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Noise and Instabilities in Quantum Mechanics

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Dynamics of entanglement in the Heisenberg model

Simone MONTANGERO
Scuola Normale Superiore
Piazza dei Cavalieri 7
56126 Pisa
ITALY

Dynamics of Entanglement in the Heisenberg Model

Simone Montangero,
Gabriele De Chiara, Davide Rossini, Matteo Rizzi, Rosario Fazio

Scuola Normale Superiore
Pisa

NATIONAL ENTERPRISE FOR NANOSCIENCE AND NANOTECHNOLOGY

Outline

- Ground state entanglement in Spin Chains

  I.Latorre, E.Rico, G.Vidal, Quant. Inf. and Comp. 4, (2004)

- Dynamics of entanglement in the Ising model

  P. Calabrese, J. Cardy, JSTAT 0504 (2005)

- Numerical method: DMRG, t-DMRG

- Entanglement in critical Heisenberg model

- Entanglement dynamics in the Heisenberg model

  WORK IN PROGRESS

Entropy of Entanglement

Ground State:

$$\rho_L = tr_{N-L}(|\Psi_{GS}\rangle\langle\Psi_{GS}|)$$

$$|\Psi_{GS}\rangle$$

$$S(\rho_L) \equiv -tr(\rho_L log \rho_L)$$

L sites B

A N $\rightarrow \infty$ sites

Heisenberg Model

$$H = \sum_i J_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta\, \sigma_i^z \sigma_{i+1}^z\right)$$

$J_i = J$ constant couplings

Δ anisotropy

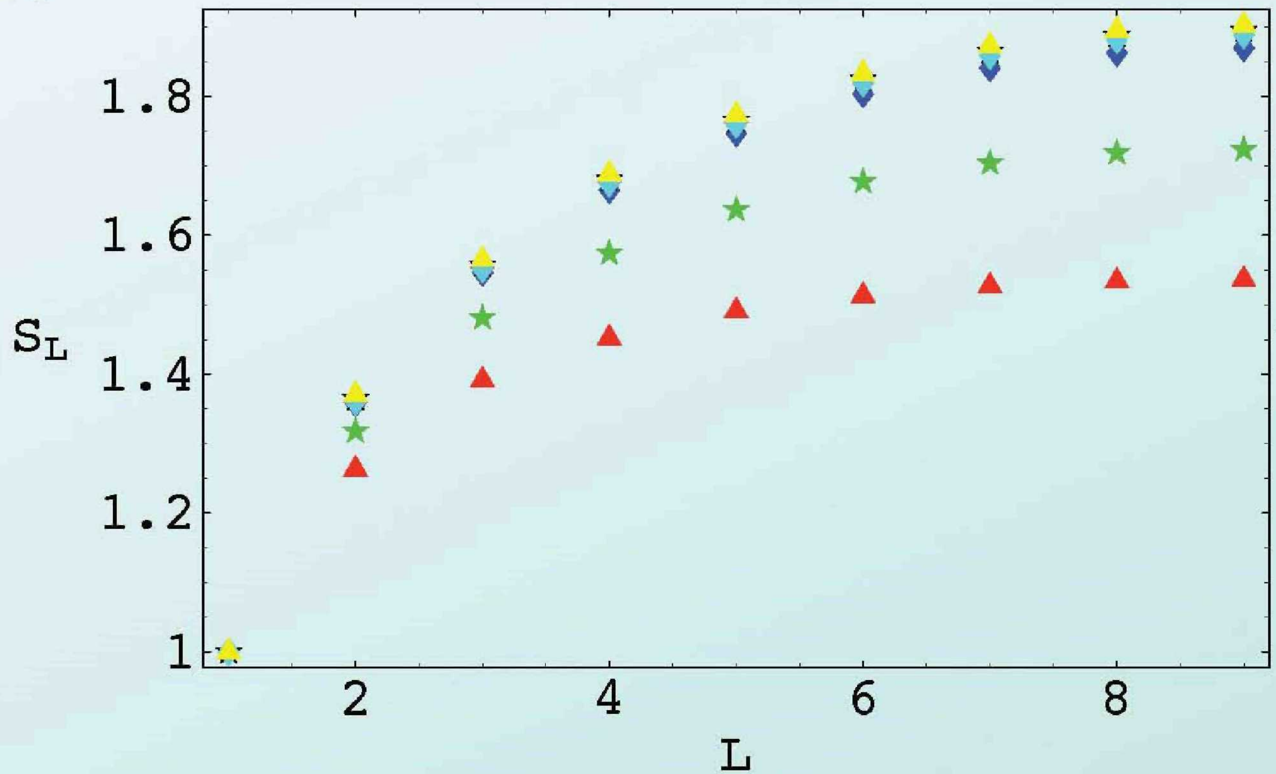
$\Delta \in [-1:1]$ Critical

$J_i \in [0, J]$ Random HM

Entropy Scaling I

N=18

$|\Delta| < 1$



$\Delta = 2$

$\Delta = 2.5$

Bethe Ansatz, integration of non-linear equations

I.Latorre, E.Rico, G.Vidal, Quant. Inf. and Comp. 4, (2004)

