Polarized Neutrons

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Topics

- Need for polarized neutrons
- Production of polarized neutrons
- Implementation of polarization analysis
- Magnetic scattering cross section: EuS
- Rules for polarization analysis
- Examples: coherent/incoherent: NbD background suppression: Fe separation of modes: EuS, Cr chirality: MnSi, trilayers
- Summary

Need for Polarized Neutrons 1

• nuclear interaction:

$$\frac{2\pi\hbar^2}{m}b\delta(\mathbf{r}-\mathbf{r}_j)$$

isotopic incoherence \Leftrightarrow spin incoherence

Need for Polarized Neutrons 1

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$$\frac{2\pi\hbar^2}{m}b\delta(\mathbf{r}-\mathbf{r}_j)$$

isotopic incoherence \Leftrightarrow spin incoherence Ni $(I \cong 0)$ $\overline{b} = \sum_{i} f_{i} b_{i}$ $\rightarrow \sigma_{coh} = 4\pi (\overline{b})^{2}$

does not depend on spin

Need for Polarized Neutrons 1



Need for Polarized Neutrons 1



Possibility to distinguish between coherent and incoherent scattering

Need for Polarized Neutrons 2

• magnetic moments: $I_{Bragg} \propto b^2 + \frac{2}{3}p^2$

Ni: $b = 1.03 \cdot 10^{-12} \text{ cm}$ $p = 0.164 \cdot 10^{-12} \text{ cm} \rightarrow 1.7\% \text{ effect!}$

Need for Polarized Neutrons 2

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$$I^{++} \propto (b+p)^2$$

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$$\frac{I^{++}}{I^{--}} = \frac{(b+p)^2}{(b-p)^2} = \frac{b^2 + 2bp + p^2}{b^2 - 2bp + p^2} \cong \frac{1 + 2p/b}{1 - 2p/b} \cong 1 + 4\frac{p}{b} \cong 1.64$$

Useful Application

• polarizer with *b* = *p*:

$$\frac{I^{++}}{I^{--}} = \frac{(b+p)^2}{(b-p)^2} = \frac{4b^2}{0} = \infty$$

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Possibility to produce polarized beams: Heusler alloy Cu₂MnAl



Spectrometer TASP at SINQ (PSI)

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• inelastic neutron scattering from Ni: what is what?



Need for Polarized Neutrons 3

• inelastic neutron scattering from Ni: what is what?



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Need for Polarized Neutrons 3

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What is available?

- Polarized triple axis spectrometers at several reactors (ILL, HMI, NIST, FRM-II, ..) and SINQ (spallation source!)
- TOF at the ILL (D7)
- Neutron Spin Echo: ILL, Jülich, HMI, NIST, LLB, FRM-II
- pulsed sources: OSIRIS at ISIS
- planned: HYSPEC at SNS



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How Can Neutrons Be Polarized

• diffraction from Heusler alloys: already discussed

$$\frac{I^{++}}{I^{--}} = \frac{(b+p)^2}{(b-p)^2} = \frac{4b^2}{0} = \infty$$

- cold and thermal neutrons
- maintenance free
- monochromatic beams!
- however: difficult to grow

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- however: difficult to grow
- reflection from magnetized multilayers: supermirrors (F. Mezei)
- polarized ³He
- polarized protons
- Fe-filters, Dy etc.

• diffraction from multilayers: Bragg's law

 $\lambda = 4 \text{ Å} \rightarrow \theta_{c} = 0.4^{0}$



choose:
$$G_{-} = G_{nm}$$

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 $G_{nm} = n_{nm} b_{nm}$

• non-magnetic layer:

choose: $G_{-} = G_{nm}$

• multilayer FeCoV/TiN_x: $G_{-} = G_{nm} \cong 0$

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• multilayer FeCoV/TiN_x: $G_{-} = G_{nm} \cong 0$



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Idea: F. Mezei

Slow variation of the thickness of the bilayers.



q₀

q₀

 $\int d$

0.8

1.0

1.2

12

1.4

- cold and thermal neutrons
- maintenance free
- not so good for area detectors



³He-Polarizers

• absorption is strongly spin dependent





- homogeneous: SANS
- no change of phase space
- "*E*-independent": TOF
- complicated technique

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³He-Polarizers

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• homogeneous: SANS

 $\sigma \cong 0$

 σ large

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Polarized Proton Target

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• scattering is strongly spin dependent



scattering cross section σ large

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Polarized Proton Target

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Implementation: Primary Spectrometer

• transport of neutron by means of neutron guides



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Implementation: Primary Spectrometer



Implementation straight forward!

