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Time correlations in models of self-organized criticality

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Time correlations in self-organized criticality (SOC)

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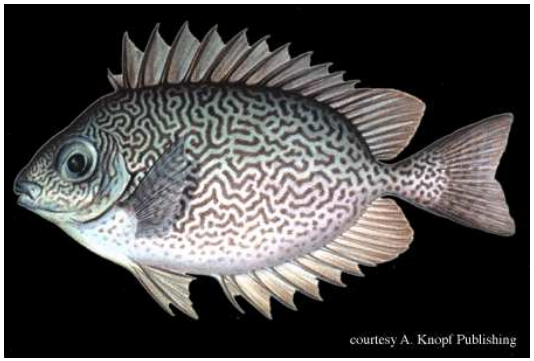
Overview
self-organization
cellular automata, “sandpiles”
SOC paradigm
time correlations in the activity
some criticisms to SOC
new sandpile with correlations
“running” BTW model
conclusions
Is it true that SOC models
do not have correlated avalanches?

Self-organization

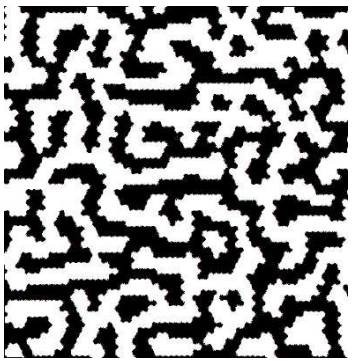
"Self-organization refers to a process in which the internal organization of a system, normally an open system, increases automatically without being guided or managed by an outside source. Self-organizing systems typically (though not always) display emergent properties." [Wikipedia]

In biology: The origin of life itself, from self-organizing chemical systems.



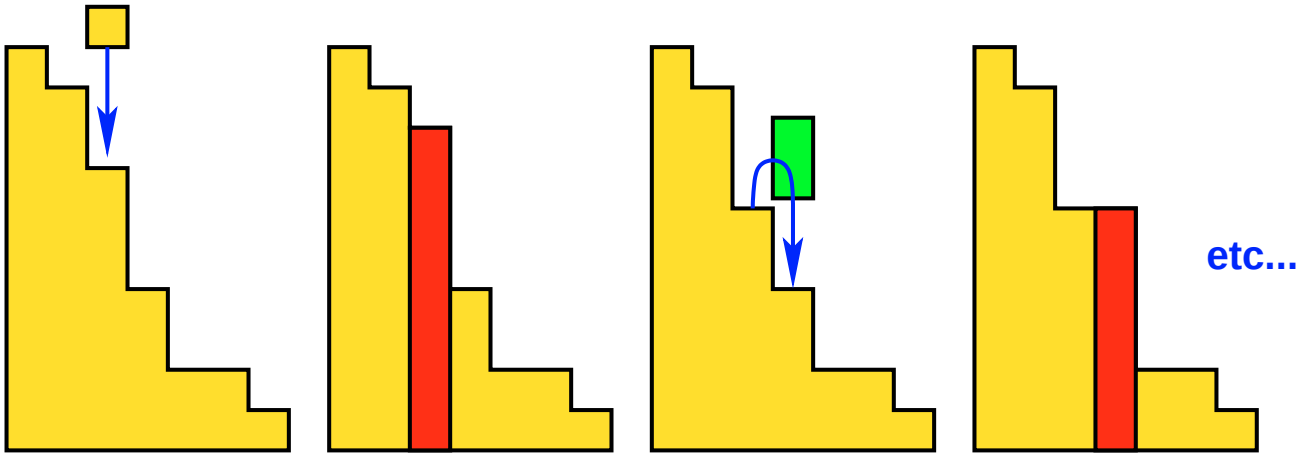
Cellular automata can self-organize

[Conway, *Game of Life*]



Sandpiles

Sandpile cellular automaton



Avalanches: emerging properties, not directly related to microscopic rules

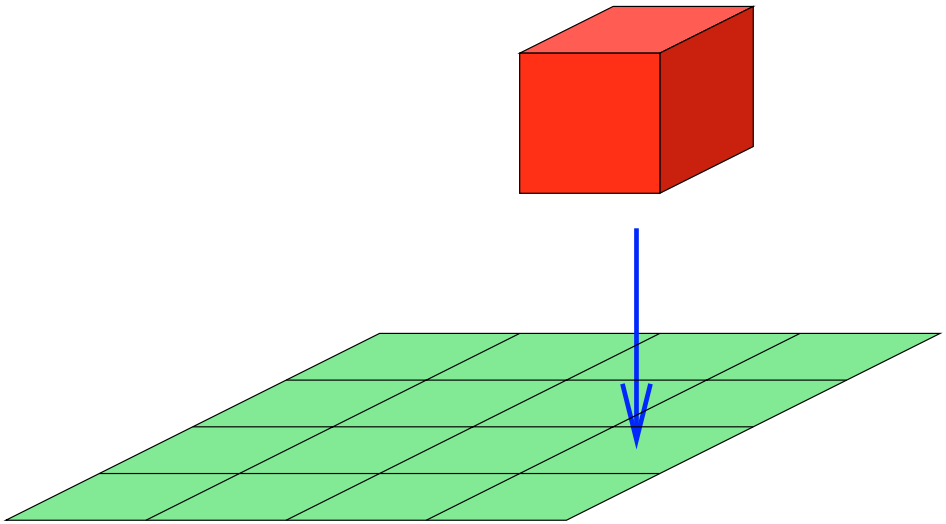
size = number of topplings

area = number of toppling sites (area $\leq$ size)

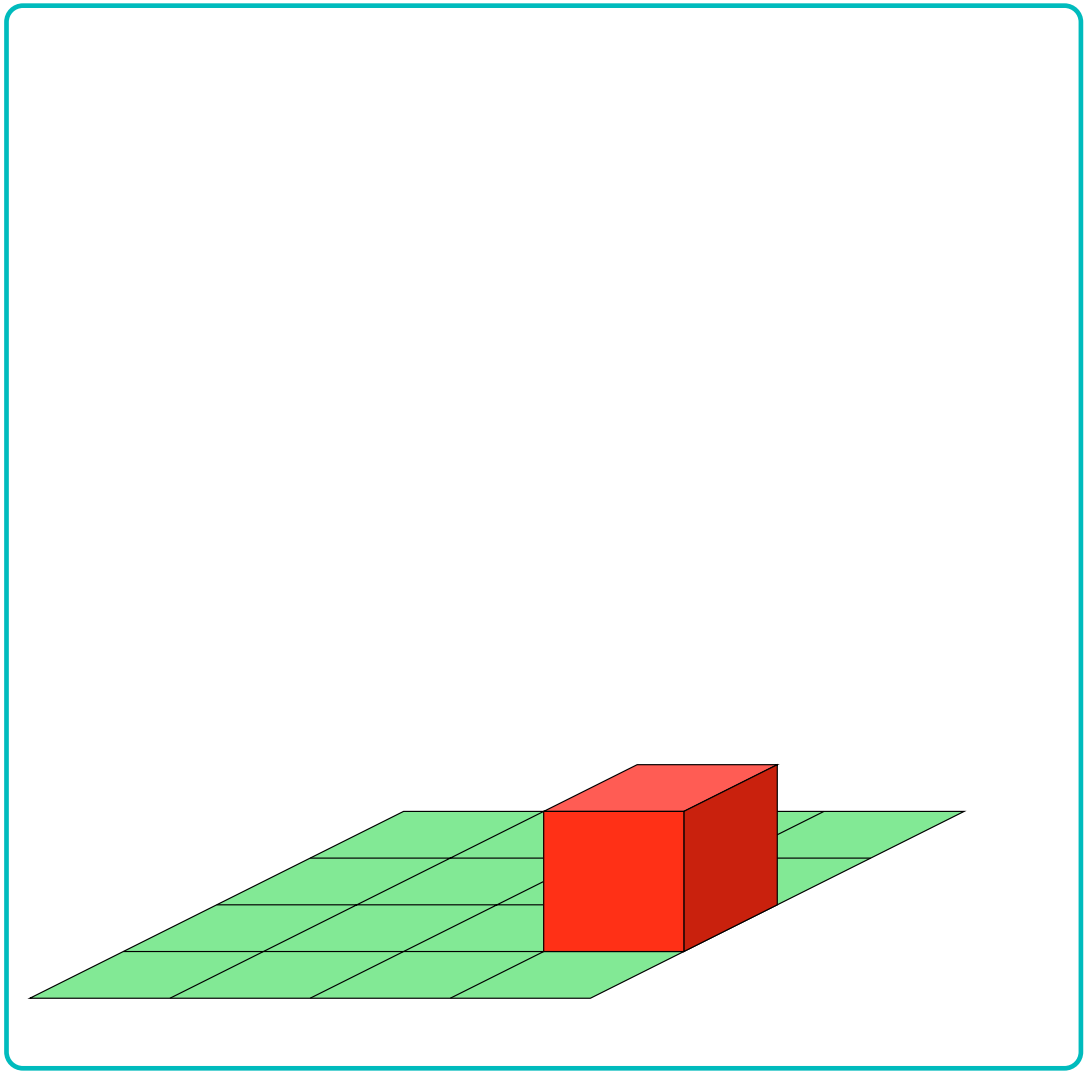
duration = number of micriscopic parallel updates till stable again

