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Quantum dephasing and decay of classical correlation functions in chaotic systems

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We discuss the dephasing induced by the internal classical chaotic motion in the absence of any external environment. We relate the dephasing to the decay of the quantum Loschmidt echo which, in the semiclassical limit, is expressed in terms of an appropriate classical correlation function. Our results are derived analytically for the example of a nonlinear driven oscillator and then numerically confirmed for the kicked rotor model.

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The study of the quantum manifestations of classical chaotic motion has greatly improved our understanding of quantum mechanics in relation to the properties of eigenfunctions and eigenvalues as well as to the time evolution of complex systems [1, 2]. However, the relation between classical dynamical chaos and quantum dephasing is still an open important problem. In order to elucidate this problem, we consider in this paper the quantum Loschmidt echo. This quantity is a measure of the stability of quantum motion under perturbations and its behavior has been already extensively investigated in different parameter regimes and in relation to the nature of the corresponding classical motion (see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and references therein). The purpose of the present paper is to show that the internal dynamical classical chaos, even in the absence of any external environment, may suppress the quantum interference and make the quantum phases irrelevant.

For a pure coherent quantum state $|\overset{\circ}{\alpha}\rangle$ the Loschmidt echo or fidelity is defined as

$$F_{\overset{\circ}{\alpha}}(t) = |f_{\overset{\circ}{\alpha}}(t)|^2 = |\langle\overset{\circ}{\alpha}|\hat{U}_0^\dagger(t)\hat{U}_\varepsilon(t)|\overset{\circ}{\alpha}\rangle|^2 = \left|\mathrm{Tr}\left[\hat{f}(t)\overset{\circ}{\rho}\right]\right|^2, \tag{1}$$

where $\overset{\circ}{\rho} = |\overset{\circ}{\alpha}\rangle\langle\overset{\circ}{\alpha}|$ is the initial density matrix. The unitary operators $\hat{U}_0(t)$ and $\hat{U}_\varepsilon(t)$ describe the unperturbed and perturbed evolutions of the system, corresponding to the Hamiltonians H_0 and $H_\varepsilon = H_0 + \varepsilon V$, respectively. Therefore, the echo operator $\hat{f}(t) = \hat{U}_0^\dagger(t)\hat{U}_\varepsilon(t)$ represents the composition of a slightly perturbed Hamiltonian evolution with an unperturbed time-reversed Hamiltonian evolution [13].

For a mixed initial state $\overset{\circ}{\rho} = \sum_{\overset{\circ}{\alpha}} p_{\overset{\circ}{\alpha}}|\overset{\circ}{\alpha}\rangle\langle\overset{\circ}{\alpha}|$ ($\sum_{\overset{\circ}{\alpha}} p_{\overset{\circ}{\alpha}} = 1$), fidelity is usually defined as [3]

$$F(t) = \frac{1}{\mathrm{Tr}(\overset{\circ}{\rho}{}^2)}\mathrm{Tr}\left[\rho_0(t)\rho_\varepsilon(t)\right] = \frac{1}{\mathrm{Tr}(\overset{\circ}{\rho}{}^2)}\mathrm{Tr}[\hat{f}^\dagger(t)\overset{\circ}{\rho}\hat{f}(t)\overset{\circ}{\rho}]. \tag{2}$$

We stress that the decay of this quantity has nothing to do with dephasing and is just due to transitions, induced by the echo operator $\hat{f}$, from the initially populated states to all initially empty states. Indeed $F(t) = \frac{1}{\mathrm{Tr}(\overset{\circ}{\rho}{}^2)}\sum_{\overset{\circ}{\alpha},\overset{\circ}{\alpha}'} p_{\overset{\circ}{\alpha}}p_{\overset{\circ}{\alpha}'}W_{\overset{\circ}{\alpha}\overset{\circ}{\alpha}'}$, with transition probabilities $W_{\overset{\circ}{\alpha}\overset{\circ}{\alpha}'} = |\langle\overset{\circ}{\alpha}|\hat{f}|\overset{\circ}{\alpha}'\rangle|^2$.

