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Unified Laws in Seismicity
(copies of transparencies)

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Unified Scaling Laws in Seismicity

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Outline

- motivation
- examples
- existence & shape
- multifractality
- optimal spatial scaling

Scale free distributions

- power laws: GR, Omori
- unified scaling laws (Bak et al, 2002)

ξ_Δ - statistics

Δ - scale (space or time or both)

$\exists\, a_\Delta : \quad a_\Delta \xi_\Delta$ is independent of Δ

$$\Downarrow$$

p.d.f. of $\xi_\Delta = a_\Delta f(a_\Delta x)$

where $f(x)$ = p.d.f. of $\eta = a_\Delta \xi_\Delta$

$$a_\Delta = c / E\xi_\Delta$$

Reasons for consideraton:

- evidence of complex spatio-temporal behaviour of seismic regime
- a tool for
  - renormalization of precursors
  - de-clustering of EQ catalogs
- understanding of limits in predictability

Problems

- existence of unified scaling laws
- shape
- real scale range
- new information contained in a unified law

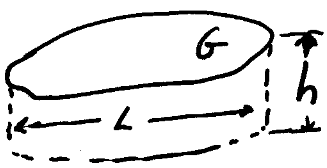
Examples

The Gutenberg-Richter law

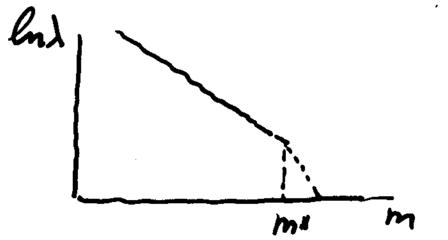
rate of $\{M > m\}$ in $G = \lambda_G(m)$

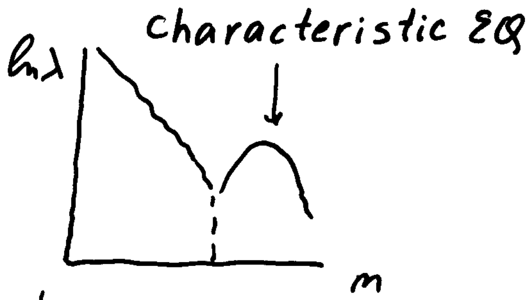
$$= A_G 10^{-bm}$$

limitation: EQ linear size $\ll \min(L, h)$



Evidence:





$m^* = 6$ Mid-Atlantic Ridge, $h = 10$ km

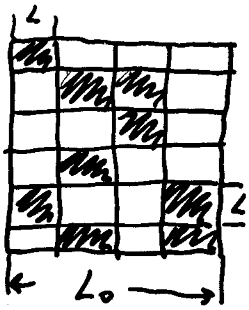
$m^* = 7.5$ Subduction zone, $h = 60$ km

$m^* - ?$ Aftershocks

some segments of $L = 100$-150 km in California

success in prediction $\begin{cases} m \geq 7.5 \\ \text{strongest aftershock} \end{cases}$

Fractality of epicenters



$\lambda(dg)$ - rate of events of $M > m_c$ in an area dg

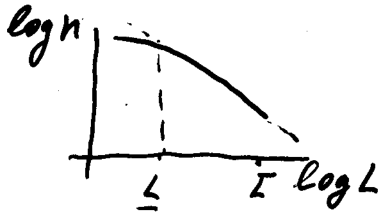
▨ - $\lambda(L \times L) > 0$

$\# \{ ▨ \} = n_L$

Definition: d_f is the Box-dimention of support of measure $\lambda(dg)$

if

$$\log n_L \simeq d_f \log L_0/L \quad , \quad L \ll 1$$



California: $d_f = \begin{cases} 1.8 & m>2 \quad L=10\text{-}100\,km \\ ? & m>3.5 \\ <1 \text{ (apriori)} & m>6 \end{cases}$

L=100-1000

Other estimations: 1.6 (Corral)

1.2 (Kagan, Kossobokov

Consequence (Keilis-Borok et al, 1989)

$$\langle \lambda(L \times L) \rangle \approx \mathrm{const}\ 10^{-bm} (L/L_0)^{d_f}$$

↑

$\lambda > 0$

Problems: range of L

d_f is not unique ; $d_f = d_f(m)$ - ?

Correlation of events

Baiesi & Paczuski, 2004

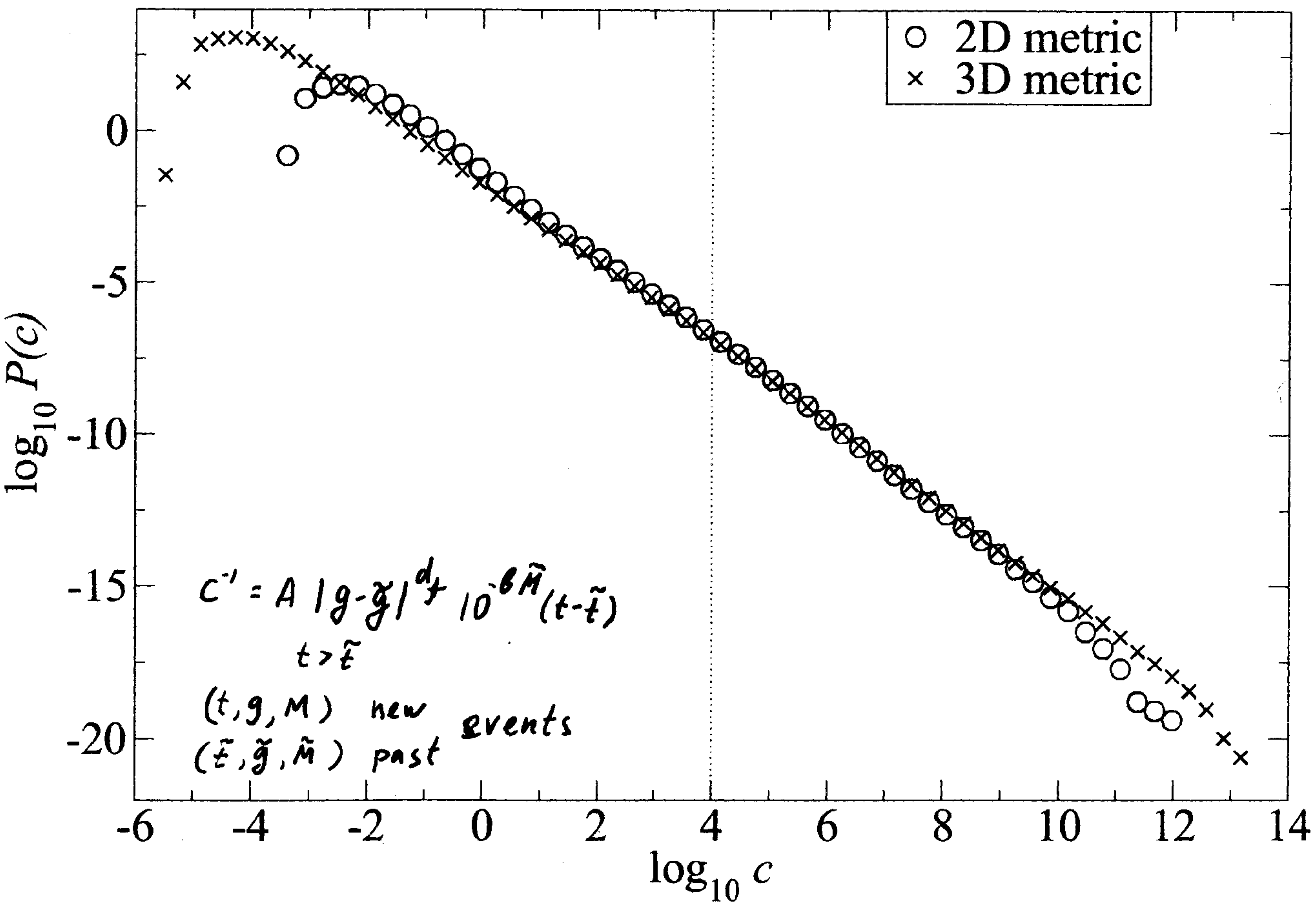
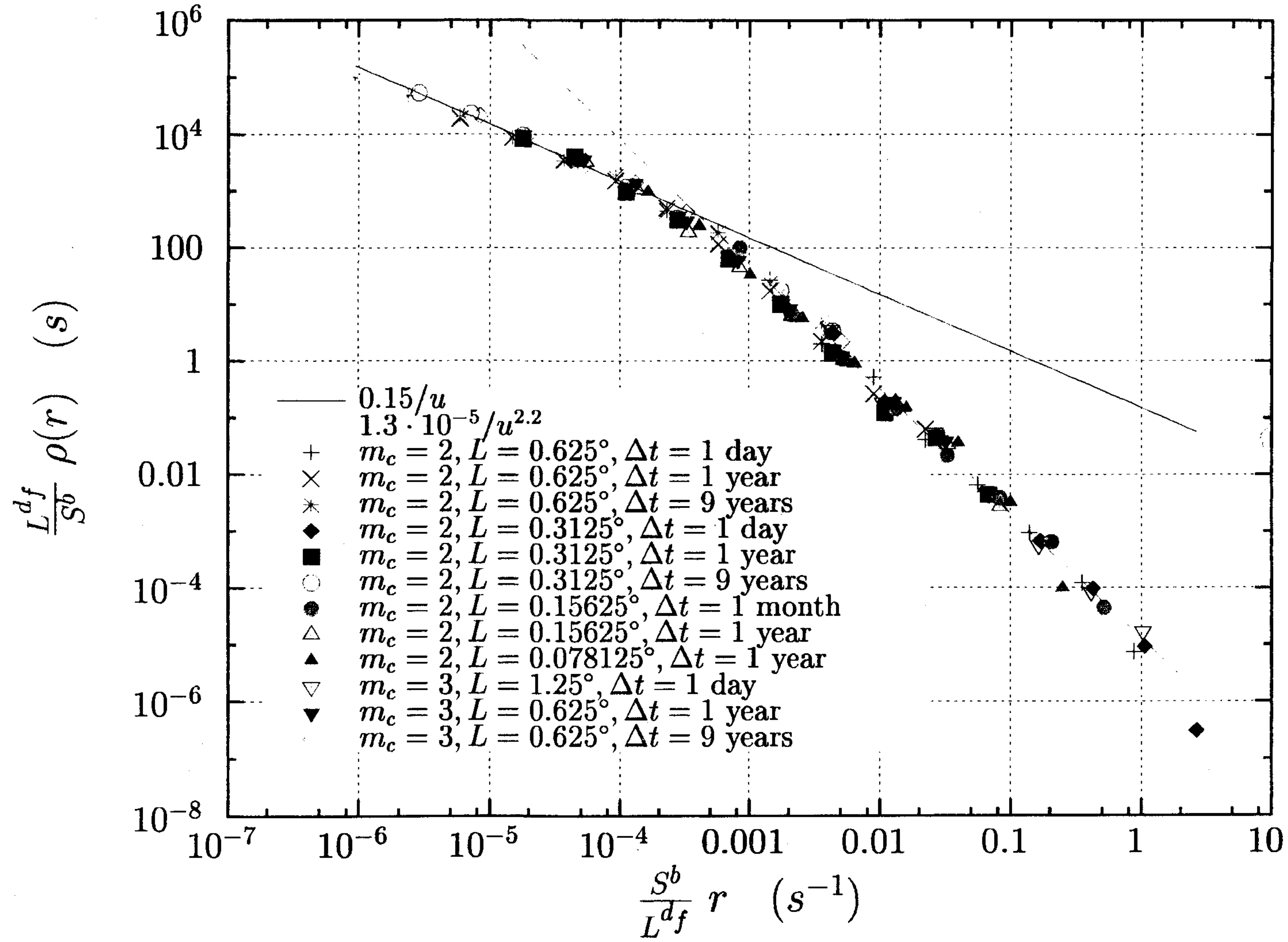


