



The Abdus Salam
International Centre for Theoretical Physics



SMR.1676 - 35

8th Workshop on Non-Linear Dynamics and Earthquake Prediction

3 - 15 October, 2005

Earthquake Prediction (a Physical Approach) Parts I & II

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These are preliminary lecture notes, intended only for distribution to participants

EARTHQUAKE PREDICTION A PHYSICAL APPROACH

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PREDICTION OF NATURAL HAZARDS

Predicting natural hazards resembles the game of croquet in *Alice in Wonderland*, where the ball was a live hedgehog who would not stand still or go where the players intended. We can make statistics about the habits of hedgehogs, but we are still far from understanding the rules of the game.

(Cinna Lomnitz)

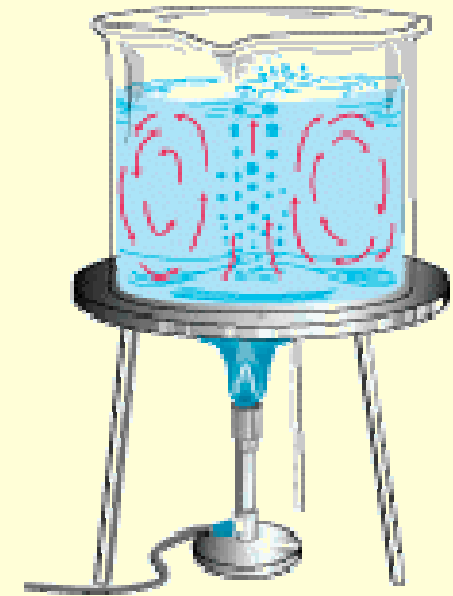
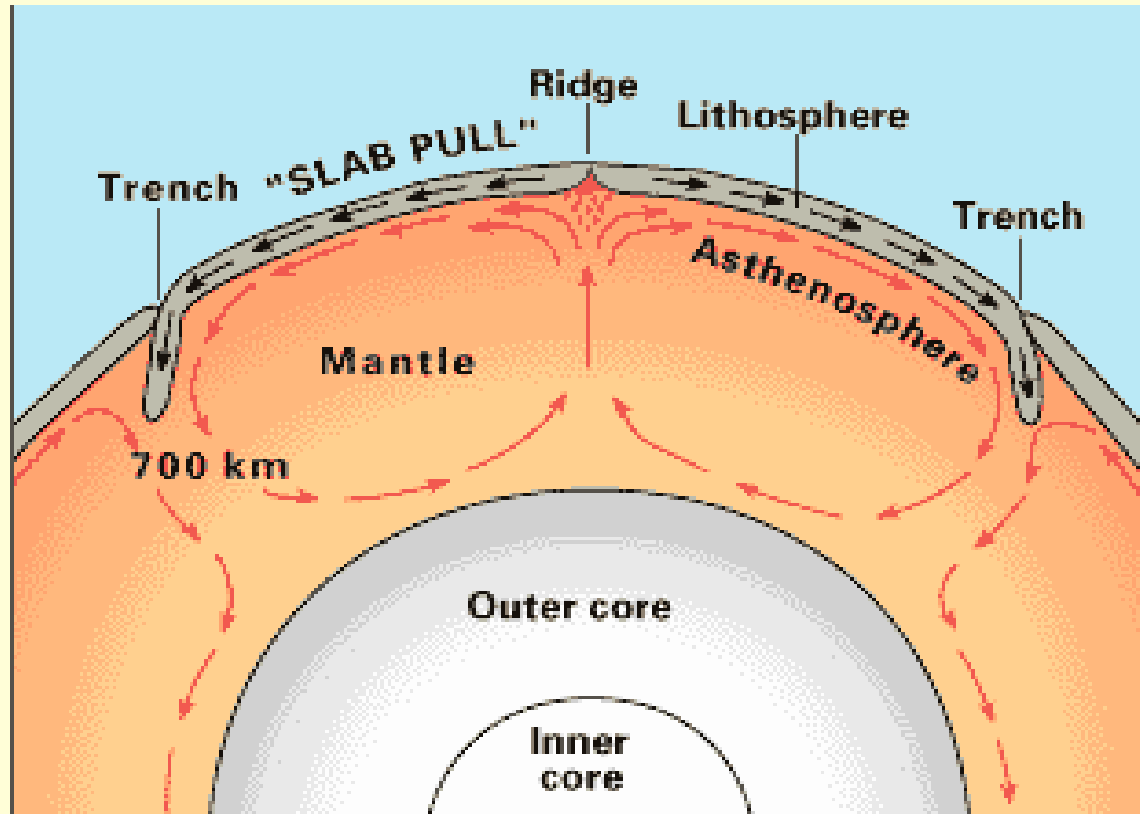


EARTHQUAKES AS CATASTROPHES

- From a mathematical point of view, an earthquake may be associated to a **catastrophe**, a sudden change that appears as the response of a system to a smooth change in the external conditions (Arnold, 1984).
- The sudden change that appear in some parameters can be modeled as a step transition, a discontinuity or a bifurcation.
- We can also talk about a volcanic eruption as a **catastrophic event**, due to the looses it may cause. Here the need for its **forecasting**.



DYNAMICS OF THE MANTLE

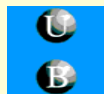


Rayleigh-Benard convection

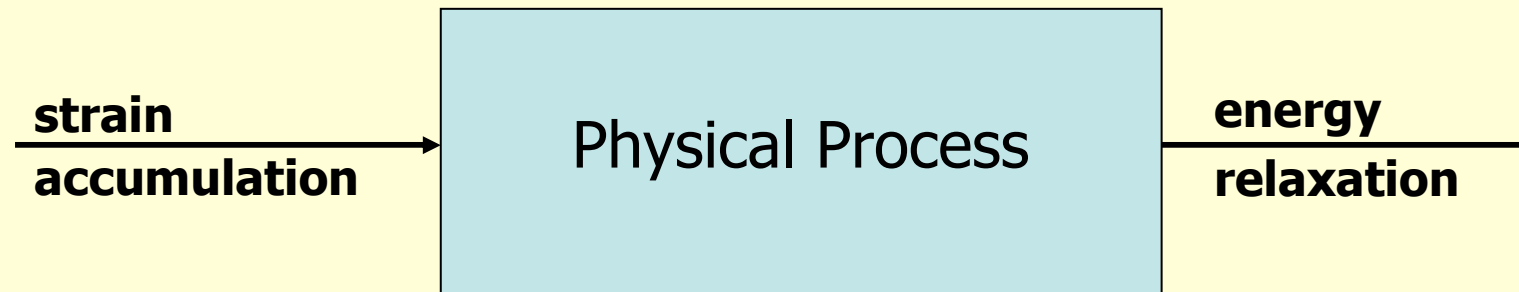


PHYSICAL SCENARIO

- Plate Tectonics provides a mechanism to explain earthquake occurrence as a **recurrent phenomenon**, defining cycles.
- The physical scenario of earthquake occurrence is that of a **dissipative complex system**, with an input (at a variable rate) of energy, that is suddenly released (*failure*). This process repeats again and again, defining earthquake cycles.
- The framework of earthquake processes is that of **far from equilibrium systems**.



THE LITHOSPHERE AS A COMPLEX SYSTEM



Physical process: **medium** response to strain accumulation. The process ends when a failure occurs, i.e., when a threshold in the stress-strain field has been reached. Strength is controlled by a great multitude of interdependent mechanisms (**which ones ?**), which brings the system to an **instable** equilibrium.

On a time scale relevant to earthquake prediction, 10^2 years or less, these factors turn the lithosphere into a **hierarchical dissipative complex system**. A prominent feature of complex systems is the persistent reoccurrence of abrupt overall changes called **critical transitions** or **critical phenomena**: the occurrence of **earthquakes**.



