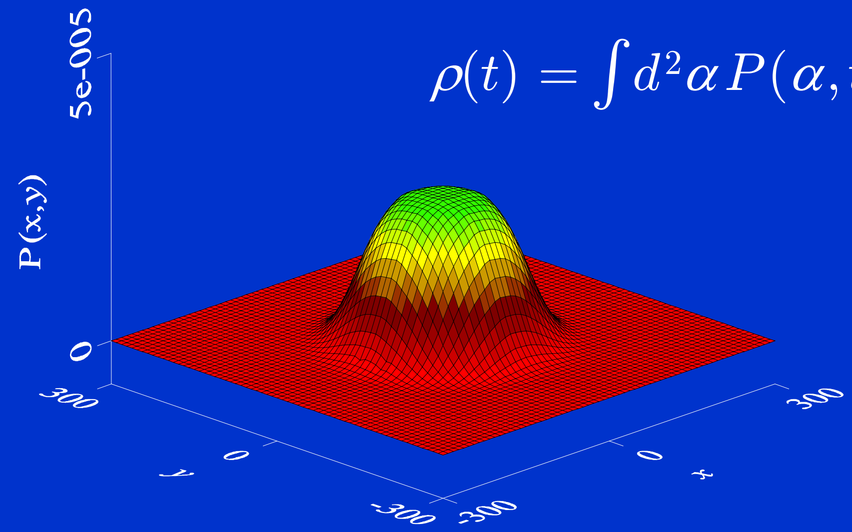
$$g^{(2)}(\tau) = 1.0 + e^{-2\kappa(1\text{-}p)|\tau|}$$



$$g^{(2)}(\tau) = 1.0 + \frac{n_{\text{spon}}}{n_{\text{sat}}(p-1)^2}\, e^{-2\kappa(p\text{-}1)|\tau|}$$



classical and non-classical fields

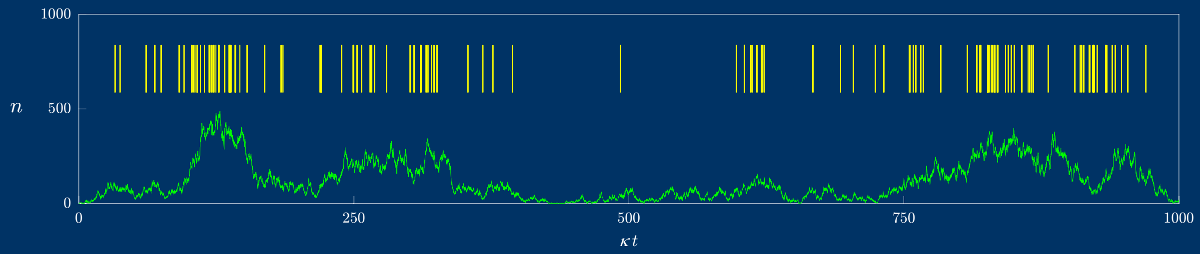


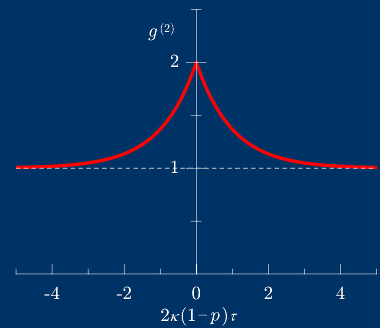
$$\rho(t) = \int d^2\alpha \, P(\alpha, t) \, |\alpha(t)\rangle\langle\alpha(t)|$$

quantum average

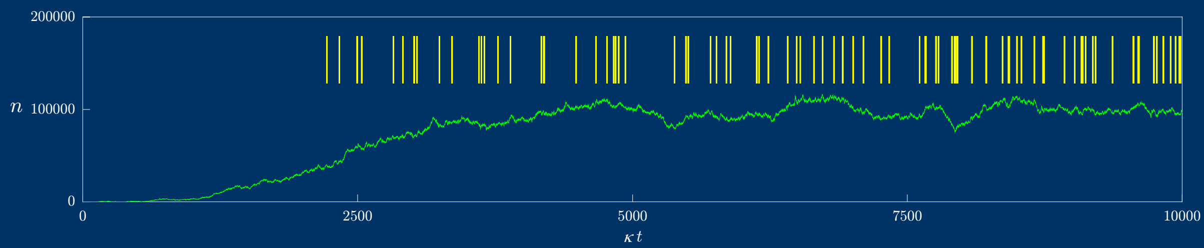
time average

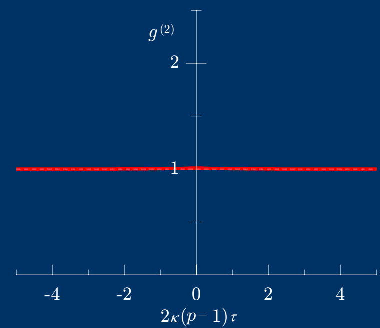
$$g^{(2)}(\tau) = \frac{\langle a^\dagger(t) a^\dagger(t+\tau) a(t+\tau) a(t)\rangle}{\langle a^\dagger(t) a(t)\rangle^2} = \frac{\langle |\alpha(t)|^2 |\alpha(t+\tau)|^2\rangle}{\langle |\alpha(t)|^2\rangle^2}$$



