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Characterizing Temporal Variability of an Earthquake Sequence

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МИТПАН

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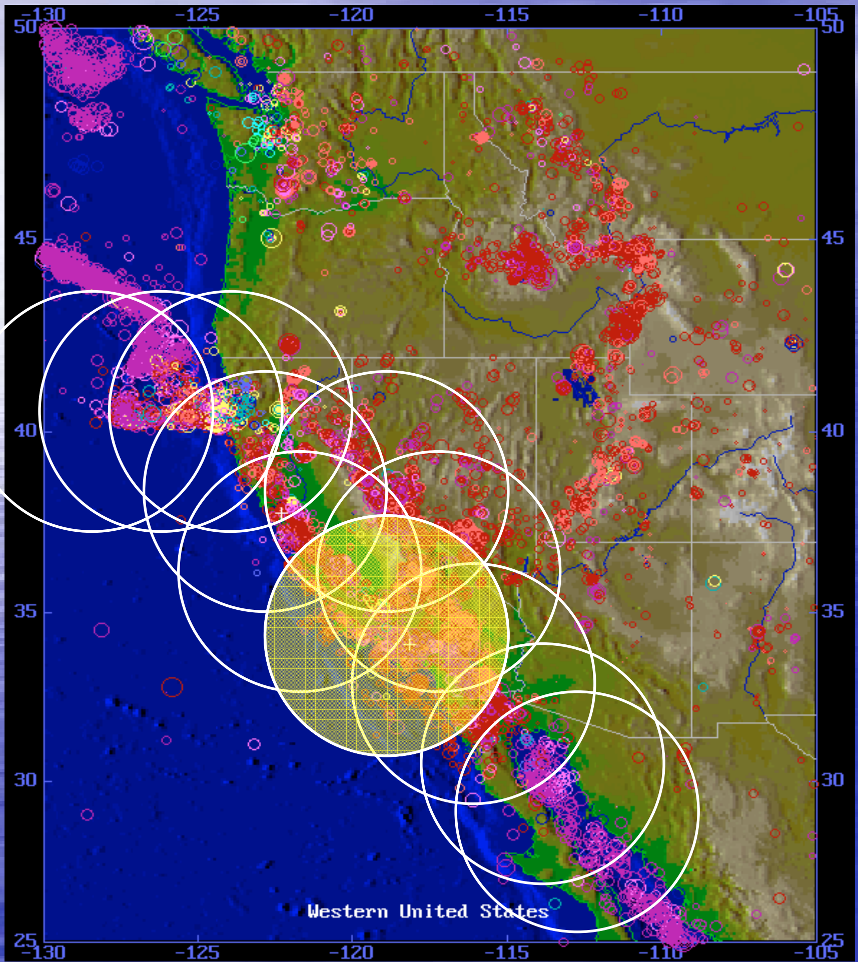
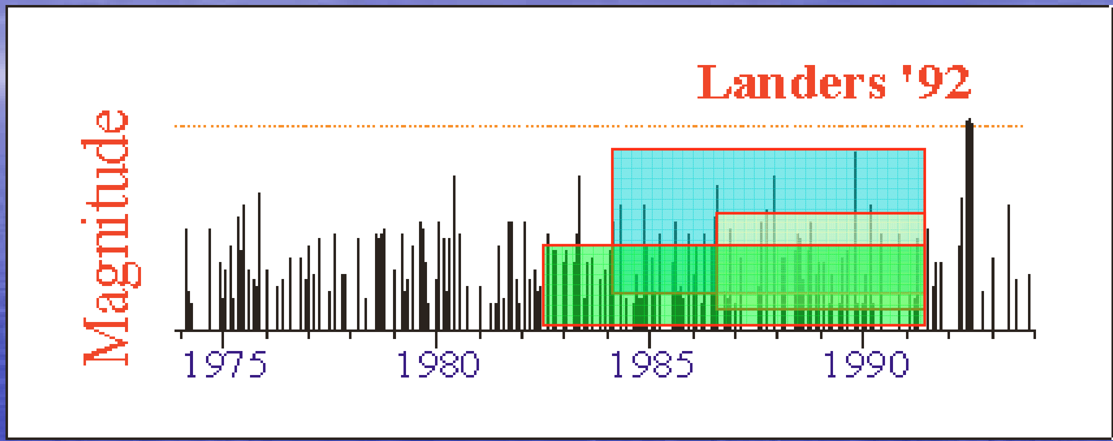
Outline

