"Periodisches System" der Elementarteilchen

Materieteilchen = Fermionen, Spin 1/2

Ladung	QUARKS			LEPTONEN			Ladung
2/3	up	charm	top	ν_e	ν_μ	ν_τ	0
					Neutrinos		
-1/3	down	strange	bottom	e Elektron	μ Myon	τ Tau	-1
	Farbkraft			keine Farbkraft			
	1.	2.	3.	1.	2.	3.	Familie

Kraftteilchen = Bosonen, Spin(S) ganzzahlig

S = 1	S = 1		S = 0	S = 2
Gluonen	γ Photon	W^+, Z^0, W^- Intermediäre Vektorbosonen	H^+, H^0, H^0, H^- (Geister) Higgsteilchen	g Graviton
Farbkraft	Elektroschwache Kraft			Gravitation

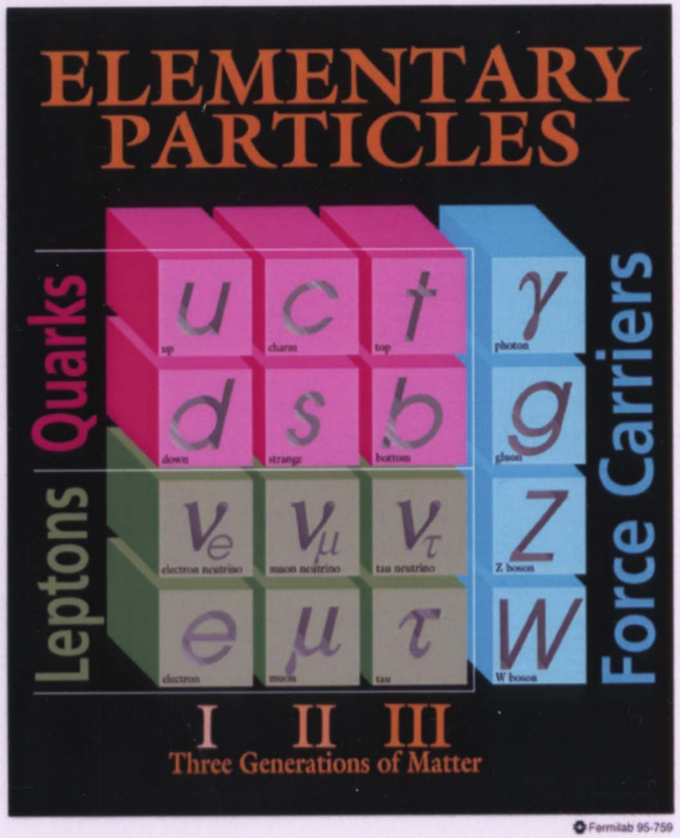
Is the muon just a heavy electron ?

Discovered in cosmic rays by
Neddermeyer and Anderson (1936)

Appears identical to electron but is
200 times as heavy. (1/9 mass of proton)

S=1/2, Magnetic moment $\mu_m \approx 3\ \mu_p$
very sensitive magnetic probe

Charge +e
Positive muons are repelled by the nuclei.
They probe magnetic fields in the
interstitial regions between the atoms



