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The Density Distributions and the Correlations of an Earthquake's Source Parameters

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The Density Distributions and the Correlations of an Earthquake's Source Parameters

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Abstract—The data analysis of the source parameters of five sets of earthquake sequences, aftershocks and earthquakes scattered in a region shows that the scalar seismic moment is correlated with the linear size of the fault and the static stress drop. We tentatively imply that a correlation also exists between the radius of the faults and the static stress drops and it is suggested that the static stress drop may be decreasing with increasing radius of the source. It is shown that the density distribution of the source radius, calculated through the source rupture duration obtained from the body wave pulse (BOATWRIGHT, 1980) using the time T between the P-wave onset and the first zero crossing on the seismogram, may be represented by a power law as the density distribution of the stress drops and of the moment which are also computed. It is also suggested, and tentatively verified, that the density distribution of the areas of the broken barriers on the faults is similar to that of the density distribution of the static stress drops. It is finally suggested that the seismicity of a region may be studied two-dimensionally as a function of the stress drop and the radius of the source instead of the classic b and b_0 values. Concerning the discussion on the range of the values of the static stress drop, whether it is almost constant in a seismic region and varies only from one region to another, it is seen that in the aftershocks of the 1994 Northridge earthquake it covers a range of almost 5 orders of magnitudes. Finally it is ascertained that the density distribution of the source parameters does not give equipartition of seismic moment release.

Key words: Stress drop, source dimension, statistical analysis.

Glossary

l	radius of the circle approximating the source
p	static stress drop
M	magnitude
M_0	scalar seismic moment
f_c	conventional corner frequency
T	time between P wave arrival and first zero crossing
P	P wave velocity
S	S wave velocity
Sk	fracture propagation velocity
$<s>$	average displacement on the fault
B	area of the surface of the broken barrier on the fault
m	rigidity of rocks

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1. Introduction

The first step following the production of catalogues of earthquakes recorded instrumentally is that of the analysis of the data. A thorough analysis is currently done of coordinates, magnitude, and also concerning the formation of clusters and patterns of occurrence, the latter especially in view of earthquake prediction. Favorable attention has also been given to the possible correlation of the parameters. Lesser attention has been given to the density distribution of the single source parameters, other than the magnitude M and the seismic moment M_0 , which could be of interest in determining the characteristics of the different seismic regions and of the aftershocks of earthquakes of different type, or for forecasting the possible accelerations of the ground (CAPUTO, 1981), or for the study of the excitation of the Chandler wobble (O'CONNELL and DZIEWONSKY, 1976), or to be used as precursors of earthquakes (CAPUTO, 1982), or for the estimate of the elastic energy stored in the crust of the Earth (CAPUTO, 1987), or for the study of a possible equipartition of the elastic energy or of the seismic moment, or, finally, for a better understanding of the world seismicity.

The analysis of the static stress drops has recently attracted the attention of several seismologists such as BOATWRIGHT (1980), SMITH and PRIESTLEY (1993), ABERCROMBIE (1995), HAUKSSON *et al.* (1995), JONES and HELMBERGER (1996), BERESNEV and ATKINSON (1997), HARDEBECK and HAUKSSON (1997), SONG and HELMBERGER (1997), BERESNEV (2001) and MORI *et al.* (2003). Some support the idea that p has a limited variation in the same seismic region (e.g., HANKS and MCGUIRE (1981), ABERCROMBIE and LEARY (1993)) and that it may vary from one region to another. Others do not set limits on the size of p, except those imposed by the properties of rocks, and support the idea that it also may vary in the same region and that the variations may be of several orders of magnitude (e.g., CAPUTO, 1981).

Assuming that the seismic moment is known, in order to determine the other source parameters one usually determined the source dimension through BRUNE's (1970) formula

$$l = 2.34S/2\pi f_c \tag{1}$$

where l is the linear dimension of the fault or the source radius, f_c is the corner frequency and S is the shear-wave velocity, and the factor 2.34 includes the average value of the radiation pattern.

However it has been noted (HANKS, 1982) that when f_c is near, or larger, than the corner frequency caused by the site response, and/or the quality factor is improperly taken into account, then the value of f_c is masked and the value assumed for it may be affected by significant errors in the case of earthquakes with small source area. In order to avoid these difficulties the linear dimension of the source l , in many cases, is properly measured through the source rupture duration obtained from the body-wave pulse (e.g., FRANKEL and KANAMORI, 1983) who used the time T between the P-wave onset and the first zero crossing on the seismogram; this is obtained from (BOATWRIGHT, 1980) formula

$$l = T/[1/kS - \sin\theta/P], \qquad (2)$$

where k is a factor, generally assumed in the range from 0.75 to almost unity, to obtain the rupture velocity kS, P and S are the velocities of the P and S waves and θ is the angle between the normal of the fault plane and the direction of the outgoing seismic ray.

Considering the uncertainties in θ , those in the velocities of the waves and in k one may reasonably assume that l may be affected by an error up to 0.3 in the log scale, comparable to experimental errors sometimes accepted in the magnitude and scalar moment.

The static stress drop is computed from the formula

$$p = 7M_0/16l^3 \qquad (3)$$

obtained introducing the following relation $\langle s\rangle = 16pl/7\mu\pi$ (ESHELBY, 1957), into the classic definition of the moment for circular faults $M_0 = \pi\mu l^2\langle s\rangle$. The effect of the error in l implies a possible error of 0.5 in the log scale of p which is larger than the experimental errors accepted in the magnitude and the scalar moment.

In spite of these uncertainties, the statistical analyses of the parameters of the earthquakes, of the aftershocks and earthquake sequences of several regions, give rather consistent density distribution and correlation with the other parameters in agreement with the theory.

We have been recently gratified by a few important sets of source parameters computed also with new methods by SMITH and PRIESTLEY (1993), ABERCROMBIE (1995), HARDEBECK and HAUKSSON (1997), MORI *et al.* (2003), to quote only the data used in this note, which allow updating of the existing catalogues (e.g., CAPUTO, 1998) of the correlations and density distribution of p and l of different regions and of the different type of data sets: foreshocks, aftershocks, scattered and sequences of earthquakes.

