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Spherical Block Model

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The Spherical Block Model: (i) Different Modifications; (ii) Simulation of Dynamics and Seismicity of the Global System of Tectonic Plates

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 Russian Federation www.mitp.ru Abstract. The spherical block model is used to study dynamics and seismicity of the global system of tectonic plates by means of numerical simulation. A brief description of the model and its different modifications (with and without discretization of segments by depth) is presented. Results of numerical experiments include the qualitative information on displacements of plates, and on the character of their interaction along boundaries. Synthetic earthquake catalogs reveal some patterns of observed seismicity. Model frequency-magnitude cumulative (FM) plots are approximately linear on definite magnitude intervals. The analysis of numerical simulations shows the dependence of synthetic seismic properties on model parameters. Future studies directed to improvement of the model and numerical algorithms are outlined.

Key words: block-and-fault models, spherical geometry, global system of tectonic plates.

1 Introduction

Study of seismicity with the statistical and phenomenological analysis of real earthquake catalogs has the disadvantage that the reliable data cover is, in general, a time interval of about one hundred years or less. This time interval is very short in comparison with the duration of tectonic processes responsible for the seismic activity. Therefore, the patterns of the earthquake occurrence identifiable in a real catalog may be only apparent and may not repeat in the future. In this connection, mathematical models of seismicity, i.e., of earthquake sequences, are important tools that yield synthetic catalogs, which may cover very long time interval that allows us to acquire a more reliable estimation of the parameters of seismic flow and to search for premonitory patterns preceding large events. The model should be adequate in the sense that it reproduces properties of observed seismicity (primarily the Gutenberg-Richter law on frequency-magnitude (FM) relation, migration of events, seismic cycle and so on). Only then it is possible to use a synthetic catalog, a result of numerical simulation, for obtaining some estimates of the characteristics of an earthquake flow.

The present paper continues the investigations of Melnikova et al. (2000) and Rozenberg et al. (2005), who described in detail an approach to the construction of the spherical modification of the block model and the first results of simulation of dynamics both of a relatively small and global systems of tectonic plates. In the existing block models, a seismically active region is represented as a system of rigid blocks that form a layer of a fixed thickness between two horizontal planes or concentric spheres. Lateral boundaries of the blocks consist of segments of infinitely thin plane faults. The system of blocks moves as a consequence of the prescribed motion of the boundary blocks and of the underlying medium. The displacement of a block may be described by three parameters (the so called two-dimensional model) as well as by six parameters (the three-dimensional model). The displacements of blocks at any point in time are defined so that the system is in a quasi-static equilibrium state. Because the blocks are perfectly rigid, all deformations take place in the fault zones and at the block bottoms. The interaction between the blocks is visco-elastic (a "normal state"), so long as the ratio of the stress to the pressure is below a certain strength level. When this level is exceeded in some part of a fault, a stress-drop (a "failure") occurs in accordance with the dry friction model. The failures represent earthquakes. Immediately following the earthquake for some period of time, the corresponding parts of the faults are in a "creep state". This state differs from the normal one because of the more rapid growth of inelastic displacements and continues until the stress falls below a given level. A synthetic earthquake catalog is produced as a result of the numerical modeling. The information on displacements of blocks and their interaction along boundaries is obtained. A detailed description of block models is given, e.g., in Gabrielov et al. (1986), Soloviev and Maksimov (2001), and Soloviev and Ismail-Zadeh (2003).

The two-dimensional plane block model has been the most extensively studied. Models approximating the dynamics of lithosphere blocks of real seismic regions have been built on its basis (e.g., Sobolev et al., 1999; Soloviev and Ismail-Zadeh, 2003). It has been used to study the dependence of properties of seismic flow on the geometry of faults and specific motions (e.g., Keilis-Borok et al., 1997; Rundquist and Soloviev, 1999). In the three-dimensional plane model (Rozenberg and Soloviev, 1997), a vertical component of displacements has been taken into account introducing three additional degrees of freedom. The spherical geometry (Melnikova et al., 2000; Rozenberg et al., 2005) has been involved after significant distortions were revealed, while trying to simulate the motion of a system of global tectonic plates with plane block models.

In this paper, the emphasis is on discussing the results of simulating the dynamics of the global system of tectonic plates by means of different modifications of the spherical block model. In addition, directions for future investigations are outlined.

2 Brief description of the model

Let us briefly describe the basic constructions of the spherical block model.

2.1 Block structure geometry, block motion

A spherical layer of a depth H bounded by two concentric spheres is considered. The outer sphere represents the Earth's surface and the inner one represents the boundary between the lithosphere and the mantle. A block structure is a limited and simply connected part of this layer (see Fig. 1).



Figure 1: A model block structure on the sphere.

Partition of the structure into blocks is defined by faults intersecting the layer. Each fault is a part of a cone surface characterized by the following two properties. Firstly, the intersection of the fault with the outer sphere (a fault line) is an arc of a great circle; the direction is specified for the fault line. Secondly, a vertex of the cone lies on a straight line, which is perpendicular to the great circle plane and passes through the center of the sphere. For such a definition of a fault, its dip angle with the outer sphere has the same value at all points of the fault line. We denote the dip angle (measured to the left of the fault line) as α . Thus, the geometry of a block structure is described by a system of the fault lines on the outer sphere enclosing the layer, and by dip angles. Faults intersect along curves, which meet the outer and inner spheres at points called vertices. A part of such a curve between two respective vertices is called a rib. Fragments of faults limited by two adjacent ribs are called segments. The common parts of blocks with the limiting spheres are spherical polygons, those on the inner sphere are called bottoms. The block structure may be a part of the spherical shell and be bordered by boundary blocks, which are adjacent to boundary segments. Another possibility is to consider the structure including the whole spherical shell (covering the whole surface of the Earth) without boundary blocks. It should be noted that the possibility of considering the closed structure is a peculiarity of the spherical model.

The blocks are assumed to be perfectly rigid. All block displacements are assumed to be negligible, compared with block sizes. Therefore, the geometry of the block structure does not change during the simulation, and the structure does not move as a whole. The gravitation forces remain essentially unchanged by the block displacements and, because the block structure is in a quasi-static equilibrium state at the initial time moment, gravity does not cause motion of the blocks. All vertices on the outer sphere are defined by geographic coordinates (latitude φ , and longitude ψ) in a spherical coordinate system with origin at the Earth's center (both this system and corresponding Cartesian system are called "System-O", Fig. 2).



Figure 2: System-O.

