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International Centre for Theoretical Physics

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Workshop on
Noise and Instabilities in Quantum Mechanics

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Nonlinear interaction for bose-Einstein condensates in a 1D optical lattice

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Nonlinear intereaction for Bose-Einstein condensates in a 1D optical lattice

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BEC's in a potential

Feshbach resonance

- Gross-Pitaevskii equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\psi + V(\vec{r})\psi + \left[\frac{4\pi n\hbar^2 a_s}{M}|\psi|^2\right]\psi$$

=n=density

Rescaled Gross-Pitaevskii equation

- Gross-Pitaevskii equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2M}\frac{\partial^2 \psi}{\partial x^2} + U_0 \sin^2(k_L x)\psi + \frac{4\pi n \hbar^2 a_s}{M}|\psi|^2 \psi$$

- rescaled:

$$i\frac{\partial \psi}{\partial \tilde{t}} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial \tilde{x}^2} + V_0 \sin^2(\tilde{x})\psi + C|\psi|^2 \psi$$

- where

$$V_0 = \frac{U_0}{E_{rec}}$$

$$E_{rec} = \frac{\hbar^2 k_L^2}{2M}$$

$$k_L = \frac{\pi}{d_L}$$

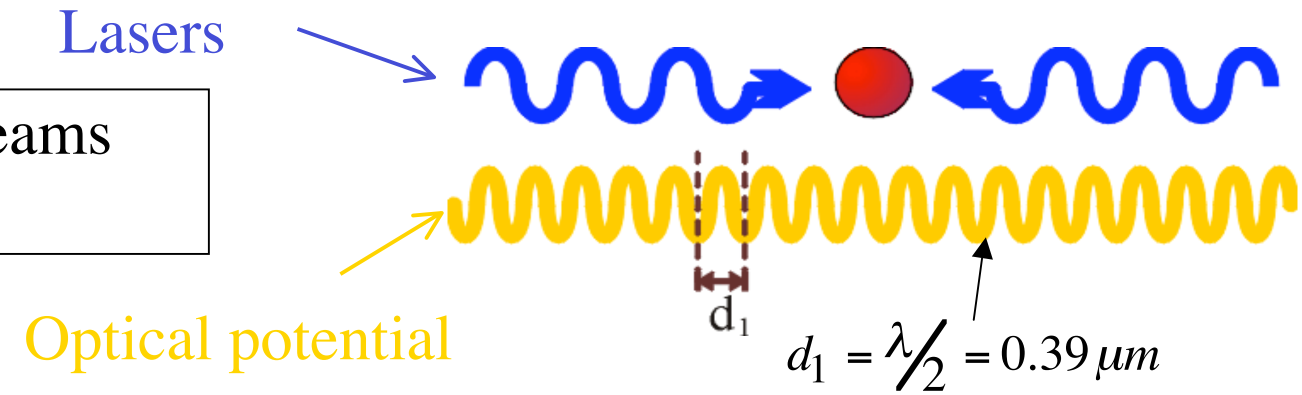
$$C = \frac{\pi n a_s}{k_L^2} = \frac{n a_s d_L^2}{4\pi}$$

Outline

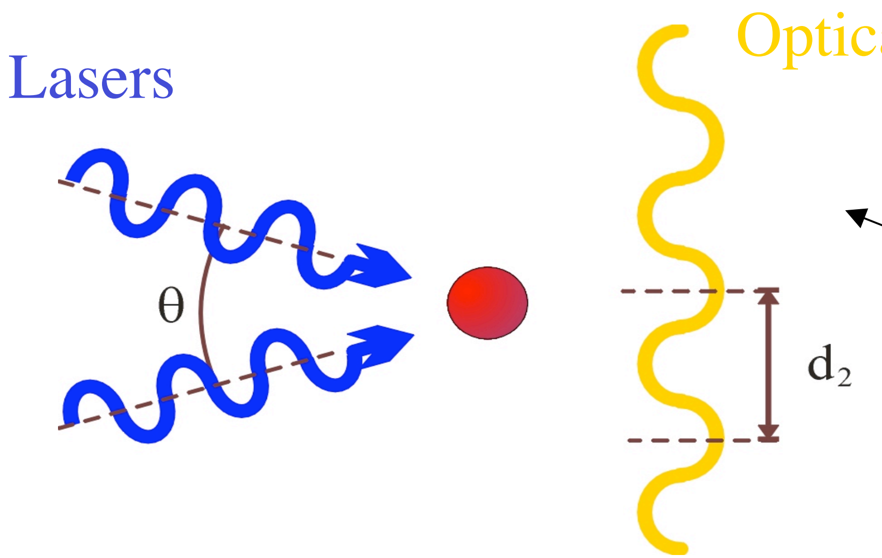
- Dynamic evolution of a condensate inside a 1D lattice
- Asymmetric Landau-Zener tunneling
- Energy bands modified by the nonlinear atomic interaction
- Resonant tunneling
- Condensate instabilities
- δ-kicked rotor
- Conclusions

1-D Light Wave Gratings for Matter Waves

- two interfering laser beams form periodic potential



$$d_1 = \lambda/2 = 0.39\,\mu m$$



$$d_2 = \frac{\lambda}{2\sin(\vartheta/2)} = 1.56\,\mu m$$

Angle tuned lattice: large spacing and large nonlinearity.

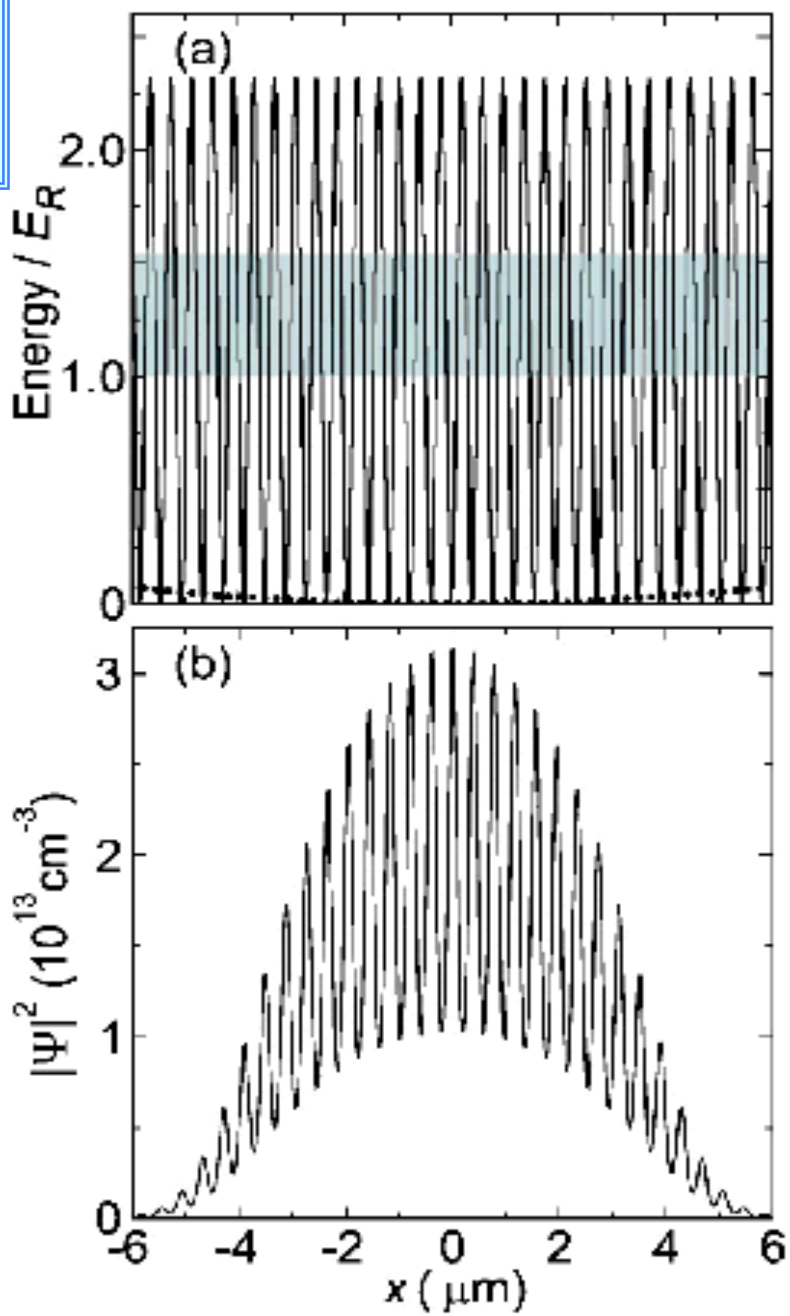
M. Cristiani, et al, Phys. Rev. A, 65, 063612 (2002)

Optical lattices

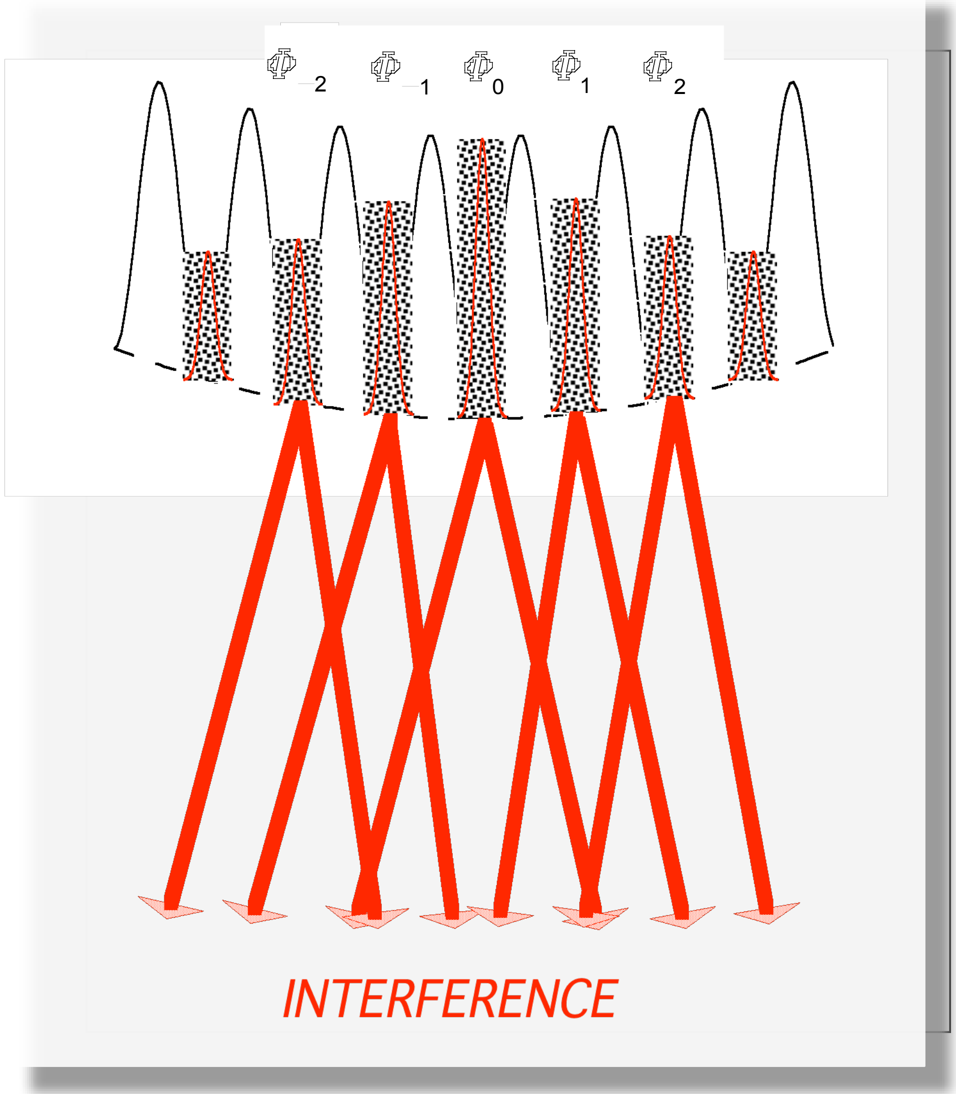
Potential

Energy bands

Wavefunction

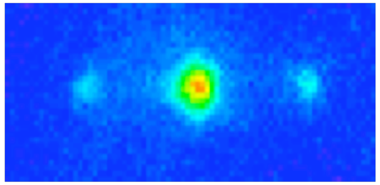


Observable

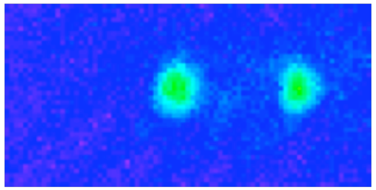


-Interference of BEC released from 1D lattice

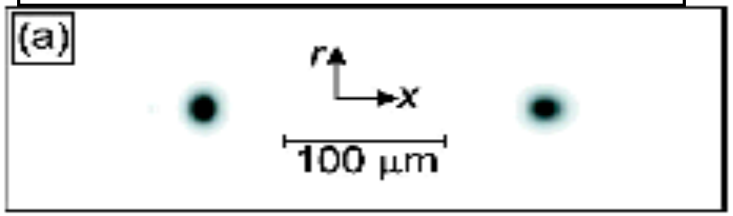
-obtain **information about phase coherence** between lattice wells



-accelerate lattice to get **two-peaked pattern**

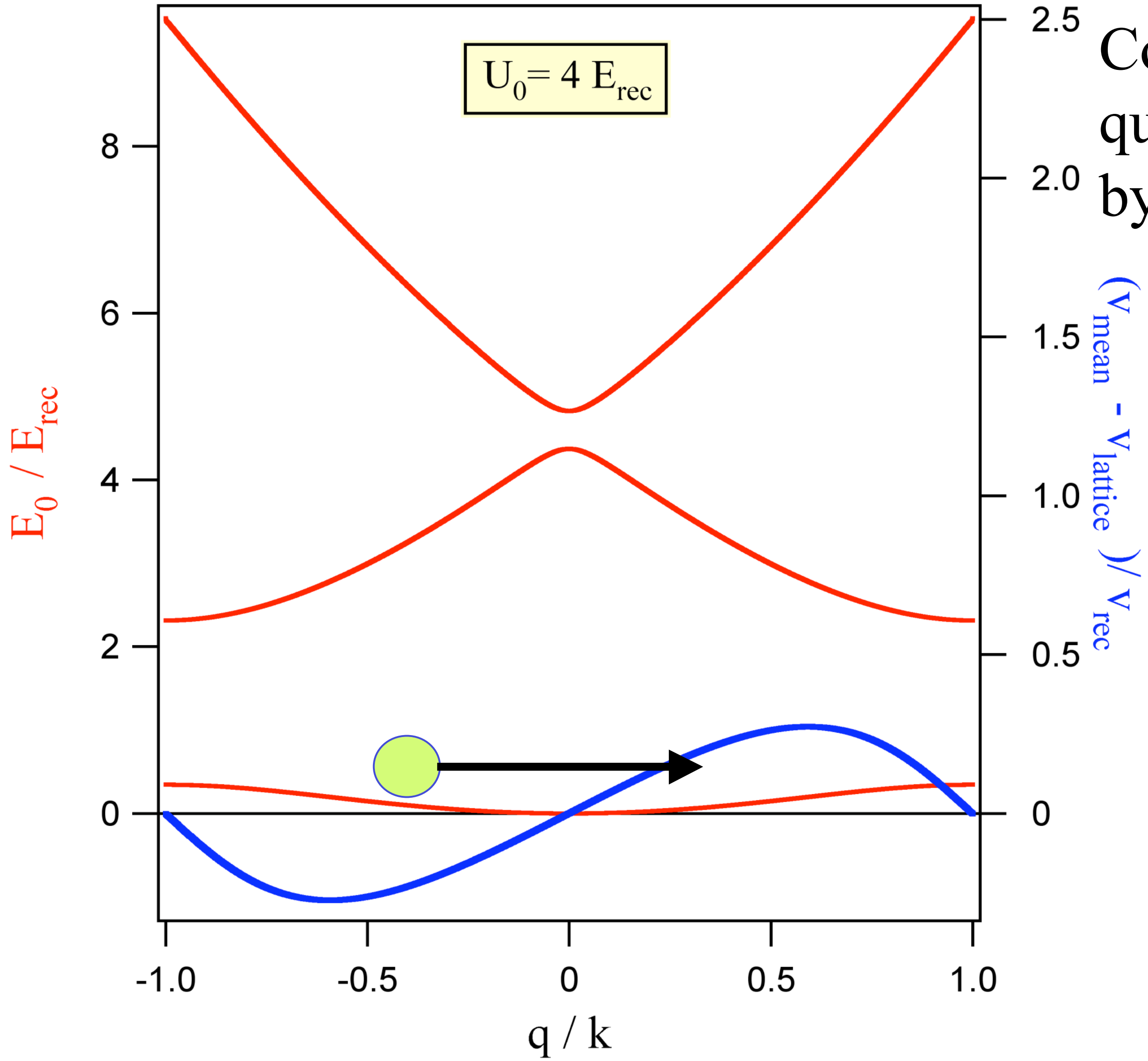


Theoretical predictions*



**R. Scott et al, Phys. Rev. A 69, 033605 (2004)*

Dynamics in a periodic potential



Continuous scan of quasi-momentum by moving the optical lattice

At band edge:

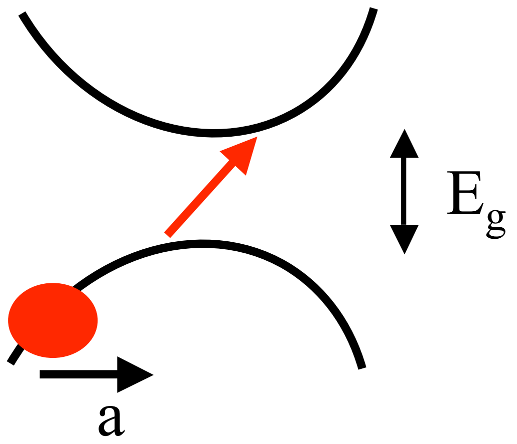
adiabatic

Bloch oscillation

non-adiabatic

tunneling

Non-adiabatic regime and Landau-Zener tunneling at the band edge



For a large acceleration a, the system undergoes a tunneling into the first excited band with a probability P_{LZ} given by the Landau-Zener formula depending on the gap E_g

The probability P_{LZ} of Landau-Zener tunneling into the first excited band is

$$P_{LZ} = \exp^{-\frac{a_c}{a}}$$

$$a_c = \frac{\pi E_g^2}{8\hbar^2 M\mathrm{v}}$$

Linear Landau-Zener tunneling from ground to excited

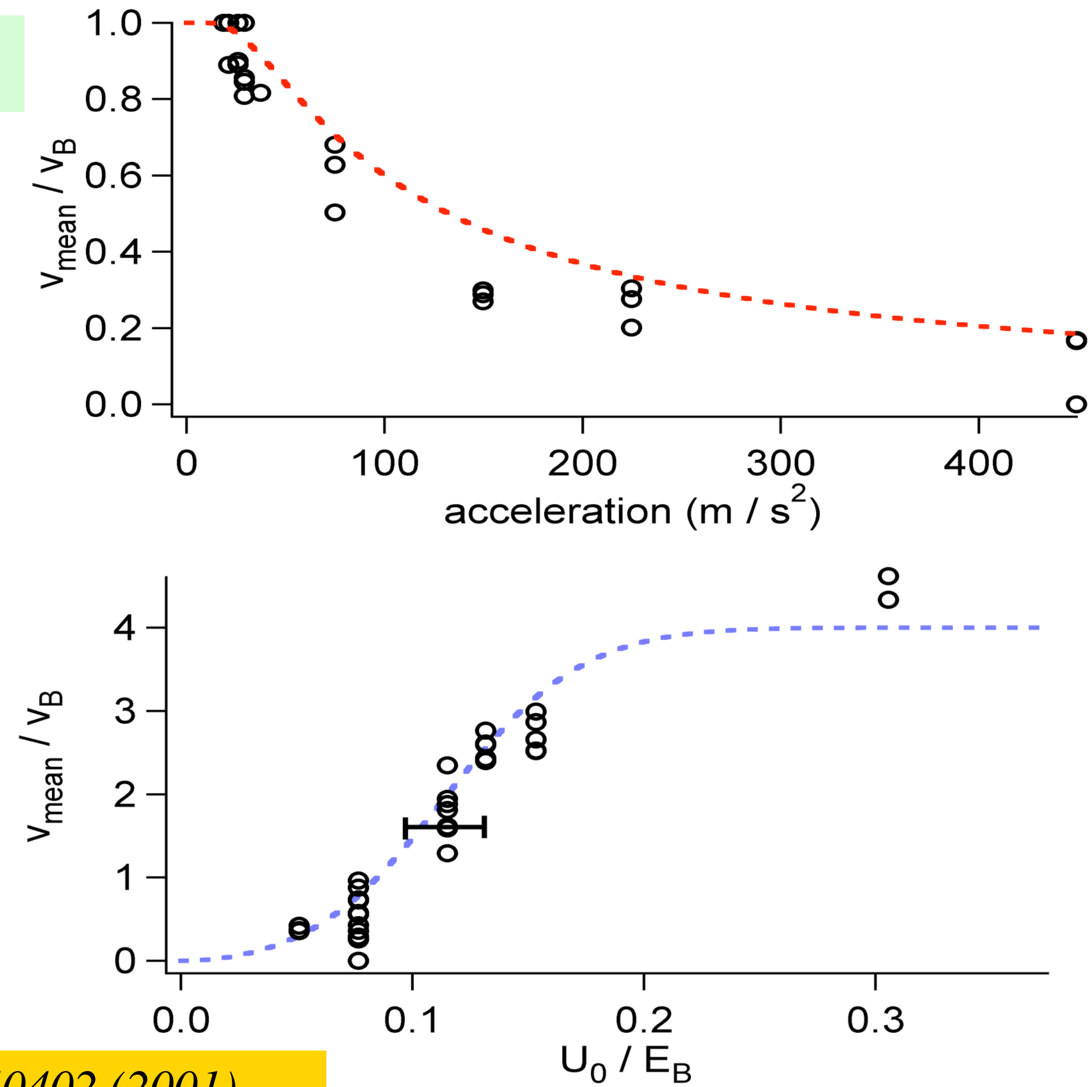
Landau-Zener formula:

$$P_{LZ} = \exp(-a_c/a)$$

$$a_c = \frac{\pi U_0^2}{16\hbar^2 k}$$

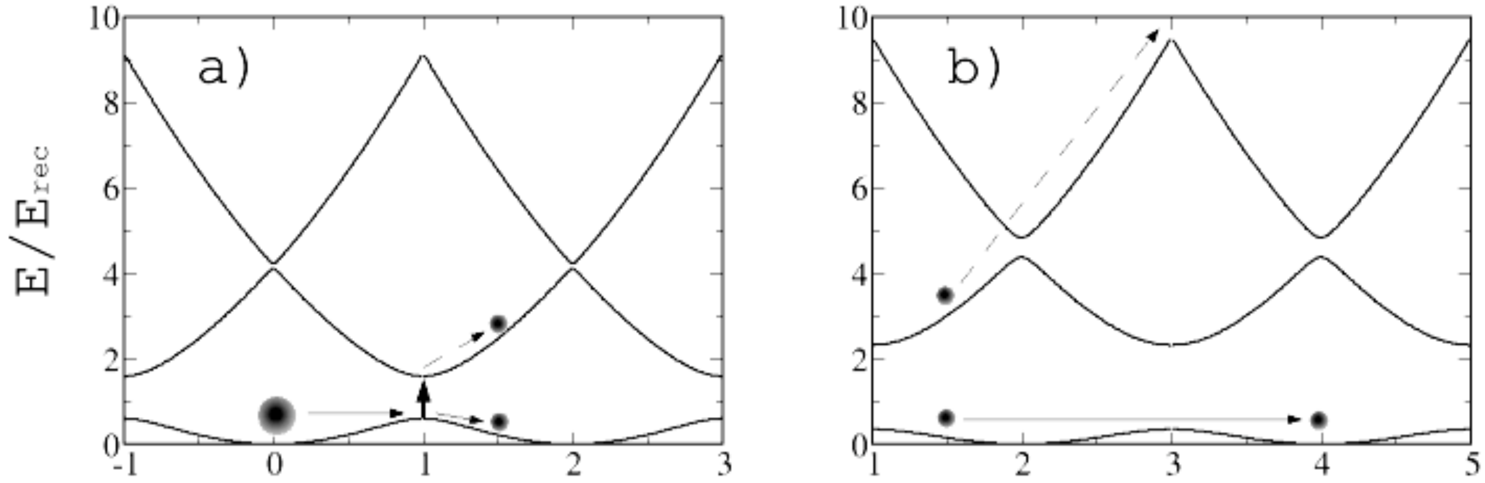
For instance for
$\mathrm{v}_{\mathrm{lattice}}/\mathrm{v}_B = 1$
we get

$$\mathrm{v}_{\mathrm{mean}} = (1 - P_{LZ})\mathrm{v}_B$$

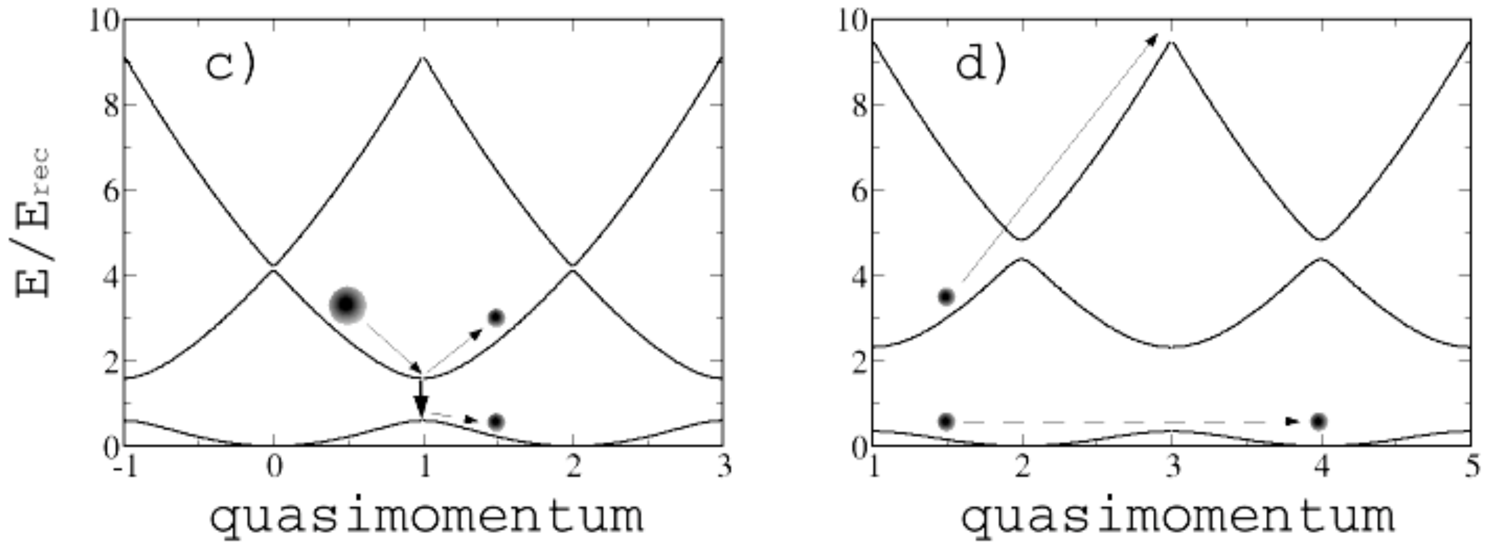


O. Morsch et al, Phys. Rev. Lett. 87, 140402 (2001)

Two-way Landau-Zener tunneling in the experiment



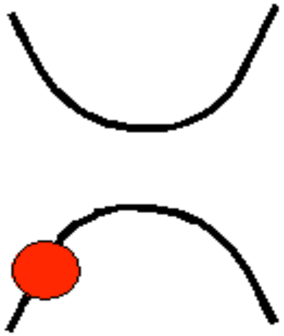
tunneling
from ground band
to excited band

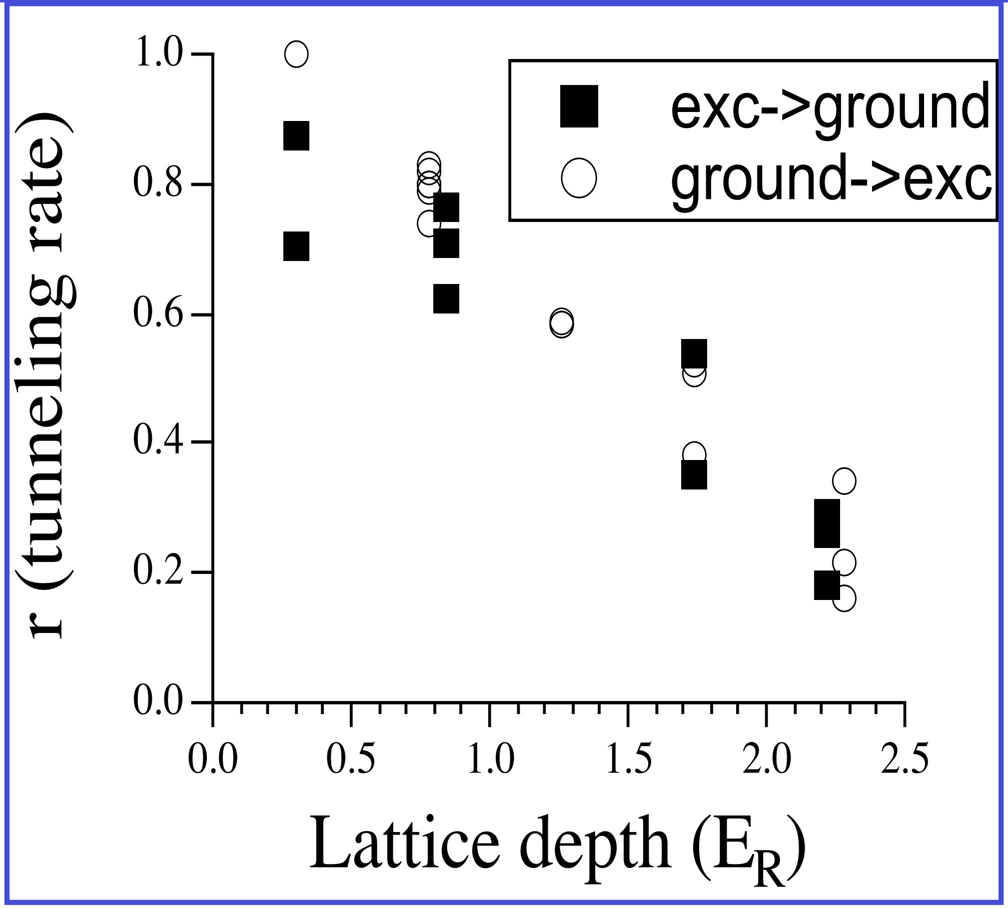


tunneling
from excited
to ground

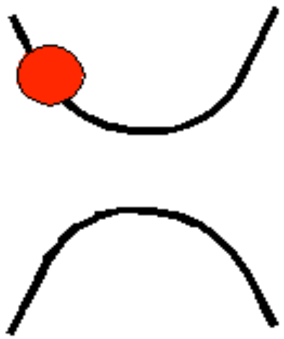
Landau-Zener tunneling: symmetric for linear system

