The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1675 - 7

Workshop on
Noise and Instabilities in Quantum Mechanics

3 - 7 October 2005

Static vs dynamical imperfections in a (quantum computing) spin system

Saverio PASCAZIO
Dipartimento di Fisica
Universita' di Bari
I-70126 Bari
ITALY

Static vs dynamical imperfections in a (quantum computing) spin system

Paolo Facchi, *Saverio Pascazio*
Dipartimento di Fisica, Università di Bari, Italy

Simone Montangero, *Rosario Fazio*
Scuola Normale Superiore, Pisa, Italy

ICTP, Trieste
3 October 2005

Collaborations (related topics, last 10 years)

G. Badurek (Vienna)
P. Facchi (Bari)
G. Falci (Catania)
R. Fazio (Pisa)
G. Florio (Bari)
V. Gorini (Como)
H. Hradil (Olomouc)
D. Lidar (Toronto)
G. Marmo (Napoli)
M. Namiki (Waseda)
S. Montangero (Pisa)

H. Nakazato (Waseda)
I. Ohba (Waseda)
J. Perina (Olomouc)
H. Rauch (Vienna)
J. Rehacek (Olomouc)
A. Scardicchio (MIT)
L. S. Schulman (Clarkson)
E. C. G. Sudarshan (Texas)
S. Tasaki (Waseda)
K. Yuasa (Waseda)

Static vs dynamic imperfections

Benenti, Casati, Montangero, Shepelyansky 2001

"… static imperfections […] are therefore more dangerous for quantum computation."

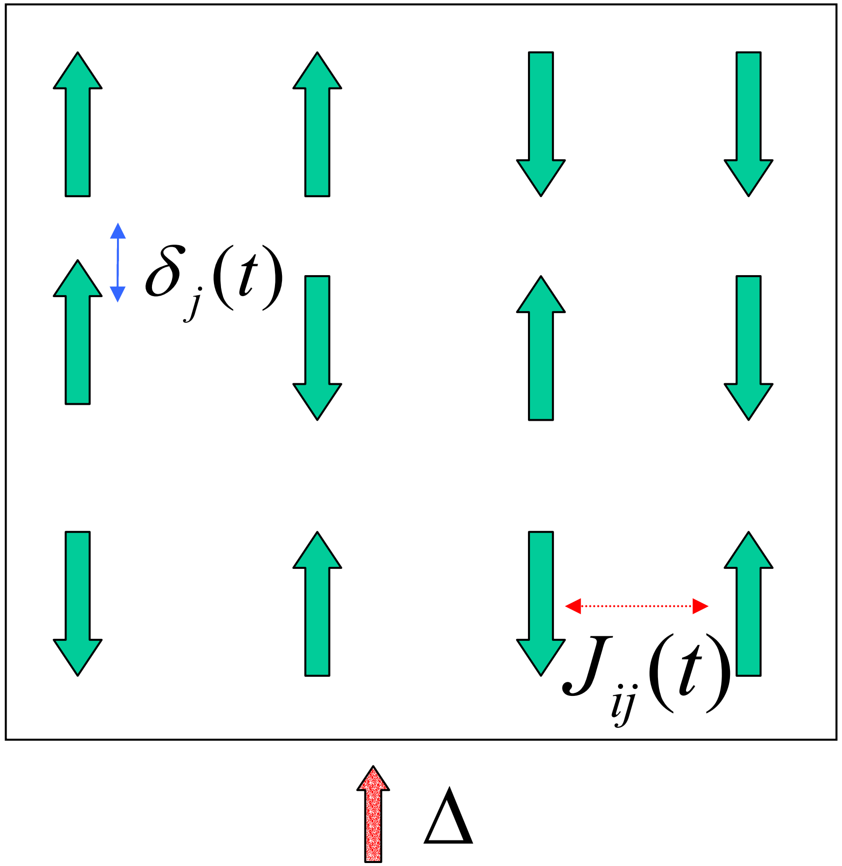
Montangero - Benasque (Spain) – Facchi

Meaning of static/dynamic: **TIMESCALES ?**

Static vs dynamic imperfections

$$H(t) = \Sigma_{j=1}^{n}[\Delta_0 + \delta_j(t)]\sigma_z^{(j)} + \Sigma_{\langle i,j\rangle} J_{ij}(t)\sigma_x^{(i)}\sigma_x^{(j)}$$

$\delta_i(t)$'s uniformly distributed in interval $[-\delta/2, \delta/2]$ and $J_{ij}(t)$'s in interval $[-J, J]$ (zero means and variances $\delta^2\sigma^2$ and $4J^2\sigma^2$, respectively, $\sigma^2 = 1/12$)



Initial state:

$$\sum_j \sigma_z^{(j)}|\Psi(t=0)\rangle = 0$$

(central band)

Dynamics: definitions

Fidelity

$$F \equiv \langle \Psi(t) | \Psi(t=0) \rangle$$

Error

$$E \equiv 1 - F$$

Total (final) time

$$T = N\tau = 25$$

contains static imperfections!