For the purpose of the present paper we consider instead the quantity $\mathcal{F}(t)$, obtained by directly extending formula (1) to the case of arbitrary mixed initial states $\overset{\circ}{\rho}$. We have:

$$\begin{aligned}\mathcal{F}(t) &= |\sum_{\overset{\circ}{\alpha}} p_{\overset{\circ}{\alpha}} f_{\overset{\circ}{\alpha}}|^2 \\ &= \sum_{\overset{\circ}{\alpha}} p_{\overset{\circ}{\alpha}}^2 F_{\overset{\circ}{\alpha}}(t) + \sum_{\overset{\circ}{\alpha},\overset{\circ}{\alpha}'}(1-\delta_{\overset{\circ}{\alpha}\overset{\circ}{\alpha}'})p_{\overset{\circ}{\alpha}}p_{\overset{\circ}{\alpha}'}f_{\overset{\circ}{\alpha}}(t)f^*_{\overset{\circ}{\alpha}'}(t).\end{aligned} \tag{3}$$

The first term in the second line of this equation is a sum of fidelities $F_{\overset{\circ}{\alpha}} = |f_{\overset{\circ}{\alpha}}|^2$ of the individual pure initial states with weights $p_{\overset{\circ}{\alpha}}^2$, while the second term depends on the relative phases of the fidelity amplitudes. If the initial density matrix $\overset{\circ}{\rho}$ encompasses an area $\Delta \gg \hbar$ in the phase space then the number M of pure states $|\overset{\circ}{\alpha}\rangle$ which form the initial mixed state is large, $M \gg 1$. Then, roughly, $p_{\overset{\circ}{\alpha}} = O(1/M)$ for $\overset{\circ}{\alpha} \leqslant M$ and zero otherwise and therefore the first term is $O(1/M)$, much smaller than the second, interference term, which is $O(1)$ and, therefore, determines the decay of $\mathcal{F}$.

Notice that both functions $F(t)$ and $\mathcal{F}(t)$ reduce to (1) for a pure state ($\overset{\circ}{\rho}{}^2 = \overset{\circ}{\rho}$). However the function $\mathcal{F}$, contrary to F in (2), *accounts for quantum interference* and is expected to retain quantal features even in the deep semiclassical region. The fidelity (2) instead has a well defined classical limit which coincides with the classical fidelity [4, 14, 15] and decays due to the phase flow out of the phase volume initially occupied [16]. In particular, if we start from a uniform distribution over the whole phase space the fidelity (2) never decays. It should also be noticed that the fidelity $\mathcal{F}$ of an initial mixed state is different from the *incoherent* sum of fidelities

$$\overline{F}(t) = \sum_{\overset{\circ}{\alpha}} p_{\overset{\circ}{\alpha}}|f_{\overset{\circ}{\alpha}}|^2 \tag{4}$$

which is typically considered in the literature.

In this paper we show that, due to dephasing induced by the underlying chaotic classical dynamics, the decay of $\mathcal{F}$ can be directly connected to the decay of an appropriate classical correlation function. We would like to stress that, contrary to decoherence produced by an external noise, in our case dephasing is of purely dynamical nature.

Notice that $\mathcal{F}(t)$ is just the quantity which is measured in the Ramsey type experiments performed on cold atoms in optical lattices [17] and in atom optics billiard [18] and proposed for superconducting nanocircuits [19]. Indeed, in these interference experiments one directly accesses the fidelity amplitudes (see [20]), so that $\mathcal{F}(t)$ is reconstructed after averaging these amplitudes over several experimental runs (or many atoms). Each run may differ from the previous one in the external noise realization and/or in the initial conditions drawn, for instance, from a thermal distribution [18].

In order to illustrate the mechanism of dephasing, we consider a nonlinear oscillator driven by a periodic multimode external force $g(t)$. The system's Hamiltonian reads

$$H_0 = \hbar\omega_0 n + \hbar^2 n^2 - \sqrt{\hbar}(a + a^\dagger)g(t), \qquad (5)$$

where $n = a^\dagger a$, $[a, a^\dagger] = 1$. In our units, the time and parameters $\hbar, \omega_0$ as well as the strength of the driving force are dimensionless. The period of the driving force is set to one.