Ising Model

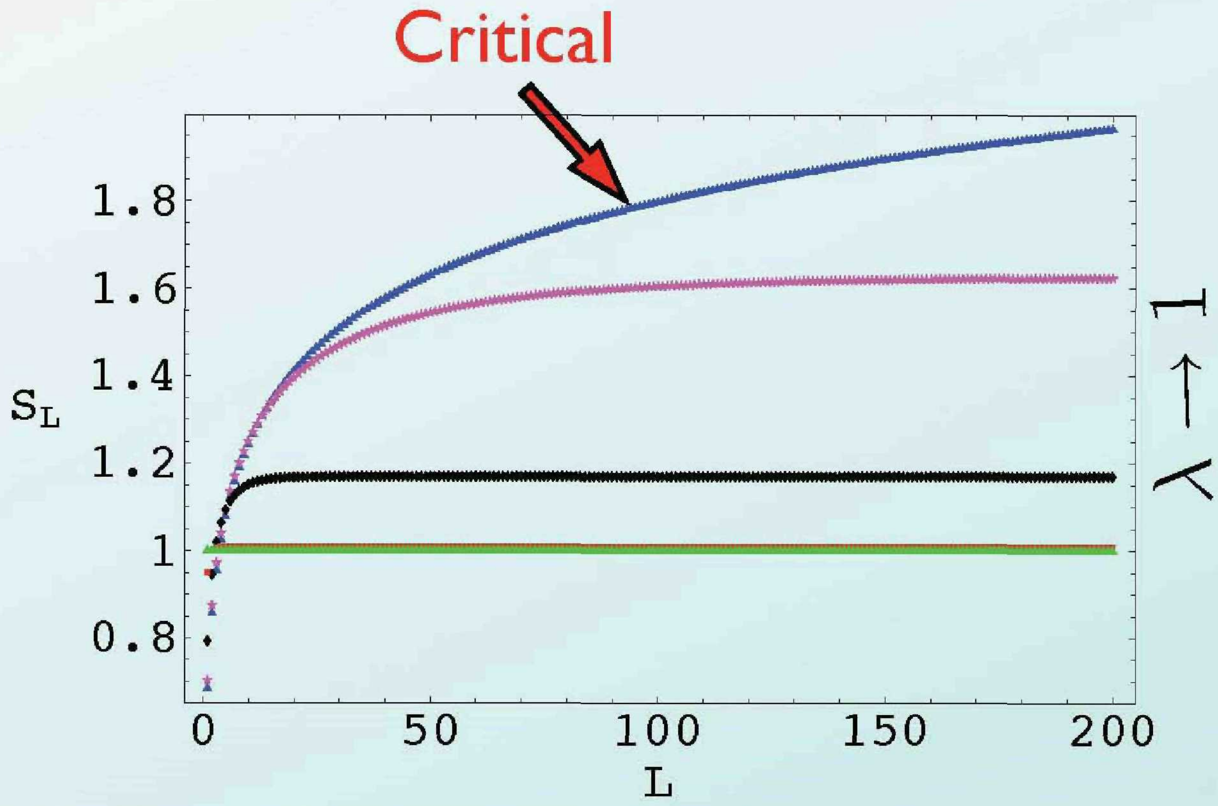
$$H = -\sum_i \left(\sigma_i^x \sigma_{i+1}^x + \lambda \sigma_i^z\right)$$

For $\lambda = 1$ the system is critical

Correlations diverge in the system ground state

System can be solved via
Jordan-Wigner + Fourier + Bogoliubov transformation

Entropy Scaling II



$$S_L = \frac{c}{3} log_2 L + a$$

central charge

c=1/2

I.Latorre, E.Rico, G.Vidal, Quant. Inf. and Comp. 4, (2004)