- General scheme of analysis
- Functions on a sequence of earthquakes
- Different ways to normalize functions

General scheme



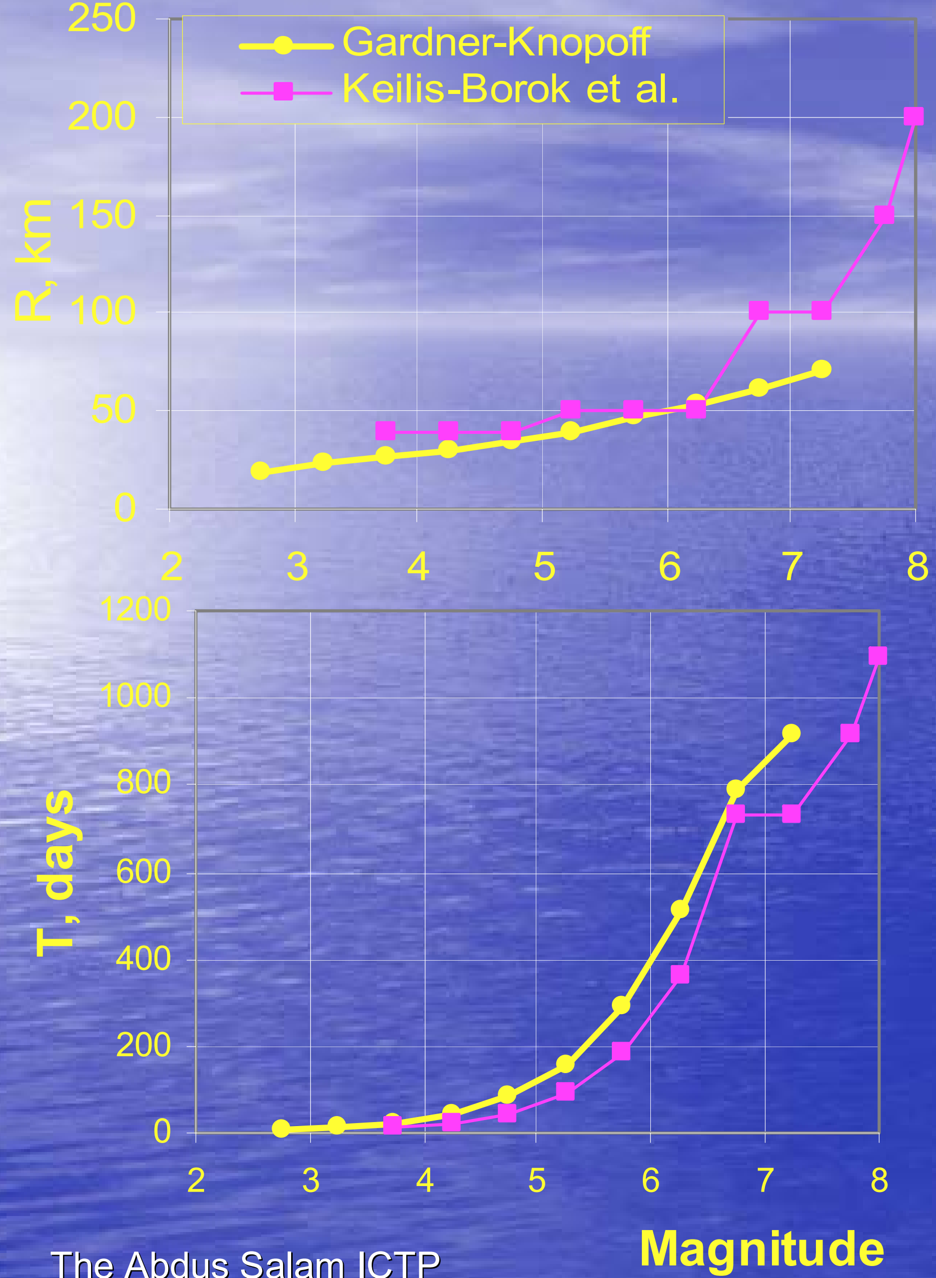


F(t)

The general scheme, as applied today (e.g., in existing earthquake prediction algorithms)

- Is restricted to seismic data and remains open to other relevant observations.
- Reduces information on spatial correlations at scales exceeding the linear dimension of an area of investigation in course independent analysis of sampled areas.
- Uses simple (if not oversimplified) and natural counts as descriptors.

Earthquakes or main shocks?



Earthquakes cluster in space and time and aftershocks of a single event may obscure characterization of seismic dynamics in the area of investigation. To avoid it aftershocks should not be counted when simple counts are derived from a given sequence of earthquakes.

Algorithms that de-cluster earthquake catalogs exist (although are far from the ideal).

Counts on earthquake sequence

Thus, after reduction of space an earthquake sequence is denoted either as

$$\{t_i, m_i\} \text{ or } \{t_i, m_i, b_i(e)\},$$

where $b_i(e)$ is the number of aftershocks attributed to the i-th main shock in the period from t_i to $t_i - e$.

Note that the number of aftershocks, $b_i(e)$, *per se* is one of the parameters of main shocks { i } inherited after de-clustering of a given earthquake catalog.

Functions of seismic activity rate

$N(t \mid m,s)$ - the number of earthquakes with $M \geq m$ in time interval from $(t\text{-}s)$ to t, i.e., the number of events of certain size per unit time, *rate of activity*.

A dual function is the time that accommodates the most recent n events with $M \geq m$,

$TI(t \mid m,n)$.

Function K

The increment of activity expressed by the difference between the number of earthquakes in the two successive intervals of time (t-s, t) and (t-2s, t-s)

$$K(t \mid m,s) = N(t \mid m,s) - N(t-s \mid m,s)$$

has a physical meaning of *acceleration*.