FIG. 2: The probability distribution of the correlation, c, between all earthquake pairs in the data base, with $m_< = 3$, using both the 2D metric and the 3D one. They are scale free distributions over many orders of magnitude. The threshold $c_< = 10^4$ where correlations are considered significant and links are made is indicated in the figure. Note that, with that threshold, most links are eliminated from the network, giving a reduced data set to examine seismic properties.

$[g]$ meter, $[t]$ sec, $\lg A = -4(1+\dim g)$

Unified scaling law for rate, $\lambda(dg)$ (California)

Corral, 2003



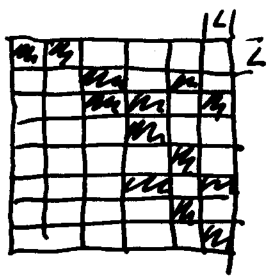
Scaled distributions of rates, $\lambda(L \times L)/a_L$, for several Δt, L, and m_c.

$d_f = 1.6$, $b = 0.95$

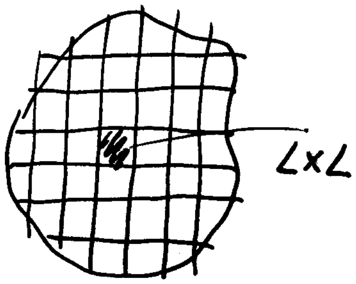
$$a_L = 10^{-bm}(L/L_0)^{d_f}$$

ΔL = 8-140 km

Δt = 1day – 9 years



Waiting-time distribution: unified law by Bak et al (2002)



Statistics $\bar{\tau}_L$: time between consecutive EQs of $M > m_c$ in a random cell $L\times L$

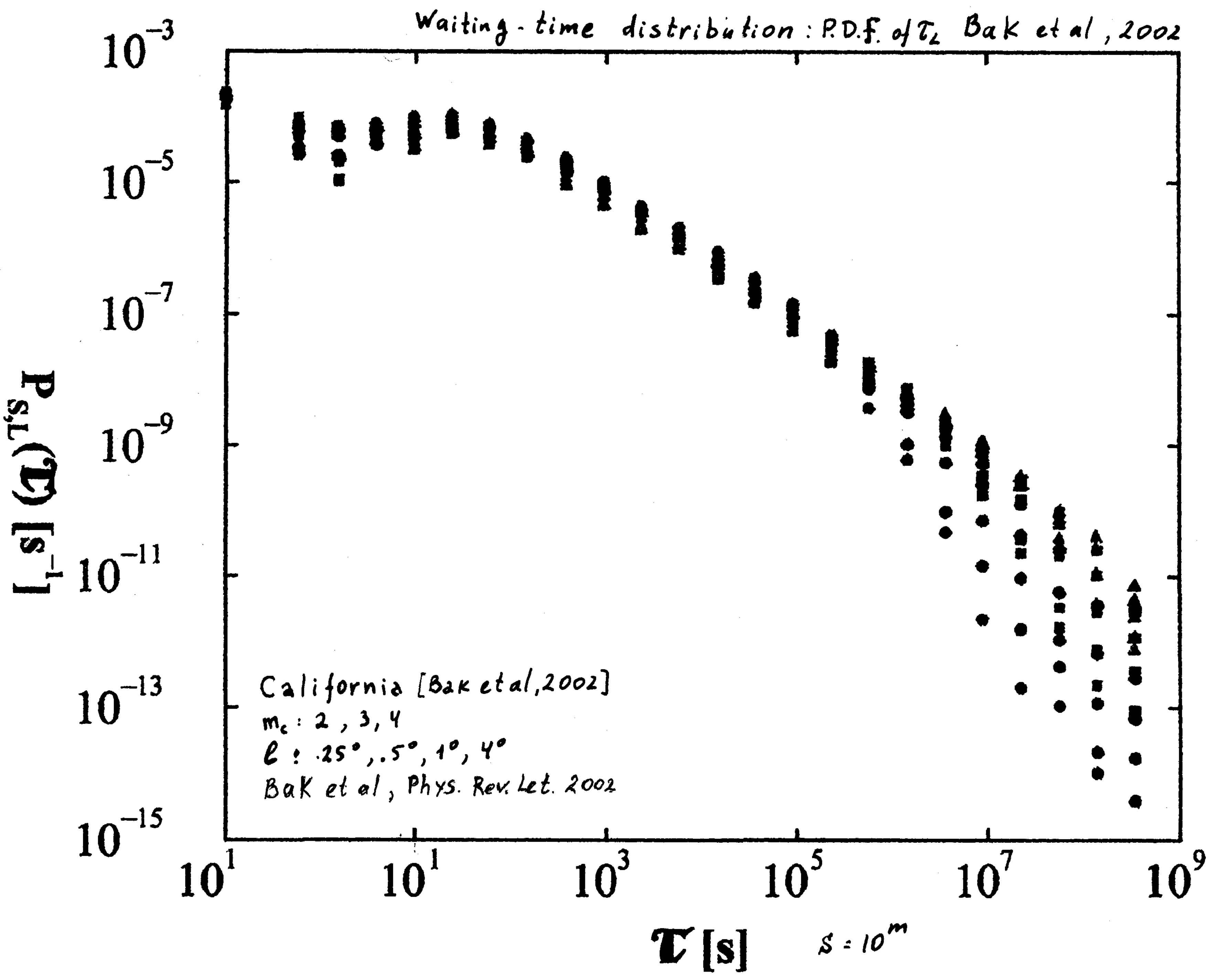
$$\mathrm{Prob}\{\text{cell } L\times L\} = \lambda(L\times L)/\lambda(G)$$