MEDIUM

The medium, the lithosphere, can be viewed as a complex hierarchical system. The complexity is due to:

The hierarchical structure of the lithosphere, composed of

- Blocks
- Boundary zones
- Nodes

The systems of boundaries and nodes are usually known as **faults networks**.

Apart of its hierarchical structure (geometric characteristic), the lithosphere is highly **inhomogeneous** in its rigidity (dynamical aspect), defining patches of different characteristic lengths.



THE LITHOSPHERE AS A HIERARCHICAL SYSTEM

Boundary zone	Block size (km)
Fault zone	$10^4 - 10^2$
Fault	$10^1 - 10^{-2}$
Crack	$10^{-3} - 10^{-5}$
Microcrack	$10^{-6} - 10^{-7}$
Interface	$10^{-8} - \dots$



INSTABILITIES

Physical Instabilities:

Rehbinder effect or stress corrosion

Nonlinear filtration

Fingers of fluids

Dissolution of rocks

Petrochemical transitions

Sensitivity of dynamic friction to local physical environment

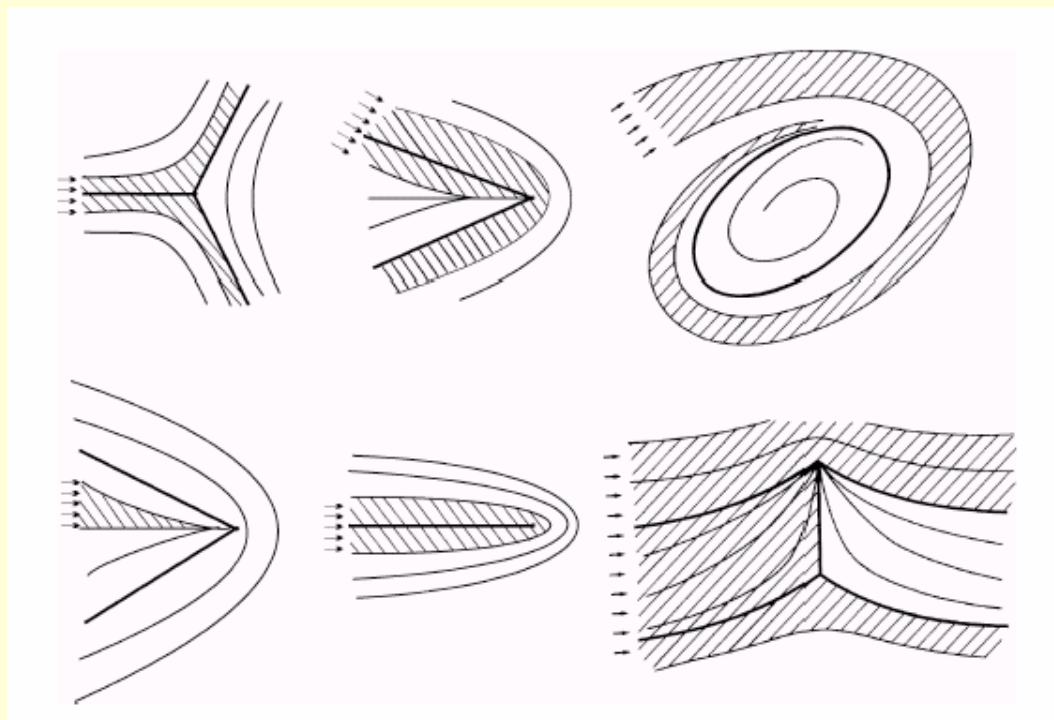
The impact of pressure and temperature on the above mechanisms

Geometrical Instabilities:

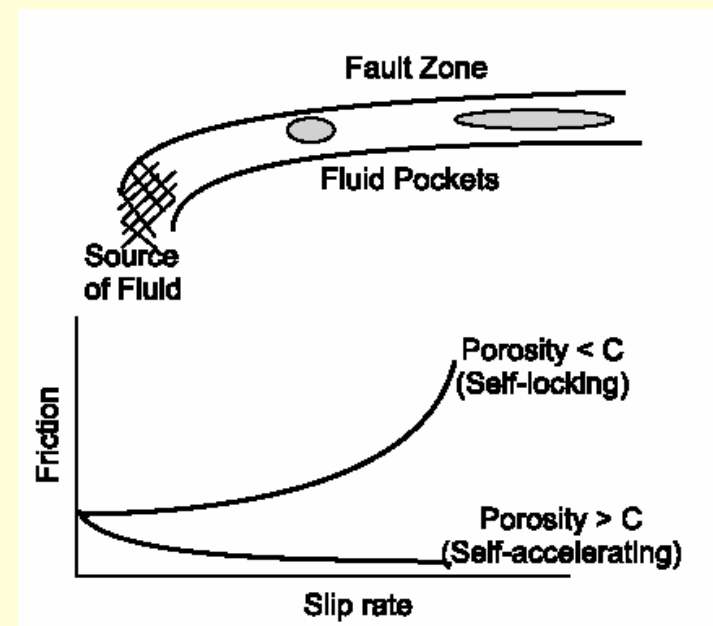
The evolution of the deformation has to be such that no “holes” (absence of matter) or penetration of matter can occur.



PHYSICAL INSTABILITIES



Instability caused by stress corrosion



Instability caused by the infiltration of a lubricating fluid

(Keilis-Borok & Soloviev, 2003)



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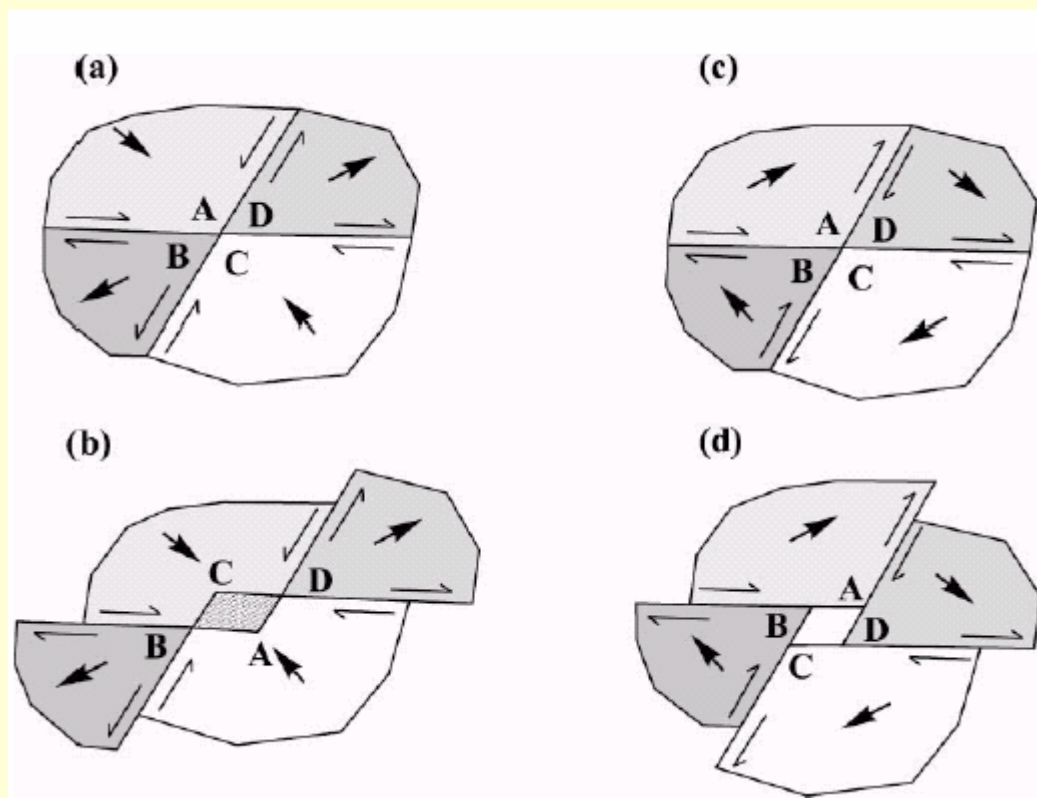
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GEOMETRICAL INCOMPATIBILITIES