'Who ordered that?' - I I Rabi

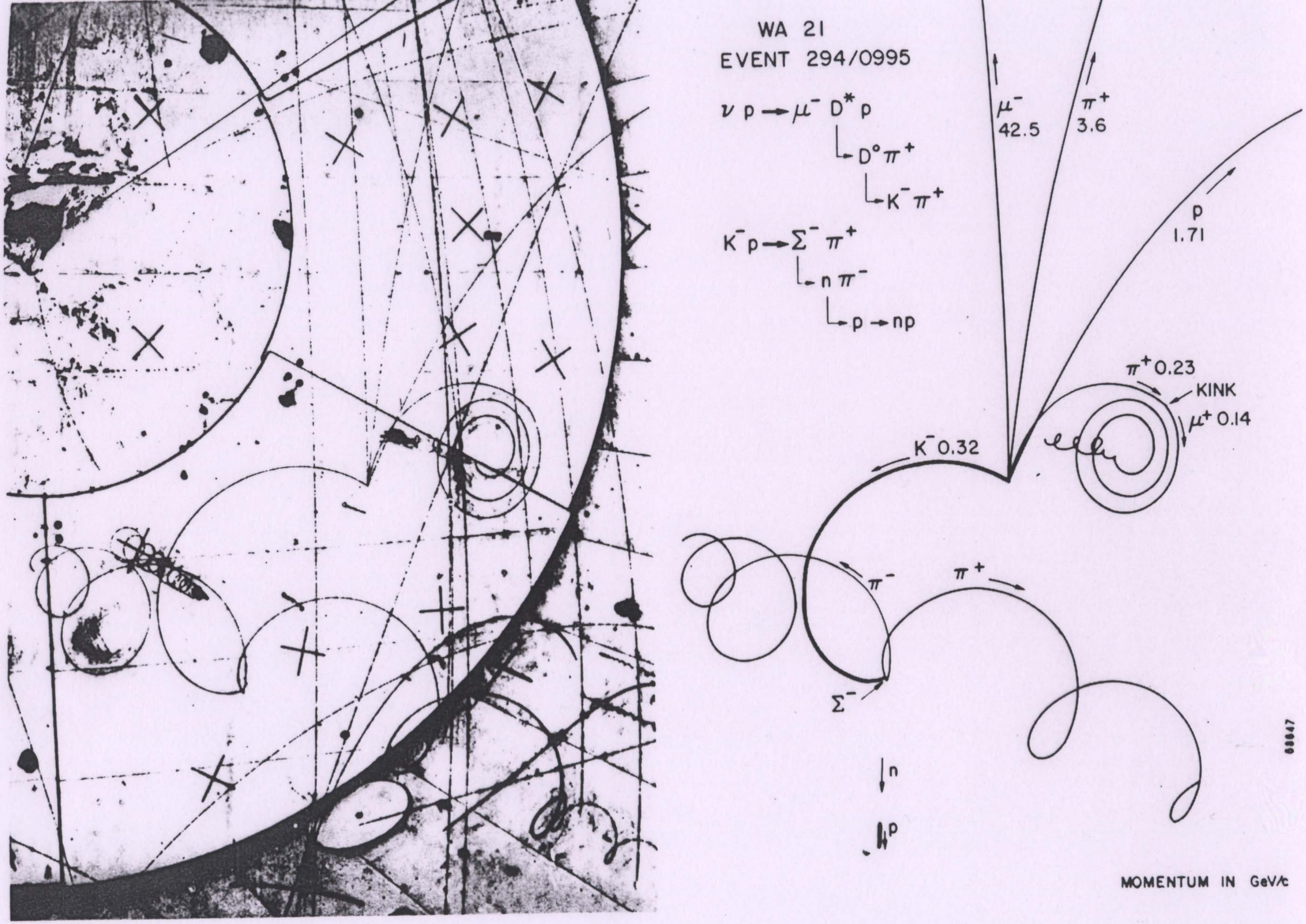
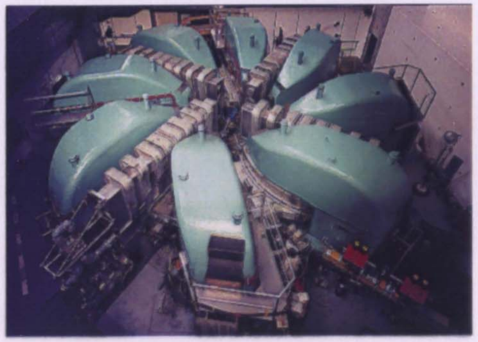
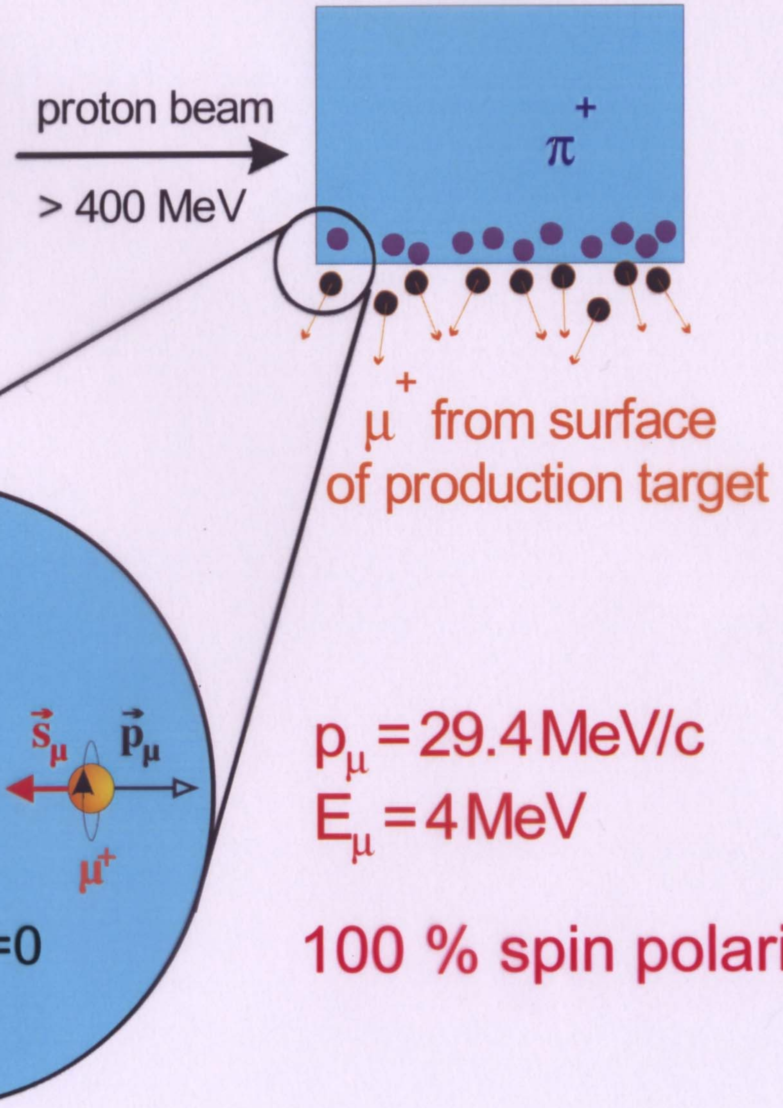


