The Abdus Salam
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Noise and Instabilities in Quantum Mechanics

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Simulation of many-body physics with trapped ions

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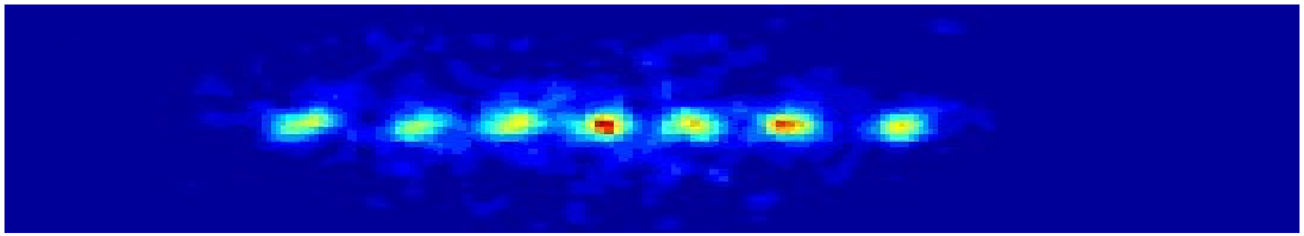


Outline

1. Motivation: Trapped ions for quantum simulation of many-body physics.

2. The physics of trapped ions: internal states (effective spins) and vibratinal modes (phonons).

3. Simulation of quantum interacting spins:
    - Effective spin-spin interactions
    - How to reach ground states
    - Spin quantum phases in linear ion traps

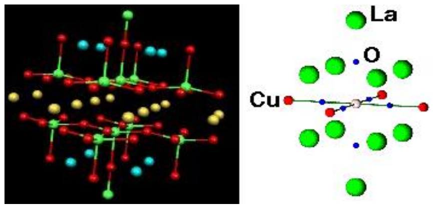
4. Interacting phonon models: 'Phonon Hubbard model', phonon superfluidity
    - Analogy between phonons and bosons in an optical lattice
    - How to reach ground states, how to prepare/measure phonon states
    - Mott Insulator-Superfluid quantum phase transition of phonons

5. Conclusions

Introduction:
Interacting quantum systems and trapped ions

Analog simulation of quantum phases

Quantum many-body physics has traditionally been aimed to the understanding of properties of materials:



Example: High-Tc superconductivity

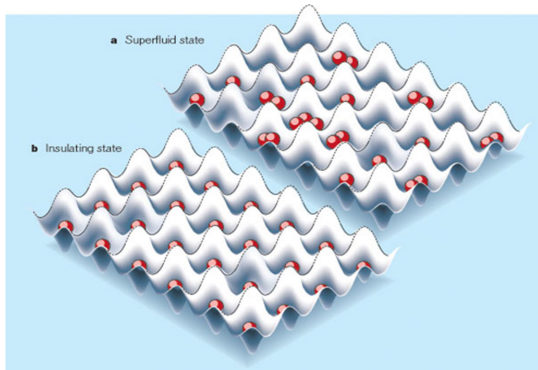
$$H = \sum_{k,s} \varepsilon_k c^{\dagger}_{k,s} c_{k,s} + U \sum_{j} n_{j,\uparrow} n_{j,\downarrow}$$

Hubbard Hamiltonian

Quantum optical experimental systems offer us the posssibility to control matter at the microscopic scale and tune interactions. The process is, thus, inverted and a variety of many-body models can be realized in a controllable way in „quantum simulators“.

Some experimental and theoretical effort is aimed to the implementation of this idea with atoms in optical lattices.

Bose-Hubbard Mode Fermionic-Hubbard Model Quantum spin interacting systems

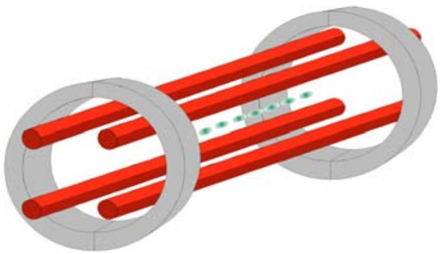


- *Allows us to study models that cannot be simulated numerically*
- *Experimental study of quantum correlation/entanglement in many-body systems*

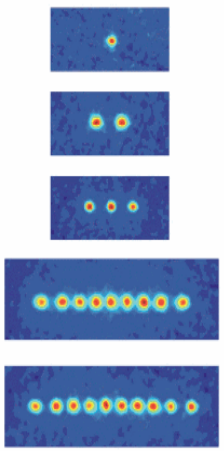
Trapped ions and quantum information

In the last years quantum information science has motivated accurate experimental techniques for the manipulation and measurement of trapped ions:

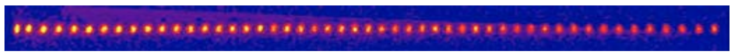
- Single qubit gates
- Several proposals and implementations of two-qubit gates (effective qubit-qubit interactions)
- Efficient preparation and measurement of internal quantum states at single ion level
- Trapping technology