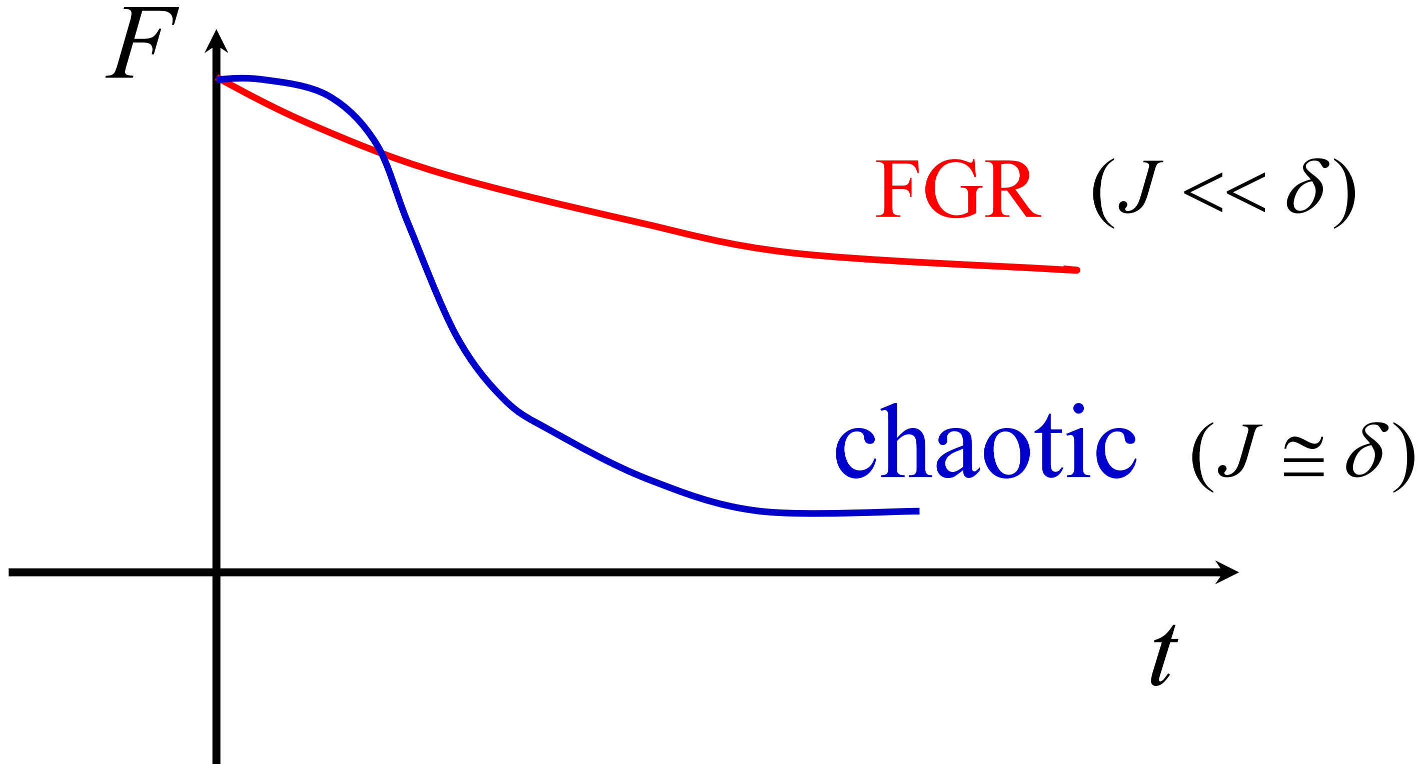
Average (over time or realizations)