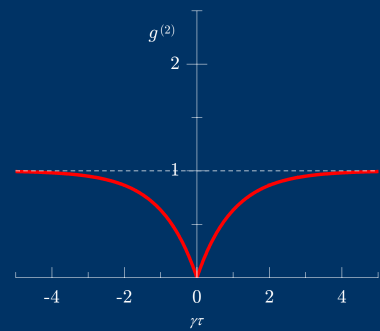


photon bunching





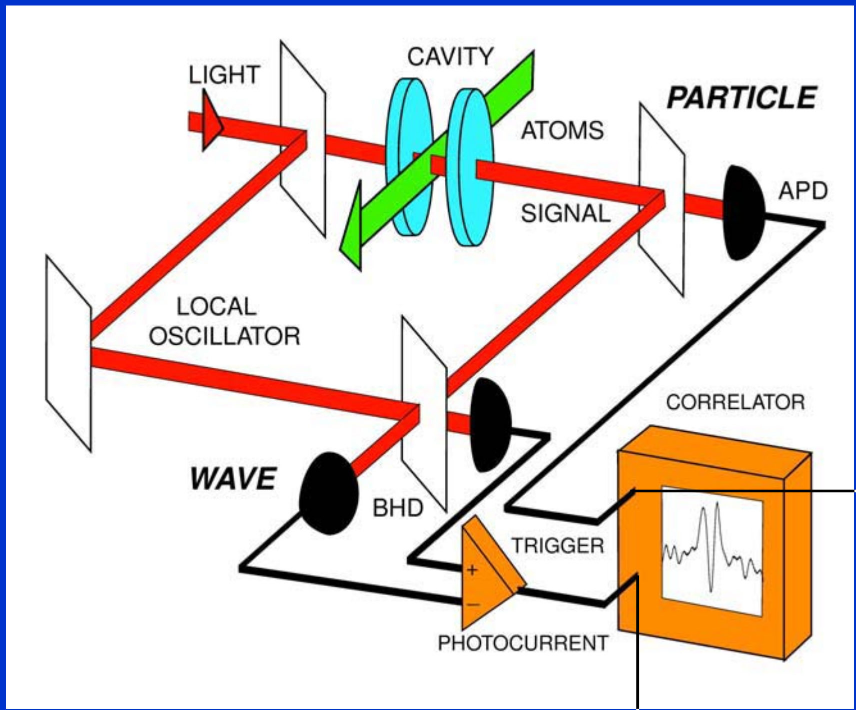
Poisson statistics

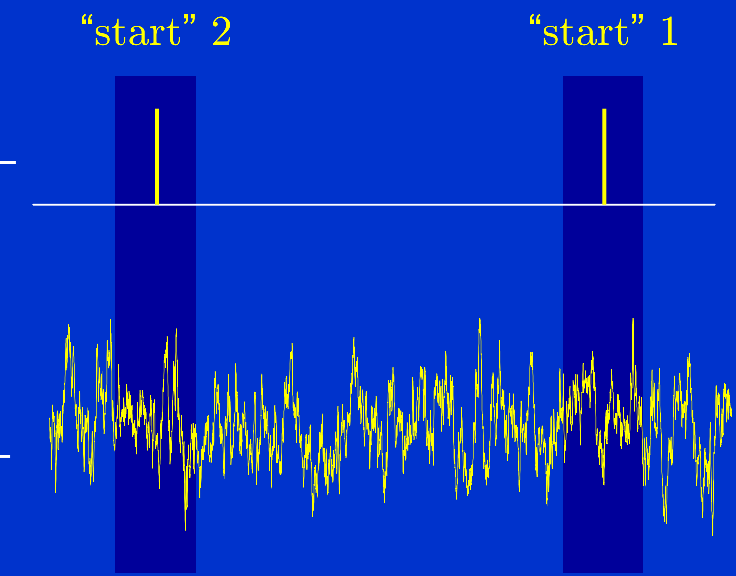


photon antibunching

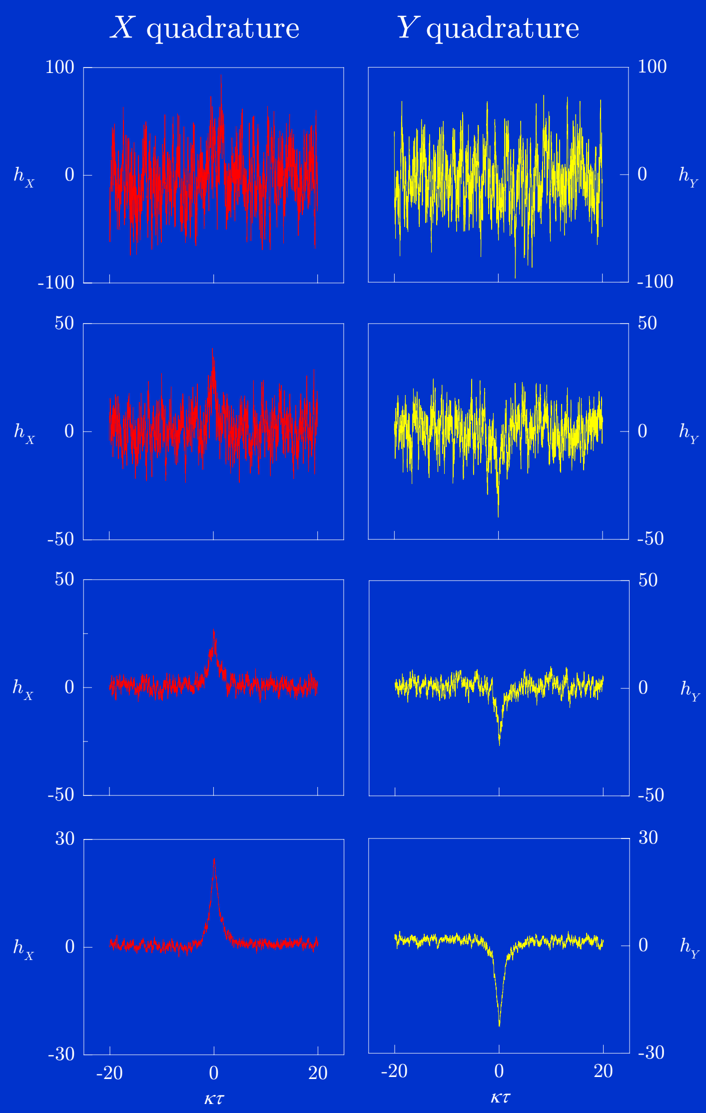
conditional homodyne detection

G.T. Foster, L.A. Orozco, H.M. Castro-Beltran, and H.J. Carmichael,
Phys. Rev. Lett. 85, 3149 (2000)





squeezed light



$$\frac{a+a^\dagger}{2} \qquad \frac{\alpha+\alpha^*}{2}$$

$$h_X(\tau)-1 = \frac{\langle :\hat{X}(t)\hat{X}(t+\tau): \rangle}{\langle a^\dagger(t)a(t)\rangle} = \frac{\langle X(t)X(t+\tau)\rangle}{\langle |\alpha(t)|^2\rangle}$$

