(High-Tc) Superconductivity and Vortex Lattice Structures

Joël Mesot
Laboratory for Neutron Scattering
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5232 Villigen, PSI

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1. Introduction
2. Low-T_c superconductors
3. High-T_c cuprate superconductors
4. Electronic and magnetic excitations
5. The Abrikosov phase

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Ann. Henri Poincaré 4 (2003) 1 – 22

Many-Body Physics: Unfinished Revolution

Piers Coleman

Abstract. The study of many-body physics has provided a scientific playground of surprise and continuing revolution over the past half century. The serendipitous discovery of new states and properties of matter, phenomena such as superfluidity, the Meissner, the Kondo and the fractional quantum hall effects, have driven the development of new conceptual frameworks for our understanding about collective behavior, the ramifications of which have spread far beyond the confines of terrestrial condensed matter physics- to cosmology, nuclear and particle physics. Here I shall selectively review some of the developments in this field, from the cold-war period, until the present day. I describe how, with the discovery of new classes of collective order, the unfolding puzzles of high temperature superconductivity and quantum criticality, the prospects for major conceptual discoveries remain as bright today as they were more than half a century ago.

Measuring excitations: what for?

Macroscopic Measurements

Transport, Specific heat,
Magnetisation
…

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Theoretical Model

Hamiltonian

$H=H_e+H_{mag}+H_{ph}+H_{e\text{-}ph}$
$+\dots.$

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Eigenvalues E_i
Eigenstates $|\Gamma_i>$

Measuring excitations: what for?

<u>Macroscopic Measurements</u>

Transport, Specific heat,
Magnetisation
…

<u>Theoretical Model</u>

Hamiltonian

$H=H_e+H_{mag}+H_{ph}+H_{e\text{-}ph}$
$+\ldots.$

$$M_\alpha = \frac{1}{k_B T}\frac{\partial \ln Z}{\partial H_\alpha} = g\mu_B \sum_i p_i \langle \Gamma_i | J_\alpha | \Gamma_i \rangle$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V = k_B\left[\sum_i \left(\frac{E_i}{k_B T}\right)^2 p_i - \sum_i \left(\frac{E_i}{k_B T} p_i\right)^2\right]$$

Eigenvalues E_i
Eigenstates $|\Gamma_i>$

Z=partition function

$$U = F - T(\partial F/\partial T)_V \qquad F = -k_B T \ln Z$$

Neutron scattering, Photoemission

yield direct information about eigenvalues and wave functions !

Neutron scattering, Photoemission

yield direct information about eigenvalues and wave functions !

Neutrons

$$\frac{d^2\omega}{d\Omega d\omega} \approx \left| \left\langle \Gamma_m \left| \hat{\mathbf{J}}_\perp \right| \Gamma_n \right\rangle \right|^2 \delta(\hbar\omega + E_{\Gamma_n} - E_{\Gamma_m}) \ ,$$

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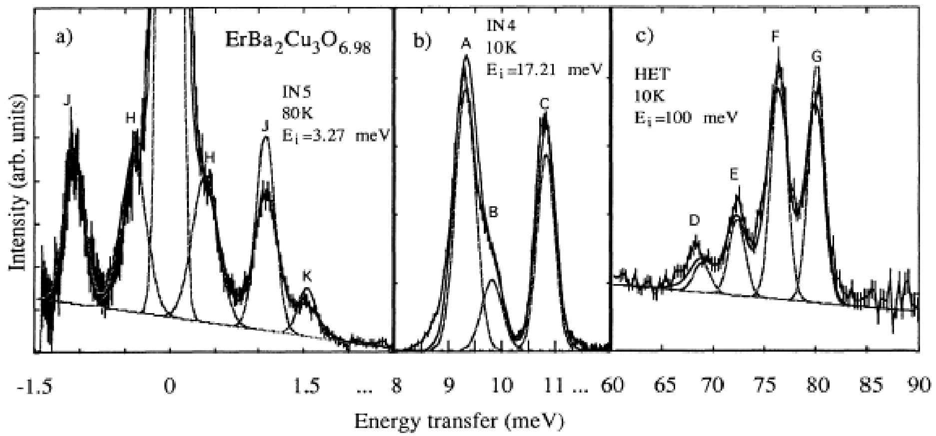

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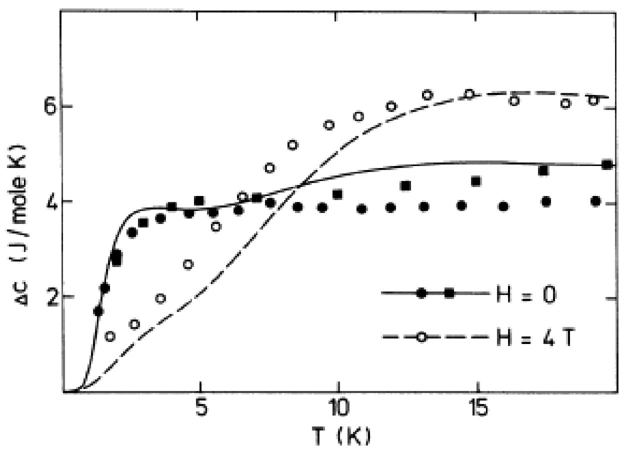

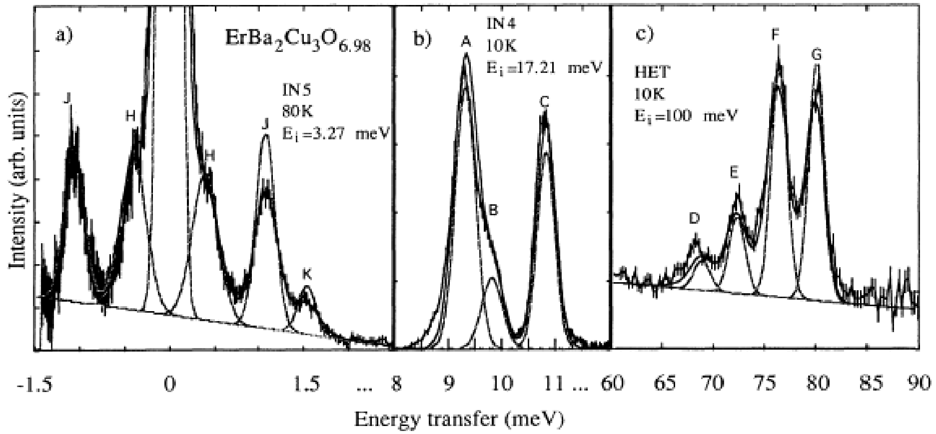

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New materials
(superconductors,
giant-magnetoresistance
Quantum phase transitions,
…)

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↔

Macroscopic properties
(electronic, magnetic…)

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New materials (superconductors, giant-magnetoresistance Quantum phase transitions, …)

Macroscopic properties (electronic, magnetic…)

Microscopic measurements (Inelastic neutron scattering Photoemission, raman, NMR…)

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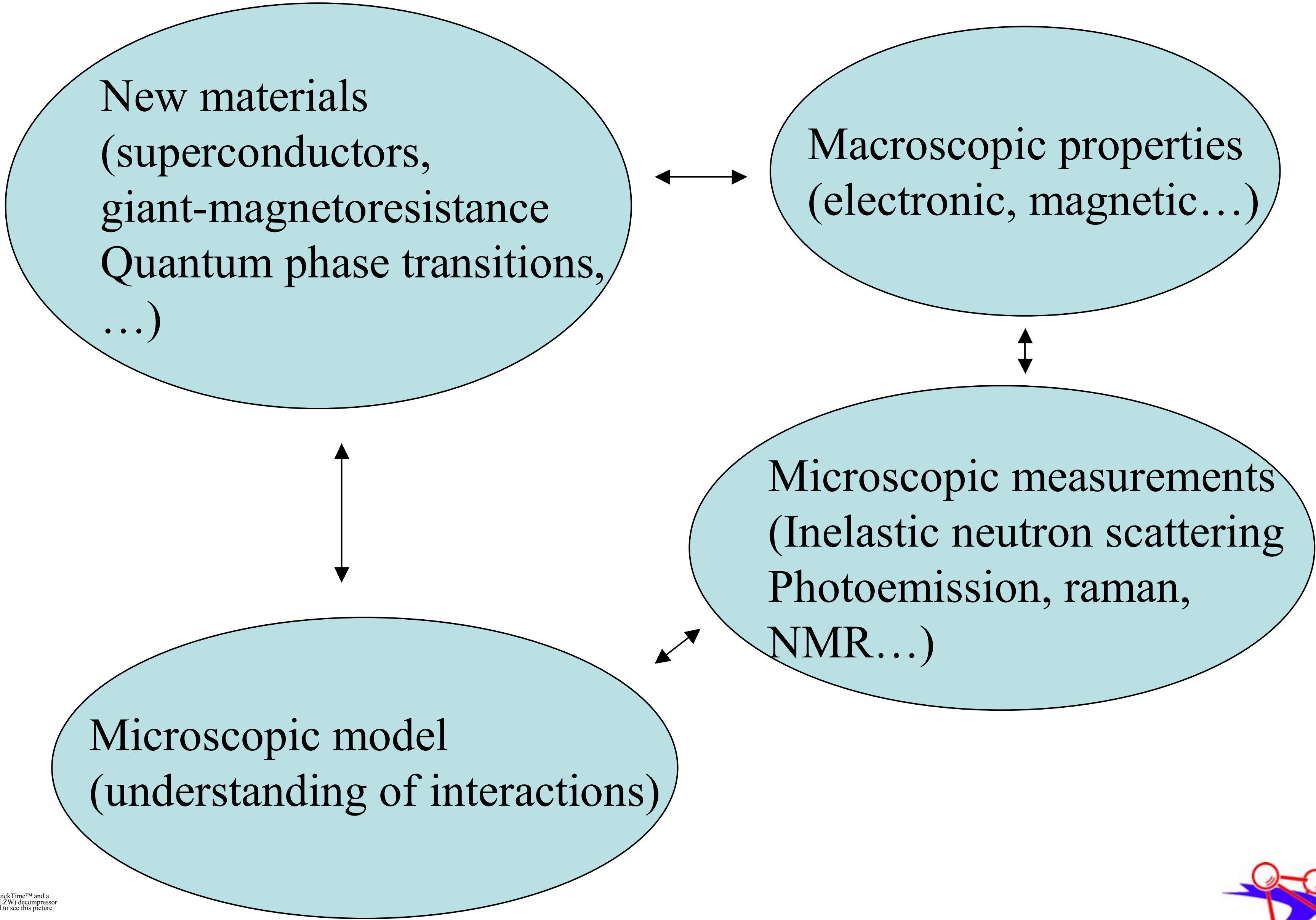
New materials (superconductors, giant-magnetoresistance Quantum phase transitions, …)
Macroscopic properties (electronic, magnetic…)
Microscopic measurements (Inelastic neutron scattering Photoemission, raman, NMR…)
Microscopic model (understanding of interactions)

Normal metals

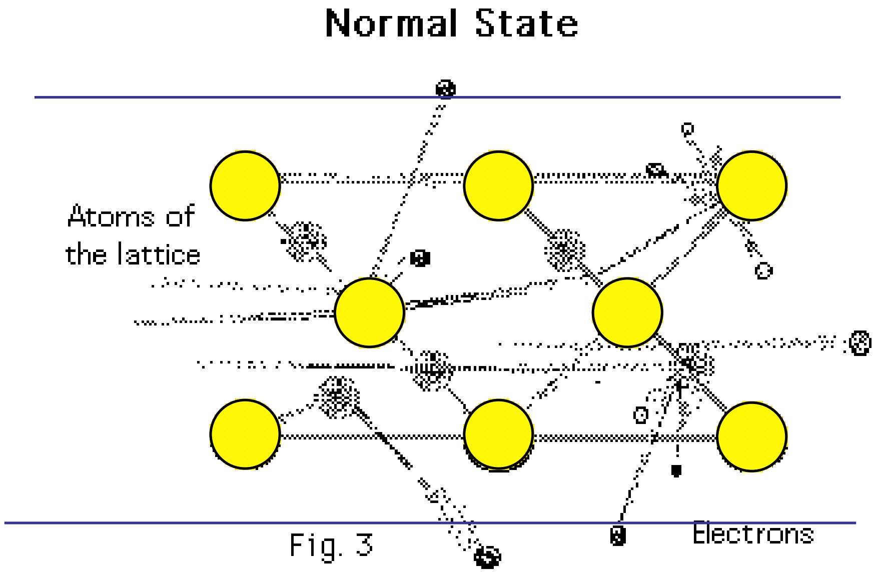

Fig. 3

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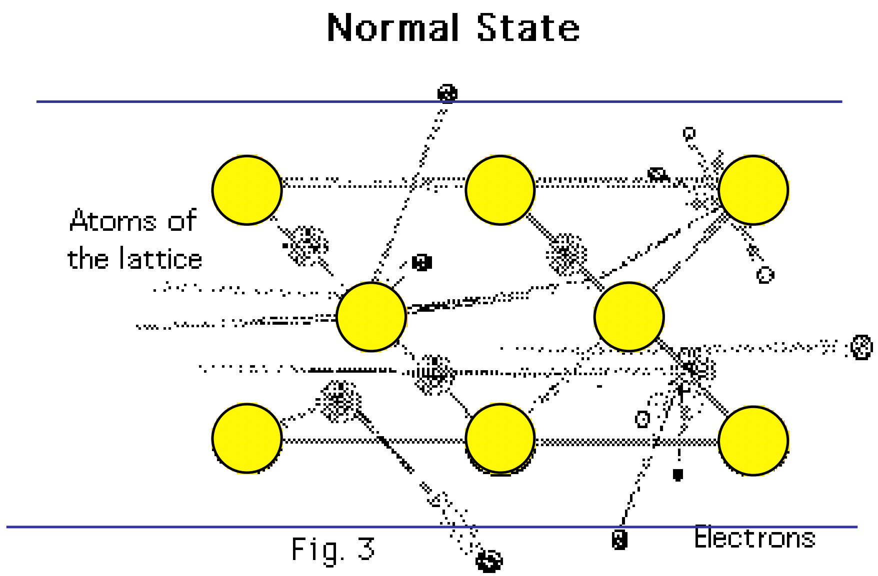
Normal metals

resistance = losses = bad efficiency



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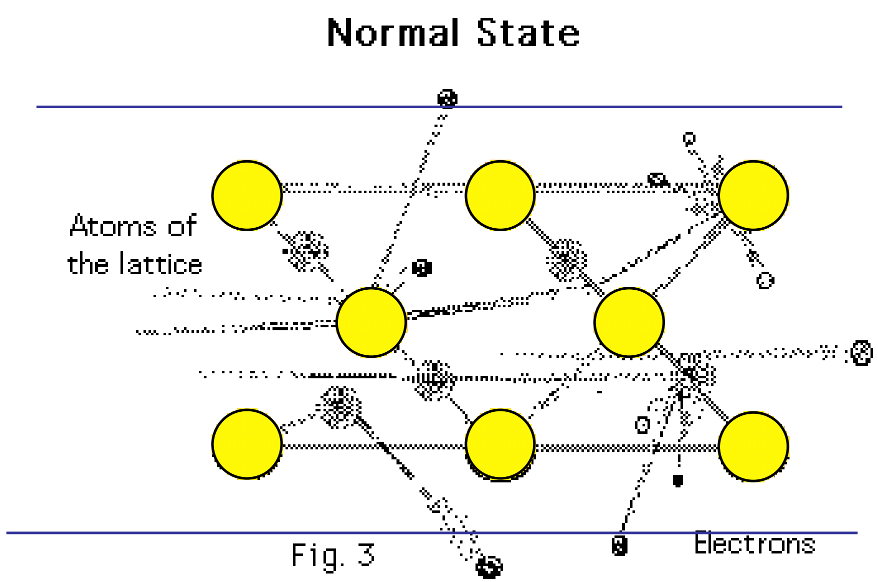

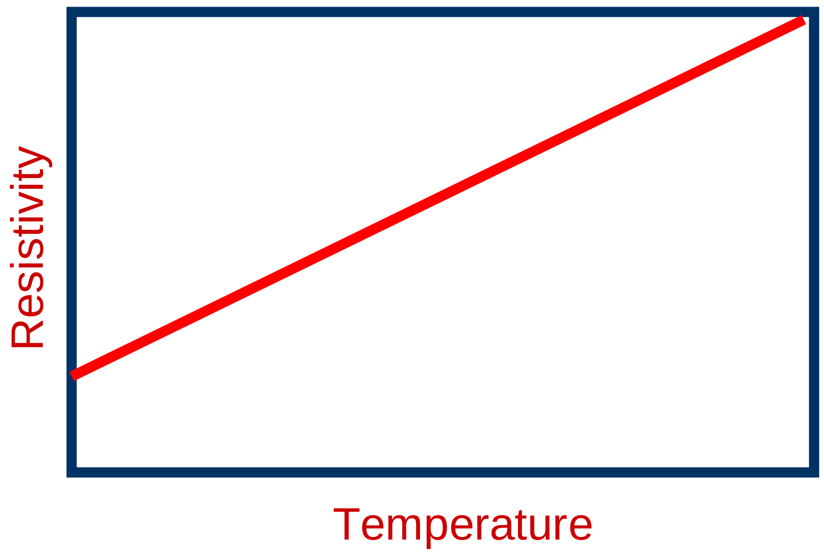

Normal conductors

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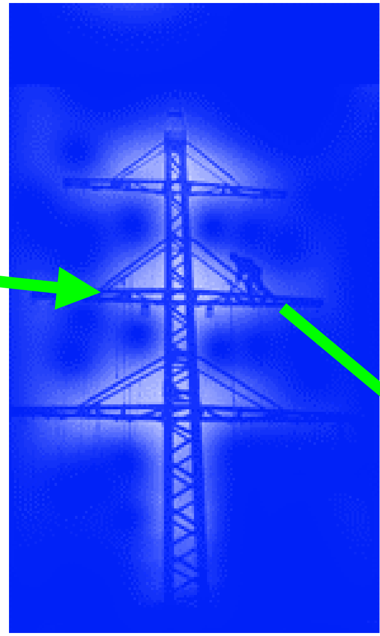
Normal conductors