$$H_0 = \langle H(t) \rangle$$

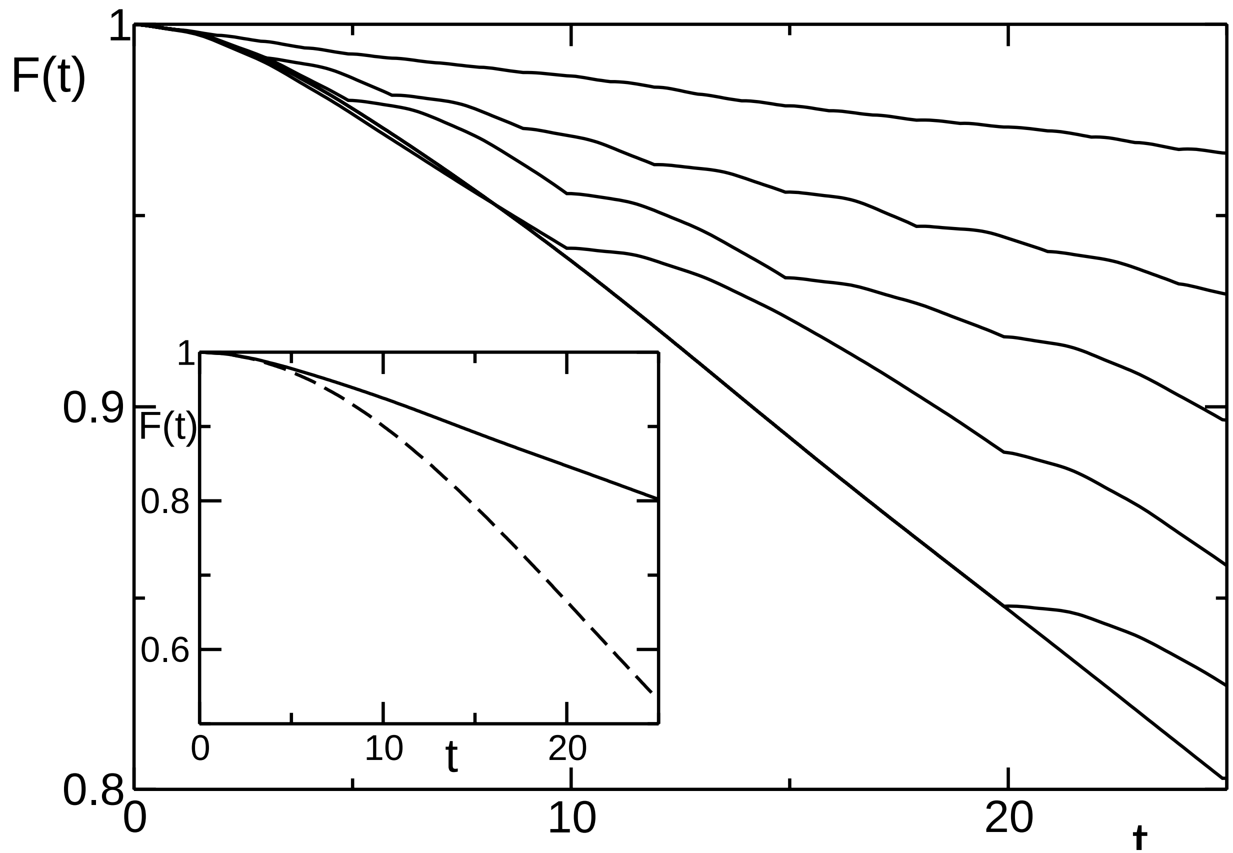
Evolution (theorem)!

$$U_\tau(t) \xrightarrow{\tau \to 0} e^{-iH_o t}$$ (in probability)

Static case (memorandum)



Fidelity vs t



Fidelity as a function of time for $n = 14$ qubits in the FGR regime ($J = 2 \cdot 10^{-2}, \delta = 4 \cdot 10^{-1}$) and from top to bottom $\tau = 1, 3, 5, 10, 20, 25$ (static imperfections). Inset: Fidelity as a function of time in the ergodic ($J = \delta = 2 \cdot 10^{-2}$, dashed line), and in the FGR regime (full line): note the (common) short-time quadratic law.

Main result

Explicit calculation of the error to order J^2:

$$\boxed{E_t(\tau) = 4J^2\sigma^2\left(Ng(\tau) + g(\Delta t)\right),} \tag{3}$$

where $\boxed{t = N\tau + \Delta t,}$ with N integer, $0 \leq \Delta t < \tau$, and

$$\begin{aligned} g(\tau) &= 2\int_0^\tau ds \int_0^s du\, \mathrm{sinc}^2(\delta u)\left[n_{\uparrow\downarrow} + n_{\uparrow\uparrow}\cos(4\Delta_0 u)\right] \\ &= n_{\uparrow\downarrow} f(\delta\tau)/\delta^2 + n_{\uparrow\uparrow} D_{\delta\tau} f(2\Delta_0\tau)/\delta^2\ , \end{aligned} \tag{4}$$

$n_{\uparrow\uparrow}$ ($n_{\uparrow\downarrow}$) being the number of nearest-neighbor parallel (antiparallel) pairs in the initial state, $\mathrm{sinc}(x) = (\sin x)/x$, $D_y f(x) = [f(x+y) - 2f(x) + f(x-y)]/2$ and

$$f(x) = \mathrm{Ci}(2x) + 2x\mathrm{Si}(2x) - \ln(2x) + \cos(2x) - \gamma - 1, \tag{5}$$

$\mathrm{Ci}(z), \mathrm{Si}(z)$ and $\gamma \simeq 0.577$.
Due to the convexity of $g(\tau)$, the error $\boxed{E_t(\tau) \leq 4J^2\sigma^2 t g(\tau)/\tau,}$ the inequality is saturated when $t/\tau = N$, thus providing a simple interpolation of (3).

varying τ

For $\tau\delta \ll 1$

$$\boxed{g(\tau) \simeq \tau^2 \left[n_{\uparrow\downarrow} + n_{\uparrow\uparrow}\mathrm{sinc}^2(2\Delta_0\tau)\right] \ ,} \tag{6}$$