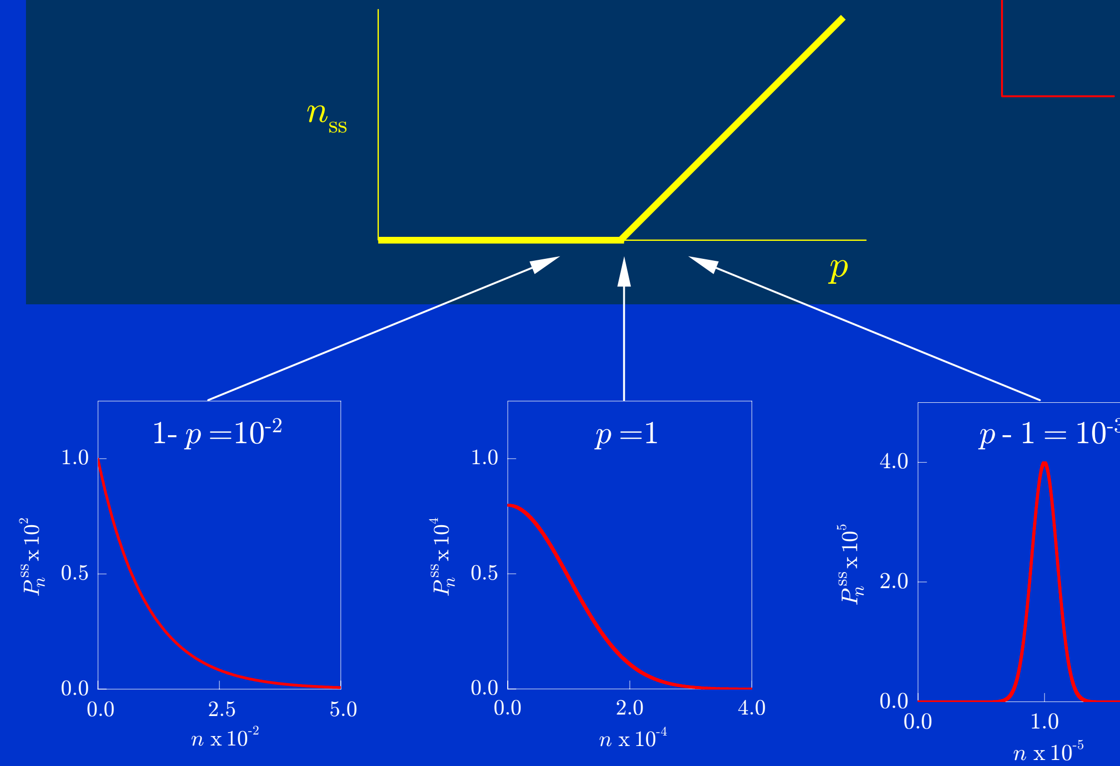
$$\frac{a-a^\dagger}{2i} \qquad \frac{\alpha-\alpha^*}{2i}$$

$$h_Y(\tau)-1 = \frac{\langle :\hat{Y}(t)\hat{Y}(t+\tau): \rangle}{\langle a^\dagger(t)a(t)\rangle} = \frac{\langle Y(t)Y(t+\tau)\rangle}{\langle |\alpha(t)|^2\rangle}$$

strong coupling and system size

$$d\bar{\alpha} = -\bar{\alpha}(1 - p + p|\bar{\alpha}|^2)dt + \sqrt{\frac{n_{\rm spon}}{n_{\rm sat}}}(dW_1 + i dW_2)$$

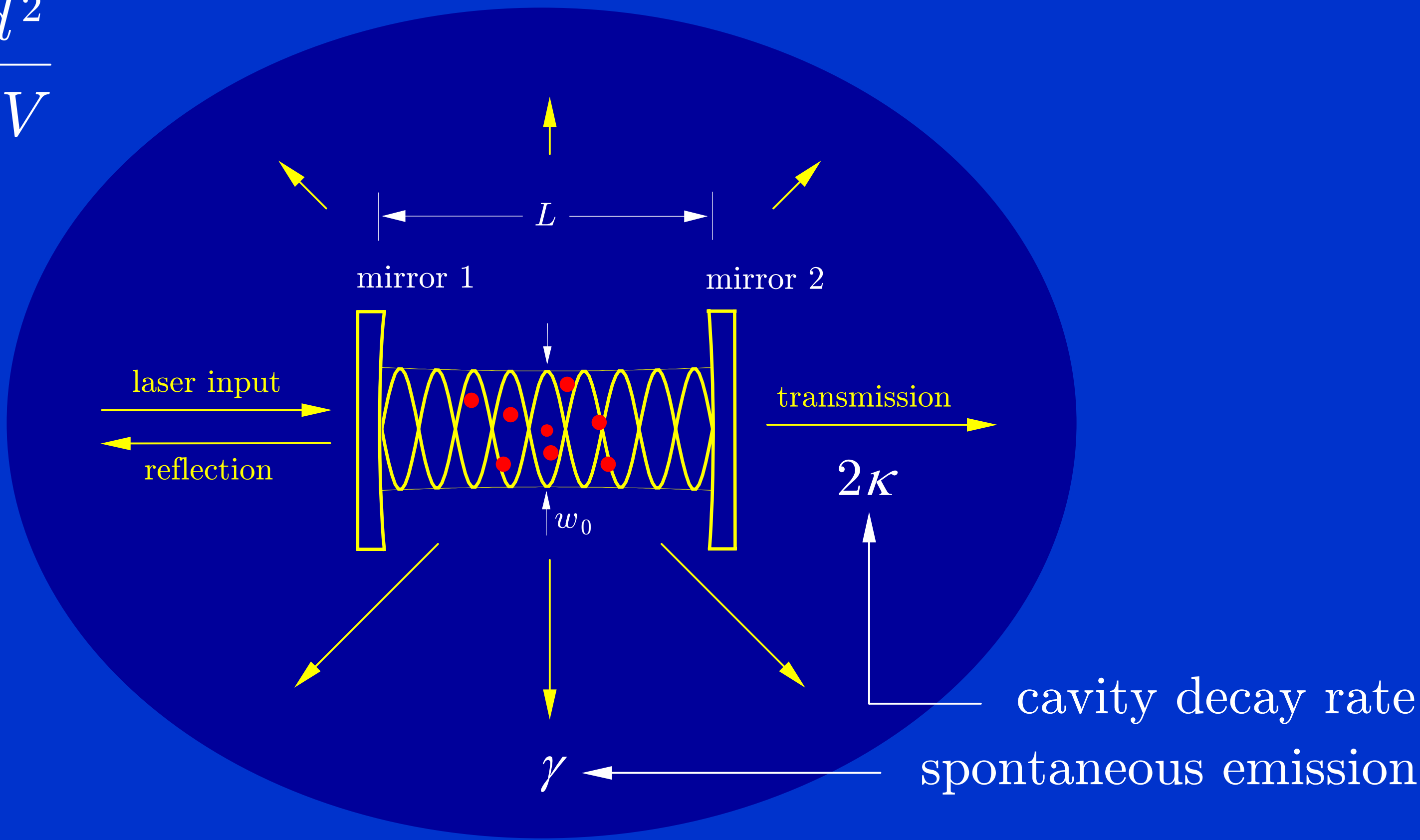
$$n_{\rm sat} = \frac{\gamma_h(\gamma_\uparrow + \gamma_\downarrow)}{8g^2}$$



cavity QED

dipole coupling strength:

$$g = \sqrt{\frac{\omega_C d^2}{2\hbar\varepsilon_0 V}}$$



Background:

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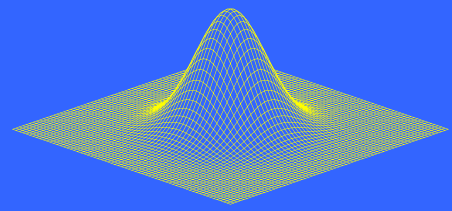
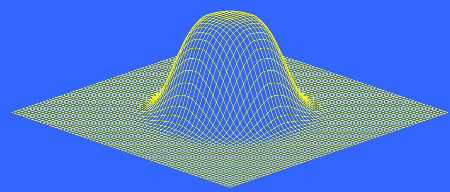
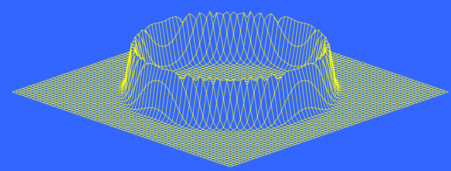
- classical and non-classical fields
- squeezed light
- strong coupling/system size





Cavity QED:

- optical bistability
- non-classical intensity noise
- squeezing/amplitude noise
- spontaneous dressed-state polarization



optical bistability

$$\dot{\bar{\alpha}} = -\kappa\bar{\alpha} + \sum_j g(r_j)\bar{\beta}_j + \bar{\mathcal{E}}_0$$

$$\dot{\bar{\beta}}_j = -\frac{\gamma}{2}\bar{\beta}_j + g(r_j)\bar{\alpha}\bar{m}_j$$

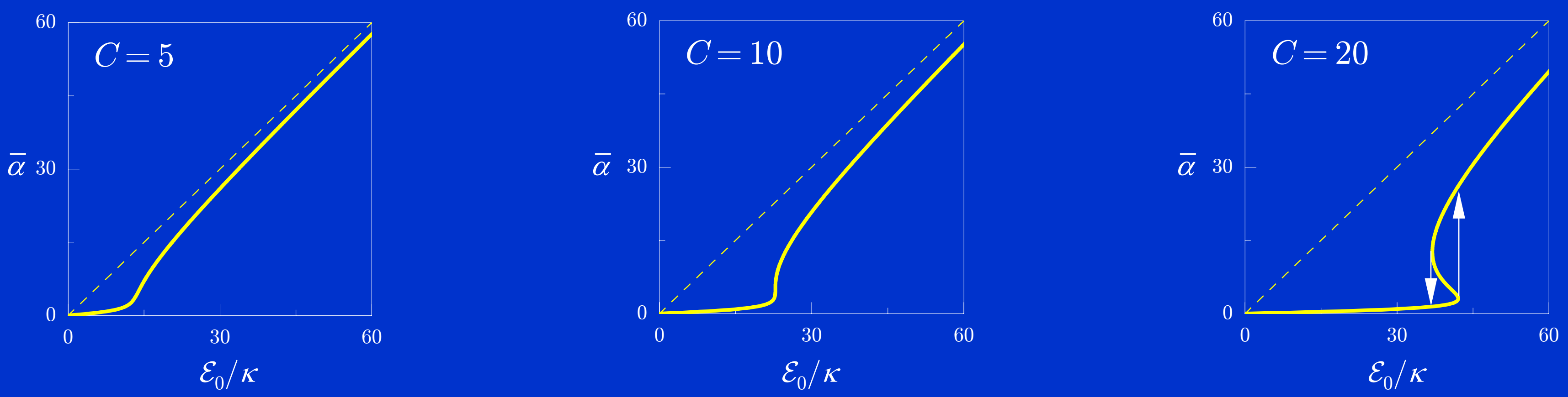
$$\dot{\bar{m}}_j = -\gamma(\bar{m}_j + 1) - 2g(r_j)(\bar{\beta}_j^*\bar{\alpha} + \text{c.c.})$$

input field

intracavity field

$$\mathcal{E}_0/\kappa = f(\bar{\alpha}; C)$$

$$2C = \frac{\alpha_0 l}{1-R}$$



quantum trajectories

non-unitary Schrödinger evolution:

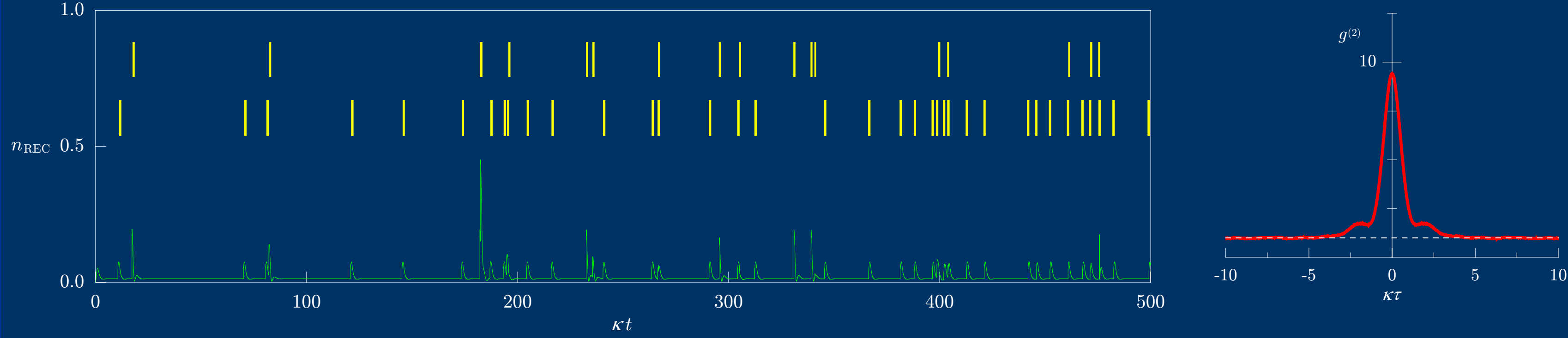
$$\frac{d\,|\bar{\psi}_{\mathrm{REC}}\rangle}{dt} = \frac{1}{i\hbar}\hat{H}_B(t)\,|\bar{\psi}_{\mathrm{REC}}\rangle$$