In the spherical modification, all blocks (both internal and boundary (if specified)) have six degrees of freedom and can leave the spherical surface. The displacement of each block consists of translation and rotation components. The translation component is determined by a translation vector (x, y, z). The rotation component is described by means of three angles γ , β , λ with respect to an immovable Cartesian coordinate system, (X, Y, Z), with the origin at the mass center of the block, point C. The X axis is directed along the parallel (latitude), the Y axis along the meridian (longitude), the Z axis is in the direction of the Earth's radius vector outwards. Denote this "System-C" (see Fig. 3).



Figure 3: System-C.

Assume that the coordinate system with axes X_1 , Y_1 , Z_1 is connected to the mass center of the block (it coincides, in the absence of block displacements, with the immovable system of axes X, Y, Z, in which we consider all block motions). Rotation of the block and its corresponding system (X_1, Y_1, Z_1) with respect to the system (X, Y, Z) is given in Fig. 4. The first angle, γ , is defined as the angle of rotation of axes Y and Z around axis X such that if axis Z_2 is the intersection of planes XOZ_1 and YOZ, then axis Z is mapped into axis Z_2 and Y into Y_2 . The second angle, β , is defined as the angle of rotation of axes X and Z_2 around axis Y_2 providing transformation of axis Z_2 into axis Z_1 (Z_1 is in the plane of XOZ_2) and X into X_2 . And the third angle, λ , is defined as the angle of rotation of axes X_2 and Y_2 around axis Z_1 such that X_2 and Y_2 transform into X_1 and Y_1 correspondingly.



Figure 4: Definition of rotation angles γ , β , and λ in System-C.

According to the definition of the rotation angles, components Δ_x , Δ_y and Δ_z of displacement at a block point with spherical coordinates (φ, ψ, r) have the following form in System-C:

$$\Delta_x = x - \hat{Y}\lambda + \hat{Z}\beta, \quad \Delta_y = y + \hat{X}\lambda - \hat{Z}\gamma, \quad \Delta_z = z - \hat{X}\beta + \hat{Y}\gamma, \tag{1}$$

where (x, y, z) is a block shift, $(\hat{X}, \hat{Y}, \hat{Z})$ are coordinates in System-C of the vector which is directed from the mass center of the block to the point (φ, ψ, r) , the angles (γ, β, λ) are assumed to be small.

2.2 Interaction between blocks, equilibrium equations

The translation vector and the angles of rotation are found from the condition that the sum of all forces acting on the block, and the total moment of these forces, are equal to zero at every point in time (the structure is assumed to be in a quasistatic equilibrium state). The interaction of the blocks with the underlying medium takes place on the inner sphere. The motions of the boundary blocks (if they are specified) and of the underlying medium, considered as an external action on the structure, are assumed to be known. As a rule, they are described as rotations on the sphere, i.e., axes of rotation and angular velocities (Euler vectors) are given. Another possibility consists in specifying a field of velocities (by some law or pointwise) for points belonging to the boundary blocks and/or the underlying medium.

Depending on the way of treating the depth of the spherical layer, two modifications of the model are worked out. Since this depth is significantly less than the linear dimensions of a block structure, it seems reasonable to consider only points belonging to a fault line on the Earth's surface while computing the properties of block interaction. Thus, it is assumed that all characteristics are described only by coordinates (φ , ψ) and do not depend on H. This version of the model is called the "modification without depth". Its advantage consists in essential saving of running time during simulations. The cons are obvious: (i) actually, dip angles are not properly taken into account; (ii) studying the mechanism of spreading an earthquake along a fault is impossible; (iii) a range of changing the model magnitude is significantly narrowed. That is why the second modification ("modification with depth", more complicated but more adequate) is designed.

Consider a point with coordinates (φ, ψ) belonging to some fault separating blocks with numbers *i* and *j*, with block *i* on the left, and block *j* on the right. Denote by $\vec{e_t}$ the unit vector tangent to the fault line at this point and directed along the fault. Let it have coordinates $\vec{e_t} = (e_1, e_2, 0)$ in the rectangular coordinate system with origin at the point (φ, ψ) and axes introduced analogously to those of system-C (call it "system-P"). Define the vector $\vec{e_l} = (-e_2 \cos \alpha, e_1 \cos \alpha, -\sin \alpha)$, which lies on the plane tangent to the fault's surface at the given point and is perpendicular to the vector $\vec{e_t}$ (α is a dip angle of the fault). Introduce also the vector $\vec{e_n} = (-e_2 \sin \alpha, e_1 \sin \alpha, \cos \alpha)$ that is perpendicular to this plane. Let the righthanded triple $(\vec{e_t}, \vec{e_l}, \vec{e_n})$ define the rectangular coordinate system with the origin at the point (φ, ψ) , "system-T", see Fig. 5.

Let $(\Delta_x^r, \Delta_y^r, \Delta_z^r)$ be the vector of relative displacement of blocks at the point (φ, ψ) in system-P. Components of displacement on the plane tangent to fault's surface at this point in system-T are correlated with Δ_x^r, Δ_y^r and Δ_z^r as follows:

$$\Delta_t = \Delta_x^r e_1 + \Delta_y^r e_2, \quad \Delta_l = -\Delta_x^r e_2 \cos \alpha + \Delta_y^r e_1 \cos \alpha - \Delta_z^r \sin \alpha,$$
$$\Delta_n = -\Delta_x^r e_2 \sin \alpha + \Delta_y^r e_1 \sin \alpha + \Delta_z^r \cos \alpha.$$



Figure 5: System-T.

The elastic force per unit area (f_t, f_l, f_n) applied to the point of the fault is defined by

$$f_t = K_t(\Delta_t - \delta_t), \quad f_l = K_l(\Delta_l - \delta_l), \quad f_n = K_n(\Delta_n - \delta_n).$$
 (2)

Here, δ_t , δ_l , δ_n are corresponding inelastic displacements, the evolution of which is described by the equations

$$\frac{d\delta_t}{dt} = W_t f_t, \quad \frac{d\delta_l}{dt} = W_l f_l, \quad \frac{d\delta_n}{dt} = W_n f_n. \tag{3}$$

The coefficients K_t , K_l , K_n , W_t , W_l , and W_n in (2) and (3) may be different for different faults.