(Keilis-Borok & Soloviev, 2003)



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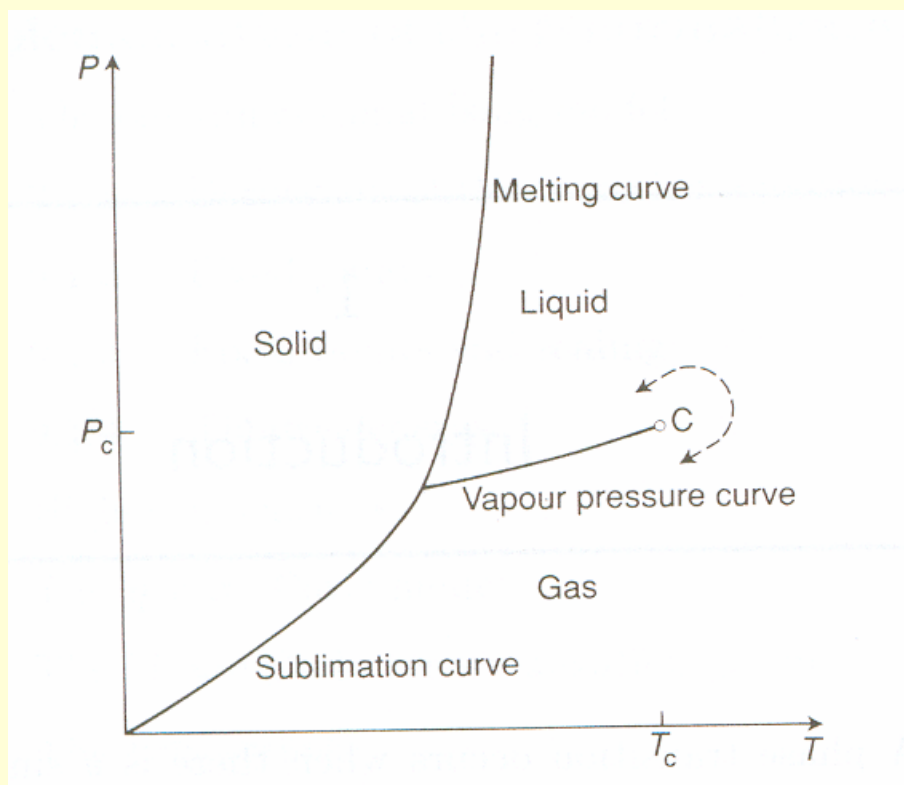
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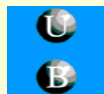
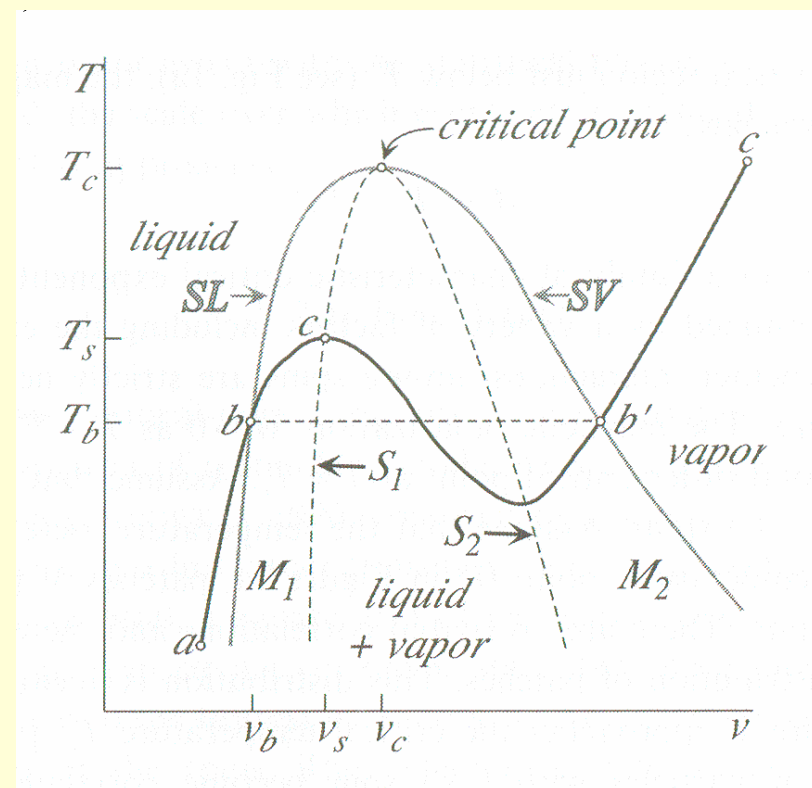
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CRITICAL PHENOMENA. 1

2nd. order (continuous) phase transition



metastability



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CRITICAL PHENOMENA. 2

The term **critical phenomena** refers to the peculiar behavior of a system when it is at or near the point of a continuous-phase transition, known as critical point. A continuous phase transition, in turn, may be defined as a point at which the system changes from one state to another without a discontinuity or jump of a given property (the density in the liquid/vapor phase change or the stress/friction in earthquakes).

On a generic way we may consider a the occurrence of an earthquake as a phase transition order/disorder, the system in a constrained state to an unconstrained one. The critical point would be the threshold “accumulated stress = resistance of the material”.

Close to the critical point, the evolution of the system is governed by the order parameter and the correlation length.

Critical phenomena are characterized by scaling (power law behavior) and (very often) universality.



ORDER PARAMETER AND CORRELATION LENGTH

The **order parameter Q** can be defined as the difference of a state variable characterizing the system in a given state (for example, the difference in densities in a fluid/vapor continuous phase transition). In the present case it could be defined as the difference between the stress concentration and the resistance of the material, the critical stress σ_c . (**Unfortunately, this parameter is not an observable, so that some other relater parameter should be found**).

The medium is inhomogeneous, so that the stress concentration will also be inhomogeneous. Consider the medium subdivided in patches and assume the patches characterized as **+1** if the stress concentration is $\sim \sigma_c$ or **-1** otherwise. (in this way we reduce this model to an Ising one). The **correlation length ξ** can be roughly defined as the linear dimension of the largest correlated spatial structure (i.e. the size of the largest +1 or -1 island). From another point of view, it can be defined as the distance over which then effects of a disturbance spread.

Close to the critical point the order parameter and the correlation length show a power law behavior

$$Q \propto |\sigma_r|^\beta, \quad \xi \propto |\sigma_r|^{-\nu}, \quad \sigma_r = (\sigma - \sigma_c) / \sigma_c$$