Fig. 23. Neutrino interaction in hydrogen bubble chamber BEBC [WA 21].

Muon Production



$p + p \rightarrow p + n + \pi^+$



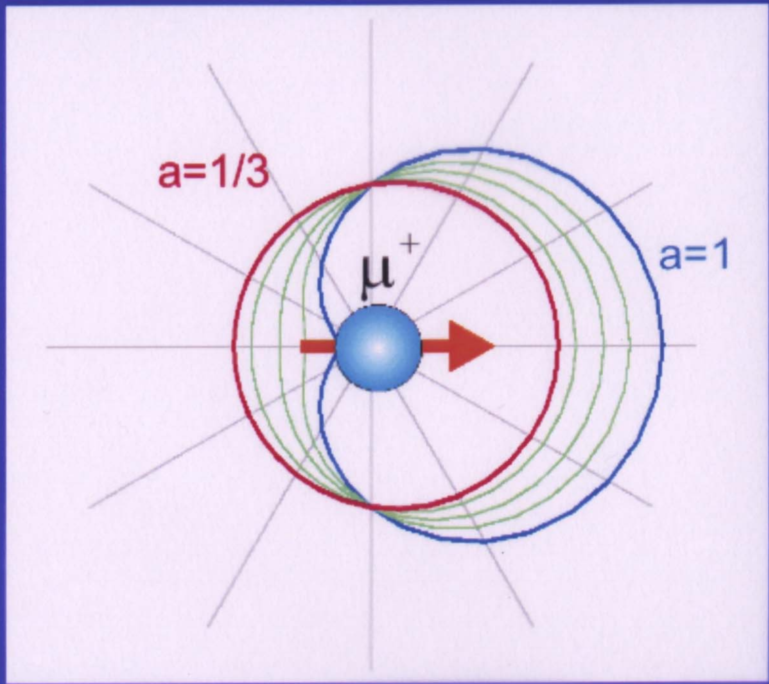
$$\pi^+ \xrightarrow{26\ \text{ns}} \mu^+ + \nu_\mu$$

$p_\mu = 29.4\,\text{MeV/c}$

$E_\mu = 4\,\text{MeV}$

100 % spin polarization

μ^+-Decay Asymmetry



Angular distribution of positrons from μ^+-decay. The asymmetry is $a = 1/3$ when all positron energies are detected with equal probability.

http://musr.triumf.ca/intro/musr/muSRBrochure.pdf by Jeff E. Sonier

MUON (A HEAVY ELECTRON)

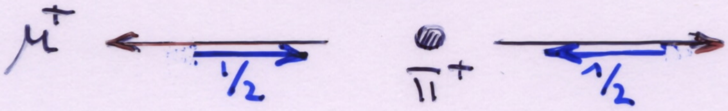
PRODUCTION PROCESS:

$$p + \binom{p}{n} \longrightarrow \binom{n}{p} + p + \binom{\pi^+}{\pi^-}$$

(NUCLEUS)

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$
$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$
$$(\pi^\pm \longrightarrow e^\pm + \overset{(-)}{\nu}_e)$$ DEPRESS

SPIN 0^-



$\tau_\pi \sim 26$ms

DUE TO $m_\nu = 0$

MUON POLARIZED ⟵ LEFT HANDED

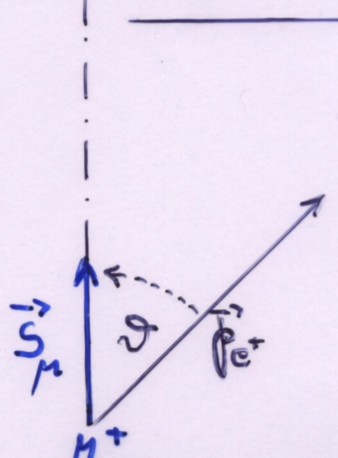
MUON DECAY PROCESS: $\tau_\mu \simeq 2.2\,\mu s$

$$\mu^+ \longrightarrow e^+ + \bar{\nu}_\mu + \nu_e$$
$$\mu^- \longrightarrow e^- + \nu_\mu + \bar{\nu}_e$$

