NATURE OF THE PROBE AND ITS COUPLINGS

$\longrightarrow$ DEFINES KIND OF PHYSICS WHICH MAY BE INVESTIGATED

NEUTRONS —— MASS DENSITY, MAGN. DENSITY

MUONS —— MAGN. DENSITY

X-RAYS —— CHARGE DENSITY (MAGN.-)
(THOMSON)

} FLUCTUATIONS EXPLORED

STRENGTH OF INTERACTION AND ITS ENERGY DEPENDENCE

CROSS-SECTION ON INDIVIDUAL SCATT. CENTERS

NEUTRONS ~ const.

MUONS –

X-RAYS ~ $\sigma_T \simeq$ const, $\sigma_s(E)$, $\sigma_a(E)$

MILD DECREASE WITH E

! $\boxed{\sigma_a \text{ DOMINANT}}$

ELEMENTAL FINGER PRINTS
K, L, – EDGES

! SPECIAL FEATURES OF X-RAYS

$\boxed{\text{SCATTERING FUNCTION } S(\vec{\kappa}, \omega)}$

$\vec{\kappa}$: MOMENTUM } TRANSFER
ω: ENERGY –

1) LINEAR RESPONSE: (BORN APPROX.) NO MULTIPLE SCATTERING —— INDEP. OF PRIMARY ENERGY

2) ISENTROPIC PROCESS —— SAMPLE NOT DISTURBED

3) EXPERIMENT : TEMPORAL
THEORY : ENSEMBLE
} AVERAGE —— ERGODIC ASSUMPTION

WAVE LENGTH $\lambda \sim 1 Å$ (0.1 nm)

X - RAY :

$$E = \hbar\omega = \hbar c k = \frac{2\pi\hbar c}{\lambda} = \frac{12398\ eV}{\lambda[Å]}$$

NEUTRON :

$$E = \frac{(\hbar k)^2}{2M_n} = \frac{(2\pi\hbar)^2}{2M_n}\frac{1}{\lambda^2} = \frac{81.8\ meV}{\lambda^2\ [Å]}$$

THAT IS :

$\underline{\lambda = 1 - 10 Å}$

- X - RAY : $E = 12.4 - 1.24$ keV
- NEUTRON : $E = 82 - 0.82$ meV

PROBE	λ [Å]	E [eV]	RANGE IN RECIPR. LATTICE $[Å]^{-1}$
PHOTON			
LIGHT	5000	2	10^{-3}
UV	911	13.6	10^{-2}
X-RAY	1	$12.4 \cdot 10^3$	10
NEUTRON	1	0.08	10
ELECTRON	1	150	10

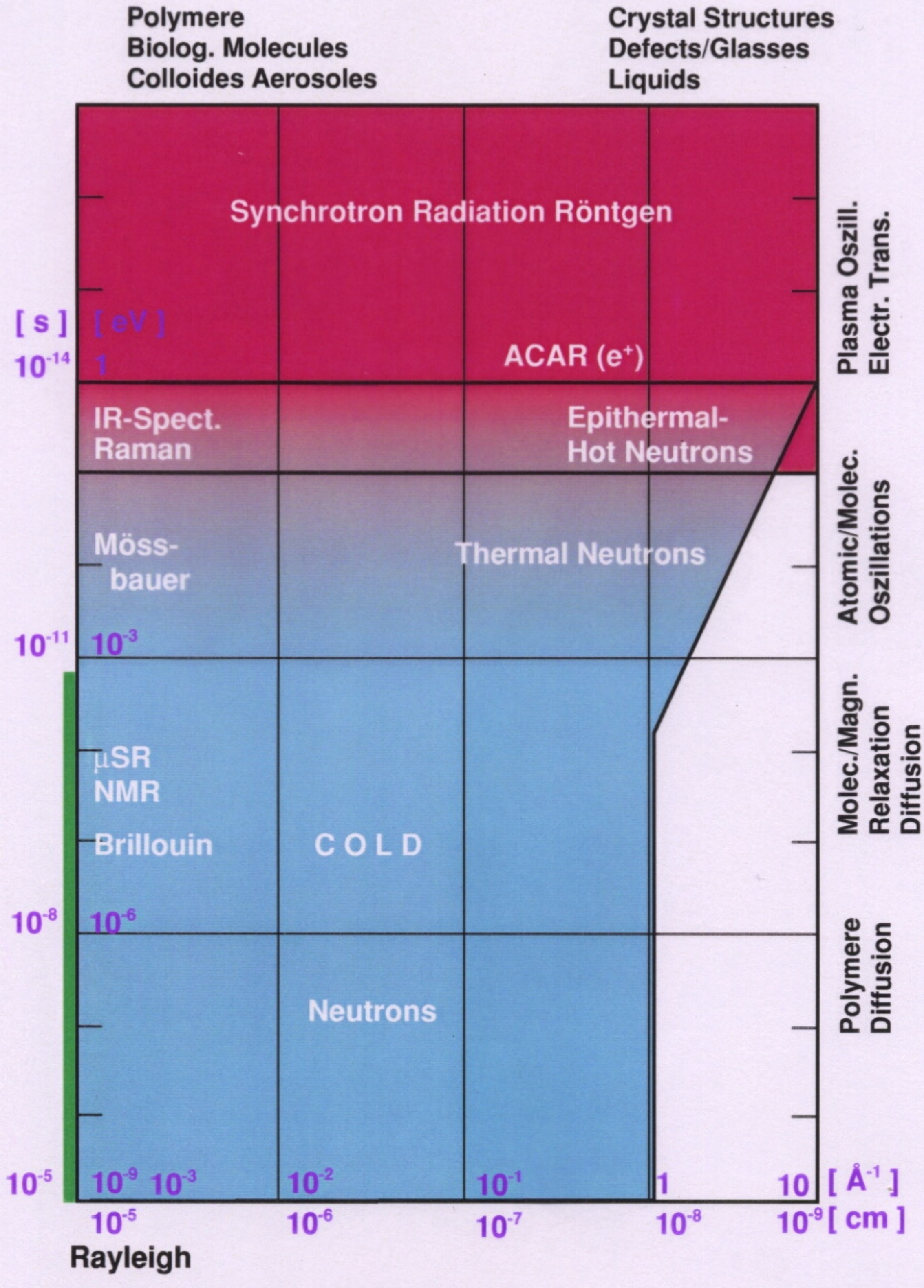
$\left.\begin{array}{l}\text{X-RAY}\\ \text{NEUTRON}\\ \text{ELECTRON}\end{array}\right\}$ CAN COVER FULL FIRST BRILLOUIN ZONE