Loss: typically factor of two due to polarization
→ most powerful thermal beam at FRM-II


Spallation Neutron Source in Oak Ridge (USA)

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New concepts:

- translation of guides (POWGEN at SNS, TRISP at FRM-II)
- Elliptic guides (not yet implemented as polarizer)

Implementation: Secondary Spectrometer 1

- monochromatic energy band
- low high energy
- single detector- area detector (dimension of detector!)
- trade-off of polarization vs. intensity

Implementation: Secondary Spectrometer 1

- monochromatic energy band
- low high energy
- single detector- area detector (dimension of detector!)
- trade-off of polarization vs. intensity
- space
- price
- maintenance
- personal opinions (my opinion: the simpler the better)

Implementation: Secondary Spectrometer 2



area detector



Implementation: Secondary Spectrometer 2



Implementation: Secondary Spectrometer 3



Magnetic Moment of Neutrons

- nuclear interaction:
 - form factor = 1: large **Q** accessible
 - coherent incoherent scattering
 - no charge \rightarrow large penetration depth



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Properties of Neutrons



Sample environment is essential (not only the neutrons)!

Scattering Cross Section

(hear W. Fischer's talk!)

• Fermi's golden rule (= 1st Born approximation):

$$\frac{d^2\sigma}{d\Omega dE_f} = \left(\frac{m}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i} \sum_{\lambda_i} p_{\lambda_i} \sum_{\lambda_f} \left| \left\langle k_f, \lambda_f \right| \mathbf{\overline{U}} \right| k_i, \lambda_i \right\rangle \right|^2 \delta \left(E_{\lambda_i} - E_{\lambda_f} + \hbar \omega \right)$$

- $|\lambda_i\rangle$: initial state of sample
- $p_{\lambda i}$: probability that initial state is occupied

• $|\lambda_f\rangle$: final state of sample

• U: interaction potential neutron-sample

$$\frac{2\pi\hbar^2}{m}b\delta(\mathbf{r}-\mathbf{r}_j)$$

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Message: \rightarrow scattering given by Fourier transform of potential

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- $|\lambda_i\rangle$: initial state of sample
- $p_{\lambda i}$: probability that initial state is occupied

• $|\lambda_f\rangle$: final state of sample

- U: interaction potential neutron-sample
- matrix element for nuclear scattering:

$$\frac{2\pi\hbar^2}{m}b\delta(\mathbf{r}-\mathbf{r}_j)$$

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$$\left\langle \mathbf{k}_{f}, \boldsymbol{\lambda}_{f} \middle| \mathbf{\overline{U}} \middle| \mathbf{k}_{i}, \boldsymbol{\lambda}_{i} \right\rangle = \left\langle f \middle| \mathbf{\overline{U}} \middle| i \right\rangle = \left\langle \boldsymbol{\lambda}_{f} \middle| \int \sum_{j} e^{-i\mathbf{k}_{f}\mathbf{r}} V_{j} (\mathbf{r} - \mathbf{r}_{j}) e^{i\mathbf{k}_{i}\mathbf{r}} dr \middle| \boldsymbol{\lambda}_{i} \right\rangle$$

Message: \rightarrow scattering given by Fourier transform of potential

Magnetic Interaction

• magnetic interaction operator:

$$\vec{\mathbf{U}}_{m} = -\mathbf{\mu} \cdot \mathbf{B} = -\gamma \mu_{N} \boldsymbol{\sigma} \cdot \mathbf{B}$$

$$\gamma = -1.913: \qquad \text{gyromagnetic ratio} \\ \mu_{N}: \qquad \text{nuclear magneton} \\ \mu: \qquad \text{magnetic moment of neutron} \\ \boldsymbol{\sigma}: \qquad \text{Pauli spin operator} \qquad \text{spin}$$