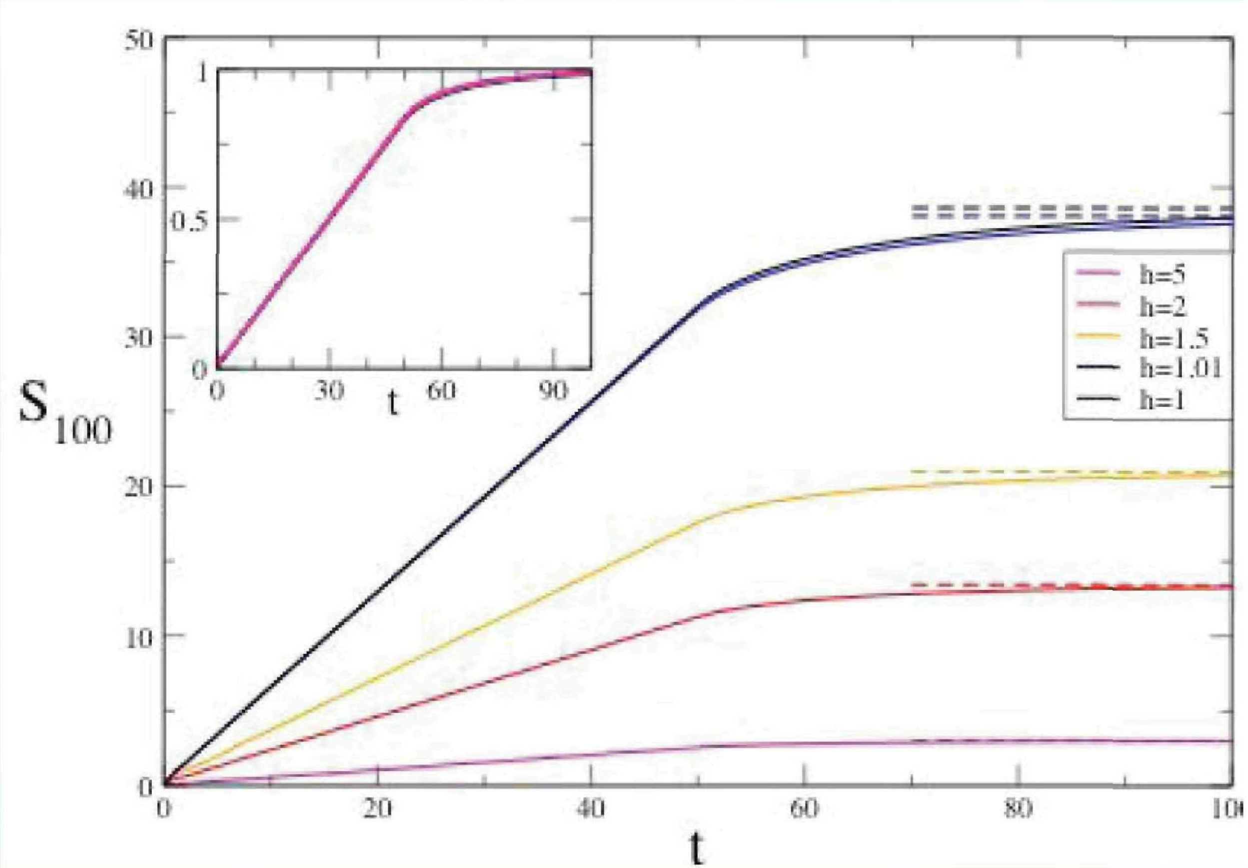
Evolution of Entanglement I

Instantaneous Quench

$$\lambda_0 = \infty \rightarrow \lambda_1$$

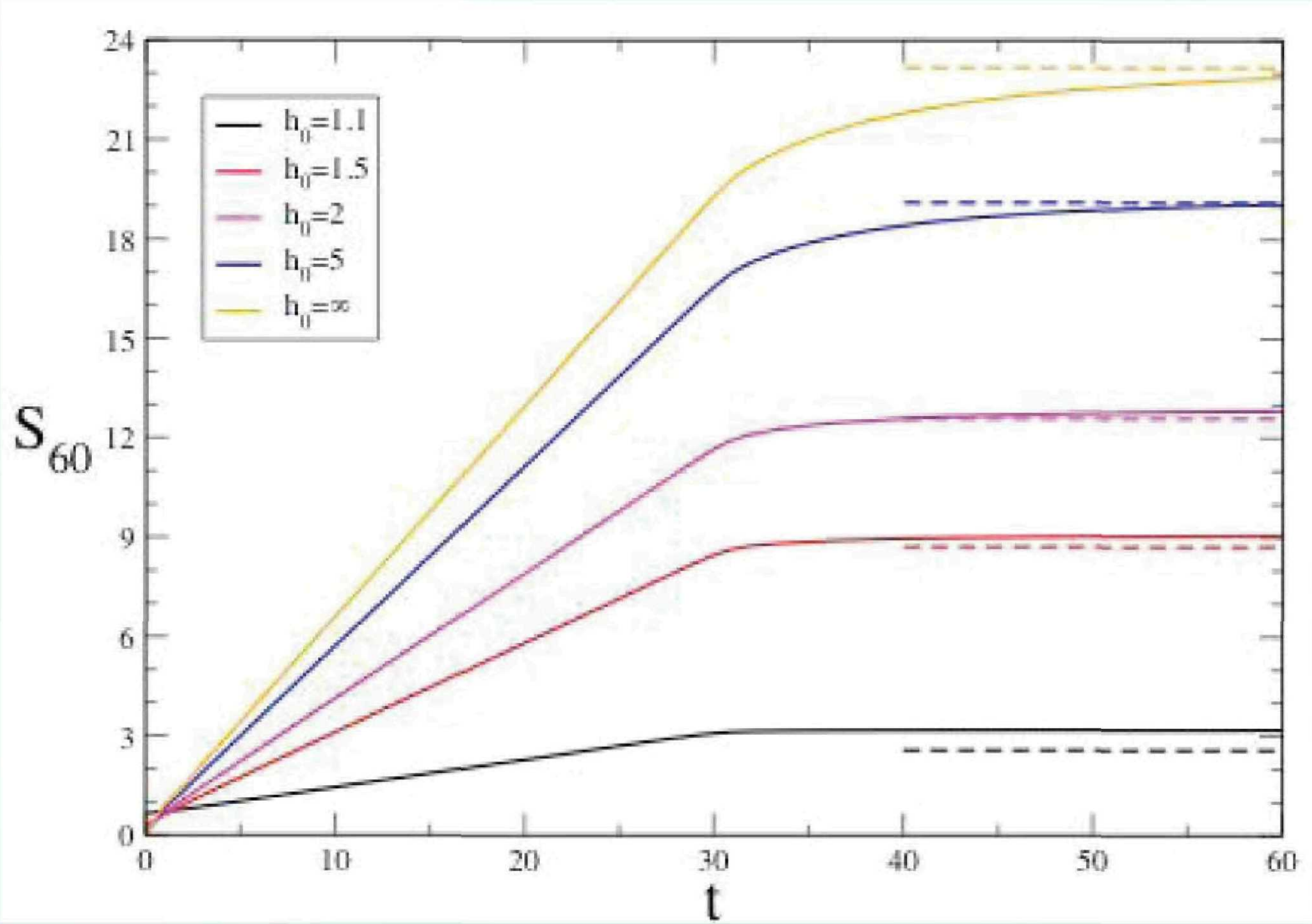
t=0 GS: separable state

$$vt^* = L/2$$



P. Calabrese, J. Cardy, JSTAT 0504 (2005)

Evolution of Entanglement II



Instantaneous Quench $\quad \lambda_0 \to \lambda_1 = 1$

P. Calabrese, J. Cardy, JSTAT 0504 (2005)

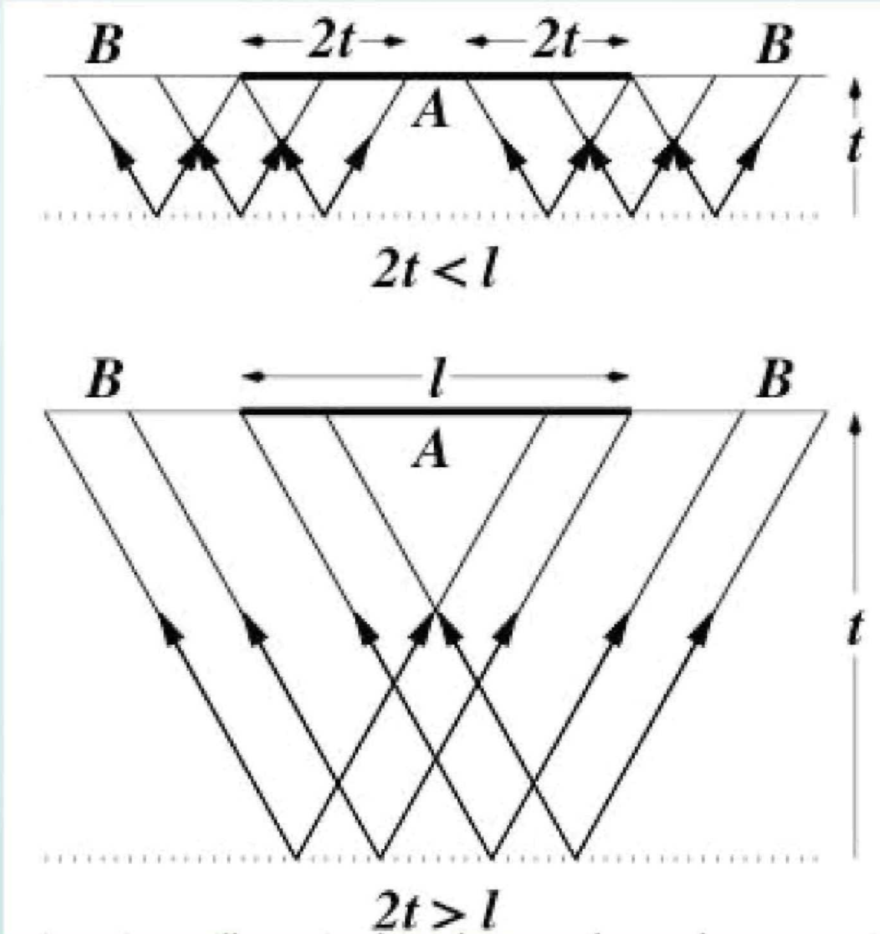
Physical Interpretation

Simple Model

$$\lambda_0 \rightarrow \lambda_1$$

$$S_L \propto t \qquad t < t^*$$

$$S_L \propto L \qquad t > t^*$$



P. Calabrese, J. Cardy, JSTAT 0504 (2005)

Half Time Summary

Static: critical scaling

$$S_L \sim \frac{c}{3}\, log_2 L$$

Dynamic:

Entropy increase is proportional to quench

Entropy saturates at t^*

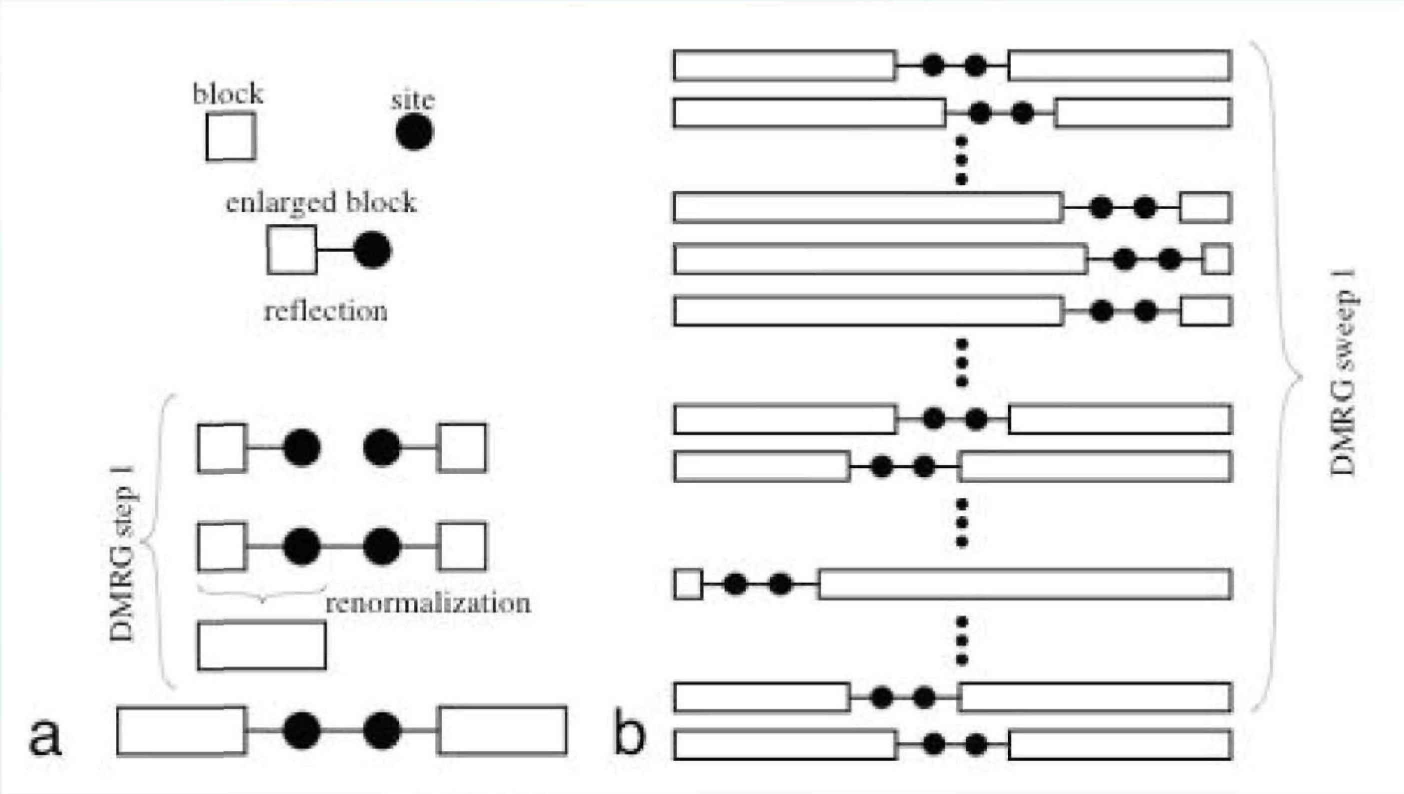
t^* depends on L and velocity

Numerical Simulation

- DMRG, White PRA (1992)