Now, calculate the components of relative displacement, Δ_x^r , Δ_y^r and Δ_z^r , with the use of formulae (1). We obtain

$$\Delta_x^r = \Delta_x^i - \Delta_x^j, \quad \Delta_y^r = \Delta_y^i - \Delta_y^j, \quad \Delta_z^r = \Delta_z^i - \Delta_z^j, \tag{4}$$

where $(\Delta_x^i, \Delta_y^i, \Delta_z^i)$ and $(\Delta_x^j, \Delta_y^j, \Delta_z^j)$ are vectors of displacement (in system-P) of the point (φ, ψ) as a point of blocks *i* and *j*, respectively. In order to obtain the components of these vectors, one should multiply the displacements in system-C (defined by (1)) by the transformation matrix from system-C (corresponding to the

block) to system-P (omitted here due to length). Let us note only that in this way one can determine the displacements both for points on any fault and on the block bottom.

In system-P (associated with a point (φ, ψ) of the block bottom) the elastic force per unit area, (f_x^u, f_y^u, f_z^u) , is of the form:

$$f_x^u = K_u(\Delta_x^u - \delta_x^u), \quad f_y^u = K_u(\Delta_y^u - \delta_y^u), \quad f_z^u = K_u^n \Delta_z^u, \tag{5}$$

where δ_x^u , δ_y^u are the corresponding inelastic displacements, the evolution of which is given by the equations:

$$\frac{d\delta_x^u}{dt} = W_u f_x^u, \quad \frac{d\delta_y^u}{dt} = W_u f_y^u. \tag{6}$$

It is assumed that there is no inelastic displacement in the vertical direction (along axis z). The coefficients K_u , K_u^n , and W_u in (5) and (6) may be different for different blocks. The vector $(\Delta_x^u, \Delta_y^u, \Delta_z^u)$ of relative displacement of the block and the underlying medium at the point (φ, ψ) considered in system-P is defined by (1) and (4) analogous to the case of finding the displacement at a fault point.

As mentioned above, components of the translation vectors of the blocks, and angles of their rotation around the mass centers of the blocks, are found from the condition that the total force and the total moment of forces acting on each block (written in system-C corresponding to the block) are equal to zero. This is the condition of quasi-static equilibrium of the system, and at the same time the condition of energy minimum.

It is important that the dependence of forces and moments on displacements and rotations of blocks is linear. Therefore, the system of equations for determining these values must be linear:

$$A\boldsymbol{w} = \boldsymbol{b}.\tag{7}$$

Here, the components of the unknown vector $\boldsymbol{w} = (w_1, w_2, \dots, w_{6n})$ are components of translation vectors of blocks and angles of their rotation (*n* is the number of blocks), i.e., $w_{6m-5} = x_m$, $w_{6m-4} = y_m$, $w_{6m-3} = z_m$, $w_{6m-2} = \gamma_m$, $w_{6m-1} = \beta_m$, $w_{6m} = \lambda_m$ ($m = 1, 2, \dots, n$). The elements of matrix A ($6n \times 6n$) and of vector \boldsymbol{b} (6*n*) are determined from rather complicated formulae, which are deduced from (1)– (6) with the transformation of forces and moments to system-C. For brevity sake, these formulae are omitted here. It should be noted that the matrix A does not depend on time and its elements are calculated only once, at the beginning of the process. The components of vector \boldsymbol{b} depend on time, explicitly, because of motions of the underlying medium and boundary blocks and, implicitly, because of inelastic displacements.

2.3 Discretization

The model uses dimensionless time. When interpreting the results, some realistic value should be given to one unit of dimensionless time. For computational purposes, time discretization is performed by introducing a time step Δt . The state of the block structure under consideration is determined at discrete times $t_i = t_0 + i\Delta t$ (i = 1, 2, ...), where t_0 is the initial time. The transformation from the state at t_i to the state at t_{i+1} is made as follows: (i) new values of inelastic displacements δ_x^u , δ_y^u , δ_t , δ_l , δ_n are calculated from equations (3) and (6); (ii) translation vectors and rotation angles at t_{i+1} are calculated for the boundary blocks (if they are specified) and the underlying medium; (iii) components of **b** in system (7) are found, and this system is used to determine the translation vectors and rotation angles for the blocks.

For the calculation of various curvilinear integrals, one should discretize (divide into cells) the spherical surfaces of the block bottoms and fault segments. The values of forces and inelastic displacements are assumed to be equal for all points of a cell. Note that, according to the assumption, in the modification without depth segments are not subject to discretization by depth. Another important remark is the following: in the case when in the modification with depth we tend its value to zero, we should not expect the closeness of simulation results to corresponding results obtained by the modification without depth. The reason is in the essential distinction from the "calculative viewpoint": in the modification without depth, curvilinear integrals over fault segments are integrals over the line of intersection between the segment and the outer sphere, whereas in the modification with depth, these integrals are taken over segment surfaces.

2.4 Earthquake and creep

For every time moment, we calculate the value of a quantity κ by the following formula

$$\kappa = \frac{\sqrt{f_t^2 + f_l^2}}{P - f_n},\tag{8}$$

where P is the parameter, which may be interpreted as the difference between the lithostatic and the hydrostatic pressure (P has the same value for all faults).

For every fault, three levels of κ are specified. They satisfy the inequalities

$$B > H_f \ge H_s. \tag{9}$$

It is assumed that the initial conditions for numerical simulation of the block structure dynamics satisfy the inequality $\kappa < B$ for all cells of the fault segments. If, at some time t_i , the value of κ in some cell of a fault segment reaches level B, a failure ("earthquake") occurs. By failure we mean slippage by which the inelastic displacements δ_t , δ_l , δ_n in the cell change abruptly to reduce the value of κ to the level H_f . The new values of the inelastic displacements are calculated from

$$\delta_t^e = \delta_t + \gamma^e \xi_t f_t, \quad \delta_l^e = \delta_l + \gamma^e f_l, \quad \delta_n^e = \delta_n + \gamma^e \xi_n f_n, \tag{10}$$

where δ_t , δ_l , δ_n , f_t , f_l , f_n are the inelastic displacements and the components of the elastic force vector per unit area just before the failure. The coefficients $\xi_t = K_l/K_t$ $(\xi_t = 0 \text{ if } K_t = 0)$ and $\xi_n = K_l/K_n$ $(\xi_n = 0 \text{ if } K_n = 0)$ account for inhomogeneities of displacements along the plane tangent to the fault (in various directions), and normal to that plane (they account for the possibility that the same value of the elastic force per unit area can result in different changes of different inelastic displacements). The coefficient γ^e is given by

$$\gamma^{e} = \frac{\sqrt{f_{t}^{2} + f_{l}^{2}} - H_{f}(P - f_{n})}{K_{l}\sqrt{f_{t}^{2} + f_{l}^{2}} + K_{n}H_{f}\xi_{n}f_{n}}.$$
(11)

It follows from (2) and (8) through (11) that after the calculation of new values of the inelastic displacements and elastic forces, the value of κ in the cell is equal to H_f . It should be noted that after the calculation according to (2) and (10), the signs of the elastic forces must be the same as just prior to the failure. For this reason, some cases require additional processing. Only then are the new components of vector \boldsymbol{b} computed, and the translation vectors and angles of rotation for the blocks are found from (7). If for some cell(s) of the fault segments $\kappa > B$, the entire procedure is repeated. This is done until all cells satisfy the condition $\kappa < B$, at which point the state of the block structure at time t_{i+1} is determined as described in Section 2.3.