In this note we tentatively estimate the density distributions of the parameters and their correlations in the different types of earthquake sets, and also investigate the possibility of a two-dimensional representation of the statistics of earthquakes for which it would be necessary to have available at least two independently measured source parameters covering a sufficiently wide range of values.

Theoretically the most rigorous way would be to use two parameters independently determined such as M_0 and l. However, in spite of the fact that p is obtained through M_0 and l, instead of l one may consider p for its tectonic and physical significance.

In this paper we will tentatively focus more on l and p, considering their correlation, range of variation and density distributions in the light of the new, more accurate, data produced with the new methods developed, for instance by BOATWRIGHT (1980), LI and THURBER (1988) for the determination of l which in turn allows estimation of p through formula (3).

Before proceeding we note that the statistics of earthquakes have long been characterized by the b or the b_0 value representing respectively the density distributions of the magnitude M and of the log of the scalar seismic moment M_0 of the earthquakes. In turn the b_0 value is related to the exponent ν, of the power law representing the density distributions of the linear size of the areas of the faults or to the exponent $-1+\alpha$ of the supposed power law representing the density distribution of the static stress drops of earthquakes. The formulae expressing the relations between the source parameters and their density distribution are (e.g., CAPUTO, 1998)

$$n_p(p) \propto p^{-1+\alpha} \tag{4}$$

$$n_l(l) \propto l^{-\nu} \tag{5}$$

$$\log n_0(M_0) = a_0 - b_0 \log M_0 \tag{6}$$

$$b_0 = (\nu + 2)/3 \quad \text{when } l_2^3\, p_1 > l_1^3\, p_2 \tag{7}$$

$$b_0 = -1 + \alpha \quad \text{when } l_2^3\, p_1 < l_1^3\, p_2 \tag{7'}$$

where $n_0(M_0)$, $n_p(p)$ and $n_l(l)$ are the density distributions of the seismic moment M_0, of the stress drop p and of the radius a of the fault, respectively and l_1, l_2, p_1, p_2 are the lower and upper limits of the parameter's ranges. The parameters M_0 and l are determined from the spectra of the seismograms, and p is inferred through equation (3).

The formulae (4) and (5) imply theoretically the validity of equation (6) which is also verified experimentally and generally accepted. Equations (3) and (6) have been discussed earlier (e.g., KANAMORI and ANDERSON, 1975; CAPUTO, 1976). As shown in the note of CAPUTO (1987) formula (7) is to be used when the range of the source radii l is sufficiently large relative to that of p, while formula (7′) is to be used when this range is small relative to that of p.

Unfortunately the data sets considered do not satisfy the conditions (7) or (7′) in a sufficiently wide range of M_0 to allow a rigorous comparison of the value of ν resulting from the density distribution of l computed by means of equations (1) or (2) with that resulting from b_0.

The analysis of the world values of M_0 (Caputo, 1987), collected and processed at Harvard University (DZIEWONSKY *et al.*,1987), gives $b_0 = 1.61$ to which corresponds $\nu = 2.83$, giving the density distribution $l^{-2.83}$.

The validity of the power law (4) of the density distribution of p seems confirmed by the analysis of 15 data sets, appearing in CAPUTO (1998), concerning different regions of the world and gathered by different authors. Most values of p used in the preceding studies have been computed from the source radius estimated through the measurement of the conventional corner frequency (BRUNE, 1970). The estimated values of $-1+\alpha$ in these data sets are in the range [−2,−1], however some of the spectra from which these stress drops were estimated, especially in the case of small events, were probably contaminated by attenuation through the so called $f_{\max}$ effect (HANKS, 1982).

These parameters are also important because it has been suggested that the variations of any of the parameters b, b_0, ν , $-1+\alpha$ characterizing the statistics of the earthquakes could be precursors of an incoming large earthquake. Often the parameter observed is b_0 , however $-1+\alpha$ also has been computed in several regions of the world (CAPUTO, 1998) and its variation, observed before and after two moderate earthquakes in California, has been suggested as a precursor to an incumbent earthquake (CAPUTO, 1982).

In this note we will also test the hypothesis that area B of the surface of the barrier broken during the earthquake be related to the earthquake parameters by (CAPUTO, 1998)

$$l^2 p = qBH, \tag{8}$$

where H is the shear strength of the rock, q a proportionality factor depending on the condition of the rock and B the area of the surface of the broken barrier. A typical value for H for granite is 300 bar (FARMER, 1968; JAEGER and COOK, 1984) while q results around unity. If the earthquakes were due to the breaking of a barrier and H were known at the depth of the source, formula (8) would give an estimate of the area broken in the barrier. In any case, since it is difficult to retrieve the values of H and q in the practical cases of the rocks of the broken barriers, we considered them constant for all the events which implies that they would not influence the density distribution of B which then depends only on l and p.

Equation (8) implies that, in the same stress conditions, large faults are more prone to apply a larger force on a barrier than smaller faults and therefore smaller faults, with the same barriers, would preferably generate larger stress drops.

This relation, as well as relations (4) and (5), when a multiple set of earthquake parameters l and p is available for a given region, will allow an estimate of the density distribution of the size of the barriers fractured during the earthquakes. In fact the density distribution of B (CAPUTO, 1998) is

$$\log n(B) = (\alpha - 1)\log B + K_2 \tag{8'}$$

or

$$\log n(B) = -\upsilon \log B + K_1, \tag{8''}$$

where (8″) is to be used, when in the available catalogue of earthquakes, $l_2 p_1^{0.5} > l_1 p_2^{0.5}$, and (8′) when $l_2 p_1^{0.5} < l_1 p_2^{0.5}$; K_1 and K_2 are constant.