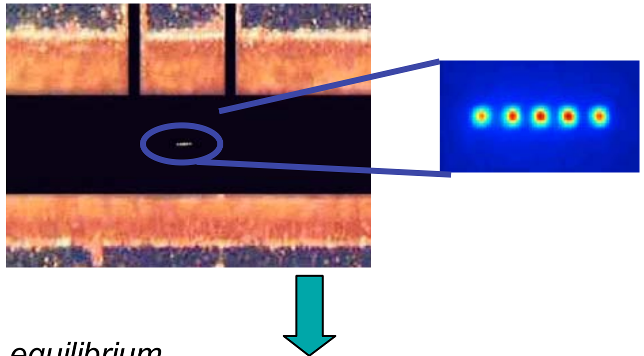
Traditional Paul Trap: *ions arrange themselves in a Coulomb Crystal*



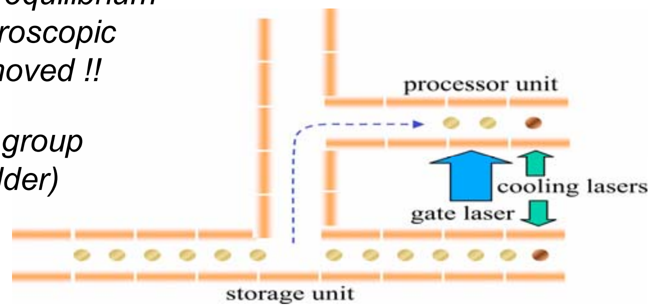
Blatt's group (Innsbruck)



Walther's group (MPQ, Garching)



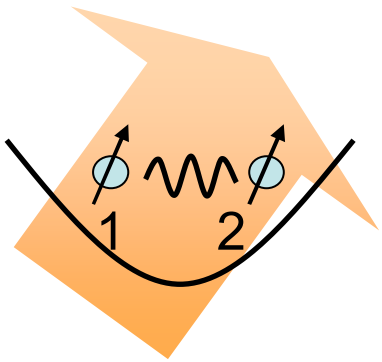
Segmented Microtraps: *equilibrium positions are fixed by microscopic electrodes. Ions can be moved !!*



Wineland's group (NIST, Boulder)

Trapped ions and many-body physics (motivation)

The basic idea behind proposal for quantum computation with trapped ions (Cirac & Zoller, 95) is that internal states (qubits) are coupled by the motion (phonons)



Example: A laser couples internal states to the motion...

$$\left.\begin{aligned} H_I &= F\sigma_1^z(a+a^\dagger) \\ &F\sigma_2^z(a+a^\dagger) \end{aligned}\right\} \rightarrow U = e^{-iJ\sigma_1^z\sigma_2^z t}$$

... which results in an effective qubit-qubit interaction.

By using similar experimental techniques, one can use trapped ions to simulate interacting quantum models. A few advantages are that:

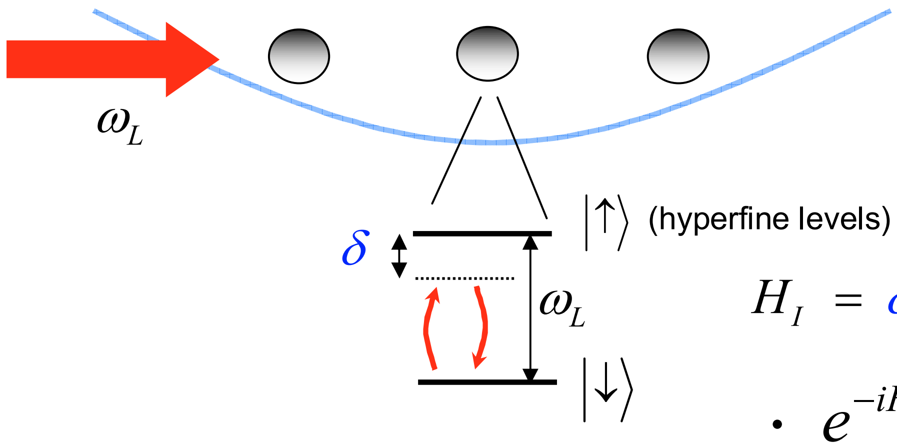
•Internal states can be prepared and measured at single particle level (separation between ions >> wavelength of light, advantage with respecto to optical lattices).

•Our proposal is much less stringent than, for example, quantum computation, (no need for quantum gates, lower fidelity is required).

The physics of trapped ions

The physics of trapped ions: (1) Internal states

Internal transitions are two-level systems (qubits), and can be manipulated with lasers

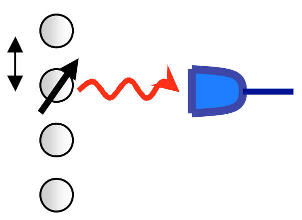


$$H_I = \delta \sum_j \sigma_j^z + \frac{\Omega}{2} \sum_j \sigma_j^x = B^z \sum_j \sigma_j^z + B^x \sum_j \sigma_j^x$$

- $e^{-iH_I t} \longrightarrow$ single qubit quantum gates (preparation)
- "Allways on" $H_I \longrightarrow$ effective magnetic fields

Efficient measurement of averages and quantum correlations at single ion level (rutine for experimentalists)