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μ_N :	nuclear magneton
μ:	magnetic moment of neutron
σ:	Pauli spin operator
	`spin

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• field of unpaired electron at position \mathbf{r}_i :

$$\mathbf{B}_{j} = \nabla \times \left\{ \frac{|\mathbf{\mu}_{e} \times \mathbf{r}_{j}|}{|\mathbf{r}_{j}|^{3}} \right\} + \frac{(-e)}{c} \frac{\mathbf{v}_{e} \times \mathbf{r}_{j}}{|\mathbf{r}_{j}|^{3}}$$

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Magnetic Scattering Length

• Fourier transform yields magnetic scattering length:

$$p = -\gamma r_0 \boldsymbol{\sigma} \cdot \left(\hat{\mathbf{Q}} \times (\mathbf{S} \times \hat{\mathbf{Q}}) + \frac{i}{\hbar |\mathbf{Q}|} (\mathbf{p}_e \times \hat{\mathbf{Q}}) \right) = -\gamma r_0 \frac{g}{2} \boldsymbol{\sigma} \cdot (\hat{\mathbf{Q}} \times (\mathbf{S} \times \hat{\mathbf{Q}}))$$
$$r_0 = 0.2818 \cdot 10^{-12} \text{ cm: classical radius of electron}$$
$$\hat{\mathbf{Q}} = \mathbf{Q} / |\mathbf{Q}|$$

• important: *p* depends on vector quantities *p* is comparable to *b*

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$$\hat{\mathbf{Q}} = \mathbf{Q} / |\mathbf{Q}|$$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

• important: *p* depends on vector quantities *p* is comparable to *b*

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Magnetic Scattering Function

• selection rule for magnetic scattering:

$$\left(\frac{d\sigma}{d\Omega dE_{f}}\right)_{mag} = \frac{k_{f}}{k_{i}} \left(\gamma r_{0} \frac{g}{2} F(\mathbf{Q})\right)^{2} \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \hat{Q}_{\alpha} \hat{Q}_{\beta}\right) \left(S_{\alpha\beta}(\mathbf{Q}, \omega)\right)$$

$$S_{X} = S_{X} = S_{$$

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Magnetic Scattering Function

• magnetic scattering function (phonons neglected!):

$$S^{\alpha\beta}(\mathbf{Q},\omega) = \frac{1}{2\pi\hbar} \int \sum_{jj'} \left\langle S_{j'\alpha}(0) S_{j\beta}(t) \right\rangle e^{i\mathbf{Q}\cdot(\mathbf{r}_j - \mathbf{r}_{j'})} e^{-i\omega t} dt$$

(hear W. Fischer's talk!)

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 $S^{\alpha\beta}(\mathbf{Q},\omega)$ corresponds to the **Fourier transform** of the magnetic pair correlation function that gives the probability to find a magnetic moment at position \mathbf{r}_j at time *t* with a spin component $S_{j\beta}(t)$ and the same or another moment at position $\mathbf{r}_{j'} = 0$ at time t = 0 with a component $S_{j'\alpha}(0)$.

(hear W. Fischer's talk!)

• from the fluctuation-dissipation theorem (\rightarrow W. Fischer's talk):

$$S^{\alpha\beta}\left(\mathbf{Q},\omega\right) = \frac{\hbar}{\pi} \frac{1}{1 - e^{-\hbar\omega/k_{B}T}} \Im\chi^{\alpha\beta}\left(\mathbf{Q},\omega\right)$$

imaginary part

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Interpretation: The magnetic moment of the neutron acts on the sample like a frequency and wavevector dependent magnetic field $B(Q,\omega)$.

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$$H(r,t) \uparrow M_{\alpha}(\mathbf{Q},\omega) = \sum_{\beta} \chi^{\alpha\beta}(\mathbf{Q},\omega) \mathbf{B}_{\beta}(\mathbf{Q},\omega)$$

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• energy and momentum conservation:

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Spin Waves

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Local Moment System

$$\mathbf{H} = -\sum_{jj'} J_{jj'} \mathbf{S}_{j} \cdot \mathbf{S}_{j'}$$

• linear spin wave theory: $\hbar \omega_q = 2S(J(0) - J(\mathbf{q}))$
where: $J(\mathbf{q}) = \sum_{jj'} J_{jj'} e^{i\mathbf{q} \cdot (\mathbf{r}_j - \mathbf{r}_{j'})}$

small q

 $E_a = 4JS(1 - \cos qa)$

 $E_{q} = Dq^{2}$ Fourier transform (compare with diffraction!)