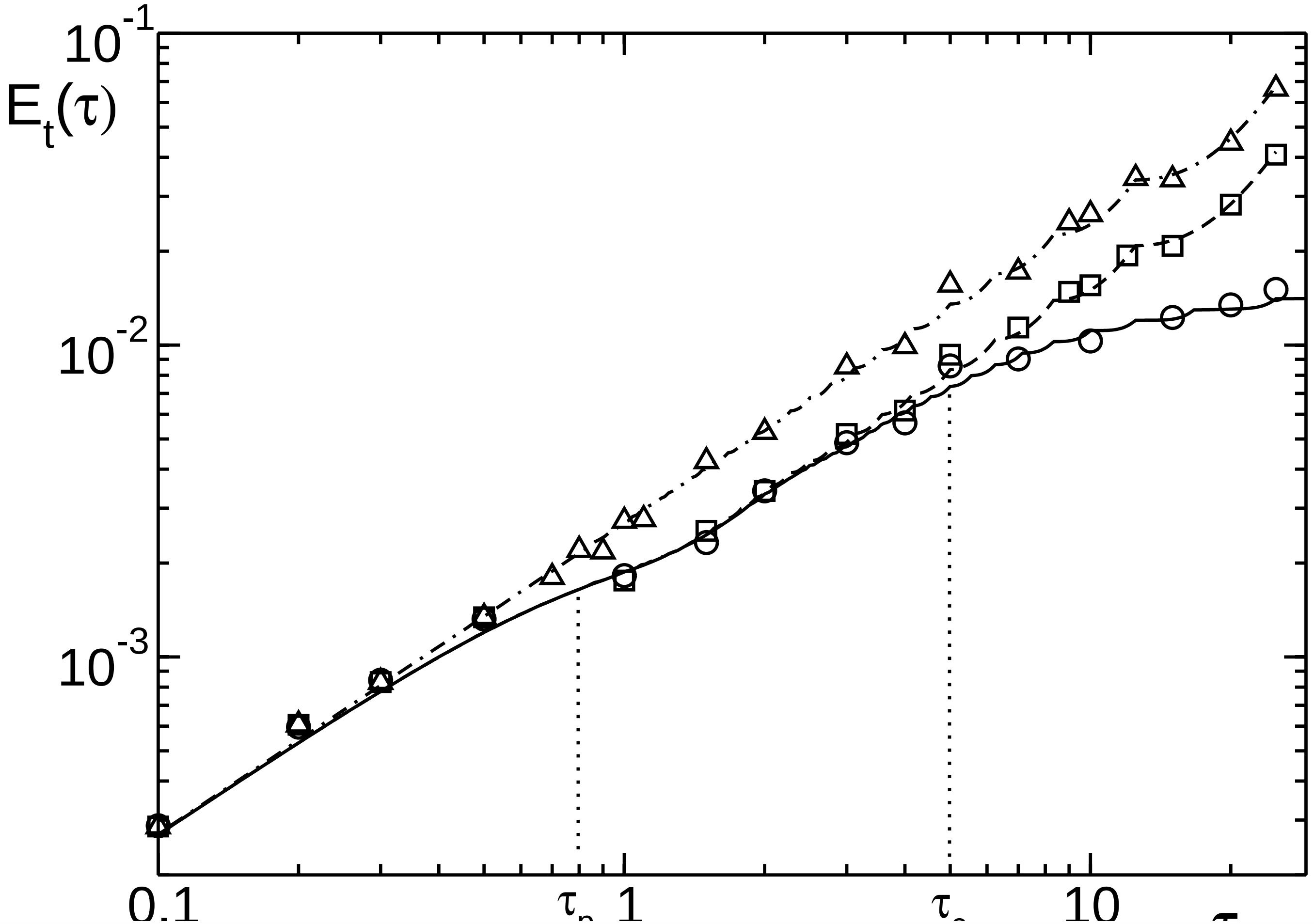
which yields $g(\tau) \simeq n_c\tau^2$ for $\tau \lesssim \tau_p = \pi/4\Delta_0$ and $g(\tau) \simeq n_{\uparrow\downarrow}\tau^2$ (ergodic regime) for $\tau \gtrsim \tau_p$, where the total number of links $\boxed{n_c = n_{\uparrow\downarrow} + n_{\uparrow\uparrow}.}$

On the other hand, when $\tau\delta \gg 1$,

$$\boxed{g(\tau) \simeq \frac{n_{\uparrow\downarrow}}{\delta^2}\left[\pi\delta\tau - \ln(2\delta\tau) - \gamma - 1\right]}\ . \tag{7}$$

(limit $\tau\delta \gg 1$ within the range of applicability of Eq. (3) only in the FGR regime.)