We use below the basis of coherent states $|\alpha\rangle$ which minimize in the semiclassical domain the action-angle uncertainty relation. These states are fixed by the eigenvalue problem $a|\alpha\rangle = \frac{\alpha}{\sqrt{\hbar}}|\alpha\rangle$ where α is a complex number which does not depend on $\hbar$. Since the scalar product equals $\langle\alpha'|\alpha\rangle = \exp\left(-\frac{|\alpha'-\alpha|^2}{2\hbar} + \frac{i}{\hbar}\mathrm{Im}(\alpha'^*\alpha)\right)$, the coherent states become orthogonal in the classical limit $\hbar \to 0$. The Hamiltonian matrix $\langle\alpha'|H_0|\alpha\rangle$ is diagonal in this limit and reduces to a classical Hamiltonian function $H_0^{(c)} = \omega_0|\alpha_c|^2 + |\alpha_c|^4 - (\alpha_c^* + \alpha_c)g(t)$. The complex variables $\alpha_c, i\alpha_c'^*$ are canonically conjugated and are related to the classical action-angle variables I_c, θ_c via $\alpha_c = \sqrt{I_c}e^{-i\theta_c}, \alpha_c^* = \sqrt{I_c}e^{i\theta_c}$. The action satisfies a nonlinear integral equation

$$I_c(t) = \left|\overset{\circ}{\alpha}_c + i\int_0^t d\tau g(\tau)e^{i\varphi_c(\tau)}\right|^2 \equiv |a_c(t)|^2, \qquad (6)$$

where $\alpha_c(t) = a_c(t)\, e^{-i\varphi_c(t)}$ and $\varphi_c(t) = \int_0^t d\tau[\omega_0 + 2I_c(\tau)]$. We have numerically verified that, when the strength of the driving force exceeds some critical value, the classical motion becomes chaotic, the phase $\varphi_c(t)$ becomes random and its autocorrelation function decays exponentially with time:

$$\left|\int d^2\overset{\circ}{\alpha'}_c \mathcal{P}_{\overset{\circ}{\alpha}_c}(\overset{\circ}{\alpha'}{}^*_c, \overset{\circ}{\alpha'}_c)\, e^{i[\varphi_c(t)-\varphi_c(0)]}\right|^2 = \exp\left(-t/\tau_c\right) . \qquad (7)$$

Here we consider the Gaussian distribution

$$\mathcal{P}_{\overset{\circ}{\alpha}_c}(\overset{\circ}{\alpha'}{}^*_c, \overset{\circ}{\alpha'}_c) = (\pi\Delta)^{-1}\exp\left(|\overset{\circ}{\alpha'}_c - \overset{\circ}{\alpha}_c|^2/\Delta\right) \qquad (8)$$

of initial conditions near the point $\overset{\circ}{\alpha}_c$ in the phase plane. Numerical results also show that, as expected from (6, 7), the action grows diffusively: $< I_c(t) >= \overset{\circ}{I}_c + Dt$.

We now analytically evaluate both $F_{\overset{\circ}{\alpha}}(t)$ and $\mathcal{F}(t)$ by treating the unperturbed motion semiclassically. This allows us to compute these two quantities even for quantally strong perturbations $\sigma = \varepsilon/\hbar \gg 1$.

The semiclassical evolution $|\psi_{\overset{\circ}{\alpha}}(t)\rangle = \hat{U}_0(t)|\overset{\circ}{\alpha}\rangle$ of an initial coherent state when the classical motion is chaotic has been investigated in [21]. With the help of Fourier transformation one can linearize the chronological exponent $\hat{U}_0(t)$ with respect to the operator n and finally arrives to the following Feynman's path-integral representation in the phase space

$$\begin{array}{c} |\psi_{\overset{\circ}{\alpha}}(t)\rangle = \int \prod_\tau \frac{d\lambda(\tau)}{\sqrt{4\pi i\hbar}} \\ \times \exp\left\{\frac{i}{4\hbar}\int_0^t d\tau\lambda^2(\tau) - \frac{i}{\hbar}\mathrm{Im}[\beta_\lambda(t)]\right\}|\alpha_\lambda(t)\rangle\ . \end{array} \qquad (9)$$