- t-DMRG, White, Feigun, PRL (2004)
- Approximate method to study many-body quantum system (ground state properties, time evolution)
- Open boundary conditions
- Finite size scaling

DMRG scheme

$H_{SB} = H_E + H_{E'} + H_{int}$



$$|\psi_G\rangle = \sum \psi_{\alpha a b \beta} \Rightarrow \rho_L = Tr_R |\Psi_G\rangle\langle\Psi_G|$$

m states

$$H_B(2) = O_{1\to 2} H_E O^{\dagger}_{2\to 1}$$

t-DMRG scheme

Time evolution operator Trotter expansion

$$H = \sum_{even} F_{i,i+1} + \sum_{odd} G_{i,i+1}$$

$$exp(-\imath Ht) = \left(e^{-\imath F dt/2} e^{-\imath G dt} e^{-\imath F dt/2}\right)$$

$$= \prod exp(-\imath F_{i,i+1} dt/2) \prod exp(-\imath G_{i,i+1} dt) \prod exp(-\imath F_{i,i+1} dt/2)$$

F, G even/odd Hamiltonan operator

$$\tilde{\Psi} = O_{\ell \to \ell+1} O^{\dagger}_{N-\ell-3 \to N-\ell-2} \Psi$$

DMRG Parameters

- N sistem size
- m size of truncated basis
- P discarded
- dt Trotter approx (second order).