BTW model

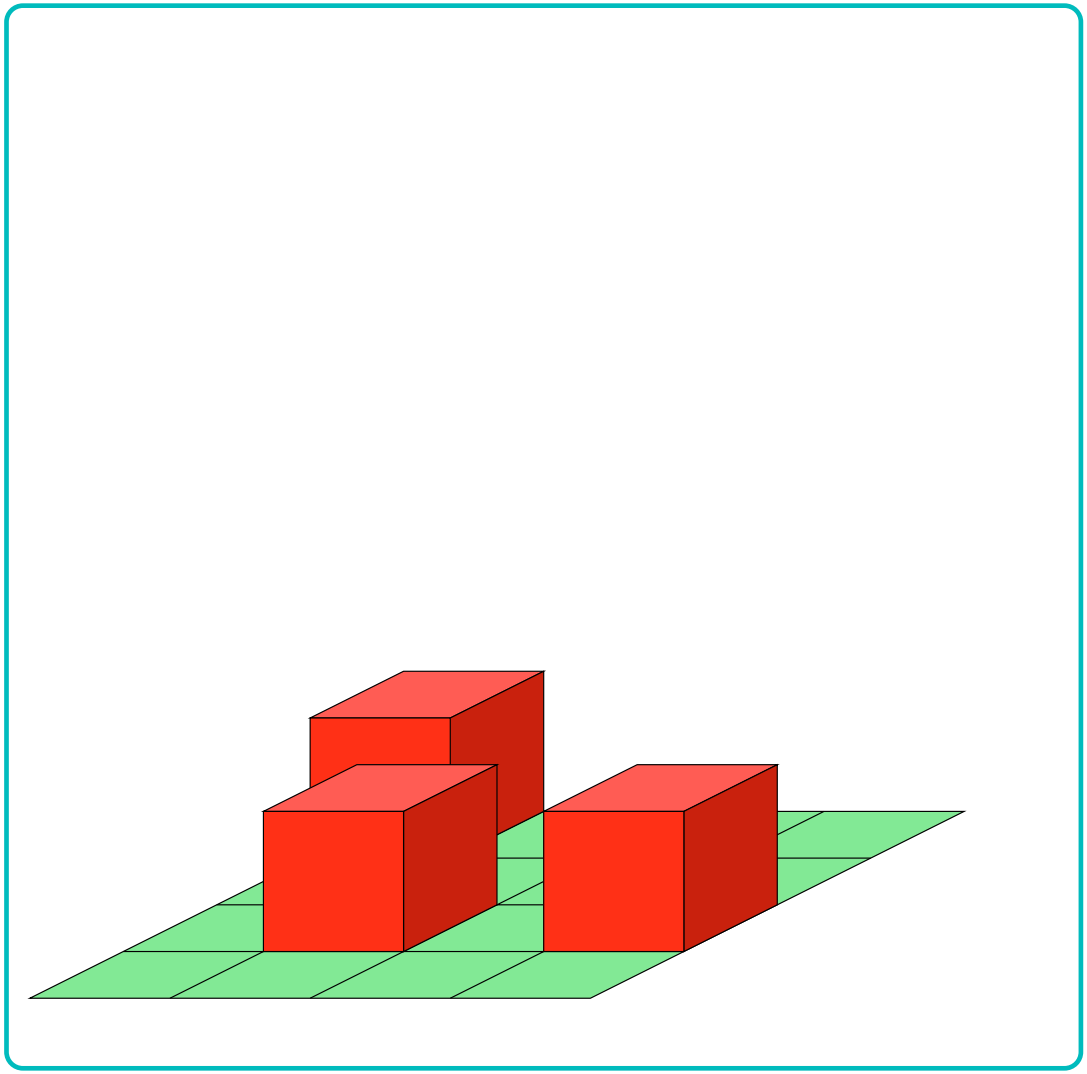
DRIVING = GRAIN DEPOSITION



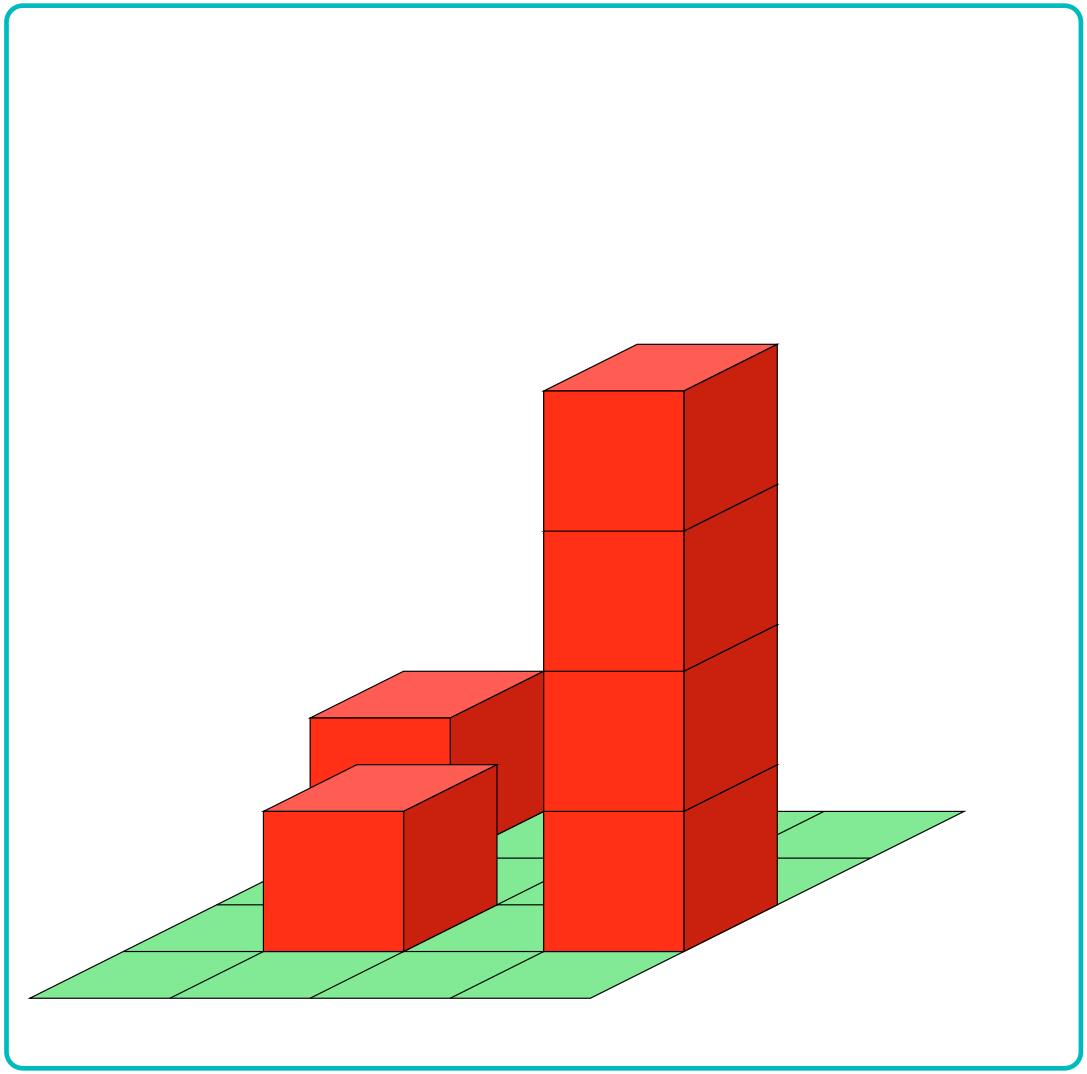
BTW model



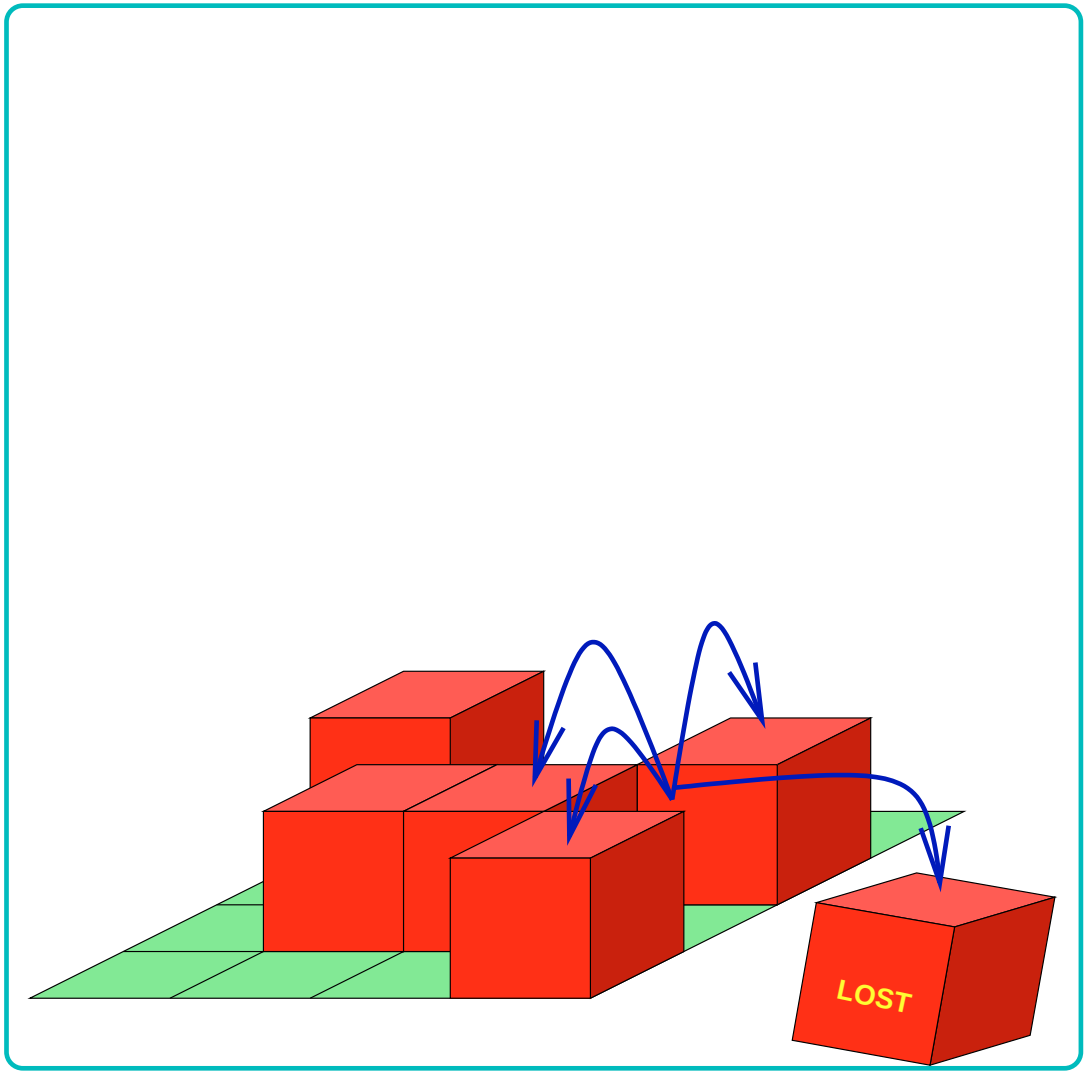
BTW model

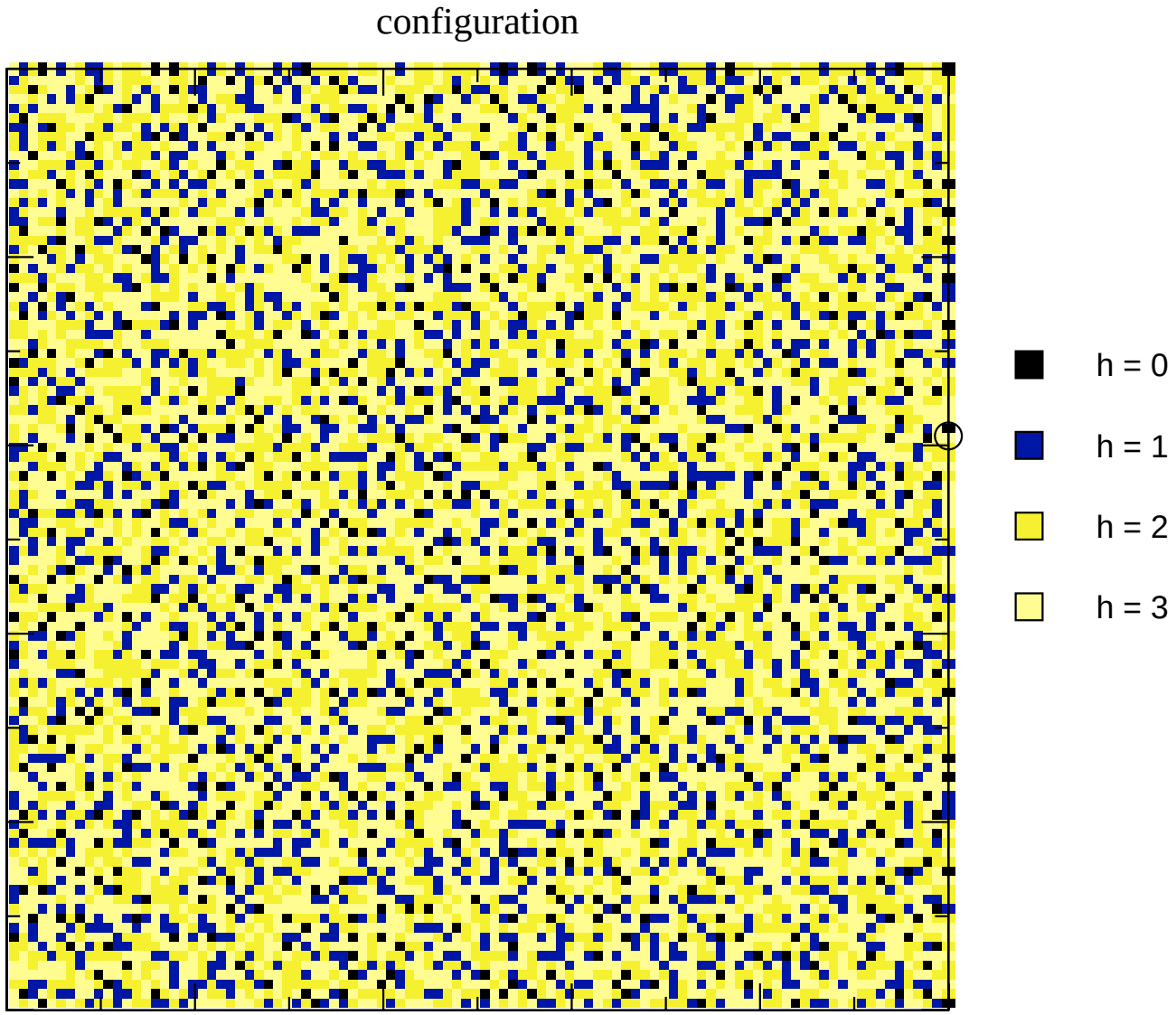


BTW model

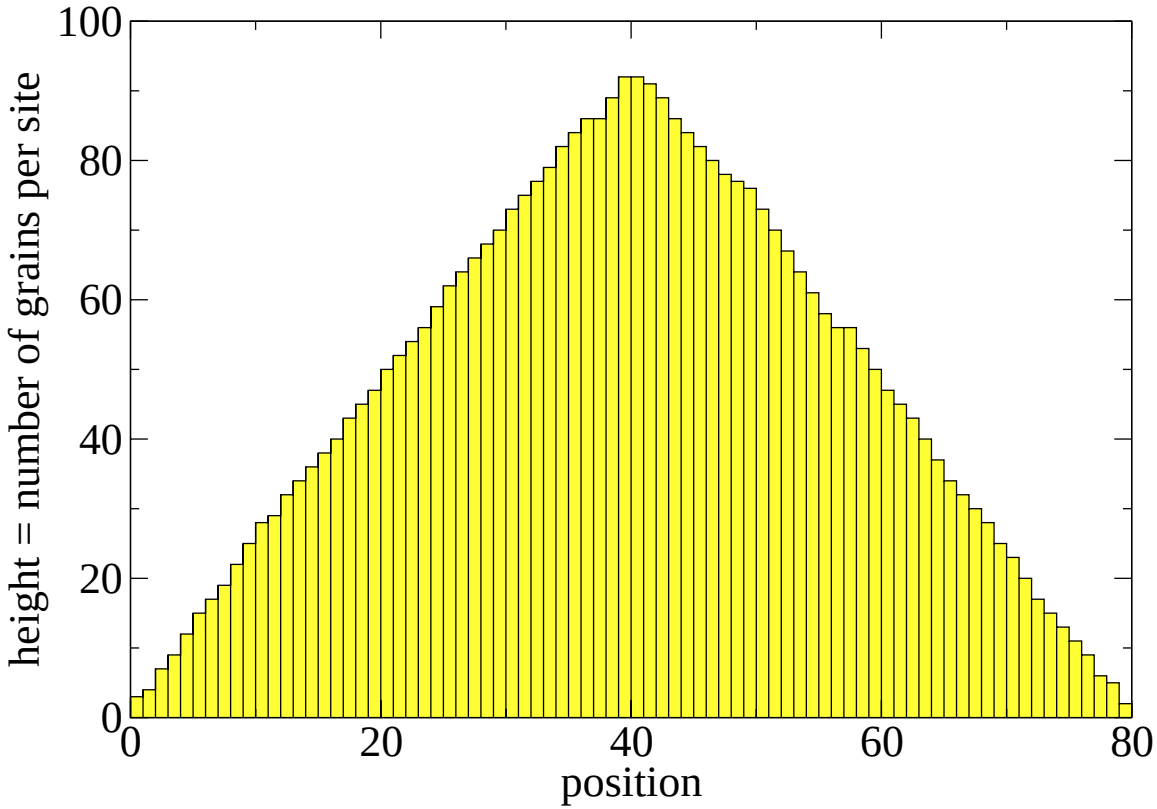


BTW model




configuration
h = 0
h = 1
h = 2
h = 3

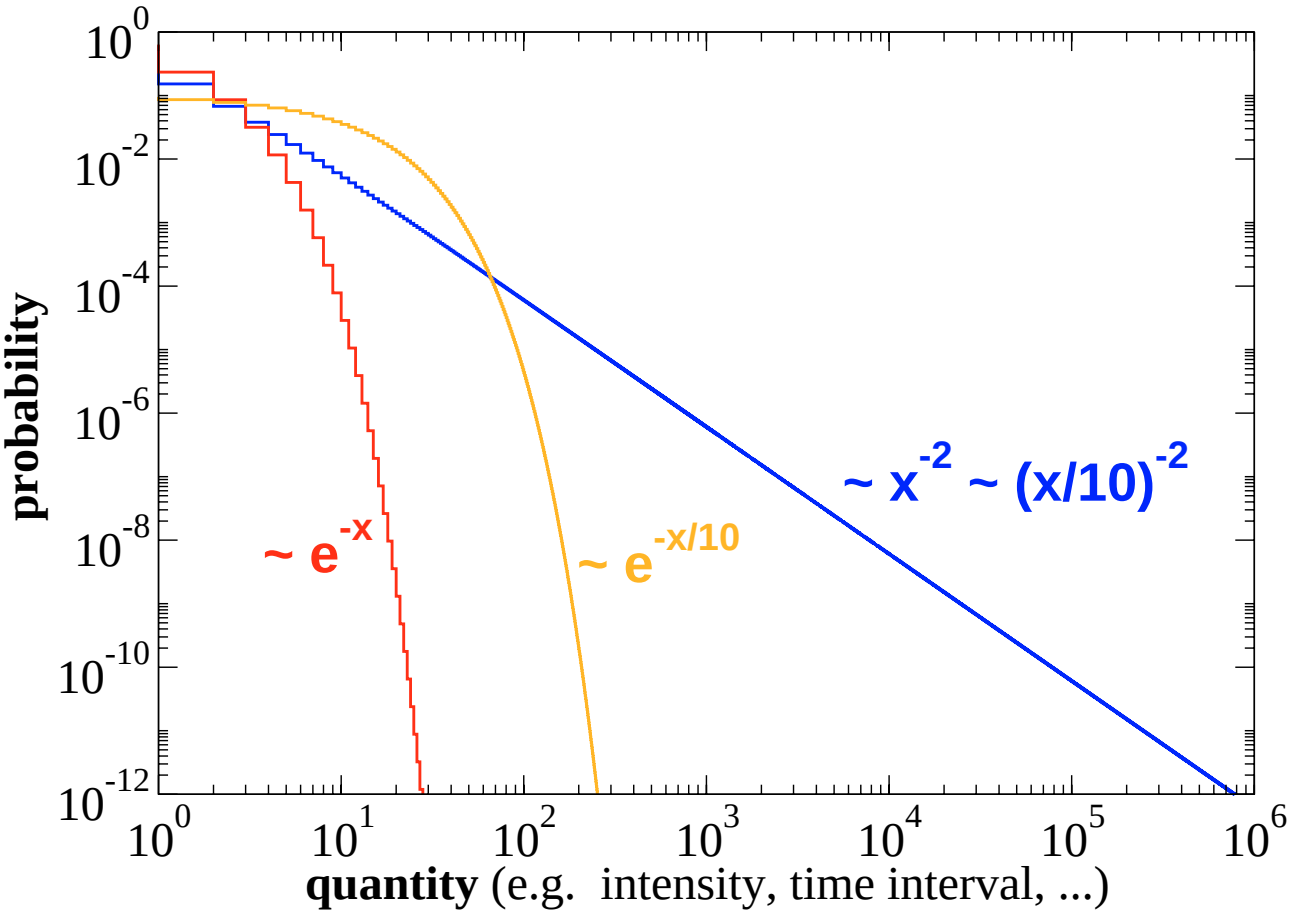
Criticality



scale-free avalanches: $P(\text{area}) \sim \text{area}^{-\gamma}$

Appealing: natural / simple / economic way of generating power-law distributions

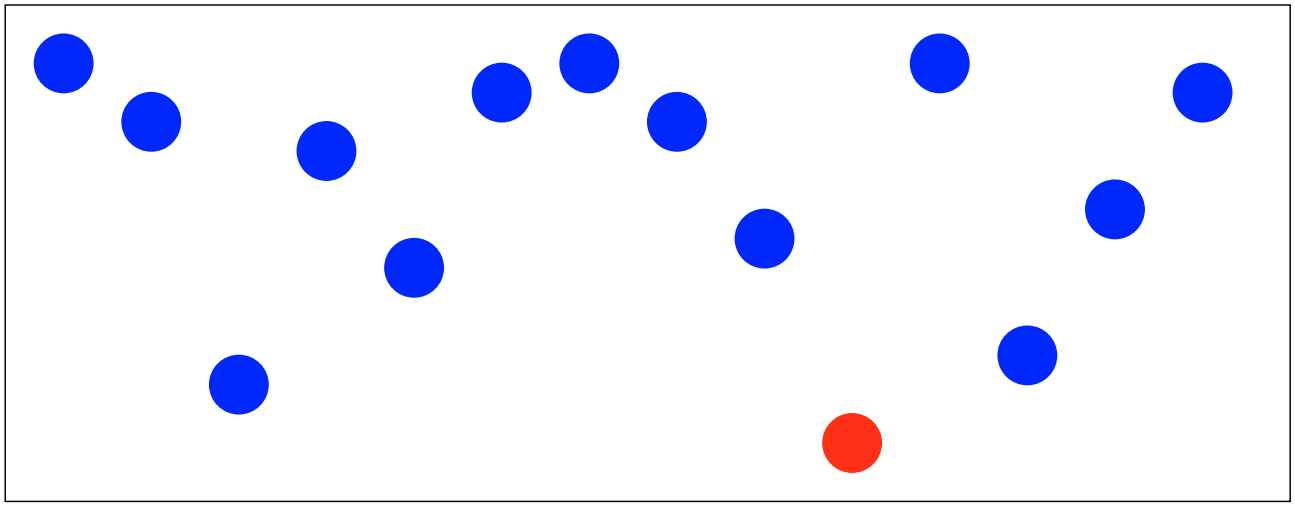
Criticality (self-similatity, scale invariance)



change scale: power-law unaffected

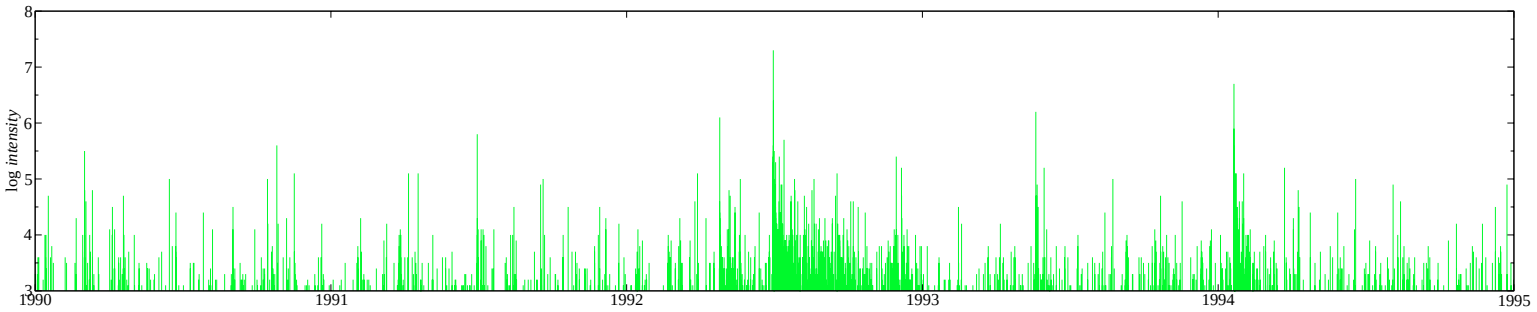
Some classes of models with scale-free avalanches

- Cellular automata with conservation (sandpiles)
- Cellular automata with dissipation (forest-fire)
- Extremal dinamics
  - coupled-maps (OFC model)
  - extremal fitness (Bak-Sneppen model)



Critical Phenomena

- Piles of sand or rice
- Earthquakes
- Solar flares
- Black hole accretion disks
- Confined plasma
- Superconduction exper.
- Cellular glasses creep exp.
- Forest fires