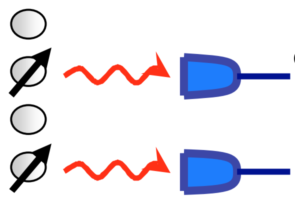
$d_0 \approx$
$2-5\mu m$
$> \lambda_{laser}$



average photon number $\Rightarrow \langle \sigma_j^z \rangle$

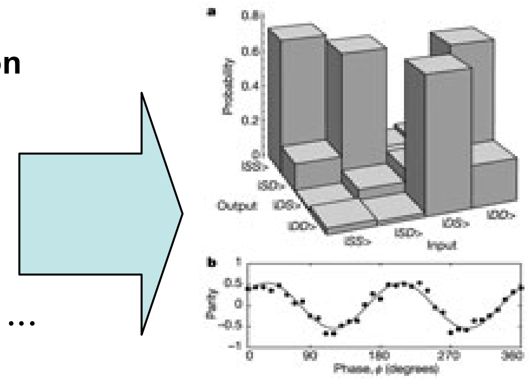
(+ qubit rotations)

$\langle \sigma_j^x \rangle, \langle \sigma_j^y \rangle$



photon-photon number correlations

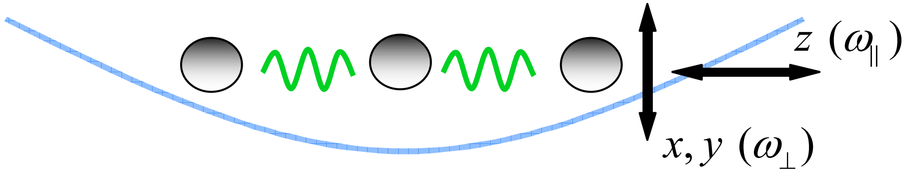
$\langle \sigma_i^z \sigma_j^z \rangle$ ...



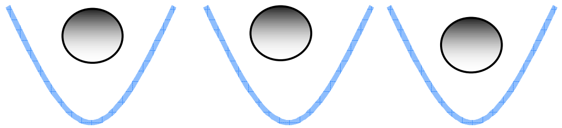
Blatt's group (Innsbruck)

The physics of trapped ions: (2) Vibrational modes

Ions are trapped by harmonic potentials created by electromagnetic forces:



Linear traps: weak axial confinement



Arrays of ion microtraps: individual trapping potentials

The motion of the ions around the equilibrium positions is described by a quadratic potential (coupled harmonic oscillators):

$$V(\vec{r}_j) = V_{conf}(\vec{r}_j) + V_{Coul}(\vec{r}_j)$$

radial trapping

$$V_{conf}(\vec{r}_j) = \frac{1}{2} m\omega_{\parallel}^2 \sum_j z_j^2 + \frac{1}{2} m\omega_{\perp}^2 \sum_j \left(x_j^2 + y_j^2\right)$$

axial trapping

Coulomb interaction couples the ions - 'Spring constants'

$$V_{Coul}(\vec{r}_j) = \sum_{i>j} \frac{e^2}{\left|\vec{r}_i - \vec{r}_j\right|} \xrightarrow[2^{nd}\ ORDER]{} \sum_{i,j} \frac{e^2}{2\left|z_i^0 - z_j^0\right|^3} \left(z_i - z_j\right)^2 - \sum_{i,j} \frac{e^2}{\left|z_i^0 - z_j^0\right|^3} \left(\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2\right)$$

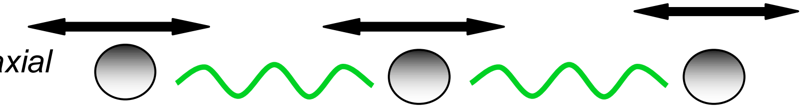
diagonalization $\rightarrow$ collective modes

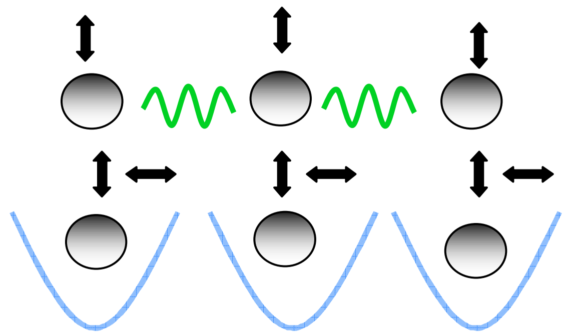
$$H = \sum_{n,\alpha=x,y,z} \hbar\Omega_n a_{\alpha,n}^{\dagger} a_{\alpha,n}$$

The physics of trapped ions: (2) Vibrational modes

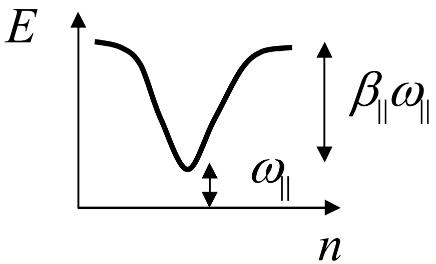
The characteristics of the modes (phonons) in each direction depend on the ratio between spring constants (Coulomb interactions) and trapping frequencies:

$$\beta = \frac{e^2}{d_0^3} / m\omega^2 \quad \begin{cases} \beta \gg 1 & \text{soft limit: collective (axial modes in a linear trap)} \\ \beta \ll 1 & \text{hard limit: close to localized (radial modes and microtraps)} \end{cases}$$