Formulae (8) through (8′) will be tested with the data sets of SMITH and PRIESTLEY (1993), ABERCROMBIE (1995), HARDEBECK and HAUKSSON (1997), JIN *et al.* (2000) and MORI *et al.* (2003) which, generally, are more numerous than the sets used previously for the same purpose (CAPUTO, 1998). In all data sets the values of the areas of B, assumed circular, are obtained from πl^2.

Finally we note that the density distributions of M_0 and p, the latter related to the pressure acting on the side of the fault, are as important as the density distribution of the magnitudes of the Gutenberg-Richter law and of the moments of the gas molecules in the Boltzmann statistics; in fact in some cases (CAPUTO, 1987), they may lead to the estimate of the state of stress in the crust or, tentatively, to the accelerations of the ground (CAPUTO, 1981).

2. *The Data*

In this note we will examine and compare the statistical properties and density distributions of the values of the source parameters of five sets of earthquake sequences, aftershocks and earthquakes scattered in a region; with the exception of the data sets of JIN *et al.* (2000) and ABERCROMBIE (1995), who used the corner frequency, l was determined with the first zero crossing of the waveform of the recorded earthquake and the analyses are performed referring to formulae (4), (5) and (6). This will also allow a comparison with the statistical properties of p determined using the value of l obtained from the conventional corner frequency.

In order to homogenize the analysis of the parameters, since the seismic moment is usually considered in its log version, we will then use the log version of the other parameters l, p and B. In Table 1 are presented the ranges of the parameters and the *log* of the ratio of the extremes of the ranges in order to obtain an estimate of the orders of magnitude covered which in turn will also allow a comparison of the ranges of physically different parameters.

The correlations between the parameters are found in Table 2. The low value of the correlation of some couples of parameters, in some cases, is not ideal for concluding a possible relation between the parameters, however it is the consistency in the different sets and different regions that implies acceptance that the parameters are correlated.

Finally we will test some basic formulae; for instance M_0 should scale as l^2 according to the classic formula $M_0 = \pi\mu l^2 \langle s \rangle$, however eliminating $\langle s \rangle$ through formula (3') leads to $M_0 = 16\, l^3 p/7$ where M_0 scales as l^3. The tentative test of the hypothesis $M_0 \propto l^3\, p$ by fitting the formula $l^x\, p^y$ to the five data sets, since p was computed using M_0 and l, would only mean that it does not contradict the hypothesis. The direct correlation between M_0 and l or between M_0 and p with the tentative determination of the relation between the two parameters is only indicative for the reason given above: M_0 is a function of two parameters simultaneously, however the data resulting from the five sets examined are more in favor of a scaling not very far from l^2.

Table 1

Ranges of the parameters of the five sets of earthquakes, of different seismic regions and different types of earthquakes, studied in this note

Authors	Range of l (m)	log ratio	Range of p (bar)	log ratio	Range of log M_0	log ratio	Number of events
SMITH and PRIESTLEY (1993)	88–789	0.95	1–282	2.45	19.30–22.23	2.93	85
ABERCROMBIE (1995)	2.2–215	1.99	0.009–9913	6.04	15.91–22.30	6.39	111
HARDEBECK and HAUKSSON (1997)	106–1098	1.02	0.02–40	3.30	24.75–26.85	2.10	279
JIN *et al.* (2000)	54–380	0.85	0.17–122	2.86	17.61–21.47	3.86	102
MORI *et al.* (2003)	450–8700	1.29	4–1307	2.51	21.32–24.08	2.76	55

Table 2

Correlation and slopes of the couples of parameters of the five data sets analyzed in this note. The independent variable is on top.

Authors	Number events	log l* log M_0	Slope	log p * log M_0	Slope	log l* log p	Slope
ABERCROMBIE (1995) (scattered)	111	0.63	2.39	0.49	0.76	−0.17	−0.08
HARDEBECK and HAUKSSON (1997) (aftershocks)	279	0.37	1.32	0.63	0.66	−0.45	−0.12
JIN *et al.* (2000) (scattered)	102	0.50	2.31	0.78	0.91	−0.13	−0.03
MORI *et al.* (2003)(aftershocks)	55	0.62	1.67	0.33	0.36	−0.53	−0.21
SMITH and PRIESTLEY (1993) (aftershocks)	85	0.67	2.13	0.49	0.40	−0.36	−0.14

We note that in order to obtain significant values of the slopes of the density distributions there is a requirement to have complete catalogues of the parameters under examination; however the events with smaller moments are often neglected or have poor records and therefore the catalogues are generally incomplete not only for the smaller moments but, consequently, also for the smaller stress drops and fault sizes.

As is generally done for the estimates of the b and b_0 values we will neglect the smaller values of the parameters and, assuming that the smaller values of the parameters are more numerous, we will consider the log of the density distributions beginning where they appear to decrease linearly.

Concerning the density distribution of the parameters, for simplicity of notation, the same functional symbol $n(x)$, where x is the parameter value, is adopted although the variable implies that the functions are different.

3. The data of Smith and Priestley (1993)

SMITH and PRIESTLEY (1993) studied 85 aftershocks along active fault planes of the 1984 Round Valley, California, earthquake sequence. They determined source dimensions and stress drops using an adaptation of the initial P-wave pulse width time-domain deconvolution technique. Among others they found that l increases systematically with M near four and that the stress drops are not systematically correlated with magnitude or depth.

The correlation between log l and log M_0 has value 0.67, the slope is 2.13 and tentatively implies that the two parameters determined independently are correlated as they should be theoretically.

From the values of l and M_0 we computed p, using the formula (3). The correlation between log p and log l, shown in Figure 1, is – 0.36 and the slope is – 0.14; that between log p and log M_0 has value 0.49, the slope is 0.40.

The slope of density distributions of log M_0 is –0.89 far from the world average (–1.61), that of log p is – 1.54 within the range of the previously determined values [–1,–2] (CAPUTO, 1998) when p was obtained using the corner frequency as in this set of data.