- Rain falls
- Loop rivers
- Traffic jams
- Mass extintions
- Brain activity
- . . .



Avalanches: also physically sound

Self-organized criticality (SOC) [Bak, Tang & Wiesenfeld, PRL 1987]

"The basic laws of physics are simple, so why is the world complex? The theory of self-organized criticality posits that complex behavior in nature emerges from the dynamics of extended, dissipative systems that evolve through a sequence of meta-stable states into a critical state, with long range spatial and temporal correlations. Minor disturbances lead to intermittent events of all sizes. These events organize the system into a complex state that cannot be reduced to a few degrees of freedom. This type of "punctuated equilibrium" dynamics has been observed in astrophysical, geophysical, and biological processes, as well as in human social activity."

[Paczuski & Bak, *Self-Organization of Complex Systems*, Proceedings..., cond-mat/9906077]

Self-organized criticality (SOC) [Bak, Tang & Wiesenfeld, PRL 1987]

Non-equilibium systems with a slow driving and evolving though avalanches may self-organize to a critical state.

- ⋆ Open / dissipative systems (self-organization)
- ⋆ scale-invariant properties (criticality), in space / time / intensities
- ⋆ instability thresholds $\longrightarrow$ chain reactions (avalanches)

Useful to think in terms of “avalanches”

Properties that one wishes to see

- $1/f$ power spectrum

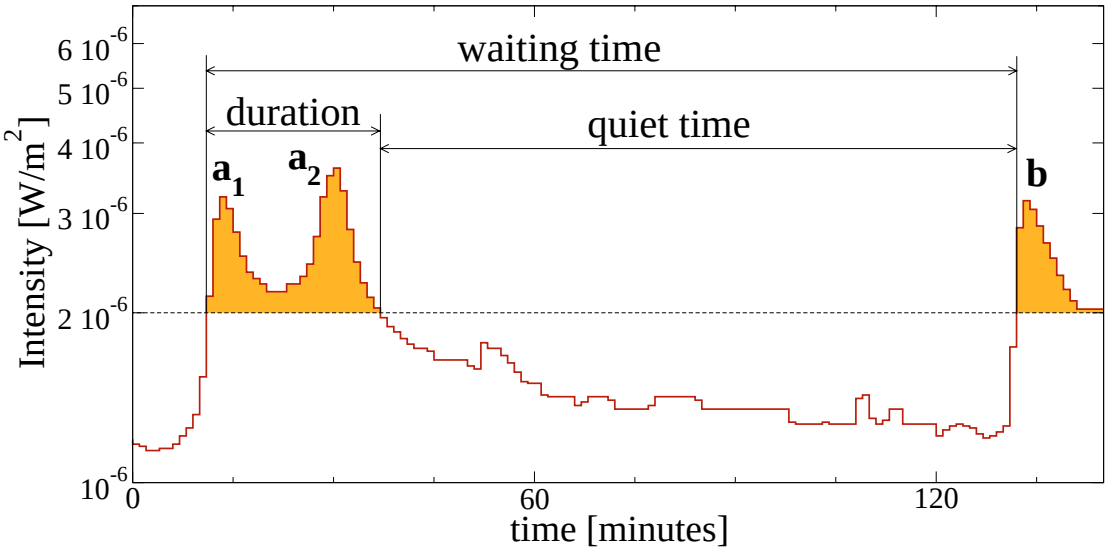
  (Fourier transform of the signal)$^2 \sim$ frequency^{-1}

  or $\sim$ frequency$^{-\gamma}$, with γ as far as possible

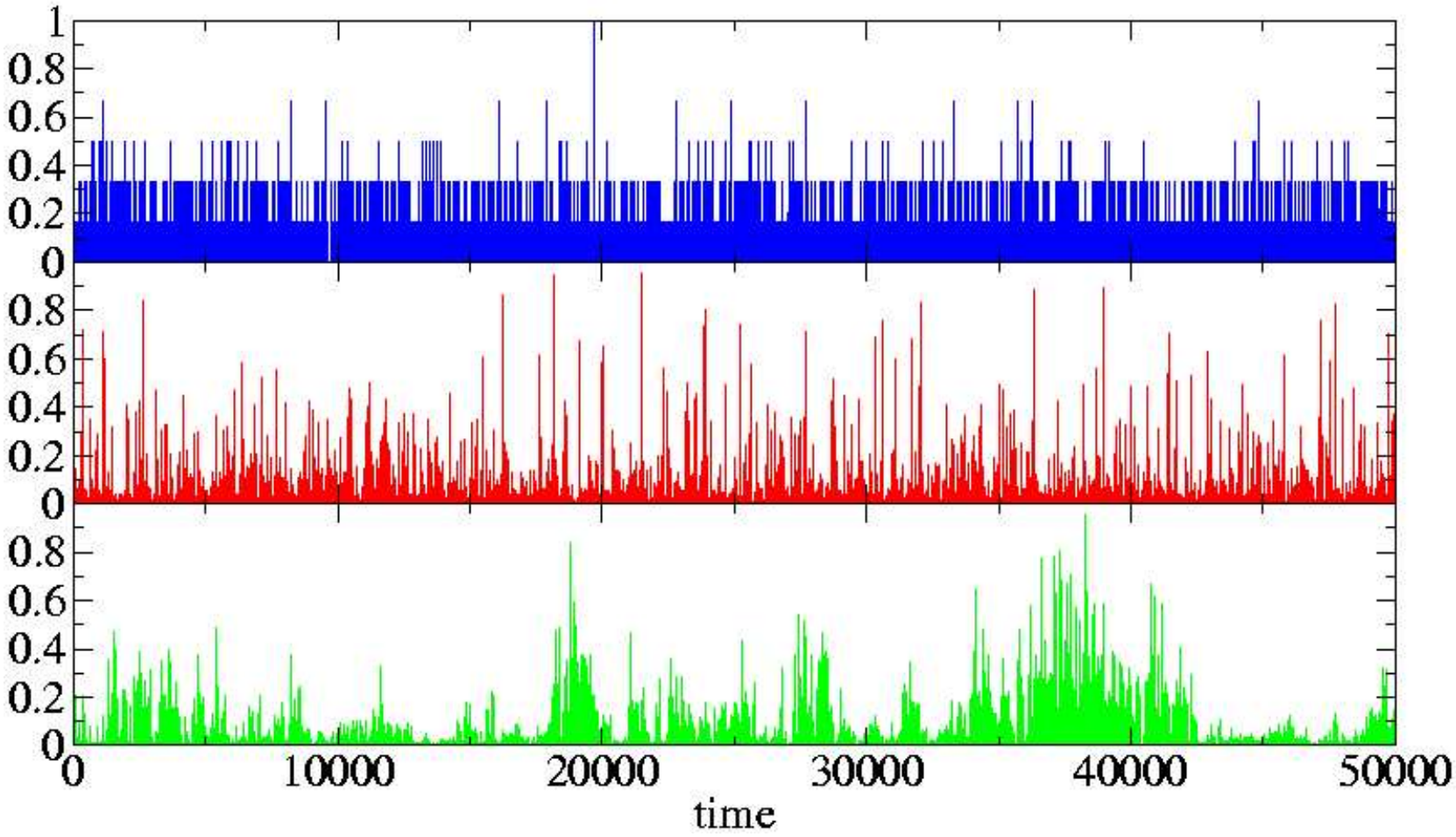
  form 0 (white noise) and 2 (random walk)

- power-law tail in the distributions of waiting-times between events

  (independent events $\longrightarrow$ exponential tail)



Time series: exponential, scale-free in intensity, and critical



against SOC hypothesis

- no SOC in solar flares

  G. Boffetta et al, Phys. Rev. Lett. (1999)

- no SOC for earthquakes

  X. Yang, S. Du, and J. Ma, Phys. Rev. Lett. (2004).

Argument:

1) SOC cellular automata do not have correlated events, indeed interoccurrence times have an exponential distribution

2) flares / earthquakes have power-law tail in the distribution

1) $+$ 2) $\longrightarrow$ No SOC...