$$\frac{1}{i\hbar}\hat{H}_B(t) = \underbrace{\mathcal{E}(\hat{a}^\dagger - \hat{a})}_{\text{laser input}} + \underbrace{\sum_j g(r_j(t))(\hat{a}^\dagger\hat{\sigma}_{j\text{-}} - \hat{a}\hat{\sigma}_{j+})}_{\text{dipole interaction}} - \underbrace{\kappa\hat{a}^\dagger\hat{a}}_{\text{cavity loss}} - \underbrace{\frac{\gamma}{2}\sum_j \hat{\sigma}_{j+}\hat{\sigma}_{j\text{-}}}_{\text{spontaneous emission}}$$

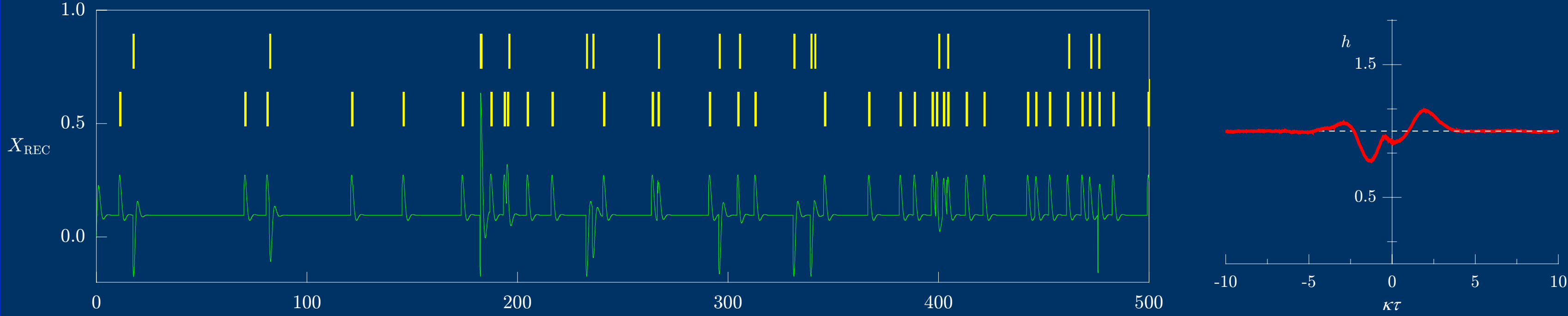
quantum jumps:

$$|\bar{\psi}_{\mathrm{REC}}\rangle \longrightarrow \begin{cases} \hat{a}\,|\bar{\psi}_{\mathrm{REC}}\rangle & \text{at rate} \quad 2\kappa\langle(\hat{a}^\dagger\hat{a})(t)\rangle_{\mathrm{REC}} \\ \hat{\sigma}_{j\text{-}}\,|\bar{\psi}_{\mathrm{REC}}\rangle & \text{at rate} \quad \gamma\langle(\hat{\sigma}_{j+}\hat{\sigma}_{j\text{-}})(t)\rangle_{\mathrm{REC}} \end{cases}$$

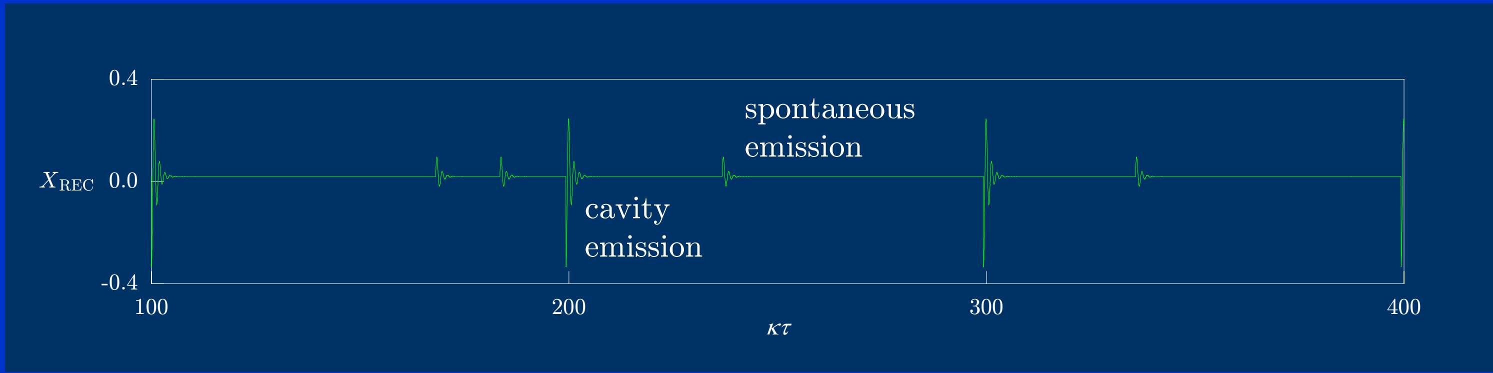
photon number expectation



field amplitude expectation



weak-excitation limit



$$g^{(2)}_{\{r_j\}}(\tau) = \left[h_{\{r_j\}}(\tau) \right]^2$$

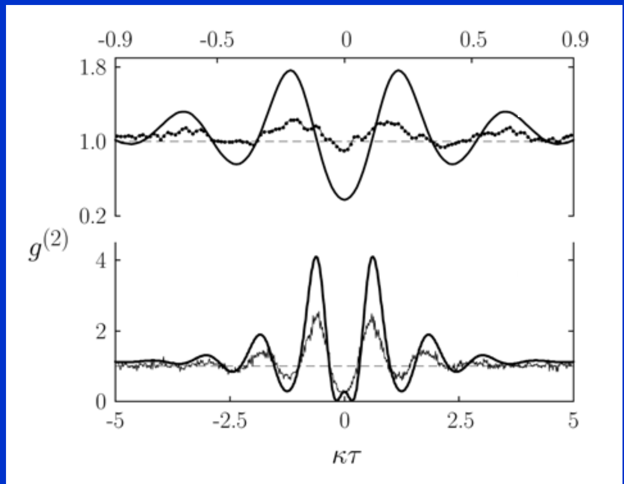
$$h_{\{r_j\}}(\tau) = 1.0 - A_{\{r_j\}} e^{-\frac{1}{2}(\kappa+\gamma/2)|\tau|} \left[\cos(\Omega_{\{r_j\}}\tau) + \frac{\frac{1}{2}(\kappa+\gamma/2)}{\Omega_{\{r_j\}}} \sin(\Omega_{\{r_j\}}\tau) \right]$$