The functions with the subscript λ are obtained by substituting $2I_c \Rightarrow \lambda$ in the corresponding classical functions: $\alpha_\lambda(t) = \left[\overset{\circ}{\alpha} + i\int_0^t d\tau g(\tau)e^{i\varphi_\lambda(\tau)}\right]e^{-i\varphi_\lambda(t)}$ and $\beta_\lambda(t) = -i\int_0^t d\tau g(\tau)\alpha_\lambda(\tau)$, where $\varphi_\lambda(t) = \int_0^t d\tau[\omega_0 + \lambda(\tau)]$. The initial coherent state $|\overset{\circ}{\alpha}\rangle$ occupies a cell with the volume $\sim \hbar$ in the phase plane (α^*, α). The corresponding normalized density equals $\rho_{\overset{\circ}{\alpha}}(\alpha^*, \alpha) = \frac{1}{\pi\hbar}|\langle\alpha|\overset{\circ}{\alpha}\rangle|^2 = \frac{1}{\pi\hbar}e^{-\frac{|\alpha-\overset{\circ}{\alpha}|^2}{\hbar}}$ and reduces to the Dirac's δ-function in the limit $\hbar \to 0$, thus fixing a unique classical trajectory starting from the point $\overset{\circ}{\alpha}$.

We now compute the fidelity $F_{\overset{\circ}{\alpha}}$ for the case in which the perturbation is a time-independent variation of the linear frequency: $\omega_0 \to \omega_0 + \varepsilon$ [22]. For convenience, we define the fidelity operator in a more symmetric way: $\hat{f}(t) = \hat{U}^\dagger_{(+)}(t)\hat{U}_{(-)}(t)$, where the evolution operators $\hat{U}_{(\pm)}(t)$ correspond to the Hamiltonians $H_{(\pm)} = H_0 \pm \frac{1}{2}\varepsilon n$, respectively. Using Eq. (9) we express $f_{\overset{\circ}{\alpha}}(t)$ as a doubled path integral over λ_1 and λ_2. A linear change of variables $\lambda_1(\tau) = 2\mu(\tau) - \frac{1}{2}\hbar\nu(\tau)$, $\lambda_2(\tau) = 2\mu(\tau) + \frac{1}{2}\hbar\nu(\tau)$ entirely eliminates the Planck's constant from the integration measure. After shifting $\nu(t) \to \nu(t) - \varepsilon/\hbar$ we obtain

$$\begin{array}{c} f_{\overset{\circ}{\alpha}}(t) = \int \prod_\tau \frac{d\mu(\tau)d\nu(\tau)}{2\pi}\exp\left\{i\sigma\int_0^t d\tau\mu(\tau)\right. \\ \left. -i\int_0^t d\tau\mu(\tau)\nu(\tau) + \frac{i}{\hbar}\mathcal{J}\left[\mu(\tau),\nu(\tau)\right] - \frac{1}{2\hbar}\mathcal{R}\left[\mu(\tau),\nu(\tau)\right]\right\} \end{array}$$

where the fuctionals $\mathcal{J}, \mathcal{R}$ equal

$$\begin{array}{c} \mathcal{J} = \hbar\int_0^t d\tau\nu(\tau)|a_\mu(\tau)|^2 + O(\hbar^3), \\ \mathcal{R} = \hbar^2|\int_0^t d\tau\nu(\tau)a_\mu(\tau)|^2 + O(\hbar^4), \end{array} \qquad (10)$$

and vanish in the limit $\hbar = 0$. The quantities with the subscript μ are obtained by setting $\nu(\tau) \equiv 0$ (in particular, $a_\mu(t) = \alpha_\mu(t)e^{i\varphi_\mu(t)}$, with $\alpha_\mu(t) = \left[\overset{\circ}{\alpha} + i\int_0^t d\tau g(\tau)e^{i\varphi_\mu(\tau)}\right]e^{-i\varphi_\mu(t)}$ and $\varphi_\mu(t) = \int_0^t d\tau[\omega_0 + 2\mu(t)]$). In the lowest ("classical") approximation when only the term $\sim \hbar$ from (10) is kept, the ν-integration results in the δ function $\prod_\tau \delta\left[\mu(\tau) - |a_\mu(\tau)|^2\right]$, so that $\mu(t)$ coincides with the classical action $I_c(t)$ [see eq. (6)]. The only contribution comes from the classical trajectory which starts from the point $\overset{\circ}{\alpha}$ and the corresponding fidelity amplitude is simply $f_{\overset{\circ}{\alpha}}(t) = \exp\left[i\sigma\int_0^t d\tau I_c(\tau)\right]$.