The values of l are constrained in a narrow range [2.3, 215] m, however we estimated that the slope of the density distribution of log l is –1.43 substantially smaller than the world average (–2.83) (CAPUTO, 1987), based on the Harvard catalogue (DZIEWONSKI *et al.*,1987), and smaller than the experimental values around –2.5 (e.g.,CAPUTO, 1987; WALLACE, 1976, 1989) of the size of the faults inferred from surface observations. However, we should note that the slope obtained from the Harvard catalogue is valid for large earthquakes and is not necessarily valid for small ones as in this set of earthquakes.

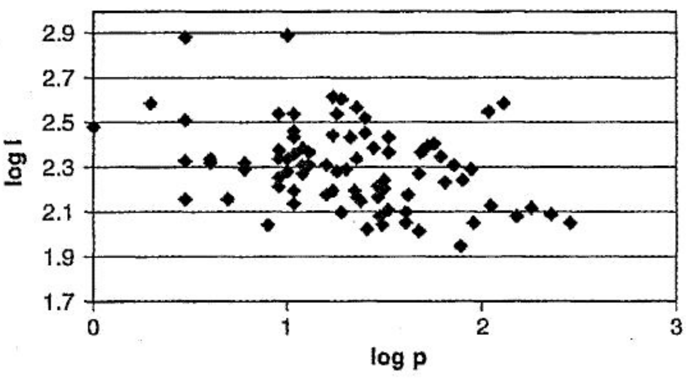

Figure 1
The figure indicates that, in the data of SMITH and PRIESTLEY (1993), log l is a decreasing function of log p. l is meters, p is bar.

Since the number of parameters of this set is relatively large, with the values of l and p we may tentatively obtain the density distribution of the areas B of the barriers fractured during the earthquake, in fact we may write equation (8)

$$\log B = 2\log l + \log p - \log qH. \tag{9}$$

We find that $\log n(B) \propto -0.84 \log B$ to be compared with the slope of $\log n(p)$ given above (−1.54) which is somewhat smaller. We note then that the ratio between the number of smaller and larger barriers broken by the earthquakes is larger than that of the corresponding stress drops.

4. The data of Abercrombie (1995)

ABERCROMBIE (1995) analyzed 111 earthquakes, some with very small magnitude, recorded in a well at 2.5 km depth, in granite, in the Cajon Pass scientific drill hole, Southern California. ABERCROMBIE (1995) computed M_0 and used several models, based on the Brune ω^{-2} model, to compute the corner frequency and infer the source radius l from P and S waves. The model which is to be considered as basic is her model 1 which assumed that the Q varies in the realistic ranges: [581–1433] for the P waves and [879–1323] for the S waves.

From the values of l and M_0 of ABERCROMBIE (1995) we computed p, using equation (3).

We note that four values of p in this set are almost or larger than 1000 bar which seem anomalous according to the current points of view, moreover all the other 107 values of p in the set are smaller than 12 bar except one which is 540 bar . If we consider these four larger values as outliers then the range of p in this set is [0.09–540]bar and the log of the ratio of maximum to minimum values is 3.8. These four large values have not been considered when computing the density distribution of log p and of the correlations of p with the other parameters of the set.

The correlation between log l and log M_0 is 0.63, has slope 2.39 and is seen in Figure 2.

The correlation between log p and log M_0 , shown in Figure 3, is 0.49 with slope 0.76.

The correlation between log p and log l is − 0.17 implying a very low correlation between the two parameters; the slope is −0.08.

The density distribution of log l has slope − 0.66, which would imply that, for the set of faults involved in this set of aftershocks, the density distribution is proportional to $l^{-0.66}$ whose exponent is far from the value −2.83 of the world average (CAPUTO, 1987). One cause of this discrepancy may be in the limited range of l, where more than 85% of the values of l are in the limited range [10,100] m.

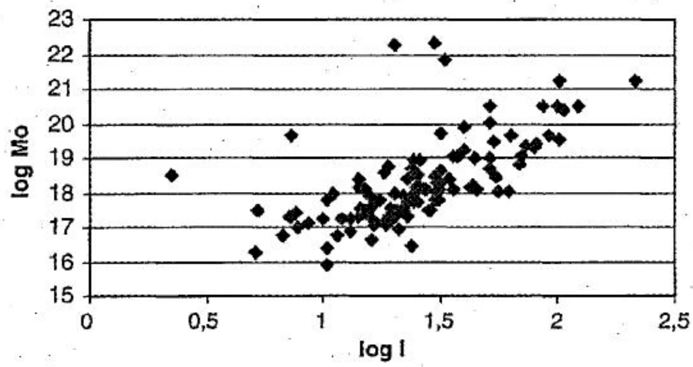

Figure 2
In spite of the scatter the figure is indicative that, in the data of ABERCROMBIE (1995), log M_0 is an increasing function of log l. l is meter, M_0 is dyne cm.

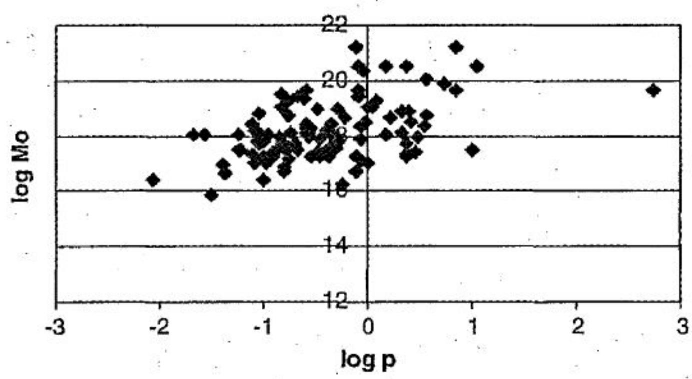

Figure 3
In spite of the scatter the figure is indicative that, in the data of ABERCROMBIE (1995), log M_0 is an increasing function of log p. p is bar, M_0 is dyne cm.

The density distribution of log M_0 has slope -0.31 which is short of the value -1.61 of world average (CAPUTO,1987) and implies that in this set the ratio of smaller to larger faults is smaller than the world average.