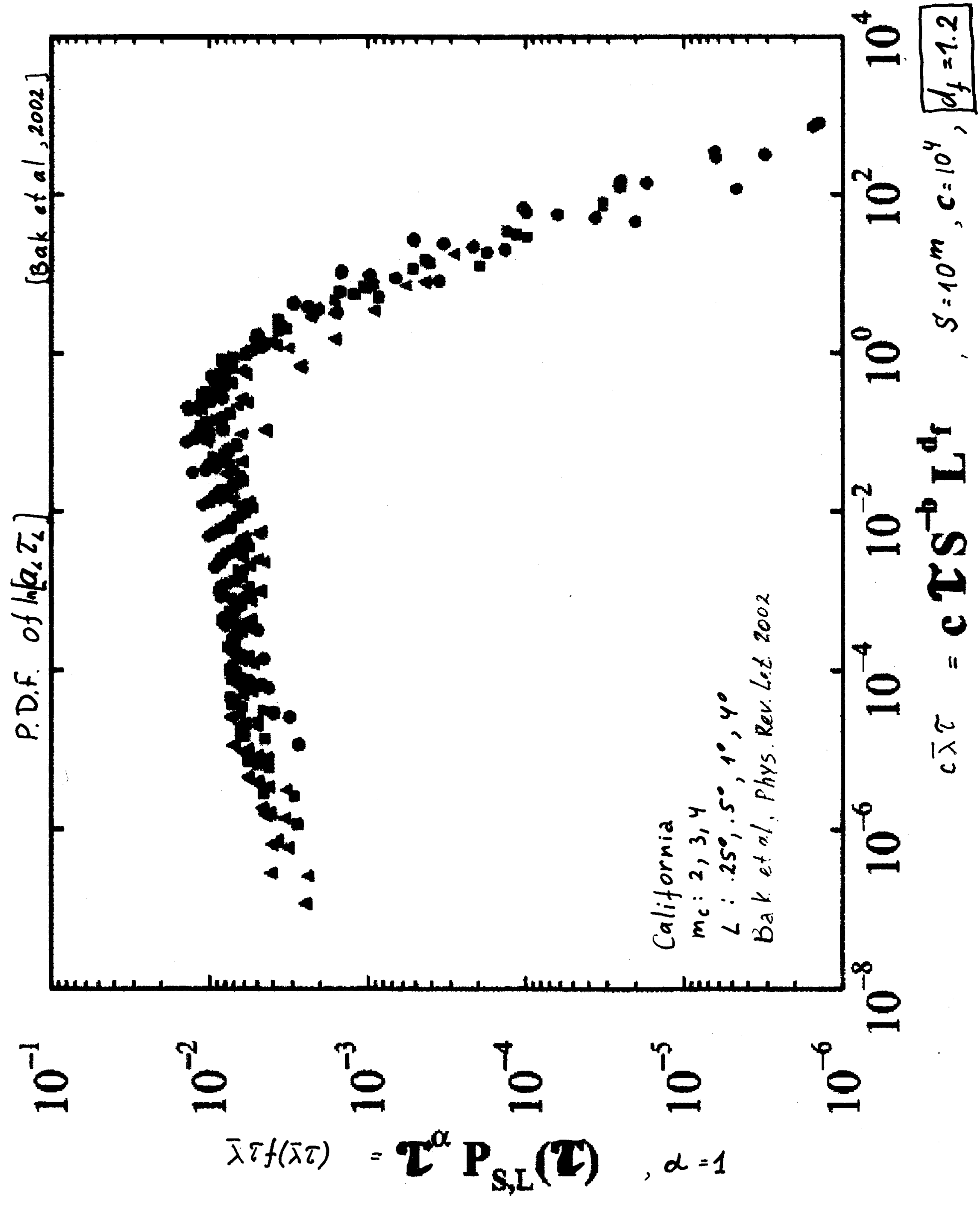
Normalization: $E\, a_L \bar{\tau}_L = 1$

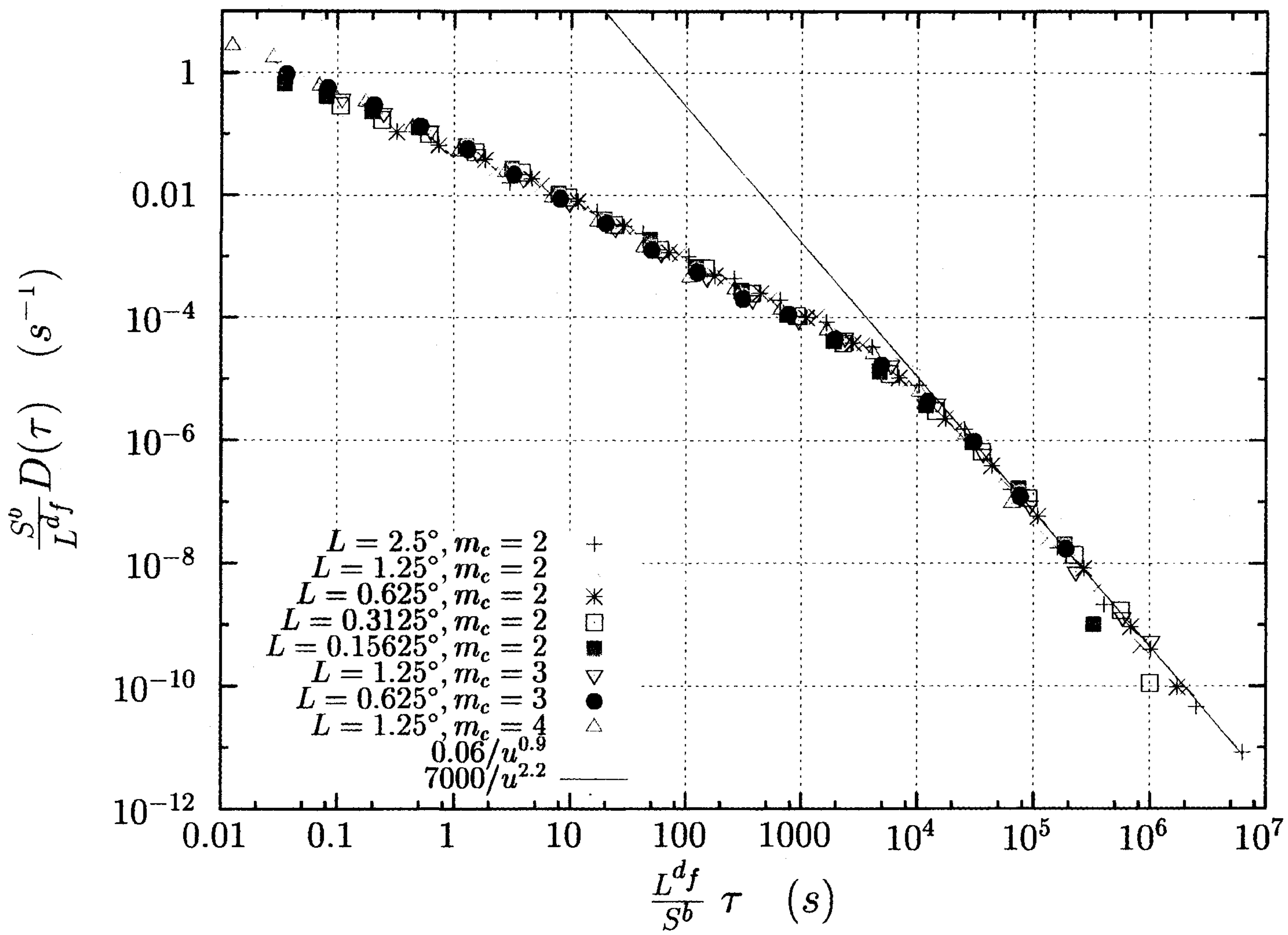
$$\Downarrow$$

$$a_L = \langle \lambda(L\times L) \rangle_{\lambda>0} = c\, 10^{-bm} \left(\frac{L}{L_0}\right)^{d_f}$$

d_f - ?


Waiting-time distribution : P.D.f. of τ_L Bak et al, 2002
$P_{S,L}(\tau)$ [s^{-1}]
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{1}
10^{1}
10^{3}
10^{5}
10^{7}
10^{9}
τ [s]
$S = 10^{m}$
California [Bak et al, 2002]
m_c : 2 , 3 , 4
ℓ : .25°, .5°, 1°, 4°
Bak et al, Phys. Rev. Let. 2002


P.D.F. of ln a_L τ_L
[Bak et al, 2002]
California
m_c : 2, 3, 4
L : .25°, .5°, 1°, 4°
Bak et al, Phys. Rev. Let. 2002
c λ̄ τ = c τ S^{-b} L^{d_f} , S = 10^m , c = 10^4 , d_f = 1.2
λ̄ τ f(λ̄ τ) = τ^α P_{S,L}(τ) , α = 1



Scaled distributions of τ_L (Corral, Phys. Rev E 68, 2003)

$d_f = 1.6$, $b = 0.95$, $\tau > 38s$

ΔL : 15 - 250 km

Δm_c = 2-4

5° Unified Scaling Law by Bak

Bak's philosophy applied to EQs:

Don't care about

- the tectonic environment

(a subdivision of seismogenic region is independent of it)

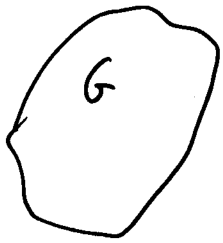
- after-, for- or main-shocks

(there is nothing in the seismograms that differentiates these events)

- space and temporal heterogeneity

Waiting-time distribution:

Universal scaling law by Corral



statistics τ_G : time between consecutive EQs of $M>m$ in a region G

Normalization : $\Sigma\, a_L \tau_L = 1$

$$a_L \propto 10^{-bm} (L/L_0)^{d_f}$$

Unified scaling law by Corral

Corral, Phys Rev E 68 (2003)