$$\Omega_{\{r_j\}} = \sqrt{\sum_j g^2(r_j) - \frac{1}{4}(\kappa-\gamma/2)^2}$$

experiments 1

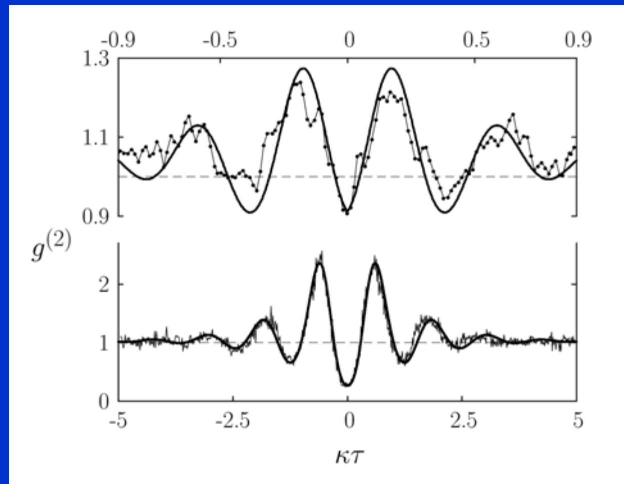
G. Rempe, R.J. Thompson, R.J. Brecha, W.D. Lee, and H.J. Kimble, Phys. Rev. Lett. **67**, 1727 (1991)

G.T. Foster, S.L. Mielke, and L.A. Orozco, Phys. Rev. A **61**, 053821 (2000)



Rempe *et al.*

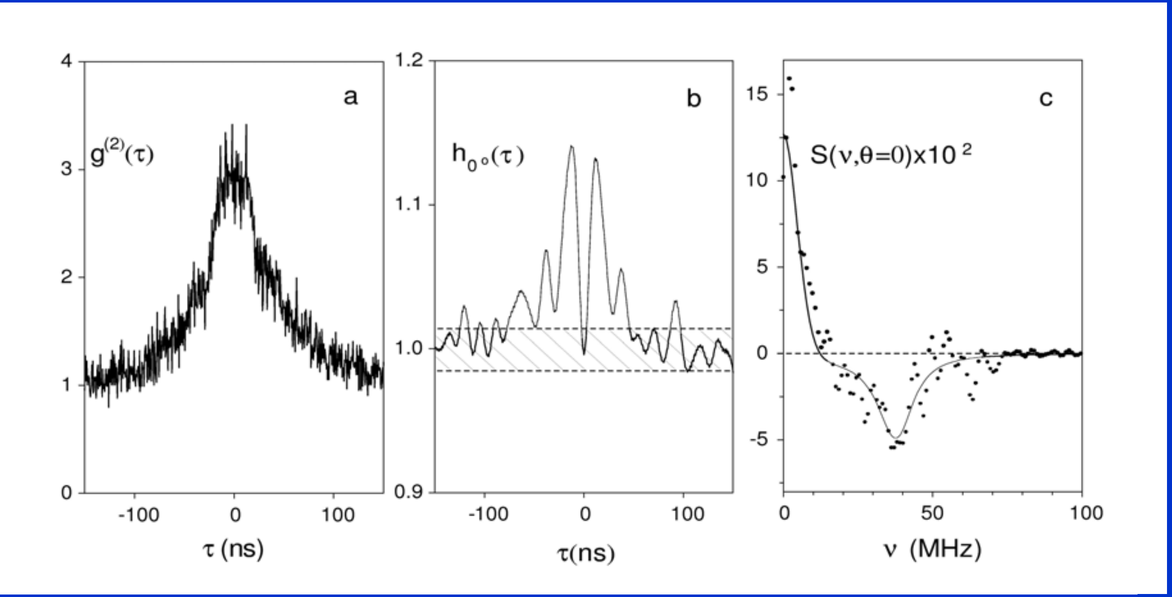
Foster *et al.*



non-classical intensity noise

experiments 2

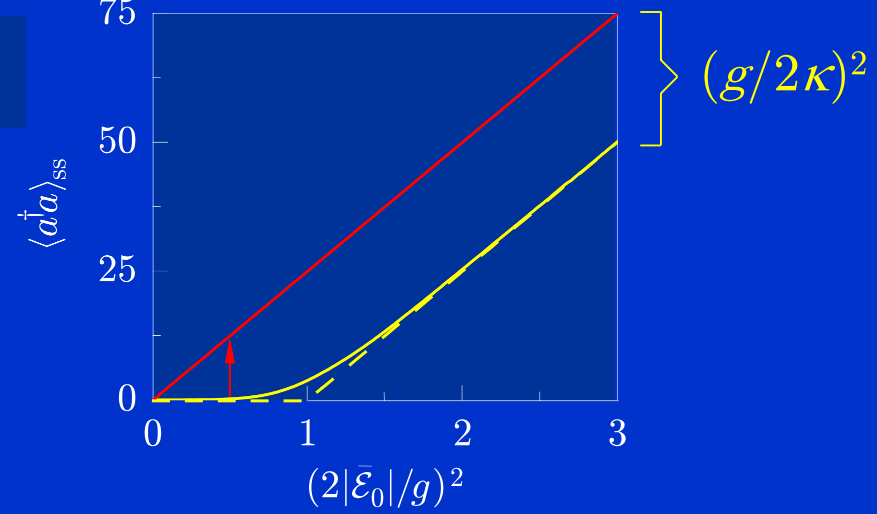
G.T. Foster, L.A. Orozco, H.M. Castro-Beltran, and H.J. Carmichael, Phys. Rev. Lett 85, 3149 (2000)