SELF-ORGANIZATION

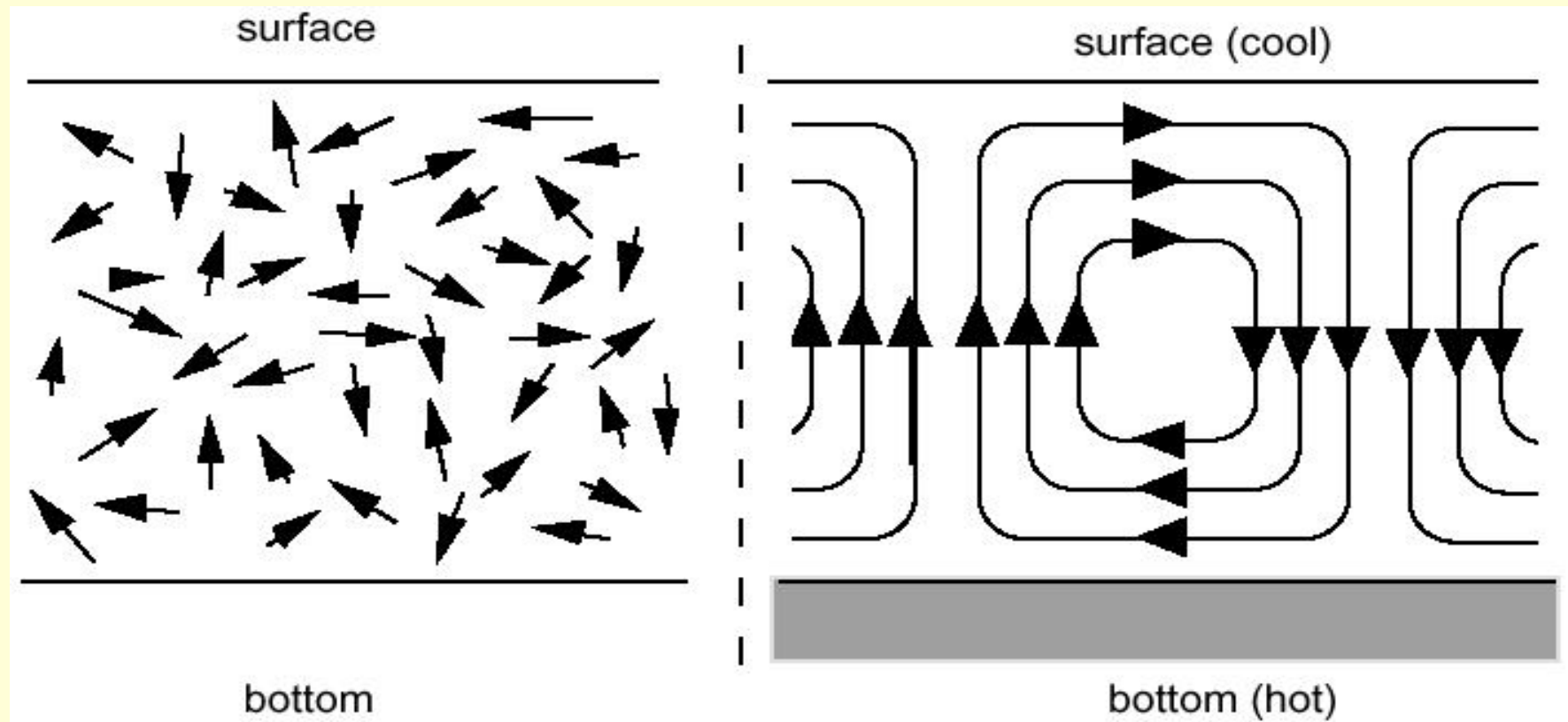
If we accept earthquake occurrence as critical phenomena, we should formulate the evolution of the system in terms of the order parameter and the correlation length.

As the correlation length grows, the patches self-organize in several geometrical and dynamical structures.

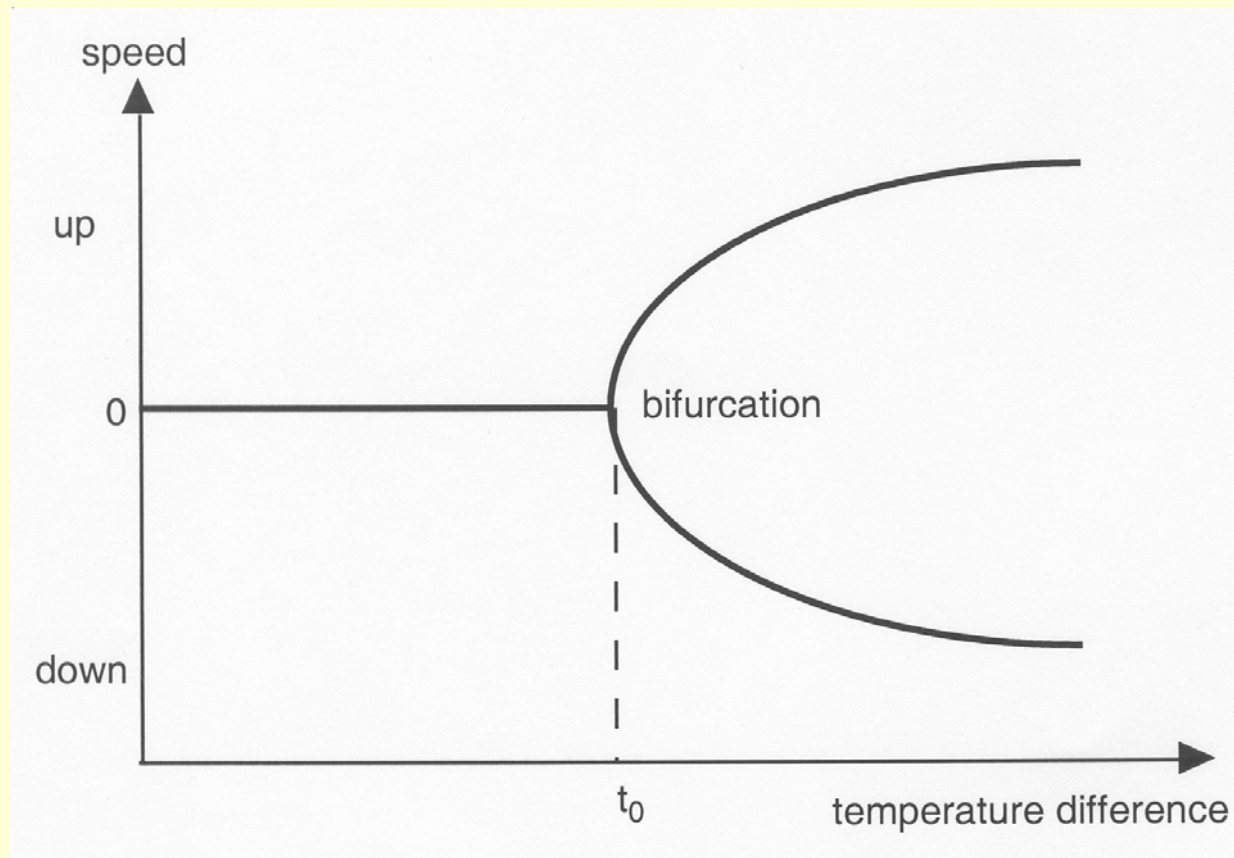
The paradigmatic example of self organization is Rayleigh-Benard convection.