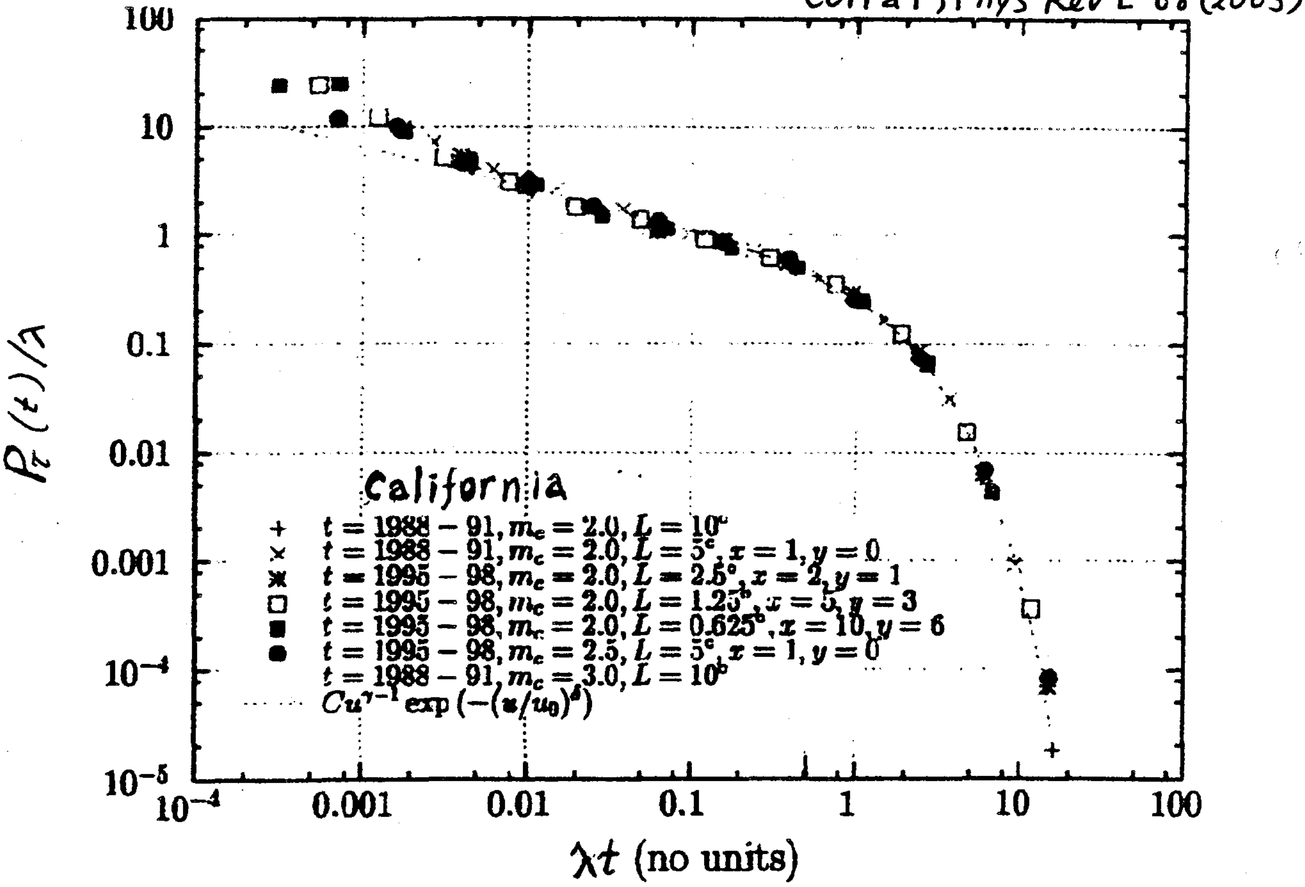
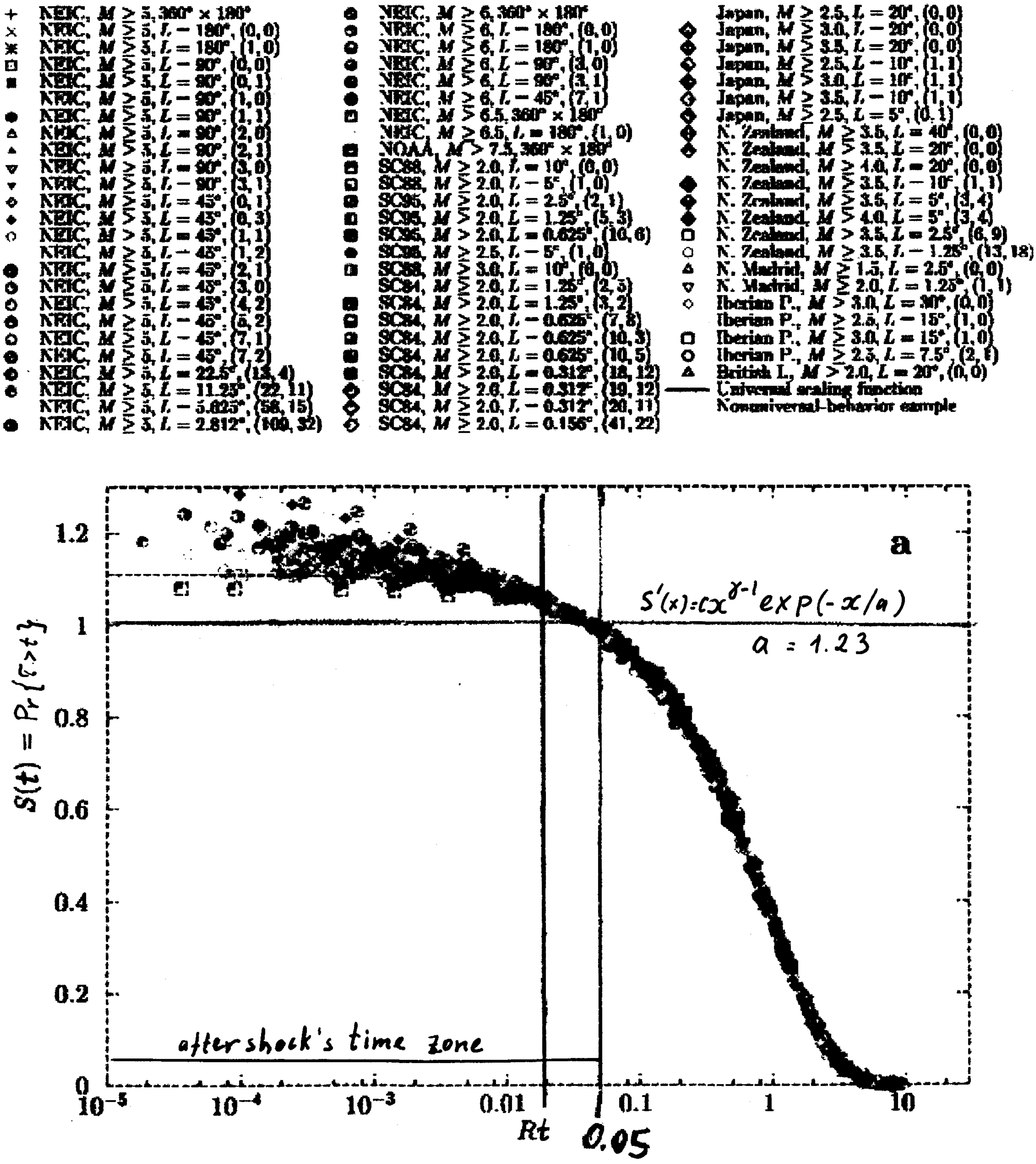


FIG. 1. Local distributions of interoccurrence times for several stationary periods and different regions, after scaling by the rate.

Catalog: NEIC, SC: 88,95 ; Japan, N. Zeland, N. Madrid, Iberian, British
m: 2; 2.5; 3.5; 5; 6
L: .15° .31° .62° 1.25° 2.8° 5.6° 11.2° 22.5° 45° 90°, 180°



Corral: arXiv: cond-mat/0310407 17 Oct 2003

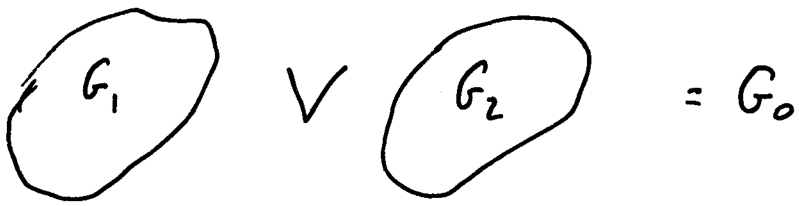
Theoretical Results (Molchan, 2005)

1° The Shape of Corral's universal scaling law

If there are two areas with independent seismicity then

a universal p.d.f. of $\tau\lambda$, $E\tau\lambda = 1$, is exponential

more exactly:



$N_i(dt)$ · stationary sequence of events i G_i

$N_0 = N_1 + N_2$

If

- N_1 and N_2 are stochastically <u>independent</u> (!)
- the rescaled τ-distribution for G_i, $i=0,1,2$ are the same with p.d.f. $f(x)$
- $f(x) < c\,x^{-\theta}$, $0<\theta<1$, $x \ll 1$

then

$$f(x) = \exp(-x)$$

Conclusion: observed $f(x) \neq \exp(-x)$

$\Downarrow$

Corral's universal scaling law can't exist in the whole range of time

2° Main peculiarities of the rescaled τ_G distribution

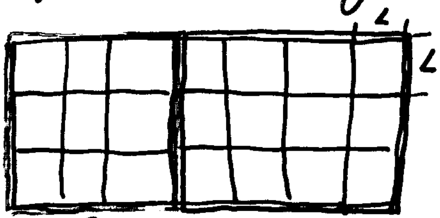
In the general Poissonian cluster model with Omori law $\Lambda(t) \uparrow \infty$, $t \ll 1$

$$\text{p.d.f. of } a_G \tau_G (t) \propto \begin{cases} \exp(-t/a) & t \gg 1 \\ \Lambda(t) & t \ll 1 \end{cases} \quad (*)$$

where a is fraction of main events among all seismic events

- (*) is supported by observations
- What other information can be extracted from the "universal" law by Corral ?