The slope of the density distribution of log p is -0.54 and is outside the range $[-2,-1]$ of the previous set of data (CAPUTO, 1981).

With the values of l and p we may tentatively obtain the density distribution of the linear dimension of area B of the barrier fractured during the earthquake using equation (8) and estimate the density distribution of a. We find that log $n(B) \propto -1.27$ log B to be compared with the slope of log $n(p)$ given above -1.54. The two values are not close. We note that the ratio between the number of smaller and larger barriers broken by the earthquakes is larger than that of the corresponding stress drops.

We note also that the value of the slope of log $n(p)$ of the data of ABERCROMBIE (1995) is smaller than the value resulting in the previous analysis of the data (CAPUTO, 1998). This is due to the fact that, in the previous estimate (CAPUTO, 1998), the smaller values of M_0 and p were neglected because of the presence of a detection threshold for smaller earthquakes at relatively longer distances which at that time seemed important to do.

5. The Data of Hardebeck and Hauksson (1997)

HARDEBECK and HAUKSSON (1997) made an exhaustive analysis of 279 aftershocks, with magnitudes covering the range [2.5–3.9], of the 1994 Northridge, California, earthquake and collected at four TERRAscope stations; they used the duration of the time function of the rupture velocity and found p, M_0 and l; they also noted an increase of the log average of p at about 15 km depth, possibly controlled by material properties, and also an apparent increase of p with magnitude (see also CAPUTO, 1987). The correlation between log l and log M_0 has value 0.37. Its linear regression has slope 1.32 which indicates a possible correlation between the two parameters.

The correlation between log p and log M_0, shown in Figure 4, has value 0.63 and slope 0.66.

The correlation between log p and log l, shown in Figure 5, is −0.45 and the slope −0.12 suggesting a possible correlation between the two parameters.

The density distribution of log M_0, shown in Figure 6, has slope −1.90, approaching the world average (−1.61); the density distribution of log p has the slope −1.27, within the range of the data [−2,−1] previously examined (CAPUTO,1981).

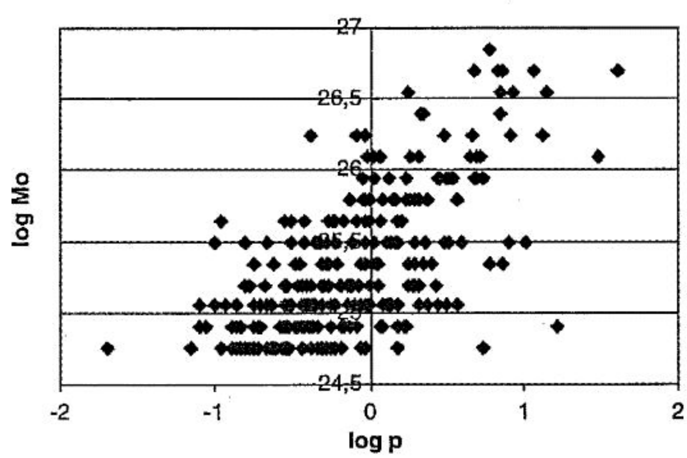

Figure 4
The figure indicates that, in the data of HARDEBECK and HAUKSSON (1997), the large values of M_0 occur preferably with large values of p not excluding that small values of M_0 are possible also with large values of p. p is bar and M_0 is dyne cm.

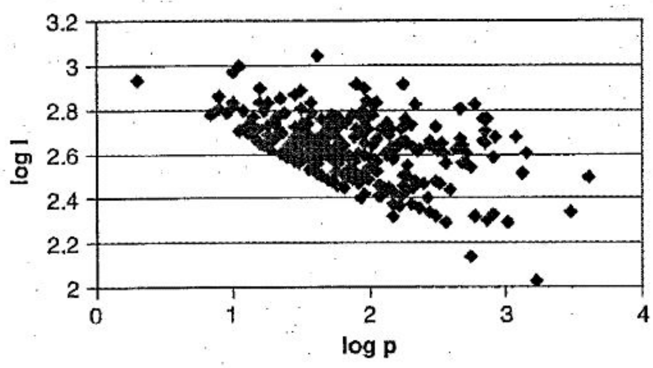

Figure 5
The figure indicates that, in the data of HARDEBECK and HAUKSSON (1997), in spite of the scatter, log l may be a decreasing function of log p . p is bar, l is meters.

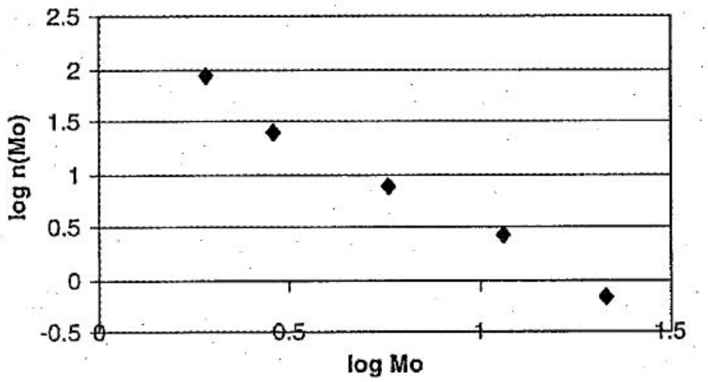

Figure 6
The figure shows the density distribution of M_0 in the aftershocks of the 1994 Northridge California earthquake (HARDEBECK and HAUKSSON, 1995). In order to obtain the value of M_0 in dyne cm in abscissa add 25.

The density distribution of log l has slope − 3.29, which would imply that, for the set of faults involved in these aftershocks, the density distribution is of the type $l^{-3.29}$ whose exponent is smaller than the value (−2.83) of the world average and implies that the ratio between the number of small faults to that of large ones is larger than in the world average.