Function L

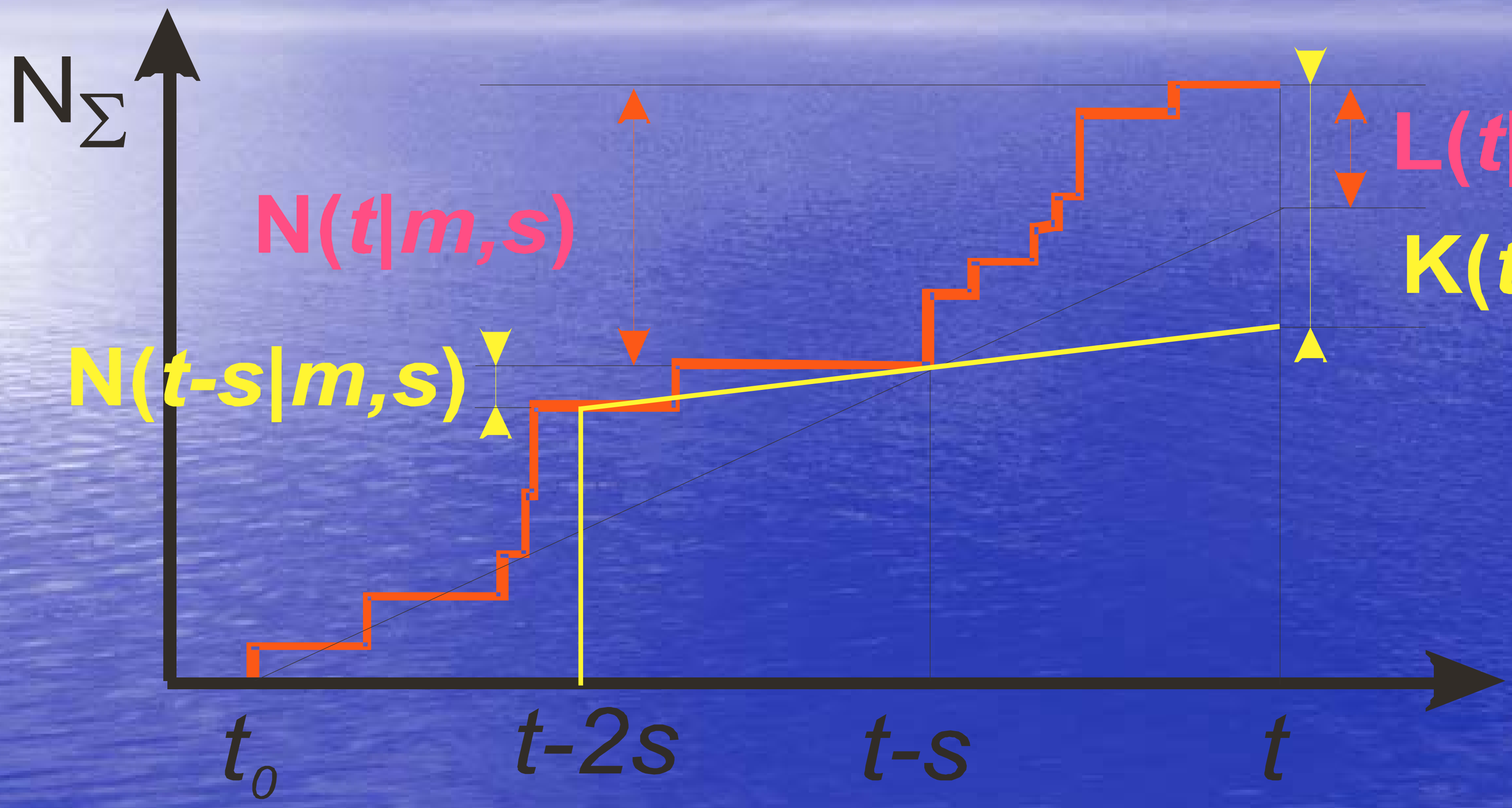
$L(t \mid m, s, t_0)$ - deviation of activity from a longer-term trend over the period from t_0 to t.

$$L(t \mid m, s, t_0) = N(t \mid m,s) - N(t-s \mid m,t-s-t_0) \times s/(t-s-t_0)$$

Usually t_0 is the beginning of the catalog unless its duration allows to use a pre-fixed trailing window

$t-t_0 >> s$ to characterize a longer-term *differential* of seismic activity rate (e.g., since the beginning of the new millennium $t-t_0 = 5s$ in the on-going Global Test of M8 algorithm).

Functions of activity



The necessity of trailing t_0 in function L

When t_0 becomes far from t (which is the inevitable case of all long lasting experiments), the longer-term rate L tends to a constant. Therefore, to preserve a physical meaning of the rate differential one should use the definition of L with a trailing t_0. This might be of crucial importance when catalogs of extended times are considered (e.g., in model catalogs).

$\Sigma(t)$

$\Sigma(t \mid m, M', s, \alpha, \beta)$ is the weighted number of earthquakes in time interval from (t-s) to t and magnitude ($m \leq M_i < M'$):

$$\Sigma(t \mid m, M', s, \alpha, \beta) = \Sigma 10^{\beta(M_i - \alpha)}$$

This function may estimate different properties depending on the choice of β.