$\left.\begin{array}{l}\text{X - RAY}\\ \text{ELECTRON}\end{array}\right\}$ — PROBE —— CHARGE DISTRIBUTION

NEUTRON —— PROBE

- DENSITY DISTRIB. (NUCLEAR SCATT.)
- DISTRIB OF MAGNETISATION (MAGN. SCATT.)

Access to space-time correlations by various experimental methods.



NEUTRON - NUCLEUS INTERACTION

STRONG INTERACTION : INVARIANCE ——

- SPACE/TIME TRANSL. —— MOMENTUM - ENERGY
- SPATIAL ROTATION —— ANGULAR MOMENTUM
- ISOSPIN ROTATION —— ISOSPIN
- SPATIAL REFLECTION —— PARITY
- TIME REVERSAL —— -

VERY LOW ENERGY SCATTERING : $\ell = 0$ ONLY

(NEGLECTING ISOSPIN - DEPENDENCE)

TWO INVARIANT AMPLITUDES ——

NEUTRON SPIN $\frac{1}{2}\vec{\sigma} = \vec{S}_n$

NUCLEAR SPIN $\vec{I}$

$$\boxed{A\,\mathbb{1} \quad ; \quad B(\vec{S}_n \vec{I})}$$

FROM POTENTIAL - SCATT. :

$$\boxed{U(r) \sim \frac{\alpha}{r^s}} \qquad \ell < \frac{1}{2}(s-3)$$

THAT IS, FOR $\ell = 0 \longrightarrow \boxed{s > 3}$

$r \to \infty$: DECREASE "SUFFICIANTLY" FAST

PARTIAL WAVE :

$$\boxed{\lim_{k \to 0} k^{2\ell+1} \operatorname{ctg} \delta_\ell = -\frac{1}{a_\ell}} \qquad a_\ell \in \mathbb{C}$$

LOW ENERGY CROSS SECTION :

$$\boxed{\lim_{k \to 0} \sigma = 4\pi a_0^2}$$

a : FREE NUCL. SCATT. LENGTH

a : USUALLY DIFFERENT FOR DIFFERENT ISOTOPES

NUCLEUS ——— BOUND CHEMICALLY AT A MOLECULE OR CRYSTAL.