Entropy and CFT

Entropy of a spin block in a critical infinite chain:

$$S_L \sim \frac{c}{3} log_2 L$$

Entropy of a block L in a critical chain of size N

$$S_L^B = \frac{c}{6} log_2 \left[\frac{L}{\pi} sin \left(\frac{\pi L}{N} \right) \right] + a$$

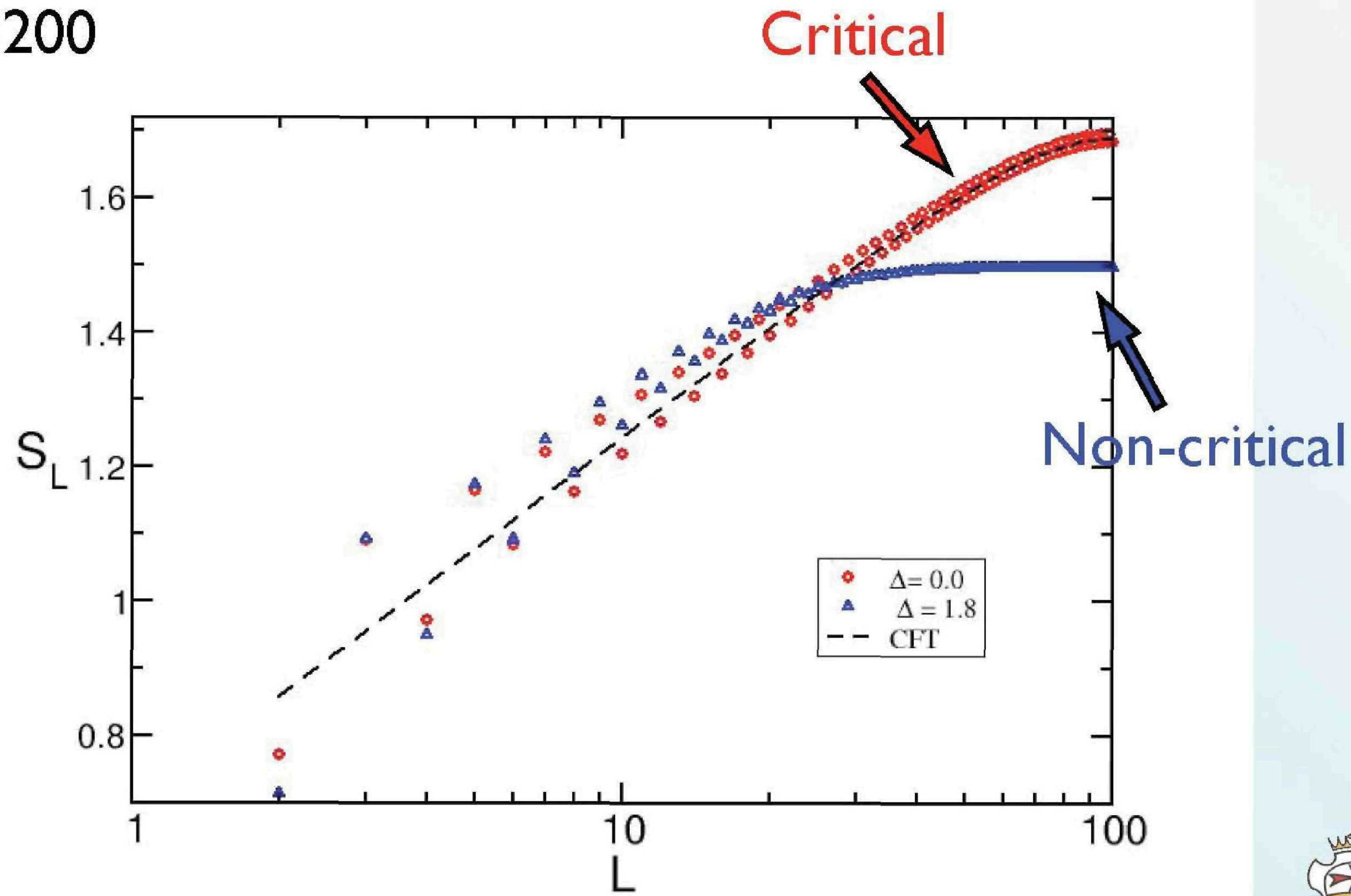
N sites

L sites B

P. Calabrese, J. Cardy, JSTAT 1 (2004)

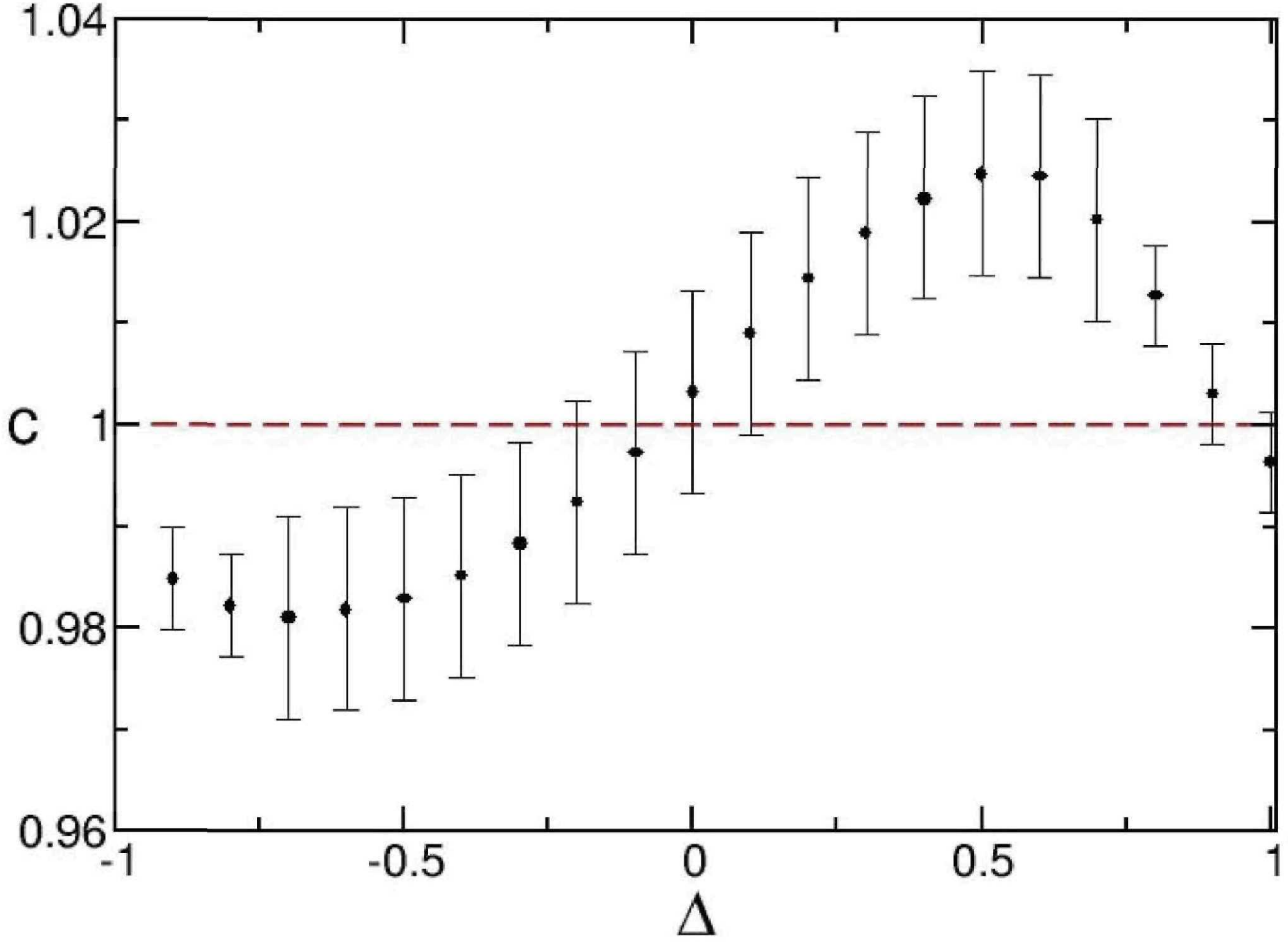
Numerical Results

N=200



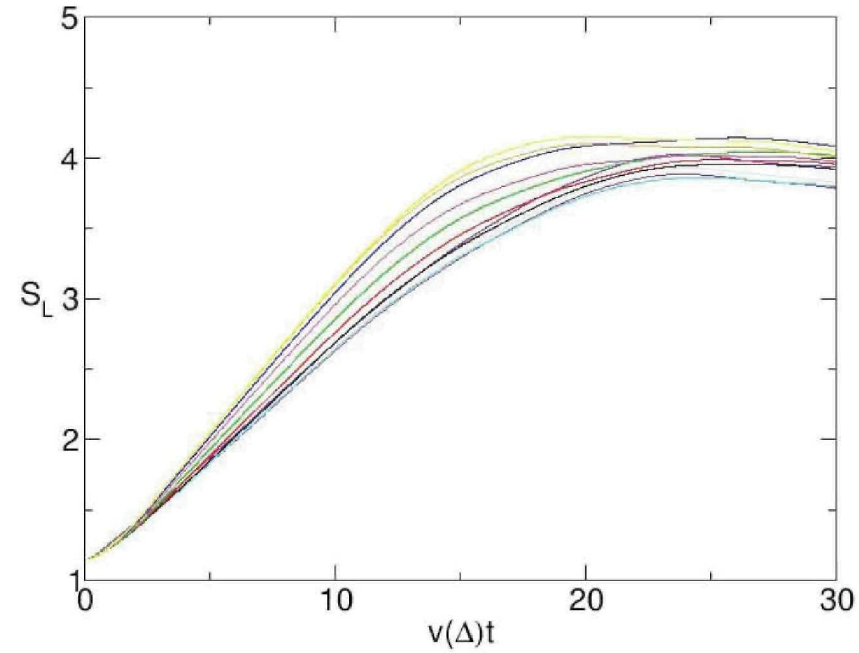
Central Charge

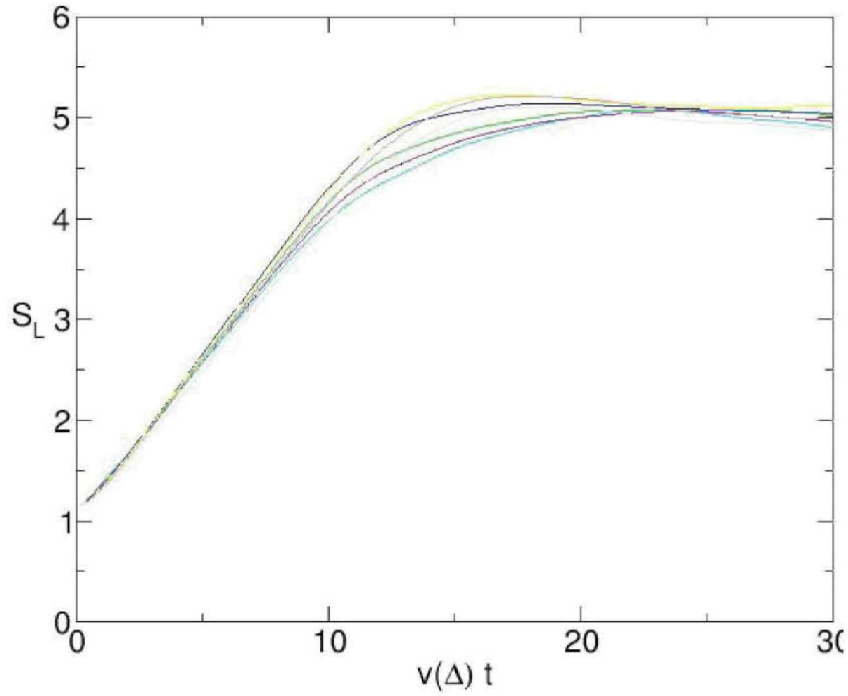