Σ is proportional to

- the total linear size of earthquake sources

  if β=B/3
- the total area of the earthquake sources

  if β=2B/3
- the total energy of earthquakes

  if β=B.

where B comes from the relation between the energy E and magnitude M

$$\log E = A + BM$$

Normalized counts of $\Sigma(t)$

- Averages –

$$\Sigma(t)/N(t)$$

- Concentrations –

$$(\ \Sigma(t)/N(t)\ )\ /\ (\ N(t)\ )^{d/3}$$

where d is concentration dimension and $\beta = d \cdot B/3$

Function G

The ratio of the number of earthquakes from two magnitude ranges –

$$G(t \mid m_1, m_2, s) = 1 - N(t \mid m_2, s) / N(t \mid m_1, s)$$

- The values m_1 and m_2 may result from inspecting the frequency-magnitude statistics of earthquakes.

An alternative to G: Function G'

The logarithm of the ratio of the number of earthquakes from two magnitude ranges normalized to difference of magnitudes –

$$G'(t \,|\, m_1, m_2, s) = \log\left(N(t \,|\, m_2, s) \,/\, N(t \,|\, m_1, s) \right) / (m_2 - m_1)$$

In such a definition G' is the tangent of the slope of the frequency-magnitude graph.

Another alternative of this kind is the trailing average magnitude, $\sum m_i / N$.

Variation of seismic activity

$V(t \mid m,s,u) = var\ N(t \mid m,s)$ is variation of the number of earthquakes on the time interval $(t-u,t)$,

$var\ N(t \mid m,s) = \Sigma \mid N(t_i \mid m,s) - N(t_{i+1} \mid m,s) \mid$

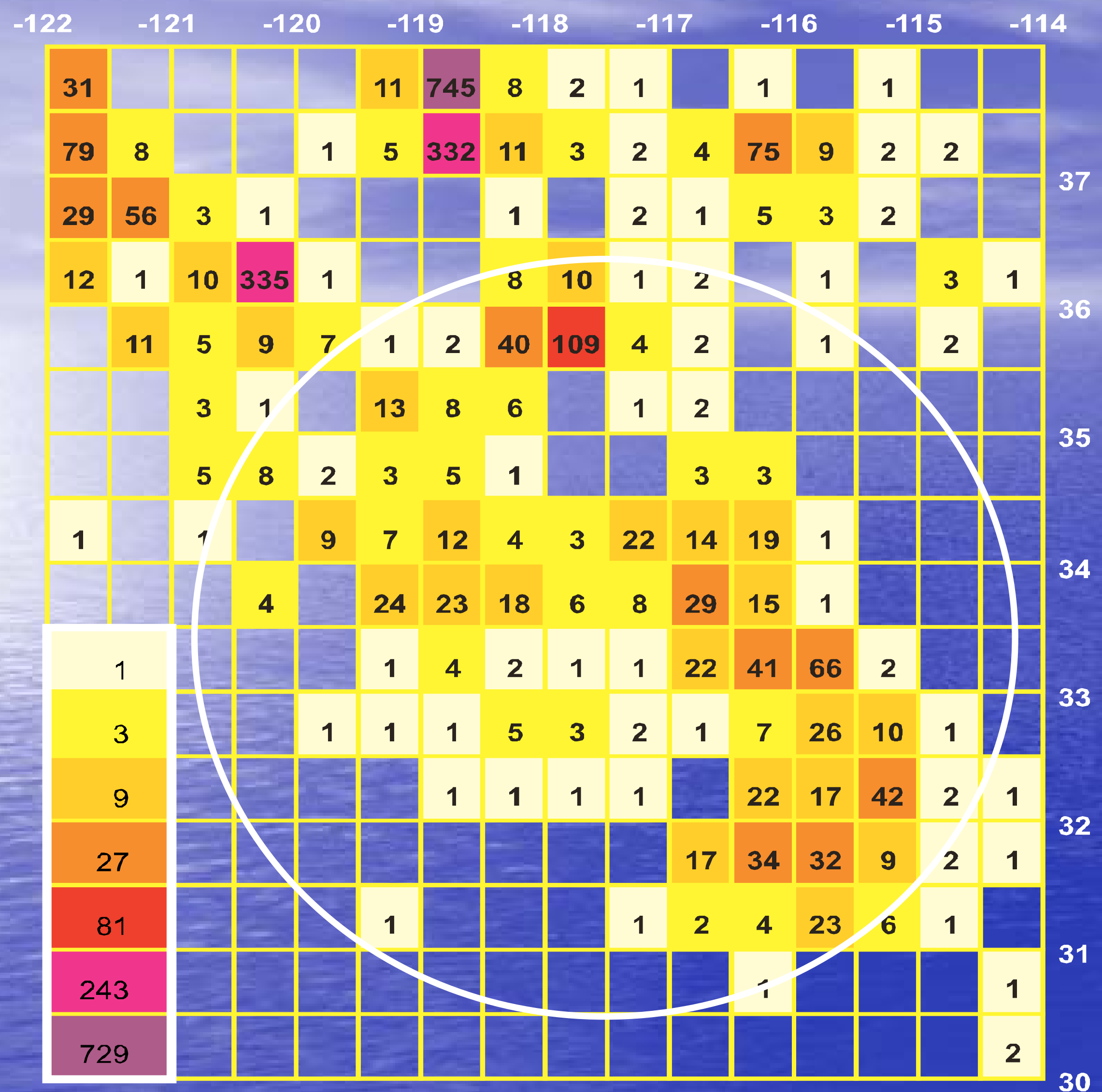
which is the total of the absolute differences between $N(t \mid m,s)$ at two consecutive times t_{i+1} and t_i from the time interval $(t-u,t)$.