Error vs τ



Error E as a funct
5×10^{-3}, $n_{\uparrow\downarrow} = 8$ (
(circles). The fits
$\Delta_0 = 1$ and $(n_{\uparrow\downarrow},$
at τ_c is shown onl

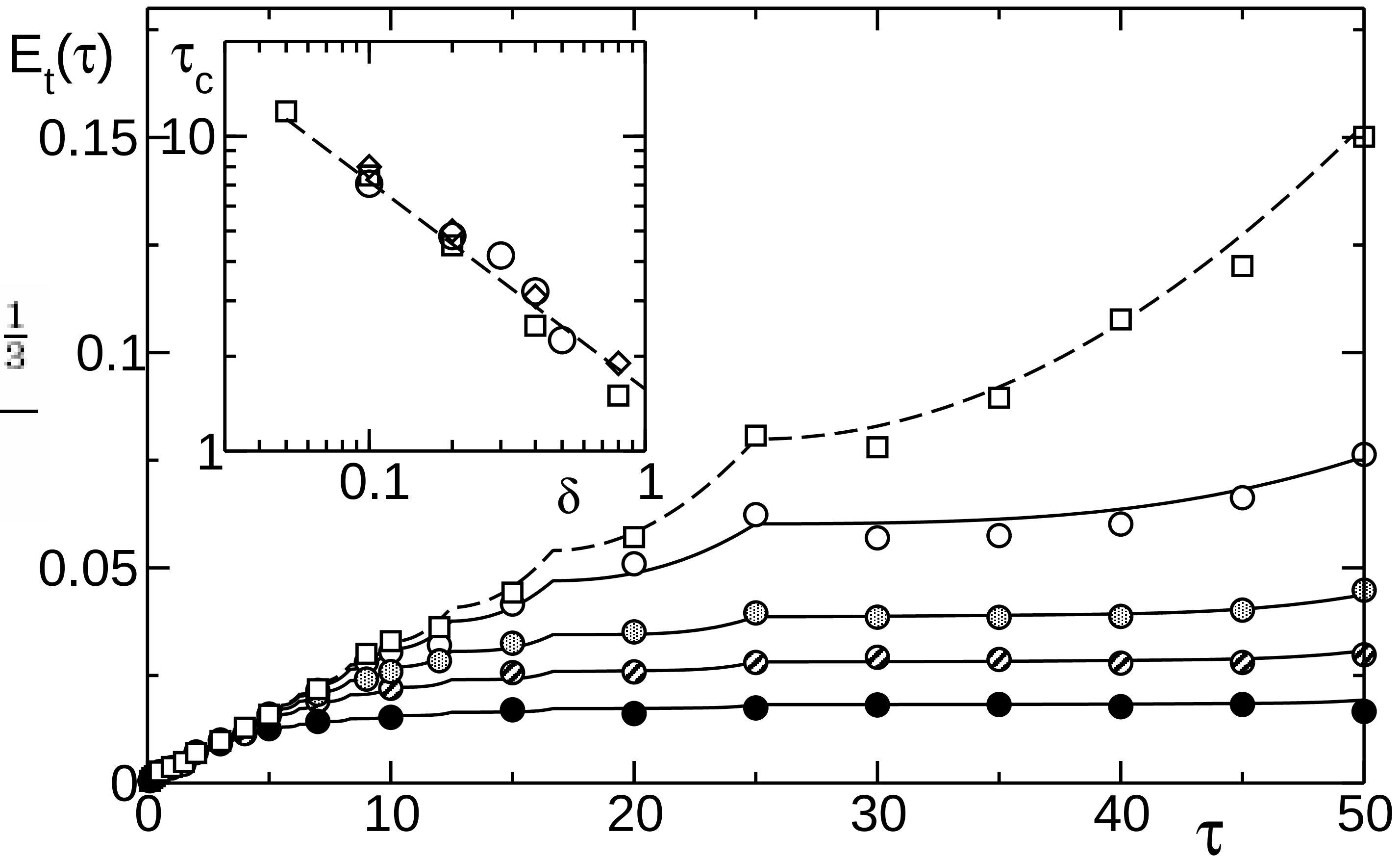
$$E_t(\tau) \simeq 4J^2\sigma^2 t \begin{cases} n_c\tau & \tau < \tau_p \quad \text{(all regimes)} \\ n_{\uparrow\downarrow}\tau & \tau_p < \tau < \tau_c \quad \text{(all regimes)} \\ n_{\uparrow\downarrow}\tau & \tau > \tau_c,\ J \simeq \delta \quad \text{(ergodic)} \\ n_{\uparrow\downarrow}\pi/\delta & \tau > \tau_c,\ J < \delta/n \quad \text{(FGR)} \end{cases}$$

A few comments

- Error scales like τ
- Threshold at τ critical (def: dynamical vs static)
- In the *static* case fidelity is a good chaos indicator
- In the *dynamical* case fidelity ceases to be a chaos indicator

Error at t

$$\tau_c = \frac{(18\varepsilon t)^{\frac{1}{3}}}{\delta^{\frac{2}{3}}}$$



Error at time $t = 50$, for $n = 10, J = 5 \cdot 10^{-3}$ and different δ values. The squares represent the ergodic regime $\delta = J$. The FGR regime is plotted for $\delta = 1, 2, 3, 5 \cdot 10^{-1}$ (empty, pointed, dashed, full circles respectively). Inset: τ_c as a function of δ for $n = 10, 12, 14$ (circles, squares and diamonds respectively). The dashed line is proportional to $\delta^{-2/3}$.