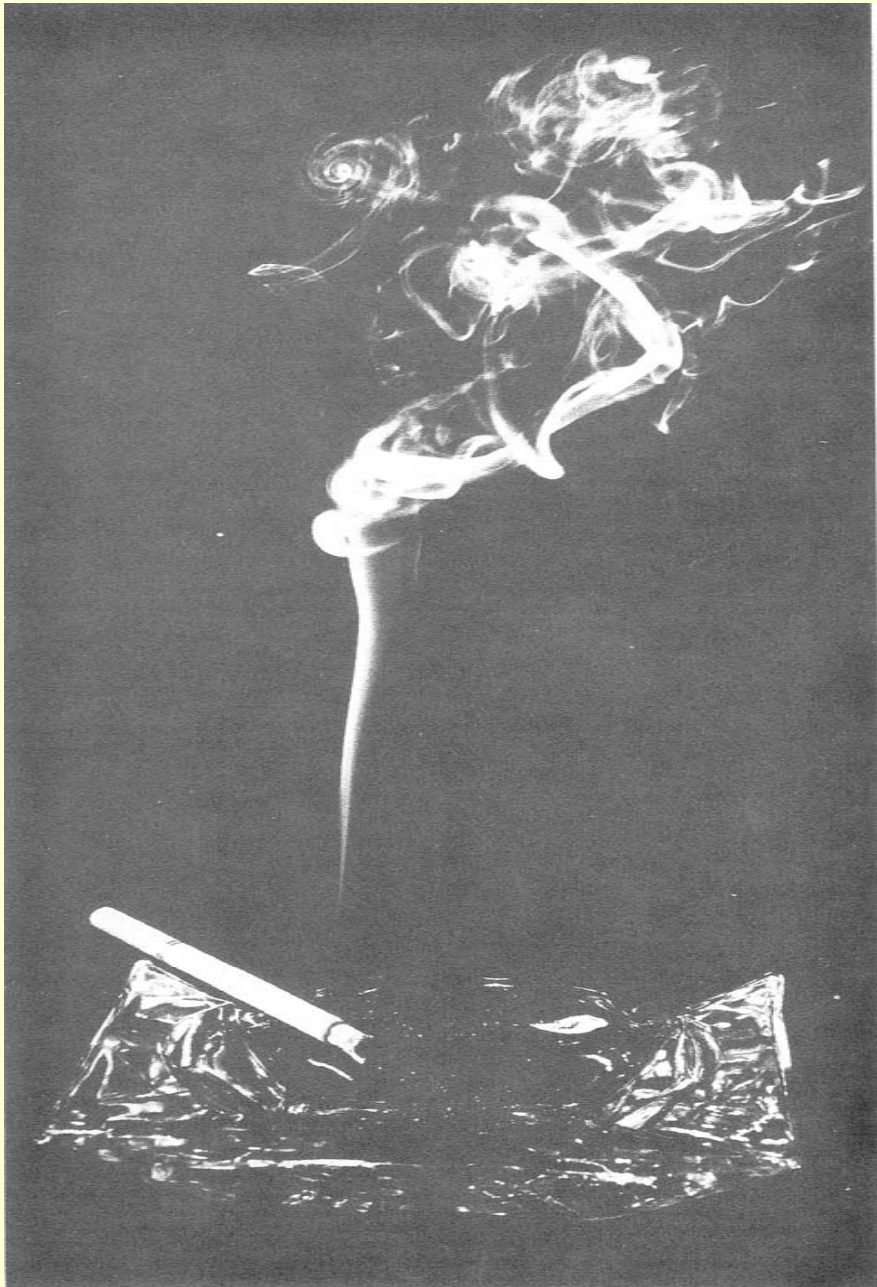
PARADIGMATIC EXAMPLE OF FAR FROM EQUILIBRIUM SYSTEMS: RAYLEIGH-BENARD CONVECTION



CONVECTION THRESHOLD: A BIFURCATION POINT



PHASE TRANSITION ORDER/DISORDER



The phase change corresponds to the evolution of a laminar to turbulent flux (Taylor-Couette convection) in an ascensional column of cigarette smoke.

Ford, J. *How random is a coin toss?*
Physics Today, April 1983, 1



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THE THRESHOLD OF TURBULENCE AS A PHASE TRANSITION

For a constant heating, the cell structures are stationary. However, if heat grows the structure of the cells evolves is continuously evolving towards a more complex structures until the threshold of turbulence is reached. The transition from convective to turbulent regimes can again be viewed as a phase transition.

Two critical points can thus be defined:

1. The beginning of convection with the meaning of the **emergence of self-organization** defining a bifurcation point
2. The transition from convection to turbulence that can be assimilated to a **phase transition order/disorder**.

The occurrence of earthquakes can be assumed to be a temporal point- process with respect to the characteristic timed scale of stress accumulation. Hence, we can talk about the occurrence of events as an instantaneous phase transition.

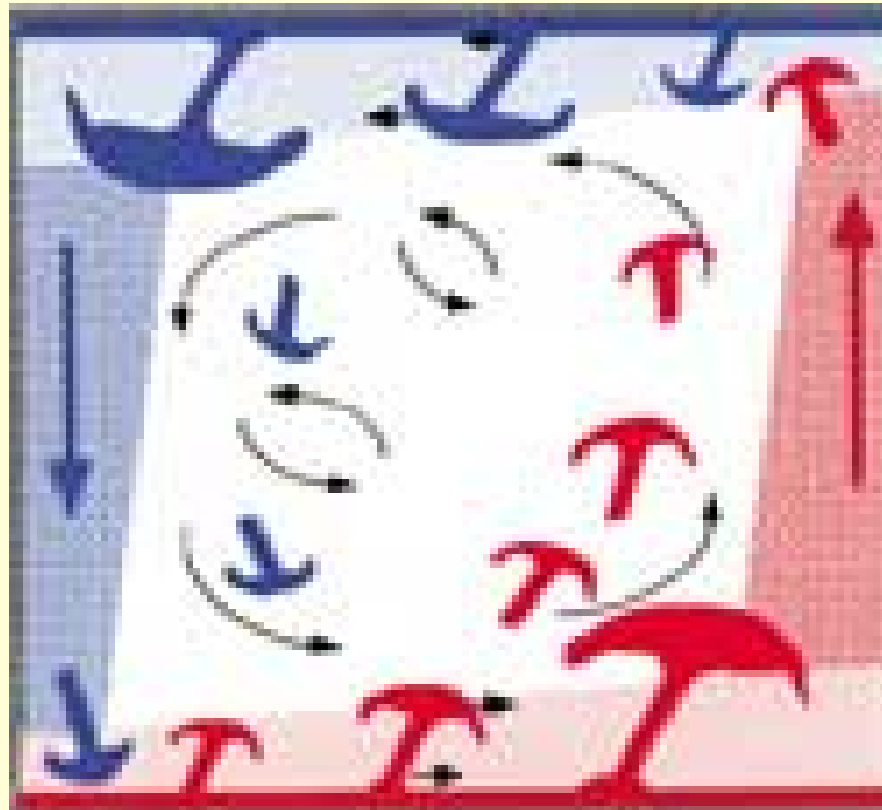


TURBULENT CONVECTION

- Turbulence is associated to transport of energy by means of a cascading process, direct and inverse. In this process all structures, at all scales, interact non-linearly.
- When a phase transition is reached, energy is liberated. Part of this energy is emitted as elastic waves, part is absorbed by the medium and part is used to account for a process of adaptation/self-organization of the system (the stress field) to a new environment.
- Is precisely this adaptive process that could manifest as a precursory activity.



TURBULENT CONVECTION



(Kadanoff, Physics Today, August 2001, p.34)



STRESS ACCUMULATION. 1

Since the beginning of the stress accumulation until its failure (earthquake occurrence), two different process can be distinguished (Bernard, 1999):

- i. the generation of crustal instabilities (transients)
- ii. the appearance of precursory activity (self-organizing process)

Relevant observations of crustal instabilities:

- 1) Silent and slow earthquakes
- 2) Fluid migration instabilities in the crust
- 3) Seismicity is not a Poisson process
- 4) Earthquake sizes have power law distributions
- 5) Size and roughness of fault segment follow power law distributions
- 6) Continuous or transient aseismic fault slips



STRESS ACCUMULATION. 2

According to the previous observations, two different physical models can be defined, that would act sequentially:

1. **Preparation-zone paradigm in seismogenesis.** Process reported in 1) to 4), and their subsequent effects (such as ground deformation and electromagnetic effects) can sometimes be recognized as being precursors to large earthquakes.
2. **Observations 5) and 6)** provides the basis for **self-organized critical models for the crust (SOC)**, or similar models leading to a chaotic system with a large degree of freedom, in which earthquakes are inherently unpredictable in size, space and time (such as cascade or avalanche process).

Models **1** and **2** not necessarily are incompatible. At the contrary, they could be merged in a general SOC model.

However, there is no general agreement in accepting both models.



SELF-ORGANIZED CRITICALITY MODELS?

There are several important observational arguments against the applicability of SOC to the earthquake problem (Knopoff, 1999).