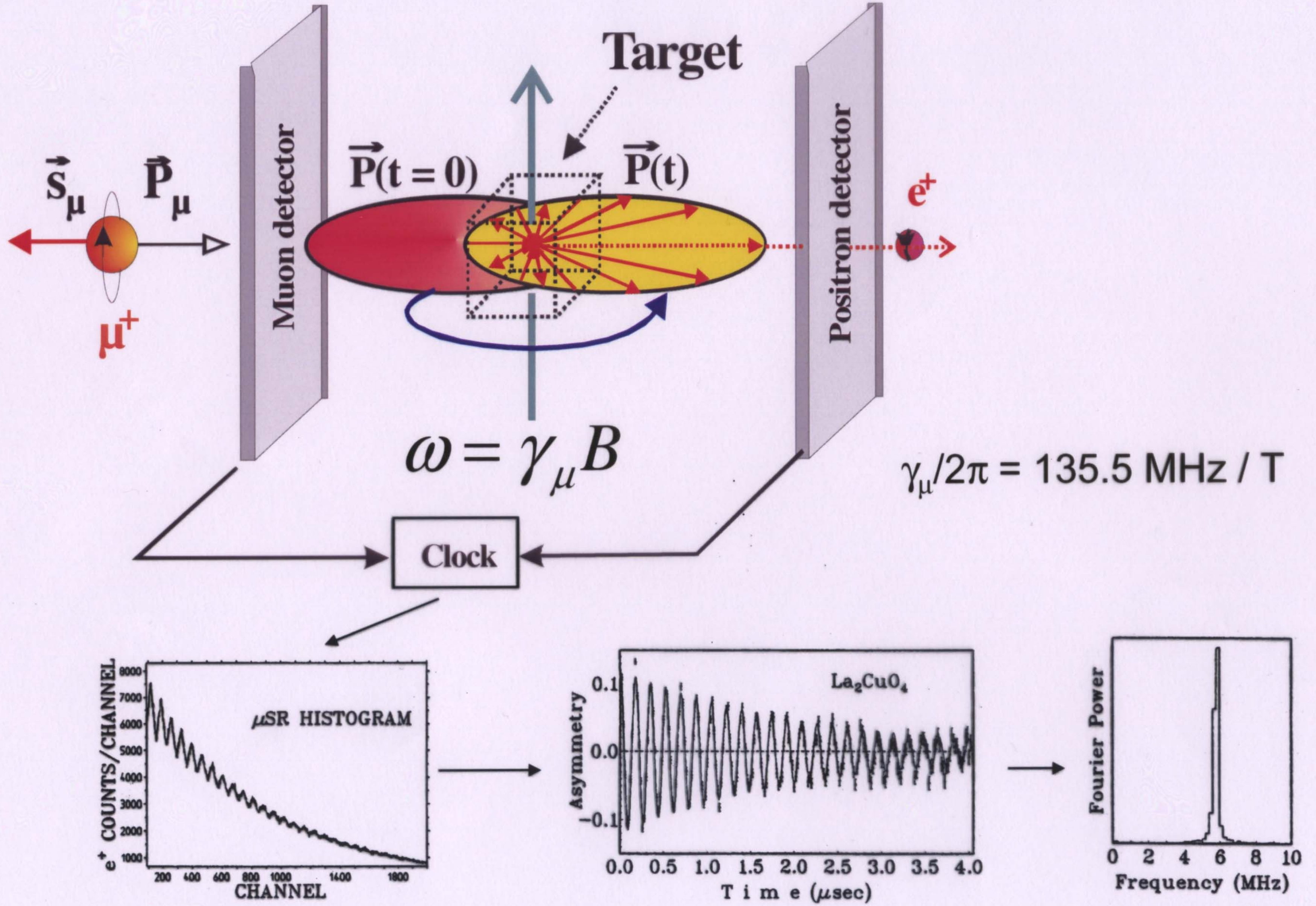
DUE TO PARITY - VIOLATION (WEAK INTERACTION!)

DECAY DISTRIBUTION:

$$I(\vartheta) \sim (1 + A\cos\vartheta)$$



$\vec{p}_e$ IN MUON-SP
DIRECTION

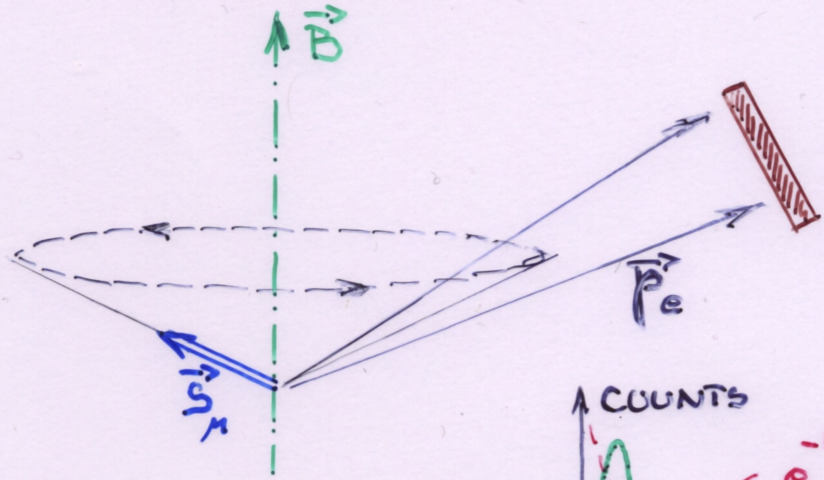

Target
$\vec{S}_\mu$
$\vec{P}_\mu$
μ^+
Muon detector
$\vec{P}(t = 0)$
$\vec{P}(t)$
Positron detector
e^+
$\omega = \gamma_\mu B$
$\gamma_\mu/2\pi$ = 135.5 MHz / T
Clock
e^+ COUNTS/CHANNEL
μSR HISTOGRAM
CHANNEL
Asymmetry
La_2CuO_4
T i m e (μsec)
Fourier Power
Frequency (MHz)

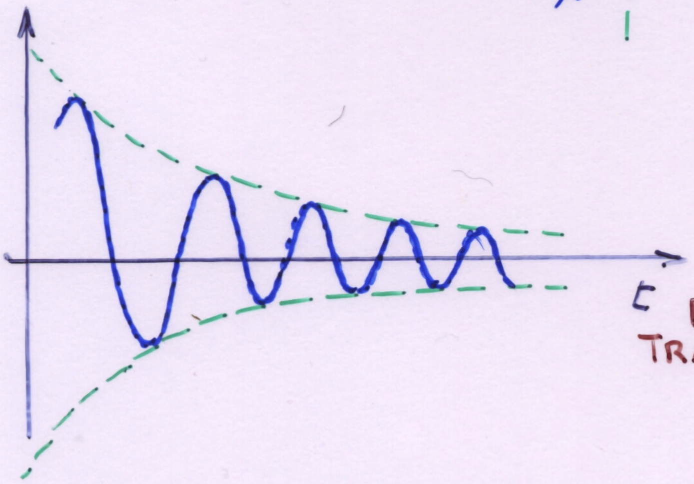
μSR (PRINCIPLE)

MUON (HEAVY ELECTR.) — MAGNETIC MOMENT

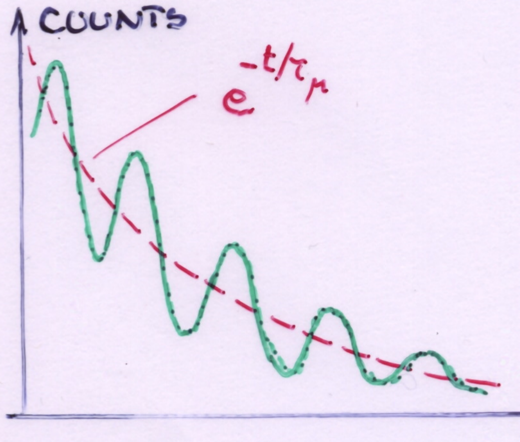
PRECESSION IN A MAGN. FIELD (A LA NMR)

DIRECT OBSERVATION BY ITS DECAY (DECAY ASYMMETRY)





DECAY TRANSF. OFF



PRECESSION FREQUENCY — $\omega_\mu \sim |\vec{B}|$

INTERNAL MAGNETIC FIELD

RELAXATION OF PRECESSION SIGNAL — INTERACTION WITH ENVIRONMENT

FIELD FLUCTUATI

POINT CHARGE IN AN ELECTRO-MAGNETIC FIELD

CLASSICAL :

$$m\ddot{\vec{x}} = F(\vec{x}, \dot{\vec{x}}; t) = e\left[\vec{E}(\vec{x},t) + \frac{\dot{\vec{x}}}{c} \wedge B(\vec{x},t)\right]$$

$(\vec{E}, \vec{B})$ POTENTALS $(\phi, \vec{A})$

LAGRANGE EQUAT.

$$L(\vec{x}, \dot{\vec{x}}; t) \xrightarrow[\text{LEGENDRE TRF.}]{} H(\vec{x}, \vec{p}; t)$$

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + e\phi$$

"MINIM." COUPLING : $\vec{p} \qquad \vec{p} - \frac{e}{c}\vec{A}$

QUANTUM - MECHANICS : SEMI-RELATIVISTIC

$i\hbar\, \partial_t \psi = H\psi$ SCHRÖDINGER WITH

$H = c\vec{\alpha}\left(\vec{p} - \frac{e}{c}\vec{A}\right) + \beta mc^2 + \phi$ DIRAC

AS

TWO-COMPONENT SPINOR-EQUTIONS (PAULI-EQUS.