All cells of the same fault, in which failure occurs at the same time, are considered as a single earthquake. The parameters of the earthquake are defined as follows: (i) the time of the event is t_i ; (ii) the epicentral coordinates are the weighted sums of the corresponding coordinates of the cells involved in the earthquake (the weight of each cell is given by its length (in the modification without depth) or area (in the modification with depth) divided by the sum of lengths/areas of all cells involved in the earthquake); (iii) the magnitude is calculated in the first modification by the formula proposed in Wells and Coppersmith (1994):

$$M = 1.16 \, \lg L + 5.08,\tag{12}$$

where L is the total surface rupture length of cells (in km) involved in the earthquake; and in the second modification by the formula proposed in Utsu and Seki (1954):

$$M = 0.98 \, \lg S + 3.93,\tag{13}$$

where S is the total area of cells (in km^2) involved in the earthquake. In the modification without depth, it is possible to attribute the same depth to all earthquakes. An argument to use formulae (12), (13) as a definition of the model magnitude is the fact that the energy released through an earthquake depends mainly on the total size (area) of fault's part covered by this earthquake.

Immediately after the earthquake, it is assumed that the failure cells are in the creep state. This implies that, for these cells, parameters W_t^s ($W_t^s > W_t$), W_l^s

 $(W_l^s > W_l)$, and W_n^s $(W_n^s > W_n)$ are used instead of W_t , W_l , and W_n in equations (3). They may be different for different faults. The failure cells are in the creep state so long as $\kappa > H_s$; when $\kappa \leq H_s$, the cells return to the normal state, after which W_t , W_l , and W_n are used in (3).

Thus, a synthetic earthquake catalog is produced as a result of the simulation.

3 Numerical simulation: modification without depth

To compare two modifications of the spherical block model, we consider results of numerical modeling of the dynamics and seismicity of the global system of tectonic plates. This block structure is a closed one including the whole spherical shell (covering the whole surface of the Earth). It does not have lateral boundaries and therefore boundary blocks are not specified for it. Previous analysis (Rozenberg et al., 2005) proved that the dynamics and seismicity of the global system of tectonic plates is more accurately modeled by means of the closed block structure than with the structure for that boundary blocks are specified. Therefore, in this study we restrict ourselves by considering the closed system of plates only. In this section, we analyze numerical results obtained by means of the modification without depth.

Specifically, the structure contains the following plates: South America, Nazca, Cocos, Caribbean, Africa, Arabia, Somalia, India, Philippines, Australia, North America, Eurasia, Antarctica, Pacific, and Juan de Fuca, see Fig. 6. The structure includes 15 blocks, 186 vertices, and 199 faults. We use dip angles of faults to consider flat gradient of subduction zones in comparison with other plate boundaries. Thus we specify a dip angle of 50° for faults corresponding to clearly observed subduction zones (for example, at the boundary South America/Nazca; totally 26 faults) and of 90° for other faults. It is obvious that in the modification without depth a dip angle is rather artificial characteristic of a fault. The motion of the closed structure is caused only by the motion of the underlying medium. The parameters of the latter are taken from the model HS2-NUVEL-1 (Gripp and Gor-

don, 1990) with the Somalia plate added (Jestin et al., 1994). The values of the coefficients in formulae (2), (3), (5), and (6) are specified using the experience of the previous studies with the two-dimensional plane block models (e.g., Soloviev and Ismail-Zadeh, 2003) and taking into account specificity of the spherical block model.



Figure 6: The global system of tectonic plates and results of simulation of the character of plate boundaries and spatial distribution of the strongest earthquakes: divergent plate boundaries (spreading, light shading), convergent plate boundaries (subduction, dark shading), transform plate boundaries (sliding, toothed shading), epicenters of model events occurred at boundaries (circles). Here and in the tables below, the following notation for the plates is used: NA— North America, SA— South America, N— Nazca, Af— Africa, Ca— Caribbean, Co— Cocos, P— Pacific, S— Somalia, Ar— Arabia, E— Eurasia, I— India, An— Antarctica, Au— Australia, Ph— Philippines, F— Juan de Fuca.

Values of the coefficients for the block bottoms are the same in all experiments described below: $K_u = 10$, $K_u^n = 20$, $W_u = 0.1$. The step of space discretization for block bottoms is equal to 0.5° and is not changed. In this case, the largest block bottom (Pacific) is divided into 90,000 cells. The step of time discretization is also constant: $\Delta t = 0.01$. And the coefficients for the faults are changed in the experiments to reproduce in the model some features of observed seismicity.

Two series of experiments are carried out. The first series investigates the dependence of model dynamics and seismicity on values of the model coefficients in the equations for forces and inelastic displacements. The second series is designed to study the influence of the space discretization step for fault segments; this parameter determines the range of magnitude of model events. In all variants we examine the parameters of the Gutenberg—Richter law for seismicity on the global scale, the seismic flow intensity, the characteristics of the interactions between plates on the most active seismic boundaries, and the spatial distribution of the strongest events.

variant	space step	parameters of faults
	for segments	
1.1	10 km	For all faults:
		$K_t = K_l = K_n = 0.03,$
		$W_t = W_l = W_n = 0.01.$
1.2		For faults that form boundaries Af/SA, N/P, Co/P,
		south, east and north of S:
		$K_t = K_l = K_n = 0.001, W_t = W_l = W_n = 0.1.$
		For other faults:
		$K_t = K_l = K_n = 0.01, W_t = W_l = W_n = 0.01.$
1.3		For faults that form boundaries Af/SA, N/P, Co/P,
		south, east and north of S, E/NA, south of P:
		$K_t = K_l = K_n = 0.001, W_t = W_l = W_n = 5.$
		For other faults:
		$K_t = K_l = K_n = 0.01, W_t = W_l = W_n = 0.1.$
1.4		For faults that form boundaries NA/SA, Af/SA, N/P, Co/P,
2.1		I/Au, E/NA, south of P:
2.2	4 km	$K_t = K_l = K_n = 0.001, W_t = W_l = W_n = 5.$
		For faults that form boundaries SA/An, Af/An, Af/S,
		north-east of Af, south, east and north of S, south of Au:
		$K_t = K_l = K_n = 0.005, W_t = W_l = W_n = 1.$
2.3	1 km	For faults that form boundaries north of Af, north of Au,
		California region:
		$K_t = K_l = K_n = 0.02, W_t = W_l = W_n = 0.05.$
		For other faults:
		$K_t = K_l = K_n = 0.01, W_t = W_l = W_n = 0.1.$