3° Unified scaling law by Bak



$G_1 \vee G_2 = G_0$

$\bar{\tau}_L(G_i) = \tau_L(\text{random } L\times L \text{ cell in } G_i)$

$a_L(G_i) = \langle \lambda(L\times L) \rangle$, $L\times L \subset G_i$, $\lambda > 0$

- If $\bar{\tau}_L(G_i)\, a_L(G_i)$, $i = 0, 1, 2$ have the same distribution then $a_L(G_1) = a_L(G_2)$ (!)

- if unified scaling law by Bak holds for G and $\lambda(\tilde{G})$ continuously depends on subarea $\tilde{G}$ then

$$\lambda(dg) = c|dg| \text{ is homogeneous}$$

Conclusion: the observed $\lambda(dg)$ is non-homogeneous

$\Downarrow$

the scaling law by Bak can hold approximately only

Spatial scaling of seismicity rate

$\lambda(L\times L)$ is rate of $\{M>m_c\}$ in cell $L\times L$

Scaling:
$$\lambda(L\times L) \propto L^{+d}$$
$$\tau(L\times L) = O(1/\lambda(L\times L)) \propto L^{-d}$$

Practice: $d = d_0$ (Box dimension)
d_2 (Correlation dim)
d_p (generalized dim by Grassberger-Procaccia)

Problem: which d is better?

Example (Pisarenko 1996)

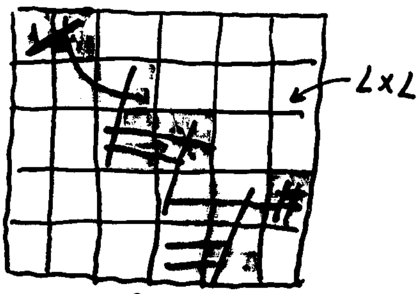
If $\lambda(dg)$ is a sample of a random Levy measure of index $0<\alpha<1$ then
$$P(\lambda(L\times L)/L^{2/\alpha} < x)$$
is independent of L, i.e. $\boxed{d = 2/\alpha > 2}$

Here $E\lambda(L\times L) = \infty$!

$d_p \neq const$ ⇔ $\lambda(dg)$ - multifractal measure
? ?

The rate $\lambda(dg)$ as a multifractal

1° Box-dimension of set S = support of $\lambda(dg)$



G

$\boxdot \cap S \neq \emptyset$

$n(L) = \# \{ \text{sell of type } \square \}$

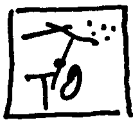
$\log n(L) \simeq -d_0 \log L \quad , L \to 0$

$d_0 = \mathbf{B}\text{-dim}\,(S)$

H-dim (Hausdorff dimension) is more strict analogue of B-dim

2° Singularity of type α / Hoelder's exponents

$g \in L \times L$



$$\log \lambda(L \times L) \simeq \alpha \log L \quad , \quad L \to 0$$

$$S_\alpha = \{ g : g \text{ is of type } \alpha \}$$

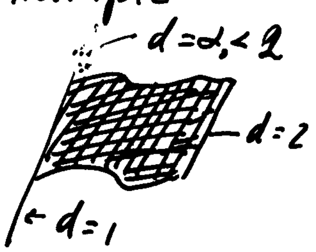
3° $\lambda(dg)$ is multifractal if

$$S = \{\text{support of } \lambda(dg)\} = \bigcup S_\alpha \quad , \quad S_\alpha \neq \emptyset$$

Multifractal Spectrum : $(\alpha, f(\alpha))$

$$f(\alpha) = \dim S_\alpha$$

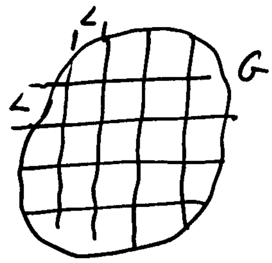
Example



trivial mixture of monofractals (no similarity)

$f(\alpha) = \alpha \quad , \alpha = \alpha_0, 1, 2$

4° $\tau(q)$ function



Renyi function

$$R_L(q) = \sum_{\lambda>0} \left[\frac{\lambda(L\times L)}{\lambda(G)}\right]^q \qquad |q|<\infty$$

$$\log R_L(q) \approx \underline{\underline{\tau(q)}} \log L \ , \quad L\to 0$$

Multifractal formalism:

$$\tau(q) = \min_\alpha (q\alpha - f(\alpha)) := \mathcal{L}f$$

If $\tau(q)$ is strictly concave and smooth then

$$f(\alpha) = \min_q (q\alpha - \tau(q)) := \mathcal{L}\tau$$

($\mathcal{L}$ - the Legendre transformation)

- $\min_q \to \alpha = \dot{\tau}(q) \to$ α is unique (monofractal) if $\tau(q) = \alpha(q-1)$ is linear
- Box-counting method give H-dim(S_α) !

5° The Renyi dimensions / the generalized Grassberger-Procaccia dimensions

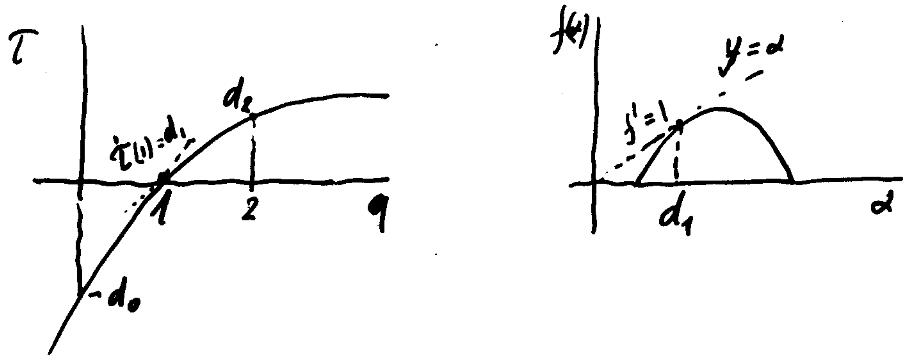
$$d_q = \frac{\tau(q)}{q-1} = \frac{\tau(q)-\overset{0}{\tau(1)}}{q-1} = \underbrace{\dot{\tau}(q^*)}_{\text{Hoelder's exponent}}$$

$d_0 = -\tau(0)$ - Box-dim

$d_1 = \dot{\tau}(1)$ - the information dimension

$d_2 = \tau(2)$ - the correlation dimension

Typical τ, f functions



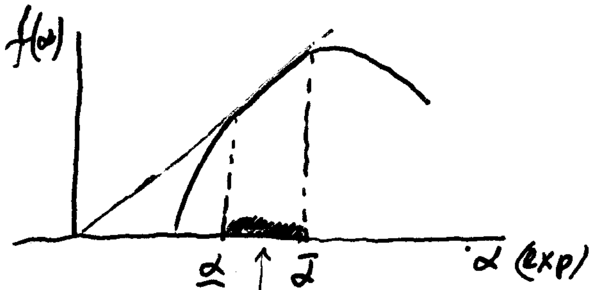
- $d_0 \geq d_1 \geq d_2 \dots \quad , d_p \downarrow$

6° Hoelder's exponents vs dimensions

(Young, 1982) If $\lambda(S_\alpha) > 0$ then

$$\alpha = H\text{-}\dim S_\alpha = f(\alpha)$$

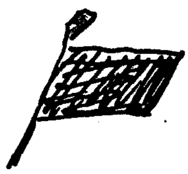
i.e. $f(\alpha) < \alpha \Rightarrow \lambda(S_\alpha) = 0$



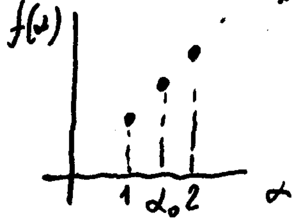
$\underline{\alpha} = \dot{\tau}(1+)$ $\quad \bar{\alpha} = \dot{\tau}(1-)$

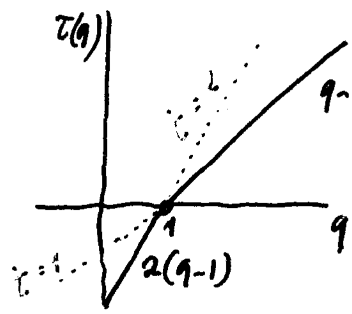
exponents related to dimensions of the support of $\lambda(dg)$

Example



mixture of monofractals





no information on d_0: $1 < d_0 < 2$

- $\dot{\tau}(1)$ exists $\Rightarrow$ d_0, d_1 are dimensions of $\operatorname{supp}\lambda(dg)$
  other d_p are Hoelder's exponents

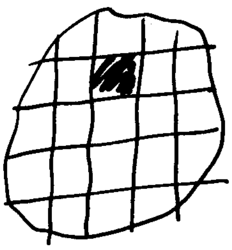
Problems:

- multifractality of real seismicity, i.e.