The first correction, given by the term $\sim \hbar^2$ in the functional $\mathcal{R}$, describes the quantum fluctuations. Now a bunch of trajectories contributes [21], which satisfy the equation $\mu(t;\delta) = |\delta + a_\mu(t)|^2 - |\delta|^2$ for all δ within a quantum cell $\sim \hbar$. This equation can still be written in the form of the classical equation (6) if we define the classical action along a given trajectory as $\tilde{I}_c(t) = |a_\mu(t)+\delta|^2 = \mu(t;\delta) + |\delta|^2 = I_c\left(\omega_0 - 2|\delta|^2; \mathring{\alpha}^* + \delta^*, \mathring{\alpha}+\delta; t\right)$. For any given δ this equation describes the classical action of a nonlinear oscillator with linear frequency $\omega_0 - 2|\delta|^2$, which evolves along a classical trajectory starting from the point $\mathring{\alpha}+\delta$. One then obtains (up to the irrelevant overall phase factor $e^{-i\omega_0 t/2\hbar}$)

$$f_{\mathring{\alpha}}(t) = \frac{2}{\pi\hbar}\int d^2\delta e^{-\frac{2}{\hbar}|\delta|^2} \exp\left\{ i\frac{\sigma}{2}\left[\tilde{\varphi}_c(t) - \tilde{\varphi}_c(0)\right]\right\}, \tag{11}$$

where the "classical" phase $\tilde{\varphi}_c(t) = \varphi_c(\omega_0 - 2|\delta|^2; \mathring{\alpha}^* + \delta^*, \mathring{\alpha}+\delta; t) = \int_0^t d\tau \left[\omega_0 - 2|\delta|^2 + 2\tilde{I}_c(\tau)\right]$. This expression gives the fidelity amplitude in the "initial value representation" [10, 23]. We stress that the fidelity $F_{\mathring{\alpha}} = |f_{\mathring{\alpha}}|^2$ does not decay in time if the quantum fluctuations described by the integral over δ in (11) are neglected [24].

In order to compute the fidelity $\mathcal{F}$, let us now consider a mixed initial state represented by a Glauber's diagonal expansion [25] $\mathring{\rho} = \int d^2\mathring{\alpha}\mathcal{P}_{\mathring{\alpha}_c}(\mathring{\alpha}^*, \mathring{\alpha})|\mathring{\alpha}\rangle\langle\mathring{\alpha}|$ whith the Gaussian weight function (8) which covers a large number of quantum cells, $\Delta \gg \hbar$. Then $\mathcal{F}(t;\mathring{\alpha}_c) = |f(t;\mathring{\alpha}_c)|^2$, where

$$\begin{gathered} f(t;\mathring{\alpha}_c) \equiv \int d^2\mathring{\alpha}\mathcal{P}_{\mathring{\alpha}_c}(\mathring{\alpha}^*, \mathring{\alpha}) f_{\mathring{\alpha}}(t) \\ \approx \tfrac{2}{\pi\hbar}\int d^2\delta e^{-\frac{2}{\hbar}|\delta|^2}\int d^2\mathring{\alpha}\mathcal{P}_{\mathring{\alpha}_c+\delta}(\mathring{\alpha}^*, \mathring{\alpha}) e^{i\frac{\varepsilon}{2\hbar}[\overline{\varphi}_c(t) - \overline{\varphi}_c(0)]}, \end{gathered}$$

with $\overline{\varphi}_c(t) = \varphi_c(\omega_0 - 2|\delta|^2; \mathring{\alpha}^*, \mathring{\alpha}; t)$. The inner integral over $\mathring{\alpha}$ is a classical correlation function. In the regime of classically chaotic motion this correlator will not sensibly depend on small variations either of the value of the linear frequency or of the exact location of the initial distribution in the classical phase space. Therefore we can fully disregard the δ-dependence of the integrand, thus obtaining

$$f(t;\mathring{\alpha}_c) \approx \int d^2\mathring{\alpha}\mathcal{P}_{\mathring{\alpha}_c}(\mathring{\alpha}^*, \mathring{\alpha}) \exp\left\{ i\frac{\sigma}{2}\left[\varphi_c(t) - \varphi_c(0)\right]\right\}. \tag{12}$$