With the values of l and p we may tentatively obtain the density distribution of area B of the barrier fractured during the earthquake. In fact using equation (8) we may estimate the density distribution of log B. We find that log $n(B) \propto -1.53 \log B$ to be compared with the slope of log $n(p)$ given above −1.27. The two values are somewhat close and the ratio between the number of smaller and larger broken barriers seems close to that of the corresponding stress drops.

6. The Data of Jin et al. (2000)

JIN *et al.* (2000) recorded at 9 local stations 102 small earthquakes with magnitudes ranging [0.6,3.6], but mostly less than 2, in the Atotsugawa fault zone in central Japan obtaining f_c, M_0 and p. They found that that p appears to vary systematically with location and that combining M_0 and f_c with those of other regions, a single relation between M and f_c does not apply, suggesting according to AKI (1981) that earthquakes are a multihierarchical system divided by characteristic sizes.

The correlation between $\log p$ and $\log l$ is −0.13 with slope −0.03 and suggests very little correlation between the two parameters. The correlation between $\log l$ and $\log M_0$ is 0.50 and the slope is 2.31; the correlation between $\log p$ and $\log M_0$ is 0.78 with slope 0.91.

The density distribution of l has slope −1.63, far from the value (− 2.83) of the world data (CAPUTO, 1987). The density distribution of p, shown in Figure 7, has slope −1.21 which falls in the range [−2,−1] of the data sets previously examined (CAPUTO, 1981), the density distribution of $\log M_0$ has slope −1.49, close to that of the world average (−1.61).

With the values of l and p we may tentatively obtain the density distribution of area B of the barrier fractured during the earthquake. In fact using equation (8) again we estimate that $\log n(B) \propto -1.50 \log B$ to be compared with the slope of $\log n(p)$ given above − 1.21. The two values are not close and the ratio between the number of smaller and larger broken barriers is smaller than that of the corresponding stress drops.

7. The Data of Mori et al. (2003)

MORI *et al.* (2003) also studied p, dynamic stress drops and the radiated energies of 55 aftershocks of the 1994 Northridge, California, earthquake with magnitudes

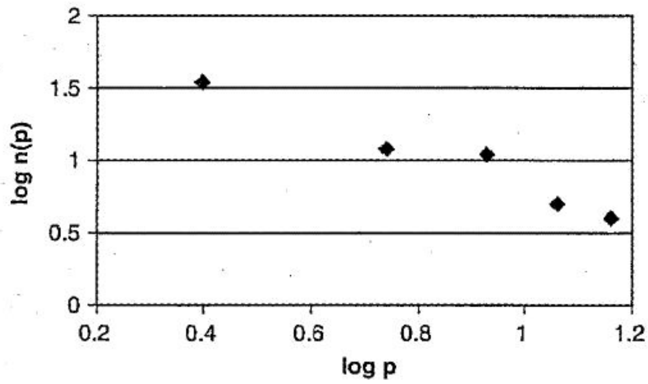

Figure 7
The figure shows the density distribution of p of 102 small earthquakes with magnitude mostly less than 2, in the Atotsugawa fault zone in central Japan recorded by JIN *et al.* (2000).

larger than four; they found that the radiated energy is relatively low compared to the p, indicating that the p and the dynamic ones are of similar magnitude; they also found an increase in the ratio of radiated energy to moment with increasing moment while there is no corresponding increase in p.

The correlation between log p and log l , shown in Figure 8, is $-$ 0.53 and the slope -0.21 which suggests a correlation of the two parameters

The correlation between log M_0 and log l, shown in Figure 9 has value 0.62, the slope is 1.67. The correlation value tentatively implies that the two parameters, although determined independently, are correlated as they should be theoretically.

The correlation between log p and log M_0 has value 0.33 and the slope is 0.36.

The density distributions of log M_0 has slope -1.34 not far from the average world datum (-1.61), that of log p has slope -1.33 within the range [-2,-1] of the data previously determined using the corner frequency.

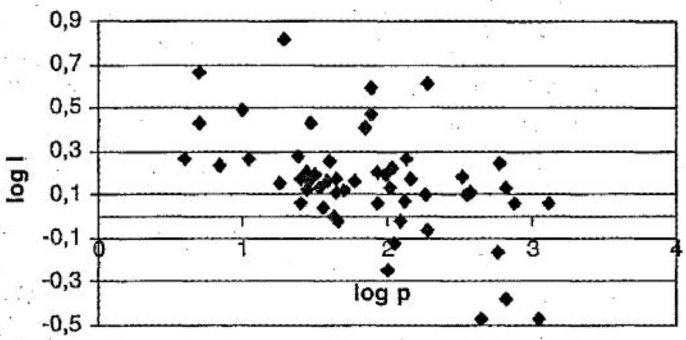

Figure 8
The figure shows the scatter of the relation between log l and log p in the data of Mori *et al.* (2003) and indicates that log l is a decreasing function of log p. p is in bar and l is km.

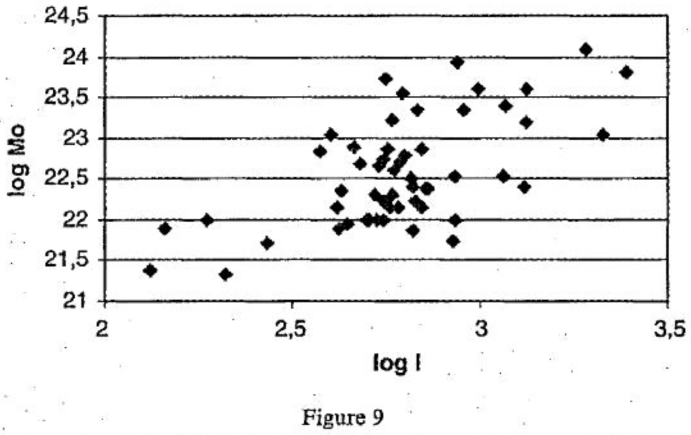

Figure 9
The figure shows that, in the data of Mori *et al.* (2003), log M_0 is an increasing function of log l. l is meters, M_0 is dyne cm.