Theorem

General framework :

$$\boxed{H(t) = H_0 + \xi(t)V,} \tag{11}$$

stochastic process with independent increments $\xi(t) = \sum_{k=1}^{N} \chi_{[k\tau-\tau,k\tau)}(t)\, \xi_k$, χ_A characteristic function of the set A and $\{\xi_k\}_k$ independent and identically distributed random variables, with expectations $E[\xi_k] = 0,\ \mathrm{Var}[\xi_k] = E[\xi_k^2] = \sigma^2 < \infty$.

Time evolution operator over the total time $t = \tau N$

$$U_N(t) = \prod_{k=1}^{t/\tau} \exp[-i(H_0 + \xi_k V)\tau] \tag{12}$$

(time-ordered product understood).

H_0 and V bounded operators, so that $U(t)$ is a norm-continuous one-parameter group of unitaries and all our subsequent estimates are valid in norm.

Theorem

Consequence of weak law of large numbers

$$U(t) \equiv P\!-\!\lim_{N\to\infty} U_N(t) = \exp(-iH_0 t), \tag{14}$$

in the following sense

$$\lim_{N\to\infty} P\left(\|U_N(t) - \exp(-iH_0 t)\| \geq \varepsilon\right) = 0, \tag{15}$$

uniformly in each compact time interval. If the term $\xi(t)V$ is viewed as exemplifying the effect of (dynamical) error-inducing disturbances, the above result physically implies that the effects of the errors are wiped out if their characteristic frequency τ^{-1} is sufficiently fast. This defines the purely dynamical regime.

Finite N ?

Central limit theorem: limiting random variable $\eta = \lim_{N\to\infty} \sum_{k=1}^{N} \xi_k/\sqrt{N}$ exists and is Gaussian: $E[\eta] = 0$ and $E[\eta^2] = \sigma^2$. For $N \gg 1$

$$U_N(t) \sim \exp\left(-iH_0 t\right) \exp\left(-i\eta V t/\sqrt{N}\right). \tag{16}$$

For *fixed* τ, effective interaction strength $\epsilon_{\text{eff}} = \sigma\|V\|/\sqrt{N} \propto \sigma\|V\|\sqrt{\tau}$.

For intermediate values of N, Eq. (16) is no longer valid. However, by assuming $V \ll H_0$ (e.g. in norm), perturbation V is replaced by

$$\bar{V}(\tau) = \frac{1}{\tau}\int_0^\tau dt\; e^{iH_0 t} V e^{-iH_0 t}\;, \tag{17}$$

so that, for $\tau\|H_0\| \gtrsim 2\pi$, the effective perturbation becomes

$$\bar{V}(\tau) \to V_Z = \sum_k P_k V P_k, \tag{18}$$

where P_k are the eigenprojections of H_0 ($H_0 = \sum \lambda_k P_k$). This phenomenon is reminiscent of the quantum Zeno subspaces.

Remark:

Extension to family of independent stochastic processes with zero mean and finite variances: straightforward.

$$H(t) = H_0 + \delta\, \boldsymbol{\xi}_0(t) \cdot \boldsymbol{V}_0 + 2J\, \boldsymbol{\xi}(t) \cdot \boldsymbol{V}, \tag{19}$$

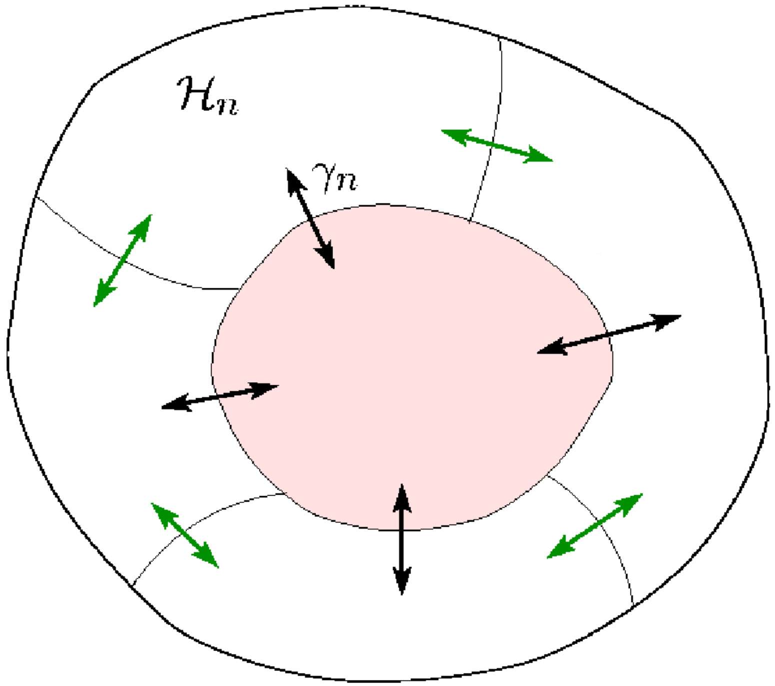
Applications in solid state physics
(Josephson junctions, quantum dots)