Table 1: Modification without depth: variation of parameters

In the first series (Table 1, variants 1.1–1.4), we consider variants, in which coefficients are varied but the discretization step for fault segments is constant. All the coefficients in the initial variant (variant 1.1) are the same for all faults. The synthetic earthquake catalog obtained in variant 1.1 contains an exceptionally large number of events, earthquakes occur on all segments, the spatial distribution of epicenters does not correspond to that observed. Hence, the necessity to take into account the characteristics of the faults and blocks (separated by them) by introducing different values of coefficients for different parts of the structure is obvious. The changes of numerical parameters are based on observed seismicity: the coefficients K_t , K_l , K_n were decreased, and the coefficients W_t , W_l , W_n were increased for faults with vastly low level of observed seismicity (as a rule, for faults that separate large-scale structures); and vice versa for active faults. These changes reflect the following considerations. First, if the same value of relative displacement of two blocks separating by a fault zone is considered then it induces a lesser force at a large-scale fault zone than at a small-scale one. This means that smaller values of coefficients K_t, K_l, K_n should correspond to large-scale fault zones. Second, the rate of growth of inelastic displacement for the same value of the force should be greater for large-scale fault zones, which are more fragmented and, consequently, less elastic and more viscous zones, than fault zones, separating small-scale structures. This means that larger values of coefficients W_t , W_l , W_n should correspond to large-scale fault zones.

Beginning from variant 1.2, the similarities between real and model data appear and improve in the following variants. Relative displacements of boundary points (e.g., at such boundaries as South America/Nazca, Pacific/Nazca, South America/Africa, India/Eurasia, surrounding Philippines, etc.) characterize qualitatively the interaction between plates along their boundaries. For variant 1.4, this information is presented in Fig. 6, where the divergent (spreading), convergent (subduction), and transform plate boundaries are marked. This figure shows also the spatial distribution of the strongest model events. These locations agree in principle with observation. A comparative analysis of the synthetic and observed

seismicity is performed. We consider events with magnitude $M \ge 5.0$ for time period 01.01.1900–31.12.2004 without any restrictions by depth and area of location selected from the global catalog NEIC (Global Hypocenters Data Base, 2004). Analysis of the spatial distribution of epicenters of the model events shows the most active synthetic seismicity at such boundaries as Nazca/South America, Cocos/Caribbean, India/Eurasia, California region, Arabia/Eurasia, south-east, east, north-east and, especially, north of Australia, and the Philippine plate margin. The level of synthetic seismicity is extremely small at such boundaries as south of Pacific plate, Nazca/Pacific, east of Africa, India/Australia, South America/Africa.

Note that simplifications accepted in the spherical block model give no opportunity to draw conclusions on the correspondence between observed and synthetic seismicity at any specific point or in relatively small regions. However, the correspondence of seismically active and quiet zones obtained by the model and the observed seismicity indicates a degree of adequacy of the model.

These compliances give us a possibility to pass to the analysis of parameters of the Gutenberg—Richter law, in particular, of the slope of the FM plot, which characterizes the ratio of the numbers of strong and weak events.

In the first series, the FM plots are constructed in steps of $\Delta M = 0.1$ for the same magnitude interval [6.9, 8.2] due to the coincidence of the ranges of the model magnitude (see Table 2). Note that the slope of the FM plot for the global seismicity observed during the period of the last 100 years is approximately equal to 1. Thus, one can see that from variant 1.2 to 1.4 the value of this characteristic does not approach to the observed value. However, the facts mentioned above induce us to use the last variant of the first series (the best one in the sense of correspondence of model interaction between blocks and spatial distribution of epicenters to real data) as a basic one for the second series.

variant	number of	magnitude	slope	approximation
	$\operatorname{synthetic}$	interval	estimate	error
	earthquakes	for approximation		
1.1	191 067			
1.2	93 875	[6.9, 8.2]	0.425	0.029
1.3	$93\ 124$	[6.9, 8.2]	0.408	0.043
1.4 = 2.1	90052	[6.9, 8.2]	0.363	0.043
2.2	$137 \ 454$	[6.3, 8.4]	0.517	0.200
2.3	181 788	[5.6, 8.6]	0.647	0.217

Table 2: Modification without depth: numerical results

Remarks.

1. All plots are approximated on specified magnitude intervals by the linear least-squares regression, $\lg N = a - bM$. A slope estimate for the plot is a "b-value" of corresponding regression. The average distance between points of the plot and the line constructed is treated as an approximation error. 2. The interval of simulation is equal to 100 units of dimensionless time for all variants.

The second series (Table 1, variants 2.1-2.3) includes variants with different discretization steps for the fault segments. Since the range of the model magnitude is varied depending on the discretization step for the fault segments, the following criterion is applied to specify a magnitude interval under investigation: it should be an interval of maximal length, on which the FM plot constructed in steps of $\Delta M = 0.1$ is "sufficiently well" approximated by the linear least-squares regression, $\lg N = a - bM$. Accordingly to formula (12), the low threshold for magnitudes of earthquakes obtained in the model is determined by the minimal length of one cell of the fault segments, which decreases when the step of discretization is decreased. Decreasing the step of spatial discretization for the fault segments enlarges the range of magnitudes of model events from [6.2, 8.9] for 10 km step to [5.0, 8.9] for 1 km step and, in addition, simultaneously the total number of events increases because events of smaller magnitudes appear. However, the number of events in the magnitude range [6.9, 8.2], which presents in all model catalogs obtained in variants 2.1–2.3, does not change essentially. It is important that a reduced step of discretization results in a wider magnitude interval where a more nearly linear FM plot is constructed and in better correspondence of its slope with the observed value (the slope of the plots in variants 2.1–2.3 increases).

From two series of numerical experiments we conclude that the best variant with respect to all aspects of our analysis is variant 2.3.

4 Numerical simulation: modification with depth

In this section, we describe results of applying the modification with depth to modeling dynamics and seismicity of the global system of tectonic plates. Since this block structure is in detail described in the previous section, recall only that it includes 15 blocks, 186 vertices, and 199 faults. Existing version of the model operates with the same depth for all blocks.