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• linear spin wave theory: $\hbar \omega_a = 2S(J(0) - J(\mathbf{q}))$

$$: \quad J(\mathbf{q}) = \sum_{jj'} J_{jj'} e^{i\mathbf{q}\cdot(\mathbf{r}_j - \mathbf{r}_{j'})}$$

• most simple case: $J_{ij} = J$ (see Kittel)

 $E_q = 4JS(1 - \cos qa)$ \longrightarrow $E_q = Dq^2$ (compare with diffraction!)

Fourier

transform

Visualization: Spin Waves

• Quantum mechanical picture:

coherent superposition of spin flips \rightarrow spin wave

Application: Spin Waves

• Classical picture:

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• Why do neutrons see spin waves?

What can we learn?

- exchange interactions: information on electronic structure
- phase transitions: critical exponents, universality

Selection rules help to distinguish spin waves from phonons.

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What can we learn?

- exchange interactions: information on electronic structure
- phase transitions: critical exponents, universality
- additional terms in interaction: anisotropies (xy-like, Ising)

Selection rules help to distinguish spin waves from phonons.





What did we really measure?



• sum of transverse and longitudinal fluctuations

"do not exist" in normal text books (extremely important in itinerant antiferromagnets!)

• distinction only possible by means of **polarized neutrons!**



What did we really measure?



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• distinction only possible by means of **polarized neutrons!**

• nuclear scattering

$$I(Q) \propto \left(\sum_{i} b_{i} e^{i\mathbf{Q}\cdot\mathbf{r}_{i}}\right)^{2}$$

sum amplitudes first!

• different isotopes:

$$b_i = \overline{b} \pm \Delta b_i$$



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$$b_{i} = \overline{b} \pm \Delta b_{i} \qquad \qquad \overline{b} = \sum_{i} f_{i} b_{i}$$
$$\sum_{i} \Delta b_{i} = 0$$

coherent scattering: waves from
different sites interfere with each other
→ correlations between nuclei
(diffraction, phonons, magnons etc.)



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incoherent scattering: only waves from same site interfere with each other \rightarrow self correlations (diffusion processes)



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Polarization analysis: Magnetic Scattering

• magnetic scattering:
$$p = -\gamma r_0 \frac{g}{2} \boldsymbol{\sigma} \cdot \left(\hat{\mathbf{Q}} \times (\mathbf{S} \times \hat{\mathbf{Q}}) \right) \propto (\boldsymbol{\sigma} \cdot \mathbf{M}_{\perp})$$

→ true magnetic interaction! → \mathbf{M}_{\perp} depends on \mathbf{Q}

\rightarrow is **not** a magnetic interaction!

 \rightarrow spin-incoherence can be detected with polarized neutrons!

Polarization analysis: Magnetic Scattering

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→ true magnetic interaction! → \mathbf{M}_{\perp} depends on \mathbf{Q}

• spin incoherent scattering: depends on polarization of neutrons

 \rightarrow is **not** a magnetic interaction!

 \rightarrow spin-incoherence can be detected with polarized neutrons!

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Pauli Spin Matrices 1

• Let us play with the matrices: $\mathbf{\sigma} \cdot \mathbf{A} = \sigma_x A_x + \sigma_y A_y + \sigma_z A_z$

$$\sigma_{x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_{y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_{z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} \sigma_{x}|\uparrow\rangle = |\downarrow\rangle & \sigma_{x}|\downarrow\rangle = |\uparrow\rangle \\ \sigma_{y}|\uparrow\rangle = i|\downarrow\rangle & \sigma_{y}|\downarrow\rangle = -i|\uparrow\rangle \\ \sigma_{z}|\uparrow\rangle = |\uparrow\rangle & \sigma_{z}|\downarrow\rangle = -i|\downarrow\rangle \end{aligned}$$