FIRST RELATIV. APPROX. :

$$H = \frac{1}{2mc}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + e\phi - \frac{e\hbar}{mc}\vec{\sigma}\cdot\vec{B}$$

SPIN AND INTRINSIC MAGNETIC MOMENT $\vec{\mu} = \frac{e\hbar}{mc}\vec{\sigma} = \frac{e}{mc}\vec{S}$; $\vec{S} = \frac{\hbar}{2}$

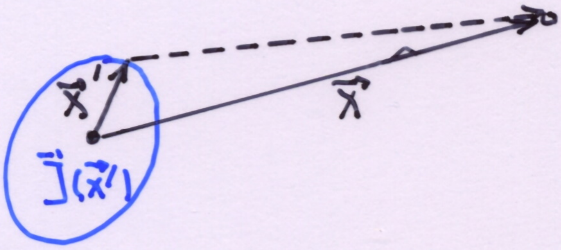
FERMIONS — ELECTRON, MUON } ELEMENTARY

NEUTRON, PROTON } EXTRA RENORM.

CURRENTS AND MAGNETIC MOMENTS

(BIOT - SAVARD / AMPERE)

$$\vec{B}(\vec{x}) = \frac{1}{c}\int \vec{J}(\vec{x}') \wedge \frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}\, d^3x'$$



CURRENT LOCAL : $\int \vec{J}(\vec{x})\, d^3x' =$

STATIONARY : $\vec{\nabla}\cdot\vec{J} =$

PART. INTEGR. : $\vec{B}(\vec{x}) = \frac{1}{c}\vec{\nabla}\wedge\vec{A}(\vec{x}) = \frac{1}{c}\vec{\nabla}\wedge\int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x}-\vec{x}'|}$

$\boxed{\vec{x} \gg \vec{x}'}$ $\qquad \frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{|\vec{x}|} + \frac{\vec{x}\cdot\vec{x}'}{|\vec{x}|^3} + \cdots\cdots$

$$A_i(\vec{x}') = \frac{\vec{x}}{c|\vec{x}|^3}\int d^3x'\, \vec{x}' J_i(\vec{x}')$$

$$\downarrow$$

$$\sum_j x_j \int d^3x'\, x_j' J_i(\vec{x}') = -\frac{1}{2} x_j \int d^3x' (x_i J_j - x_j J_i$$

SINCE $\int d^3x' (x_i J_j + x_j J_i) = 0$

DUE TO $\vec{\nabla}\cdot\vec{J} = 0$

$$-\frac{1}{2}\sum_{j,k}\varepsilon_{ijk} x_j \int d^3x' (\vec{x}'\wedge\vec{J}(\vec{x}'))_k = \int d^3x' [\vec{x}'\wedge(\vec{x}'\wedge\vec{J}(\vec{x}'))] = c\,\vec{x}\wedge\vec{m}$$

WITH : $\vec{m} = \frac{1}{2c}\int d^3x' (\vec{x}'\wedge\vec{J}(\vec{x}'))$

AS MAGNETIC MOMENT

HENCE : $\boxed{\vec{A}(\vec{x}) = \frac{\vec{m}\wedge\vec{x}}{|\vec{x}|^3}}$

CURRENT OF CHARGED PARTICLES :

$$\vec{J} = \sum_i \frac{q_i}{M_i}\delta(\vec{x}-\vec{x}_i)\vec{p}_i \xrightarrow{\frac{q_i}{M_i}\sim\frac{e}{m}} \boxed{B(\vec{x}) = -\frac{e}{mc}\frac{\vec{p}\wedge\vec{x}}{|\vec{x}|^3}}$$

$\vec{\mu} = \frac{e}{2mc}\vec{L} \qquad \vec{L} = \vec{x}\wedge\vec{v} \qquad$ ORBITAL

ELECTRONS :

MAGN. MOMENT : SPIN $\frac{e}{mc}\vec{\sigma}$

ORBITAL CURRENTS $\frac{e}{2mc}\vec{L}$

ORIGIN OF MAGNETIZATION (INTER. MAGN. FIELDS) IN CONDENSED MATTER

INTERACTION WITH EXTERNAL FIELD $\vec{B}$

$$\boxed{H_{int} = -\vec{\mu} \cdot \vec{B}(\vec{x},t)}$$

EXTERNAL FIELD : MAGN. FIELD OF PROBING PARTICLES WHICH CARRY A MAGNETIC MOMENT

e.g. MUONS, NEUTRONS

$$\vec{B}(\vec{x}) = \vec{\nabla} \wedge \vec{A}(\vec{x})$$

$$\downarrow$$

$$\frac{\vec{\mu} \wedge \vec{x}}{|\vec{x}|^3} = -\vec{\mu} \wedge \vec{\nabla}\frac{1}{|\vec{x}|} = \gamma_\mu \vec{\sigma} \wedge \vec{\nabla}\frac{1}{|\vec{x}|}$$

$$B(\vec{x}) = \vec{\nabla} \wedge (\vec{\mu} \wedge \vec{\nabla}\frac{1}{|\vec{x}|}$$

$$= -(\vec{\mu}\cdot\vec{\nabla})\vec{\nabla}\frac{1}{|\vec{x}|} + \vec{\mu}\,\Delta\frac{1}{|\vec{x}|}$$

$$\boxed{B(\vec{x}) = \vec{\nabla}\left(\vec{\mu}\vec{\nabla}\frac{1}{|\vec{x}|}\right) + 4\pi\,\delta(\vec{x})}$$