As a basic variant, we choose variant 2.3 from Table 1, since it is the best one with respect to all aspects of our analysis. The parameters that are not changed in experiments described below (except angles) are presented in Table 3.

discretization	parameters of faults
and depth	
time step	For faults that form boundaries
-0.01;	NA/SA, Af/SA, N/P, Co/P, I/Au, E/NA, south of P:
space step:	$K_t = K_l = K_n = 0.001, W_t = W_l = W_n = 5.$
for segments	For faults that form boundaries
-1 km,	SA/An, Af/An, Af/S, north-east of Af,
for block bottoms	south, east and north of S, south of Au:
$-0.5^{\circ};$	$K_t = K_l = K_n = 0.005, W_t = W_l = W_n = 1.$
layer's depth	For faults that form boundaries
-40 km.	north of Af, north of Au, California region:
	$K_t = K_l = K_n = 0.02, W_t = W_l = W_n = 0.05.$
	For other faults:
	$K_t = K_l = K_n = 0.01, W_t = W_l = W_n = 0.1.$
	For faults corresponding to subduction zones (26 faults)
	dip angles are equal to 50° .
	For other faults
	dip angles are equal to 90° .

Table 3: Modification with depth: basic parameters

Let us pass to the description of simulation results. Three series of experiments are carried out. The first series (Table 4, variants 3.1–3.4) investigates the dependence of model dynamics and seismicity on the value of the step of discretization of segments by depth (all other parameters including dip angles (their influence on simulation results essentially increases for the modification with depth) are the same). The second series (Table 4, variants 4.1–4.3) is designed to study the impact of faults' dip angles. Third one (Table 4, variants 5.1–5.4) repeats the first series but the best variant of the second series is chosen as a base. Again, as in the previous section, we examine the parameters of the Gutenberg—Richter law for seismicity on the global scale, the seismic flow intensity, etc.

variant	step of	number of	slope	approximation
	discretization	$\operatorname{synthetic}$	estimate	error
	by depth	earthquakes		
3.1	$40 \mathrm{km}$	97148	0.692	0.115
3.2	$20 \mathrm{km}$	116596	0.620	0.096
3.3	$8 \mathrm{km}$	150202	0.677	0.233
3.4	$4 \mathrm{km}$	177321	0.742	0.149
4.1 = 3.3	$8 \mathrm{km}$	150202	0.677	0.233
4.2	$8 \mathrm{km}$	208235	0.737	0.273
4.3	$8 \mathrm{km}$	376425	0.903	0.163
5.1	40 km	258507	0.879	0.166
5.2	$20 \mathrm{km}$	306271	0.900	0.219
5.3 = 4.3	$8 \mathrm{km}$	376425	0.903	0.163
5.4	$4 \mathrm{km}$	431818	0.905	0.152

Table 4: Modification with depth: numerical results

Remarks.

1. All plots are approximated on specified magnitude intervals by the linear least-squares regression, $\lg N = a - bM$. A slope estimate for the plot is a "b-value" of corresponding regression. The average distance between points of the plot and the line constructed is treated as an approximation error. 2. In variants 4.2, 4.3 dip angles being equal to 90° in the first series are substituted for 75° and 105° (depending on fault's direction) in such a way: for 26 faults adjacent to subduction zones in variant 4.2, and for almost all faults in variant 4.3 (angles being equal to 50° are not changed). 3. The interval of simulation is equal to 100 units of dimensionless time for all variants.

4. The magnitude interval for the plot is equal to $\left[6.0, 8.0\right]$ for all variants.

As one would expect, the characteristics of the interactions between plates on

seismic boundaries, and the spatial distribution of the strongest events correspond to the real data and do not have principal distinctions in all variants. Parameters inspiring interest are presented in Table 4.

Analyzing the data from Table 4, we may point out the following facts. Simulation results obtained in variants 3.1–3.4 do not allow us to assert that there exists a definite dependence of the slope of the FM plot on the value of the step of discretization of segments by depth in the case when the majority of faults have got dip angles of 90°. In addition, the slopes of the model FM plots in these variants are rather far from 1. It is likely that the matter is in the fact that failures occur simultaneously in cells belonging to faults with dip angles of 90°. Therefore, strong events prevail over others, the FM plot is more gently sloping, and the influence of the step of discretization by depth is vague.

The situation in variants 4.1-4.3 is quite different: we observe the essential expansion of range of changing model magnitude and the "improvement" of the slope estimate for the model FM plot. This happens chiefly because of cells of a fault with dip angle different from 90° come to "critical" state at different time instants (see Fig. 7); and, as a consequence, we have got a greater number of weak events.

The best correspondence of the slope estimate for the model FM plot to the real *b*-value for the global seismicity in the second series is obtained for variant 4.3. Namely for this variant the value of the step of discretization of segments by depth is varied in the third series (variants 5.1-5.4). It is easily seen that in this series the slope estimate for the model FM plot tends to 1 (although very slowly). This fact points out an essential distinction in the properties of model seismicity in the cases when dip angles are equal to 90° and are not.



Figure 7: State of cells belonging to a fault with dip angle different from 90° (boundary South America/Nazca).

The analysis of results obtained in three series leads to the conclusion that the shape of the model FM plot is to a greater degree determined by dip angles of faults in comparison with the value of the step of discretization of segments by depth. Indeed, decreasing of this value influences the intensity of the flow of model earthquakes but does not result in changes of the number of events in a magnitude interval common for all model catalogs. The total number of earthquakes increases due to appearance of weak events of lesser magnitudes defined by minimal area of a cell; at that the slope is slightly varied. Figs. 8 and 9 confirm these conclusions.

One can see from Fig. 8 that plot (5) constructed for the best variant obtained by the modification without depth has got two rather clear (in comparison with other plots) linear parts with different slopes; the intermediate zone between these parts is observed nearby magnitude point M = 7.0. Such bend of shape of the FM plot may be explained as follows: it reflects transition from earthquakes involving whole "short" segments to earthquakes involving whole "long" segments. When taking into account layer's depth, such clear bend is not observed since new cells afford more uniform filling of the magnitude range of model events. In addition, the change of the slope of plot (5) in the vicinity of maximal magnitude is sharper; it is a consequence of abnormally large number of strong events.



Figure 8: The model FM plots constructed for variants 3.1–3.4 from Table 4 ((1)–(4), respectively) and variant 2.3 from Table 1; N is accumulated number of earthquakes, M is magnitude.