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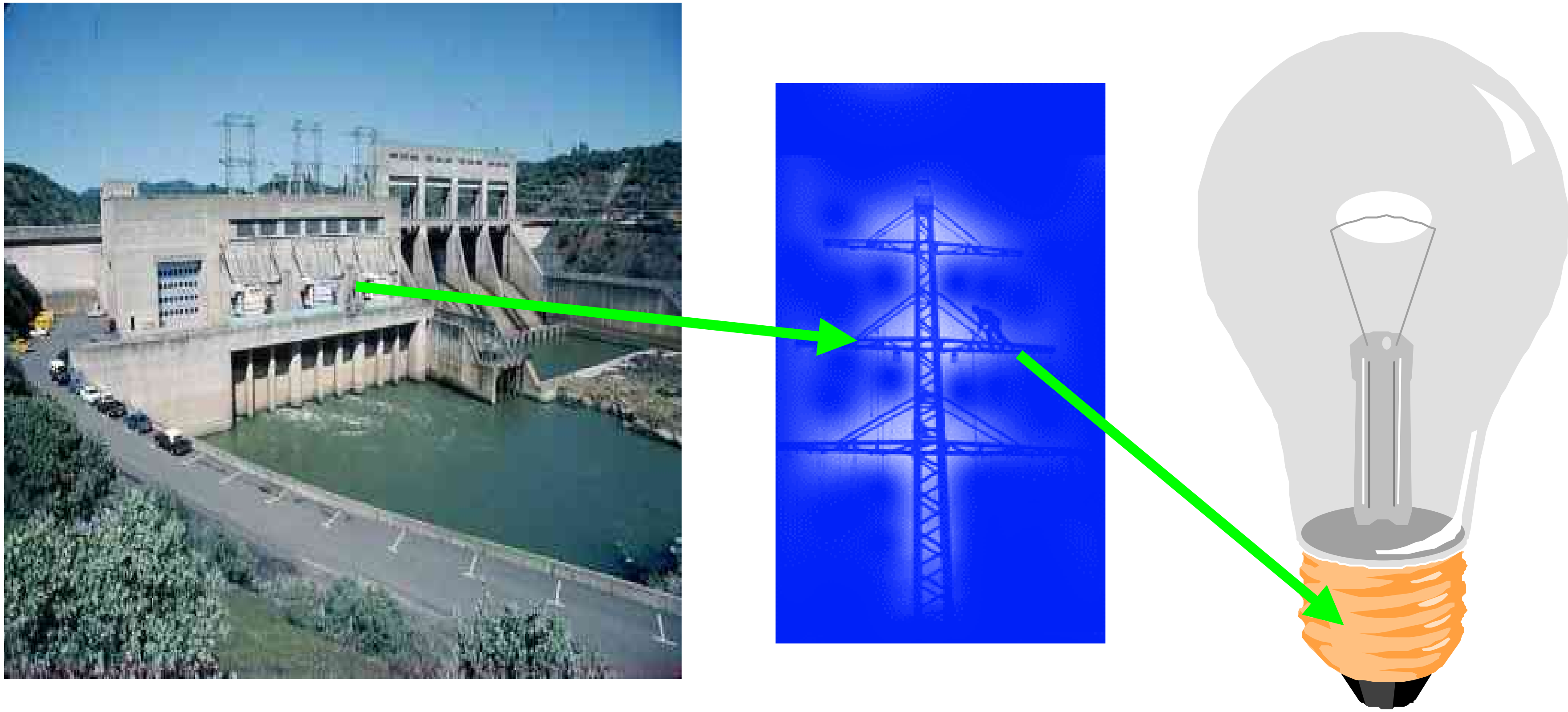
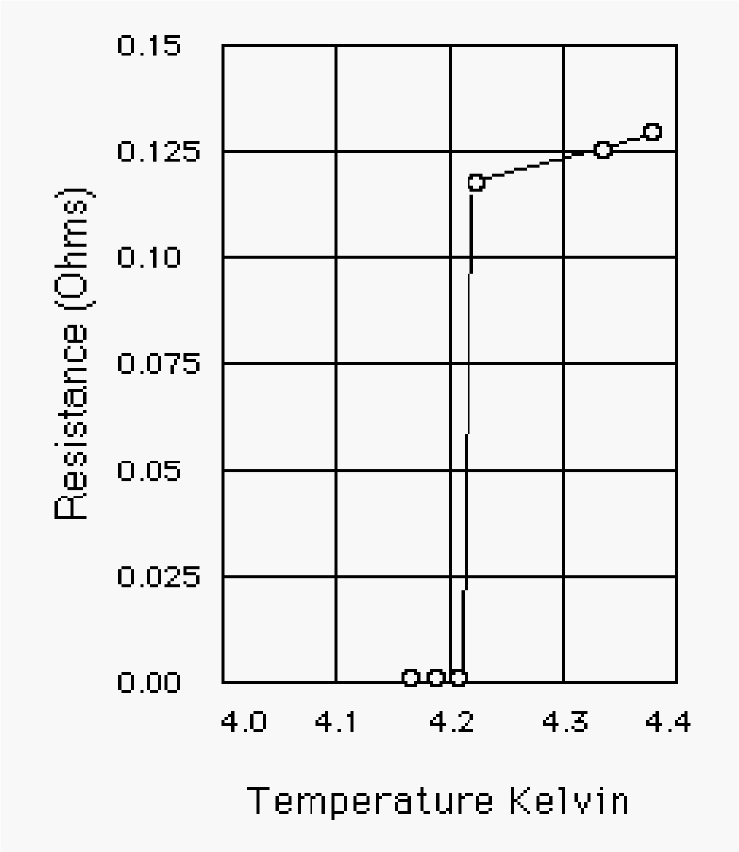
Normal conductors



Superconductivity

Kamerlingh-Onnes (1911)



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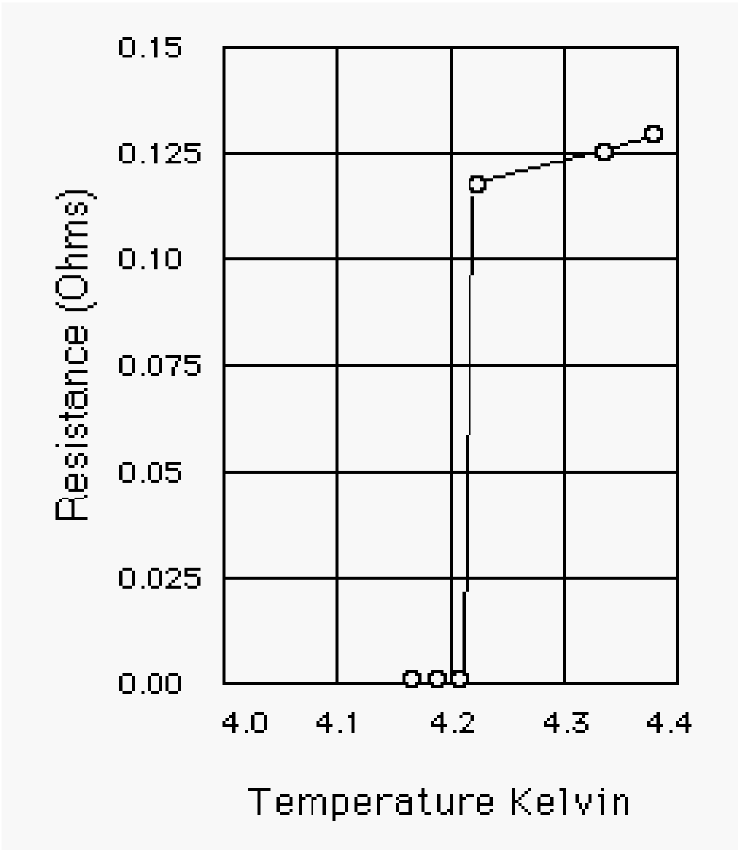
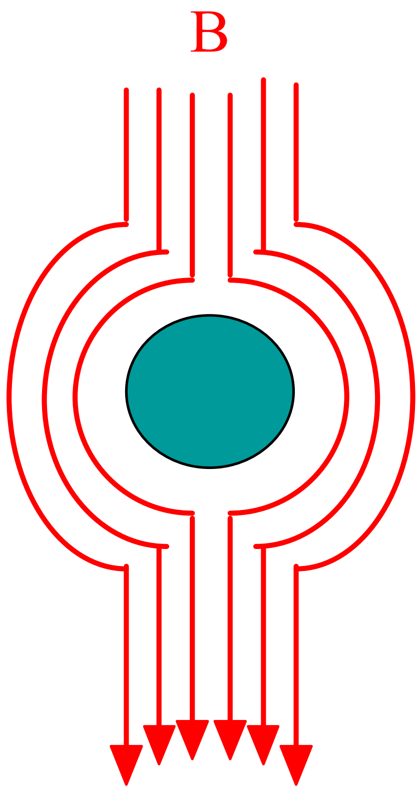


Meissner effect (1933)



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Meissner Effect
(Levitation)

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Levitation: examples

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Levitation: examples

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But what is the mechanism producing superconductivity

Isotope effect

$T_c \approx M^{-\alpha}$ ––––> phonons are involved ($\omega_{ph} \approx M^{-0.5}$)

Table 1: measured coefficients α of the isotopic effect T_c - $M^{-\alpha}$

	α		α
Hg	0.50±0.03	Cd	0.50±0.10
Tl	0.50±0.10	Mo	0.33±0.05
Sn	0.47±0.02	Ru	0.00±0.10
Pb	0.48±0.01	Os	0.20±0.05

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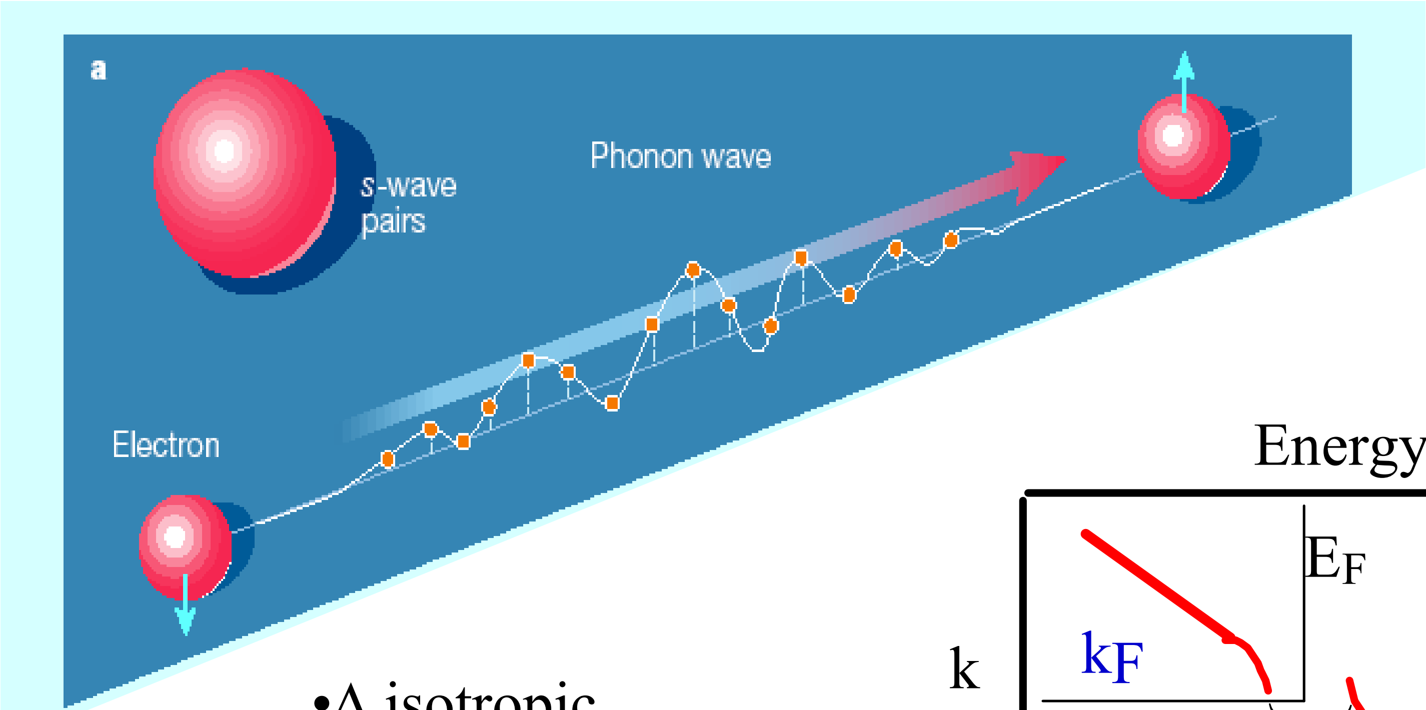
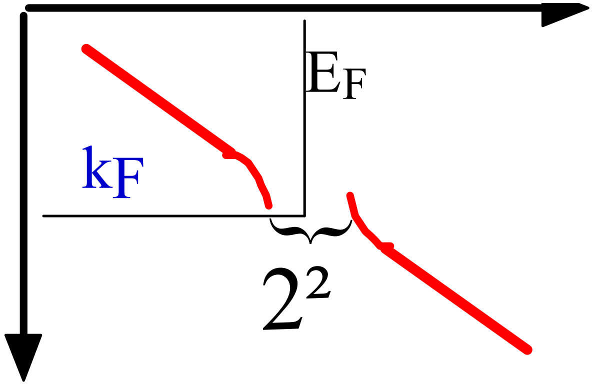
?

BCS Theory (1957)





•Δ isotropic

•$2\Delta(T=0)\approx 3.5\ k_B T_c$

• Cooper pairs (bosons) condense into a macroscopic coherent ground-sate

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Normal vs Superconducting electrons

Normal vs Superconducting electrons

“normal state”

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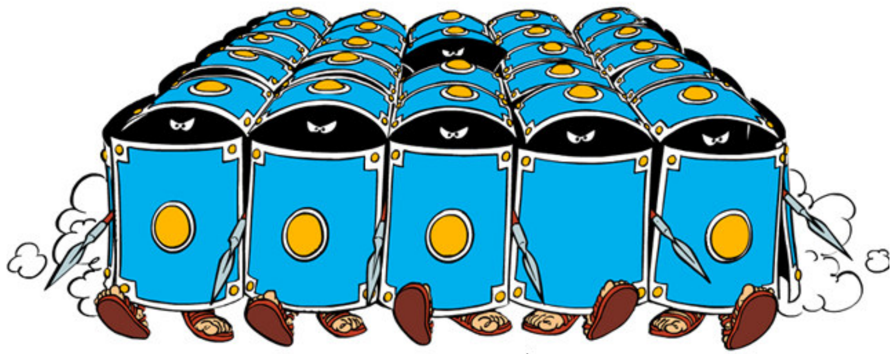
Normal vs Superconducting electrons

“normal state”

“Superconducting state”



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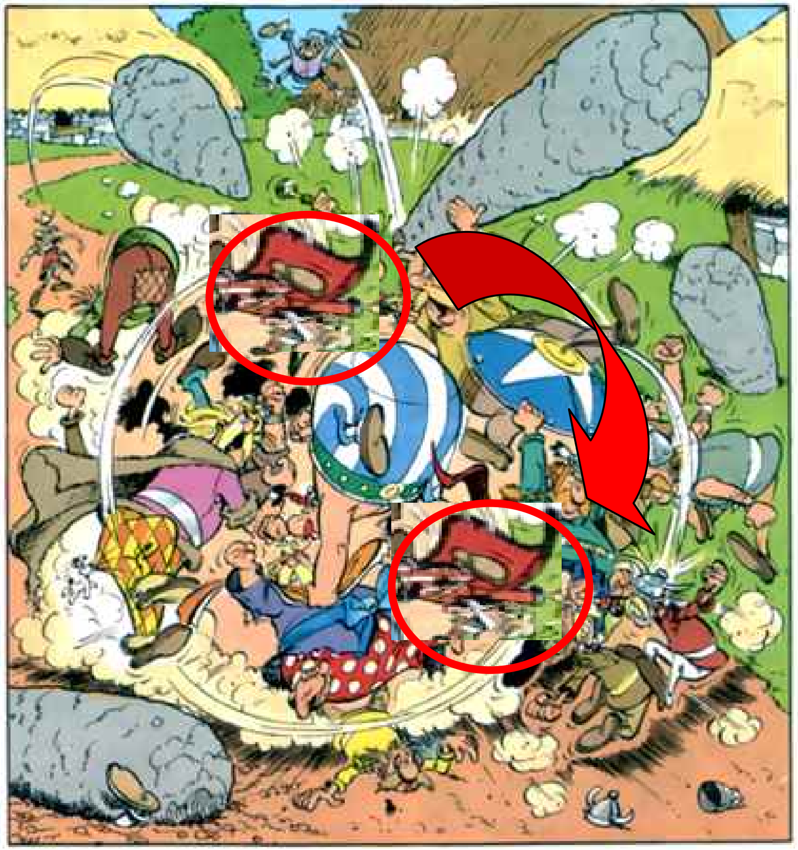
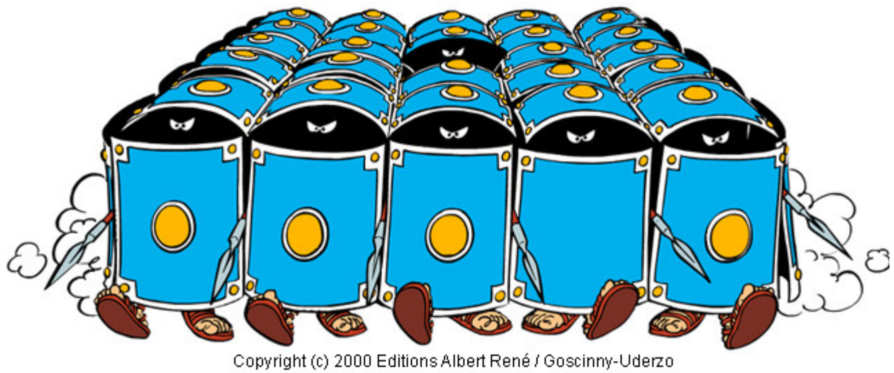


“Superconducting state”



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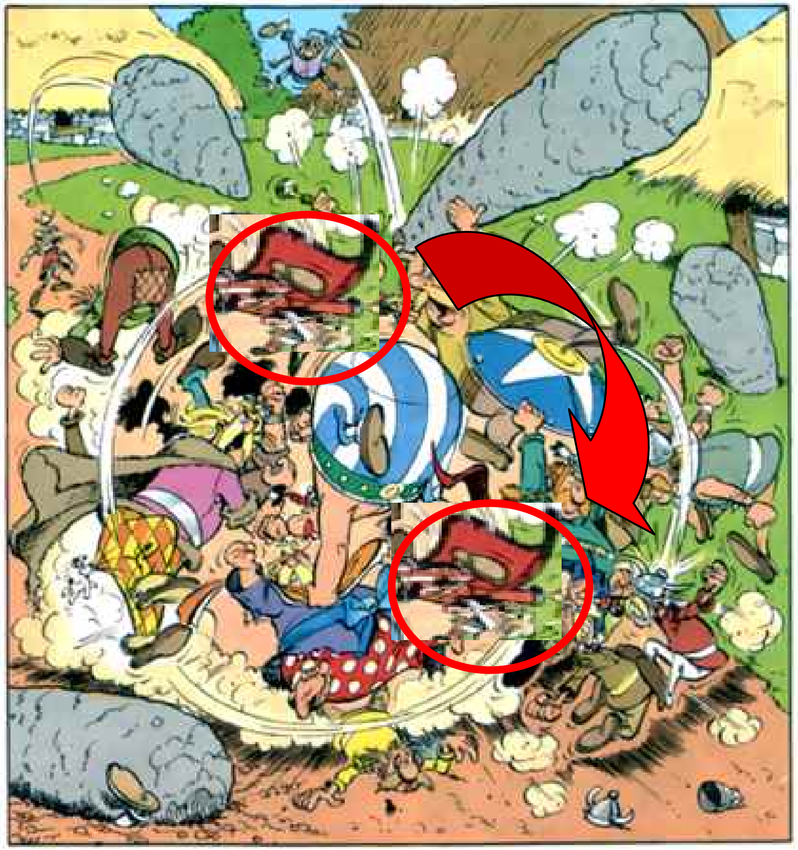

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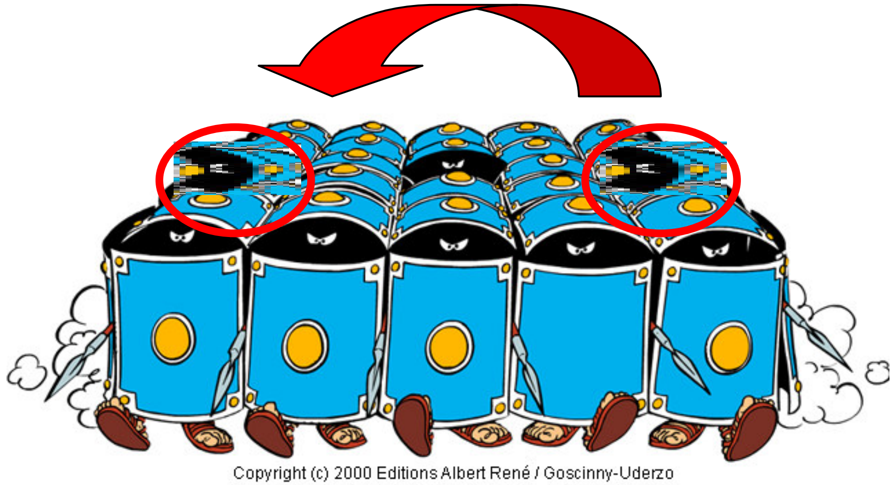