$$V(t\,|m,s,u) = \sum \updownarrow$$

Clustering of earthquakes

In case of de-clustered catalogs each main shock has its number of aftershocks, $b_i(e,m_{aft})$, of magnitude $M \geq m_{aft}$ in the first e days after its origin time t_i.

$B(t \mid m,M',s,m_{aft},e) = \max b_i(e,m_{aft})$ is the maximum calculated over the main shocks with $m \leq M_i < M'$ and time interval (t-s,t).



Earthquakes are not distributed uniformly.

Heterogeneity of seismic distribution can be measured providing additional characterization of the area under investigation

Coarse-grained number of magnitude 3 or larger earthquakes, 1980-1984 (USGS/NEIC GHDB).

Entropies

Many other measures of clustering were proposed in the literature. For example, *Shannon entropy*

$$- \Sigma \, \boldsymbol{p_i} \times \mathbf{ln} \, \boldsymbol{p_i}$$

where $\boldsymbol{q_i}$ is some time dependent probability estimate of occupation of a given cell of the grid, i.e., $\boldsymbol{p_i = n_i / N}$.

It is the limit case ($q \rightarrow 1$) of the Renyi's entropies

$$(1 - q)^{-1} \times \mathbf{ln} \, \Sigma \, \boldsymbol{p_i^q}$$

which integrals $\Sigma \, \boldsymbol{p_i^q}$ provide a rather simple one-parameter family of measures of the observed clustering in space and time.

Active zone size

Active zone size, AZS, is a measure of the extent to which seismic activity in a given region is diffused. Formally, it is $\Sigma\, \boldsymbol{p_i^q}$ at q = 0. The premonitory efficiency of AZS was first observed in generalized Burridge-Knopoff models of fault.

AZS is more directly a measure of the broadening of small to moderate size events, which leads to development of a nucleation region associated with a coming large event.