To compare the model and real data, the FM plots for synthetic seismicity in variants 3.3, 4.3, and that for observed seismicity are given in Fig. 9. The observed FM plot is constructed for earthquakes selected from the global catalog NEIC (events with magnitude not less than 5.0 for time period 01.01.1900–31.12.2004 without any restrictions by depth and area of location).

For the model plots in Fig. 9, there exist magnitude intervals where these plots are nearly linear. But they essentially differ from the real one, especially in the domain of small magnitudes; it is therefore necessary to increase the number of weak events in the model. Toward this end, we plan to carry out a number of simulations with different parameters of the underlying medium, and the dependence of properties of a synthetic catalog on these parameters determining coupling between blocks and the underlying medium will be investigated.



Figure 9: The FM plots constructed for the real (1) and synthetic ((2)— variant 3.3, (3)— variant 4.3, Table 4) catalogs; N is accumulated number of earthquakes, M is magnitude.

Studying the shape and slope of the model FM plots shows that the results obtained by the modification with depth look more adequate than those in the case of the modification without depth. Additional comparative analysis of two modifications are carried out for the purpose of establishing the relation between model (dimensionless) and real time intervals. Relative velocities of displacements of characteristic points at plate boundaries obtained in the spherical block model are analyzed and compared with those given by the model HS2-Nuvel-1 (Gripp and Gordon, 1990). The results are presented in Table 5.

Taking into account the quantitative behavior of displacements of points, we conclude that the unit of dimensionless model time corresponds to about 1 year for both modifications. It is clear that this conjecture requires careful verification, first, by further comparative analysis of real and model catalogs, and second, by study of the impact of model parameters on the period between strong events in different regions. Another obvious conclusion that can be derived from Table 5 is

that the velocities of relative displacements of boundary points in the modification with depth is much more close to HS2-Nuvel-1 velocities than in the modification without depth.

Table 5: Velocities of relative displacement of boundary points: (1)— in the model of plate motion HS2-Nuvel-1 (cm/year); (2)— in the modification without depth (variant 1.4, Table 1, cm/100 units of dimensionless time); (3)— in the modification with depth (variant 4.3, Table 4, cm/100 units of dimensionless time)

point co	ordinates	blo	ck	varian	t (1)	varia	nt (2)	varia	nt (3)					
latitude	longitude	Ι	II	Δx_1	Δy_1	Δx_2	Δy_2	Δx_3	Δy_3	$\frac{\Delta x_1}{\Delta y_1}$	$\frac{\Delta x_2}{\Delta y_2}$	$\frac{\Delta x_3}{\Delta y_3}$	$\frac{ \vec{v}_2 }{ \vec{v}_1 }$	$\frac{ \vec{v}_3 }{ \vec{v}_1 }$
-21.71 -9.63 11.19 -18.58 14.18 28.11 -49.85 -7.00 29.15	-71.44 -13.25 -89.11 -112.63 52.60 84.84 130.44 149.62 130.59	SA SA P S E An P E	N Af Co N Ar I Au Au Ph	$7.8 \\ 3.2 \\ 3.75 \\ 14.3 \\ 0.56 \\ 1.2 \\ 0.86 \\ 10.14 \\ -4.0$	$\begin{array}{c} 1.8 \\ 0.7 \\ 7.01 \\ -3.3 \\ 1.8 \\ 4.9 \\ 7.26 \\ 3.26 \\ 3.2 \end{array}$	$775 \\ 327 \\ 319 \\ 1490 \\ 59 \\ 107 \\ 80 \\ 991 \\ -410$	$\begin{array}{c} 210\\ 63\\ 504\\ -269\\ 120\\ 458\\ 692\\ 303\\ 311 \end{array}$	$813 \\ 340 \\ 374 \\ 1507 \\ 56 \\ 121 \\ 87 \\ 1014 \\ -406$	$182 \\ 72 \\ 700 \\ -344 \\ 180 \\ 490 \\ 726 \\ 326 \\ 328$	$\begin{array}{r} 4.33\\ 4.57\\ 0.53\\ -4.33\\ 0.31\\ 0.24\\ 0.12\\ 3.10\\ -1.25\end{array}$	3.69 5.19 0.63 -5.54 0.49 0.23 0.12 3.27 -1.32	$\begin{array}{r} 4.47\\ 4.72\\ 0.53\\ -4.38\\ 0.31\\ 0.25\\ 0.12\\ 3.11\\ -1.24\end{array}$	$100.4 \\ 100.9 \\ 75.0 \\ 103.0 \\ 71.1 \\ 94.1 \\ 95.4 \\ 97.8 \\ 100.9$	$\begin{array}{c} 104.1 \\ 105.3 \\ 99.8 \\ 105.2 \\ 100.3 \\ 100.9 \\ 100.2 \\ 100.0 \\ 102.3 \end{array}$
36.89	-119.87	NA	Р	-2.68	3.84	-268	368	-268	383	-0.70	-0.73	-0.70	96.9	99.4

Remark. The coordinate system, in which relative displacements of a boundary point is considered, is connected with this point (the center is in the point, the axis x is directed along the parallel to the east, the axis y is directed along the meridian to the north). The fault segment that the point belongs to separates the blocks I and II, at that the block I is considered as motionless, whereas the block II moves relative to the block I. The vector of relative velocity $\vec{v_i}$ has coordinates $(\Delta x_i, \Delta y_i)$, i = 1, 2, 3. Its direction is characterized by the value $\frac{\Delta x_i}{\Delta y_i}$.

The comparative analysis of two modifications of the spherical block model presented above leads us to the following conclusion: the modification with depth describes more accurately the dynamics and seismicity of the global system of tectonic plates in comparison with the modification without depth. It happens despite the fact that the dimensions of plates are much larger than the depth of spherical layer; and one might expect light differences in simulation results. Another piece of analysis is performed only for the simulation results obtained by the modification with depth. Summary seismic moment release rate of active boundaries obtained in the modification with depth is compared with that of observed seismicity. Generalized characteristics of observed seismicity are obtained by using the catalog CMT (CMT, 2002), which is more homogeneous than NEIC. We consider earthquakes from 1976 to 2000, with the depth of the tensor moment centroid less than or equal to 70 km and the density of seismic moment greater than 10^{17} N×m; these properties approximately correspond to events of $M_s > 5$ (the CMT catalog contains 9,121 such earthquakes). The FM relation for seismic moment is close to a linear one, the annual number of events grows slightly during the 25-year interval. The distribution of summary seismic moment release rate is given in Fig. 10.