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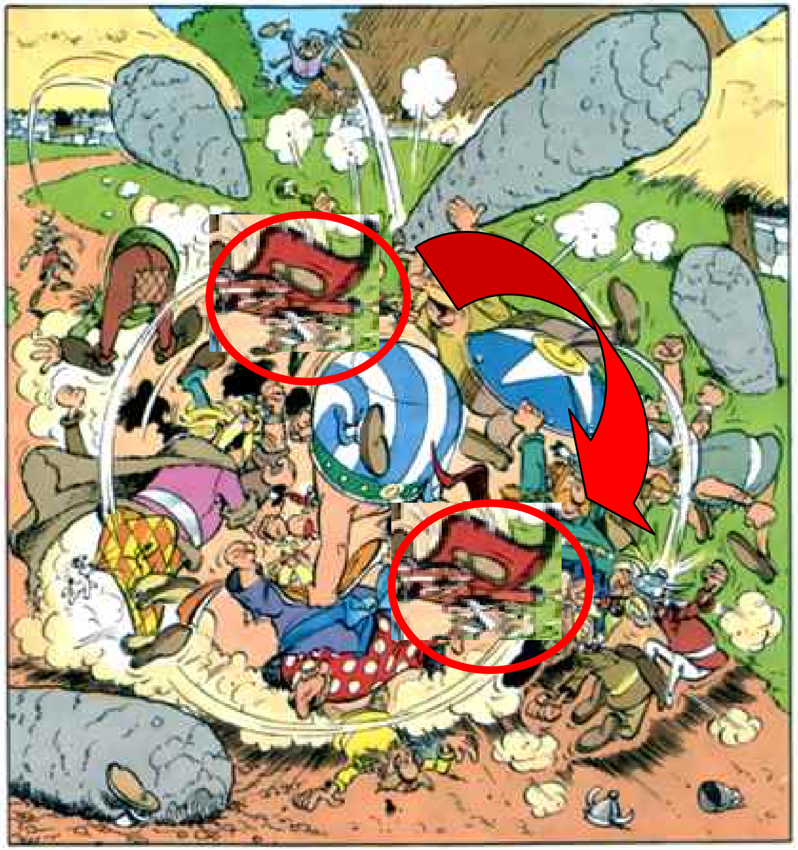

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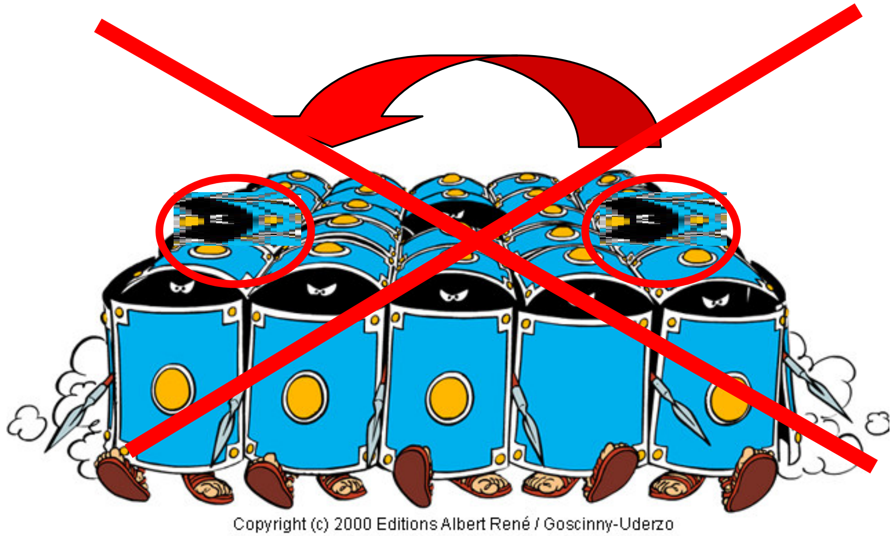


How can we experimentally prove the e-phonon interaction?

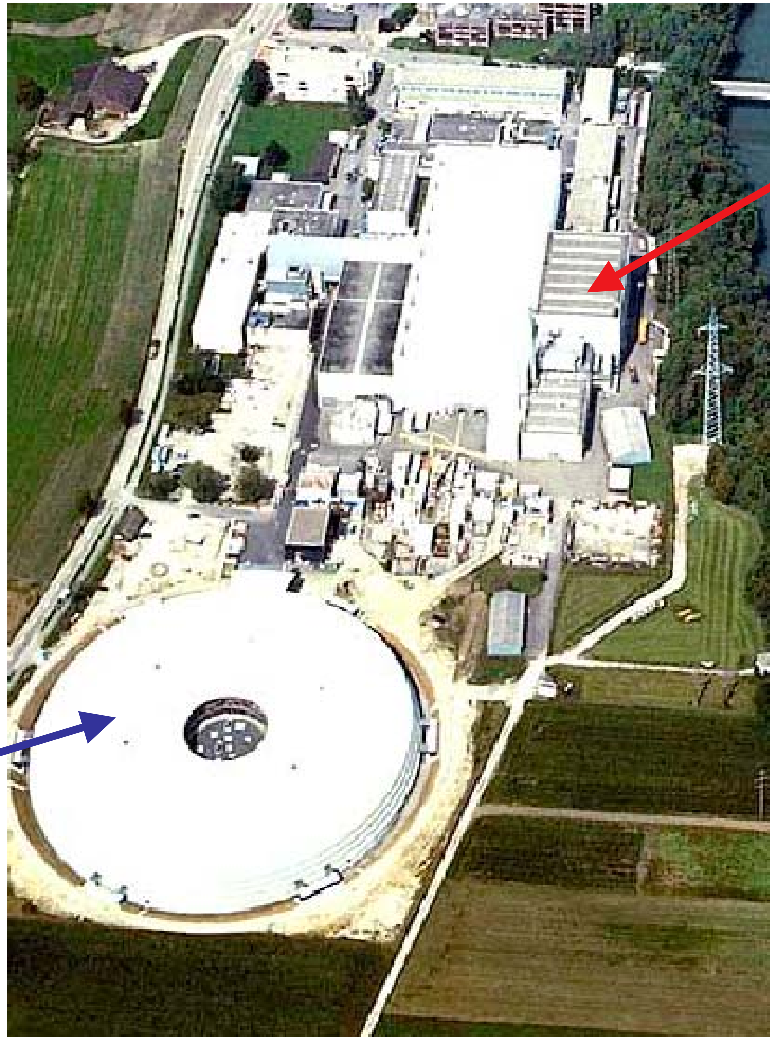
Study the dynamics of:

-electrons --> tunneling spectroscopy, photoemission

-lattice (phonons) ---> neutron scattering

Neutrons+Photons+Muons @ PSI

Neutrons
SINQ



Photons
SLS

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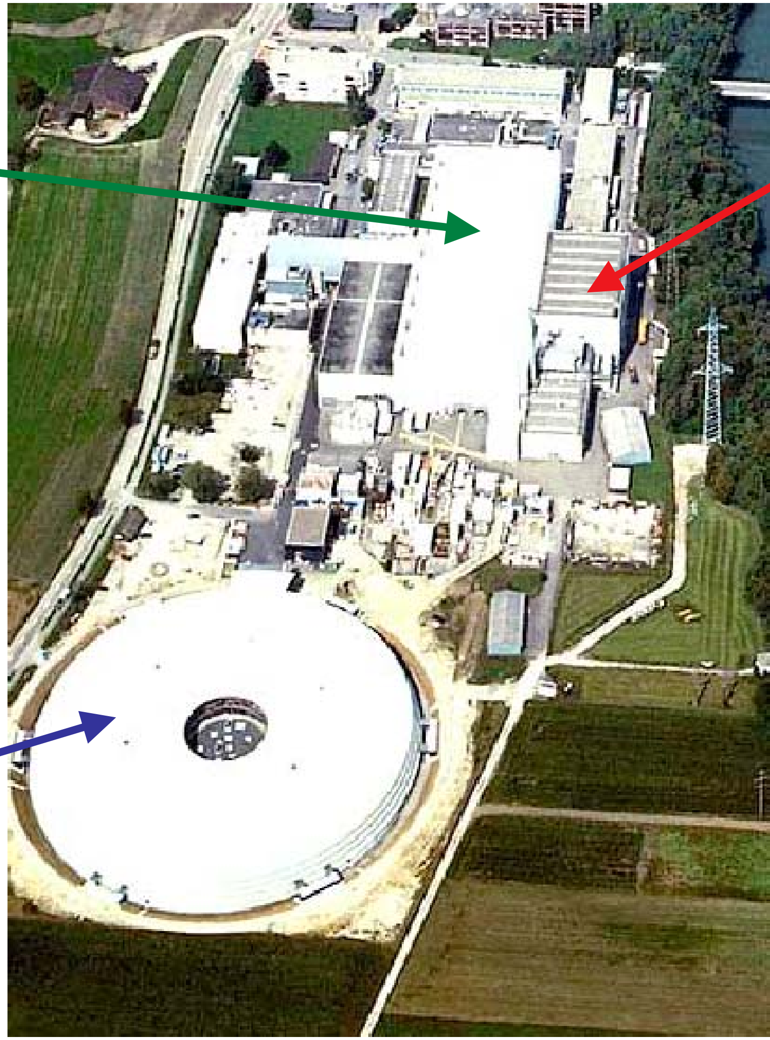
Neutrons+Photons+Muons @ PSI



Muons
SμS

Neutrons
SINQ

Photons
SLS

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Electrons in a periodic potential (metals)

(Free electron: $E=1/2mv^2=k^2/2m$)

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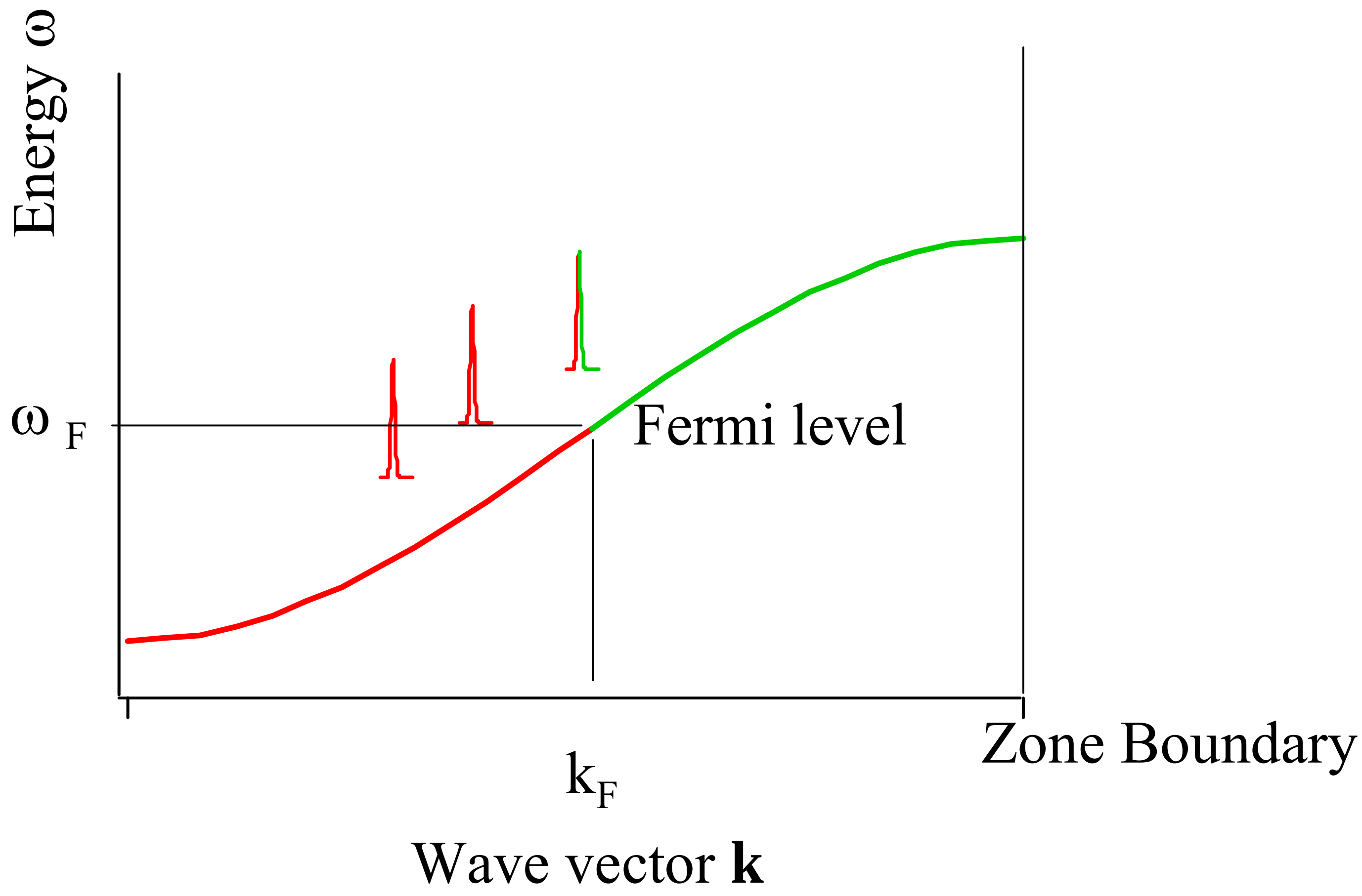
Electrons in a periodic potential (metals)

(Free electron: $E=1/2mv^2=k^2/2m$)

a) No interactions --> delta function



b) Weak interactions--> Renormalized quasiparticles

$$m^*,\ \omega(\mathbf{k}) = \omega_0(\mathbf{k}) + \Sigma'(\omega, \mathbf{k}) \qquad \tau(\mathbf{k},\omega) \sim 1/\Sigma''(\mathbf{k},\omega)$$

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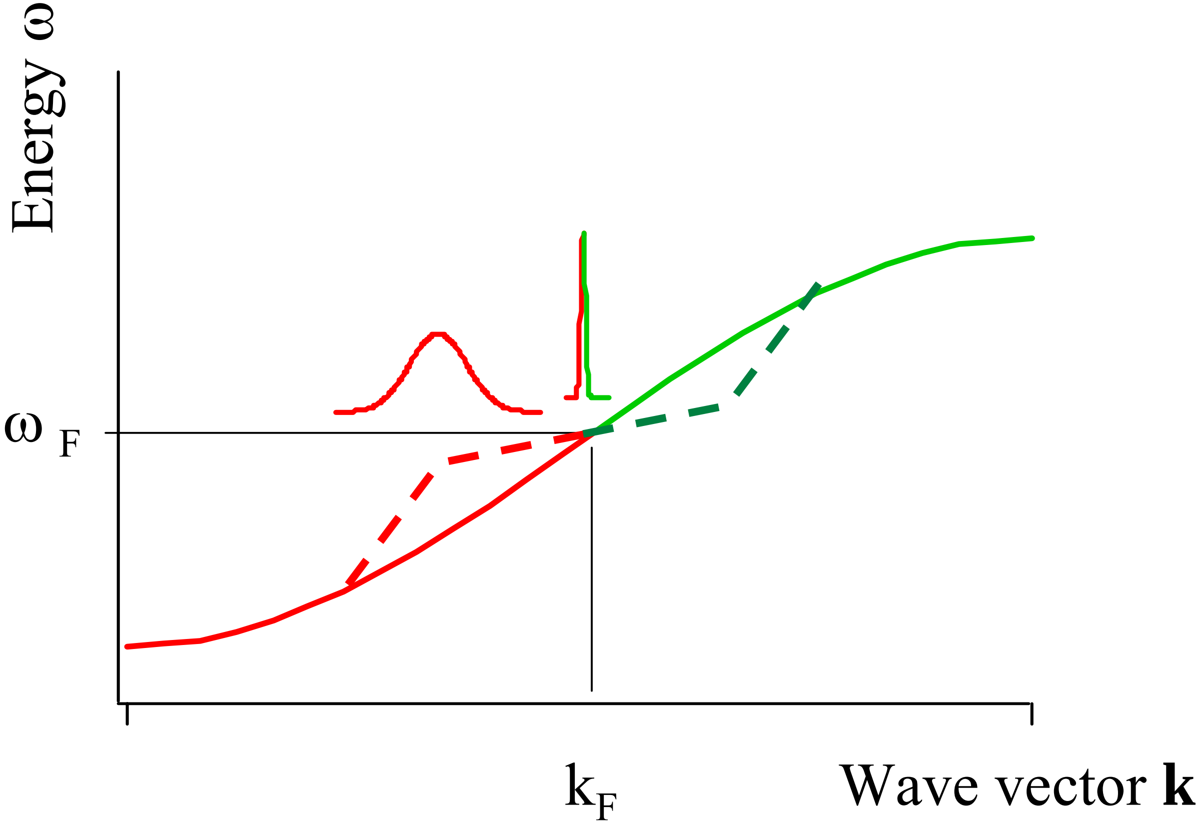

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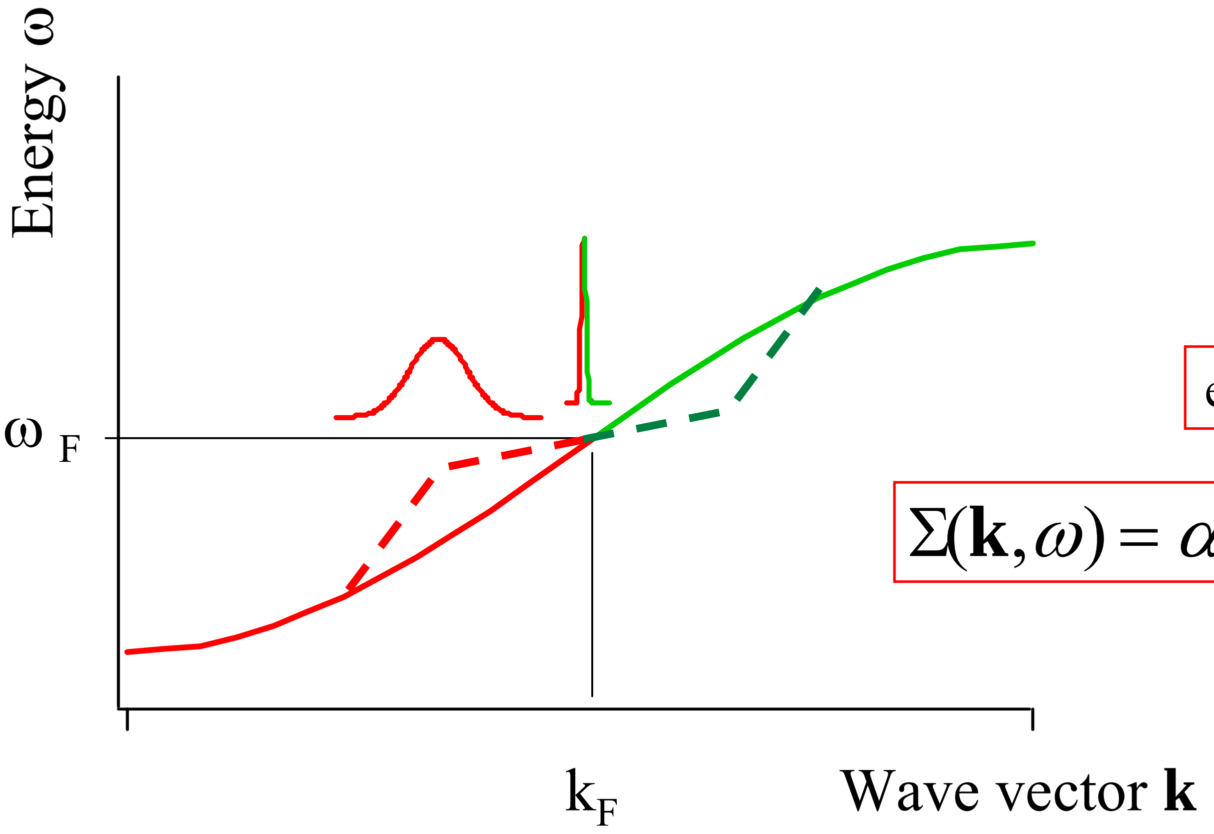

e-e interaction

$$\boxed{\Sigma(\mathbf{k},\omega) = \alpha\omega + i(\beta\omega^2 + \gamma T^2)}$$