This is the main result of our paper and directly relates the *quantum* fidelity decay to the decay of correlation functions of *classical* phases (see Eq. (7)). No quantum feature is present in the r.h.s. of (12).

The decay pattern of the function $\mathcal{F}(t) = |f(t;\mathring{\alpha}_c)|^2$ depends on the value of the parameter $\sigma = \varepsilon/\hbar$. In particular, it is easy to show from Eq. (12) that, for $\sigma \ll 1$, we recover the well known Fermi Golden Rule (FGR) regime [26]. When the strength σ, roughly, exceeds one, then the square modulus of the classical correlation function in the right hand side of Eq. (12) does not depend on σ [27]. The decay of this correlation function (and, therefore, of fidelity) is tightly related to the local instability of the chaotic classical motion. However, the decay rate is not necessarily the Lyapunov exponent (it is worth noting in this connection that the Lyapunov exponent diverges in our driven nonlinear oscillator model).

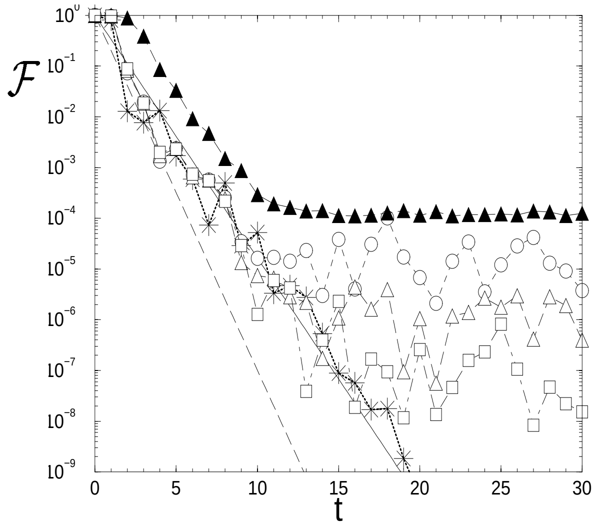

FIG. 1: Decay of the fidelity $\mathcal{F}$ for the kicked rotator model with $K = 10$, perturbation strength $\epsilon/\hbar = 1.1$, $\hbar = 3.1\times10^{-3}$ (circles), 7.7×10^{-4} (empty triangles), and 1.9×10^{-4} (squares). Full triangles show the average fidelity $\overline{F}$ for $\hbar = 7.7\times10^{-4}$. Stars give the decay of the classical angular correlation function. The straight lines denote exponential decay with rates given by the Lyapunov exponent $\lambda \approx \ln(K/2) = 1.61$ (dashed line) and by the exponent $\Lambda = 1.1$ [9] (solid line).

As a second example, we consider the kicked rotator model [28], described by the Hamiltonian $H = \frac{p^2}{2} + K\cos\theta\sum_m \delta(t-m)$, with $[p,\theta] = -i\hbar$. The classical limit corresponds to the effective Planck constant $\hbar \to 0$. We consider this model on the torus, $0 \le \theta < 2\pi$, $-\pi \le p < \pi$. The fidelity $\mathcal{F}$ is computed for a static perturbation $\epsilon p^2/2$, the initial state being a mixture of Gaussian wave packets uniformly distributed in the region $0.2 \le \theta/2\pi \le 0.3$, $0.3 \le p/2\pi \le 0.4$. In Fig. 1 we show the decay of $\mathcal{F}(t)$ in the semiclassical regime $\hbar \ll 1$ and for a quantally strong perturbation $\epsilon/\hbar \sim 1$. It is clearly seen that the fidelity $\mathcal{F}$ follows the decay of the classical angular correlation function $|\langle \exp\{i\gamma[\theta(t) - \theta(0)]\}\rangle|^2$ (with the fitting constant $\gamma = 2$) up to the Ehrenfest time scale $\propto \ln(1/\hbar)$. We remark that $\mathcal{F}$ decays with a rate Λ different from the Lyapunov exponent.