The density distribution of log l has slope $-$ 2.07, which would imply that, for the faults involved in this set of aftershocks, the density distribution of l is proportional to $l^{-2.07}$ whose exponent is larger than the value −2. 83 of world average (CAPUTO, 1987), indicating that in this set of earthquakes the larger faults are predominant.

With the values of l and p we may tentatively obtain the density distribution of B. In fact again using equation (8) we find that $\log n(B) \propto -1.21 \log B$ to be compared with the slope of $\log n(p)$ given above $-$ 1.33. The two values are close. We note that the ratio between the number of smaller and larger broken barriers seems smaller than that of the stress drops, which is contrary to the finding of HARDEBECK and HAUKSSON (1997) for the aftershocks of the same earthquakes; one possible explanation is the comparably larger size of earthquakes in the set of MORI *et al.* (2003) to the set of HARDEBECK and HAUKSSON (1997).

8. Discussion of the Ranges of the Parameters and of their Correlations

In this note we discuss and compare the correlations of the linear size of faults l, obtained from the corner frequency or the pulse duration, the moment M_0 and static stress drop p of five sets of the data of SMITH and PRIESTLEY (1993), ABERCROMBIE (1995), HARDEBECK and HAUKSSON (1997), JIN *et al.* (2000) and MORI *et al.* (2003); we will later also discuss and compare the density distributions of the seismic source parameters.

In Table 1, where we list the ranges of the five different data sets considered, it is clearly seen that the sets have a very variable number of data ranging from 55 to 279. In general the five data sets discussed are more numerous than those used in previous studies (e.g., CAPUTO, 1998).

The orders of magnitudes of the ranges covered by the parameters in each set are scattered as seen in Table 1.

From Table 1 it follows that the range of the parameters in each set, excepting the values of l in the set of ABERCROMBIE (1995), is larger than two orders of magnitude, which shows that the spread of the values used for the slopes of the correlations covers at most two orders of magnitudes; this is far from the width of the sets generally used to estimate the presently available slopes of the density distributions of the magnitude and of the scalar seismic moment of world data also not so far from the slopes of the same regional density distributions.

As we noted, all the values of p of the set of ABERCROMBIE (1995) have been considered for completeness in Table 1, if we consider the four anomalous values almost or larger than 1000 bar as outliers, the range of log (max. value of p/min. value of p) is 3.8, and the general scatter of the ranges of log (max. value of p/min. value of p) from Table 1 reduces to [2.45–3.8].

It seems also that log l is a decreasing function of log p and that the correlation between the log of the two parameters is always less than 0.55, which would not be ideal in order to accept that log l be a decreasing function of log p. However the hypothesis is confirmed, or at least not contradicted, by the consistency of the negative slopes of their correlation which is in the range [–0.03, –0.14]. It is the consistency of the negative slope in all data sets, taking into account the different regions and type of earthquakes sampled, that makes the hypothesis suggestive.

We may then tentatively consider that $\log p$ decreases with increasing $\log l$ and that they are related by a power law. We should also note that the linearity of the relation between $\log l$ and $\log p$ with slope γ implies that between l and p would exist a power law relation of the type $l \propto p^{\gamma}$.

The correlation between $\log M_0$ and $\log l$ in the five sets is more than 0.3 and the slopes are in the range [1.32, 2.39]. However, taking into account the limited number of data in the sets and different type of earthquakes and regions sampled, the scatter is reasonable. We note that in all five sets the slope is always positive, which implies that M_0 is an increasing function of l as it theoretically should be. A typical example of these correlations is shown in Figure 2.

Concerning the correlations between $\log M_0$, $\log l$ and $\log p$, it has been seen that, for the three sets of earthquakes analyzed by CAPUTO (1987) the positive values of the correlations are in agreement with those obtained in this note.

We mentioned that for two-dimensional models of seismicity it would be natural to use two completely independent parameters; this couple is obviously M_0 and l since the two parameters are estimated independently. However, the weak correlation between $\log p$ and $\log l$ for a possible 2-D statistical analysis of seismic activity, in spite of the fact that p is computed using M_0 itself, makes the two parameters l and p also good candidates, mostly for the tectonic implications of the value of p; an example is given in Figure 10.

The slopes of the correlation of $\log M_0$ versus $\log l$ and of $\log M_0$ versus $\log p$ are positive, as theoretically expected, and this could suggest that M_0, l and p , in limited ranges may be related by power laws which qualitatively would justify the assumptions in equations (5), (5′) and (6).

Excluding the outliers of p in the data of ABECROMBIE (1995), the maximum range of p is in the set of HARDEBECK and HAUKSSON (1997) however, in order to find the maximum range of p in a set of aftershock we should put together this set with that of MORI *et al.* (2003) since both concern the aftershocks of the 1994 Northridge California earthquake. One finds that the range [0.02–1307] and the log of the ratio is 4.81, that is p covers almost five orders of magnitude. The reliability of the estimates of the slopes is indicated by the correlation of the associated parameters.

9. *Discussion of the Density Distribution of the Parameters*

In Table 3 we report the slopes of the density distributions of $\log n(M_0)$, $\log n(l)$, $\log n(p)$ and $\log n(B)$

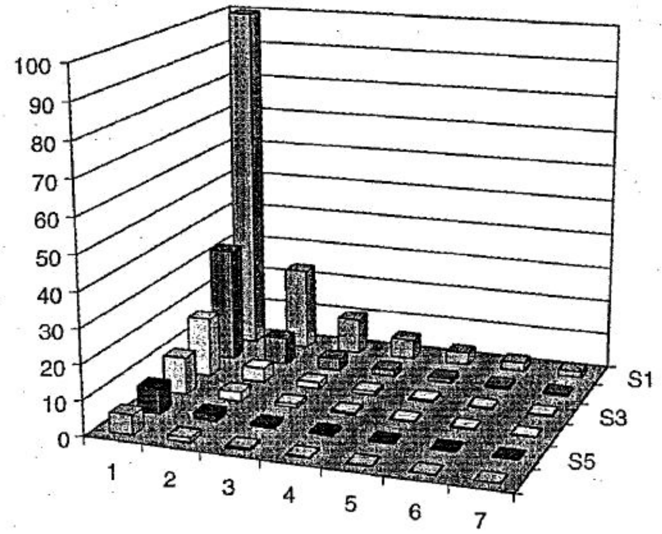

Figure 10
2-D representation of the distribution of the seismicity of the catalogue of SMITH and PRIESTLEY (1993) as a function of l (in the front abscissa) and p (in the lateral abscissa marked with S). The density distribution, according to equations (4) and (5), is proportional to $p^{-1+\alpha}l^{-v}$ with $-1+\alpha=-1.61$ and $v=-2.10$ shown in Table 3; in the figure the units of the ordinate are arbitrary.