  $\tau(q) \neq d_0(q-1)$ (evidence)

- optimal scaling of $\lambda(L\times L)$ and $\tau(L\times L)$ for random cell $L\times L$ under multifractal condition

Random cell $L\times L$:



■ - $L\times L$

$$Pr(■) = c_p[\lambda(L\times L)]^p, \; p \geq 0$$

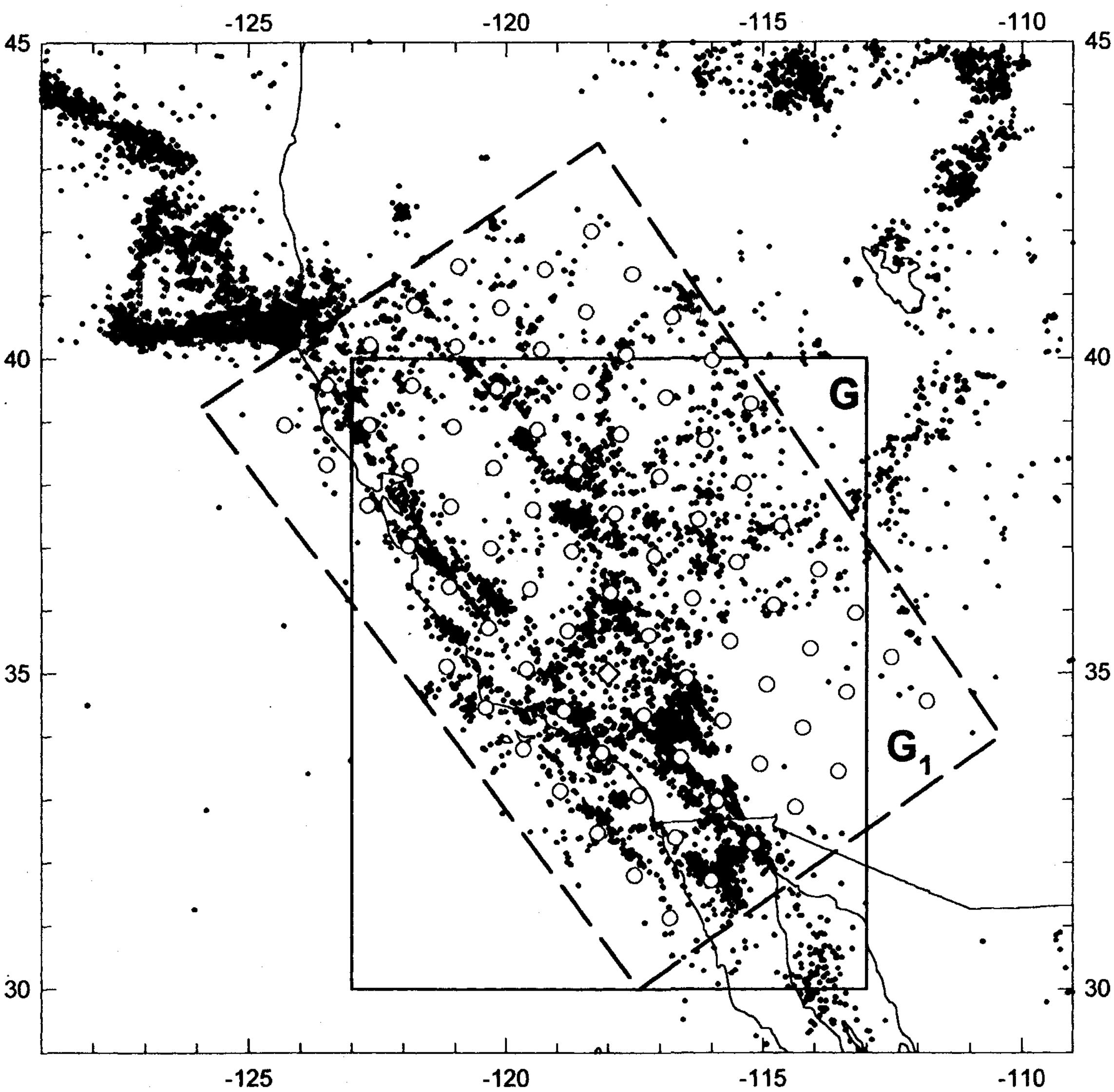
$$1/c_p = \sum[\lambda(L\times L)]^p = R_L(p)/[\lambda(G)]^p$$

$p=1$ - the Bak's case

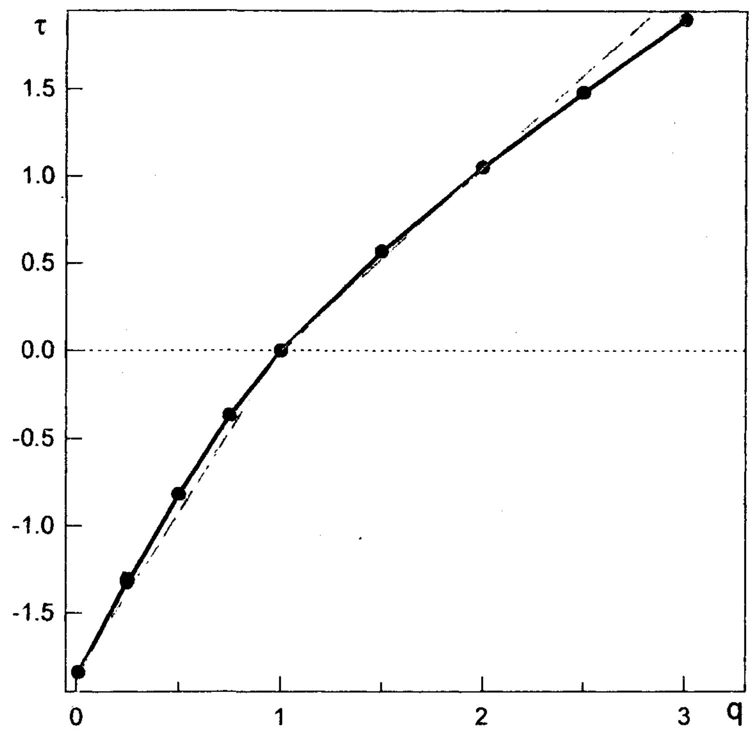
$p=0$ - the case of generalized G-R law

$$\langle\lambda(L\times L)\rangle = \frac{1}{n_L}\sum_{\lambda>0}\lambda(L\times L)$$

and of distribution of $\{\lambda(L\times L)\}$



California: seismic events with M≥3 (·); grid of spacing L=100 km (°) centered at (rombus); the main (G) and alternative (G_1) rectangular seismic region for dimension computations



Tau-function for M≥2 events in region G; it is based on the interval of scales ΔL=(10, 100) km. The broken straight line of two segments is τ(q) for a bi-fractal with the observed dimensions d_0 and d_2.