But can one draw general conclusion on SOC from the properties of a few automata? (Do all automata well represent the SOC paradigm?)

in favour of SOC

Right statement

Some cellular automata do / do not have correlated events

Models with correlations:

- $1/f$ spectrum [Zhang, PRE 2000]

  [Davidsen & Paczuski, PRE 2002]

  [Woodard, Newman, Sánchez, and Carreras, 2005]

- waiting time distributions with power law

  [Sánchez, Newman & Carreras, PRL 2002]

  [Norman et al, AJ 2002]

  [Fragos, Rantsiou, and Vlahos, A & A, 2004]

  [Hedges and Takacs, 2005]

  [Lippiello, de Arcangelis, and Godano, 2005]

- foreshocks and aftershocks (OFC model)

  [Hergarten and Neugebauer, PRL 2002]

- features of turbulence

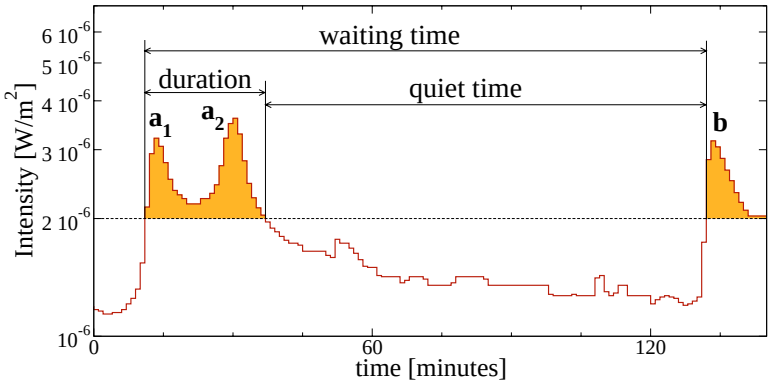
  [De Menech and Stella, Ph.A 2002]

in favour of SOC

by studying “running sandpiles”

(models of laboratory plasma)

Sánchez, Newman & Carreras (PRL 2002)



⟶ power-law tail in the distributions of waiting times if:

1: durations contaminate statistics

2: minimum thresholds in the detection

3: SOC persists with correlated driving

point 1: not relevant for earthquakes, but eventually consider quiet times

SOC + correlated driving $\longrightarrow$ correlated output

1) [Sánchez, Newman & Carreras (PRL 2002)] running 1d sandpile: $1/f^q$ drive

change of drive: $\approx$ same intensity statistics

2) [Norman et al, AJ 2002] Lu-Hamilton model (solar flares): drive strength varies as a random walk

3) [Fragos, Rantsiou, and Vlahos, A & A, 2004] Ellerman bombs (Sun): drive strength varies as a directed percolarion

4) [De Menech & Stella, Ph.A 2002] waves in BTW model are correlated

5) [Hedges and Takacs, 2005] OFC model (earthquake fault)

— OFC model has correlated epicenters [Peixoto and Prado, PRE 2005]

6) [Lippiello, de Arcangelis, and Godano, 2005] BTW + extremal dinamics drive

How simple can the driving be in sandpiles, to give a correlated output?

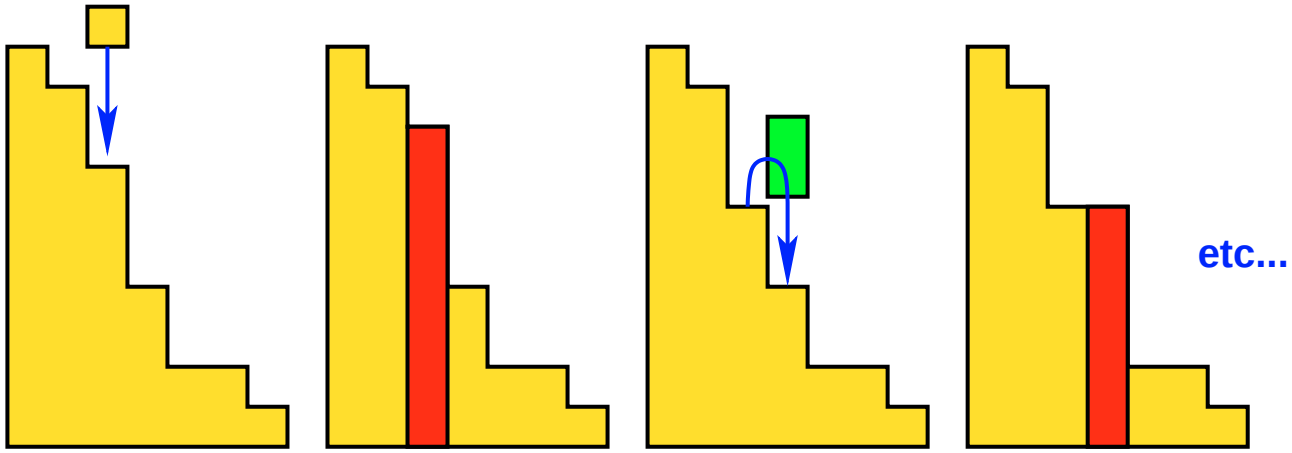
New model

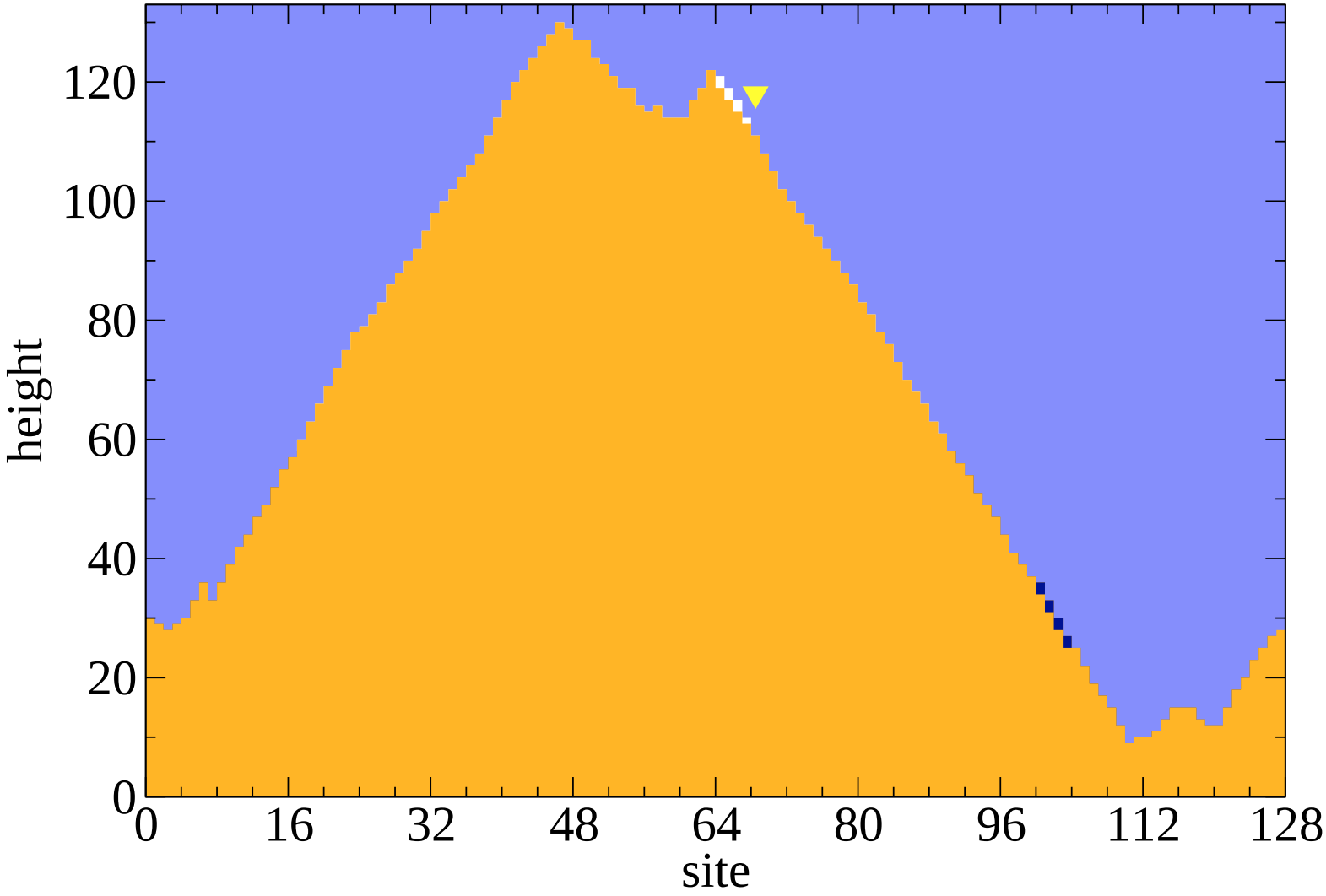
(1) Position where sand is added: random walk

$\longrightarrow$ simple spatial correlations in the driving (more realistic)