Figure 10: The distribution of summary seismic moment release rate for the catalog CMT (1976–2000).

The seismic moment is summarized on a grid of 2×2 degrees, the logarithm is taken, and the result is smoothed using a sliding window of 6×6 degrees with values weighted inversely proportional to the distance from the center of the window. The Circum-Pacific belt is distinct particularly the west and east equatorial parts. The Alpine-Himalayan folded belt is characterized by a wide zone of scattered seismicity. Other boundaries (especially divergent boundaries) are often outlined by discontinuous zones of increased rate of seismic moment release. To compare these data with the results of simulation, values of seismic moment are computed for the synthetic catalog (variant 4.3, Table 4) by recalculating the magnitude by the formula $\lg M_0 = 1.5M_s + 9.14$ (Ekstrom and Dziewonski, 1980), where M_0 is earthquake seismic moment, M_s is earthquake magnitude. The distribution of the summary seismic moment release rate is subsequently calculated as described above for the case of the real catalog. This distribution is shown in Fig. 11.



Figure 11: The distribution of summary seismic moment release rate for the synthetic catalog (modification with depth).

The absence of earthquakes in oceanic rift zones in the model is related to parameter fitting. A quantitative comparison of the distributions of model and observed seismicity is possible, but not productive at the present stage because first, the observed seismicity is rather weak on many parts of plate boundaries, and second, the range of model magnitudes $M \in [4.5, 8.5]$ is too narrow, and the magnitude takes on higher values than the real one (specifically because the areas of synthetic earthquakes are larger than the observed ones since synthetic earthquakes often cover the full depth of a fault zone). Thus, the value of summary seismic moment for the synthetic catalog is considerably larger than for the catalog CMT, and the difference may reach several exponents. To find adequate correspondence between model and real magnitudes is the subject of a separate investigation, because this correspondence may be different in different seismic regions. Nevertheless, the model reflects the most important patterns of the global seismicity distribution: (i) two large seismic belts, where most of the strong earthquakes related to subduction zones occur, (ii) extensive, but less pronounced seismicity at mid-oceanic ridges, and (iii) increased seismic activity associated with triple junctions of plate boundaries.

5 Parallel Algorithm for Numerical Simulation

Computational experiments show that the spherical block model of lithosphere dynamics and seismicity (especially the modification with depth) during performing on sequential computers requires considerable expenditures of memory and time of a processor. However, the approach applied to modeling admits effective parallelization of calculations on a multiprocessor machine, and it makes possible the use of real geophysical and seismic data in the process of simulation of dynamics of complicated block structures, including the global system of tectonic plates (Soloviev et al, 2001; Melnikova and Rozenberg, 2003).

The variant of parallel program was realized on Supercomputer MVS-1000M (768 Alpha-21264A, 667 MHz CPU; peak performance is about 1 TFlops) at Joint Supercomputer Center (Moscow, Russia) by the scheme "master-worker" ("processor farm"). For compatibility with different platforms (in the sense of fast transition), the special library MPI ("Message Passing Interface") was used, and the parallel algorithm was designed in such a way that the unique loading module was formed for all processors.

The block-scheme of the main calculative procedure is presented in Fig. 12.



Figure 12: Scheme of the main calculative procedure

Let us give necessary explanations. In the beginning of the work, the number of processor, which the program has loaded to, is detected (zero processor becomes the master). Then the information on a block structure is read, and auxiliary calculations (space discretization, calculation of the matrix A) are performed. At every time step the most time-consumable procedure is calculation of values of forces and inelastic displacements in all cells of space discretization of the block bottoms and fault segments. Since these calculations may be performed independently from each other, they are uniformly shared between all processors. The exchange of information at every time step is realized according to the following scheme (see Fig. 12, where operations carried out only by the master are marked by "M", only by the workers— by "W"). The master calculates new values of block, boundary block and underlying medium displacements, then necessary parameters are transferred to the workers. Recalculated values of the vector **b** are returned to the master, then the next time step is carried out. For processing the situation treated as an earthquake, the scheme is slightly complicated, since in this case the master should ask all the workers until cells of segments in the critical state exist. The time of calculations on each processor is much more than the time of exchange. Therefore, rather high useful loading of each processor is achieved.

For testing the dependence of time necessary for solving the problem on the number of processors and comparing with sequential algorithm, the following values were analyzed: acceleration coefficient $S_r = T_1/T_r$ and effectiveness coefficient $E_r = S_r/r$, where T_r is the time of program performance on multiprocessor computer with r processors, T_1 is the corresponding time for sequential algorithm. Both T_1 and T_r essentially depend on parameters of the structure under consideration but for all variants we obtained the similar qualitative results. In Table 6, we present the results of testing program performance on multiprocessor computer with the use of r processors (we chose variant 4.3 from Table 4, 100 time steps (or 1 unit of dimensionless time) with a considerable number of earthquakes occurred).

Table 6: Calculation time (in seconds), acceleration and effectiveness coefficients for different number of processors

r	T_r	S_r	E_r
1	10598.88		
2	5378.79	1.97	0.99
4	2697.72	3.93	0.98
8	1358.48	7.80	0.98
10	1088.34	9.74	0.97
16	684.78	15.48	0.97
32	346.72	30.57	0.96
48	233.87	45.32	0.94
64	175.66	60.34	0.94
100	144.40	73.40	0.73
200	75.70	140.01	0.70
300	56.54	187.46	0.62
400	44.15	240.06	0.60

It is appeared that at least for $r \leq 64$ acceleration coefficient S_r is slightly less than r, consequently, the effectiveness is rather high (E_r is close to 1, namely, not less than 0.94). Then the effectiveness decreases with increasing the number of processors in action but does not fall below a reasonable level even for 400 processors.

Conclusion

Some results of modeling the dynamics and seismicity of the global system of tectonic plates on the Earth's surface are given. They include qualitative information on displacements of plates, and on the nature of their interaction along boundaries. Synthetic earthquake catalogs, which reveal some patterns of observed seismicity, are created. Model FM plots are nearly linear on some magnitude intervals. Analysis of numerical simulations shows dependence of synthetic seismic properties on model parameters and directions for further investigation to match ranges of model and real magnitudes. The dynamics of the global system of tectonic plates is more accurately modeled by means of the modification, which takes the depth of the spherical layer into account, than of the modification neglecting discretization of fault segments by depth. Preliminary conclusions on the correspondence of dimensionless model time and real time are made on the basis of relative velocities of displacements of characteristic points at plate boundaries for the synthetic dynamics and for the model HS2-NUVEL-1, but will require additional analysis.

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