1. Seismicity at almost all scales is absent from most faults, before any large earthquake on that fault.
2. There is no evidence for long-range correlations of the stress field before large earthquakes.
3. Some doubts arise on the universality of power law laws for observed seismicity.
4. Faults and fault systems are inhomogeneous, and may present several scale sizes.



PRECURSORY ACTIVITY

The main problem with precursory activity is that there is no 'universal precursor'. Some precursors have been detected in seismic catalogs and in model simulations, but up to now is the whole set of observations and processes that can provide some indication (probabilistic forecasting) on the future occurrence of a large earthquake. This is the base of '**pattern recognition methods**' (Keilis-Borok and Soboliev, 2003), which are based on the analysis of fluctuations in the rates of occurrence of intermediate-magnitude earthquakes. Up to now, the developed prediction techniques are successful at about 80% level, and present an improvement over poissonian estimates of the order of 3:1.



UNIVERSAL LAWS

(generally accepted) basic and robust facts of earthquake phenomenology:

- Gutenberg-Richter law

$$\log \dot{N}_{ta} = -b m + \log a'$$

- Omori's law

$$n'(t) = \frac{k}{(t+c)^p}$$



BREAK DOWN OF SCALE INVARIANCE?

- Two significantly different branches in the scaling relation have been lately reported in the literature:
- Knopoff (PNAS, 97 (2000)11880), $M \sim 4.8$
- Kanamori & Heaton (Geophys. Mon. 120 (2000) 147), $M_W > 4.5$ and $M_W < 2$.
- Ben-Zion & Zhu (Geophys. J. Int., 148(2002), F1-F5). $M_L \sim 3.5$.



BREAK DOWN OF OMORI'S LAW?

- Fluctuations too large
- Time dependent model parameters



THE END OF SEISMIC CYCLE?

The abandon of the ingrained concept (in many seismologists' mind) of the distinction between foreshocks, aftershocks and mainshocks is an important step toward a simplification and toward an understanding of the mechanism underlying earthquake sequence.

(Helmstetter & Sornette, 2003)



TABLE I. HALLMARKS OF SELF-ORGANIZATION
Parameters

	Convection	earthquake occ.
Control parameter	Temperature	Strain
Order parameter	Amplitude of fluctuation (displacement, velocity)	Rate of released energy
Threshold	Gradient of temperature	Gradient of deformation



TABLE I. HALLMARKS OF SELF-ORGANIZATION Dynamics

Phase transition	Onset of convection (bifurcation point)	Onset of intermittency (bursts of energy release)
Self-organization	Convective cells Turbulent convection	Emergence of Spatio-temporal patterns
Phase transition	Onset of turbulence	Failure (earthquake)



OBSERVABLES

Our goal is to infer the occurrence time of the next event from observations made during the self-organizing process and the further evolution of the developed structures. We should emphasize that the main observables will be the variations of the structures, geometrical and dynamical, and their rate of change. This is what we term precursory activity, and the measured parameters as precursors. As the underlying physics of earthquake occurrence is very elusive, the planned problem is: which are the precursors we have to look for, where and when?

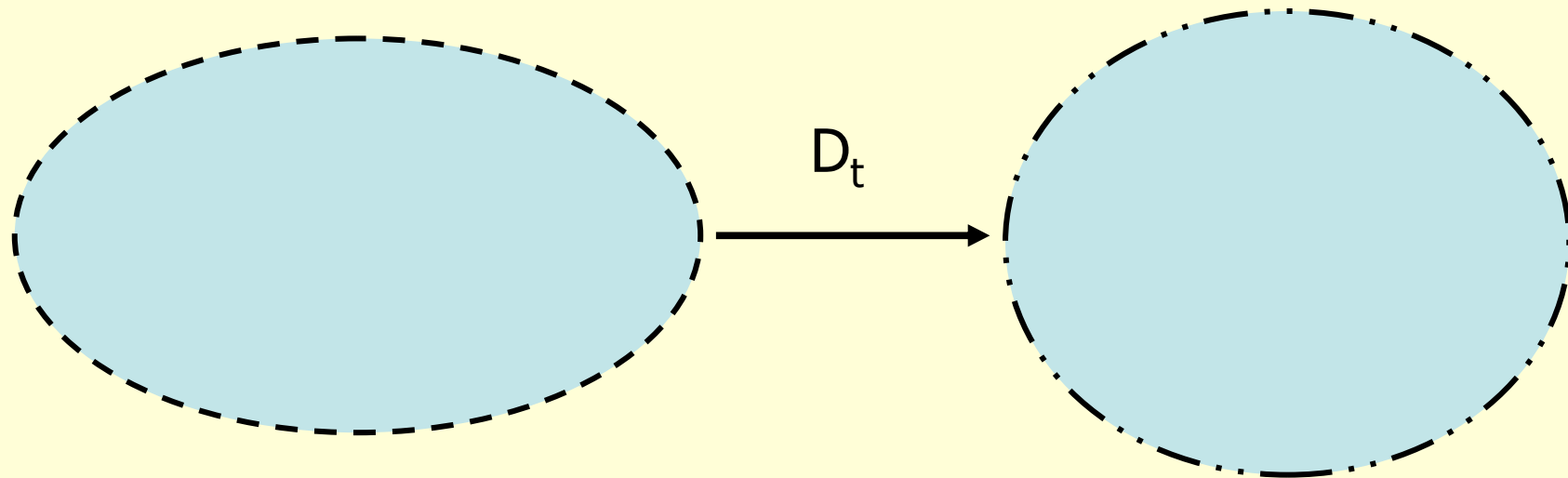
Similarly to Rayleigh-Benard convection, we are dealing with a physical system characterized by a continuous input of energy. The deformation potential will be continuously increasing, and because of medium inhomogeneities the energy will be primarily released at the weakest zones. Hence, the most promising observable will be the evolution of seismicity.



WEATHER FORECASTING

Observable Deterministic Dynamics

$$D_t\{s(\mathbf{x},t)\} \rightarrow s(\mathbf{x},t+\Delta t)$$



$s(\mathbf{x},t_0) = s_0(\mathbf{x})$: measured initial conditions

Ground level



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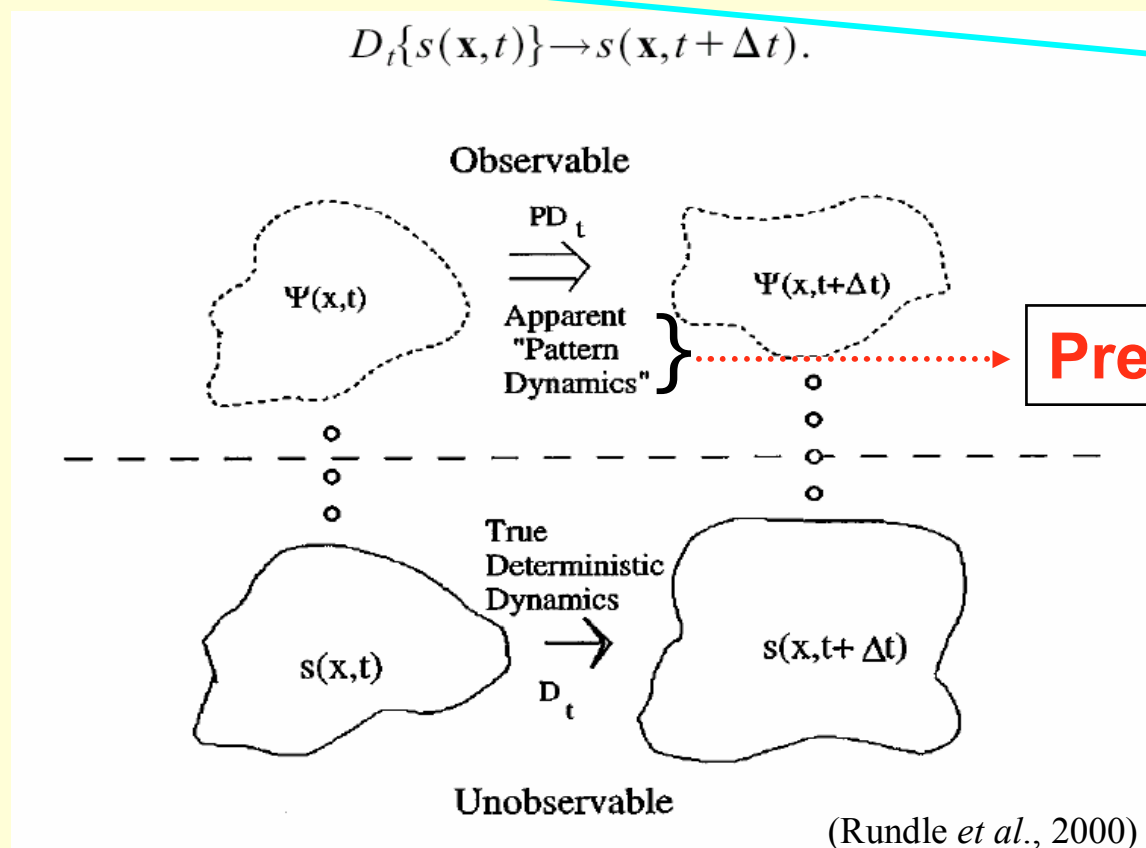
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FORECASING OF EARTHQUAKES