(2) boundary conditions: periodic

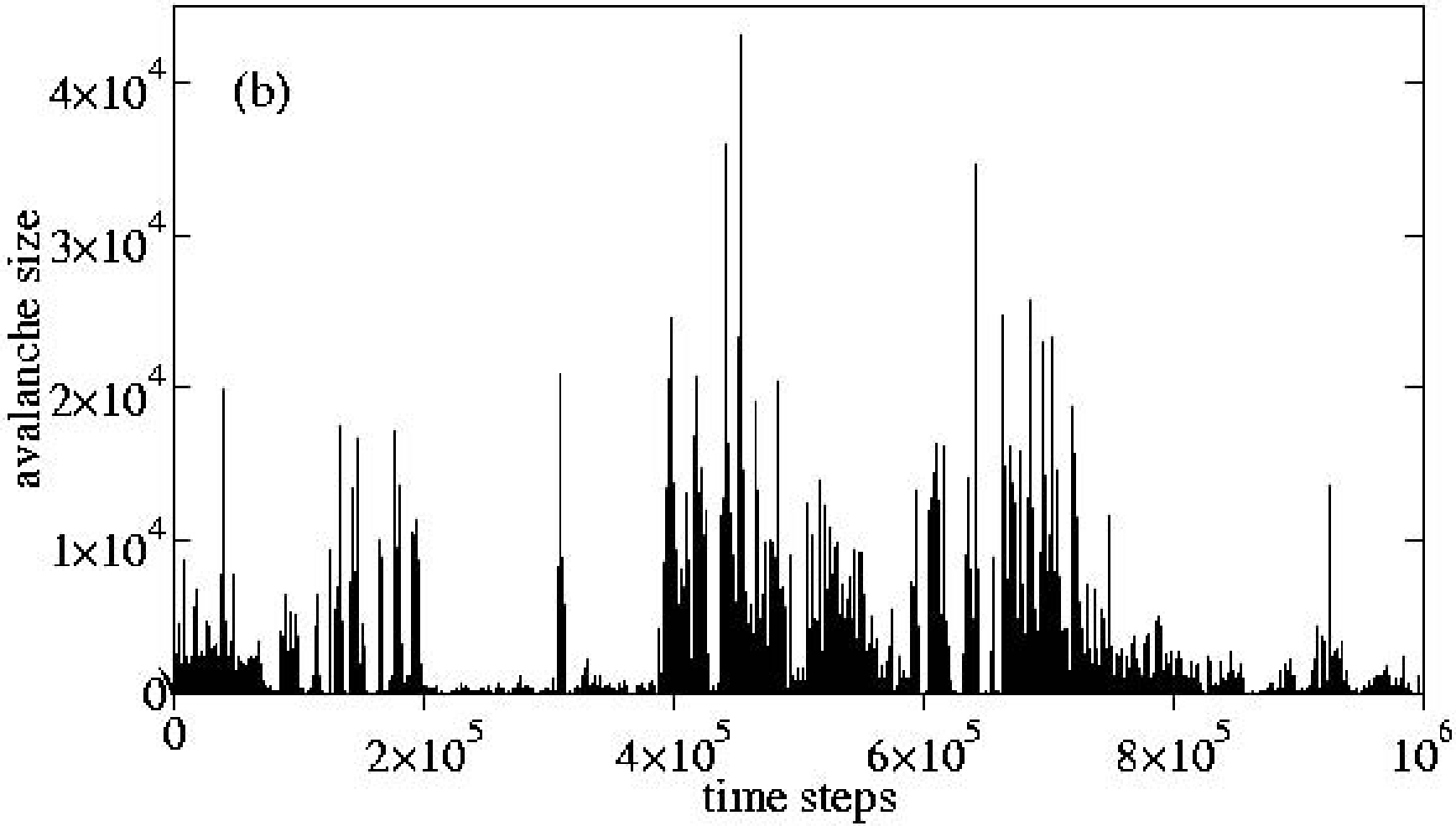
(3) instability rule



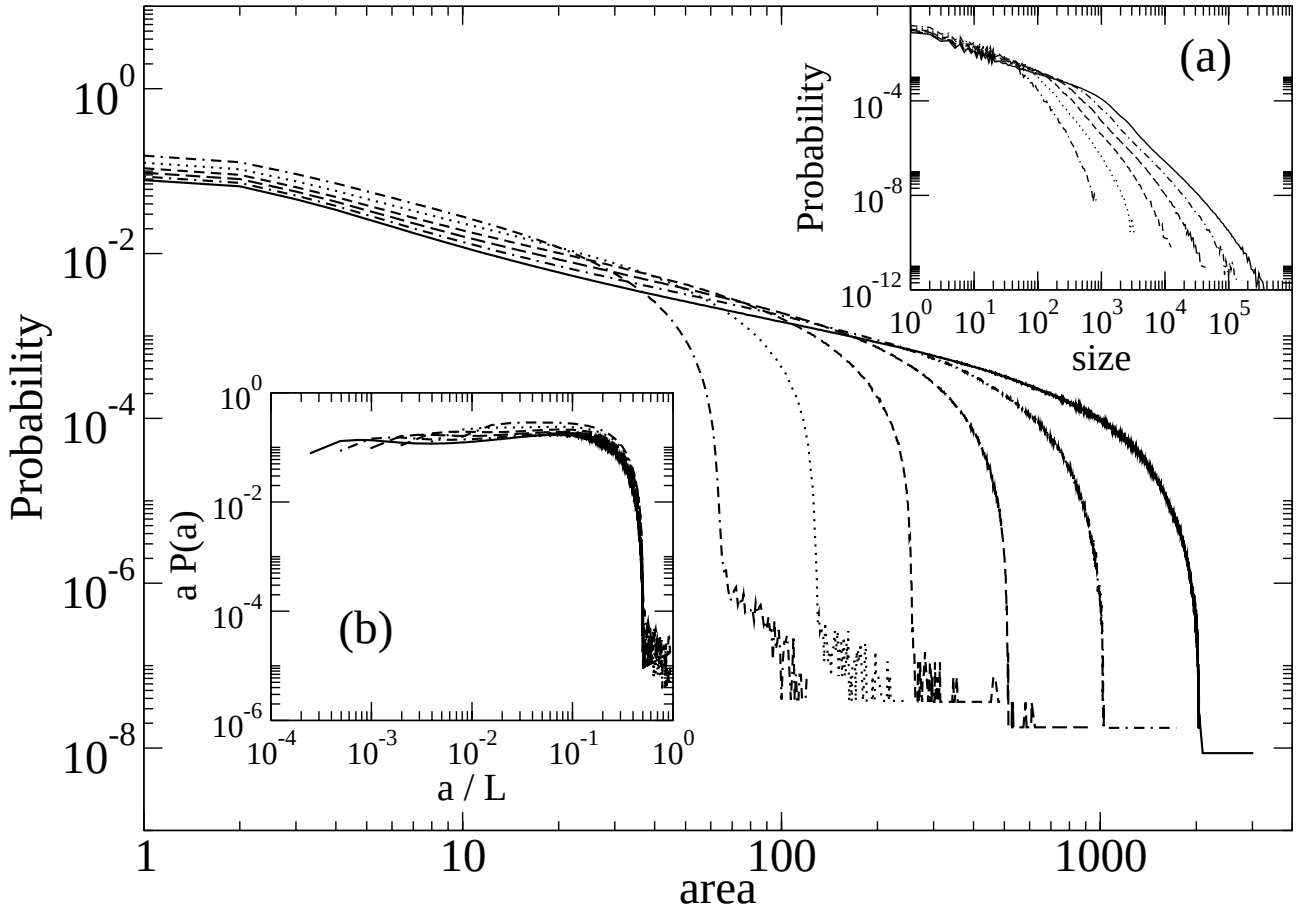


→ Still critical, but now also correlations in time

Time series of avalanche *size*