from below



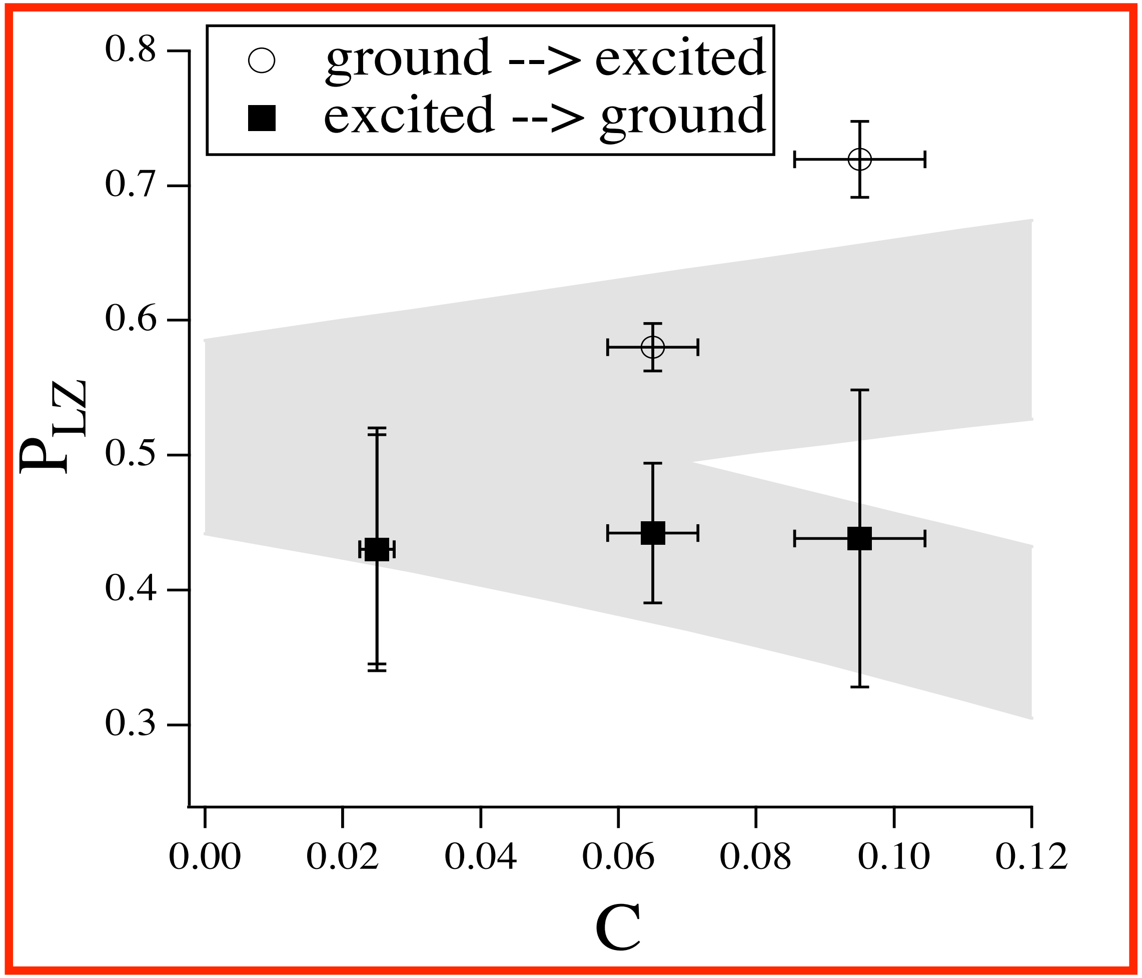


from above



- in the linear case (negligible nonlinearity due to mean-field interaction), the tunneling rates from below and above are identical!

Asymmetry in the Landau-Zener tunneling- I



M. Jona-Lasinio et al, Phys. Rev. Lett. 91, 230406 (2003)

Asymmetry in LZ tunneling-II

$$i\frac{\partial\psi}{\partial\tilde{t}} = -\frac{1}{2}\left(-i\frac{\partial}{\partial\tilde{x}} - \alpha t\right)^2\psi + V_0\sin^2(\tilde{x})\psi + \mathrm{C}|\psi|^2\psi$$

with $\qquad \alpha = \dfrac{Ma}{16E_{rec}k}$

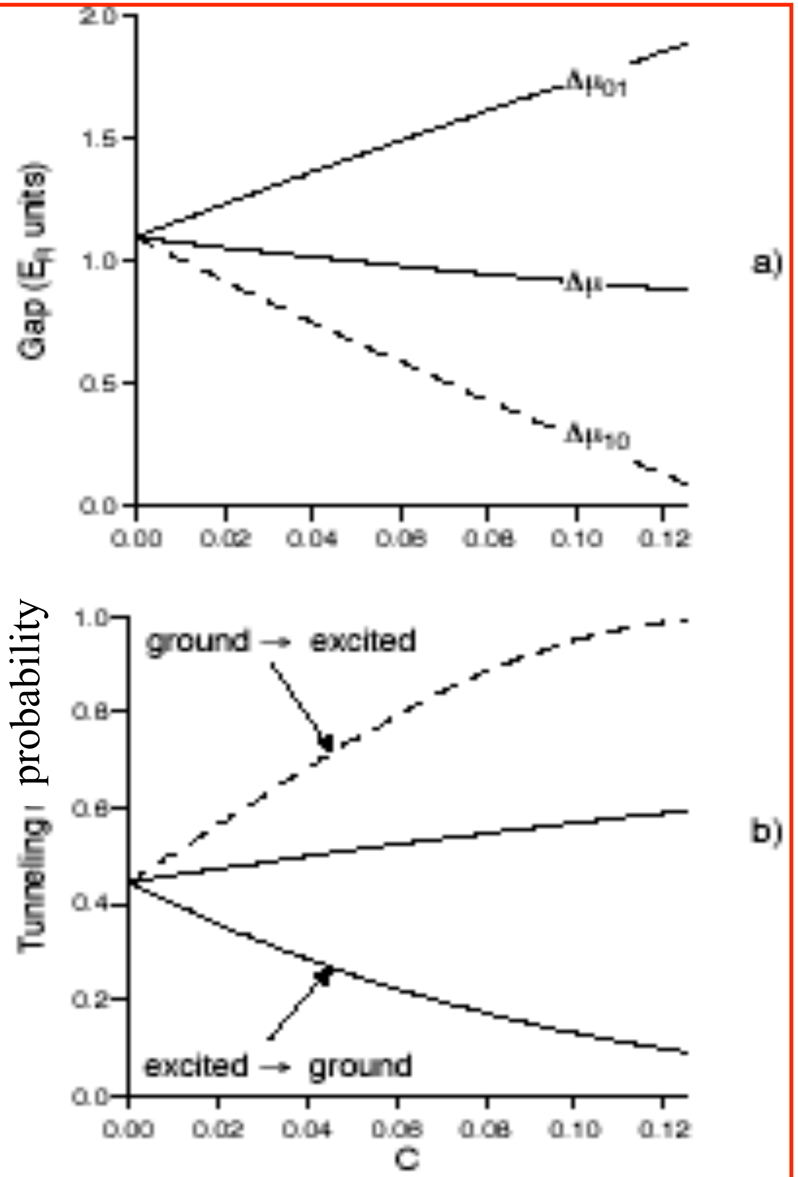
At the band edge write the wavefunction as:

$$|\psi(x,t)> = a(t)e^{iqx}\,|g> + b(t)e^{i(q-1)x}\,|e>$$

The following equation is derived:

$$i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = \left[\frac{\alpha t}{2}\sigma_3 + \frac{V_0}{2}\sigma_1\right]\begin{pmatrix} a \\ b \end{pmatrix} - \frac{\mathrm{C}}{2}\begin{pmatrix} |a|^2 & -b^*a \\ -a^*b & |b|^2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

Asymmetry in LZ tunneling-III



In a): Chemical potential gap between ground and excited Bloch bands at Brillouin zone edge versus C.
$\Delta\mu$ is calculated self-consistently assuming an atomic density in each band; $\Delta\mu_{10}$ assumes total density in the ground and $\Delta\mu_{01}$ total density in the excited.

In b), tunneling probability P_{LZ} calculated from Landau-Zener formula using the $\Delta\mu$ energy gaps in a).

Lattice with depth U_0=2.2E_R.

Effective periodic potential

- Start from the rescaled Gross-Pitaevskii equation:

$$i\frac{\partial\psi}{\partial\tilde{t}} = -\frac{1}{2}\frac{\partial^2\psi}{\partial\tilde{x}^2} + V_0\sin^2(\tilde{x})\psi + C|\psi|^2\psi$$

- In the perturbative limit, Choi and Niu [PRL **82**, 2022 (1999)] found an effective potential determined by the mean-field interaction:

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + V_{eff}\sin^2(x)\psi \qquad \text{with} \qquad V_{eff} = \frac{V_0}{1 \pm 4C}$$

- In the perturbative limit, we derive the following effective potential:

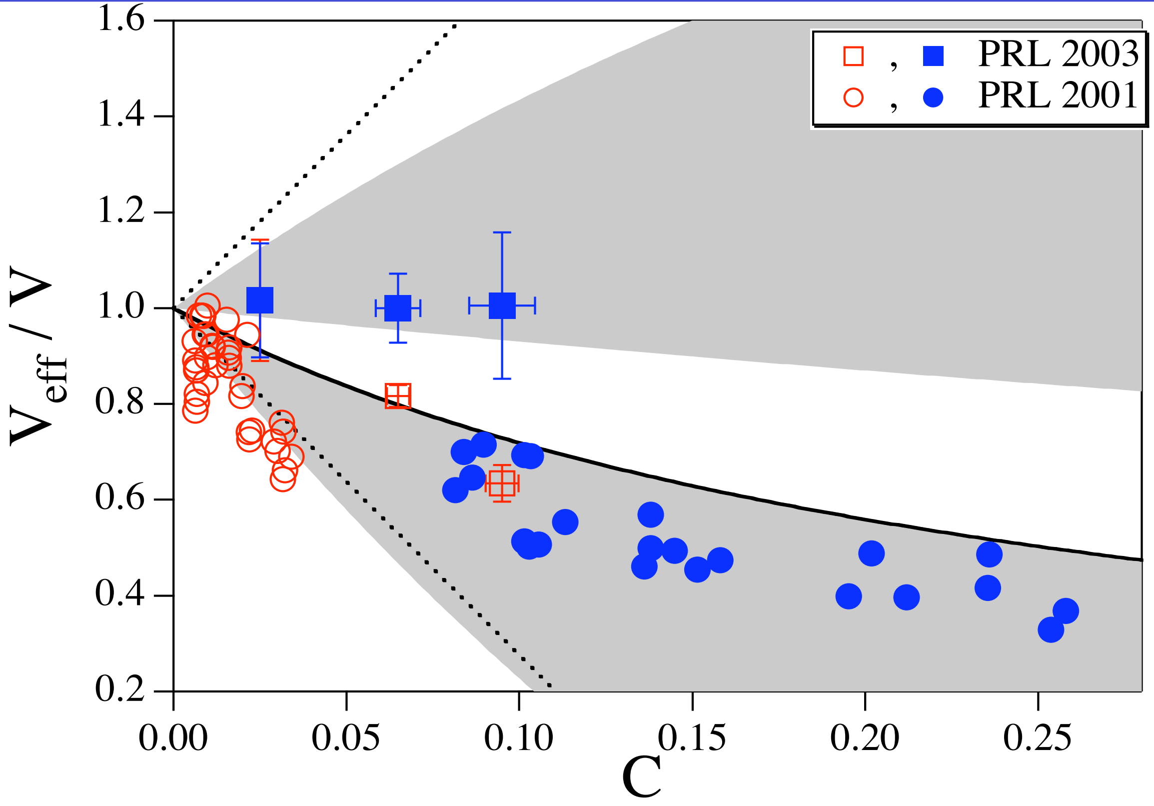
$$V_{eff} = V_0\sqrt{1 \mp \frac{2C}{V_0} + \frac{C^2}{V_0^2}}$$

- In ground state the potential experienced by the atoms is reduced owing to the nonlinearity introduced by interactions
- In excited state the potential experienced by the atoms is increased owing to the nonlinearity introduced by interactions

M. Jona-Lasinio et al, Laser Phys. Lett. 1, 147 (2004)

Effective potential

Ratio V_{eff}/V versus C.



The solid line for ground to excited is from Choi and Niu model.
The dotted lines for both directions are from chemical potentials. The shaded regions represent an average over different atomic momenta.
experiment with d=1.18 μm (filled marks) and d=0.39 μm (open marks).
U_o=0.1375 E_R and a_L=2.92 m s^{-2}.

M. Jona-Lasinio et al, Laser Physics, 15, 1180-8 (2005).

Remarks

• An alternative interpretation for the modification of the Landau-Zener tunneling is in term of the effective mass modified by the optical lattice and by nonlinear interaction.

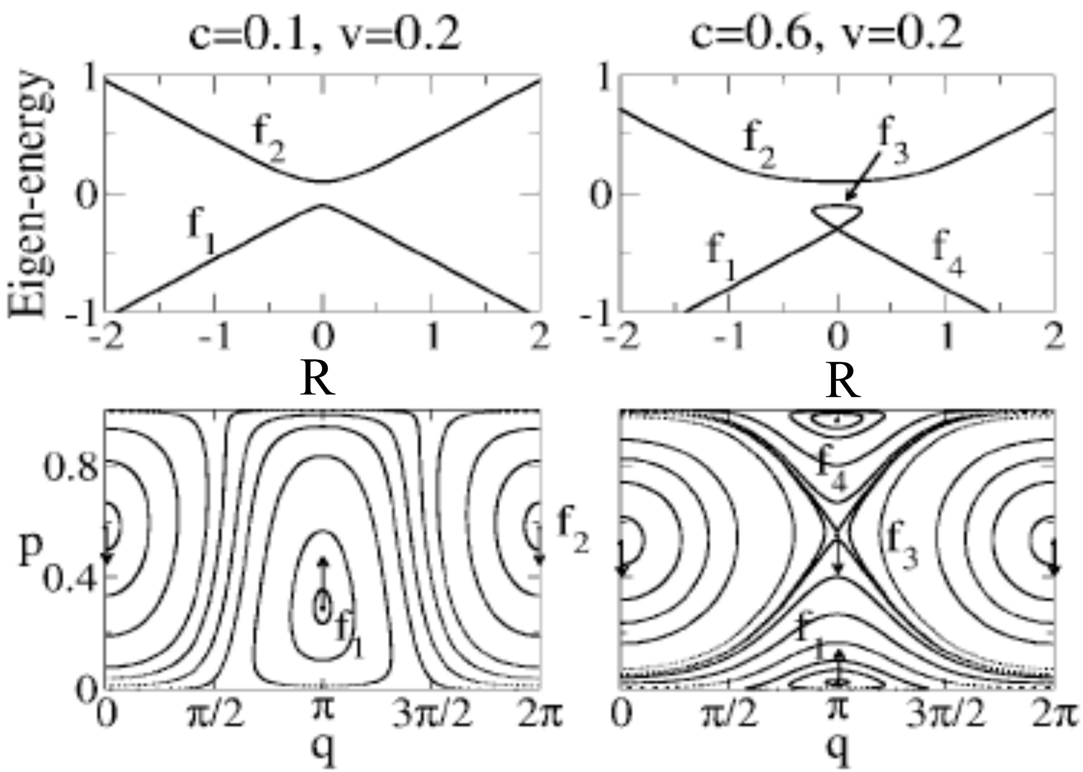
•Nonlinear LZ tunneling applies also to the photoassociation of a Bose-Einstein condensate (production of molecules) as in Ishkhanyan et al [PhysRevA **69**, 043612 (2004)].