RANGE $\sim 10^{-12}$ cm
DEPTH $\sim$ 40 MeV

RANGE $\sim 10^{-8}$ cm
DEPTH $\sim$ 1 eV

NUCL. RANGE $\ll$ VIBRATION AMPLITUDE
COLLISION TIME $\ll$ OSCILL. PERIOD

$\rightarrow$ IMPULSE APPROXIMATION

NUCL. MASS : M – BOUND TO VERY HEAVY SYSTEM

SCATT. AMPLITUDE TO EXCITED (MOLEC.) STATE $|m\rangle$

$$-\frac{M+m}{m}\, a \int d^3x\, \psi_m^*(\vec{x})\, \psi_0(\vec{x})\, e^{i\vec{\varkappa}\vec{x}}$$

a : FREE SCATT. LENGTH

$V(\vec{x})$ POTENTIAL (NEUTR. - NUCL.)
(IN BORN - APPROXIMATION)

$$-\frac{2\pi m}{\hbar^2}\int d^3x'\, V(\vec{x}')\, e^{i\vec{\varkappa}\vec{x}'} \int d^3x\, \psi_m^*(\vec{x})\, \psi_0(\vec{x})\, e^{i\vec{\varkappa}\vec{x}}$$

IDENTICAL –

FOR : $V(\vec{x}) = \frac{\hbar^2}{2\pi} \underbrace{\frac{M+m}{Mm}\, a}_{\text{BOUND SCATT. LENGTH } b} \delta(\vec{x})$

FERMI :
PSEUDO - POTENTIAL (PHENOMENOL.)
DEFINED SUCH $\rightarrow$ THAT BORN - APPROX. VALID

"ACTUAL POTENTIAL"
SHORT RANGE / VERY STRONG $\longrightarrow$ PERTURBATION THEORY NOT VALID !

$$\boxed{b = A + \tfrac{1}{2}B(\vec{\sigma}\cdot\vec{I})}$$

NON FLIP   SPIN-FLIP

$\vec{I}$: NUCLEAR SPIN

ASSUME : S - WAVE SCATTERING

$\vec{J} = \vec{I} + \vec{S}$ TOTAL ANGULAR MOMENTUM

CONSTANT OF MOTION

EIGENVALUES : $I \pm \frac{1}{2}$

IMPLIES TWO SCATT. LENGTHS $b^{(+)}, b^{(-)}$

$$b^{(+)} = A + B\tfrac{1}{2}\left[(I+\tfrac{1}{2})(I+\tfrac{3}{2}) - I(I+1) - \tfrac{3}{4}\right]$$
$$b^{(-)} = A + B\tfrac{1}{2}\left[(I-\tfrac{1}{2})(I+\tfrac{1}{2}) - I(I+1) - \tfrac{3}{4}\right]$$

THEREFOR :

$$\boxed{\begin{aligned} A &= \frac{1}{2I+1}\left[(I+1)\,b^{(+)} + I\,b^{(-)}\right] \\ B &= \frac{1}{2I+1}\left[\,b^{(+)} - b^{(-)}\right] \end{aligned}}$$

NUMBER OF CHANNEL STATES $2J+1$

THAT IS : $2(I+\frac{1}{2})+1 = 2I+2$ FOR $I+\frac{1}{2}$

$2(I-\frac{1}{2})+1 = 2I$ FOR $I-\frac{1}{2}$

ALL WITH SAME INTRINSIC PROBABILITY (UNPOL. TARGET)

HENCE :   SPIN AVERAGED SCATTERING LENGTH

$$\boxed{\bar{b} = \frac{I+1}{2I+1}\,b^{(+)} + \frac{I}{2I+1}\,b^{(-)}}$$

$$|\bar{b}|^2 = |A|^2$$

USE $(\vec{\sigma}\vec{I})^2 = \vec{I}^2 + i\vec{\sigma}(\vec{I}\wedge\vec{I}) = \vec{I}^2 - (\vec{\sigma}\vec{I})$

$$\overline{|b|^2} = |A|^2 + |B|^2\,\tfrac{1}{4}\,I(I+1)$$ LINEARS IN I VANISH

$$= \frac{1}{2I+1}\left[(I+1)\,|b^{(+)}|^2 + I\,|b^{(-)}|^2\right]$$

$$\boxed{|\delta b|^2 = \overline{|b|^2} - |\bar{b}|^2 = \tfrac{1}{4}(I+1)\,|B|^2}$$

WECHSELWIRKUNG MIT LICHT

I) KOPPLUNG AN DIE ELEKTR. LADUNG

$$\sum_j \frac{1}{2m}\left(\vec{p}_j - \frac{e}{m}\vec{A}(\vec{x}_j)\right)^2 \longrightarrow -\frac{e}{mc}\sum_j \vec{A}(\vec{x}_j)\,\vec{p}_j \quad , \quad \frac{e^2}{2mc^2}\vec{A}^2(\vec{x}_j)$$