$d_0 = 1.8 \pm 0.1$; $d_1 = 1.35 \pm .05$ $d_2 = 1.1 \pm .05$

Evaluation of $\tau(q)$

$$\log R_L(q) \simeq \tau(q) \log L$$

Problem: find the range of scale (L_*, L^*) with stable slope in $(\log R_L(q), \log L)$

Idea: • isolated points in support of $\lambda(dg)$ do not affect its multifractal spectrum

$\Downarrow$

use cell $L \times L$ with $\# \Sigma Q_s > K = 0, 1, 2 ..$

$\Downarrow$

$L > L_*$

• $n_L \gtrsim 100 \Rightarrow L < L^*$

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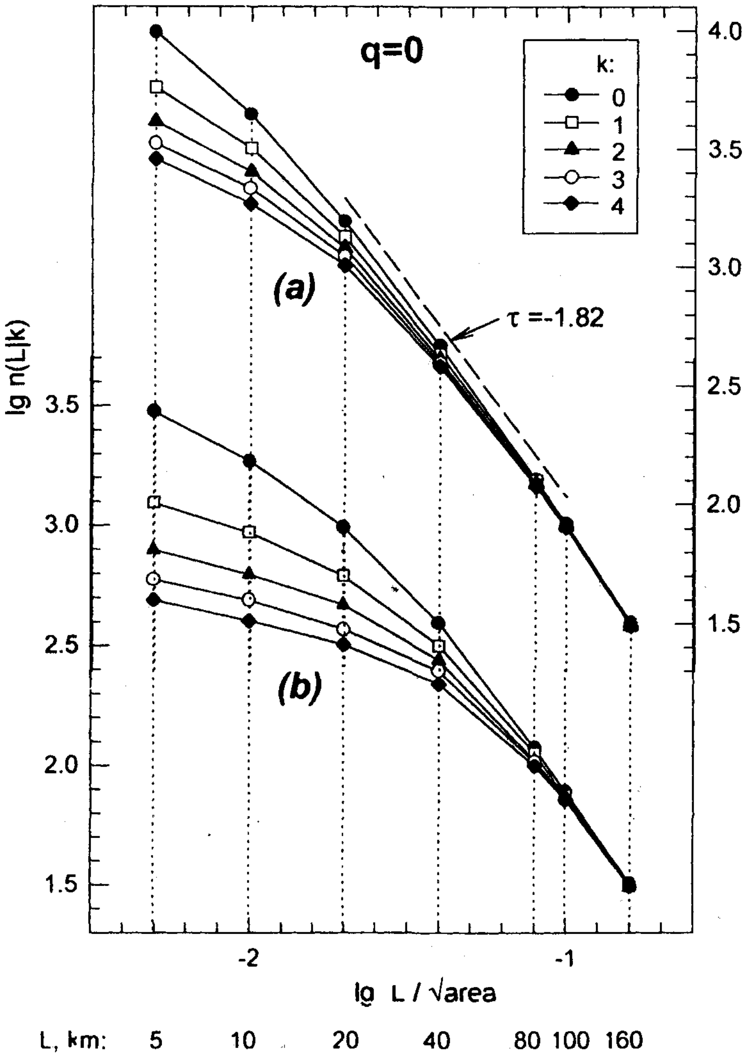
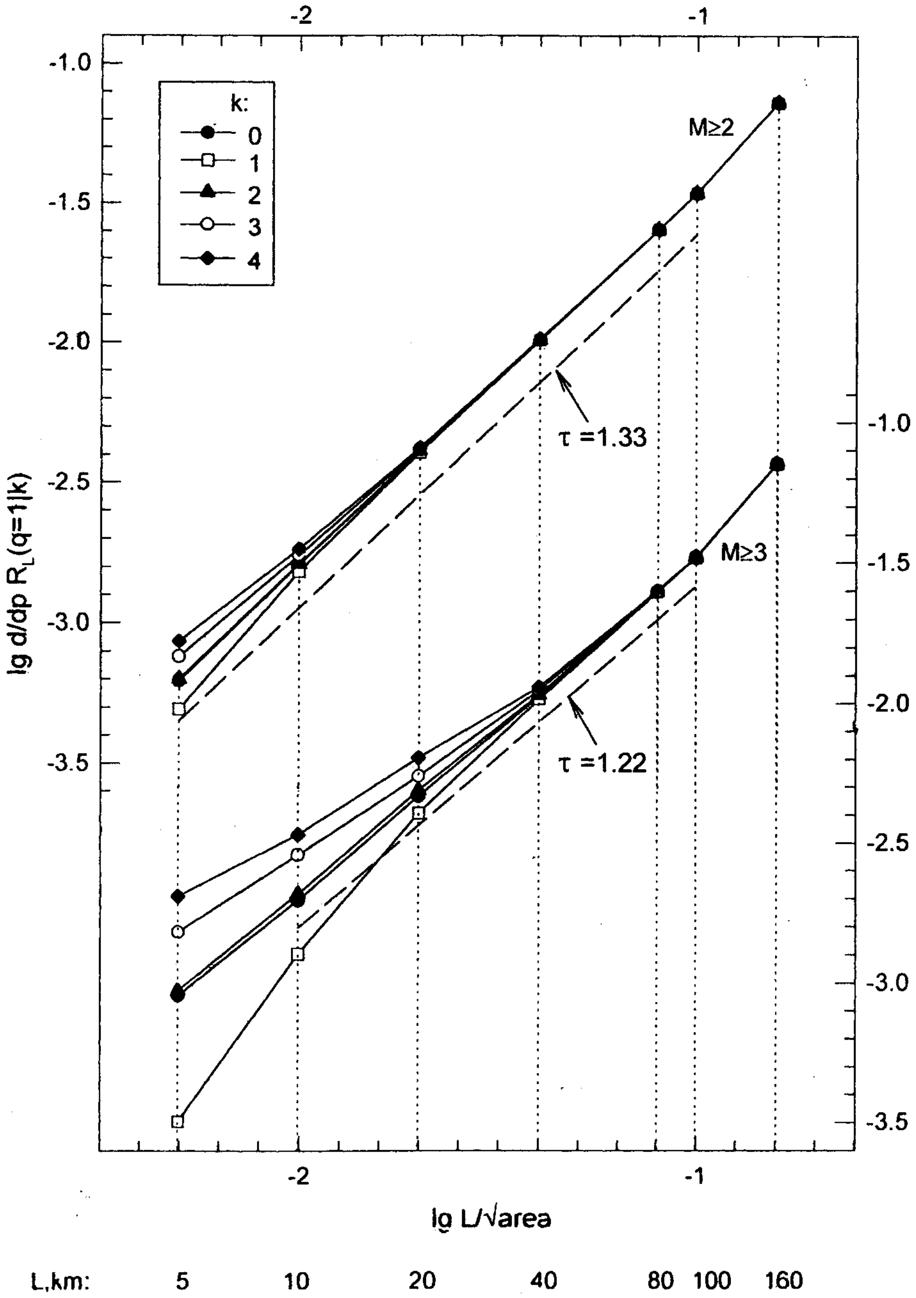


Data for estimating the box dimension $d_0=-\tau(0)$ based on $M \geq 2$ events (*a*) and $M \geq 3$ events (*b*).

The vertical axis shows the number of L×L cells, n(L|k), that have numbers of events greater than k, k = 0, 1, 2, 3, 4. The dashed line shows both the slope $\tau(0)$ and the interval of scales ΔL for estimation of d_0 by least squares using n(L|0).



Data for estimating the information dimension d_1 from M≥2 (*a*) and M≥3 (*b*) events

The vertical axis shows the derivatives d/dq $R_L(q=1|k)$ for k=0, 1, 2, 3, 4 where R_L is the modified Renyi function.

The dashed line shows both the slope d_1 and the interval of scales ΔL for estimation of slope by least squeres.

Weighed means of $\lambda(L\times L)$ and $\tau(L\times L)$: Scaling

$$W^{(p)}(L\times L) = [\lambda(L\times L)]^p \cdot c_p$$ – weigh of cell $L\times L$

1° $$\langle \lambda(L\times L)\rangle_p = \sum_{\lambda>0} \lambda(L\times L)\cdot W^{(p)}(L\times L) \sim L^d$$

$$d = \tau(p+1) - \tau(p) = \begin{cases} d_0 & p=0 \\ d_2 & p=1 \end{cases}$$

California: 1.8 ($p=0$), 1.1 ($p=1$)

2° $$\langle \tau(L\times L)\rangle_p = \sum \lambda^{-1}(L\times L)\, W^{(p)}(L\times L) \sim L^{\tilde d}$$

$$\tilde d = |\tau(p-1) - \tau(p)| = \begin{cases} |\tau(-1)|-d_0 & p=0 \quad \geq 2 \\ d_0 & p=1 \quad 1.8 \end{cases}$$

3° $$d[\langle\lambda\rangle_p] \leq \dot\tau(p+0) \leq \dot\tau(p-0) \leq d[\langle\tau_L\rangle_p]$$

Hint

$$\langle \lambda^{\varepsilon}(L\times L)\rangle_p = \lambda^{\varepsilon}(G)\, R_L(p+\varepsilon)/R_L(p) \sim L^{\tau(p+\varepsilon)-\tau(p)}$$

Optimal spatial scaling of $\tau(L\times L)$ and $\lambda(L\times L)$

Statistics $\bar{\tau}_L$: time between consecutive EQs of $M>m_c$ in w_p-random cell $L\times L$

$$\xi_L = L^d \bar{\tau}_L \quad , d-?$$

Statistics λ_L: rate of w_p-random cell $L\times L$

$$\eta_L = L^{-d} \lambda_L \quad , d-?$$

|| d is inadmissible if
|| η_{L_i}, $L_i \to 0$ can have limit (in distribution)
|| of type $\dot{0}$ or ∞