(DIPOL) (HYPERFINE) CONTACT

MUON DEPOLARISATION

$$|1/2, m=-1/2\rangle \xrightarrow[W_{-+}]{} |1/2, m=+1/2\rangle$$

N_- $\qquad$ N_+ : POPULATION

$$\frac{1}{2}\{N_-(t) - N_+(t)\} = \frac{1}{2} m(t) = \frac{1}{2} e^{-\lambda t}$$

λ: RELAXATION RAT

$$\boxed{\dot{m} = 2(W_{+-}N_+ - W_{-+}N_-)}$$

STATIONARY : $W_{+-} = W_{-+} e^{-\hbar\omega/kT}$

IF $\underline{\hbar\omega \ll kT}$ $\longrightarrow$ $\underline{\dot{m} = -2W}$

$$\boxed{\lambda = 2W}$$

H_0 : SAMPLE AND PROBE (MUON) $\quad H_0^{(s)} + H_0^{(\mu)}$

H_1 : INTERACTION BETWEEN THE TWO

$$H_1(t) = -g_\mu \mu_\mu \vec{I} \cdot \vec{B}(t)$$

$\vec{B}$: FLUCT. IN FIELD

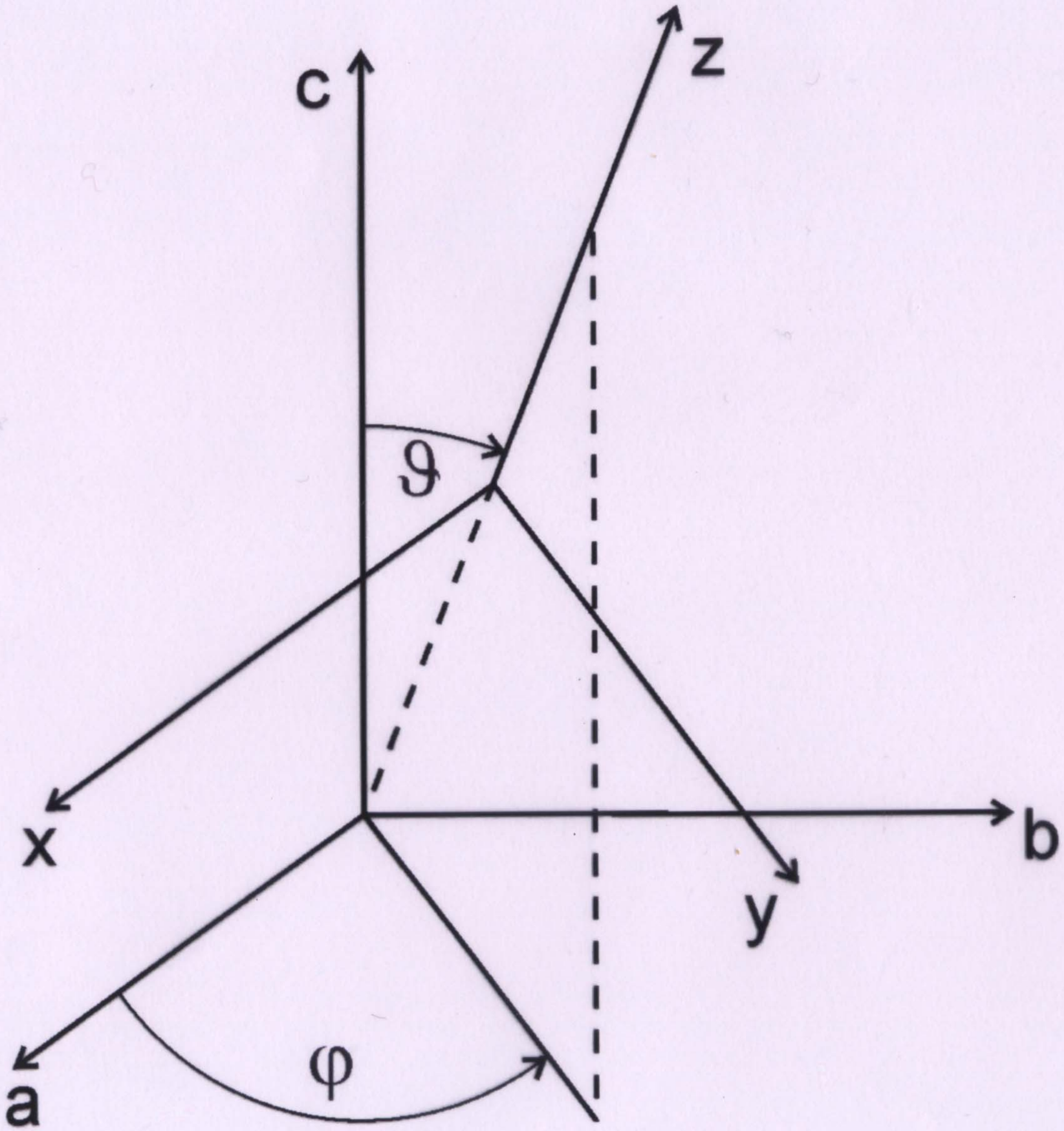
FOR $T > T_c$: $\langle \vec{B} \rangle = 0$