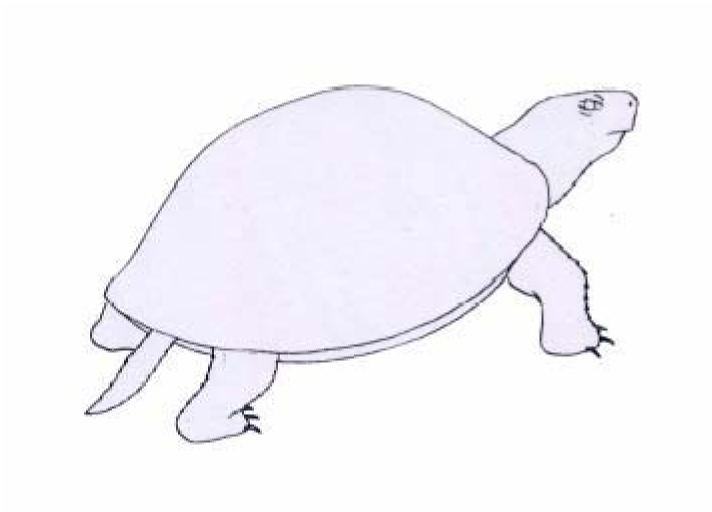
Facchi, Montangero, Fazio, Pascazio,
Phys. Rev. A **71**, 060306(R) (2005)

Zeno subspaces

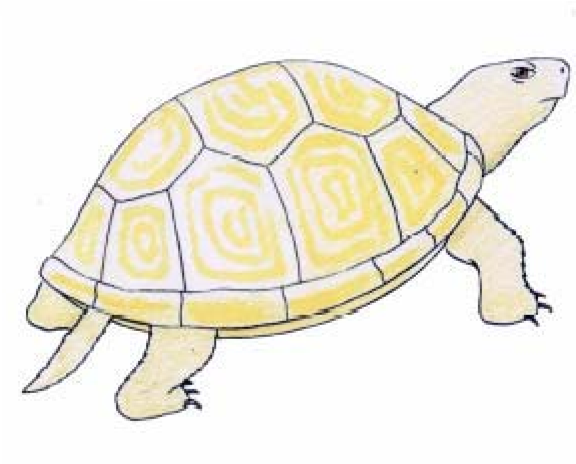
↓

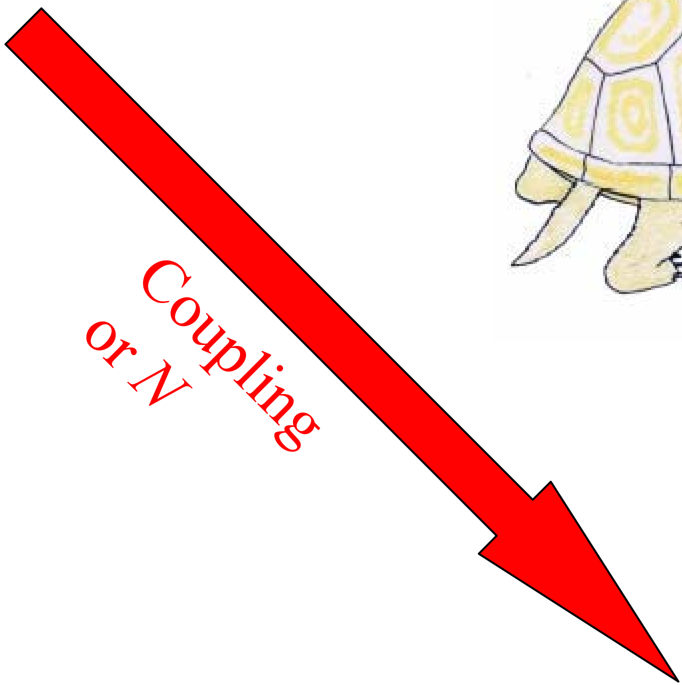
Different couplings yields different subspaces and different noises