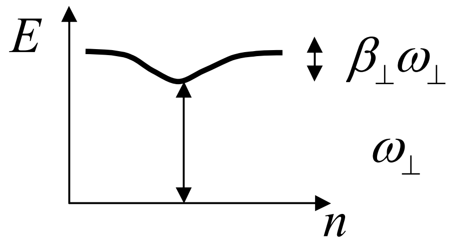




soft limit: *small energy gap (acoustic phonons)*



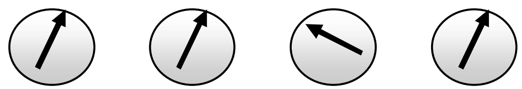
hard limit: *large energy gap, small energy dispersion (optical phonons)*



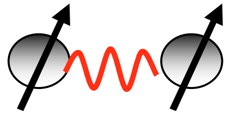
Simulation of quantum interacting spins with trapped ions

D. Porras and J.I. Cirac, Phys. Rev. Lett. (2004).

X. Deng, D. Porras and J.I. Cirac, in preparation.

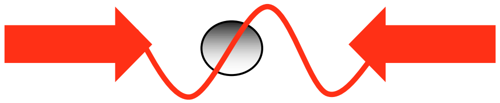


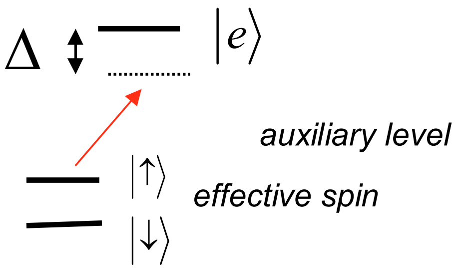
Effective spin-spin interactions



Effective spin-spin interaction can be transmited by coupling the vibrational modes.

First, we have to couple the internal states to the motion of the ions: place the ions in an off-resonant standing wave:





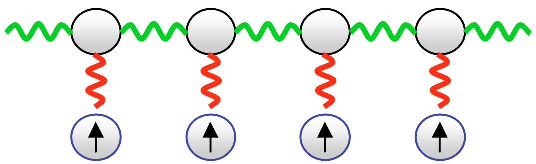
Level $|\uparrow\rangle$ experience an ac-Stark shift that depends on the position through the amplitude of the standing-wave:

$$H_{ac-Stark} = \frac{1}{\Delta}\left(\frac{\Omega}{2}\right)^2 |\uparrow\rangle\langle\uparrow| \cos^2(k\,x) \xrightarrow[linearize]{} F\sigma^z x$$

state dependent force on the ions (pushing gate, Cirac and Zoller, 2000)

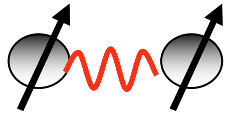
Thus, trapped ions are a system of effective spins coupled to phonons.

$$H = \sum_j \left(B^x\sigma_j^x + B^z\sigma_j^z\right) + \sum_n \Omega_n a_n^\dagger a_n + F\sum_j \sigma_j^z x_j$$



$$F\sum_{j,n} \frac{M_{j,n}}{\sqrt{2m\Omega_n}} \sigma_j^z (a_n^\dagger + a_n)$$

Effective spin-spin interactions



Under certain conditions, a system of spins coupled to phonons is equivalent to a system of interacing spins (formally: unitary transformation):

$$H = \sum_n \Omega_n a_n^\dagger a_n + \sum_j \left(B^x \sigma_j^x + B^z \sigma_j^z \right) + F \sum_j \sigma_j^z x_j$$

$$\boxed{\begin{aligned} &H \to e^{-S} H e^{S} \\ &S = \sum_{j,n} \eta_{j,n} \sigma_{j,n}^z (a_n^\dagger - a_n) \end{aligned}}$$

$$\boxed{\eta_{j,n} = \frac{F \mathcal{M}_{j,n} x_n^0}{\Omega_n}}$$

x_n^0 = ground state size of the n'th mode

$$H = \sum_n \Omega_n a_n^\dagger a_n + \sum_j \left(B^x \sigma_j^x + B^z \sigma_j^z \right) + \frac{1}{2} \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \text{ residual spin-phonon couplings } \mathcal{O}(B^x \eta)$$

We get an effective spin-spin Ising interaction which depends on the characteristics of the vibrational modes:

$$J_{i,j} = \sum_n \frac{\mathcal{M}_{in} \mathcal{M}_{nj}}{\Omega_n^2}$$

$\beta \gg 1$ *soft limit: long range interaction*

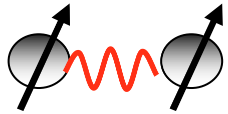
(range of the order of the chain length)