Let D_t be an (unknown) operator that describes the evolution of the system, at a given fixed point x , from a time t to a time $t+\Delta t$. The dynamics of the real system is not an observable: the only **observable** will be its projection PD_t .



Which ones?

Precursory activity

They should be determined from:

- theoretical approaches
- observations



PREDICTION

The aim on any scientist is to advance the future evolution of the system under study. Three different kinds of prediction can be distinguished:

- **Deterministic prediction** (individual events, based on –linear- physical models): when will occur the next solar eclipse, visible from Barcelona? (On 3 October 2005. It will be partial and will begin at 7 h 41 m 07 s, UT).
- **Probabilistic prediction or forecasting** (evolution of a physical system, based on -non-linear- physical models): which will be the weather for New Year? (It can be sunny or unpleasant. We will not be able to be more precise until a few days before).
- **Oracles** (not scientific): will I succeed in my job? (The conditions are favorable, but the vision of a cat can spoil it all).



PRECURSORS

IASPEI Preliminary List of Significant Precursors

Only five possible precursors, out of the forty proposed, seem to deserve further study (*Wyss, 1997*):

- one based on ground water chemistry
- one based on deformation of the crust
- three based on **seismicity patterns**



PREMONITORY SEISMICITY PATTERNS

Some changes are observed in the earthquake's flow before a large event.

These changes (a precursory activity) are akin to the general symptoms of instability of many non-linear systems before a catastrophe.

In particular, the response to a perturbation:

- becomes more irregular
- acts at long distances
- increases



PREMONITORY SEISMICITY PATTERNS

Before a strong earthquake it is very often observed observed:

- an increase of the seismic activity, clustering of the earthquakes in time and space, and spatial concentration of sources; in other words, the increase of the response to the perturbation;
- an increase of the variation of seismicity and its clustering;
- a long-range interaction of earthquakes, which can be interpreted as an increase of the range of influence of the perturbation.



SINGLE SEISMICITY PATTERNS FORMALLY DEFINED AS PREMONITORY

- the *burst of aftershocks*, which is associated to moderate magnitude events characterised by a large number of aftershocks
- the *seismic quiescence*
- the *relative increase of the b-value* for the moderate events, with respect to smaller events
- the *increase of the spatial correlation* in the earthquake flow
- the log-periodic variations of the earthquake flow on the background of its exponential *rise*



GENERALIZATION OF THE CONCEPT OF PRECURSOR

Precursors: (dynamical) self-organized pattern in observables, generated when the system approaches a critical state.

- **Observational difficulty of precursors:**

- the time series associated to natural phenomena are non-stationary and/or intermittent.

and/or

- there are some in the data, but we haven't found them yet.
- there are some, but not in the data currently available.
- there aren't any.

- **Complex systems:** for some spacio-temporal scales, natural phenomena are highly organized.



NATURAL & MAN-MADE ORGANIZED SYSTEMS

hurricanes



tornados



galaxy



thermonuclear
explosion



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TIME TO FAILURE: A PRECURSOR?

An example of a self-organizing dynamical pattern is the time evolution of the deformation field. In approaching the failure (earthquake), the temporal evolution of the deformation (sometimes) organizes as a power law in time:

$$\Omega(t) = A + B(t_f - t)^m$$

where $\Omega(t)$ is the cumulative deformation.

The deformation field $\Omega(t)$ can be estimated in terms of direct measurement of the deformation, the released seismic energy or the rate of seismicity.

Time to failure is not a universal feature



UNIVERSAL PATTERNS: AN ILLUSION?

UP TO NOW, NO UNQUESTIONED UNIVERSAL BEHAVIOR (PRECURSORY PATTERN) HAS BEEN DETECTED. DO THEY REALLY EXIST ?

The idea of universality appears in the theory of critical phenomena. Close to the critical point the observables self-organize as power laws, with the same value of the exponents for different phenomena. Is in this context that we can talk about **universality**.

Different instabilities may drive the system to failure (the critical point), and no one predominates over the others. Which one will select the system will depend on the initial conditions. Hence, we cannot pretend, in a reductionist point of view, that the precursory patterns will repeat in some pre-established way. Indeed, a premonitory phenomenon may have different manifestations in different timescales and magnitude ranges.

Hence, **the universal behavior has to be understood in the sense of the behavior of the whole set of observables.**



PREDICTION

Basically there are two methods for the forecasting of future activity:

- **Pattern Recognition**

- Time of increased probability, **TIP** (Keilis-Borok and Soboliev, 2003)
- Phase Dynamics (Rundle et al., 2000)

- **Probabilistic**

- Previous activity + some *reasonable* assumption (Rikitake, 1976; Kagan and Jackson, 2000; and a lot more ...)



TIME OF INCREASED PROBABILITY. 1

- **Physical basis:** the dynamics of the lithosphere viewed as a non-linear hierarchical dynamical system.
- **Observational basis:** the seismic activity in a wide magnitude range.
- **Methodology:** that of critical phenomena, for which prediction of complex systems requires a holistic approach *from the whole to details*, in consecutive approximations, starting with the most robust coarse-graining of the process considered..



TIME OF INCREASED PROBABILITY. 2

- **Prediction algorithms:** based on the hypothesis that prediction is as pattern recognition problem of infrequent events.
- **Major features of the patterns:** long range correlations and similarity.
- **Validation of predictive methods:** rigorous analysis of successful predictions, false alarms and failure to predict.
- The **probabilistic side of prediction** is reflected in the rates of errors, evaluated as error diagrams.



BASIC TYPES OF PREMONITORY PHENOMENA

The approach of a strong earthquake is indicated by (some of) the following **changes in the basic characteristics of seismicity**:

- a. Rise of seismic activity.
- b. Rise of irregularity in space and time.
- c. Reversal of territorial distribution of seismicity.
- d. Transformation of magnitude distribution.
- e. Rise of earthquake clustering in space and time.
- f. Rise of the earthquake correlation range.
- g. Accelerated stress-release



EQUIVALENCE TO TURBULENT CONVECTION

- a. Rise of seismic activity
development of convective cells
- b. Rise of irregularity in space and time
- c. Reversal of territorial distribution of seismicity
turbulent convection
- d. Transformation of magnitude distribution
approach to critical point
- e. Rise of earthquake clustering in space and time
- f. Rise of the earthquake correlation range
- g. Accelerated stress-release
close to critical point



SEISMICITY PATTERNS

Premonitory phenomena of each type are depicted by different seismicity patterns:

1. Measures of seismic activity.
2. Measures of earthquake clustering.
3. Measures of earthquake correlation range.
4. The measures of irregularity.
5. The measures of premonitory transformation of magnitude distribution.