N=1000



Finite Fixed Quench

N=50 L=20 m=50 dt=10^-3





$$\Delta_0 - \Delta_1 = 1$$

$$\Delta_0 - \Delta_1 = 1.5$$

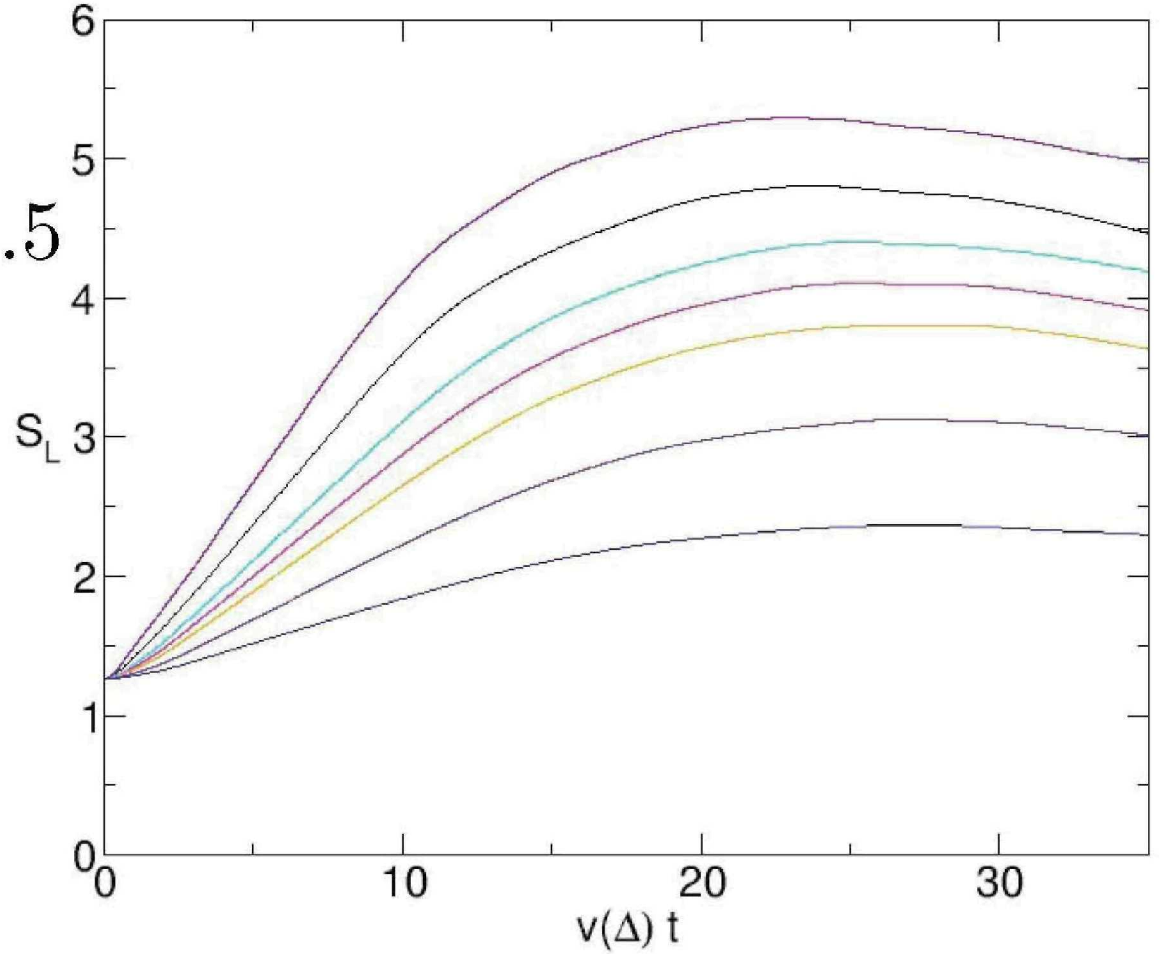
$$v(\Delta) = 2J\pi\frac{sin\theta}{\theta} \qquad cos\theta = \Delta_1$$

Eggert et.al. PRL 73 (1994)