We point out that the classical autocorrelation function in the r.h.s. of Eq. (12) reproduces not only the slope but also the overall decay of the function $\mathcal{F}$. The classical dynamical variable that appears in this autocorrelation function depends on the form of the perturbation. Therefore the echo decay, even in a classically chaotic system in the semiclassical regime and with quantally strong perturbations, is to some extent perturbation-dependent.

In Fig. 1 we also show the fidelity $\overline{F}$, obtained after averaging the fidelities $F_{\mathring{\alpha}} = |f_{\mathring{\alpha}}|^2$ of the pure Gaussian states $|\mathring{\alpha}\rangle$ building the initial mixture. As discussed above, the incoherent sum of fidelities $\overline{F}$ is very different in nature from $\mathcal{F}$. Nevertheless, due to dephasing induced by classical chaos, the decays of $\mathcal{F}$ and of $\overline{F}$ are intimately connected: both quantities decay with the same rate Λ but the decay of $\overline{F}$ is delayed by a time t_d. The function $\overline{F}$ is given by the sum of a mean value part ($\mathcal{F} = |\overline{f}|^2$) and a fluctuating part,

$$\overline{F}(t) = \mathcal{F}(t) + \overline{\left|f(t) - \overline{f(t)}\right|^2}, \tag{13}$$

and the fluctuating term (which vanishes in the FGR regime) is responsible for the delayed decay of $\overline{F}$ with respect to $\mathcal{F}$. Analytical arguments [29] as well as numerical results indicate that the delay time is $t_d \sim \frac{1}{\Lambda}\ln(\frac{\Delta}{\hbar})$, with Δ area of the initial distribution and Λ decay rate of the classical correlation function which governs the decay of $\mathcal{F}$. Note that the expected saturation values of $\overline{F}$ and $\mathcal{F}$ are $1/N$ and $1/(NM)$, respectively, where N is the number of states in the Hilbert space and M the number of quantum cells inside the area Δ. This expectation is a consequence of the randomization of phases of the fidelity amplitudes and is borne out by the numerical data shown in Fig. 1.

In this paper we have demonstrated that the decay of the quantum fidelity $\mathcal{F}$ is determined by the decay of classical correlation functions, which are totally unrelated to quantum phases. This quantum dephasing is a consequence of internal dynamical chaos and takes place in absence of any external environment. We may therefore conclude that the underlying internal dynamical chaos produces a dephasing effect similar to the decoherence due to the environment.

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$$\hat{f}(t) = T\exp\left[-i\frac{\varepsilon}{\hbar}\int_0^t d\tau\mathcal{H}(\tau)\right].$$

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$$\chi_2(t) = \int_0^t d\tau_1\int_0^t d\tau_2\langle[I_c(\tau_1) - \langle I_c(\tau_1)\rangle] \\ \times[I_c(\tau_2) - \langle I_c(\tau_2)\rangle]\rangle \equiv \int_0^t d\tau_1\int_0^t d\tau_2 K_I(\tau_1,\tau_2)\ .$$

Assuming that the classical autocorrelation function decays exponentially, $K_I(\tau_1,\tau_2) = \langle(\Delta I_c)^2\rangle\exp(-|\tau_1-\tau_2|/\tau_I)$ with some characteristic time τ_I, we obtain $\chi_2(t) = 2\langle(\Delta I_c)^2\rangle\tau_I t = 2Kt$ for the times $t > \tau_I$ and arrive, finally, to the FGR decay law $\mathcal{F}(t;\overset{\circ}{\alpha}_c) = \exp(-2\sigma^2 Kt)$ [4, 6, 7]. Here $K = \int_0^\infty d\tau K_I(\tau,0) = \langle(\Delta I_c)^2\rangle\tau_I$. The FGR approximation is valid as long as the contributions of the higher connected correlators $\chi_{\kappa\geq 4}(t)$ remain small, that is, for $\sigma \ll 1$.
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