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$$\sigma_{y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_{z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

 σ_z is already diagonal with eigenfunctions:

$$\left|\uparrow\right\rangle = \left|\begin{matrix}1\\0\end{matrix}\right\rangle \quad \text{and} \quad \left|\downarrow\right\rangle = \left|\begin{matrix}0\\1\end{matrix}\right\rangle$$

$$\begin{split} \sigma_{x}|\uparrow\rangle = |\downarrow\rangle & \sigma_{x}|\downarrow\rangle = |\uparrow\rangle \\ \sigma_{y}|\uparrow\rangle = i|\downarrow\rangle & \sigma_{y}|\downarrow\rangle = -i|\uparrow\rangle \\ \sigma_{z}|\uparrow\rangle = |\uparrow\rangle & \sigma_{z}|\downarrow\rangle = -i|\downarrow\rangle \end{split}$$

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Pauli Spin Matrices 2

$$p \propto \mathbf{\sigma} \cdot \mathbf{M}_{\perp} = \sigma_{x} M_{\perp,x} + \sigma_{y} M_{\perp,y} + \sigma_{z} M_{\perp,z}$$

only spin-flip scattering possible

$$\begin{aligned} \sigma_{x}|\uparrow\rangle = |\downarrow\rangle \quad \sigma_{x}|\downarrow\rangle = |\uparrow\rangle \\ \sigma_{y}|\uparrow\rangle = i|\downarrow\rangle \quad \sigma_{y}|\downarrow\rangle = -i|\uparrow\rangle \quad \langle\uparrow|\sigma_{x}|\uparrow\rangle = \langle\uparrow|\downarrow\rangle = 0 \quad \langle\downarrow|\sigma_{x}|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0 \\ \langle\uparrow|\sigma_{y}|\uparrow\rangle = i\langle\uparrow|\downarrow\rangle = 0 \quad \langle\downarrow|\sigma_{y}|\downarrow\rangle = -i\langle\downarrow|\uparrow\rangle = 0 \\ \rangle \quad \langle\downarrow|\sigma_{z}|\uparrow\rangle = \langle\downarrow|\uparrow\rangle = 0 \quad \langle\downarrow|\sigma_{z}|\uparrow\rangle = -\langle\downarrow|\uparrow\rangle = 0 \end{aligned}$$

only non-spin-flip scattering possible for $M_{\perp,z}$ same is true for incoherent scattering: A^2+BI_z



Pauli Spin Matrices 2

$$p \propto \mathbf{\sigma} \cdot \mathbf{M}_{\perp} = \sigma_{x} M_{\perp,x} + \sigma_{y} M_{\perp,y} + \sigma_{z} M_{\perp,z}$$

only spin-flip scattering possible



only non-spin-flip scattering possible for $M_{\perp,z}$ same is true for incoherent scattering: A^2+BI_z

Selection Rules for Polarization Analysis

Note: The polarization of the neutron defines the *z*-axis!

$$\left\langle \uparrow \left| \boldsymbol{\sigma} \cdot \breve{\mathbf{M}}_{\perp} \right| \uparrow \right\rangle = \breve{M}_{\perp,z} \qquad \qquad \overbrace{\mathbf{A}} = \mathbf{M}_{\perp,z} \qquad \qquad$$

Selection Rules for Polarization Analysis

Note: The polarization of the neutron defines the *z*-axis!

$$\left\langle \uparrow \left| \boldsymbol{\sigma} \cdot \breve{\mathbf{M}}_{\perp} \right| \uparrow \right\rangle = \breve{M}_{\perp,z}$$

$$\left\langle \downarrow \left| \boldsymbol{\sigma} \cdot \breve{\mathbf{M}}_{\perp} \right| \downarrow \right\rangle = -\breve{M}_{\perp,z}$$

$$\left\langle \uparrow \left| \boldsymbol{\sigma} \cdot \breve{\mathbf{M}}_{\perp} \right| \downarrow \right\rangle = \breve{M}_{\perp,x} + i\breve{M}_{\perp,y} = \breve{M}^{+}$$

$$\left\langle \uparrow \left| \boldsymbol{\sigma} \cdot \breve{\mathbf{M}}_{\perp} \right| \downarrow \right\rangle = \breve{M}_{\perp,x} - i\breve{M}_{\perp,y} = \breve{M}^{-}$$