NUMERICAL ALGORITHMS

Several algorithms have been devised for the quantification of the above seismicity patterns and its future evolution, *i.e.*, the prediction of the next strong earthquake. The more widely used are Algorithms M8, MSc, CN and SSE.

In all cases, prediction is aimed at earthquakes of magnitude M_0 or higher, and uses all available earthquakes of a seismic catalog with aftershocks removed. Several running averages are computed for the sequence of earthquakes for a given time window and magnitude range:

- **Seven functions** are calculated with different characteristics, and as a result the earthquake sequence is given a robust averaged description.
- **Very large** values are identified for each function from the condition that they exceed Q percentiles.
- An **alarm** or a **TIP**, time of increased probability, is declared for a give n period of time when at least six out of seven functions become “very large” within a narrow time window.



PHASE DYNAMICS. 1

- As in the case of TIP, the starting point is the variety of **spatial and temporal patterns** associated to strong earthquakes.
- **Main hypothesis:** seismic activity is highly correlated across many space scales and time scales within large volumes of the Earth's crust.
- **Observational fact:** earthquake main shock occur at quasiperiodic intervals and, for some parts of the world, average recurrence intervals are well defined.
- **Goal:** identify basis patterns for all possible space-time seismicity configurations.
- **Methodology:** the patterns are defined by the eigenstates and eigenvalues of one of an appropriately constructed family of correlation operators.



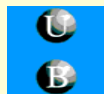
PHASE DYNAMICS. 2

- The **seismicity (firing patterns)** are emergent processes that develop from the underlying structures, parameters, and dynamics of a multidimensional nonlinear system. It can be located in both space and time with considerable accuracy.
- The firing activity, $\Psi(x,t)$, can be represented as a set of time series at all positions x , where $\Psi(x,t) = 1$ if an event occurs in the time interval between t and $t+\Delta t$, and $\Psi(x,t) = 0$ otherwise.
- The classical Karuhunen-Loeve expansion technique is extended to include the construction of pattern states that can be used to forecast events in time.
- The above procedure Involves constructing a correlation operator $C(x_i, x_j)$ for the sites that contains the spatial relationship of slip events over time. $C(x_i, x_j)$ is decomposed into the orthonormal spatial eigenmodes and their associated time series.



PHASE DYNAMICS. 3

- The presence of both large- and small-scale correlations in the data, prompts a study of the change in these modes for each year and allows the identification of modes which consistently appear over some identifiable time period prior to an event.
- A complex rate correlation operator $K(\mathbf{x}_i, \mathbf{x}_j)$ can be used to compute the probability of future events on a fault patch model.
- In this method there is no fitting, smoothing, windowing or declustering performed on the data; and no *a priori* knowledge of the location or extend of the activity/quiescence area is required.
- The method is a coarse-grained measure of the spatio-temporal variations in seismicity performed on the regional historic catalog that quantifies the change through time of a unit vector over the entire space.
- Changes in phase angle of the basic activity rate function $S(x, t_b, t)$ are then the physically meaningful quantities, rather than changes in the norm or variance. Here the name *phase dynamics*.



PHASE DYNAMICS. 4

- There are no free parameters to be determined by fits to data.
- The method cannot predict magnitudes. Up to the present, the method can forecast seismic activity for earthquakes of magnitude equal or greater to a given threshold, and this can be accomplished through the linear dimension of the greed. For example, to forecast earthquakes of magnitude $m \geq 6$ the linear size of the3 greed is selected to be $\Delta L \sim 10$ km.



PROBABILISTIC PREDICTION. 1

Rikitake's approach (Rikitake, 1999):

- The starting point is a seismic catalog of characteristic earthquakes.
- Assume the validity of the *seismic gap*.
- Main hypothesis: *the rupture time of characteristic earthquakes follow a Weibull distribution*.
- This hypothesis permits the estimation of the **expected time** of the next earthquake along with its **standard deviation**.



PROBABILISTIC PREDICTION. 2

The method (Jackson and Kagan, 1999; Kagan and Jackson, 2000) refers to the long term and short term forecast for magnitude 5.8 and larger earthquakes.

- The forecast applies to the ensemble of earthquakes during a previously established test period and it is not meant to predict any single earthquake.
- The short term forecast is based on a specific stochastic model of earthquake occurrence. Considers temporal clustering of the sort that causes foreshock-main shock- aftershock sequences.
- The long term forecast is essentially an empirical description of observed spatial clustering, able to estimate the probabilistic structure of future earthquakes. Temporal clustering is ignored.



A POSSIBLE STRATEGY FOR THE STUDY OF PRECURSORY ACTIVITY (pattern formation)



THE CUBIC GINZBURG-LANDAU EQUATION

The cubic Ginzburg-Landau equation is one of the most studied nonlinear equations. It describes on qualitative, and often in a quantitative, level a vast variety of phenomena, from nonlinear waves to second-order phase transitions, from Rayleigh-Benard convection to pattern formation, apart of others in condensed matter physics. It is specially addressed to the study of non-equilibrium phenomena in spatially extended systems.



THE COMPLEX G-L EQ. 1

The Equation is given by

$$\partial_t A = A + (1 + ib)\Delta A - (1 + ic)|A|^2 A$$

where A is a complex function of (scaled) time t and space \mathbf{x} and the real parameters b and c characterize linear and nonlinear dispersion. This equation arises as a “modulational” (or “envelope” or “amplitude”) equation. In analogy with phase transition, A is often called an **order parameter**.

The physical quantities $\mathbf{u}(t,r)$ (temperature, velocities, densities, *etc.*) are given in the form

$$\vec{u} = A' \exp[i(q_c \cdot x - \omega_c t)] U_l(z) + c.c. + h.o.t.$$

U_l is an eigenvector of the linear approximation, and ω_c and \mathbf{q}_c the corresponding eigenvalues.



THE COMPLEX G-L EQ. 2

- This equation has regimes where the behavior is intrinsically chaotic, and is often studied as a prototype equation for *spatio-temporal chaos* and *pattern-formation*.
- This equation can generate a large variety of *coherent structures*. The most interesting are *front*, *pulse*, *source* and *sink*. The precursory seismic activity would be associated to the generation and evolution of these structures, *i.e.*, to the process of **self-organization**.



THE COMPLEX G-L EQ. 3

- i. $\omega_c = 0, q_c \neq 0$ define stationary periodic instabilities, and the complex Ginzburg-Landau equation reduces to the real one

$$\partial_t A = A + \Delta A - |A|^2 A$$

also known as the *Complex Nonlinear Diffusion Equation* by analogy with the Nonlinear Schroedinger Equation. This is the case of the Rayleigh-Benard convection.

- ii. $\omega_c \neq 0, q_c = 0$ represent oscillatory uniform instabilities.
iii. $\omega_c \neq 0, q_c \neq 0$ represent oscillatory periodic instabilities.

These different possibilities agree with the observed seismicity patterns.



EQUIVALENCE TO TURBULENT CONVECTION

- a. Rise of seismic activity
development of convective cells
- b. Rise of irregularity in space and time
- c. Reversal of territorial distribution of seismicity
turbulent convection
- d. Transformation of magnitude distribution
approach to critical point
- e. Rise of earthquake clustering in space and time
- f. Rise of the earthquake correlation range
- g. Accelerated stress-release
close to critical point



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8th Workshop on NLD&EP, ICTP
3-15 Oct. 2005

Trieste,

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