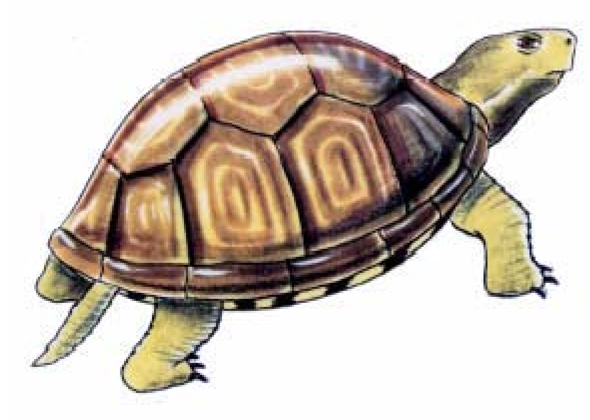




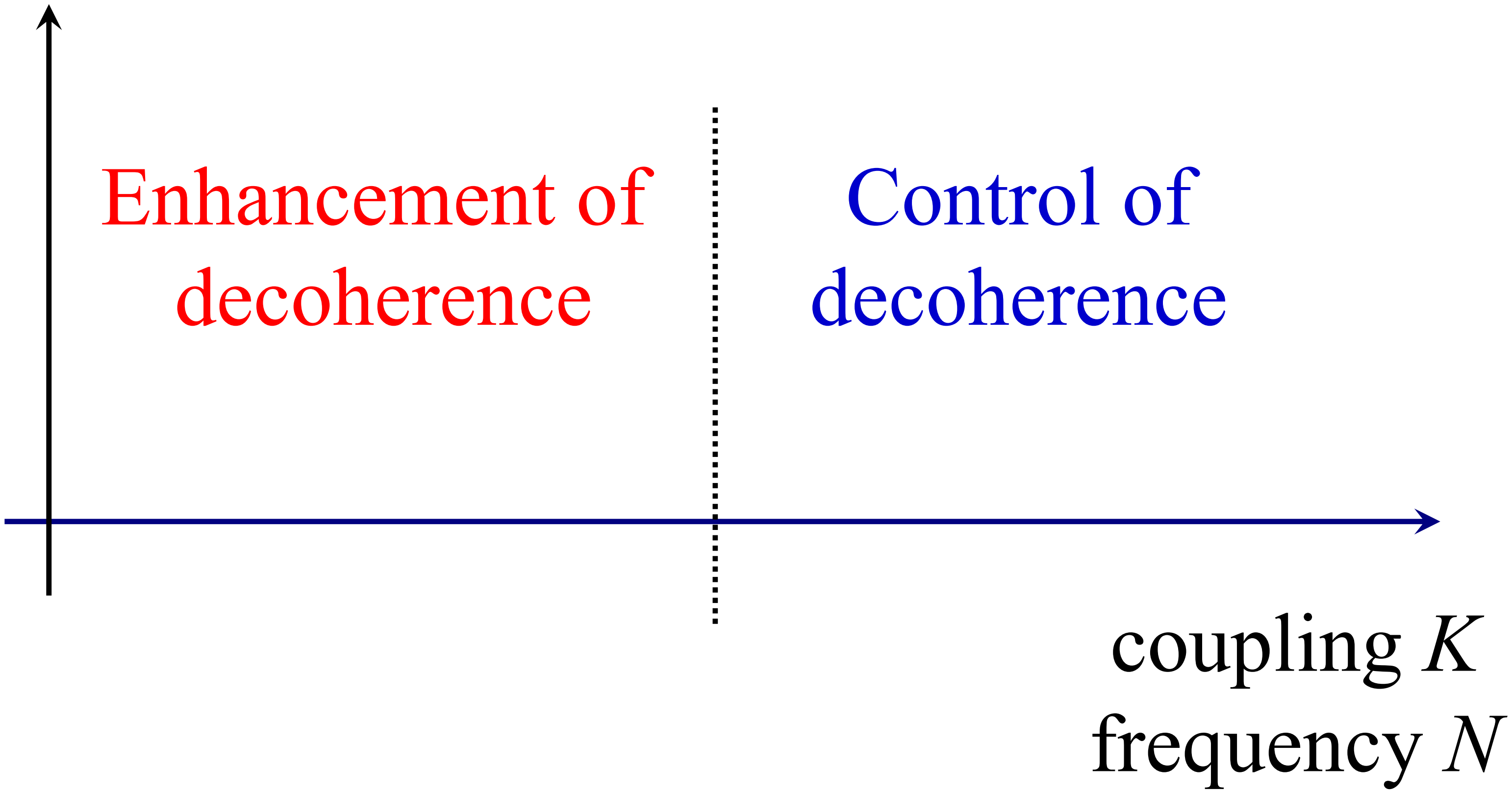
Dynamical superselection sectors







Main lesson: optimize timescales and…



Understand and suppress decoherence

Decoherence-free subspaces
Palma, Suominen and Ekert (1996)
Duan and Guo (1997)
Zanardi and Rasetti (1997)
Lidar, Chuang and Whaley (1998)
Viola, Knill and Lloyd (1999)
Vitali and Tombesi (1999, 2001)
Beige, Braun, Tregenna and Knight (2000)
…
Tasaki, Tokuse, Facchi, Pascazio (2002)
Facchi, Lidar and Pascazio (2003)
Fachi, Tasaki, Nakazato, Pascazio (2004)

Benenti, Casati, Montangero, Shepelyansky (2001)

Vitali, Tombesi, Milburn (1997, 1998)
Fortunato, Raimond, Tombesi, Vitali (1999)
Kofman, Kurizki (2001)
Agarwal, Scully, Walther (2001)

Calarco, Datta, Fedichev, Pazy, Zoller,
"Spin-based all-optical quantum computation with quantum dots: understanding and suppressing decoherence" (2003)

Falci (2003)
Falci, D'Arrigo, Mastellone, Paladino (2004)
Brion, Harel, Kebaili, Akulin, Dumer (2004)
Zhang, Zhou, Yu, Guo (2004)