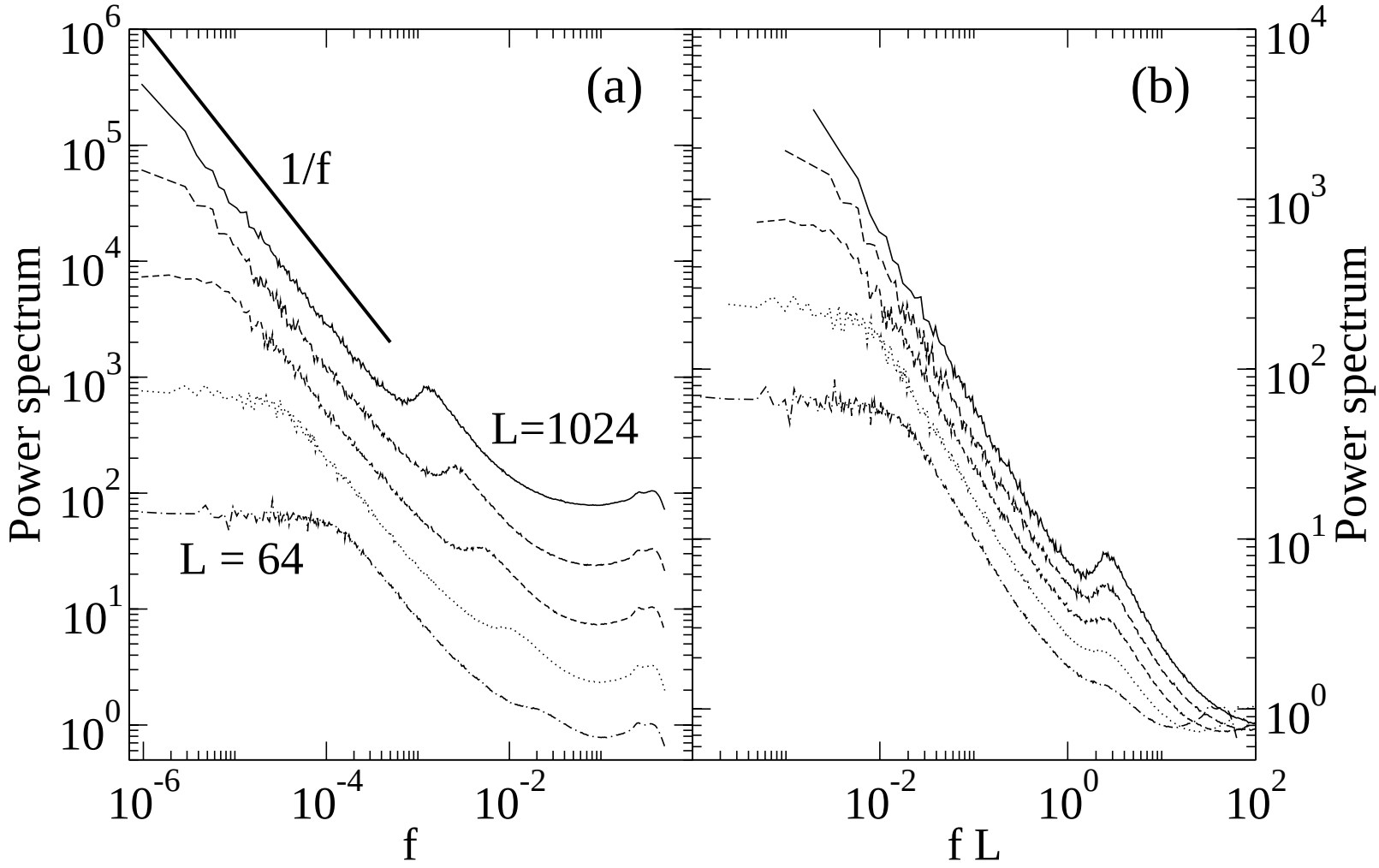


Scaling in avalanche area and size

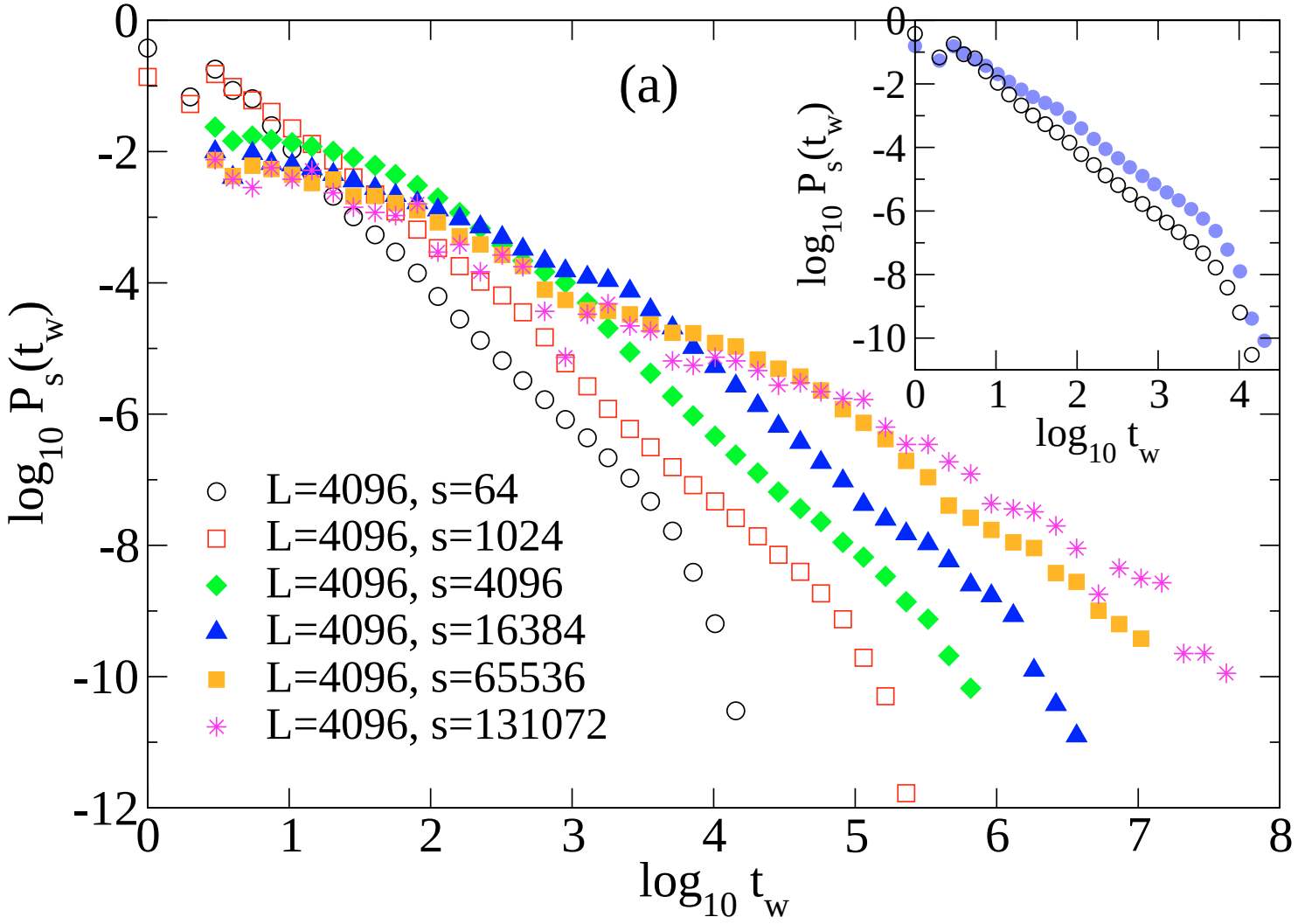


$$P_L(a) \simeq a^{-1} f(a/L)$$

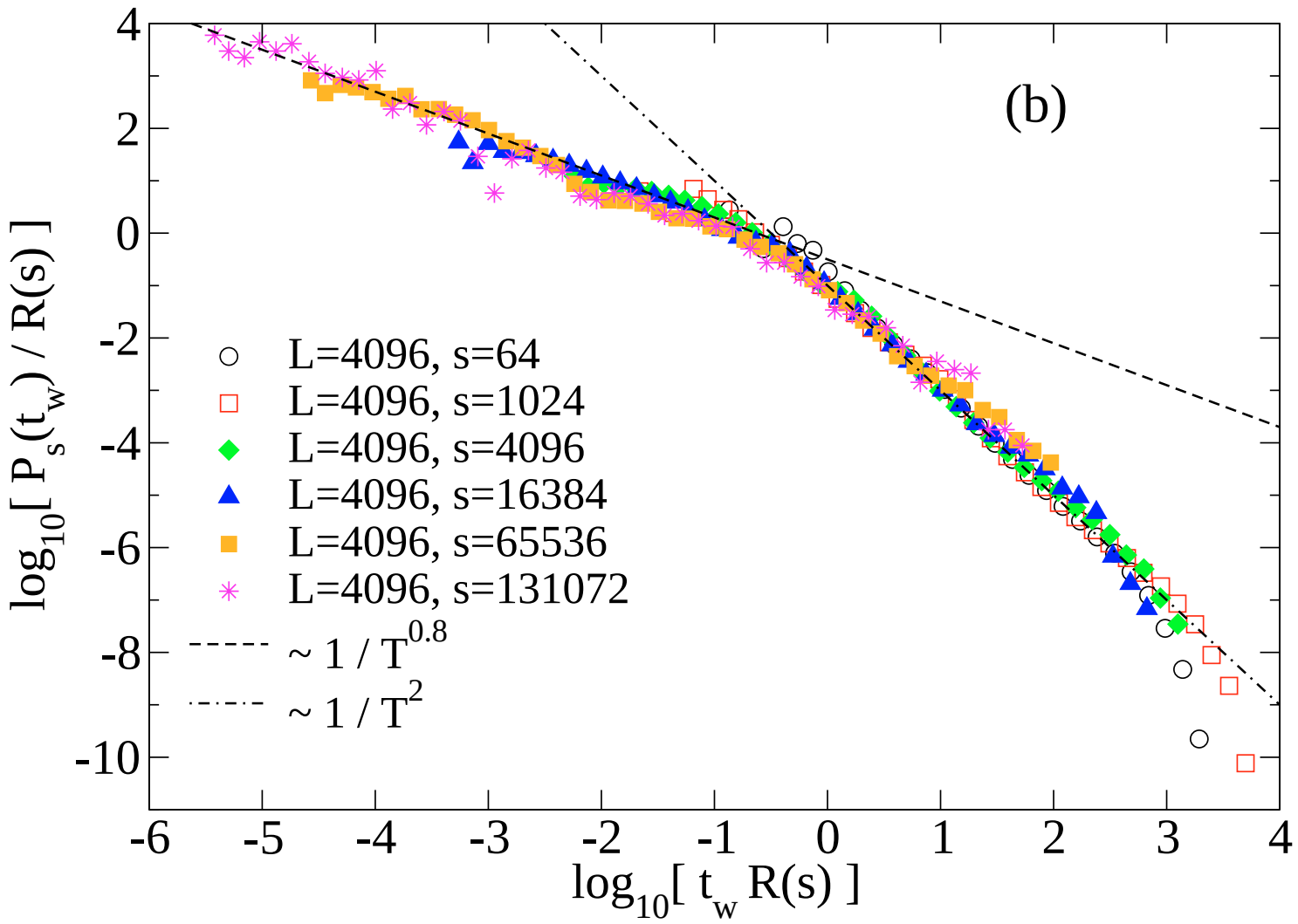
$1/f$ spectrum (long memory)