$\beta \ll 1$ *hard limit: short range interaction*

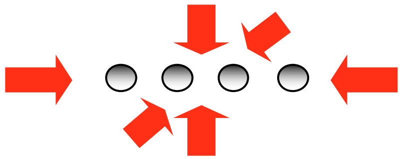
(dipolar decay law)

$$J_{i,j} = \frac{J}{\left| z_i^0 - z_j^0 \right|^3}$$

Effective spin-spin interactions



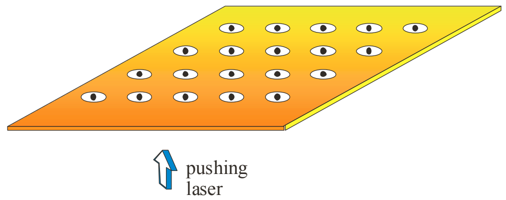
We can induce interactions between different components by coupling different spatial directions to different spin components.



In the most general case: Effective XYZ spin-spin interactions

$$F_z\sum_j \sigma_j^z z_j + F_x\sum_j \sigma_j^x x_j + F_y\sum_j \sigma_j^y y_j = \frac{1}{2}\sum_{i,j}\left(J_{i,j}^z \sigma_i^z \sigma_j^z + J_{i,j}^x \sigma_i^x \sigma_j^x + J_{i,j}^y \sigma_i^y \sigma_j^y\right)$$

Also in 2D !!: Penning traps, 2D arrays of ion microtraps.



Errors in the quantum simulation

Internal states are prepared and measured in a different basis than the one of interacting spins. Thus, averages/correlations will deviate from the ideal quantum spin model:

$$\boxed{\begin{aligned} &\left\langle \sigma^x \right\rangle \rightarrow \left\langle e^{-S}\sigma^x e^{S}\right\rangle = \left\langle \sigma^x \right\rangle + O(\eta^2) \\ &S = \sum_{j,n} \eta_{j,n}\sigma_{j,n}^z (a_n^\dagger - a_n) \end{aligned}}$$

Error = $\left(\eta_{j,n}\right)^2 =$ Adimensional parameter that quantifies the displacement of the modes in the basis of effective spins

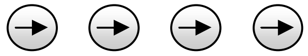
In the limit $\eta \ll 1$ errors can be neglected

How to reach effective spin ground states

Ground states can be prepared by adiabatic evolution (example quantum Ising model):

1. Choose the parameters of the effective Hamiltonian in a way that you can prepare the ground state easily.

$$H = B^x \sum_j \sigma_j^x . \qquad (J = 0)$$



paramagnetic ground state

Condition for adiabaticity:

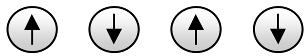
$$\frac{dJ}{dt} / J \ll \Delta \ (\text{gap})$$

$$(\Delta \approx J / N \ \text{ at q.p.t., } \mathrm{B}^{\mathrm{x}} \approx J)$$

2. Change the parameters slowly (be careful with quantum phase transitions)

3. Measure the new ground state

$$H = B^x \sum_j \sigma_j^x + \frac{1}{2} \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z . \qquad (J >> B^x)$$



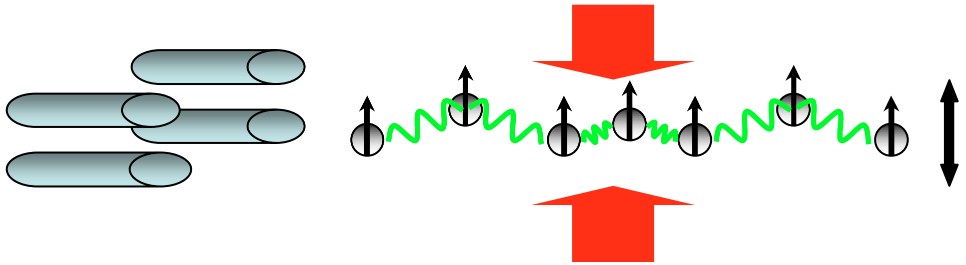
antiferromagnetic ground state

Non-adiabatic evolution !!: quantum non-equilibrium dynamics, Kibble-Zurek mechanism (creation of defects ('kinks') in the antiferromagnetic ground state).

Spin quantum phases in a linear ion trap: Ising model

Let us see how our proposal can be implemented in a concrete experimental set up: a chain of ions in a Paul trap.

Radial (transverse modes) transmit an effective spin-spin interaction (antiferromagnetic, dipolar decay)



$$H \approx \frac{1}{2}\sum_{i,j} \frac{J}{|z_i^0 - z_j^0|^3}\sigma_i^z \sigma_j^z + B^x \sum_i \sigma_i^x$$

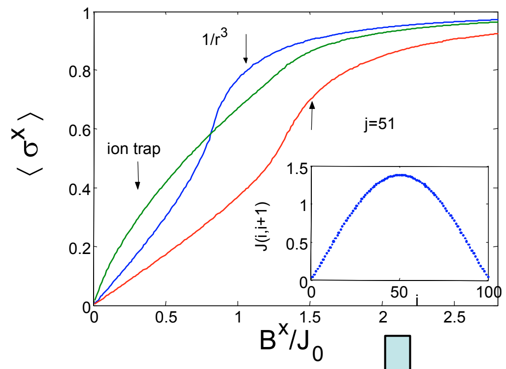
(Antiferromagnetic Quantum Ising model)

The $1/r^3$-Ising model is not exactly solvable.