Table 3

Density distribution of the parameters of the five data sets of earthquakes, of different seismic regions and different types of earthquakes, analyzed in this note. Below each datum is the standard deviation in italics

Authors	log n (M_0) slope	log n (l) slope	log $n(p)$ slope	log $n(B)$ slope	numbers of data
SMITH and PRIESTLEY	−0.89	−2.54	−1.64	−1.21	85
(1993) (aftersh.)	*0.02*	0.30	0.08	0.05	
ABERCROMBIE (1995)	−0.31	−0.66	−0.54	−1.27	111
(scattered)	*0.07*	0.04	0.15	0.11	
HARDEBECK and HAUKSSON	−1.90	−3.29	−1.27	−1.53	279
(1997) (aftersh.)	*0.09*	0.09	0.18	0.08	
JIN *et al.* (2000)	−1.49	−1.63	−1.21	−1.50	102
(scattered)	*0.20*	0.01	0.09	0.17	
MORI *et al.* (2003)	−1.34	−2.07	−1.33	−1.21	55
(aftersh.)	*0.12*	0.11	0.06	0.08	
World average, or range	−1.61	−2.83	[−2,−1]		

It is immediately verified that, as expected, in the cases of log $n(M_0)$, log $n(l)$ and log $n(p)$ the slopes are all negative, however the ranges of the independent variables are limited to a narrow band. In all cases the density distributions are markedly decreasing functions of their parameters.

The negative slopes of log $n(B)$ would imply that as for all the other variables, also the density distribution of the areas of surfaces of the barriers broken by the earthquakes B is given by a power law with negative exponent.

It also should be noted that the values of the slopes of the density distribution of log $n(p)$ determined for other sets of data, scattered earthquakes and aftershocks sequences, were generally in the range [−2, −1] (CAPUTO, 1998); we verify that the values of Table 3 fall inside this range with one exception; the slope resulting from the set of data of ABERCROMBIE (1995).

The data sets of HARDEBECK and HAUKSSON (1997) and of MORI *et al.* (2003) concern the aftershocks of the same earthquake of 1994 at Northridge, California, therefore a comparison of the results of the analyses is in order. The log $n(M_0)$, log l and log $n(p)$ we obtained from the 279 data of HARDEBECK and HAUKSSON (1997) have slopes − 0.68, − 3.23 and − 1.27 respectively with M_0 in the range [5.6–708]10^{24} dyne cm, while from the data of MORI *et al.* (2003) with M_0 in the range [2.1–1200] 10^{21} dyne cm we obtained the somewhat similar slopes −1.34, 2.07 and −1.33, respectively. The ranges are not very wide but the slopes, also taking into account the limited number of data, are not in agreement and indicate that the log of the density distributions of l and M_0 may not be linear in a relatively wide range.

The ranges of M_0 in the two above-mentioned sets of data are relatively wide but are not overlapping. It is not feasible to estimate the reliability of the slopes of the density distributions since they are computed from histogrammes whose definition has some degrees of freedom.

10. Conclusions

The limited number of sets examined prevents the deductions of firm conclusions between the characteristics of the density distributions of the parameters in different seismic regions. Other sets of data are available, however the number of earthquakes examined is too small to draw reliable conclusions. With the limited number of sets examined in this note we may only draw preliminary conclusions.

The slopes of the density distributions of the parameters shown in Table 3 are scattered however with the same sign, all confirm that the density distributions of the source parameters are decreasing function and tentatively suggest that the density distributions of the parameters, in limited ranges, may be represented by power laws in agreement with the tentative assumptions (4) and (5). The scatter of the slopes of the density distributions of the single parameters in the five sets is not small; it is the consistency of the signs of the slopes which supports decreasing density distributions and possibly power laws at least in limited ranges. We must add that the tectonics of the different regions and/or of the different type of earthquakes involved (e.g., earthquake sequences, scattered earthquakes or aftershocks) may substantially influence the scatter of the slopes. Systematic differences between sets of aftershocks, of foreshocks, of scattered earthquakes and earthquake sequences may eventually result when studying a larger number of data sets.

It is shown that the correlation of log M_0 with log l, and consequently of log p, is generally large. Although the correlations between log l and log p and their slopes are not adequately large the coherence of the signs we may suggest that the *log p* is a decreasing function of *log l* although the slope of the linear law relating *log l* to *log p* is small.

From Table 3 it would also result that the density distribution of the areas of the surfaces of the barriers broken by the earthquakes B is a decreasing function of B and that it may possibly be given by a power law with a negative exponent as is theoretically implied by the density distribution of log l and log p. No relation is apparent between the values of the slopes of the density distributions of log B and log p of the different sets.

The density distributions of the seismic source parameters clearly show that there is no equipartition of seismic moment.

An increase of the value of $2\alpha - 1$, the exponent of the power law of the dimensions of the fracture areas B of the fault, would imply that the fractal dimension of the active portions of the faults is increasing and that a larger number of smaller irregularities are being broken and therefore that an increased instability is brought into the fault. An additional consequence is that the fractal dimension of the earthquake is more likely that associated with the density distribution of the stress drops occurring on the fault, which theoretically may be associated with that of the areas of the broken barriers.

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