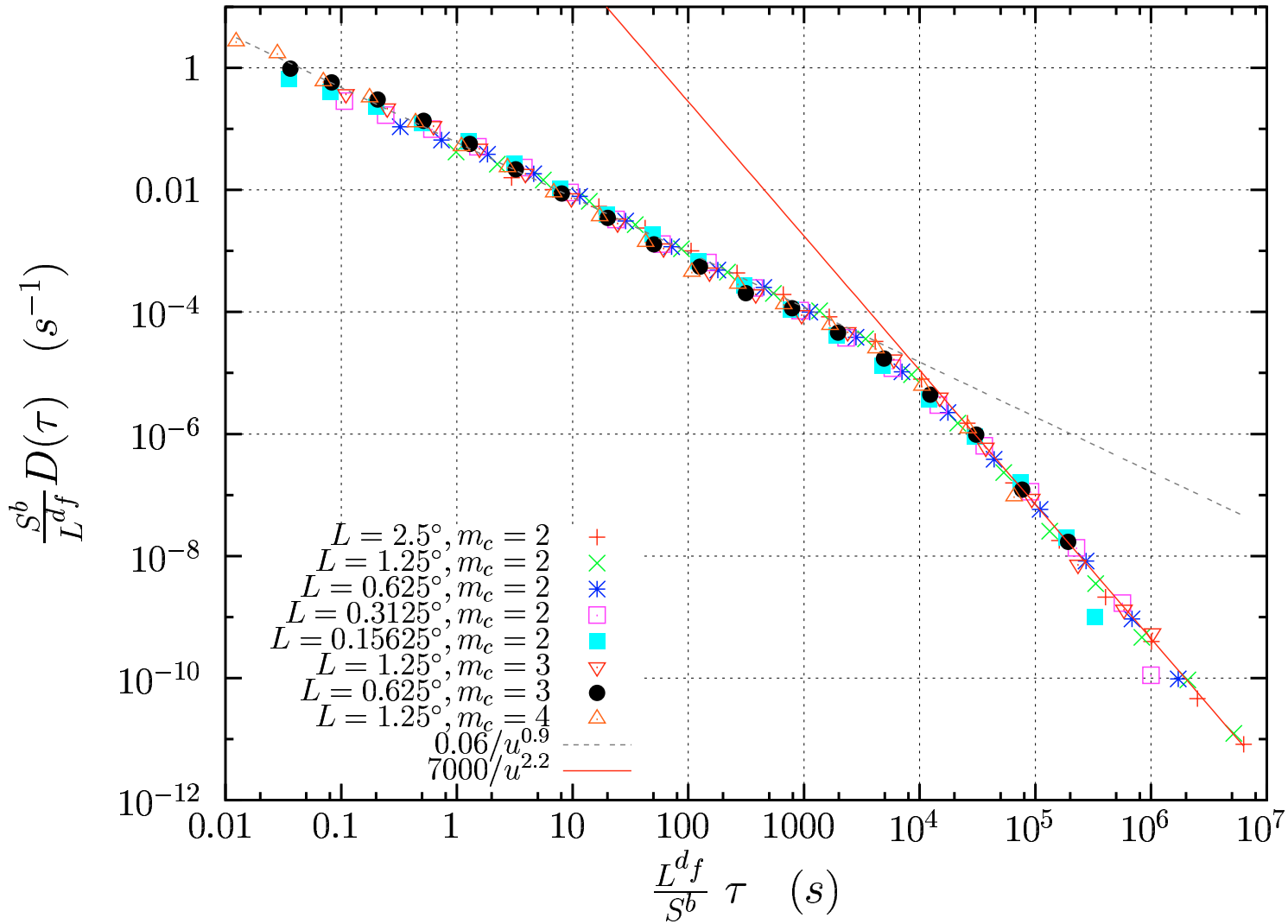
waiting-time distributions



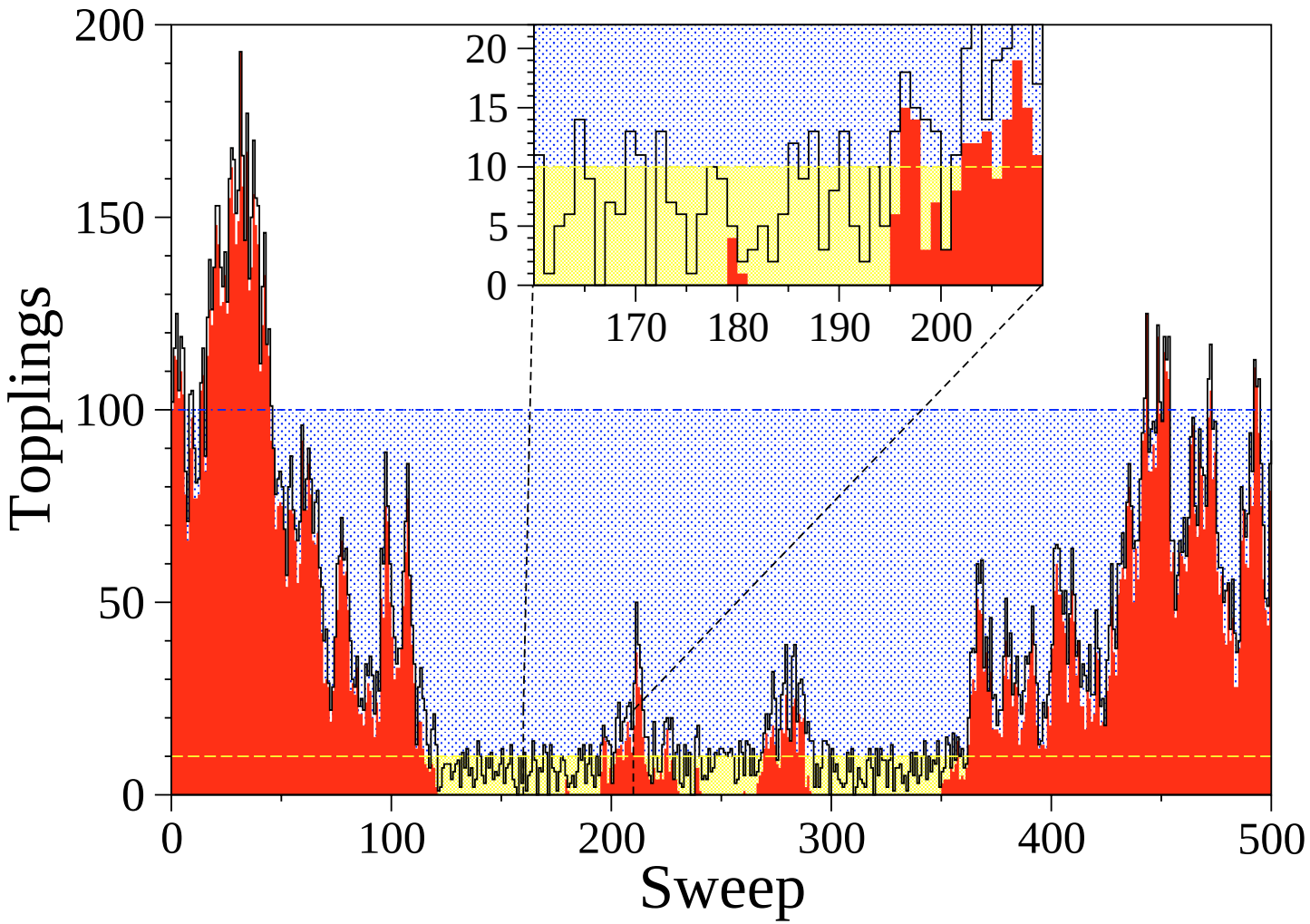
waiting-time distributions: rescaled by rates $R(s)$



Earthquakes: rescaled waiting-time distributions [Corral, Phys.Rev.E 2003]



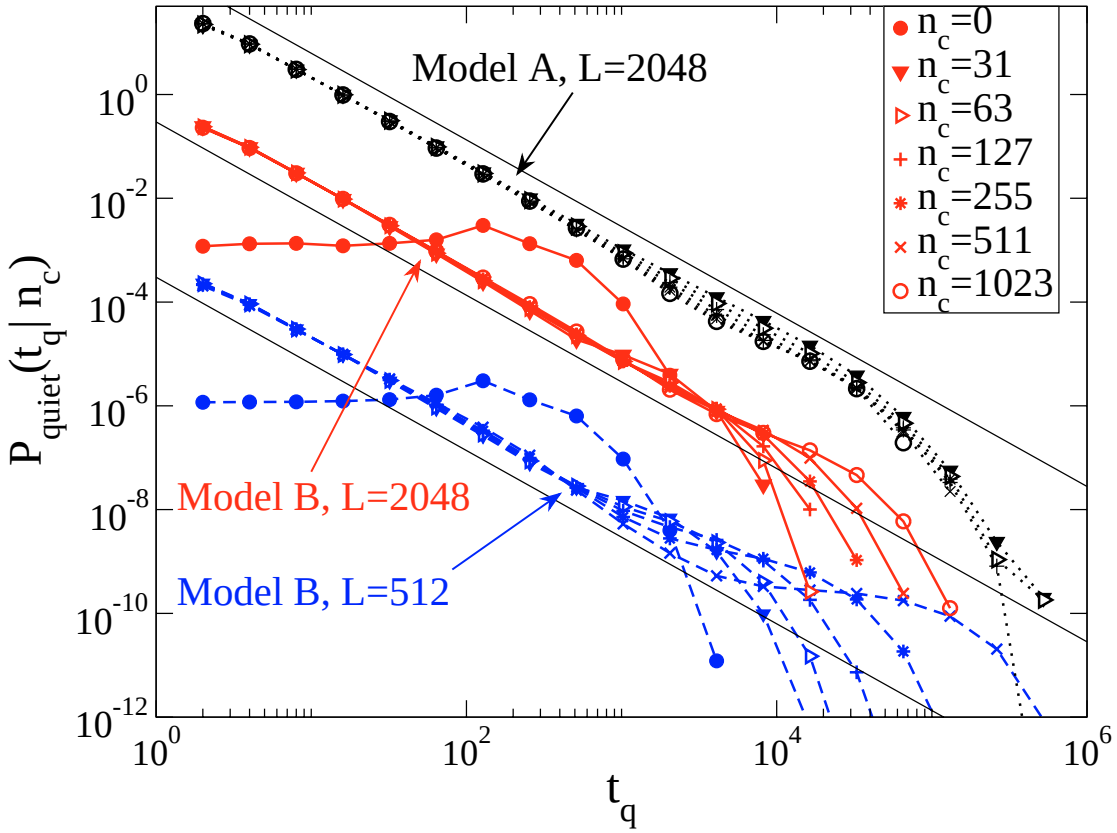
Thresholds: remove the noise



“Running” BTW model

— time step = 1 parallel update of unstable sites

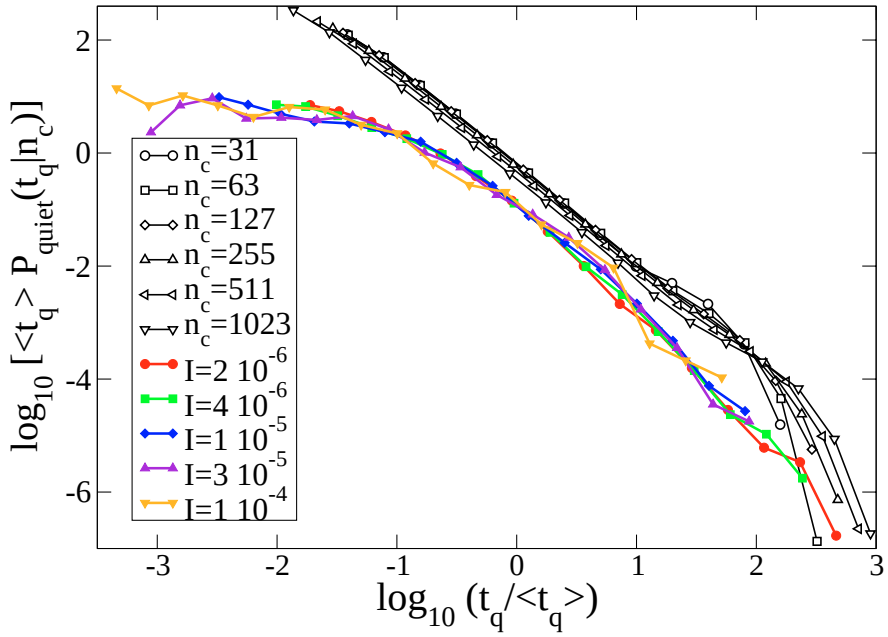
— one grain is added every ΔT parallel updates



“Running” BTW model

- Event duration & quiet times: same clock
- Thresholds: unavoidable
- Correlation of bursts inside a macro-avalanche

  again a phenomenon where space-time correlations are strong



Conclusions

- Correlations in space, time, and intensity are related to each other
- No problem with the waiting time statistics
- SOC models can yield $1/f$ noise
- SOC cellular automata can be good models of natural phenomena
- SOC in a sandpile without open boundary conditions

Baiesi and Maes,

Realistic time correlations in self-organized criticality from a random walk driving

http://www.arxiv.org/abs/**cond-mat/0505274** (not accepted in PRL)

Paczuski, Boettcher and Baiesi,

Inter-occurrence Times in the Bak-Tang-Wiesenfeld Sandpile Model...

http://www.arxiv.org/abs/**astro-ph/0606464** (accepted in PRL)