•Those authors examined different nonlinear Hamiltonians and derived the associated nonlinear LZ probability.

•For very slow scanning rate they derived a linear, and not exponential, dependence on the scanning rate α.

Nonlinear Hamiltonian

Eigenvectors of the nonlinear Hamiltonian



Phase space portraits

Liu, Wu and Niu Phys. Rev. Lett 90, 170404 (2003)

Resonant tunneling

Apply a force F

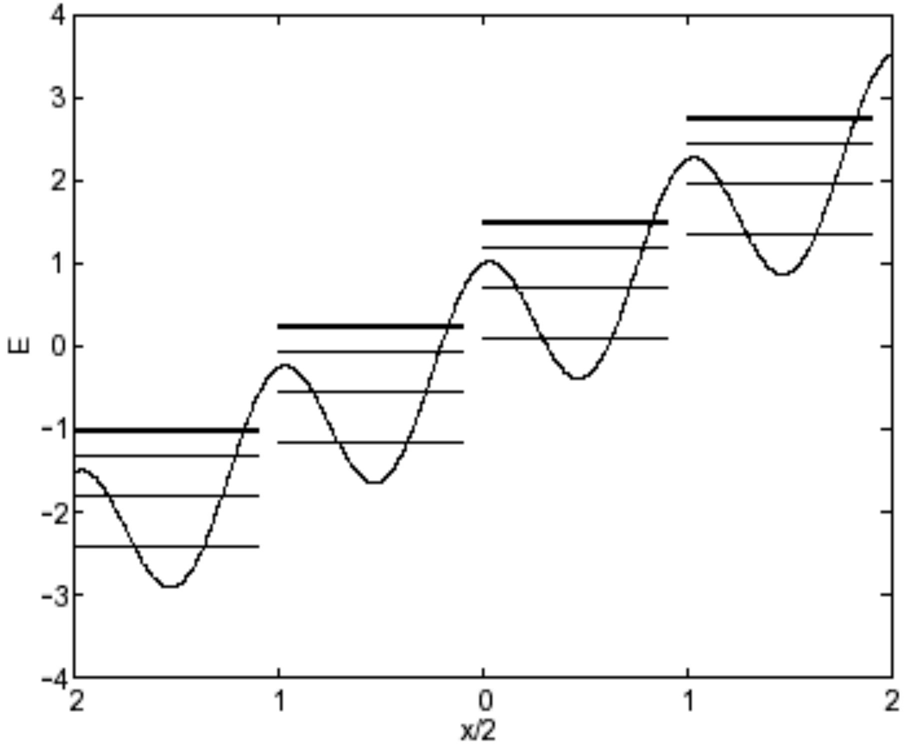


FIG. 1. Schematic drawing of the Wannier-Stark ladder of the resonances. The width of the levels is "proportional" to the resonance width Γ_n defining the lifetime of the states.

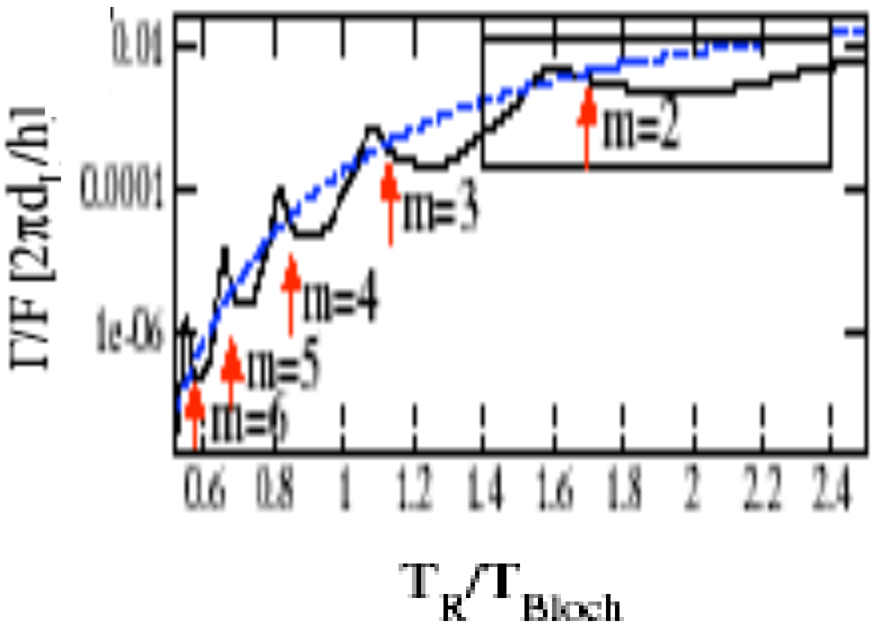
Maxima in the resonance width Γ occur when $Fd_L m$ (with m integer) is close to the difference between the first two energy band of the $F=0$ problem.

Glück et al , PRL 83, 891 (1999)

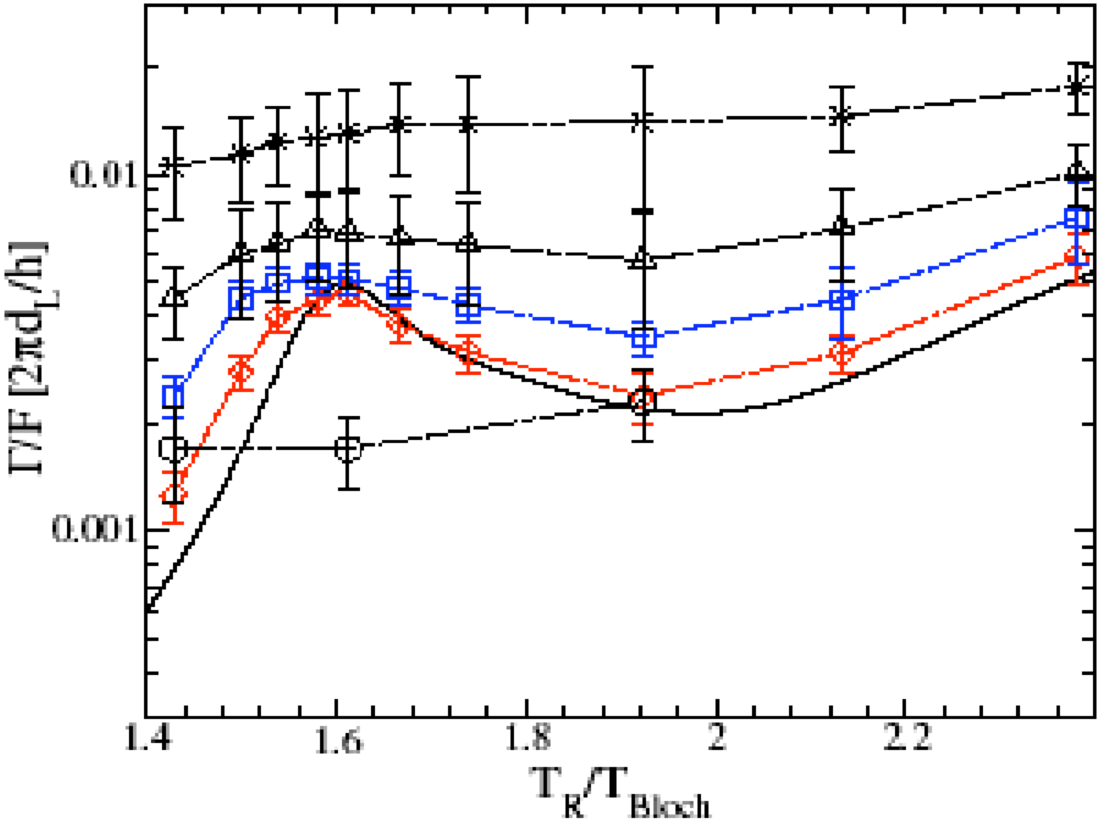
Width of the Wannier resonance

Time scales:

- Recoil period $T_R = h/E_R$
- Bloch oscillation period $T_R = h/Fd_L$
- Collapse time (for attractive interactions)



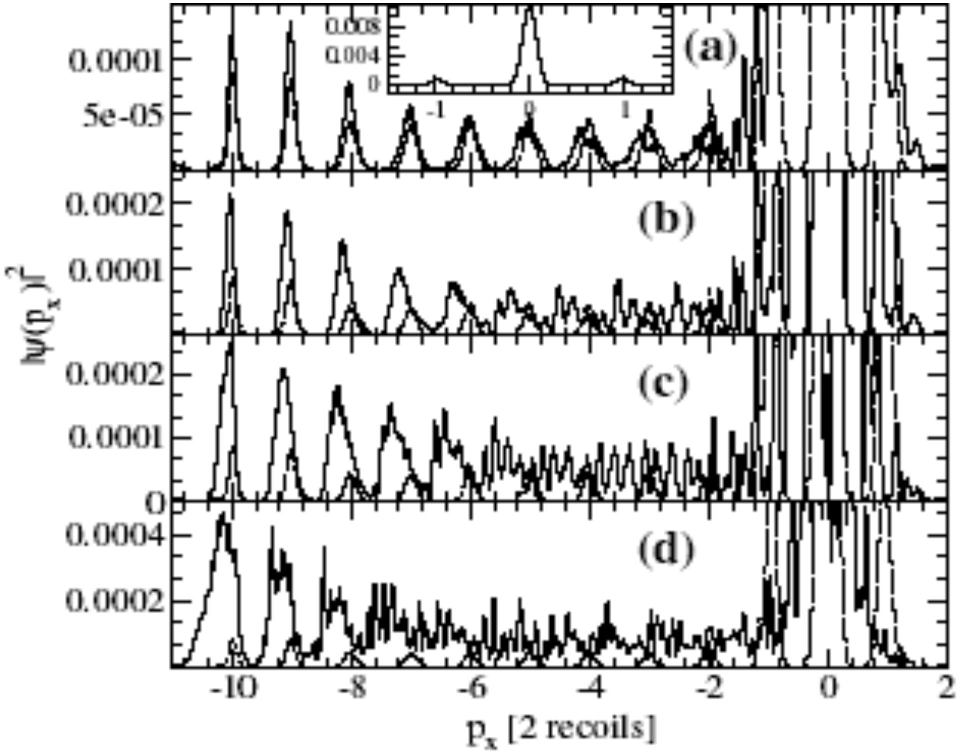
Resonant tunneling with interactions



C=0, solid line
C=0.027, diamonds
C=0.065, squares
C=0.12, pyramids
C=0.31 stars
C=-0.31, circles

Wimberger et al , arXiv:cond-mat/05????

Probes of atomic momentum distribution



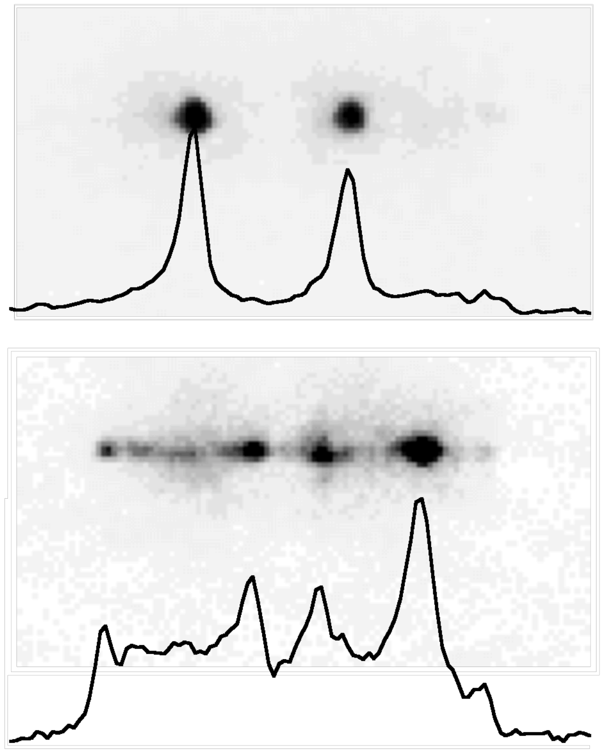
$$P_{\rm rec}(t) \equiv |\langle \psi(t) | \psi(t=0) \rangle|^2 \ .$$

$$P_{\rm sur}(t) \equiv \int_{-p_c}^{p_c} dp_x \left(\int dp_y dp_z \, |\psi(\vec{p}, t)|^2 \right) ,$$

with $p_c = 3\ p_R$

(a) C=0.027
(b) C=0.065
(c) C=0.12
(d) C=0.31

Strong nonlinear dynamics



- if the mean-field interaction is strong, instabilities arise at the edge of the Brillouin zone

Instability at the band edge: numerical simulation

- small acceleration (0.25 m/s^2): condensate starts becoming unstable as it approaches the band edge; solitons are formed and they decay into vortices

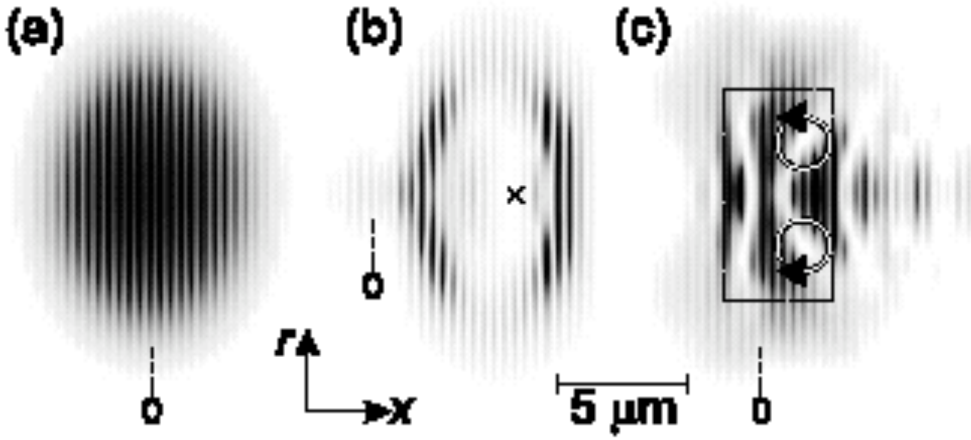
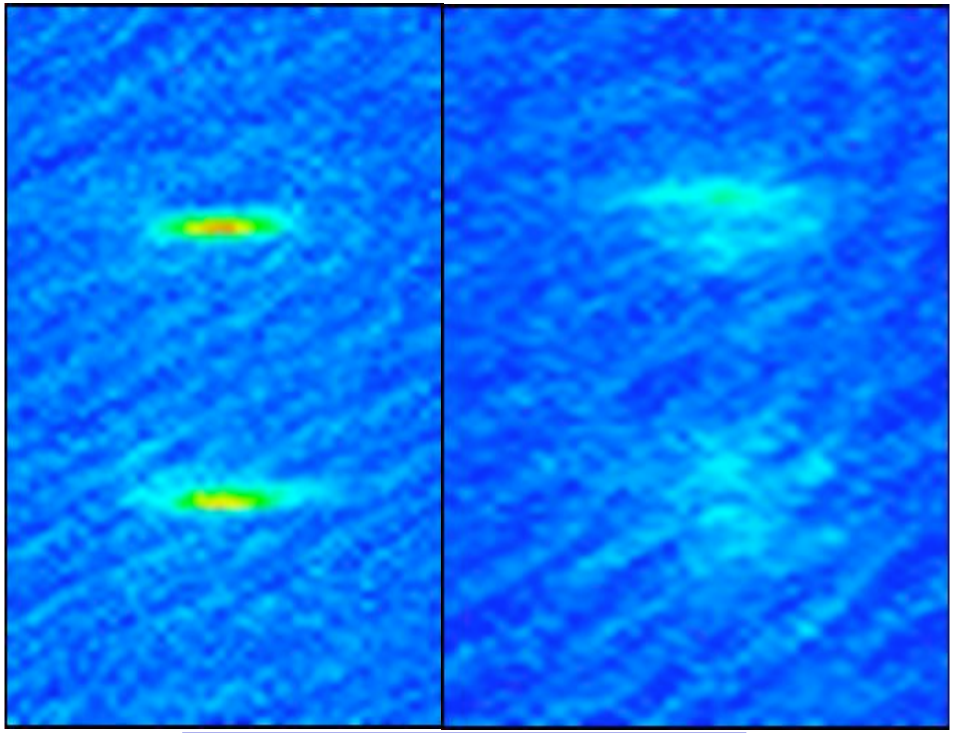