RESONANZ STR. (2. ORDNUNG) "ANOM." DISPERSION RAMAN - STR.	THOMSON - STR. (1. ORDNUNG) IM ALLGEMEINEN DOMINANT

THOMSON:

$$\langle m_0 | \sum_j e^{i\vec{\varkappa}\vec{X}_j} | m_0\rangle \vec{\varepsilon}_1\vec{\varepsilon}_0 = \vec{\varepsilon}_1\vec{\varepsilon}_0 \int d^3x \, \langle m_0 | \sum_j \delta(\vec{x}-\vec{X}_j) | m_0\rangle \, e^{i\vec{\varkappa}\vec{x}}$$

MISST LADUNGS - (ELEKTRONEN-) DICHTE

$$n_e(\vec{x}) \; : \; |\Psi_{m_0}(\vec{X}_1 \cdots\cdots \vec{X}_N)|^2$$

$$\vec{\varkappa} = \vec{k}_0 - \vec{k}_1$$

ENTHÄLT ATOMARE FORMFAKT. $f(\vec{\varkappa})$

II) KOPPLUNG ANS MAGNETISCHE MOMENT (MAGN. STREUUNG)

SPIN $\quad \frac{e\hbar}{2mc}\sum_j \vec{S}_j(\vec{\nabla}\wedge\vec{A}(\vec{x}_j))$

ORBITAL $\quad -\frac{e\hbar}{2(mc)^2}\sum_j \vec{S}_j[\dot{\vec{A}}(\vec{x}_j)\wedge\vec{p}_j]$

AUS SUBST. I BEI RELATIVISTISCHER BEHANDLUNG

$$-\frac{i\hbar\omega}{mc^2}\langle m_0 | \sum_j e^{i\vec{\varkappa}\vec{X}_j}\left(i\,\frac{\vec{\varkappa}\wedge\vec{p}_j}{\hbar\varkappa^2}\,\vec{a} + \vec{S}_j\vec{B}\right) | m_0\rangle$$

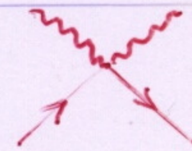
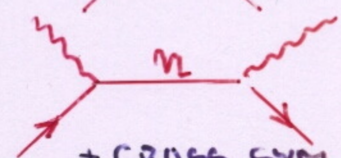
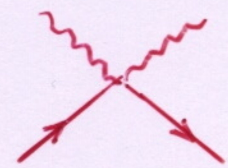
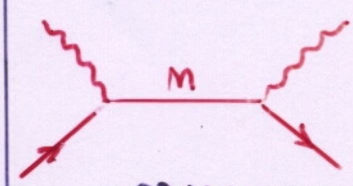
$$\vec{a} = \vec{\varepsilon}_1 \wedge \vec{\varepsilon}_0 \quad ; \quad \mathcal{B}(\vec{\varepsilon}_1, \vec{\varepsilon}_0 \, ; \vec{k}_0, \vec{k}_1)$$

ELECTRO-MAGN. COUPLING OF X-RAYS

$$\langle \,|\sum_j e^{i\vec{\varkappa}\vec{X}_j}\ \hat{O}\ |\,\rangle\ f(\vec{\varepsilon}\,',\vec{\varepsilon})$$

$\vec{\varepsilon}\,',\vec{\varepsilon}$ LIGHT POLARIZATION BEFORE/AFTER SCATTERING

$e^{i\vec{\varkappa}\vec{X}_j}$: $\mathcal{F}$-TRF. OF POSITION $\delta(\vec{x}-\vec{X}_j)$ OF THE SCATT. CENTERS

	$\hat{O}$	MAGNITUDE	$f(\vec{\varepsilon}\,',\vec{\varepsilon})$	PERTURB. ORDER
$\vec{A}^2$	$\mathbb{1}$	1 (THOMSON)	$\vec{\varepsilon}\,'\cdot\vec{\varepsilon}$	
$(\vec{p}\vec{A})$	$i\,\dfrac{\vec{\varkappa}\wedge\vec{p}_j}{\hbar\varkappa^2}$	$-i\,\dfrac{\hbar\omega}{mc^2}$	$\vec{\varepsilon}\,'\wedge\vec{\varepsilon}$	 + CROSS. SYM.
$\dot{\vec{A}}\vec{A}$	$\vec{S}_j$	$-i\,\dfrac{\hbar\omega}{mc^2}$	$\vec{\varepsilon}\,'\wedge\vec{\varepsilon}$	
$(\vec{\nabla}\wedge\vec{A})$	$\vec{S}_j$	$-i\,\dfrac{\hbar\omega}{mc^2}$	$(\hat{\vec{k}}\,'\wedge\vec{\varepsilon}\,')(\hat{\vec{k}}\,'\vec{\varepsilon})-$ $(\vec{k}\wedge\vec{\varepsilon})(\vec{k}\vec{\varepsilon}\,')-$ $(\vec{k}\,'\wedge\vec{\varepsilon}\,')\wedge(\vec{k}\wedge\vec{\varepsilon})$	 + CROSS SYM.