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$$\left\langle \uparrow \left| \boldsymbol{\sigma} \cdot \mathbf{M}_{\perp} \right| \downarrow \right\rangle = \mathbf{M}_{\perp,x} + i\,\mathbf{M}_{\perp,y} =$$

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• nuclear scattering (excluding nuclear spin incoherence):

no Pauli spin matrices involved

 \rightarrow all scattering is non-spin-flip



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• (room) **background**:

 \rightarrow contributes to all scattering channels

(special case: isotropic ferromagnet!)

• **paramagnetic** scattering in a vertical field: $\mathbf{Q} \perp \mathbf{B}$



(special case: isotropic ferromagnet!)

• **paramagnetic** scattering in a vertical field: $\mathbf{Q} \perp \mathbf{B}$



(special case: isotropic ferromagnet!)

• **paramagnetic** scattering in a vertical field: $\mathbf{Q} \perp \mathbf{B}$



• **spin incoherent** scattering:

discussed before: nuclear scattering



• at reasonable temperatures: $\langle I_x^2 \rangle = \langle I_y^2 \rangle = \langle I_z^2 \rangle = \frac{1}{3}I(I+1)$

• contribution of spin-incoherent:

$$I_{NSI}^{nsf} = \frac{1}{3}\sigma_{NSI} \qquad I_{NSI}^{sf} = \frac{2}{3}\sigma_{NSI}$$

	non-spin-flip	spin-flip
Q // B	$\sigma_{N} + 0\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	
$\mathbf{Q} \perp \mathbf{B}$	$\sigma_{N} + \frac{1}{2}\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	

Measurement of all cross sections allows the determination of individual scattering contributions.

$$\frac{1}{2}\sigma_{mag} = I_{Q\perp B}^{nsf} - I_{Q//B}^{nsf}$$

$$\frac{1}{2}\sigma_{mag} = I_{Q/B}^{sf} - I_{Q\perp 4B}^{sf}$$

	non-spin-flip	spin-flip
Q // B	$\sigma_{N} + 0\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	$\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg}$
$\mathbf{Q} \perp \mathbf{B}$	$\sigma_{N} + \frac{1}{2}\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	$\frac{1}{2}\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg}$

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Q // B	$\sigma_{N} + 0\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	$\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg}$
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Measurement of all cross sections allows the determination of individual scattering contributions.

Example: Paramagnetic scattering from ferromagnetic material:

$$\frac{1}{2}\sigma_{mag} = I_{Q\perp B}^{nsf} - I_{Q/B}^{nsf}$$

$$\frac{1}{2}\sigma_{mag} = I_{Q/B}^{sf} - I_{Q\perp AB}^{sf}$$

	non-spin-flip	spin-flip
Q // B	$\sigma_{N} + 0\sigma_{m} + \frac{1}{3}\sigma_{NSI} + \sigma_{bg}$	$\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg}$
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•e21

Polarized Triple-Axis Spectrometer

• **controlled** access to predetermined points in reciprocal space



Example 1: Deuterium in Nb

- investigation of the interstitial diffusion process
- interesting for superionic conductors (like AgI)
- deuterium: $\sigma_{coh} = 5.6$ barns, $\sigma_{inc} = 2.0$ barns

• spectrometer: D7 at Institute Laue-Langevin, $E_i = 3.52 \text{ meV}$ (TOF at a continuous source!)

Example 1: Deuterium in Nb

- investigation of the interstitial diffusion process
- interesting for superionic conductors (like AgI)
- deuterium: $\sigma_{coh} = 5.6$ barns, $\sigma_{inc} = 2.0$ barns
- 1st approach: theoretical separation, high quality data necessary
- 2nd approach: polarization analysis
- spectrometer: D7 at Institute Laue-Langevin, $E_i = 3.52 \text{ meV}$ (TOF at a continuous source!)