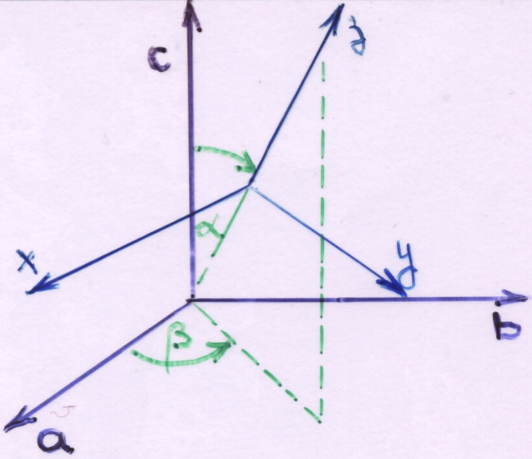
STATE POPULATION IN SAMPLE (CANONICAL)

$$p_\nu = \frac{1}{Q} e^{-E_\nu/kT} \qquad \sum_\nu p_\nu = 1$$

TRANSITION RATE :

$$\boxed{W = \lim_{t\to\infty} \frac{1}{t} \frac{1}{\hbar^2} \sum_{\nu,\nu'} p_\nu \left| \langle m'\nu' | \int_0^t H_1(\tau)\, d\tau | m\nu \rangle \right|^2 = \lambda/2}$$


c
z
ϑ
x
b
y
a
φ



(abc) CRYSTAL SYSTEM

(xyz) MAGN. SYSTEM

INITIAL CONDITIONS :

$$\langle m' | I_x | m \rangle = \tfrac{1}{2}$$
$$\langle m' | I_y | m \rangle = -\tfrac{i}{2}$$
$$\langle m' | I_z | m \rangle = 0$$

ROTATION TO CRYSTAL SYSTEM

$$\langle m' | I_a | m \rangle = \tfrac{1}{2}\,\xi(\alpha,\beta)$$
$$\langle m' | I_b | m \rangle = \tfrac{1}{2}\,\eta(\alpha,\beta)$$
$$\langle m' | I_z | m \rangle = \tfrac{1}{2}\,\zeta(\alpha,\beta)$$

$$\langle m'\nu' | H_1(t) | m\nu \rangle =$$
$$= -g_\mu \mu_\mu e^{-i\omega_\mu t} \left\{ \xi \langle \nu' | B_a(t) | \nu \rangle + \eta \langle \nu' | B_b(t) | \nu' \rangle + \zeta \langle \nu' | B_c(t)$$

WITH $\vec{B}(t) = e^{i/\hbar H_0^{(s)} t}\, \vec{B}(0)\, e^{-i/\hbar H_0^{(s)} t}$

$$H_0^{(s)} | \nu \rangle = E_\nu | \nu \rangle$$

$$= -g_\mu \mu_\mu e^{-i\omega_\mu t}\, O^{(\alpha\beta)}(t)$$

$$W = \left(\frac{g_\mu \mu_\mu}{2\hbar}\right)^2 \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau} \langle \hat{O}^+(0)\, \hat{O}(\tau) \rangle_T$$

FOR POLYCRYSTAL
OR CUBIC SYMMETRY — AVERAGE OVER MUON POLARIS. WITH RESPEC TO CRYSTAL AXIS

OFF DIAG. $\overline{\xi\eta}$, $\overline{\xi\zeta}$, $\overline{\zeta\eta}$ .....

DIAG. $\overline{\xi\xi} = \overline{\eta\eta} = \overline{\zeta\zeta} =$

$$\boxed{\lambda = 2W = \frac{1}{3}\left(\frac{g_\mu \mu_\mu}{\hbar}\right)^2 \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega_\mu \tau} \langle \vec{B}^+(0)\, \vec{B}(\tau) \rangle_T}$$

VAN HOVE CORREL. FCT