Finite Quench

N=50 L=20 m=50 dt=10^-3

$$\Delta_0 = 1.5$$

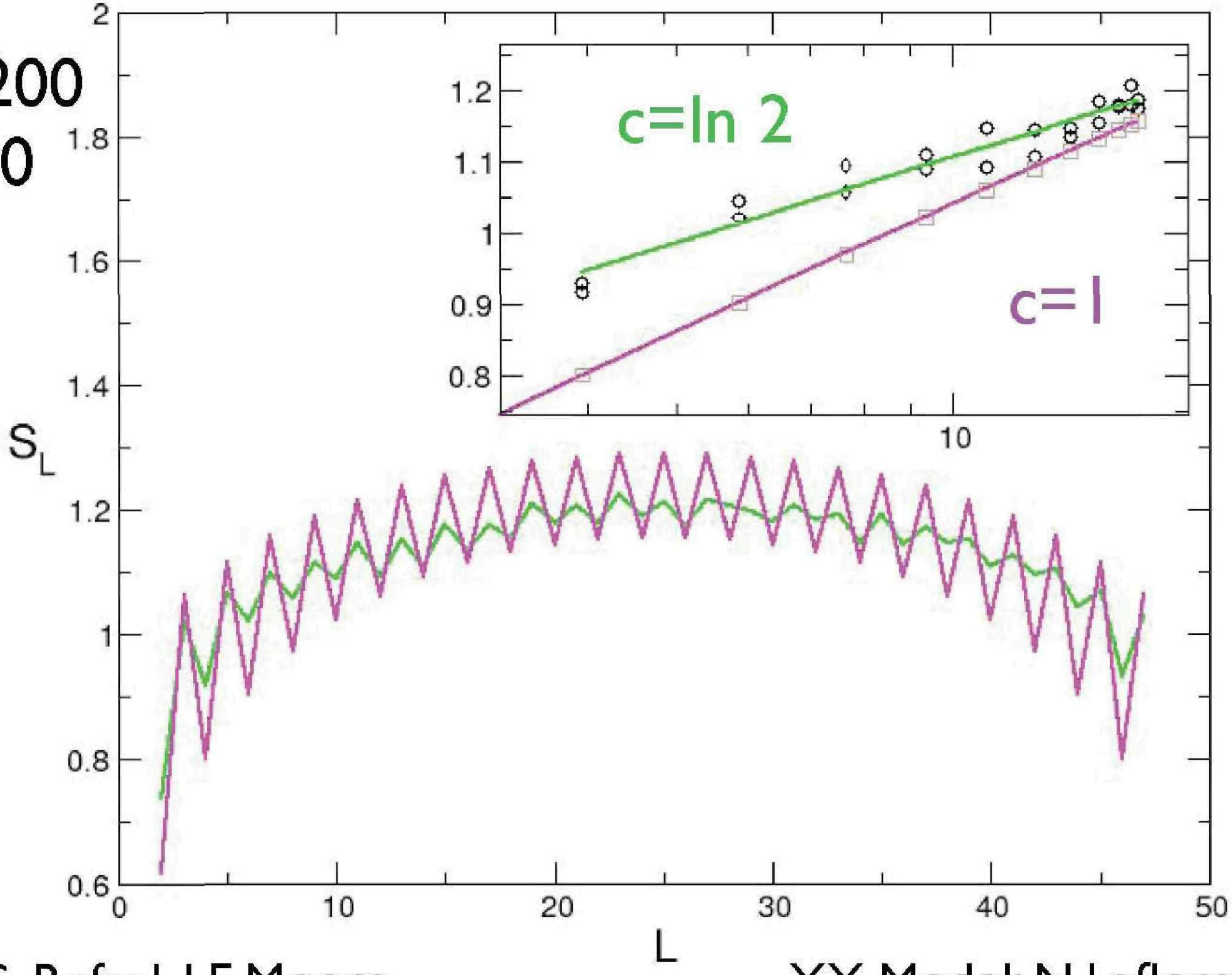


$\Delta_1 \to 0$

$$vt^* = L$$

Random Heisenberg Model

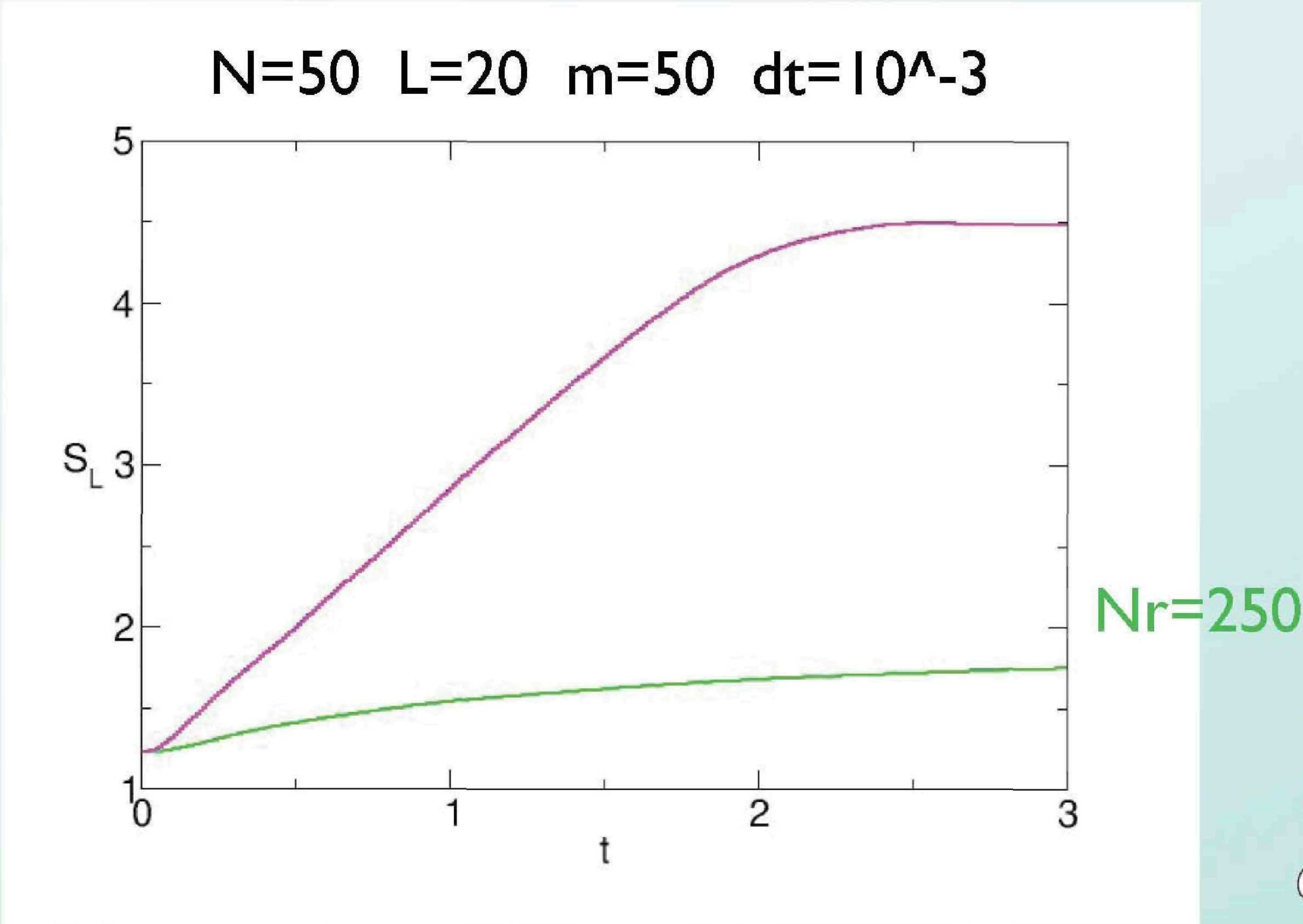
Nr=1200
m=50



CFT: G. Refael, J.E.Moore, PRL 93, 260602 (2004)

XX Model: N.Laflorencie, cond-mat/0504446

Time Evolution in RHM



Conclusions and Outlook

- Static scaling in HM confirmed.
- Central charge can be fitted from numerical simulations.
- Time evolution scheme holds in different models.
- Central charge in random Heisenberg model
- Time evolution in RH under investigation