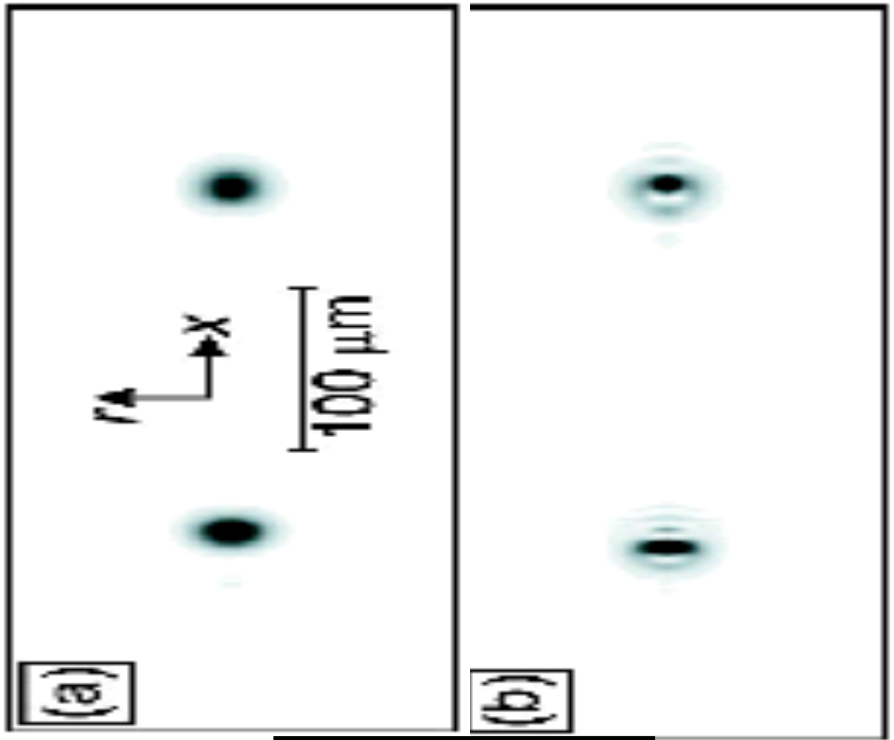
FIG. 3. Grey-scale plots of density (white = 0, black = high) in the x-r plane (axes inset) for system A with $\Delta x = 25\ \mu$m and $t = 0$ ms (a), 7.8 ms (b), 10.9 ms (c). Vertical dotted lines indicate $x = 0$ in each case. The horizontal bar shows scale. The cross in (b) marks the center of a soliton. The region within the dashed box in (c) is shown in Fig. 5(a).

R. Scott et al, Phys. Rev. A 69, 033605 (2004)

Instability at the band edge
Experiment and theory



Experiment

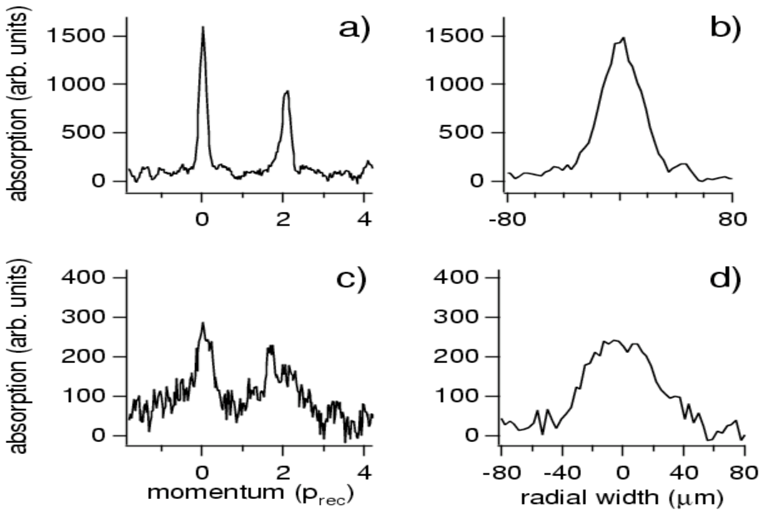


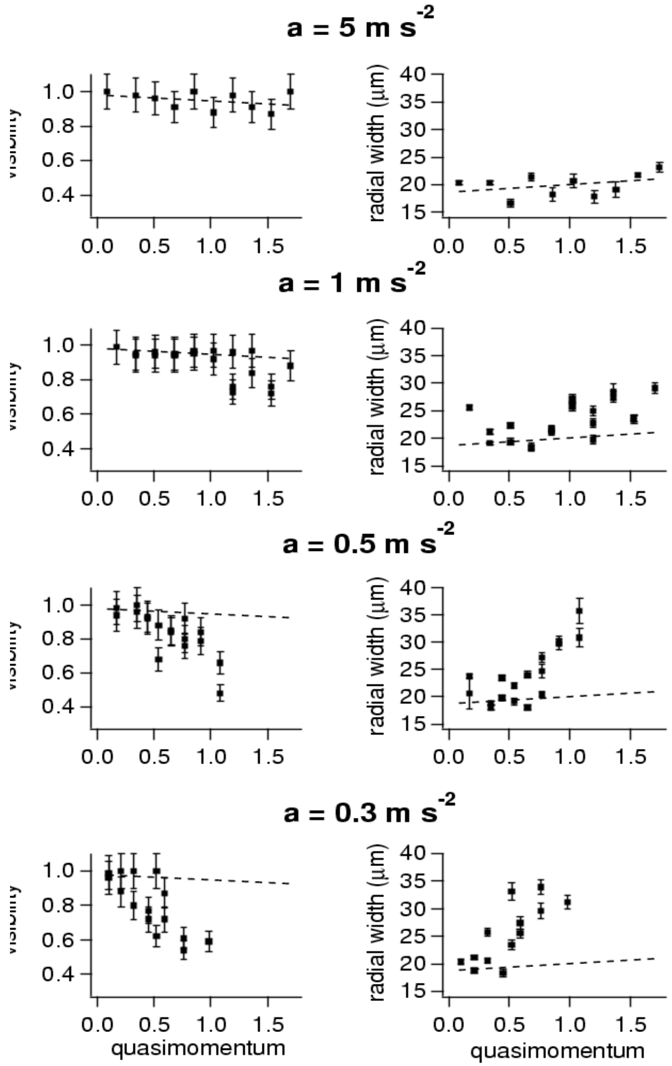
Theory

For both, a=5m/s^2 on the left, a=0.3m/s^2 on the right

Instability at the band edge: Experimental results

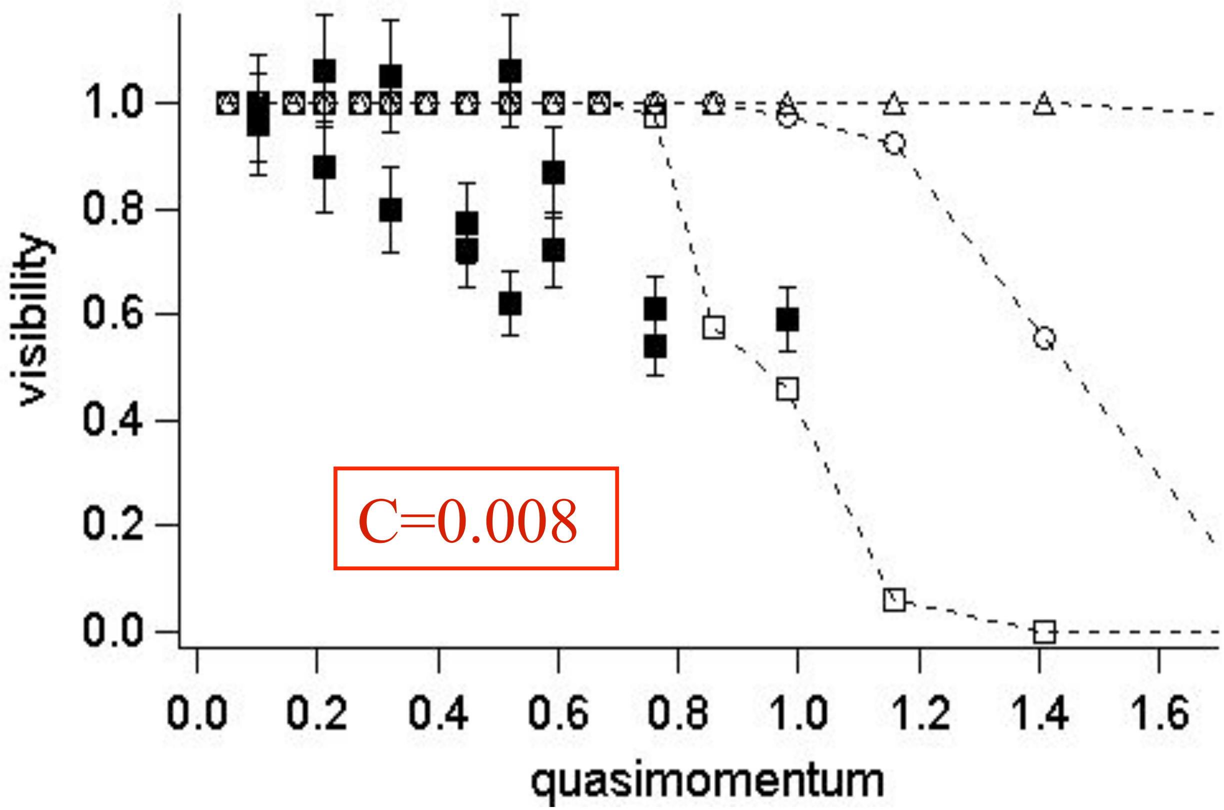
- as the acceleration is lowered, both visibility and radial width change drastically close to the zone edge





M. Cristiani et al, *Optics Express* 12, 4 (2004)

Instability:1D simulation



numerical integration of 1D GP-equation

- **instability is a consequence of nonlinearity**
- **instability occurs also in 1D**
- **question: how does transverse instability arises?**

Instability growth rate

Our growth rate: 500 s^{-1}= 0.002 ($8\omega_R$)

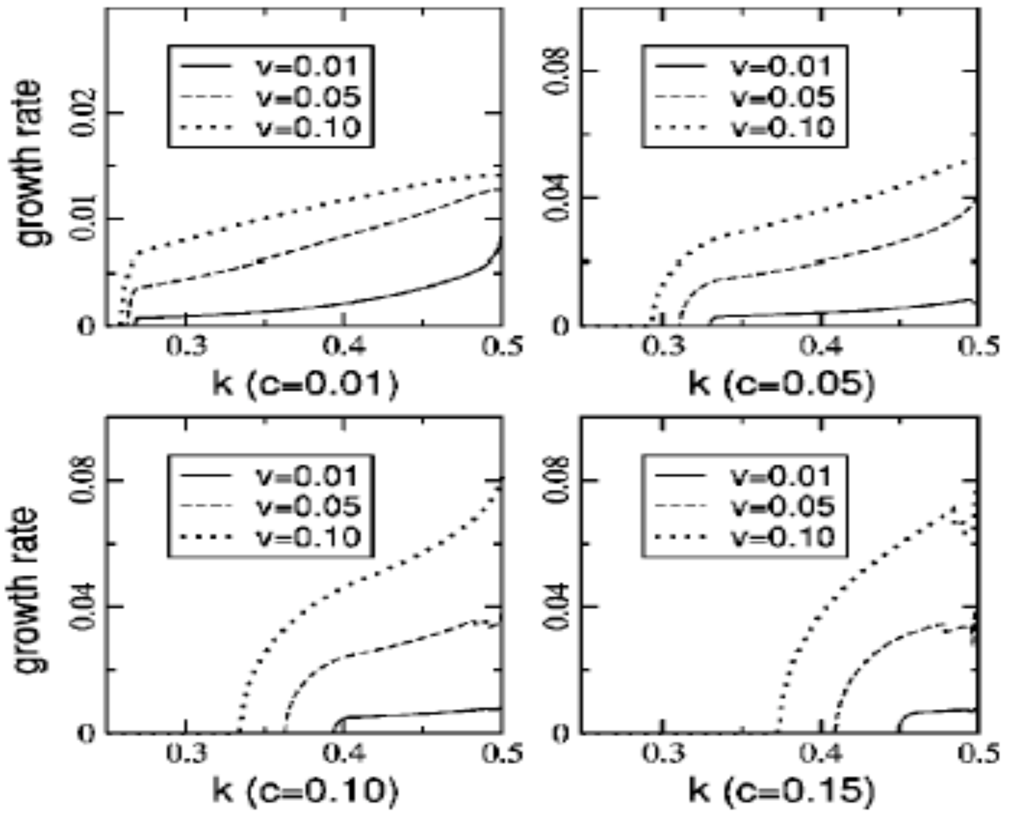


FIG. 2. Growth rates of the most unstable modes of each Bloch state ϕ_k. (units $8\omega_R$)

Wu and Niu, Phys. Rev. A 64, 061603 (2001)

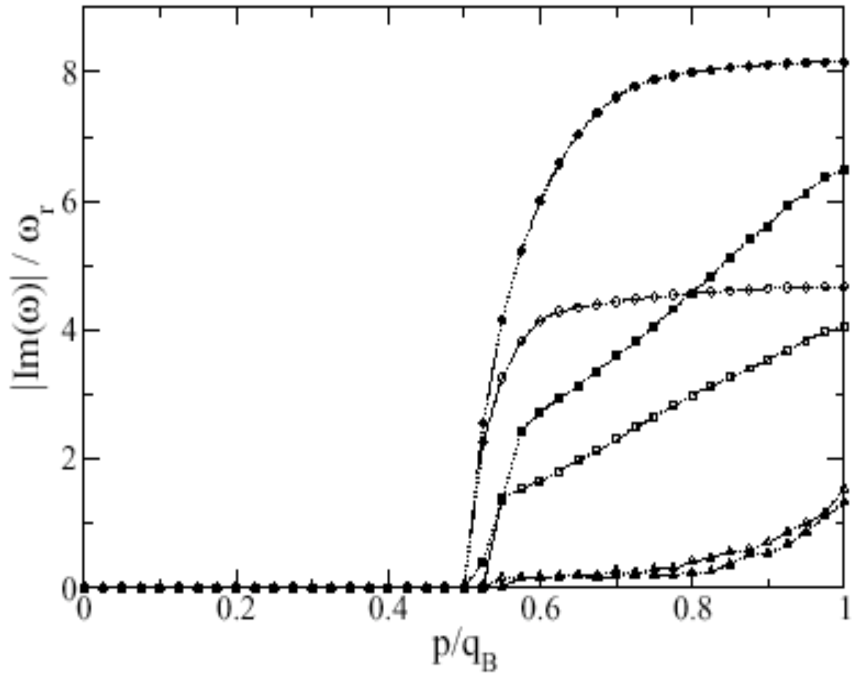


FIG. 8: Growth rates of the most dynamically unstable modes for different values of the lattice intensity s ($s = 0.1$ triangles, $s = 1$ squares, $s = 5$ circles) obtained with both GP and NPSE (filled and empty symbols, respectively).