$$\frac{\sigma_m}{\sigma_e}\simeq\left(\frac{\hbar\omega}{mc^2}\right)^2\left(\frac{N_m}{N}\right)^2\langle S\rangle^2\,\frac{f_m^2}{f^2}\xrightarrow{\hbar\omega\,\sim\,10\,\mathrm{keV}}4\cdot10^{-6}\langle S\rangle^2$$

$$T\longrightarrow T_c \qquad \langle S\rangle^2 \downarrow 0$$

HOWEVER : AMPLIFICATION BY CHOICE OF ENERGY $E\sim E_m$ (RESONANCE) FOR SECOND ORDER TERMES

$$\frac{\hbar\omega}{\Gamma_m}\sim O(10^4)$$

X-RAY COUPLING / UNIQUE FEATURES

I) $(\vec{p}\cdot\vec{A})$ TERMS (FIRST ORDER) NO SCATT. !

PHOTO-ABSORPTION → EXAFS

EL.-SCATT. IN VICINITY OF ABSORB. ATOM

ELECTRON EMISSION → PHOTO-ELECTR. SPECTROSCOPY

e.g. ARPES

II) $\vec{A}^2$, $\vec{A}\dot{\vec{A}}$ TERMS (FIRST ORDER)

CHARGE AND MAGNET. THOMSON SCATTERING

POLARIZATION ANALYSIS:

$$(\vec{\varepsilon}'^{*}\cdot\vec{\varepsilon}) \;;\; (\vec{\varepsilon}'^{*}\wedge\vec{\varepsilon})$$

III) NON RESONANT SCATTERING

$\hbar\omega_i \gg E_m - E_i$

$$r_0 \langle f | \sum_j e^{i\vec{\varkappa}\vec{x}_j} \cdot$$

$$\Big\{ (\vec{\varepsilon}_f^{*}\cdot\vec{\varepsilon}_i) \qquad \text{THOMSON}$$

$$- i\frac{\hbar\omega_i}{mc^2}\Big[\frac{mc}{e\hbar}(\hat{\vec{\varkappa}} \wedge \vec{M}_L(\vec{\varkappa}) \wedge \hat{\vec{\varkappa}})\,\vec{P}_L \qquad \text{ORBITAL-DENSITY}$$

$$+ \frac{mc}{e\hbar}\vec{M}_S(\vec{\varkappa})\cdot\vec{P}_S\Big]\Big\} | i \rangle \qquad \text{SPIN-DENSITY}$$

$\vec{P}_S \neq \vec{P}_L$: $\exists$ CONFIGUR. WHERE EITHER ONE VANISHES

→ L- AND S-CONTRIBUTION MEASURED SEPERATLY

BEWARE : $\frac{\hbar\omega_i}{mc^2}$ LARGE ω_i DOES NOT HELP, SINCE $P_L, P_S \sim \sin^2\vartheta_B \sim 1/\omega_i^2$

IV) TUNING ONTO RESONANCE

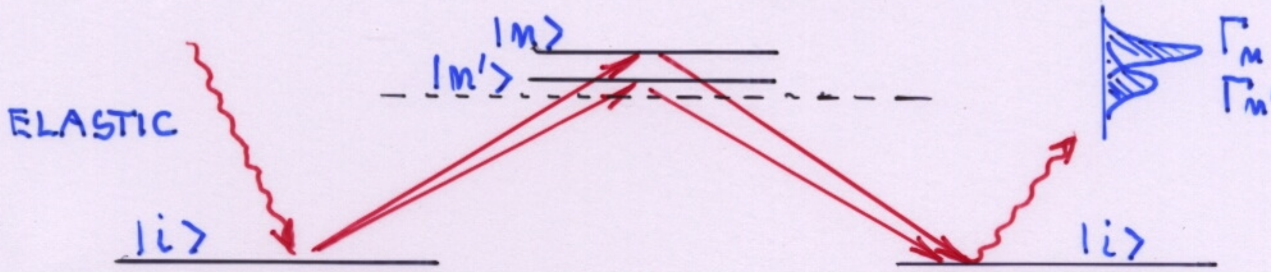
$\hbar\omega_i \simeq E_m - E_i$

$(\vec{p}\vec{A})(\vec{A}\vec{p})$ DOMINANT FOR $(\vec{k}\cdot\vec{x}_j) \ll 1$
CORE LEVEL EXCITATION

↓

DIPOL. APPROX.