Problem: find all admissible exponents d

<u>Result</u> All admissible exponents in spatial scaling of τ_L and λ_L related to w_p-random cell $L \times L$ are given by unique interval

$$[\dot{\tau}(p+0), \dot{\tau}(p-0)]$$

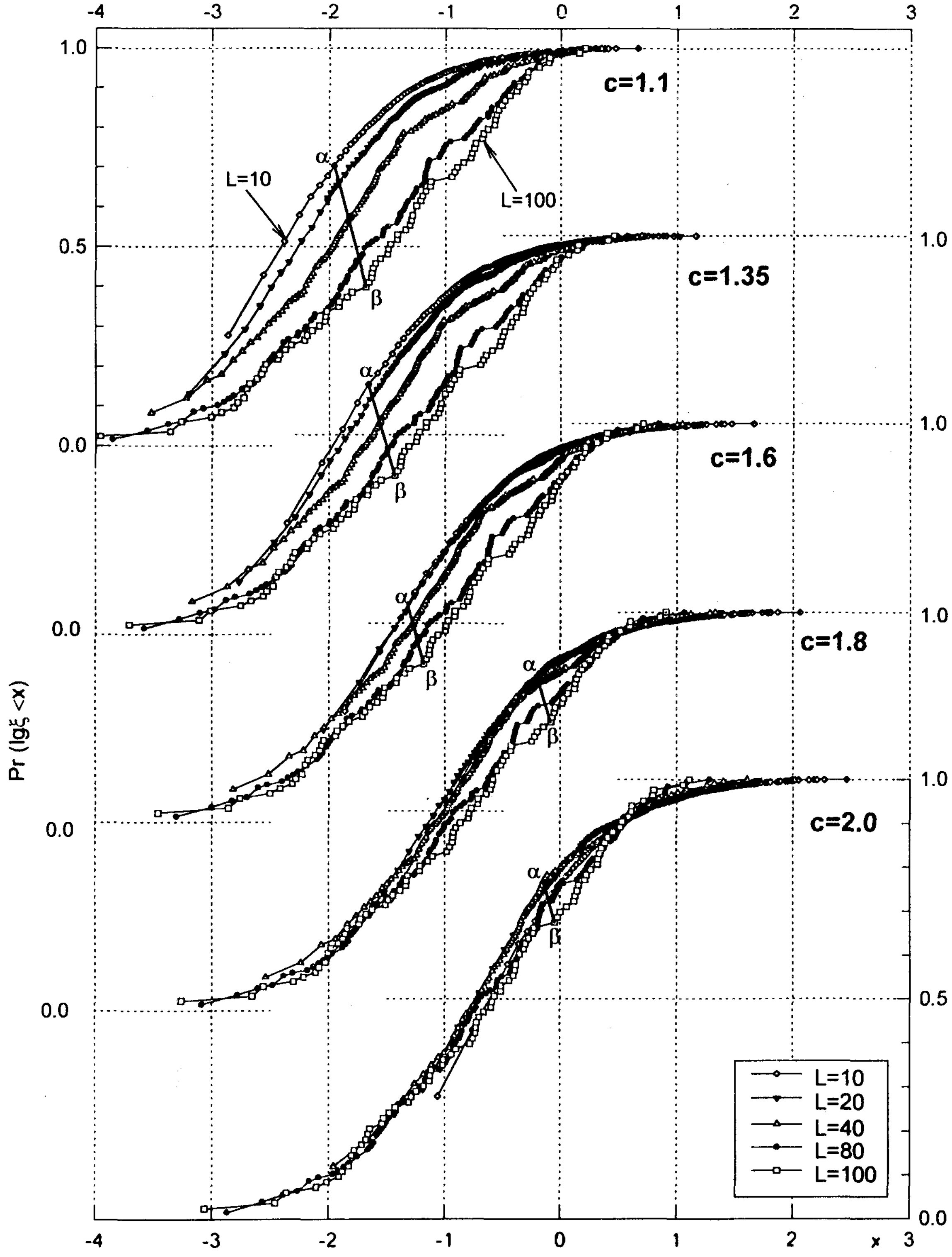
i.e. $d = \dot{\tau}(p)$ if $\dot{\tau}$ exist

$$d = \begin{cases} \dot{\tau}(0) & p=0 \\ d_1 & p=1 \\ \dot{\tau}(2) & p=2 \end{cases}$$

	California
$p=0$	2
$p=1$	1.35
$p=2$	

2-16 Scaling of λ_L, p=0

predicted d=2

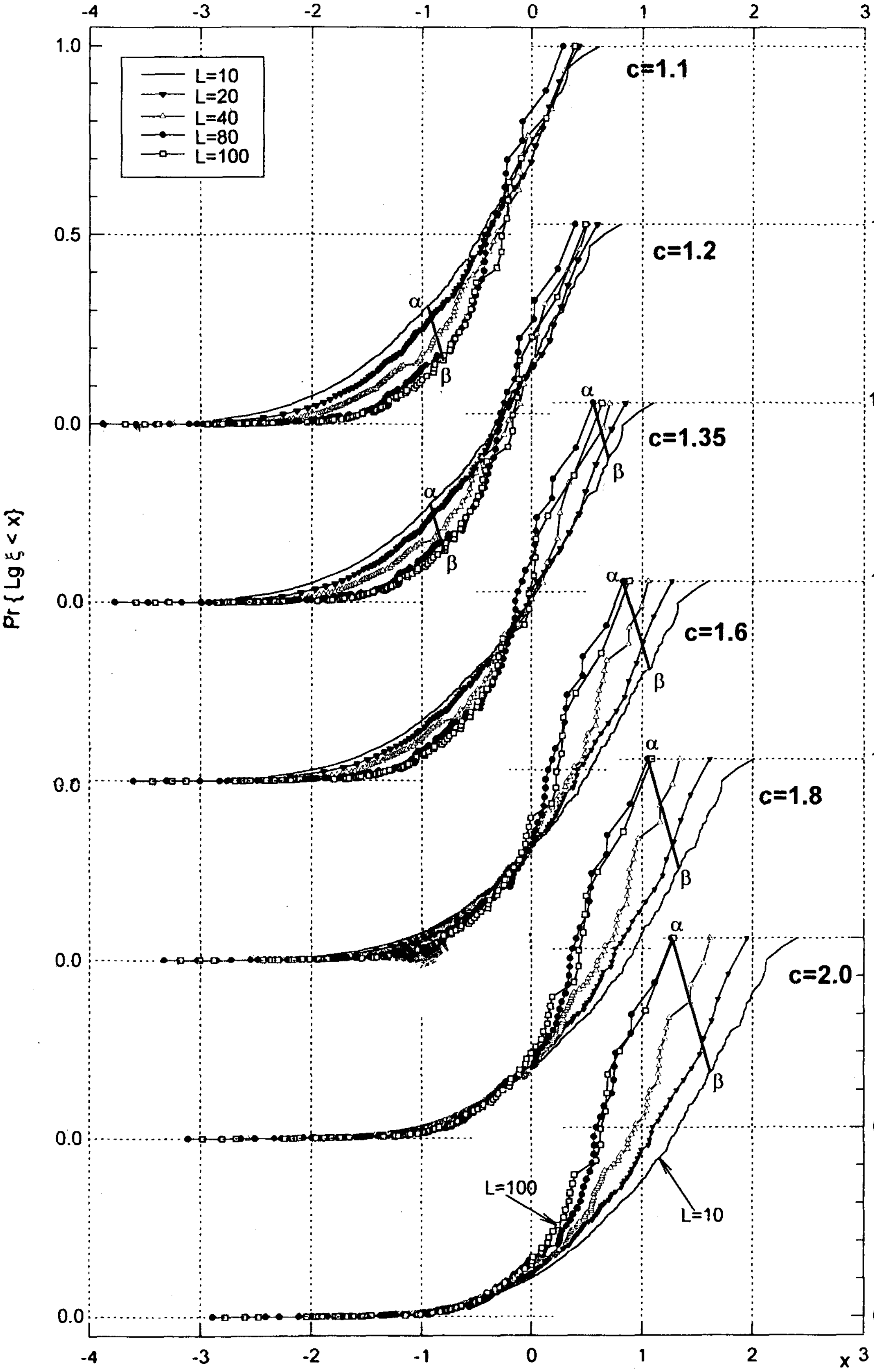


Distribution functions for lg ξ_L corresponding to the scales L=10, 20, 40, 80 and 100 km and to the scaling index c = 1.1, 1.35, 1.6, 1.8 and 2.0.

The segment (α, β) has the slope (-1), its length provides information on the scatter of the distributions of lg ξ_L at a fixed c.

The case p=0: each L×L cell enters in the distribution with the same weight

Scaling of λ_L, p=1
predicted d = 1.35



Distribution functions for lg ξ_L corresponding to the scales L=10,20,40,80 and 100 km and to the scaling index c=1.1,1.2,1.35,1.6,1.8 and 2.0. The segment (α,β) has the slope (-1), its length provides information on the scatter of the distributions of lg ξ_L at a fixed c. The case p=1: each L×L cells enters in the distribution with a weight proportional to the seismicity rate in the cell.

Conclusion

1. Unified scaling laws by Corral and by Bak et al can't exist in strict sence

2. We find a suitable exponents d in the relations

$$\tau(L \times L) \propto L^d \quad \lambda(L \times L) \propto L^d$$

assuming multifractality of rate $\lambda(dg)$;
d is not unique and depends of a problem, i.e. d is a parameter in appications

3. We confirm the multifractal property of rate in the range of scale : 10-100 km in California
We do it twice: estimating $\tau(q)$ and using prediction of exponents in scaling of τ_L and λ_L