(units radial frequency)

Modugno et al, Phys. Rev. A 70, 042365 (2004)

Remarks

- Visibility represent an indirect macroscopic evidence of the instability occurrence

- Growth rate for microscopic and macroscopic evidences

- Dynamical instability of the usual Bloch states sets in when there is a period-doubled state of the same energy and wave number.(*)

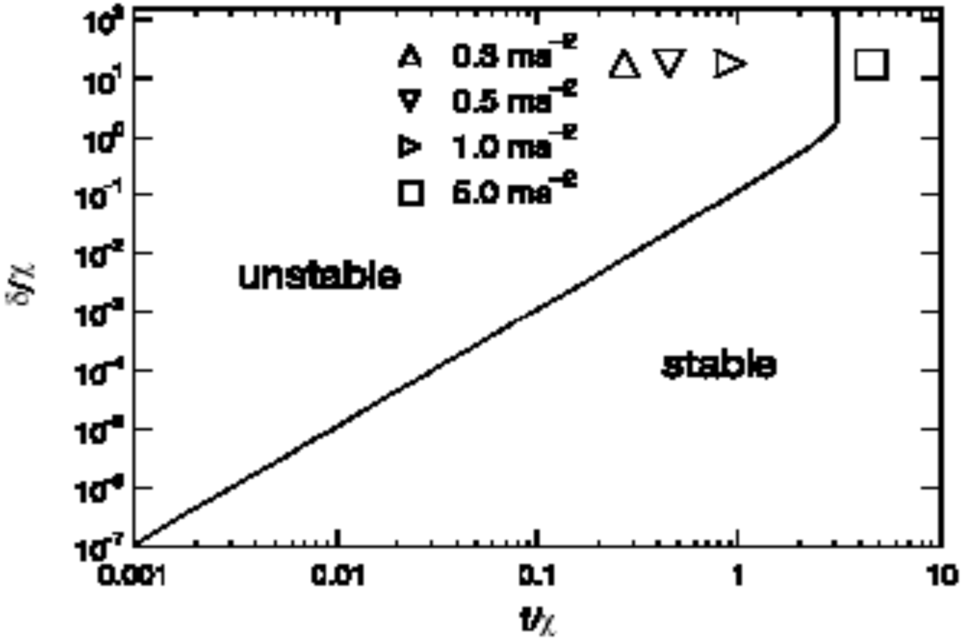
- Period doubled states produce a doubling of the observed momentum components, to be observed after time of flight.

**M. Machholm et al, Phys. Rev. A 69, 043604 (2004)*

Band edge instabilities: interpretation

- The acceleration must be small enough: the condensate must spend a sufficient amount of time in the unstable zone at the band edge.

M. Cristiani et al, Optics Express 12, 4 (2004)



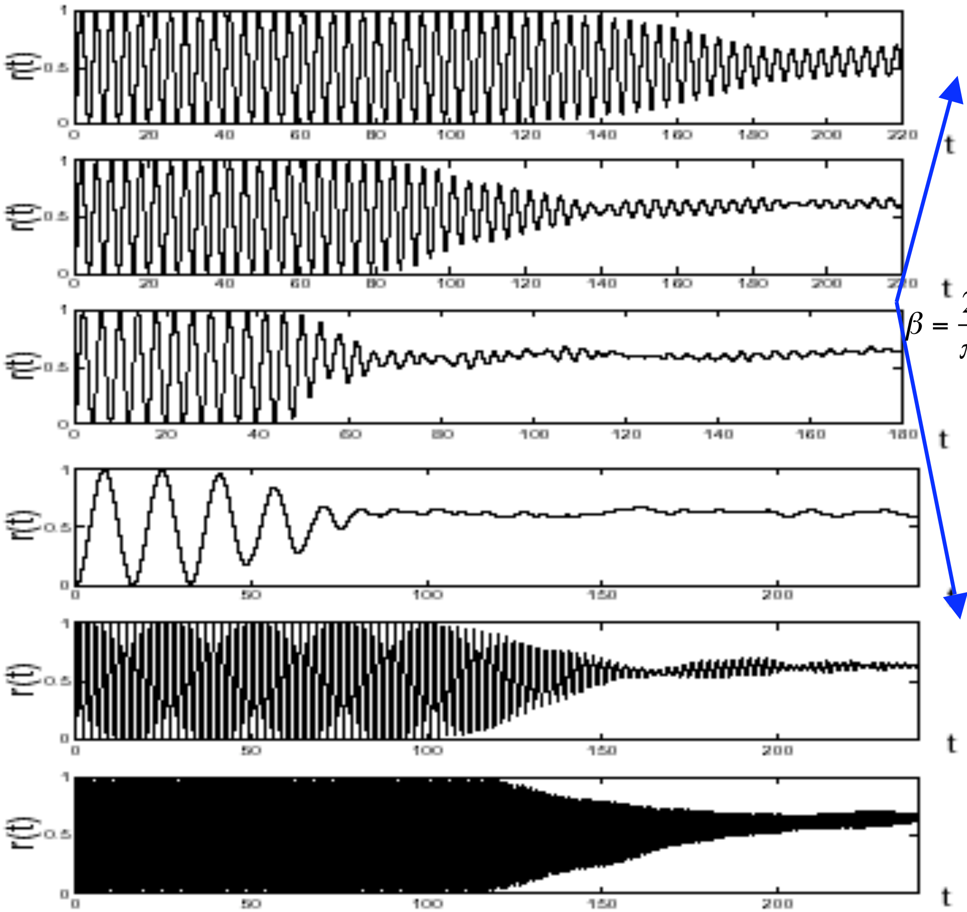
δ = band gap width

$$f = \frac{Mad}{\hbar}$$

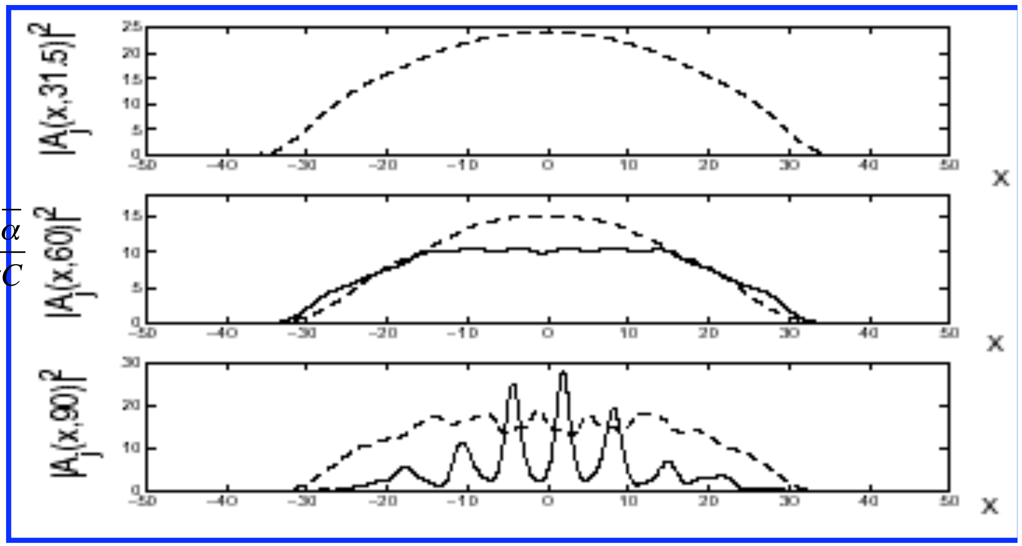
Y. Zheng, M. Kostrum and J. Javanainen, PRL 93, 230401 (2004)

Theory of Non-symmetric Landau-Zener Tunneling

Role of modulational instability on the transfer between bands.



$\beta = \frac{2\alpha}{\pi C}$



Spatial structure

Tunneling rate versus time

Konotop, Kevrekidis and Salerno, arXiv:cond-mat/0404608

Nonlinear quantum transport
δ-Kicked Bose condensate

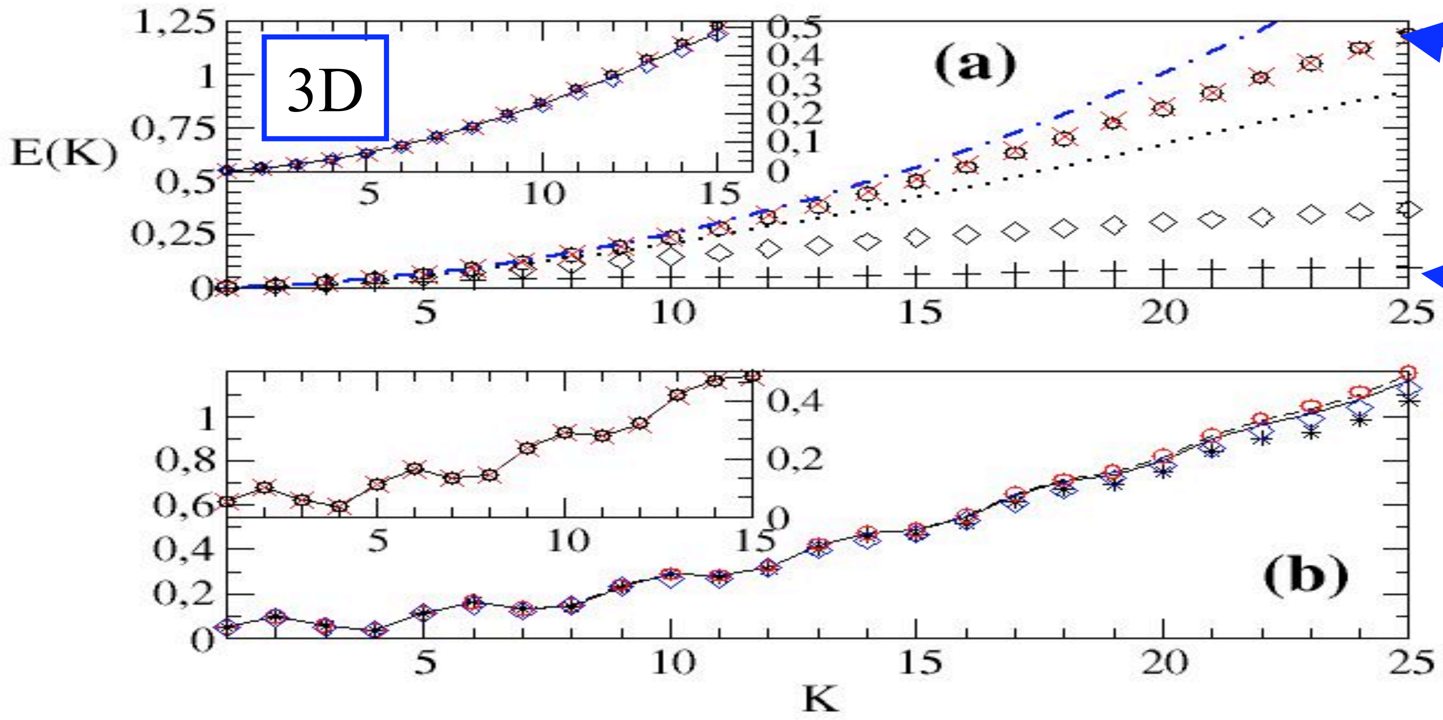
Quantum resonances occur when the pulsing time is an integer or half-integer of the Talbot time $T_T = h/(4E_{rec})$

Numerical results show that the fundamental and higher-order quantum resonances of the δ-kicked rotor are observable in state-of-the-art experiments with a Bose condensate in a shallow harmonic trap. The atomic interaction decreases the contrast of the esonances.

Wimberger et al , PRL 94, 130404 (2005)

Kicked Bose condensate

Quadratic increase of the energy gained by atoms E(K) versus kick number K



N=5x10^3

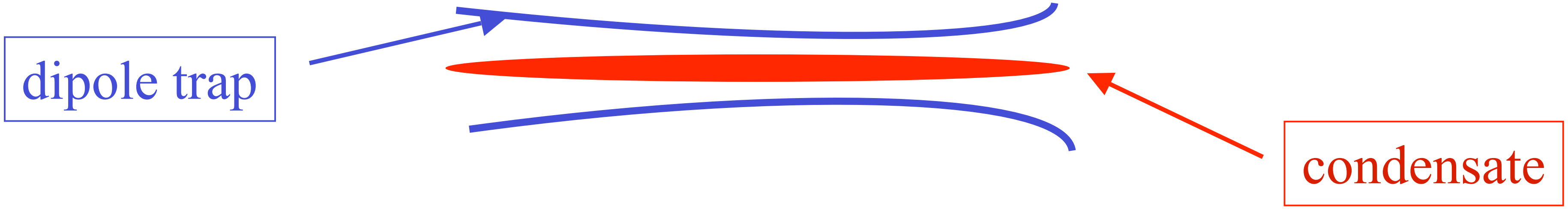
N=10^5

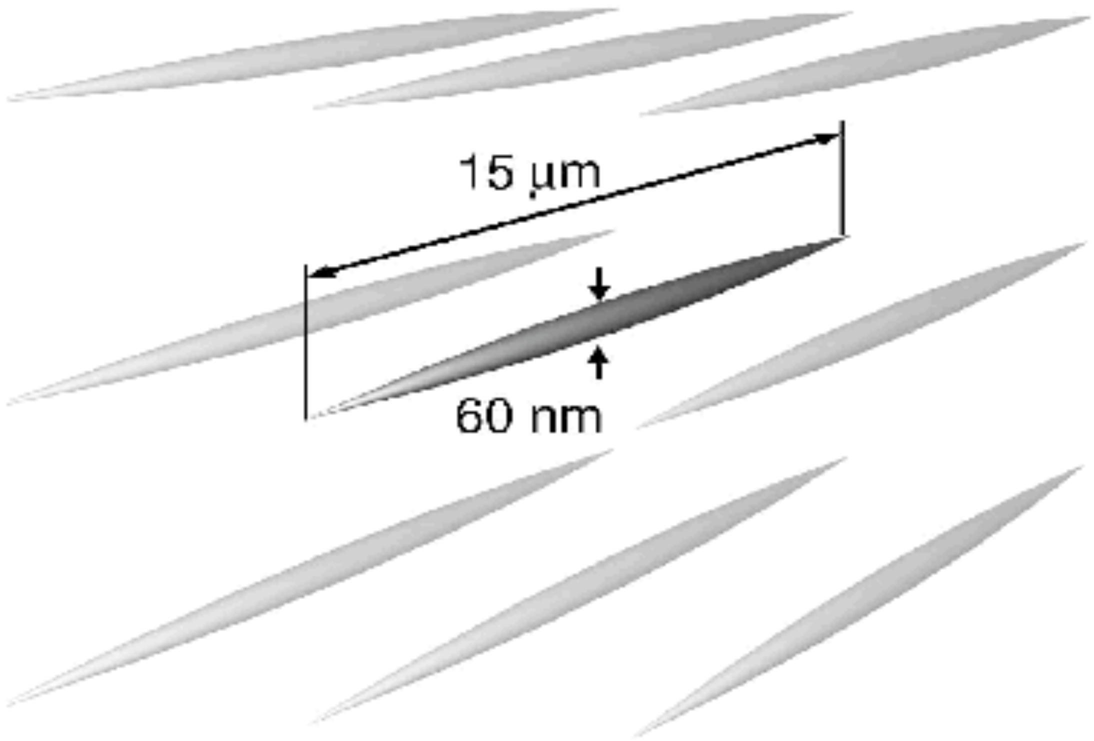
U_0=8E_R
ω/2π=10,100 Hz
in (a) at T_T
in (b) at $T_T/4$

Wimberger et al , PRL 94, 130404 (2005)

Simplification: 1D Geometry

Transverse confinement by an optical trap:





The geometry and size of trapped 1D gases in a two Dimensional optical lattice. The spacing between the 1D tubes, in the horizontal and vertical directions, is l /2.