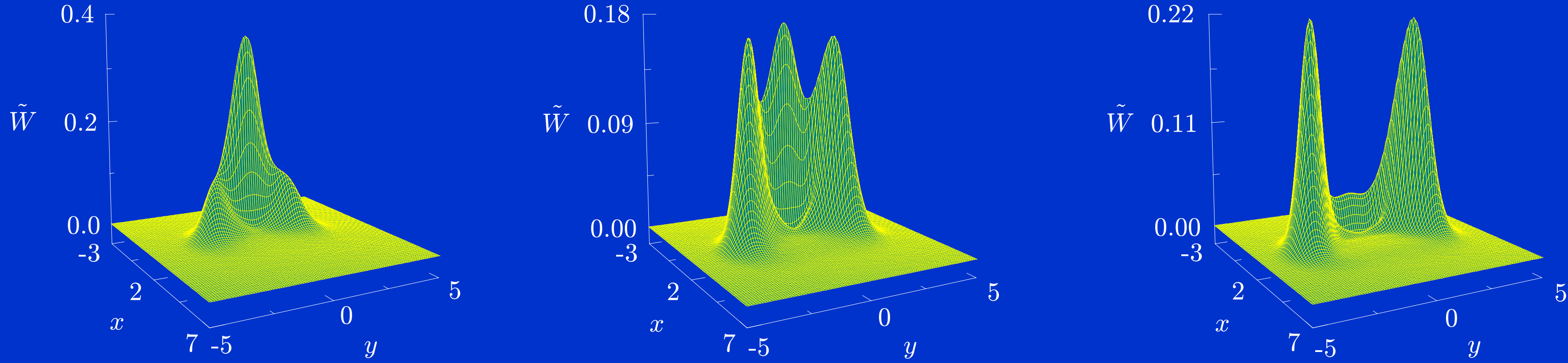


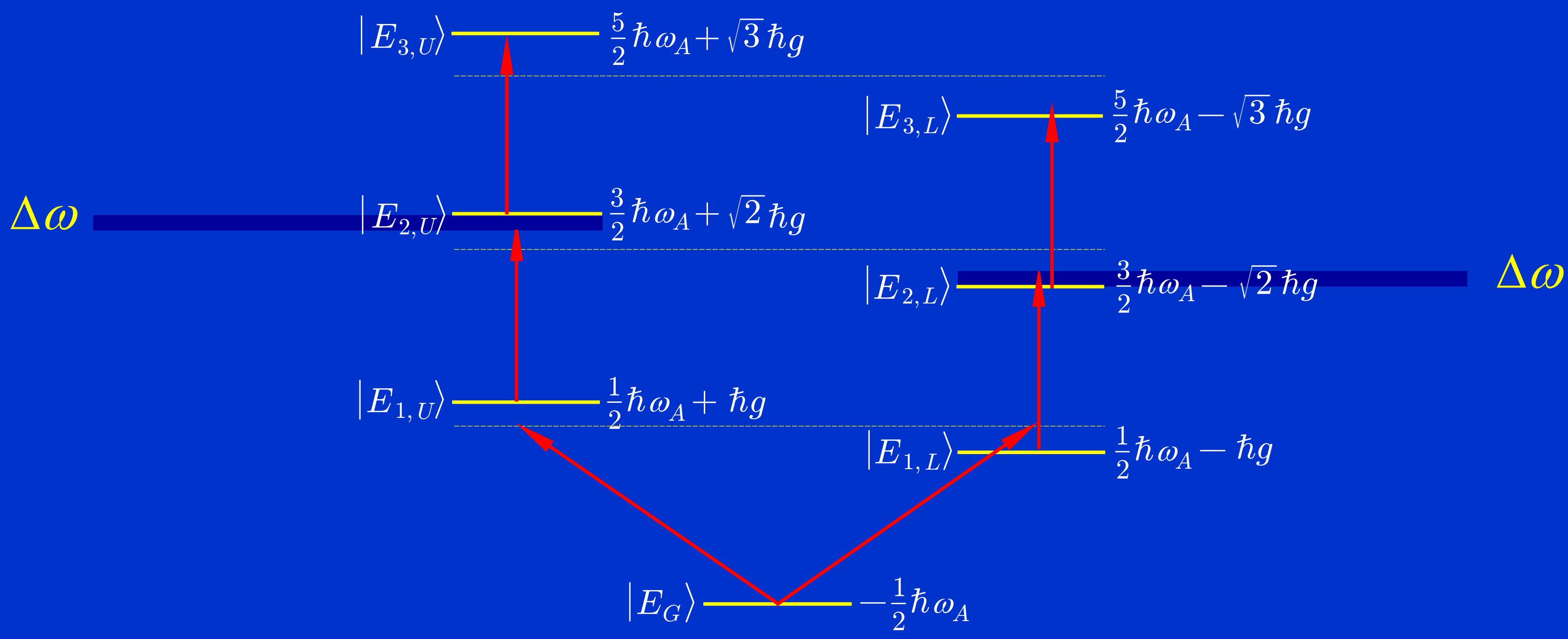
squeezing/amplitude noise

spontaneous dressed-state polarization

$g/\kappa = 10$
$\gamma/2\kappa = 0$



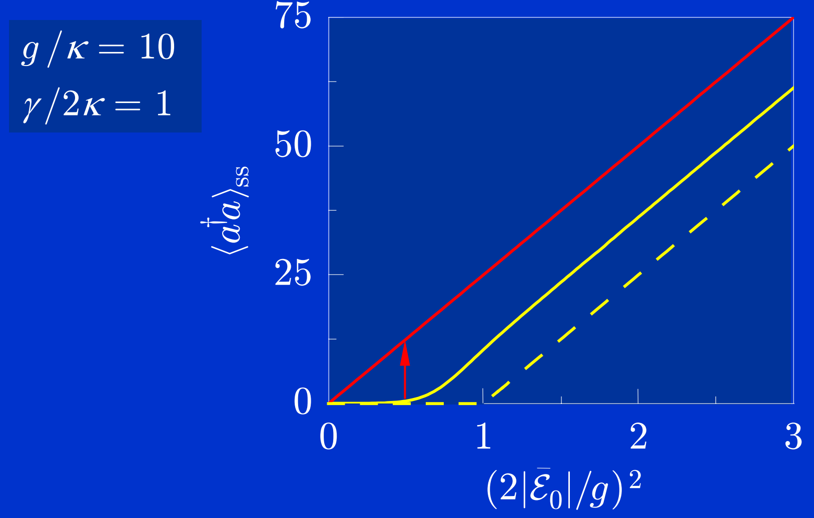


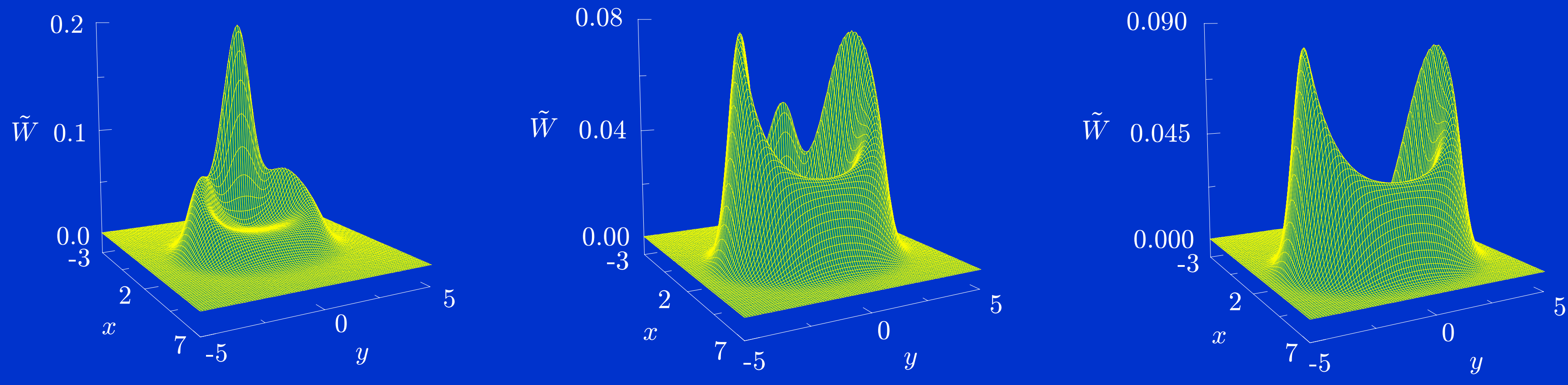


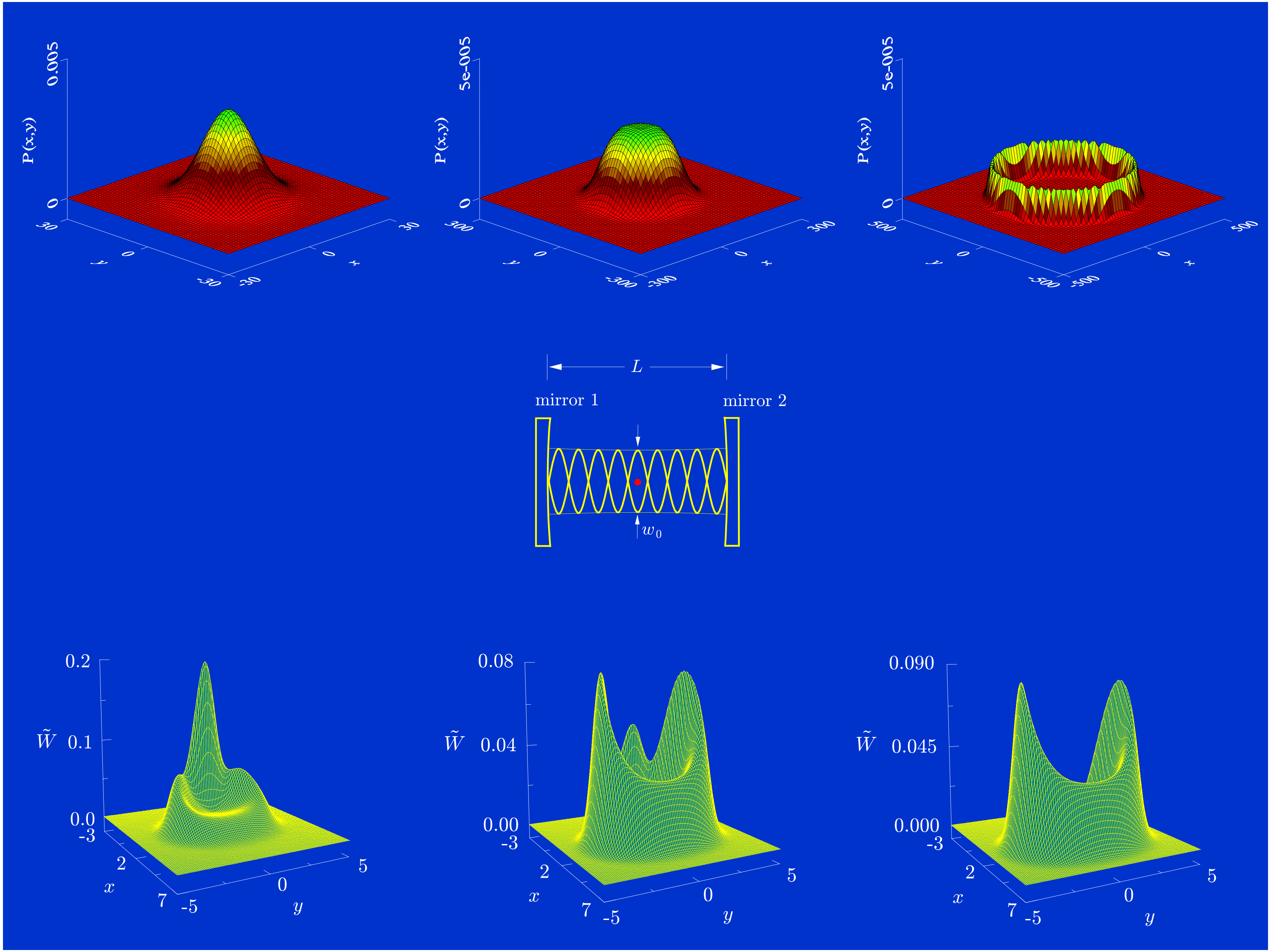
$$\Delta\omega = \pm(\sqrt{\bar{n}+1} - \sqrt{\bar{n}}\,)g \approx \pm\frac{g}{2\sqrt{\bar{n}}}$$

$$\bar{n}\left[1 - \frac{1}{1+(\Delta\omega/\kappa\,)^2}\right] \approx \bar{n}(\Delta\omega/\kappa\,)^2 = (g/2\kappa)^2$$

spontaneous emission






P(x,y)
0.005
0
30
0
-30
y
x
5e-005
300
-300
500
-500
L
mirror 1
mirror 2
w_0
$\tilde{W}$
0.2
0.1
0.0
-3
2
7
-5
5
0.08
0.04
0.00
0.090
0.045
0.000