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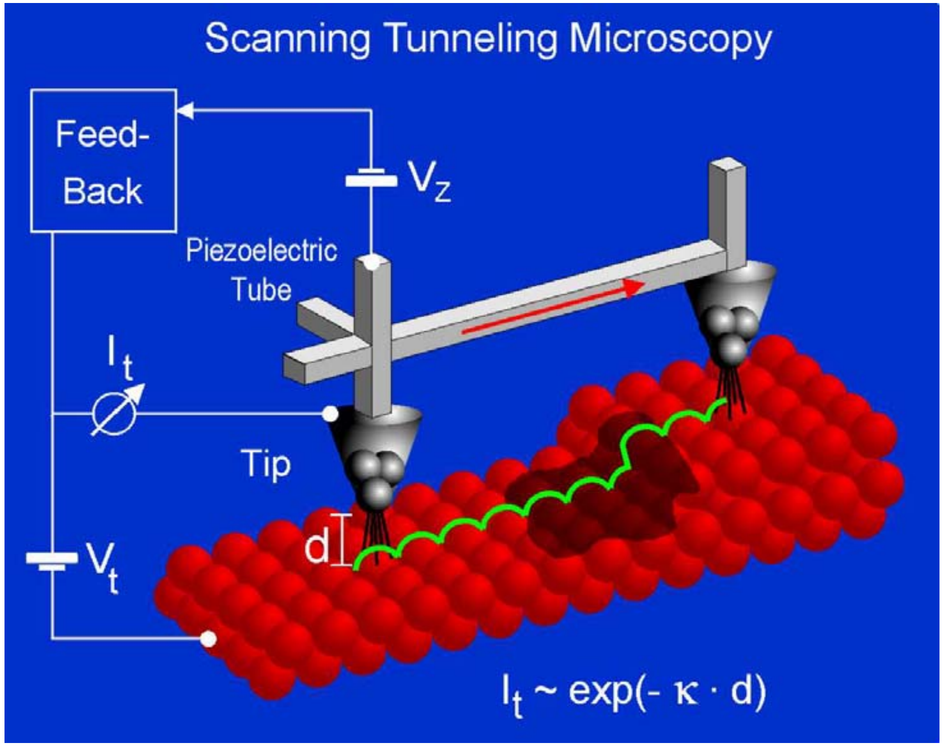
Evidence for e-Phonon Interaction: Tunneling Spectroscopy



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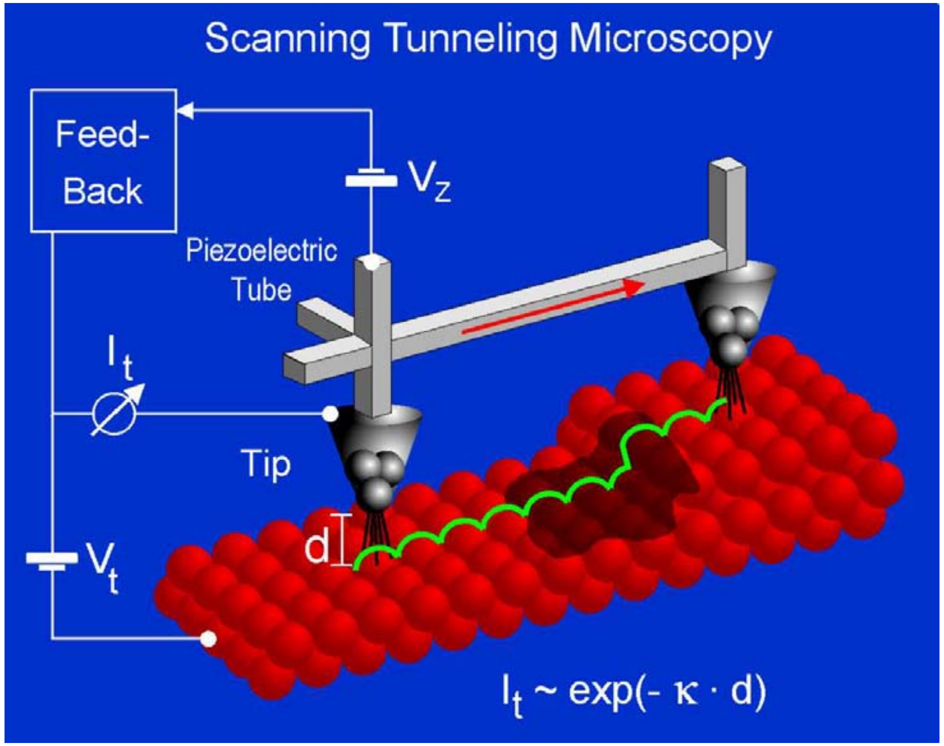

Electronic DOS

$$\frac{dI}{dV} \propto N(\omega) --> \alpha^2(\omega) g(\omega)$$

Phonon DOS

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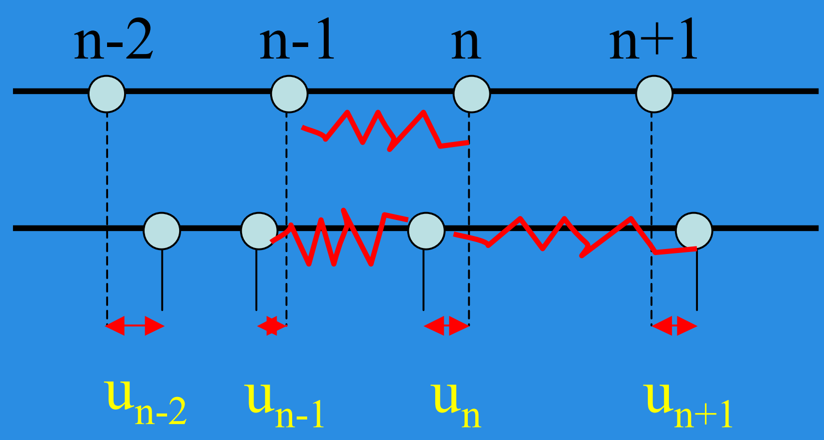
Phonons

Equilibrium



$$F_n = \beta(u_{n+1} - u_n) - \beta(u_n - u_{n-1})$$

$$M\ddot{u}_n = F_n = \beta(u_{n+1} + u_{n-1} - 2u_n)$$

$$u_n = \xi e^{i(\omega t + qna)}$$

$$\omega = \pm\sqrt{\frac{4\beta}{M}}\sin\left(\frac{qa}{2}\right)$$

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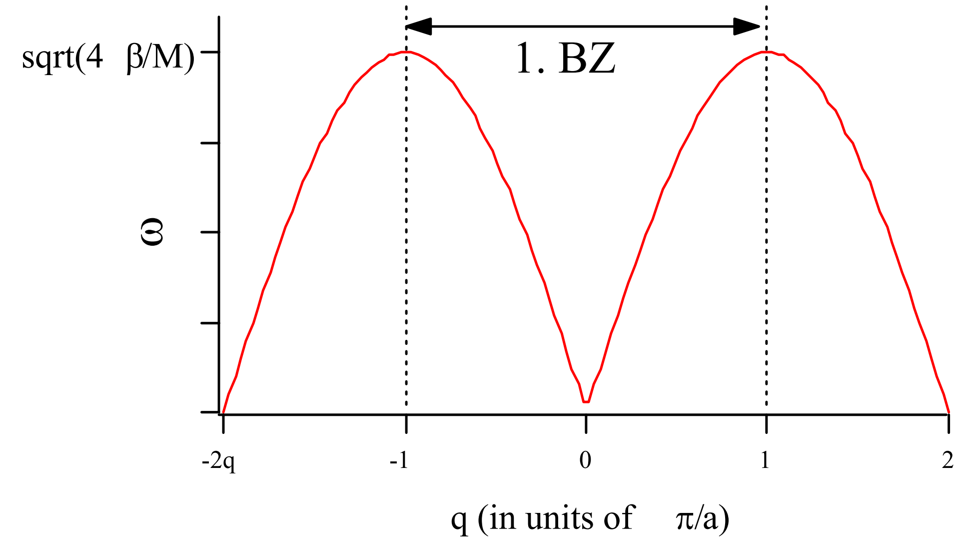
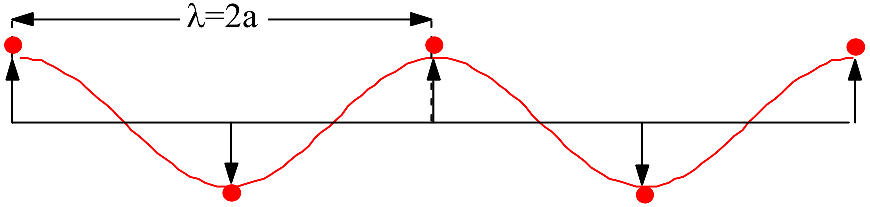
The coherent displacement of atoms can be visualized by the ratio:

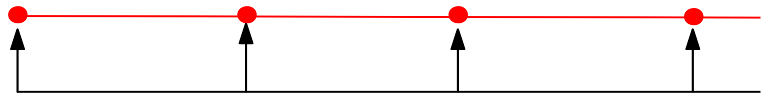
$$u_n / u_{n+1} = e^{-iqa}$$



a) Zone-boundary: q=π/a

b) Zone center: q=0





$$u_n / u_{n+1} = -1$$

$$u_n / u_{n+1} = 1 \qquad \lambda = \infty$$

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Connection to “real world” ?

Debye-Modell for small q ($q \ll \pi/a$)

$$\sin\left(\frac{qa}{2}\right) \approx \frac{qa}{2}$$

$$\omega \approx \sqrt{\frac{\beta}{M}}aq$$

“Density” $\rho = M/a$, elastic constant $c = \beta a$

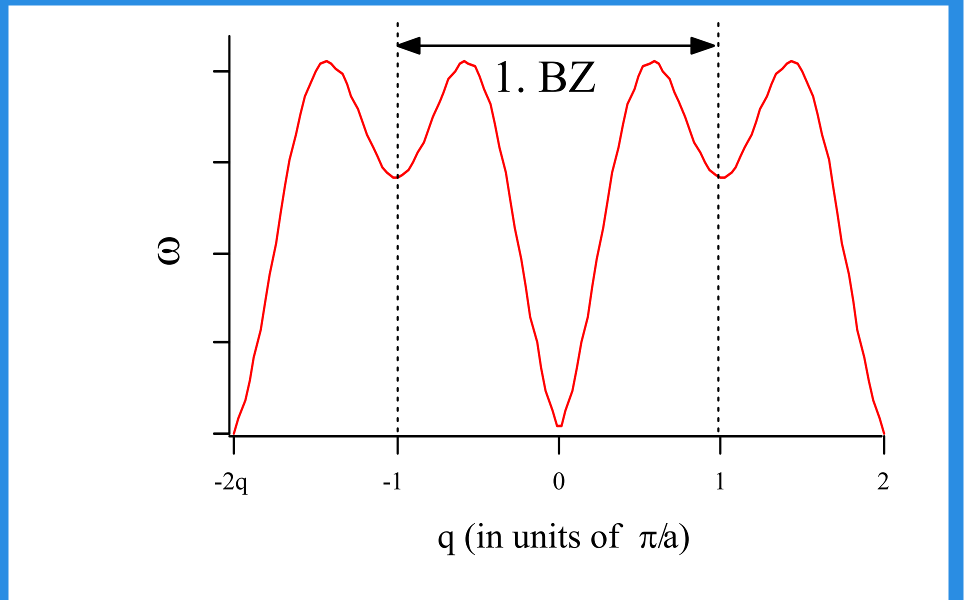
$$\omega \approx \sqrt{\frac{\beta}{M}}aq = \sqrt{\frac{c}{\rho}}aq = \mathrm{v}q$$

v= sound velocity

NN+NNN…

$$M\ddot{u}_n = F_n = \sum_j \beta_1(u_{n+j} + u_{n-j}) + \beta_0 u_n \qquad u_n = \xi e^{i(\omega t + qna)}$$

$$\omega^2 = \frac{4}{M}\sum_j \beta_j \sin^2(\frac{jqa}{2})$$



$$\beta_1 = 2\beta_2$$

$$\beta_j = 0 \; if \;\; j > 2$$

Nuclear, Inelastic, Coherent Neutron Scattering

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{4\pi^3}{v_o} \cdot \frac{k'}{k} \sum_{s,\mathbf{q}} \frac{1}{\omega_s(\mathbf{q})} \left| \sum_{\mathbf{d}} \frac{< b_{\mathbf{d}} >}{\sqrt{M_{\mathbf{d}}}} e^{-W_{\mathbf{d}}(\mathbf{Q})} e^{i\mathbf{Q}\cdot\mathbf{d}} \left[\mathbf{Q} \cdot \mathbf{e}_{\mathbf{d},s}(\mathbf{q}) \right] \right|^2$$

$$\left\{ \left[n_s(\mathbf{q}) + 1 \right] \delta\left\{ \omega - \omega_s(\mathbf{q}) \right\} \sum_{\tau} \delta(\mathbf{Q} - \mathbf{q} - \tau) \right.$$

$$\left. + n_s(\mathbf{q}) \delta\left\{ \omega + \omega_s(\mathbf{q}) \right\} \sum_{\tau} \delta(\mathbf{Q} + \mathbf{q} - \tau) \right\}$$

$$n_s(\mathbf{q}) = \left[\exp\left\{ \frac{\hbar\omega_s(\mathbf{q})}{k_B T} \right\} - 1 \right]^{-1}.$$

Bose-Einstein statistics

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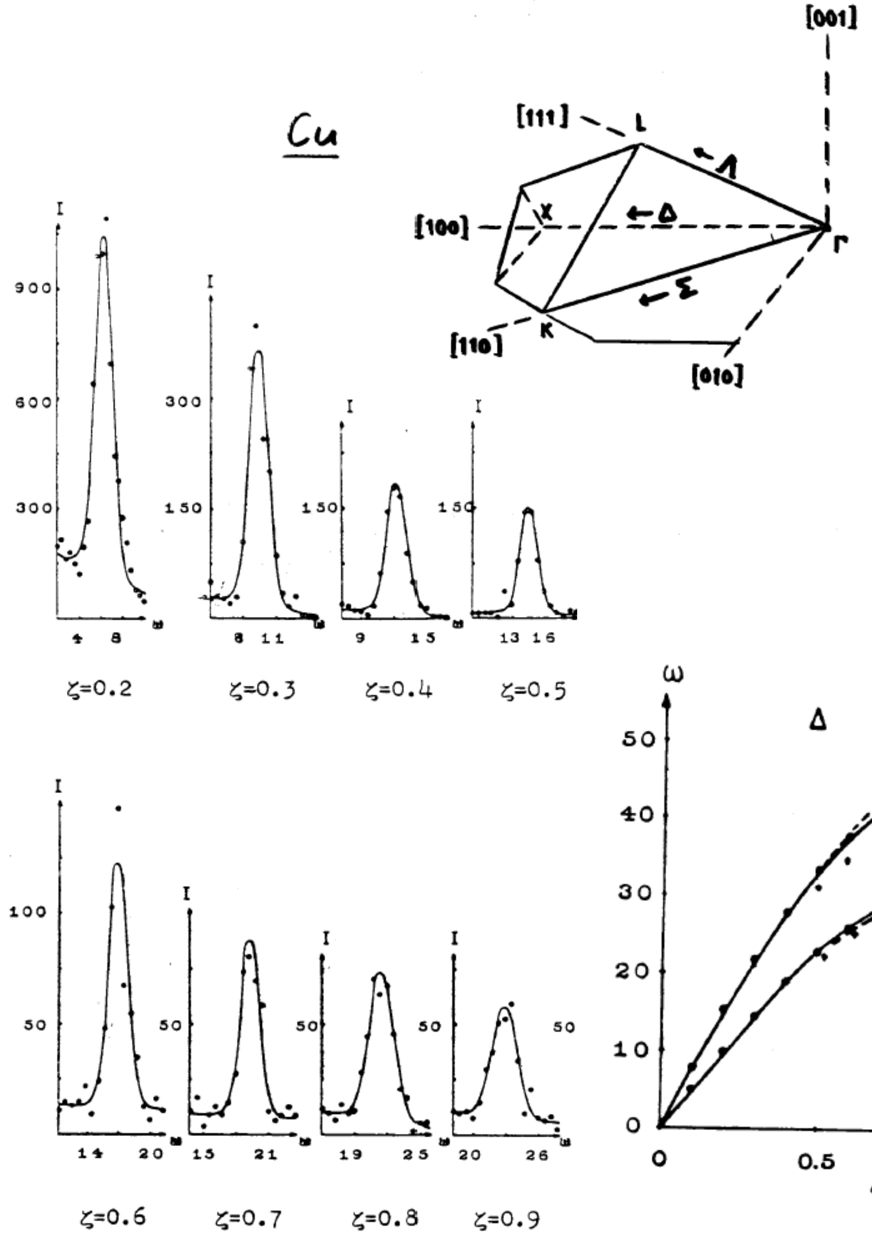
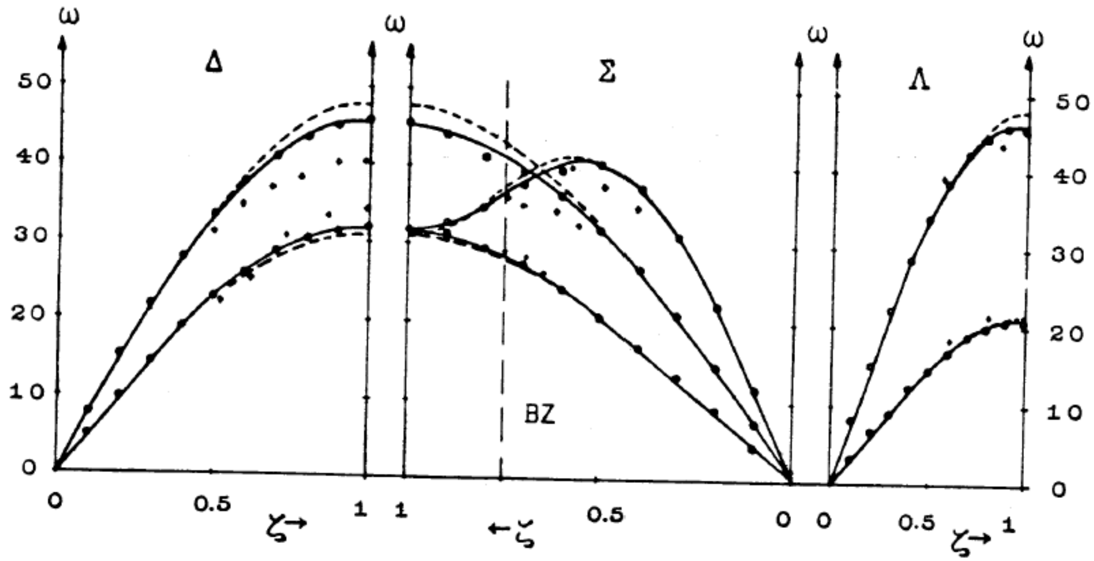
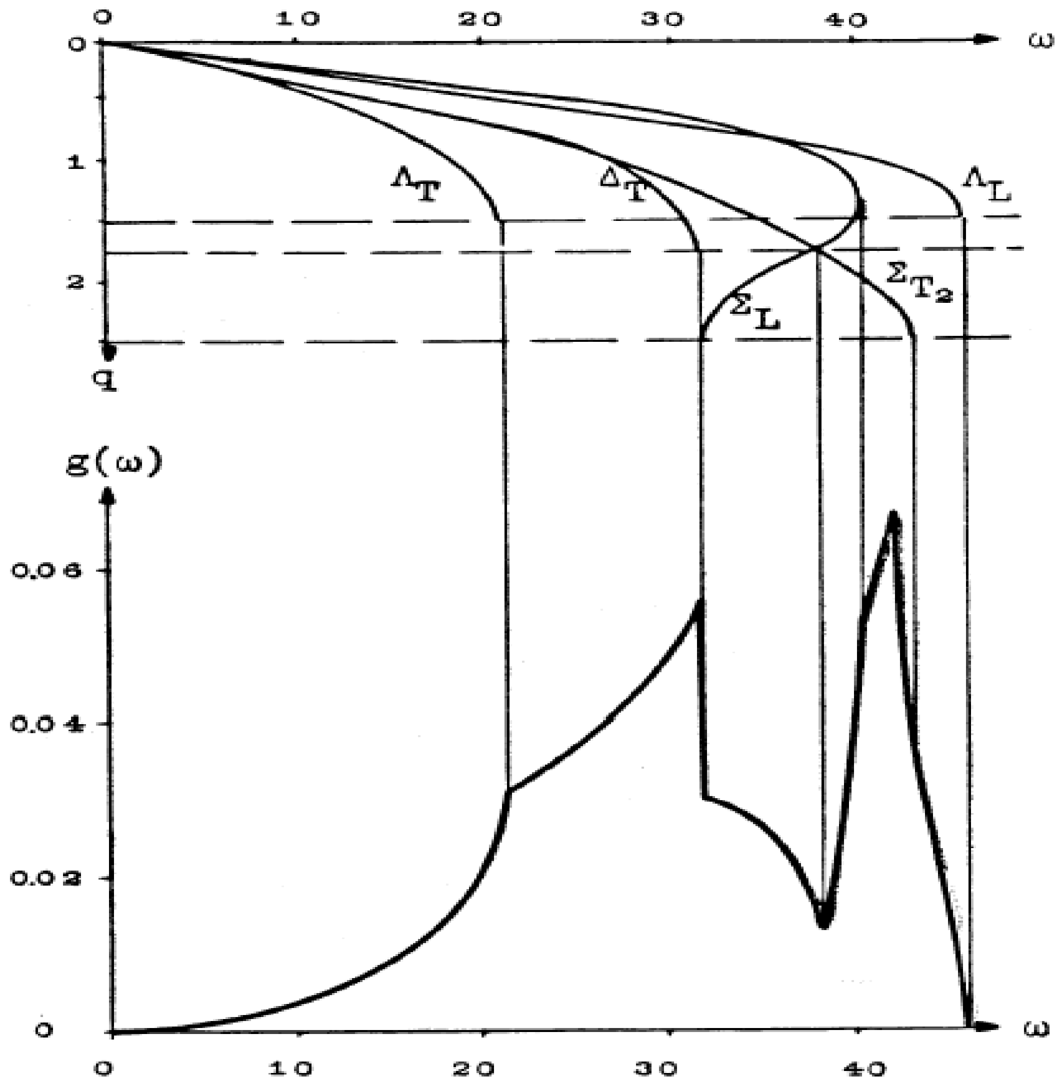