Numerical results with the Density Matrix Renormalization Group (powerful numerical method for solving one dimensional quantum problems).

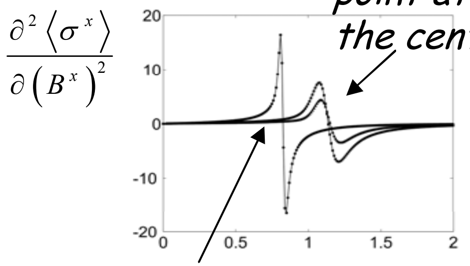
Spin quantum phases in a linear ion trap: Ising model

- Second order phase transition
- Ising-$1/r^3$ model - same universality class as nearest - neighbor model
- In a linear Paul trap, the distance between ions is not constant: coexistence of different paramagnetic phases



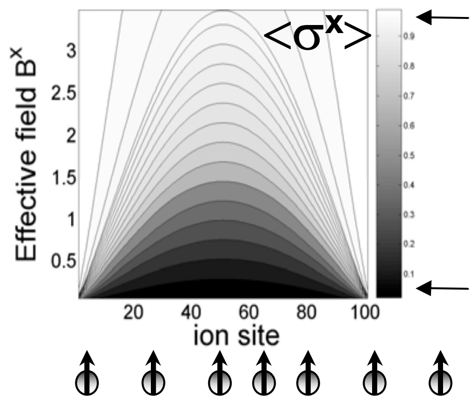
Interaction stronger at the center
CALCULATIONS WITH N=100 IONS

Shit of the critical point depending on the position



critical point at the center

critical point $1/r^3$



paramagnetic (white)

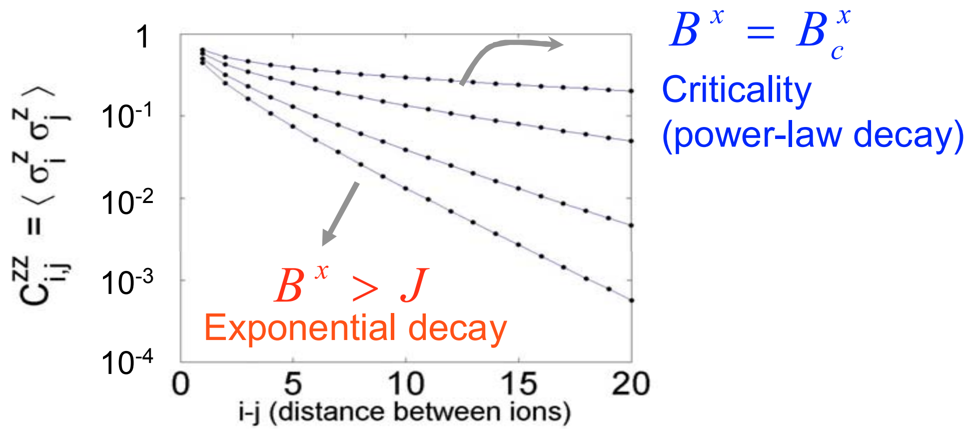
antiferro-magnetic (black)

Coexistance of quantum phases along the ion chain

Exploring quantum criticallity

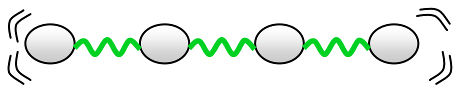
Our proposal also allows us to test experimentally a corner stone of quantum many-body theory:

- B ≠ B critical → quantum correlations decay exponentially
- B = B critical → quantum correlations follow power laws (no gap, no scale)



Bose-Hubbard model and phonon superfluidity in trapped ions

D. Porras and J.I. Cirac, Phys. Rev. Lett. (2004).

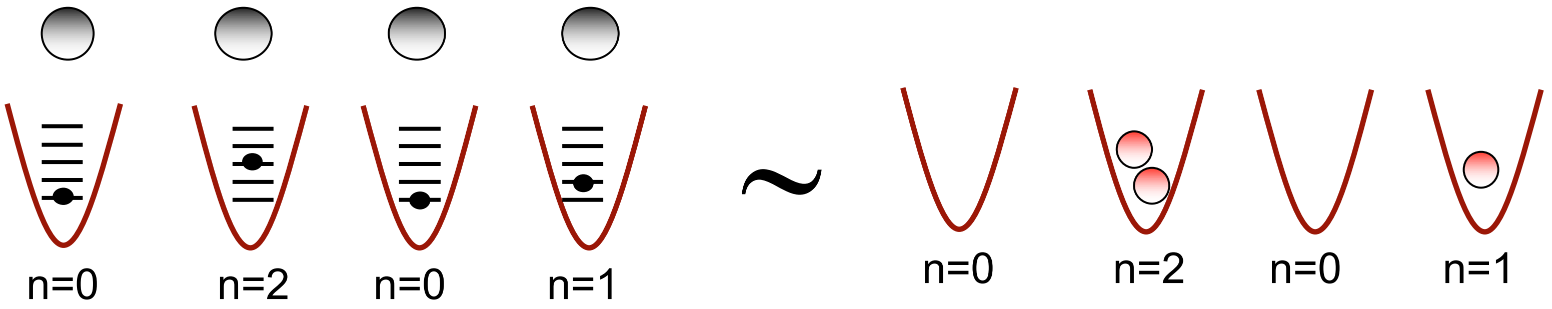


Phonons in trapped ions resemble bosons in a lattice

Trapped ions are isolated, thus phonons cannot be created/destroyed.
Let us assume that we are in the hard limit:

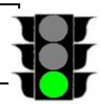
Coulomb interaction is small compared with the trapping frequencies

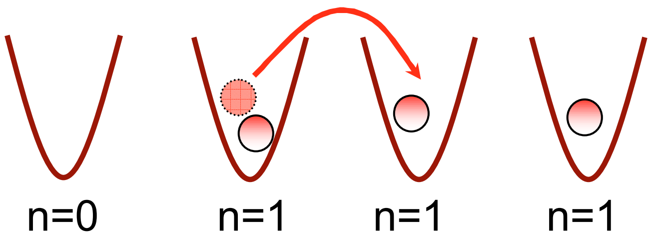
The phonon n-Fock state at each ion corresponds to having n bosons in a site of the lattice:



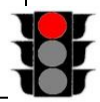
If the trapping frequency ω is much larger than any other energy scale, then the number of phonons is a conserved quantity: creation/destruction of phonons is „penalized“ by ω

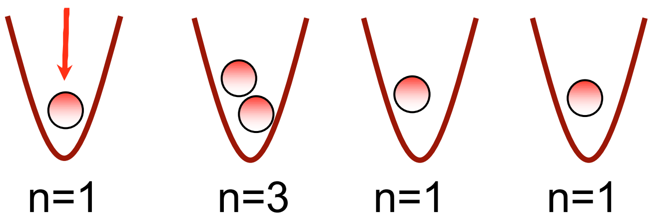
Tunneling is allowed: Energy difference $= 0$





Creation/Destruction of phonons is forbidden: Energy difference $= \omega$





Under these conditions, phonons behave in much the same way as ***bosons in an optical lattice***
(phonon number is conserved !!!)

Controlling phonon-phonon interactions

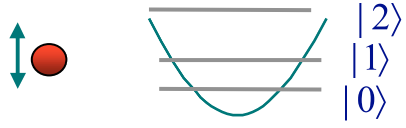
Coulomb interactions communicate different ions, thus allowing phonons to tunnel from one ion to the other. If the trapping frequency is much larger than the Coulomb couplings, then phonons satisfy a tight-binding bosonic model:

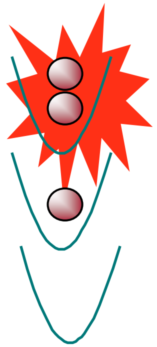
$$V \approx \sum_{i,j} \frac{e^2}{d_{i,j}^3} x_i x_j \propto \sum_{i,j} t_{i,j}(a_i + a_i^\dagger)(a_j + a_j^\dagger)$$

$$\to \cancel{a_i a_j + a_i^\dagger a_j^\dagger}$$ Fast-rotating terms (neglibible if ω>>t)

$$\to a_i^\dagger a_j + a_i a_j^\dagger$$ Resonant terms (tunneling)

Any nonlinearity is a phonon-phonon interaction:





For example, we can add nonlinear terms to the potential They can be obtained by placing the ions in a standing-wave:

$$H_{sw} \approx \frac{1}{\Delta}\left(\frac{\Omega}{2}\right)^2 \sum_j |1\rangle_j\langle 1| \cos^2(kx_j + \pi) \to \quad 1 - \frac{1}{2}(kx_j)^2 + \frac{1}{6}(kx_j)^4 + O(kx_j)^6$$

$$\to \boxed{U \ (a_i^\dagger a_i)^2}$$

How to prepare phonon ground states

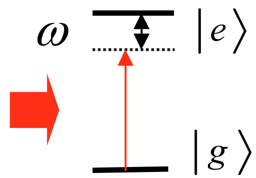
1. Choose the parameters of the effective Hamiltonian in a way that you can prepare the ground state easily. (Mott insulator)

2. Change the parameters slowly (be careful with quantum phase transitions)

3. Measure the new ground state (again with the help of an internal state)
4. allows to measure: phonon-number averages and fluctuations

(Meekhof and Wineland, 96)

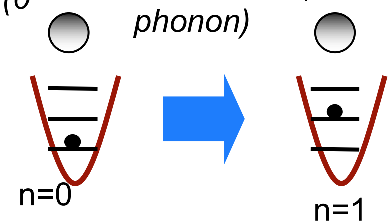
$$H \approx \sum_{i,j} t_{i,j}(a_i^\dagger a_j + a_i a_j^\dagger) + U \sum_j (a_j^\dagger a_j)^2.$$



laser is red-detuned, such that the only resonant process involves the excitaton of a phonon

cooled ion (0 phonons)