Ratchets

Optical lattice potential: $U(x,t) = U_0(t)\sin^2(\pi x/d + \phi(t))$

kicking realized through U(t) or ϕ(t)

temporal asymmetry:

$$\phi(t) = a\sin(\omega t) + b\cos(2\omega t) + \phi_0$$

þ directed diffusion *(Schiavoni et al., PRL 90, 094101 (2003)*

spatial asymmetry: add

$$U_{asymm}(x) = a\cos^2(2\pi x/d + \vartheta)$$

⇒ quantum ratchet, and quantum chaos *(Monteiro et al., PRL 89, 194102 (2002))*

Challenges:

⇒ investigate effect of nonlinearity of a BEC (due to interatomic collisions) on the quantum dynamics

Conclusions

- BECs in optical lattices are a rich system for investigating **solid-state-like phenomena**, both in the linear and nonlinear regimes.

- **non-linear effects** of BECs in optical lattices modify phase evolution, tunneling properties and stability of the condensate.

- mean-field interaction leads to **asymmetric Landau-Zener tunneling.**

- Future experimental plans

- One dimensional geometry

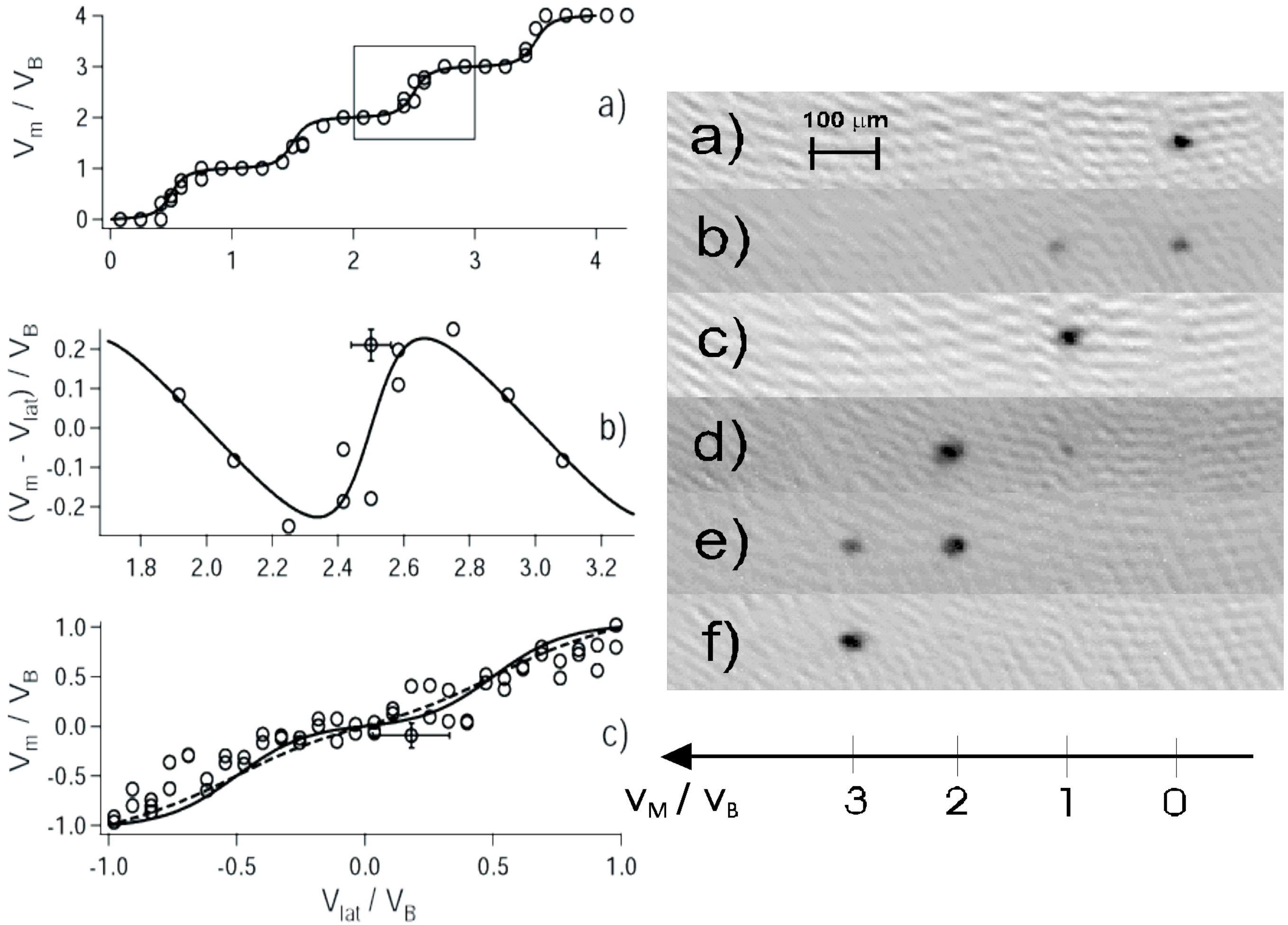
- Kicked rotor and ratchet: role of the nonlinearity

Dynamics: Bloch oscillations

- accelerate the lattice by introducing a small, linearly varying frequency difference between the lattice beams

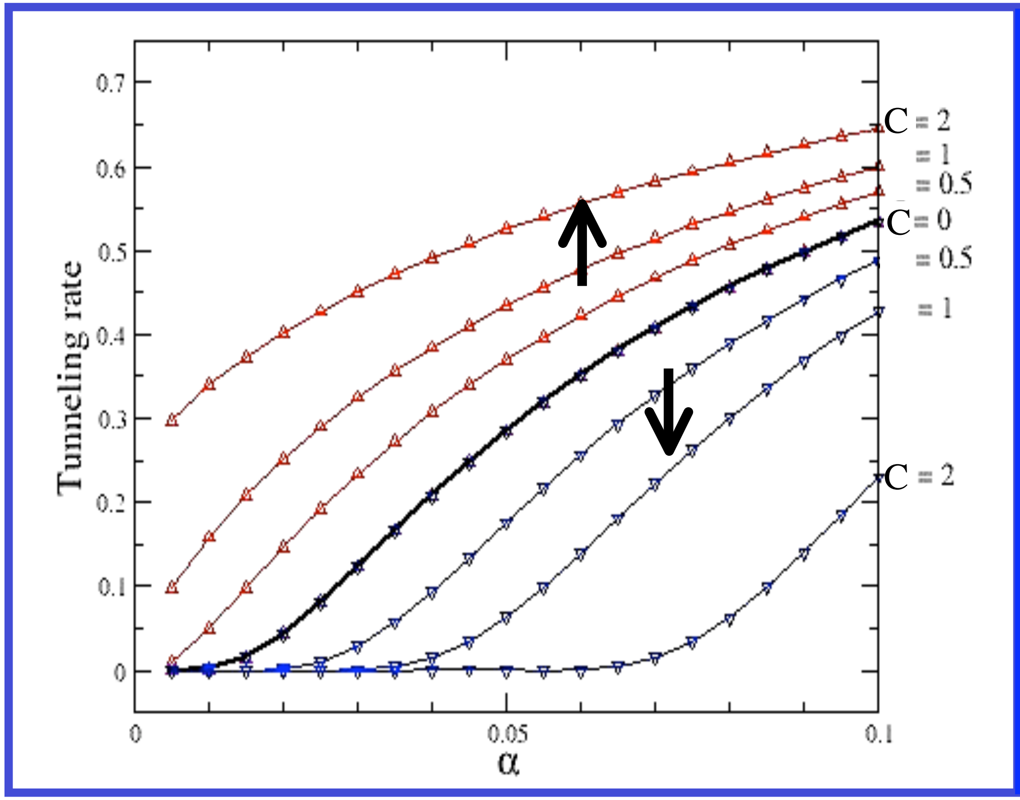
- after a time-of-flight, image momentum components of the condensate

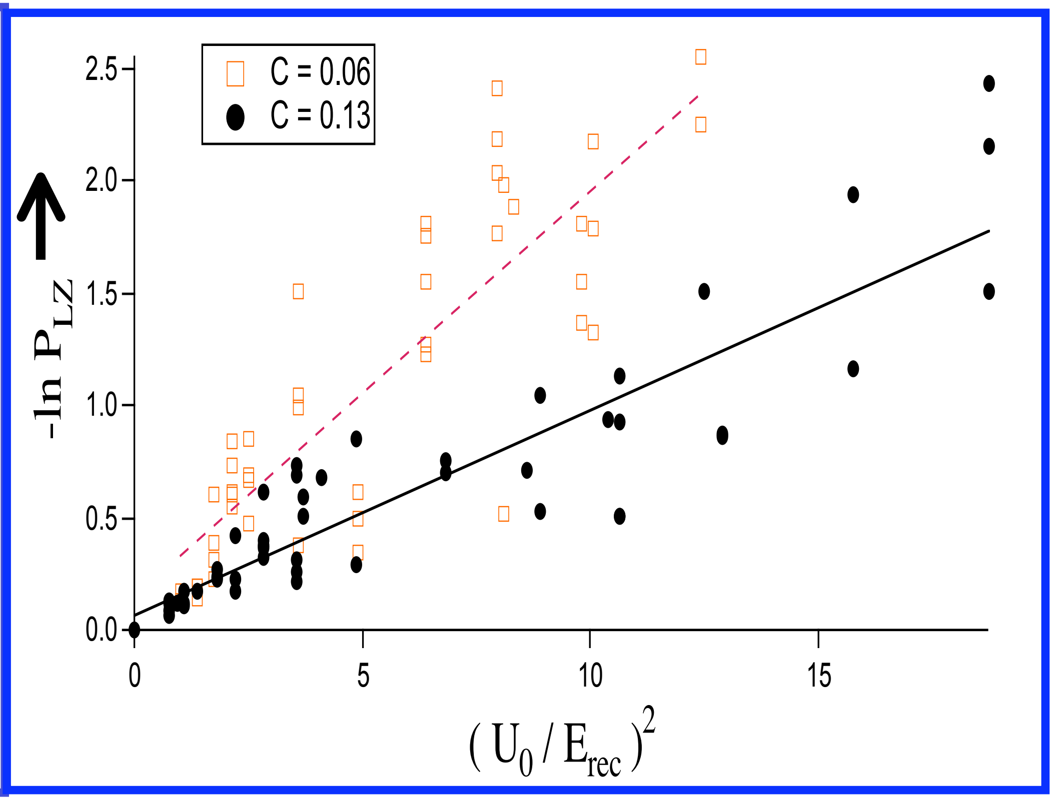
- calculate mean velocity



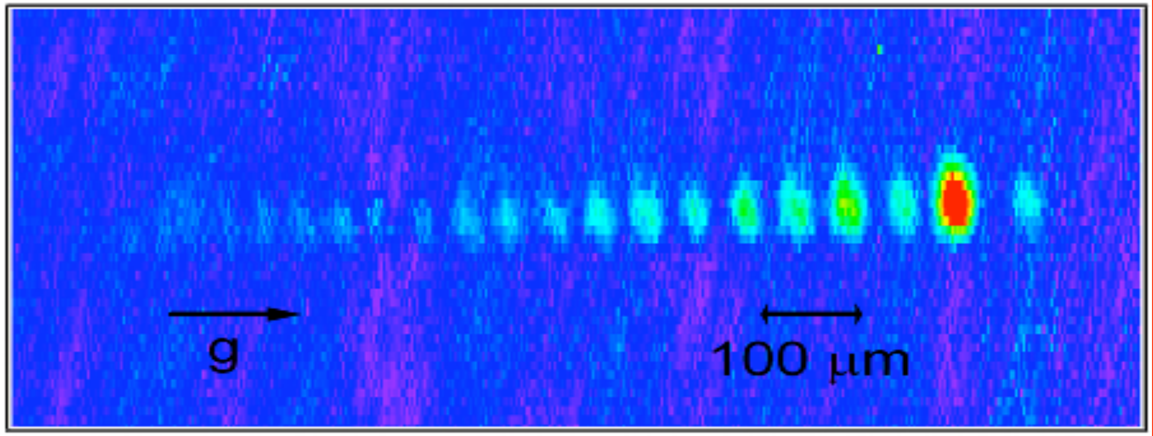
Mean-field interaction and tunneling

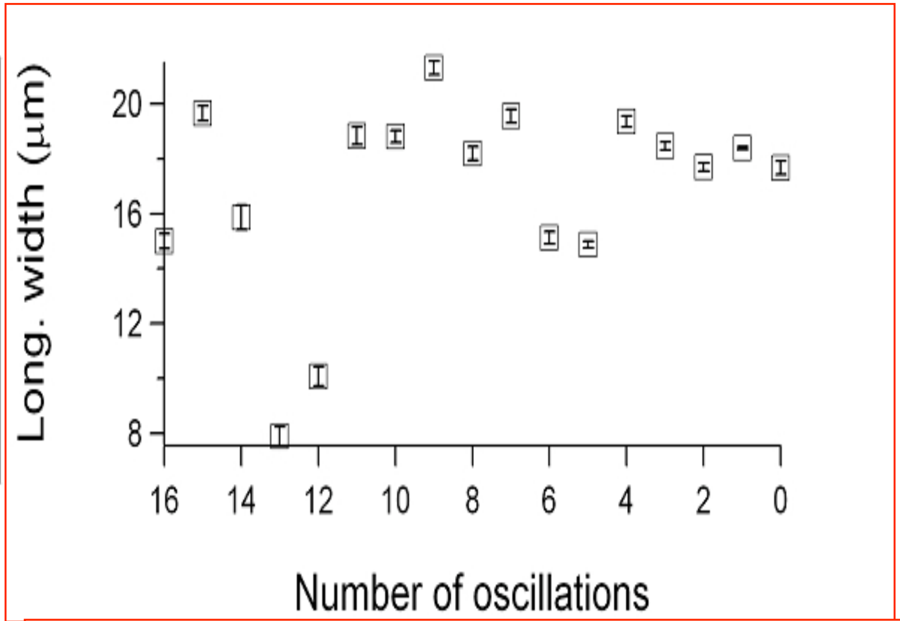
- The mean-field interaction increases the tunneling probability from ground to excited state and decreases that from excited to ground state