Phonons in Cu



Phonon density-of-state



Energy

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Nuclear Incoherent Neutron Scattering

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{1}{4M}\cdot\frac{k_f}{k_i}\left[<b^2> - <b>^2\right]e^{-2W(\mathbf{Q})}\left\langle\left(\mathbf{Q}\cdot\mathbf{e}_s(\mathbf{q})\right)^2\right\rangle\frac{g(\omega)}{\omega}\left\{\coth\left(\frac{\hbar\omega}{2k_BT}\right)\pm 1\right\}$$

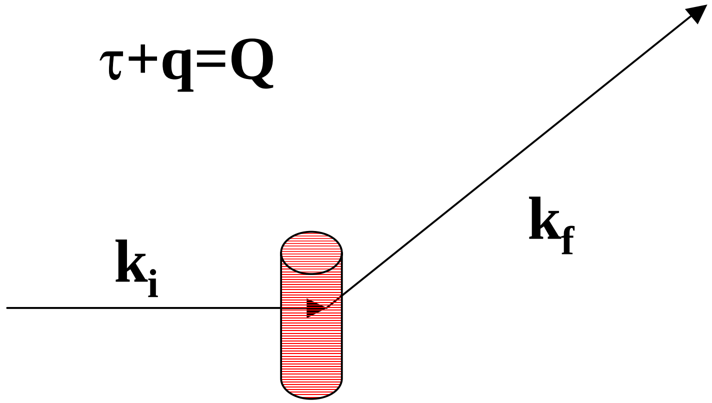
$\mathbf{Q}=\mathbf{k}_i-\mathbf{k}_f$= momentum transfer

$\hbar\omega = E_i - E_f =$ energy transfer

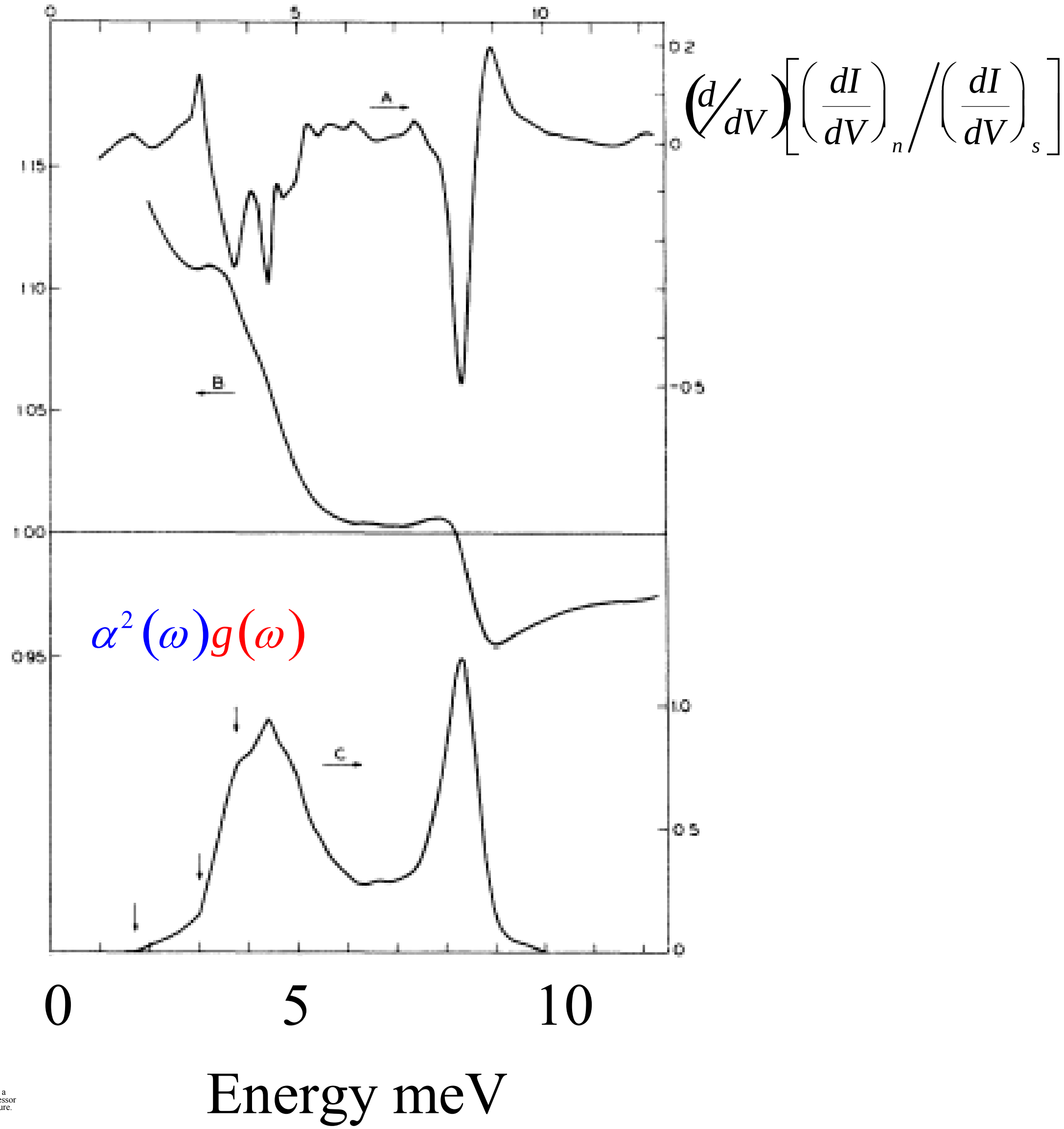
b= scattering length

$\mathbf{e}_s(\mathbf{q})$= phonon eigenvector

W($\mathbf{Q}$)=Debye-Waller factor



McMillan PRL 65- Pb-I-Pb tunneling



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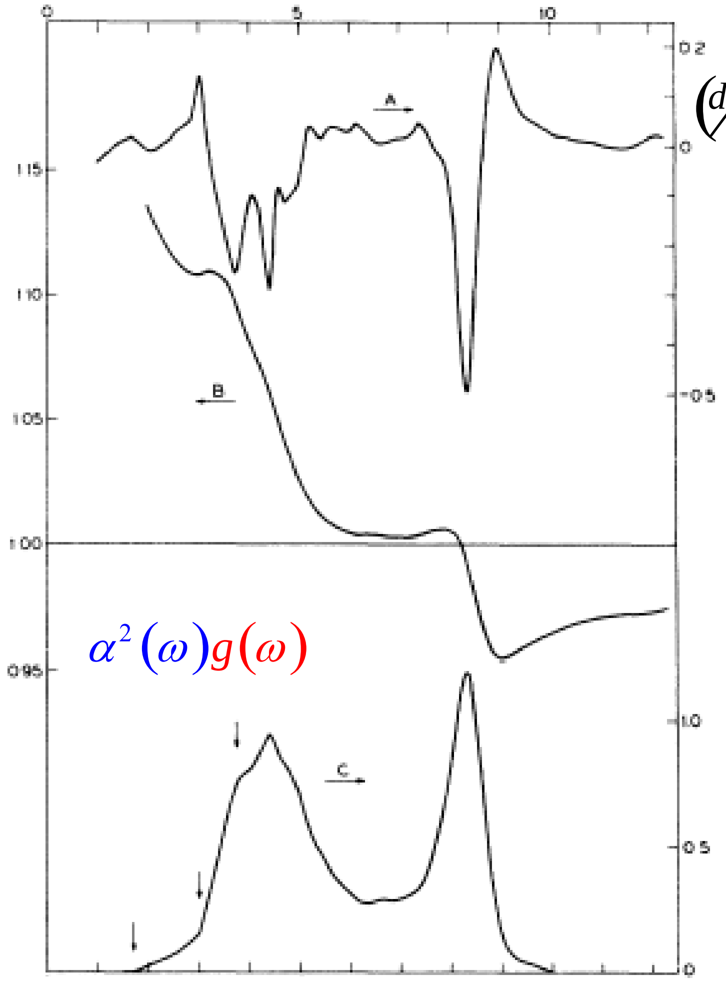
McMillan PRL 65- Pb-I-Pb tunneling



$$\left(\frac{d}{dV}\right)\left[\left(\frac{dI}{dV}\right)_n \Big/ \left(\frac{dI}{dV}\right)_s\right]$$

$\alpha^2(\omega)g(\omega)$

0 5 10

Energy meV

Stedman, PRB 67 (Neutrons)

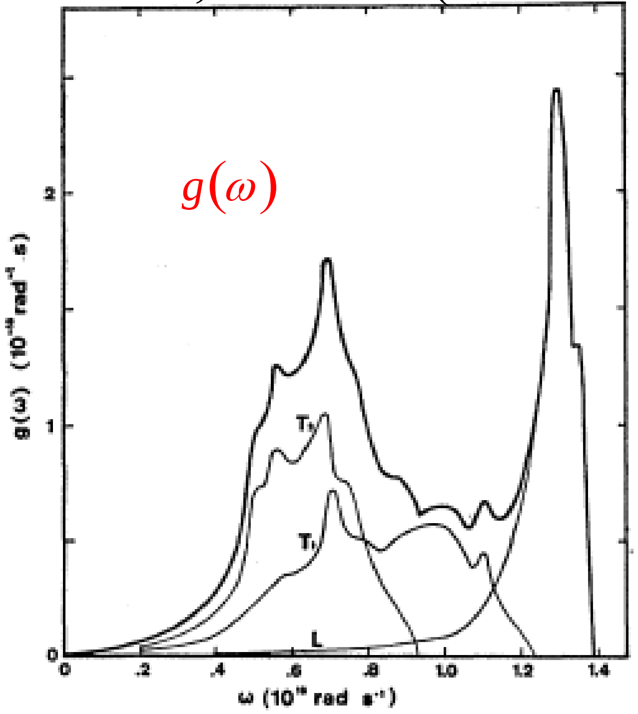


FIG. 2. The phc n-frequency distribution of lead.

10^{13} rad/s=6.6 meV

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Full understanding of superconductivity (1986)?

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J. M. Ziman (1972)

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TIFF (LZW) decompressor
are needed to see this picture.

Full understanding of superconductivity (1986)?

J. M. Ziman (1972)

“SC was long considered the most extraordinary and mysterious of the properties of metals; but the theory of Bardeen, Cooper and Schrieffer –the BCS theory- has explained so much that we can say that we now understand the superconducting state almost as well as we do the normal ‘state’.”

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TIFF (LZW) decompressor
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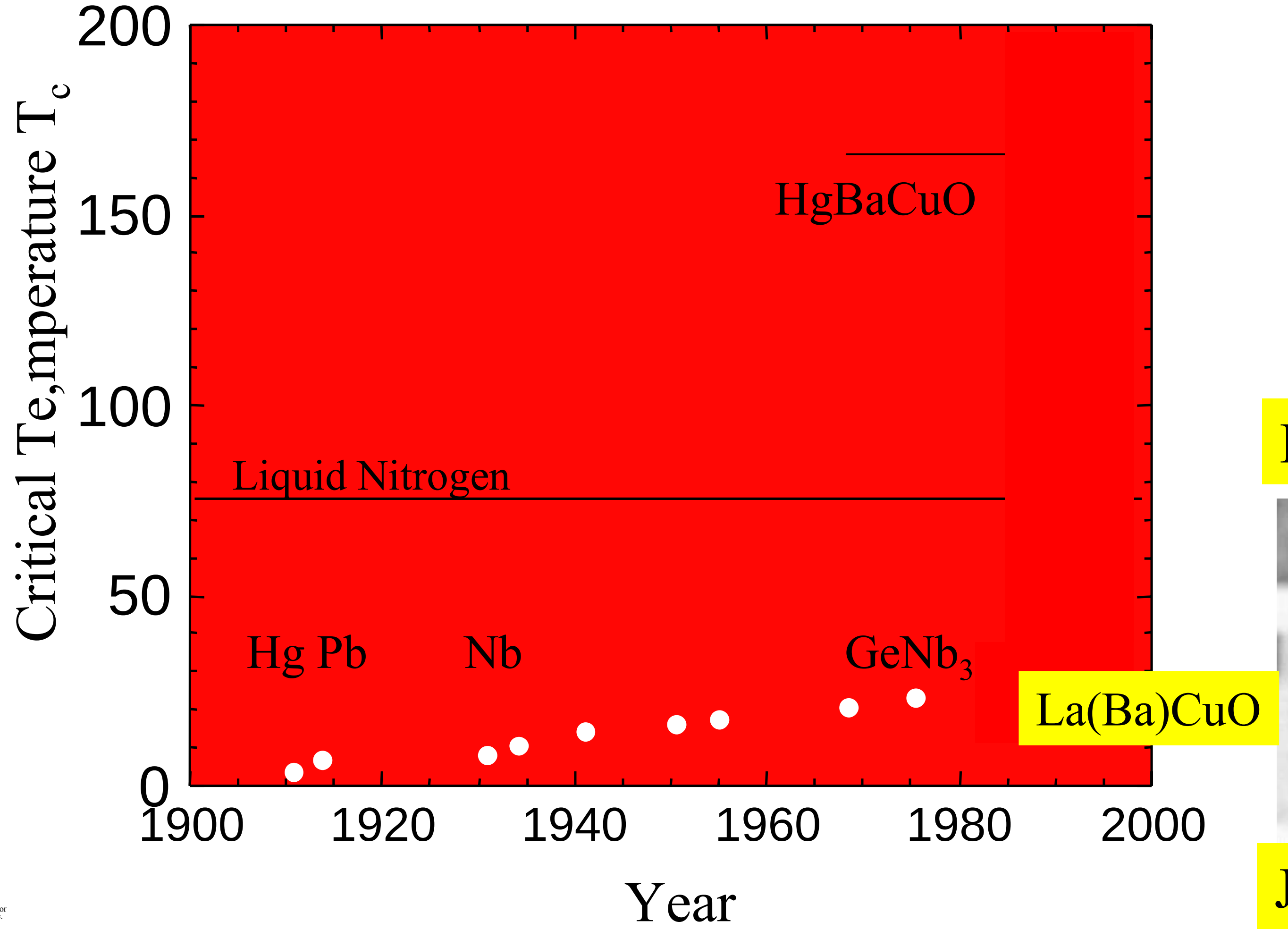
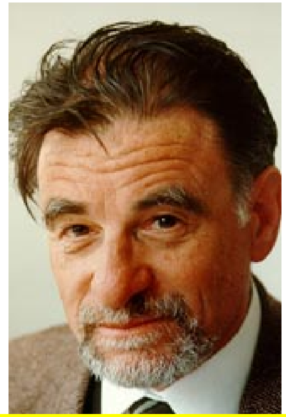
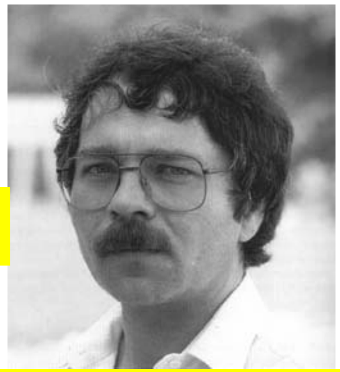
The 1986 revolution