$$\frac{m^2}{\hbar^2}(E_i - E_m)^2 \frac{\langle f|(\vec{\varepsilon}_f^{\,*}\vec{x}_j)|m\rangle\langle m|(\vec{\varepsilon}_i\vec{x}_\ell)|i\rangle}{E_i - E_m + \hbar\omega_i - i\Gamma_m/2}$$



CONNECTION TO MAGNETISM:

SELECTION RULES AND $|m\rangle$:
PAULI - EXCLUSION
HUND'S LAW

$|i\rangle$: p - LIKE CORE

EDGES $\left\{ \begin{matrix} L_2 \\ L_3 \end{matrix} \right.$ $\xrightarrow[\text{DIPOL}]{}$ $|m\rangle$ d - LIKE STATES

MAGN. PROPERTIES OF 3d - METALS

EDGES $\left\{ \begin{matrix} L_2 \\ L_3 \end{matrix} \right.$ $\xrightarrow{\text{DIPOL}}$ $|m\rangle$ 5d - BAND
$\xrightarrow[\text{QUADRUPOL}]{}$ 4f - BAND

RARE EARTH'S

↓ POLARIZ. AND WAVE VECTOR DEPENDENCE DIFFERENT

MAGN. COUPLING OF THE NEUTRON AND MUON

MAGN. FIELD OF MAGN. MOMENT :

$$\vec{B}(\vec{x}) = \underbrace{-\vec{\nabla}\left(\vec{\mu}\cdot\vec{\nabla}\frac{1}{|\vec{x}|}\right)}_{\text{DIPOL-DIPOL}} + \underbrace{4\pi\vec{\mu}\,\delta(\vec{x})}_{\text{HYPER-FINE}} = \vec{\nabla}\wedge\vec{\mu}\wedge\vec{\nabla}\frac{1}{|\vec{x}|}$$

LEEDS TO : DIPOL-DIPOL HYPER-FINE

COUPLING TO MAGNETISATION OF THE SAMPLE

i) ELECTR. SPIN $\quad \vec{\mu}_e = -\frac{|e|}{mc}\vec{S}_e$

ii) ORBIT. CURRENTS $\quad \vec{m}_e = \frac{e}{2mc}(\vec{x}\wedge\vec{p})$

HAMILTONIAN :

$$\boxed{H_1 = -\vec{\mu}\cdot\vec{B}(\vec{x},t)}$$

$\langle f | H_1 | i \rangle$ FOR $\quad |i\rangle = e^{i\vec{k}\vec{x}}\varphi_i(\vec{x}_e)|a_i\rangle$

$\quad |f\rangle = e^{i\vec{k}'\vec{x}}\varphi_f(\vec{x}_e)|a_f\rangle$

$$\langle a_f | \int d^3x \int d^3x_e\, \varphi_f^*(\vec{x}_e)\varphi_i(\vec{x}_e)\, e^{i\vec{\varkappa}\vec{x}} H_1 | a_i\rangle$$

$$= \langle a_f | \int d^3\xi\, e^{i\vec{\varkappa}\vec{\xi}} H_1 | a_i\rangle \underbrace{\int d^3x_e\, \varphi_f^*(\vec{x}_e)\varphi_i(\vec{x}_e)\, e^{i\vec{\varkappa}\vec{x}_e}}_{F(\vec{\varkappa})\ \text{MAGN. FORM FACTOR}}$$

$\vec{\xi} = \vec{x} - \vec{x}_e$

$\vec{\varkappa} = \vec{k} - \vec{k}'$

F($\vec{\varkappa}$) MAGN. FORM FACTOR

$F(0) = 1 \quad \vec{\varkappa} \simeq 0$ FOR MUON

NEUTRONS :

$$\langle a_f | \hat{H}_1 | a_i \rangle \sim \vec{S}_n \langle a_f | \hat{\vec{\varkappa}} \wedge \vec{S}_e \wedge \hat{\vec{\varkappa}} - \frac{i}{\hbar\varkappa}(\hat{\vec{\varkappa}}\wedge\vec{p}_e) | a_i\rangle$$

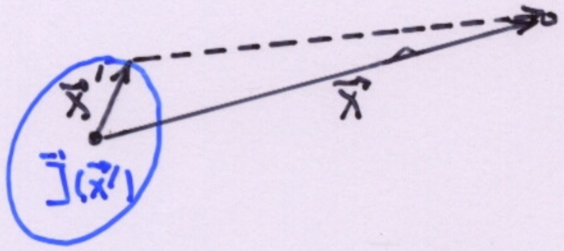
TRANSVERSE ($\perp \vec{\varkappa}$) OF $\quad \vec{S}_e - \frac{i}{\hbar\varkappa^2}(\vec{\varkappa}\wedge\vec{p}_e)$

($\parallel \vec{\varkappa}$) – TURN SAMPLE $\longrightarrow$ SECOND EXP.