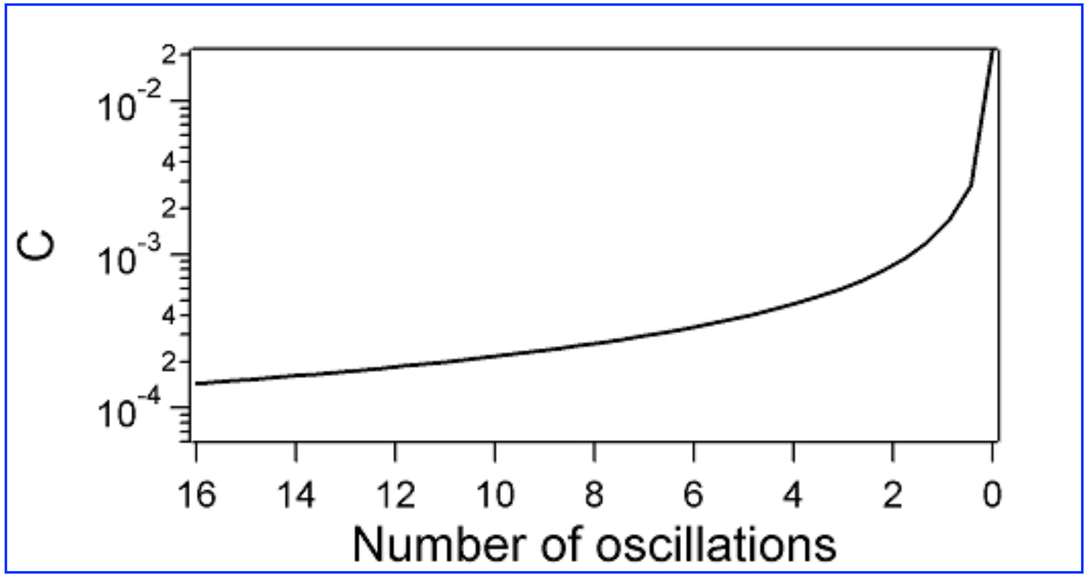


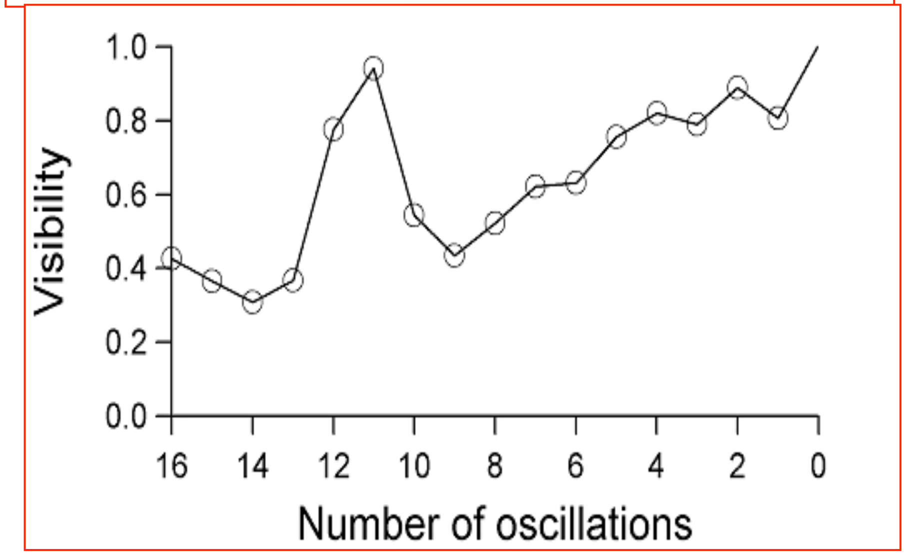


Several Bloch Oscillations

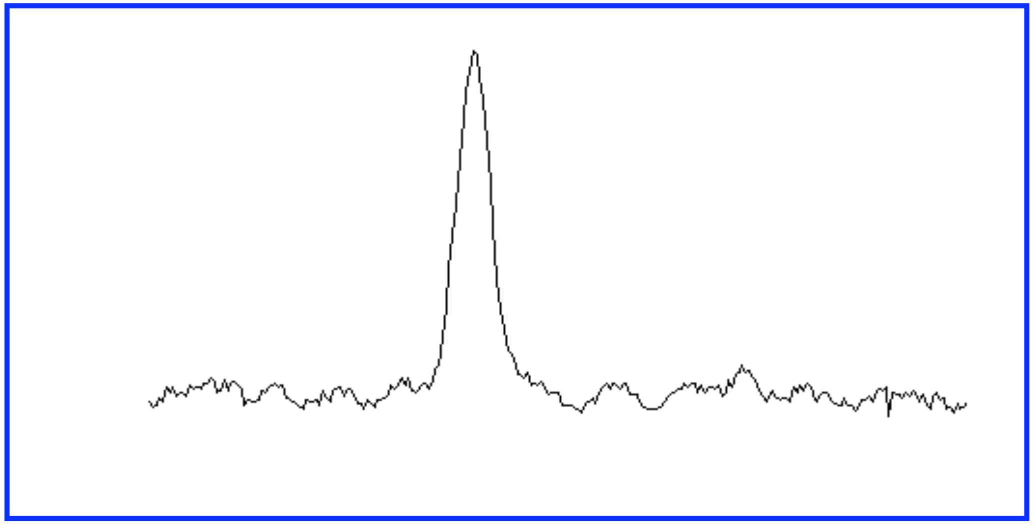




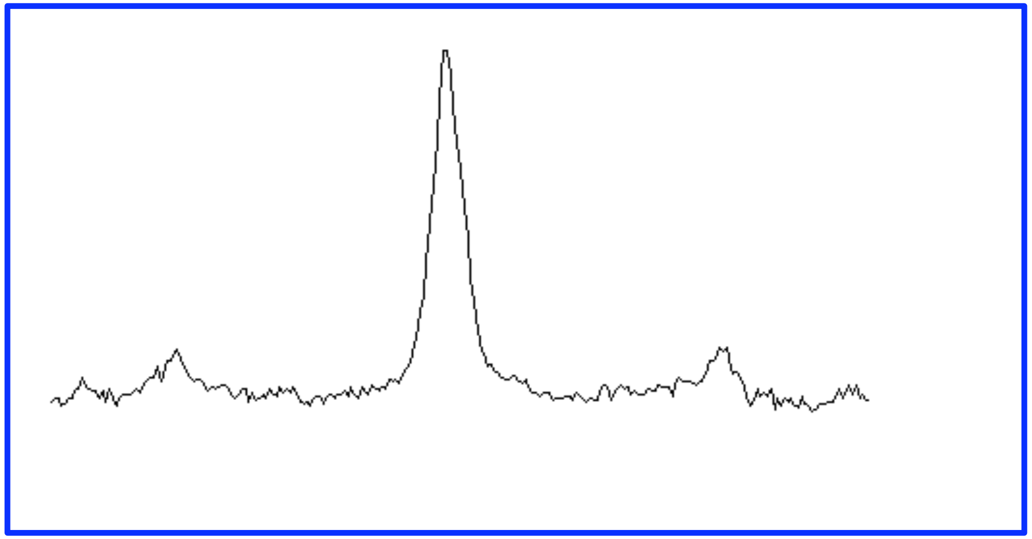




Instability at the band edge: Experimental results- I



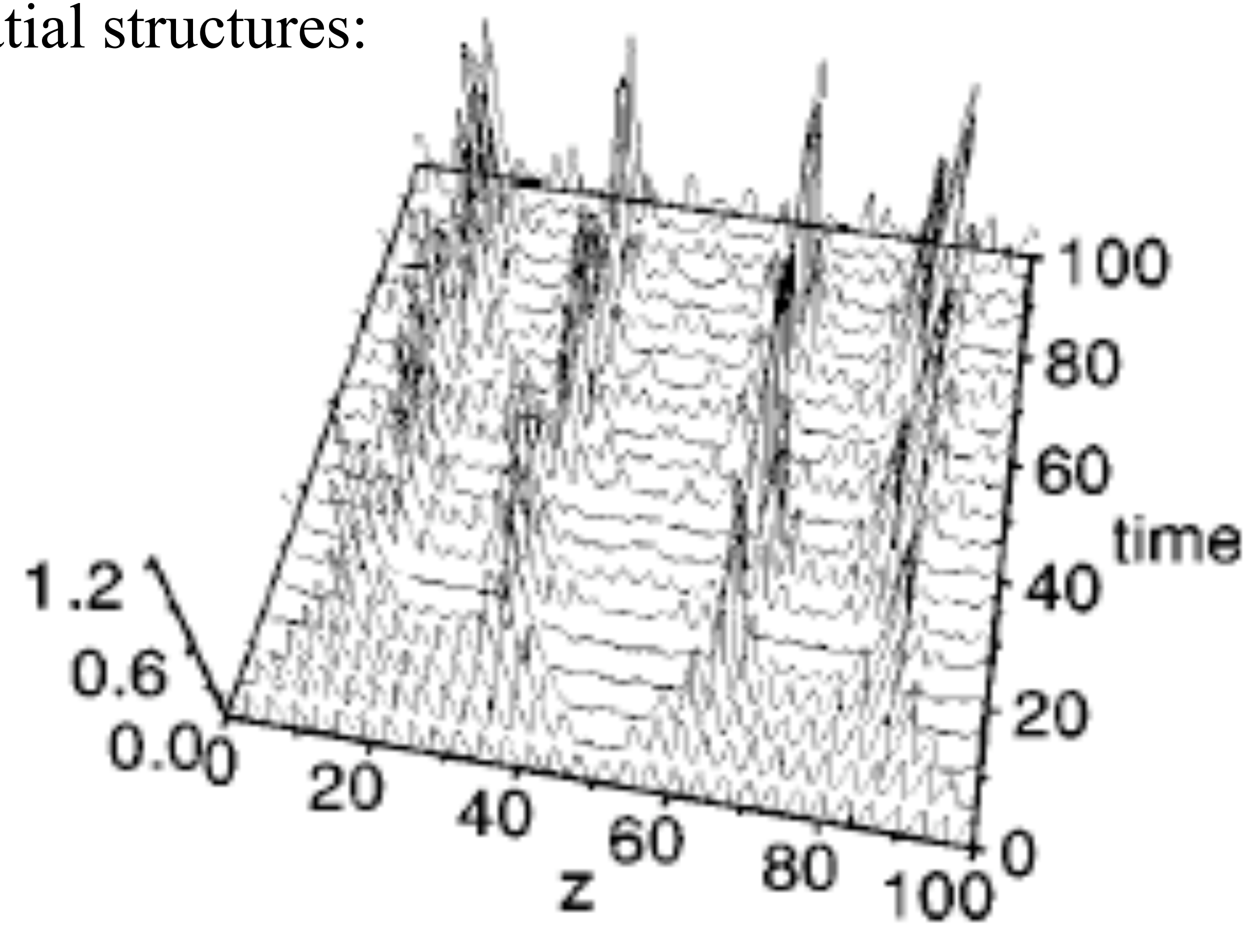
- large acceleration (5 m/s^2): as condensate performs Bloch oscillations, interference peaks change periodically in height



- small acceleration (0.3 m/s^2): as condensate reaches edge of Brillouin zone, the interference peaks start broadening and washing out

Modulation instability

Large spatial structures:



Konotop and Salerno, Phys. Rev. A 65, 0212602 (2002)

Condition for 3D → 1D crossover

$$\boxed{\mu_{3D}} << \boxed{\hbar\omega_\perp}$$

Chemical potential
(ground state energy)

Typical energy of
transverse excitations

If an optical lattice is present along the longitudinal direction:

$$\mu_{3D}^L = \mu_{3D} f(U_0)$$

For small lattice depth $f \sim 1$.
In the tight-binding approximation:

$$f(U_0) \approx \left(\frac{2}{\pi}\right)^{1/5} U_0^{1/10}$$

The 3D →1D transition
is expressed by
the condition

$$\eta = \frac{\mu_{3D}^L}{\hbar\omega_\perp} \propto N^{2/5} \omega_z^{2/5} \omega_\perp^{-1/5} U_0^{1/10} << 1$$

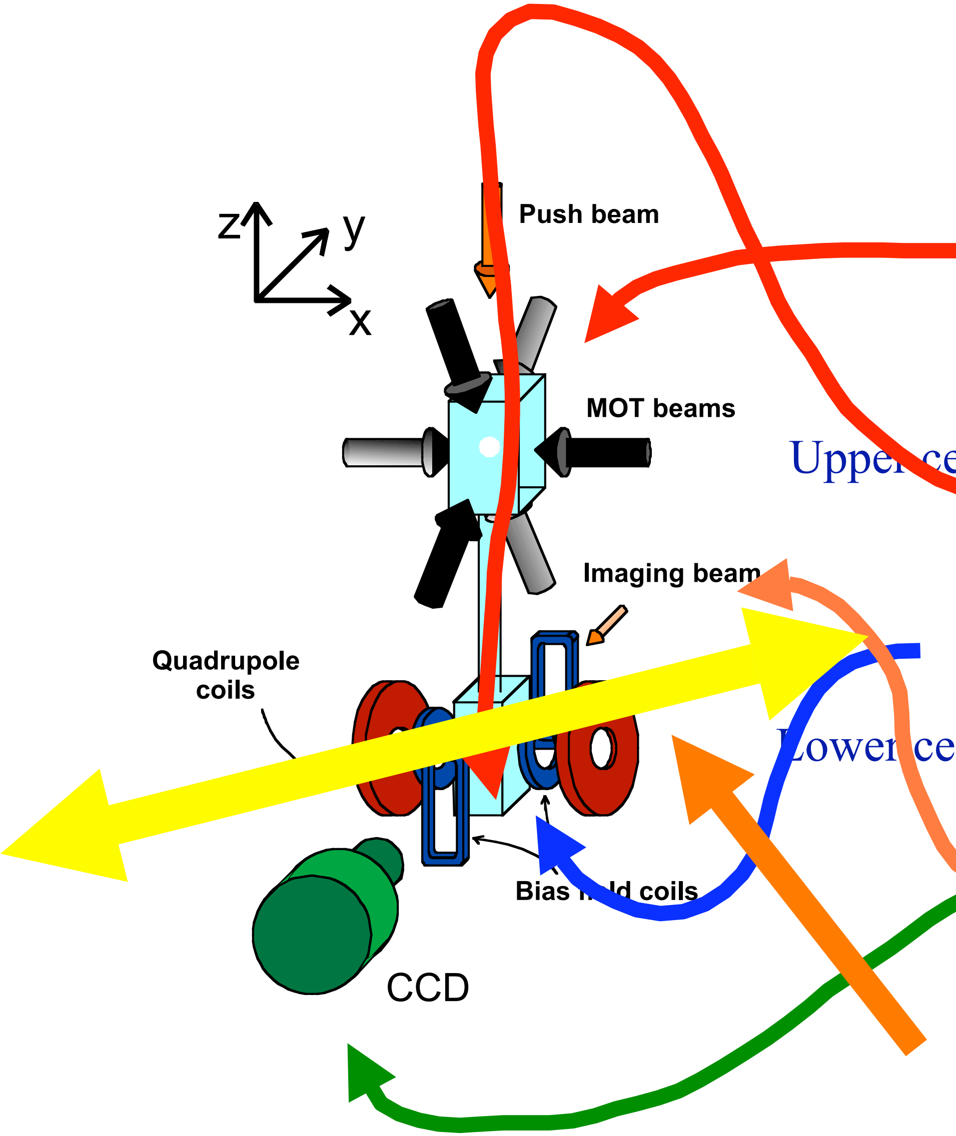
To reach the 1D regime:
- Low number of trapped atoms
- Strong transverse confinement

2D transverse lattice → tubes

Esslinger* experiment $\eta \approx 0.1$

* H. Moritz et al., PRL 91, 250402 (2003)

Experimental setup



capture and cool ^{87}Rb atoms in upper MOT

push atoms into lower MOT (better vacuum, longer lifetime)

transfer atoms into TOP-trap (rotating bias field plus quadrupole)

shadow imaging of atomic cloud/ condensate

Apply optical lattice