K. A. Müller



J. G. Bednorz

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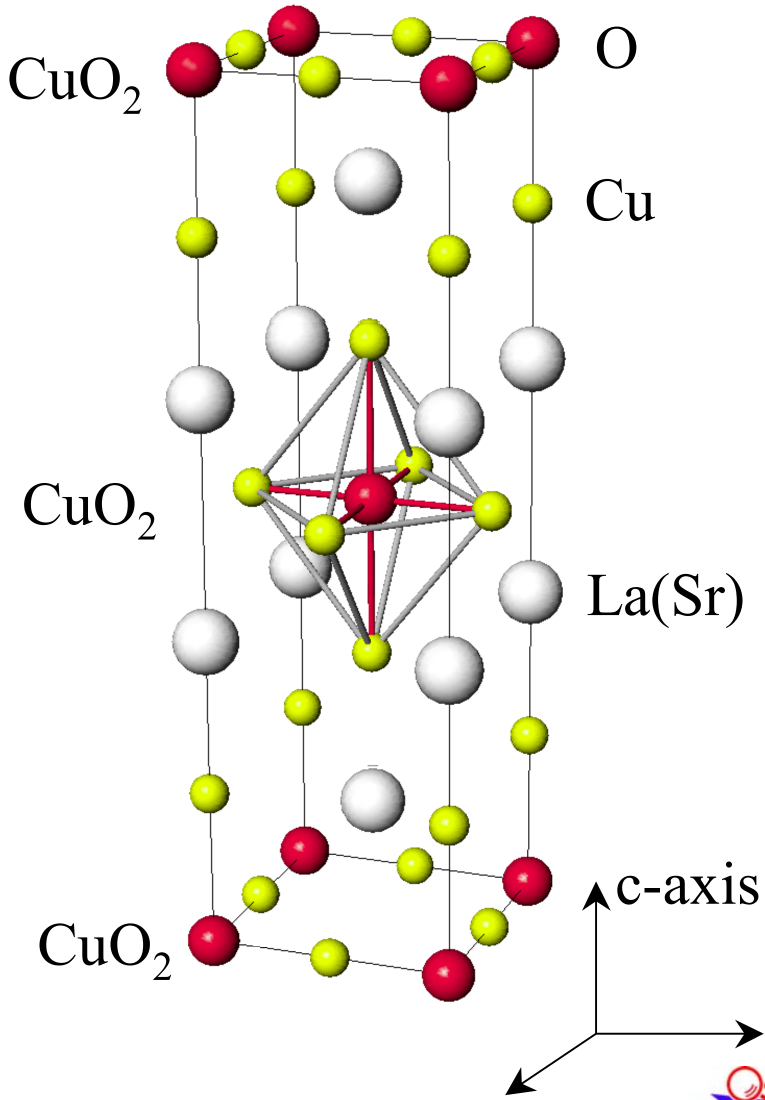
2D-Structure La(2-x)Sr(x)CuO(4)



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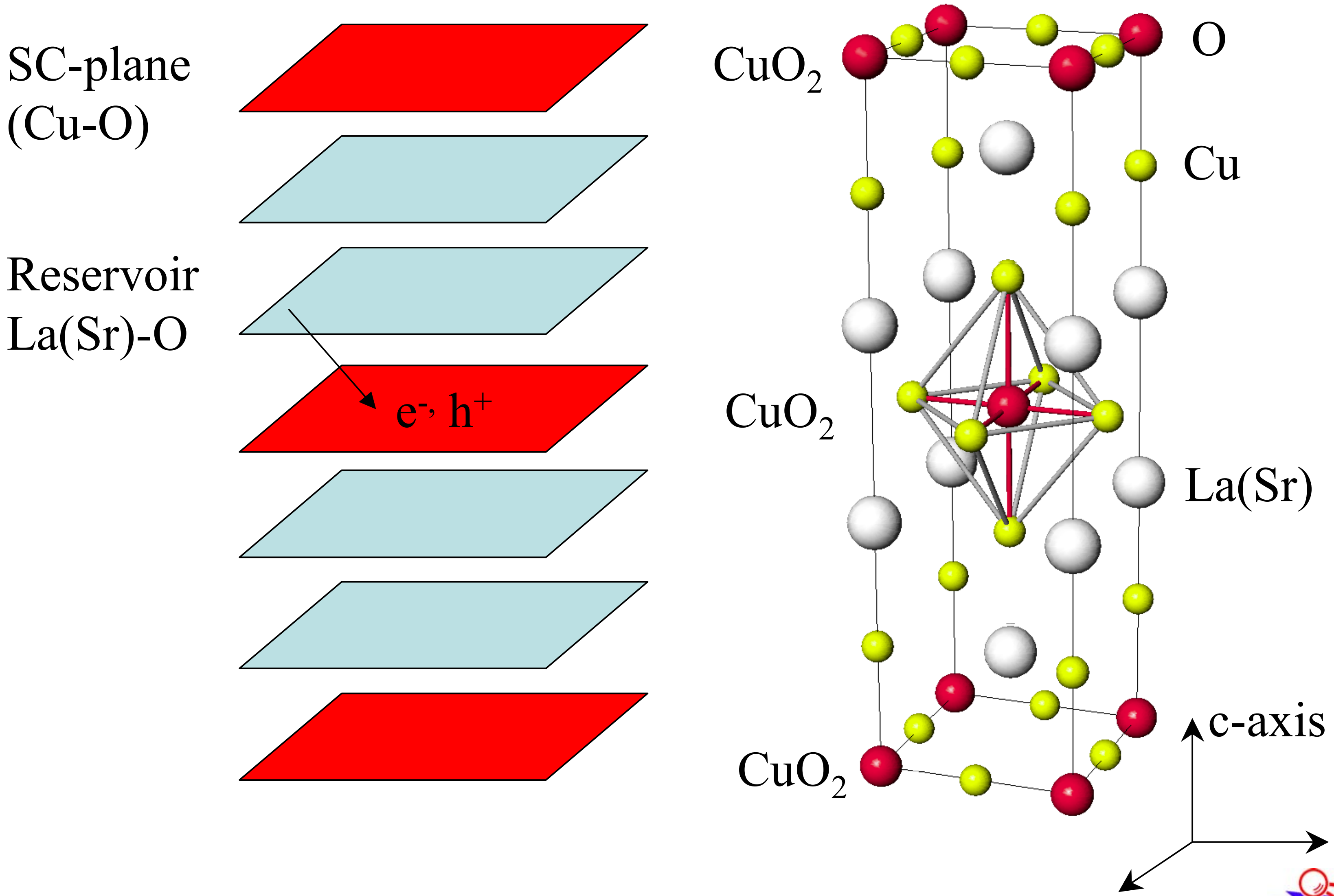
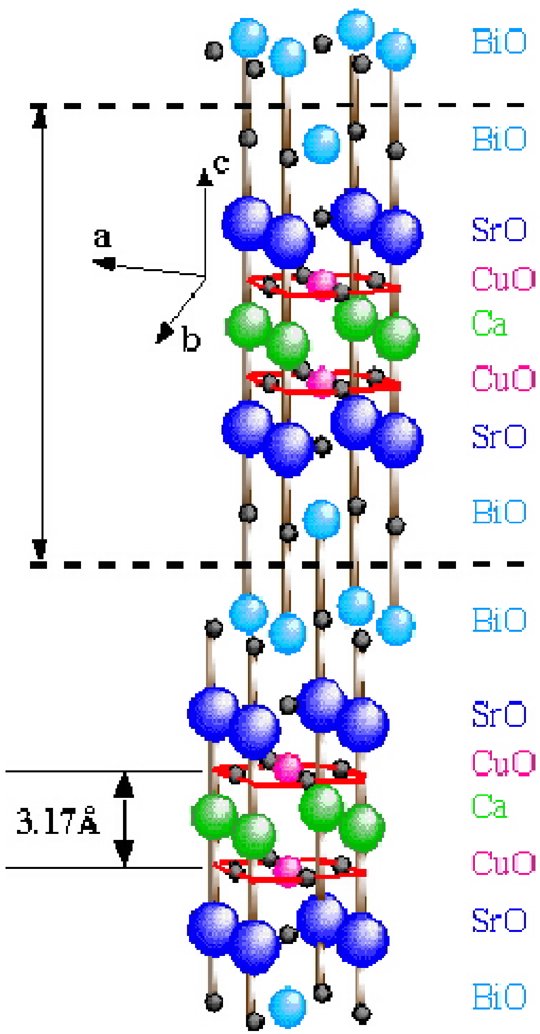
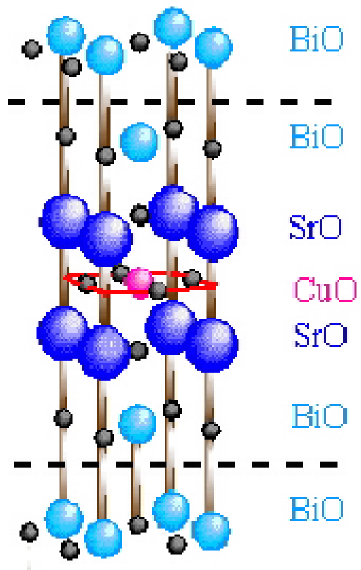


$Bi_2Sr_2Ca_nCu_{1+n}O_{6+2n}$ (n=0,1…)

Bi2212 (n=1)



Bi2201 (n=0)



QuickTime™ and a
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are needed to see this picture.

The Power of Neutron Diffraction

$$\left(\frac{d\sigma}{d\Omega}\right)_{inc} = \left[\langle b^2\rangle - \langle b\rangle^2\right]\sum_{j=j'} e^{-i\mathbf{Q}\left(\hat{\mathbf{R}}_{j'}-\hat{\mathbf{R}}_j\right)} = N\left[\langle b^2\rangle - \langle b\rangle^2\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{coh} = N_0\frac{(2\pi)^3}{v_0}\sum_{\tau}\left|\mathrm{F}_{\tau}\right|^2\delta(\mathbf{Q}-\tau);$$

Structure factor $\mathrm{F}_{\tau} = \sum_d b_d e^{i\mathbf{Qd}}$

τ=reciprocal lattice vector

d= position of atom d
in unit cell

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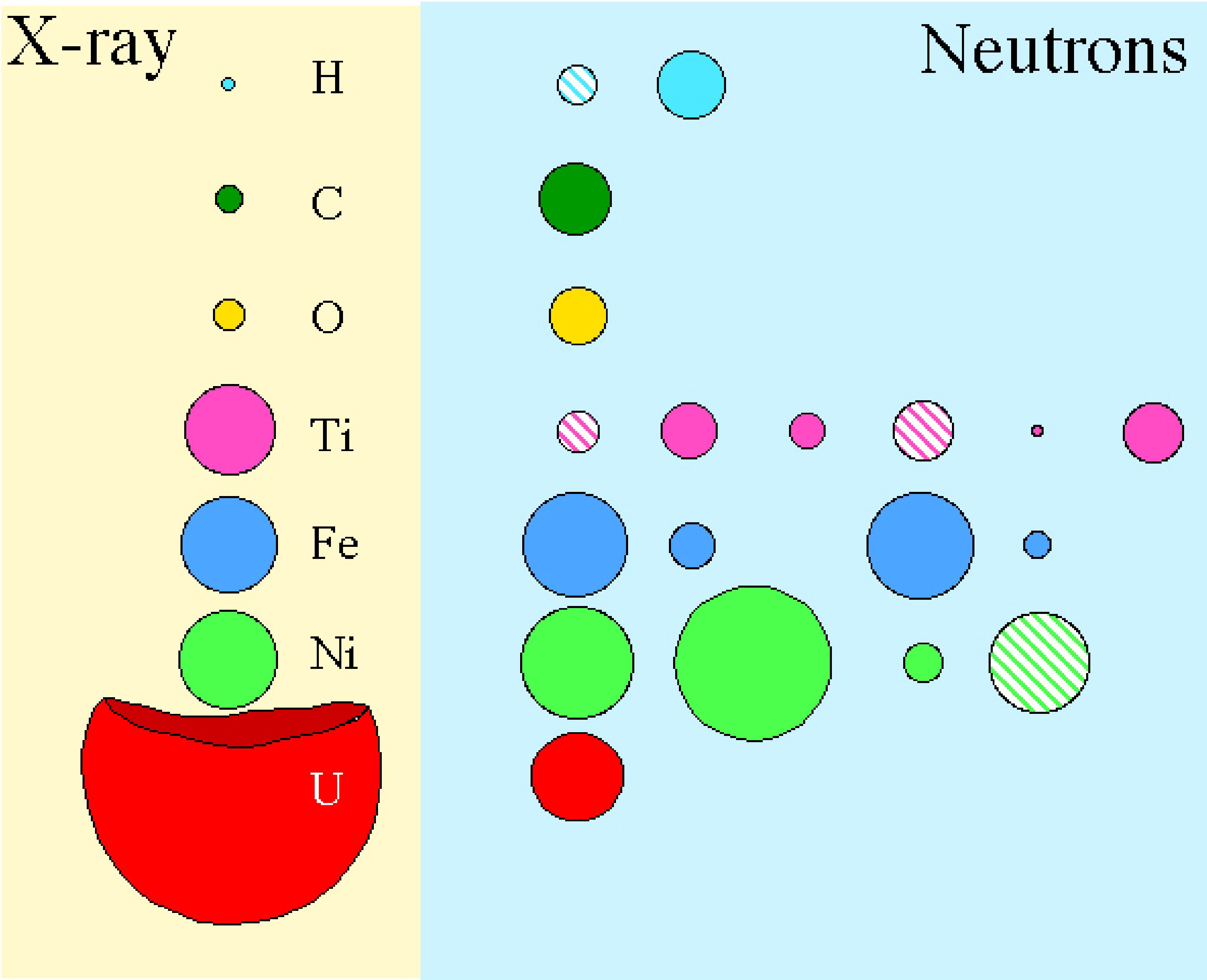
Scattering power



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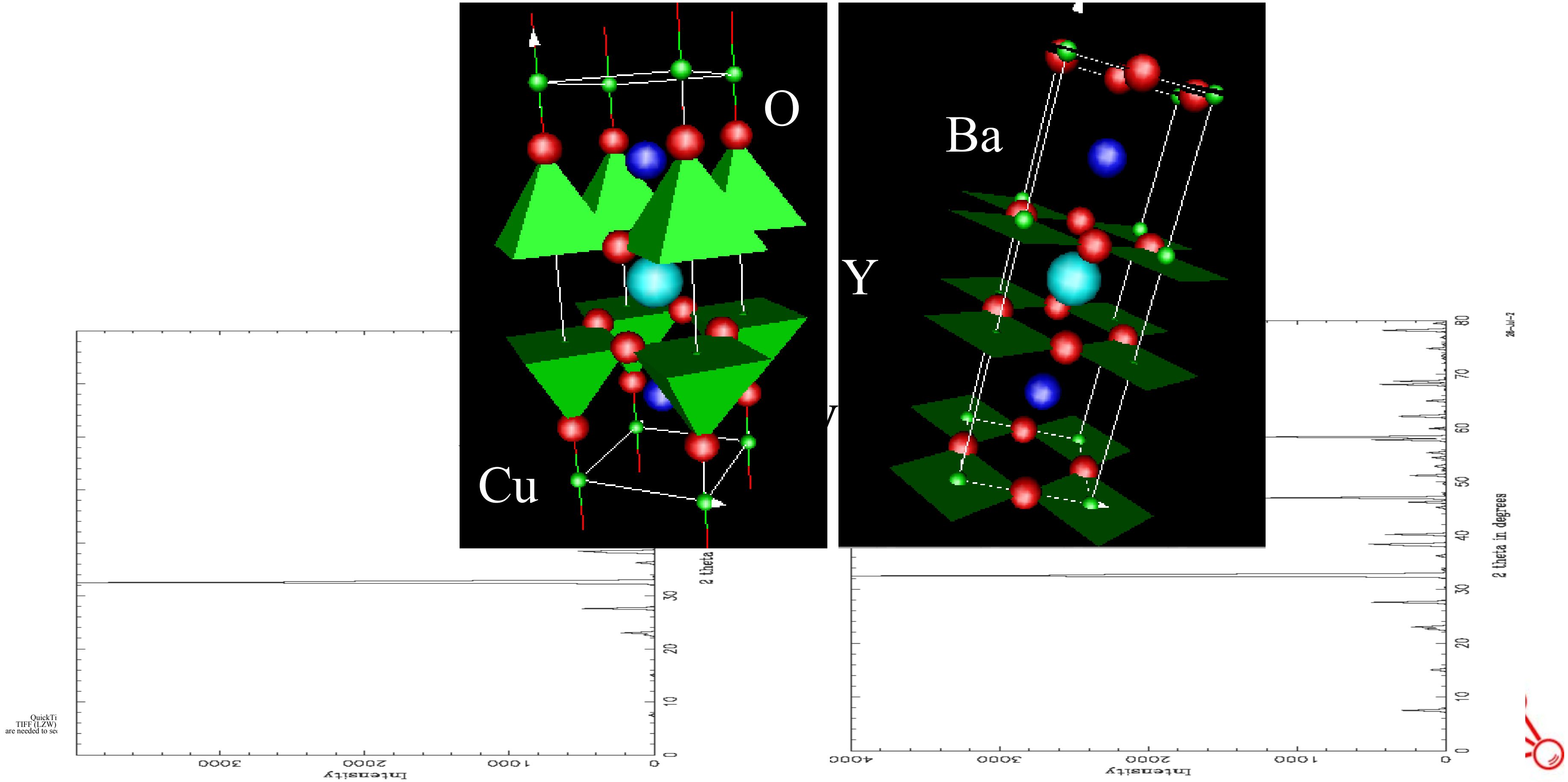
$YBa_2Cu_3O_6$