CURRENTS AND MAGNETIC MOMENTS

(BIOT - SAVARD / AMPERE)

$$\vec{B}(\vec{x}) = \frac{1}{c}\int \vec{J}(\vec{x}\,') \wedge \frac{\vec{x}-\vec{x}\,'}{|\vec{x}-\vec{x}\,'|^3}\, d^3x'$$



CURRENT LOCAL : $\int \vec{J}(\vec{x})\, d^3x' = 0$

STATIONARY : $\vec{\nabla}\cdot\vec{J} = 0$

PART. INTEGR. : $\vec{B}(\vec{x}) = \frac{1}{c}\vec{\nabla}\wedge\vec{A}(\vec{x}) = \frac{1}{c}\vec{\nabla}\wedge\int d^3x' \frac{\vec{J}(\vec{x}\,')}{|\vec{x}-\vec{x}\,'|}$

$\vec{x} \gg \vec{x}\,'$ $\qquad \frac{1}{|\vec{x}-\vec{x}\,'|} = \frac{1}{|\vec{x}|} + \frac{\vec{x}\cdot\vec{x}\,'}{|\vec{x}|^3} + \cdots\cdots$

$$A_i(\vec{x}\,') = \frac{\vec{x}}{c|\vec{x}|^3}\int d^3x'\, \vec{x}\,' J_i(\vec{x}\,')$$

$$\downarrow$$

$$\sum_j x_j \int d^3x'\, x_j' J_i(\vec{x}\,') = -\frac{1}{2} x_j \int d^3x' (x_i J_j - x_j J_i)$$

SINCE $\int d^3x' (x_i' J_j + x_j J_i) = 0$

DUE TO $\vec{\nabla}\cdot\vec{J} = 0$

$$-\frac{1}{2}\sum_{j,k}\varepsilon_{ijk}\, x_j \int d^3x' (\vec{x}\,'\wedge\vec{J}(\vec{x}\,'))_k = \int d^3x' [\vec{x}\,'\wedge(\vec{x}\,'\wedge J(\vec{x}\,'))] = c\,\vec{x}\wedge\vec{m}$$

WITH : $\vec{m} = \frac{1}{2c}\int d^3x' (\vec{x}\,'\wedge\vec{J}(\vec{x}\,'))$

AS MAGNETIC MOMENT

HENCE : $\boxed{\vec{A}(\vec{x}) = \frac{\vec{m}\wedge\vec{x}}{|\vec{x}|^3}}$

CURRENT OF CHARGED PARTICLES :

$$\vec{J} = \sum_i \frac{q_i}{M_i}\delta(\vec{x}-\vec{x}_i)\vec{p}_i \xrightarrow{\frac{q_i}{M_i}\sim\frac{e}{m}} \boxed{B(\vec{x}) = -\frac{e}{mc}\frac{\vec{p}\wedge\vec{x}}{|\vec{x}|^3}}$$

$$\boxed{\vec{\mu} = \frac{e}{2mc}\vec{L} \qquad \vec{L} = \vec{x}\wedge\vec{v}}$$ ORBITAL

ELECTRONS :

MAGN. MOMENT : SPIN $\frac{e}{mc}\vec{\sigma}$

ORBITAL CURRENTS $\frac{e}{2mc}\vec{L}$

ORIGIN OF MAGNETIZATION
(INTER. MAGN. FIELDS)
IN CONDENSED MATTER

INTERACTION WITH EXTERNAL FIELD B

$$\boxed{H_{int} = -\vec{\mu} \cdot \vec{B}(\vec{x},t)}$$

EXTERNAL FIELD : MAGN. FIELD OF PROBING PARTICLES WHICH CARRY A MAGNETIC MOMENT
e.g. MUONS, NEUTRONS

$$\vec{B}(\vec{x}) = \vec{\nabla} \wedge \vec{A}(\vec{x})$$

$$\downarrow$$

$$\frac{\vec{\mu} \wedge \vec{x}}{|\vec{x}|^3} = -\vec{\mu} \wedge \vec{\nabla} \frac{1}{|\vec{x}|} = \gamma_\mu \vec{\sigma} \wedge \vec{\nabla} \frac{1}{|\vec{x}|}$$

$$B(\vec{x}) = \vec{\nabla} \wedge (\vec{\mu} \wedge \vec{\nabla} \frac{1}{|\vec{x}|}$$

$$= -(\vec{\mu} \cdot \vec{\nabla})\vec{\nabla} \frac{1}{|\vec{x}|} + \vec{\mu} \Delta \frac{1}{|\vec{x}|}$$

$$\boxed{B(\vec{x}) = \vec{\nabla}(\vec{\mu} \vec{\nabla} \frac{1}{|\vec{x}|}) + 4\pi\vec{\mu}\delta(\vec{x})}$$

(DIPOL) (HYPERFINE) CONTACT

CROSS SECTION

MOMENTUM TRANSFER : $\vec{\varkappa} = \vec{k}_0 - \vec{k}_1$

ENERGY TRANFER : $\hbar\omega = E_0 - E_1$

1) BORN APPROXIMATION $T \simeq V$

2) KINEMATICAL APPROX.