Deuterium in Nb

J. C. Cook et al., J. Phys. Condens. Matter 2, 79 (1990).



Deuterium in Nb



- separation successful
- good agreement with theory
- incoherent-coherent residence times: $\tau_{coh} = 0.49 \tau_{inc}$

Example 2: Paramagnetic Scattering in Fe

• Fe in ferromagnetic phase



$$\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg} - (\frac{1}{2}\sigma_m + \frac{2}{3}\sigma_{NSI} + \sigma_{bg})$$

Example 2: Paramagnetic Scattering in Fe

(b) o İ പ്പം

300

 $T_{c} + 22 K$

(1.10, 1.10, 0)

ON

• Fe in ferromagnetic phase



Paramagnetic Scattering in Fe

 \rightarrow no spin waves above T_C

→ time evolution of magnetic fluctuations can be measured:

 $\rightarrow \Gamma = Aq^{2.5}$

→ behavior similar as a simple Heisenberg ferromagnet

constant-*E* scan

Overview with TOF at a pulsed source?
Paramagnetic Scattering in Fe

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Overview with TOF at a pulsed source?



So far:

- Incoherent coherent scattering
- Supression of nuclear scattering

Next:

• distinction between magnetic modes

Example 3: Longitudinal Fluctuations in EuS



Example 3: Longitudinal Fluctuations in EuS



Example 3: Longitudinal Fluctuations in EuS



Polarized Neutrons Necessary?



Polarization analysis important for measurements near T_C

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Example 4: Cr, an Itinerant Antiferromagnet



Very simple material?

complicated topology of Fermi surface



FIG. 59. Fermi-surface cross section: (100) plane (after Laurent *et al.*, 1981). Typical nesting vectors, $\tilde{Q}_{\pm} = (0, 0, 1 \pm \tilde{\delta})$, between the Γ and H surfaces are shown.

Example 4: Cr, an Itinerant Antiferromagnet



Very simple material?

complicated topology of Fermi surface



FIG. 59. Fermi-surface cross section: (100) plane (after Laurent *et al.*, 1981). Typical nesting vectors, $\tilde{Q}_{\pm} = (0, 0, 1 \pm \tilde{\delta})$, between the Γ and H surfaces are shown.









Is it all? Is I^{+-} always equal to I^{-+}

- of course not!
- non-centrosymmetric materials: I^{+-} may be $\neq I^{-+}$



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Example 5: MnSi



Magnetic Spiral in MnSi

• Ishida et al. (1985): left handed spiral







Magnetic Spiral in MnSi









 $I^{\pm} \approx \mathbf{S}_{i} \cdot \mathbf{S}_{j} \pm i \mathbf{P}(\mathbf{S}_{i} \times \mathbf{S}_{j})$

[-1-1-1]

no spin analysis necessary: only spin flip scattering

▲[111]



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no spin analysis necessary: only spin flip scattering

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Example 6: Chirality in Multilayers





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Example 6: Chirality in Multilayers





scattering geometry



 $\boldsymbol{\sigma}_{c} = i \mathbf{P}_{0} \cdot \left(\mathbf{S}_{bot} \times \mathbf{S}_{top} \right)$



Example 6: Chirality in Multilayers





scattering geometry



 $\boldsymbol{\sigma}_{c} = i \mathbf{P}_{0} \cdot \left(\mathbf{S}_{bot} \times \mathbf{S}_{top} \right)$



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Chirality in Multilayers



Chirality in Multilayers



Summary

- extraction of magnetic cross sections
- separation of magnetic modes

Missing: Polarized beam instruments at pulsed sources





Summary

- extraction of magnetic cross sections
- separation of magnetic modes
- itinerant antiferromagnets: Fermi surface topology
- chirality in itinerant magnets and multilayers
- next: 3-*d* polarization analysis of magnetic excitations

Missing: Polarized beam instruments at pulsed sources





Summary

- extraction of magnetic cross sections
- separation of magnetic modes
- itinerant antiferromagnets: Fermi surface topology
- chirality in itinerant magnets and multilayers
- next: 3-*d* polarization analysis of magnetic excitations
- not mentioned: high resolution with neutron spin echo application in particle physics

Missing: Polarized beam instruments at pulsed sources





I am sorry for the tough finish of my presentation!