AZS in seismic regions

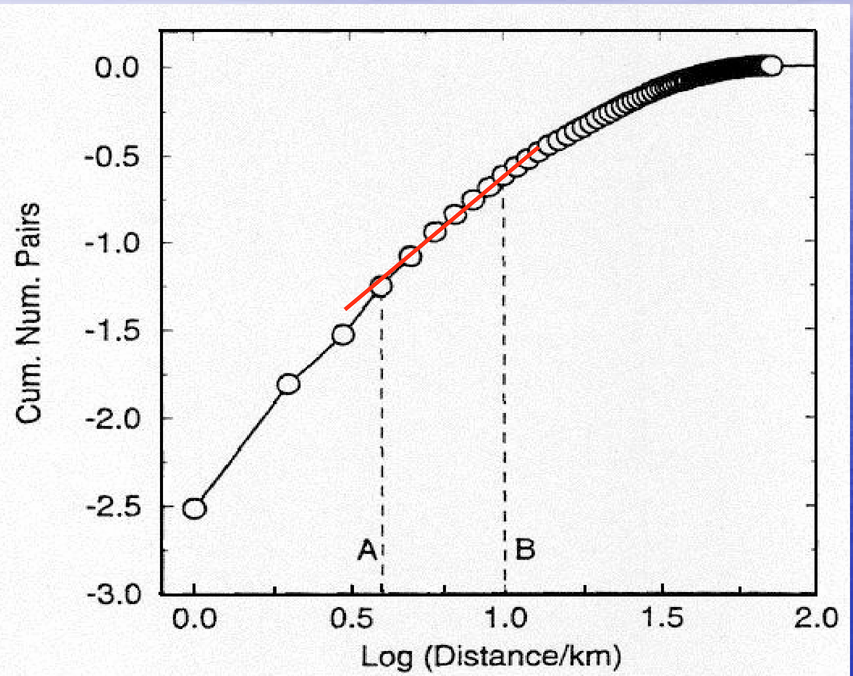
To measure active zone size in the Earth we might use a box counting algorithm, in which the large spatial regions will be subdivided into many smaller regions that contain seismic events of a certain magnitude level.

For example, AZS(t | m,s)= $\Sigma\ \delta(n_i)$,

where $\delta(x) = 0$, if $x = 0$ and $\delta(x) = 1$, if $x > 0$.

In the Earth this measure is somewhat more complex due to the variable complexity of faults networks in different regions.

Spatial correlation dimension



$C(R) = p(r < R)/P$

$= 2p(r < R)/(N(N-1))$

where $p(r < R)$ is the number of pairs separated by less than R km and $P=N(N-1)/2$ is the total number of pairs for N events.

The slope of the straight segment over distance range [A,B] is used to estimate the spatial correlation dimension $CD=d_{[A,B]}$.

Correlation dimension vs. Clustering

A correlation dimension is inversely related to the degree of spatial clustering, as the slope of the straight segment decreases when the number of event pairs with relatively small inter-event distances increases. Smaller values for CD indicate higher degrees of spatial clustering.

Long-range interaction of earthquakes

- A. Prozorov (1982) introduced a term-less precursor, which follow shortly a major earthquake but on a large distance from it. He concluded that “long-range aftershocks” mark the location of a future major earthquake.
- The two new phenomena which represent the long-range correlation were found first on a synthetic catalogue.

Average depth

Nadeau et al. (1995) suggested to use the average depth of hypocenters in a given time window claiming that deepening of seismic activity precede large earthquakes.

It is necessary to note however that the depth is one of the most inaccurate parameters of hypocenter determination: the error in depth may be large resulting seismologist attributed values in many cases.

. . .

The measures that can be useful for characterizing temporal variability of an earthquake sequence is obviously not limited to the list presented here.

Normalization

Samples used in the counts should be comparable in nature and in number.

That is why *normalization* of earthquake sequences is necessary to ensure adequate uniform application with the same set of adjustable parameters in regions of different seismic activity.

Normalizing sequences: Magnitude cutoff

We recommend to use the normalization of a sequence by choosing the minimal magnitude cutoff M_{min}, defined by one of the two conditions:

- $M_{min} = M_0 - C$,

  C being a constant

- M_{min} derived from $N(M_{min}) = A$

  A being a constant rate of activity

Normalizing counts: Percentile cutoff

The values of functions are normalized to their empirical probability distribution functions, that is performing a non-linear transformation of different ranges from different seismic environment to [0,1].

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