WITH $V(\vec{x},t) = \sum_j \frac{2\pi\hbar^2}{m_0} b_j \, \delta(\vec{x} - \vec{X}_j(t))$

$$\boxed{\frac{d^2\sigma}{d\Omega dE} \quad \frac{k_1}{k_0} \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} d\tau \, e^{-i\omega\tau} \sum_{ij} \overline{b_i^* b_j} \; \overline{e^{i\vec{\varkappa}\vec{X}_i(\tau)} e^{-i\vec{\varkappa}\vec{X}_j(0)}}}$$

SCATT. LENGTH : AVERAGE OVER SAMPLE

EXPONENTIALS : TIME AVERAGE.

ASSUME RANDOM DISTRIBUTION OF ISOTOPES AND NUCLEAR SPINS (NO CORREL. BETWEEN $i \neq j$)

$\overline{b_i^* b_j} = \overline{|b_j|^2} = \overline{|b|^2}$ FOR $i = j$

$\overline{b_i^* b_j} = \overline{b_i^*}\,\overline{b_j} = |\overline{b}|^2$ FOR $i \neq j$

$$\sum_{i \neq j} \overline{b_i^* b_j} = N(N-1)|\overline{b}|^2$$

$$\sum_{i=j} \overline{b_i^* b_j} = N\overline{|b|^2}$$

$$\sum_{ij} \overline{b_i^* b_j} = N^2 |\overline{b}|^2 + N\left(\overline{|b|^2} - |\overline{b}|^2\right)$$

$$= \underbrace{N^2 |\overline{b}|^2}_{\downarrow} + \underbrace{N\,\overline{(\delta b)^2}}_{\downarrow}$$

$$\boxed{\begin{array}{cc} |b_c|^2 & |b_{in}|^2 \\ \text{COHERENT} & \text{INCOHERENT} \end{array}}$$

THERMAL AVERAGE
– CANONICAL ENSEMBLE

ASSUME : 1) SAMPLE IN THERMODYN. EQUILIBRIUM
TEMPERATURE T , $\beta = 1/kT$

2) SAMPLE , ERGODIC SYSTEM (BIRKHOFF)

TIME AVERAGE EQUIVAL. AVERAGE OVER CANON. ENSEMBLE

PARTITION FCT. $\quad Z = \mathrm{Sp}\, e^{-\beta H} = e^{-\beta F}$

$H \; : \; H_0^{(S)}$

$F \; :$ FREE ENERGY

EXPECTATION VALUE OF AN OBSERVABLE $\hat{O}$ $\quad \langle \hat{O} \rangle_T = \frac{1}{Z}\, \mathrm{Sp}\left(e^{-\beta H}\, \hat{O} \right)$

EXPLICITLY $\quad \langle \hat{O} \rangle_T = e^{\beta F}\, \mathrm{Sp}\left(e^{-\beta H}\, \hat{O} \right)$

$$= \sum_m p_m \langle m | \hat{O} | m \rangle$$

WITH $\quad p_m = e^{-\beta(\hbar\omega_m - F)} \quad ; \quad \sum_m p_m = 1$

CROSS SECTIONS

COHERENT :

$$\left.\frac{d^2\sigma}{d\Omega\, dE}\right|_{coh} = \frac{k_1}{k_0} \frac{\sigma_c}{4\pi} \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau} \sum_{ij} \langle e^{i\vec{\varkappa}\vec{X}_i(\tau)}\, e^{-i\vec{\varkappa}\vec{X}_j(0)} \rangle_T$$

INCOHERENT :

$$\left.\frac{d^2\sigma}{d\Omega\, dE}\right|_{inc.} = \frac{k_1}{k_0} \frac{\sigma_{in}}{4\pi} \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau} \sum_{i} \langle e^{i\vec{\varkappa}\vec{X}_i(\tau)}\, e^{-i\vec{\varkappa}X_i(0)} \rangle_T$$

$$\sigma_c = 4\pi\, |\bar{b}_c|^2 \qquad \sigma_{in} = 4\pi\, \overline{|\delta b|^2}$$