“Apex” model

“Plane” model



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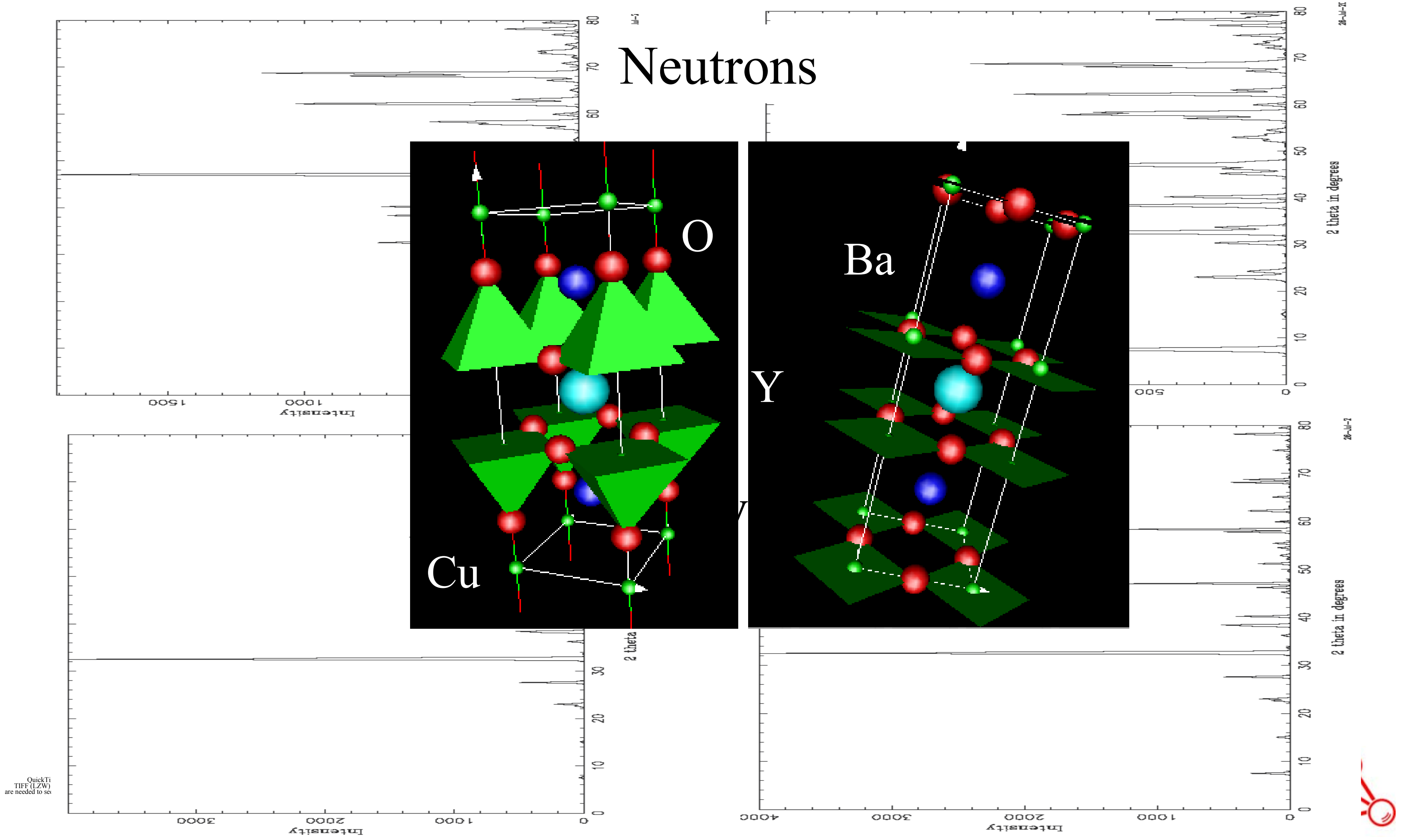
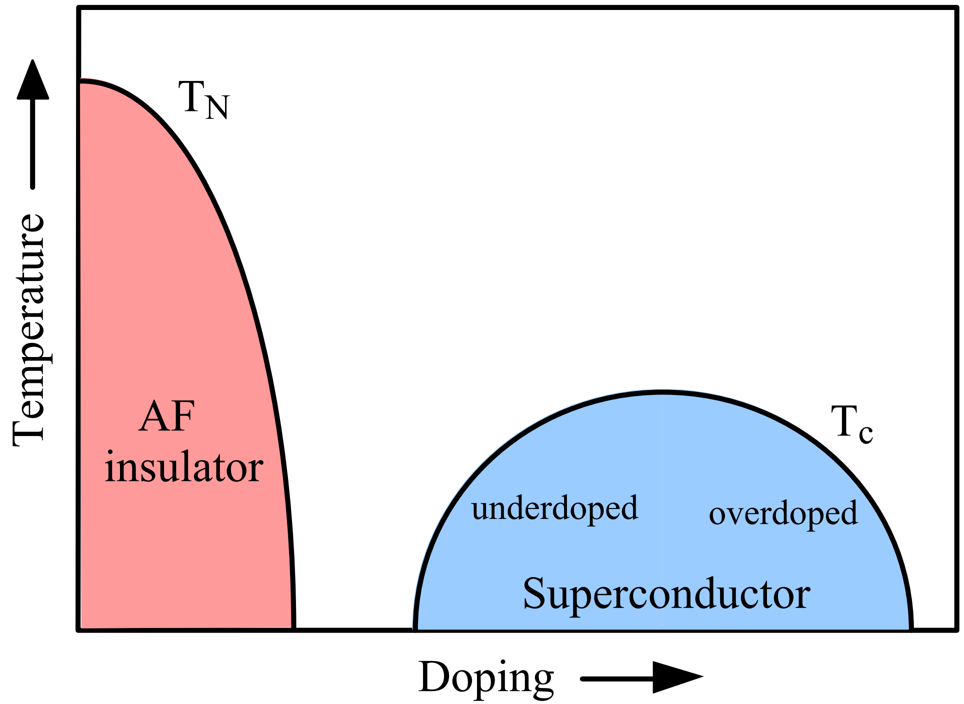


Phase Diagram of HTSC

HTSC: a doped antiferromagnet
Undoped Cu^{2+}: $3d^9$--> 1 hole/1 spin.



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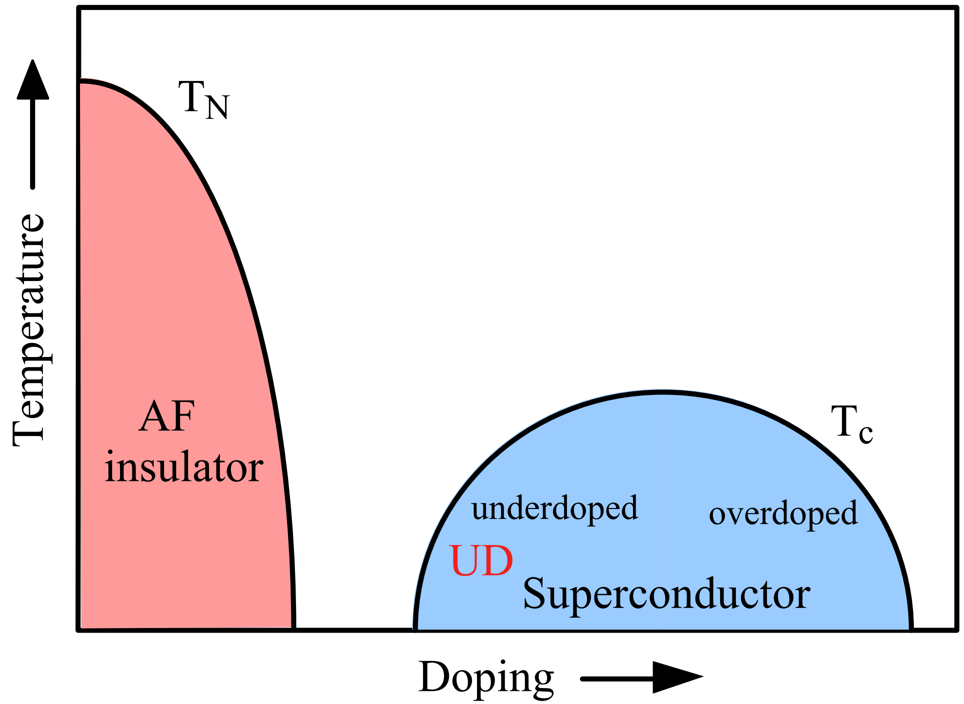

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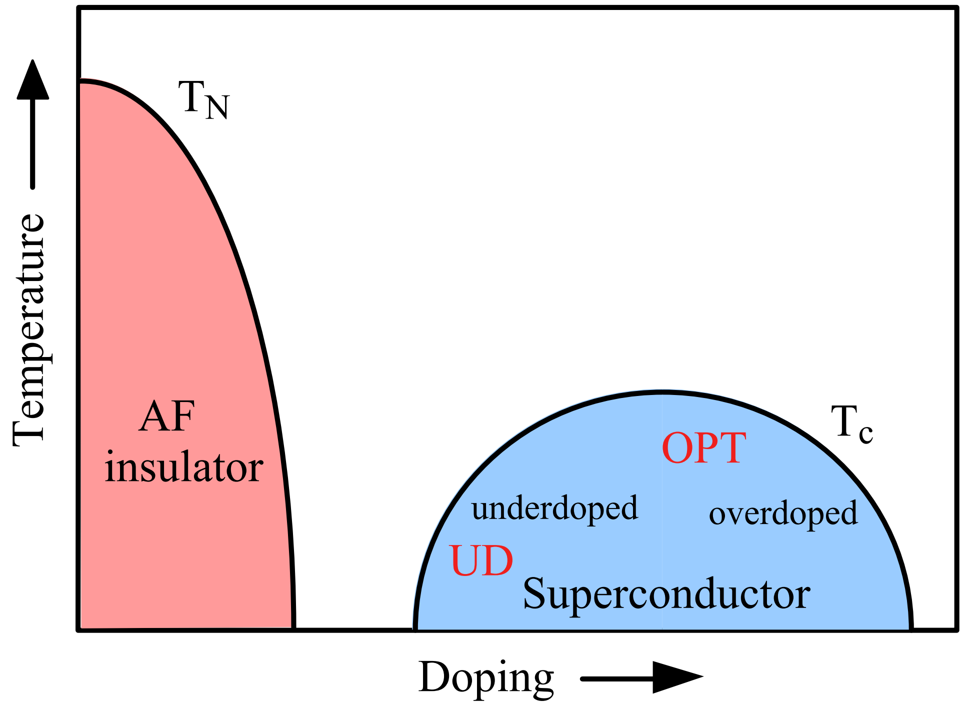

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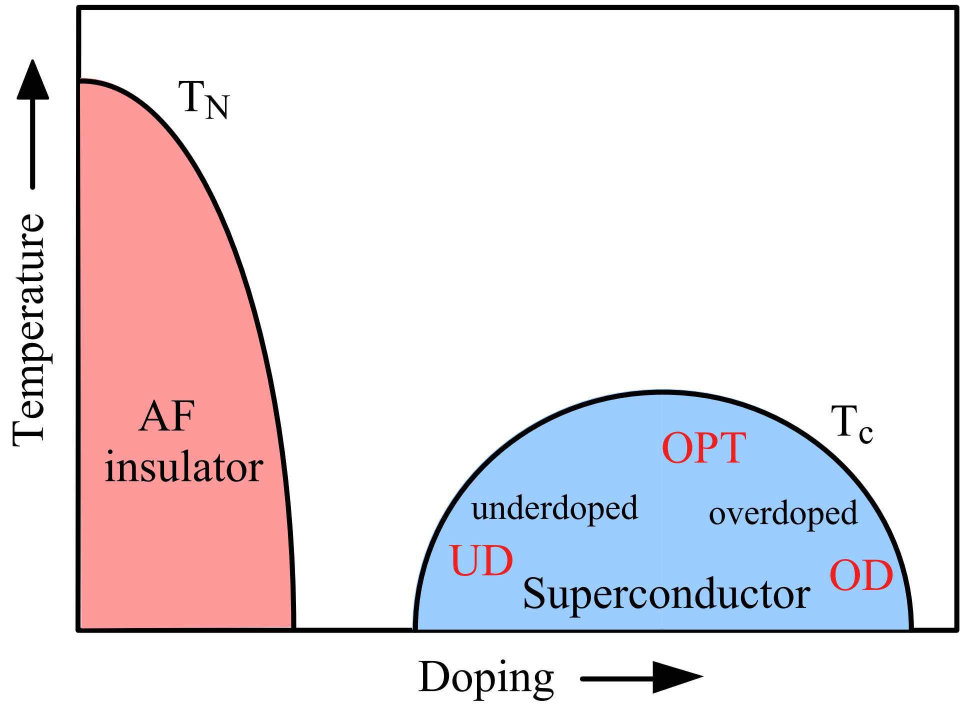

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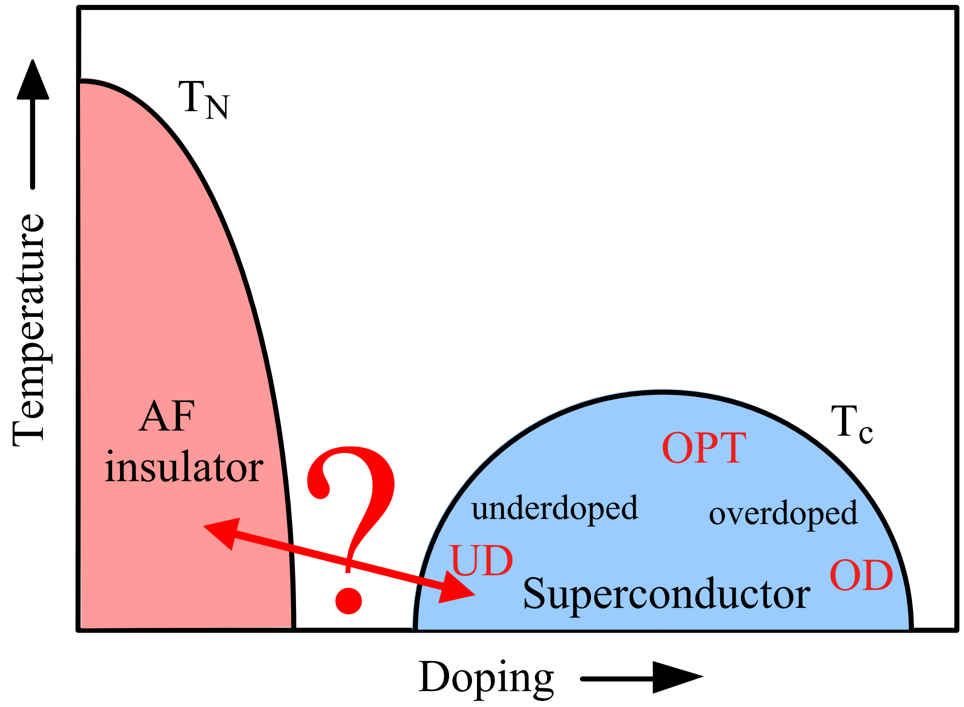

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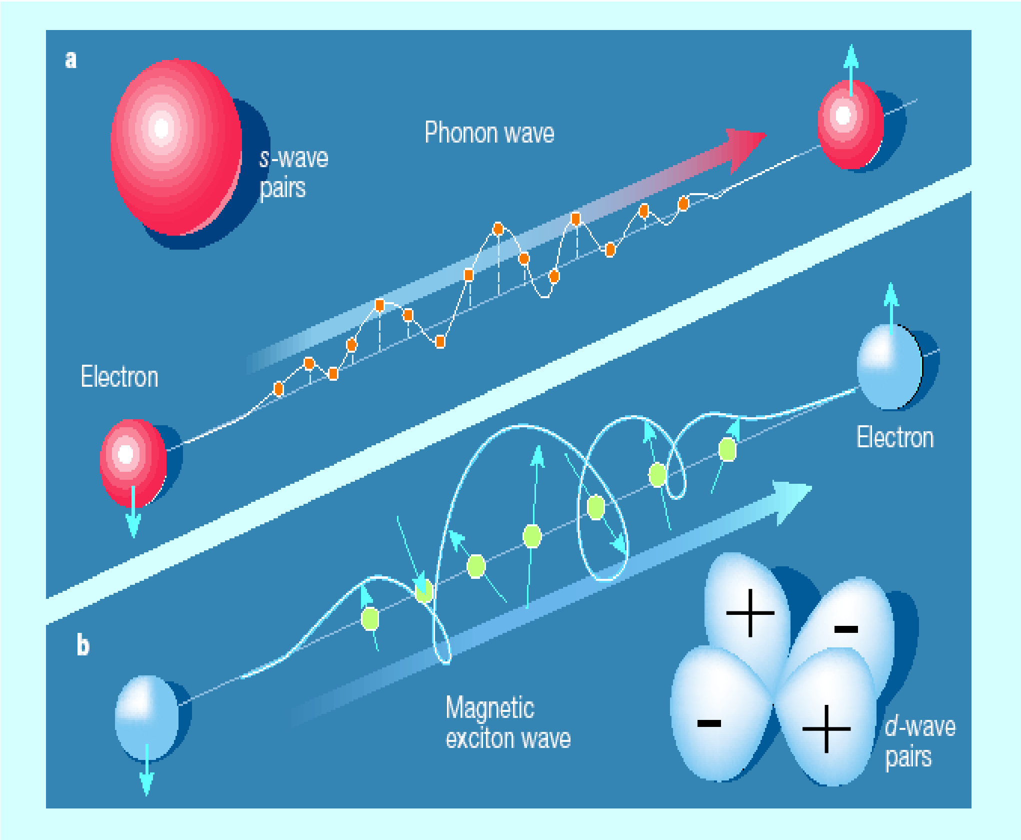
d-wave gap?

$2\Delta(0)=3.5\ k_B T_c$

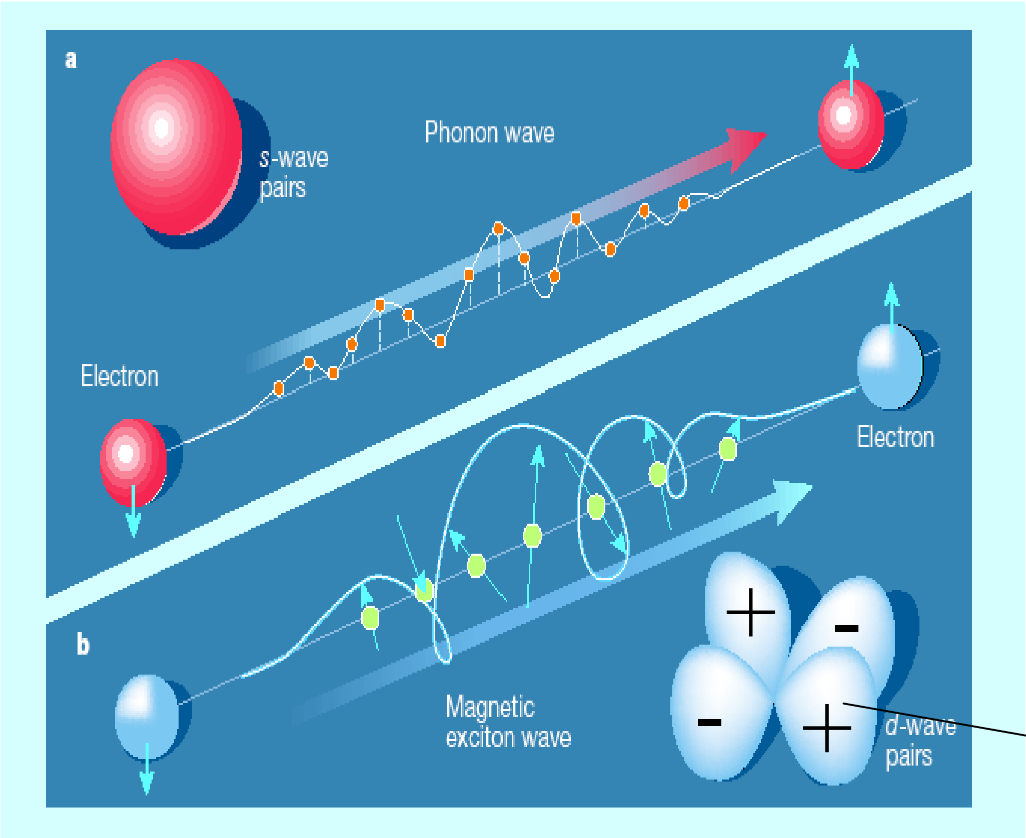


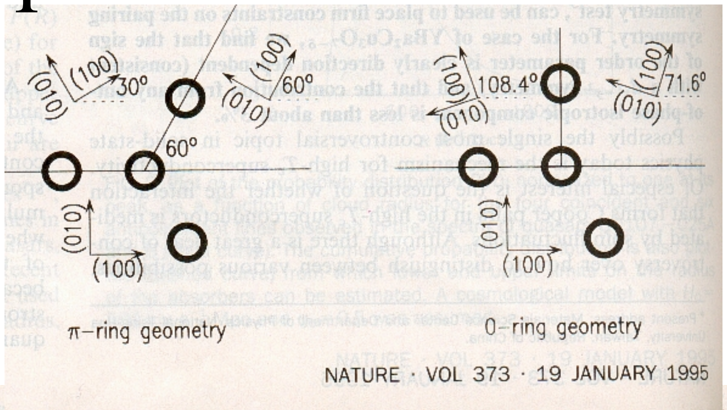
$\Delta(k)=\Delta_0\ [\cos k_y-\cos k_y]$

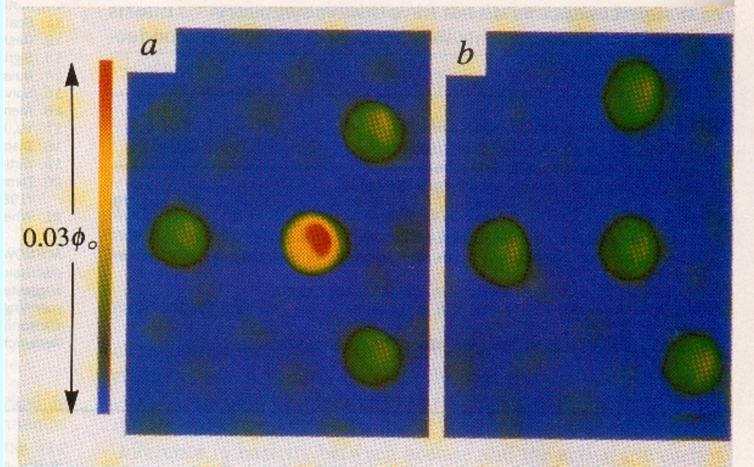
QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

d-wave gap?