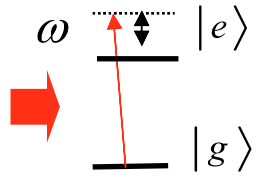
Fock state (1 phonon)



$$|\Psi_0\rangle = a_1^\dagger \dots a_N^\dagger |0\rangle$$

We prepare a ***Mott insulator Phase***

$$t \gg U$$



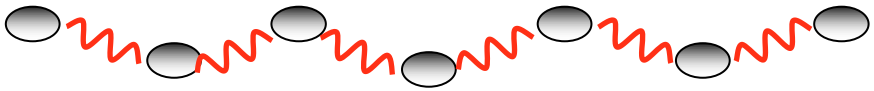
$$|\Psi_0\rangle = \left(\frac{1}{\sqrt{N}} \sum_j a_j^\dagger \right)^N |0\rangle$$

Superfluid

laser is blue detuned, such that it excites the internal state depending on the number of phonons

Superfluid-Mott insulator motional phase transition in Paul traps

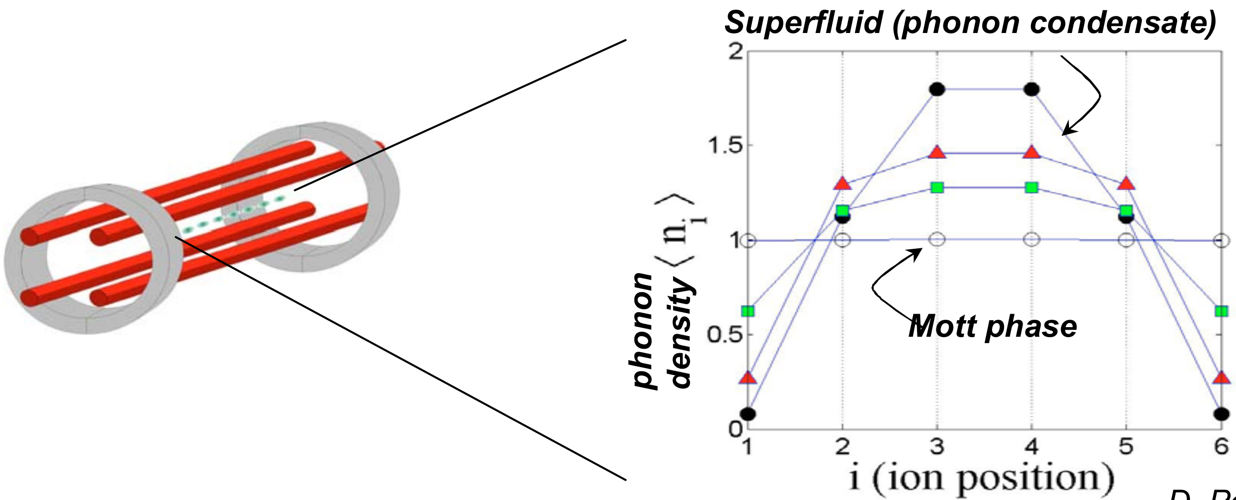
In chains of trapped ions in Paul traps, the radial modes (transverse to the axis of the trap) fulfill the condition that the trapping frequency is much larger than the Coulomb repulsion:



$\omega_{radial} \gg t$ (tunneling due to Coulomb inter.)

The Hamiltonian describing radial phonons under the action of a standing wave in a Coulomb chain in a Paul trap, is thus a Boson Hubbard Hamiltonian:

$$H \approx \hbar\omega \sum_j a_j^\dagger a_j + \sum_{i,j} t_{i,j}(a_i^\dagger a_j + a_i a_j^\dagger) + U\sum_j (a_j^\dagger a_j)^2$$



Our numerical calculations show the possibility of observing the Mott insulator-superfluid quantum phase transition.

Trapped ions offer the interesting advantage that single site measurements are possible !!

D. Porras and J.I. Cirac, Phys. Rev. Lett. (2004)

Coulomb chain with N = 6 sites

Conclusions

•Trapped ions are ideally suited to build ***quantum simulators*** and explore a variety of ***quantum phase transitions***. They have the advantage that quantum states can be prepared and measured at the single particle level.

•By using vibrational modes one can induce effective spin-spin interactions in such a way that a variety of quantum spin models are implemented (***quantum Ising, XY and Heisenberg models***).

•In ion traps quantum correlation can be studied with a degree of controllability that still is not possible in solid-state or optical lattice set-ups. In this way a number of phenomena from quantum many-body physics (***quantum correlation at criticality***) could be accessed for the first time in experiments.

•Quantum models that can be realized with trapped ions show also new remarkable features like coexistence of different quantum phases and quantum correlation induced by long-range spin-spin correlations.

•On the other hand, radial (transverse) phonons in linear ion traps are similar to bosons in optical lattices. ***Superfluidity and BEC of phonons*** could be observed for the first time in Paul traps.

•By placing the set of ions on the maximum (or minimum) of a standing-wave, on can induced effective phonon-phonon interaction and realize a ***Phonon Hubbard Model***.