$2\Delta(0)=3.5\ k_B T_c$







$\Delta(k)=\Delta_0\ [\cos k_y-\cos k_y]$

Phase sensitive experiments:

Tsuei et al. Nature **373** (1995) 225

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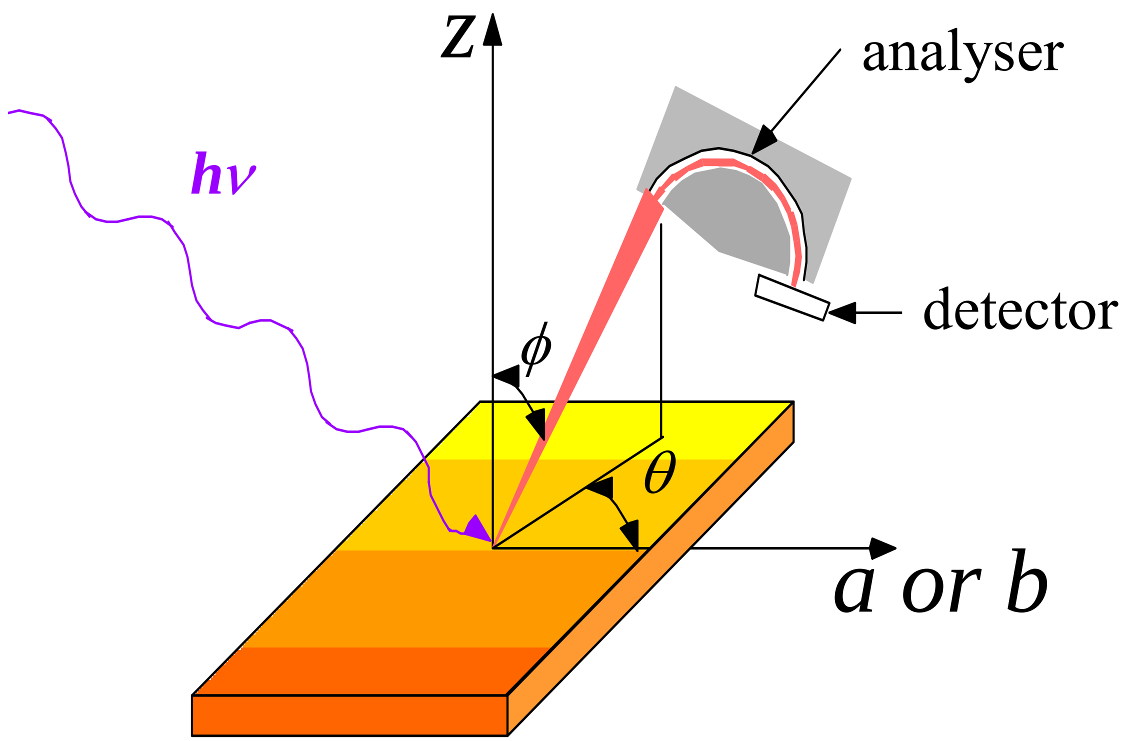
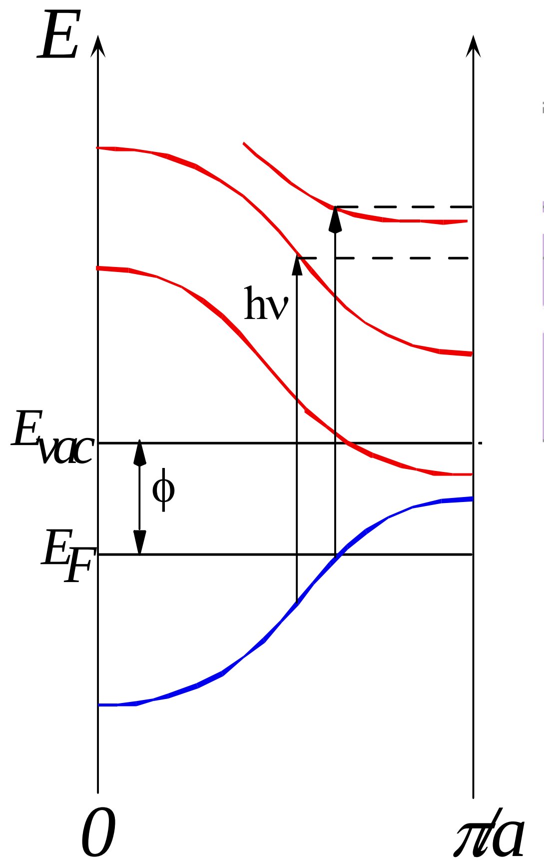
Angle Resolved Photoemission (ARPES)





$$k_{//}^{f} = k_{//}^{i} = \sqrt{\frac{2mE}{\hbar^2}}\sin(\phi)$$

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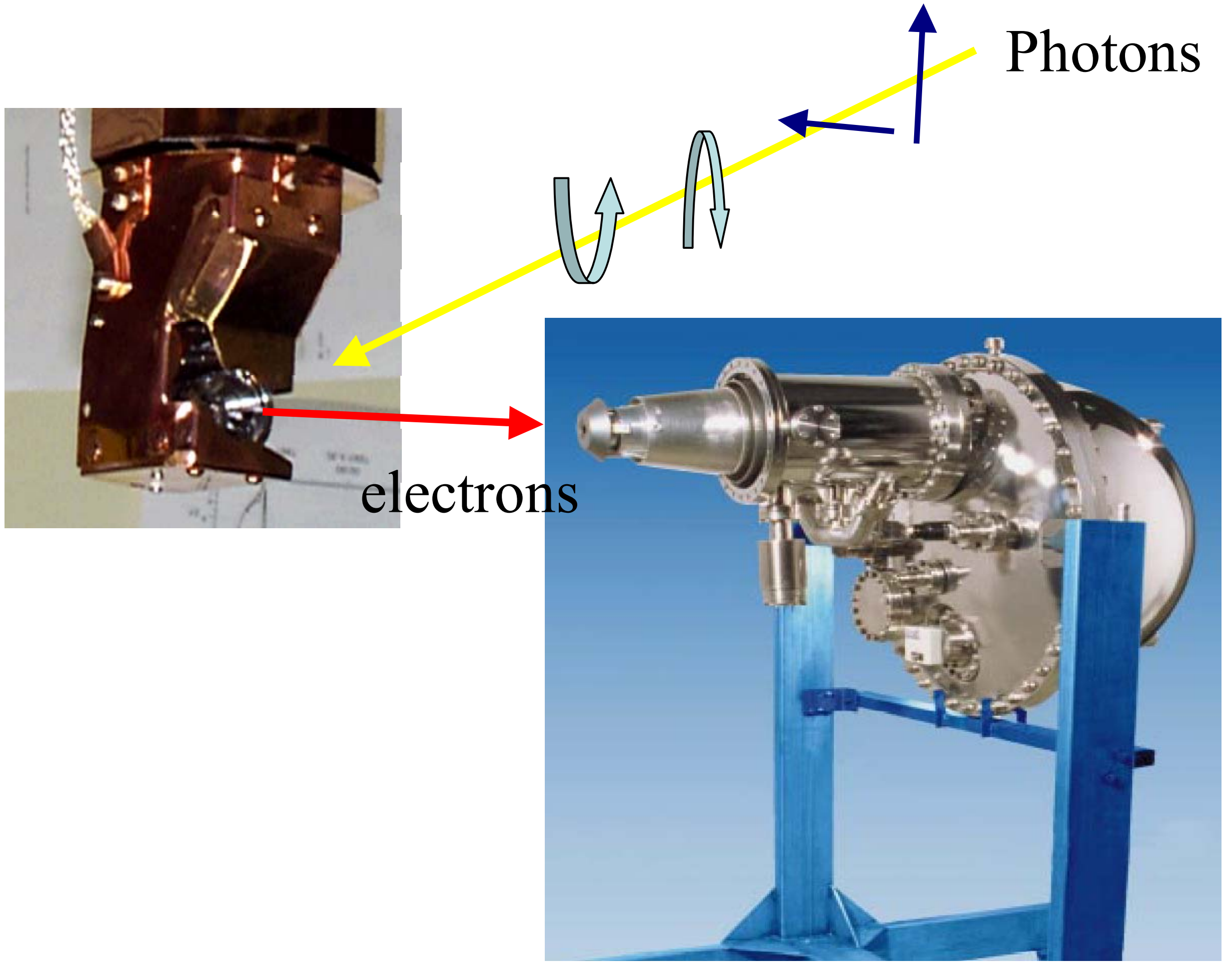
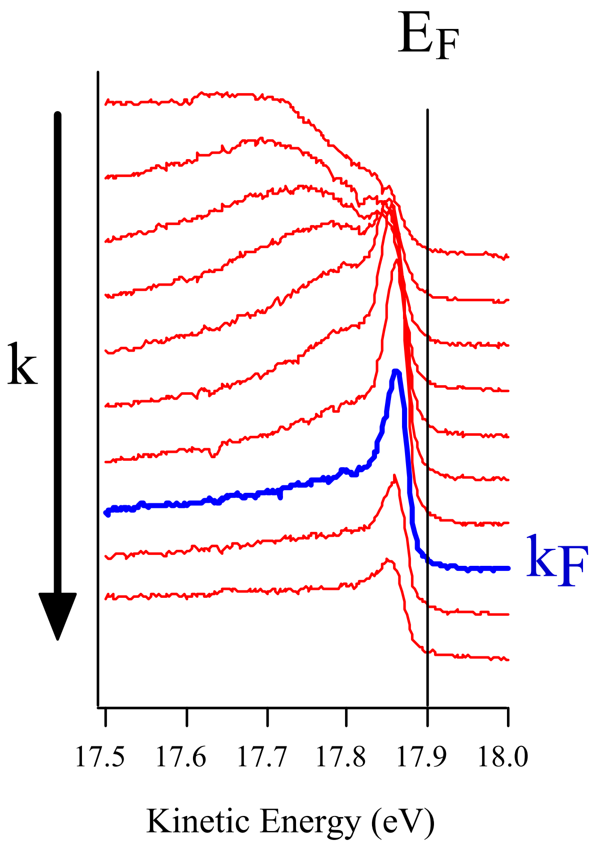
Surface-Interface-Spectroscopy beamline @ SLS-PSI

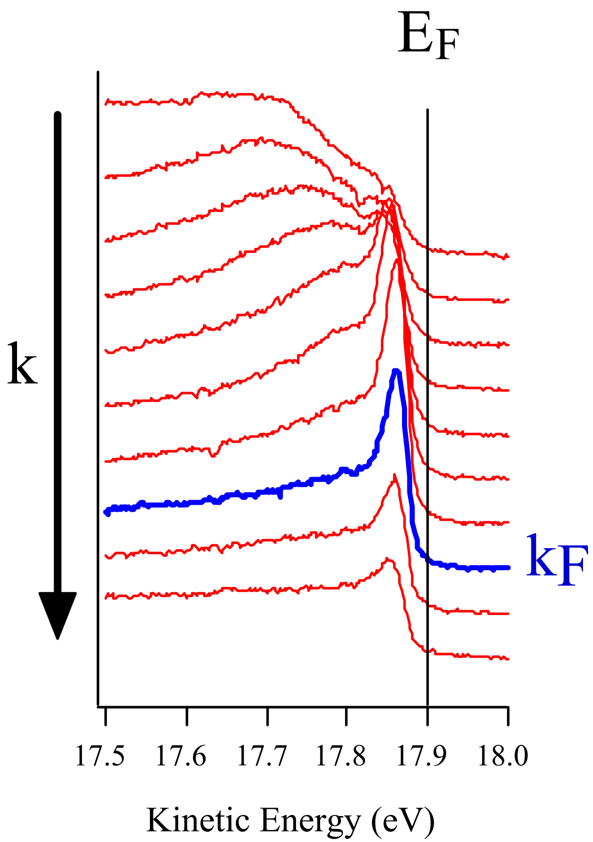


Spectral function



QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Spectral function



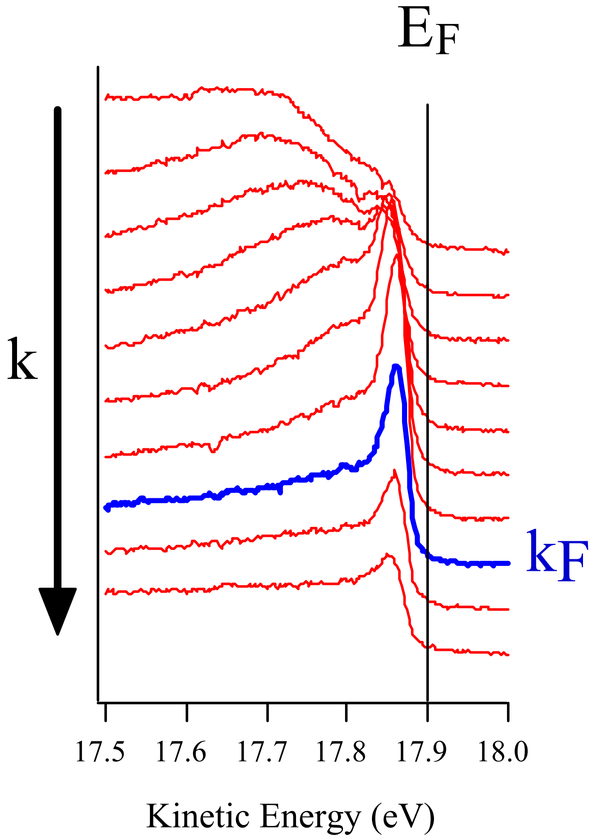
$$I(\mathbf{k},\omega)=M(\mathbf{k},\omega)f(\omega)A(\mathbf{k},\omega)$$

$A(\mathbf{k},\omega)$ = spectral function

$f(\omega)$ = Fermi function

$M(\mathbf{k},\omega)$ = matrix elements

Spectral function



$I(\mathbf{k},\omega)=M(\mathbf{k},\omega)f(\omega)A(\mathbf{k},\omega)$

$A(\mathbf{k},\omega)$ = spectral function

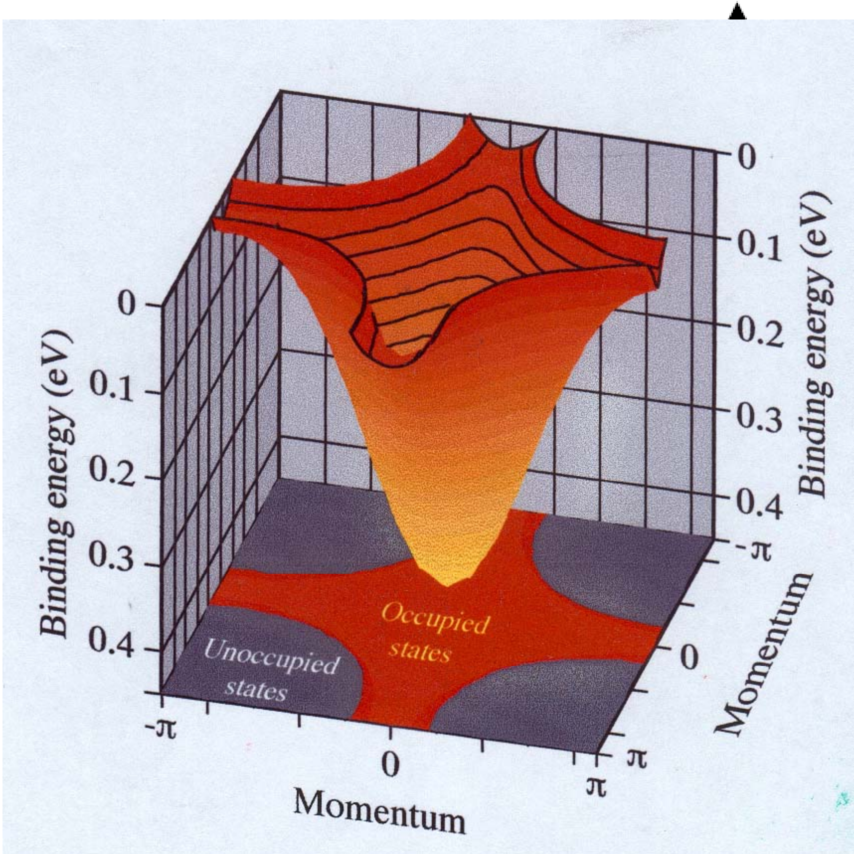
$f(\omega)$ = Fermi function

$M(\mathbf{k},\omega)$ = matrix elements

Self-energy $\Sigma(\ \ ,\omega)$

$$A(\mathbf{k},\omega)=\frac{1}{\pi}\frac{\left|\Sigma''(\mathbf{k},\omega)\right|}{\left[\omega-\varepsilon_{\mathbf{k}}-\Sigma'(\mathbf{k},\omega)\right]^2+\underbrace{\left[\Sigma''(\mathbf{k},\omega)\right]^2}_{\tau^{-1}(\mathbf{k},\omega)}}$$

Angle Resolved Photoemission (ARPES)



QuickTime™ and a
TIFF (LZW) decompressor
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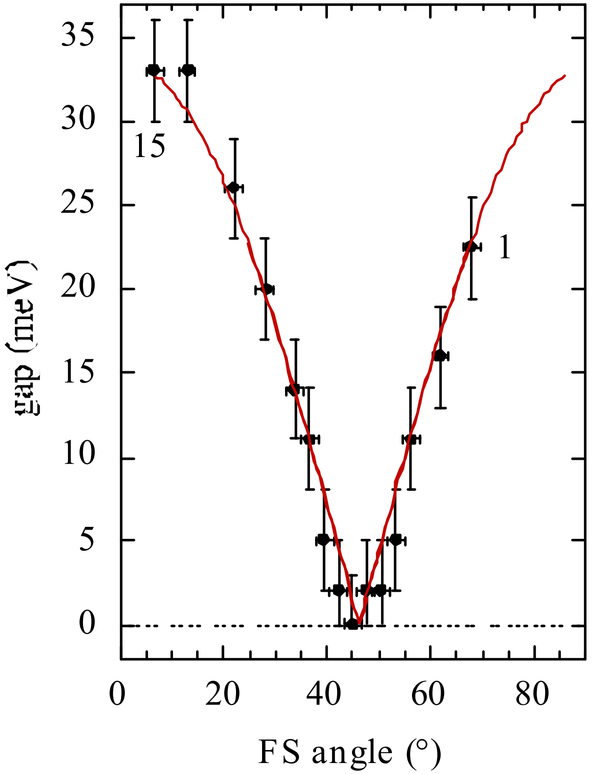
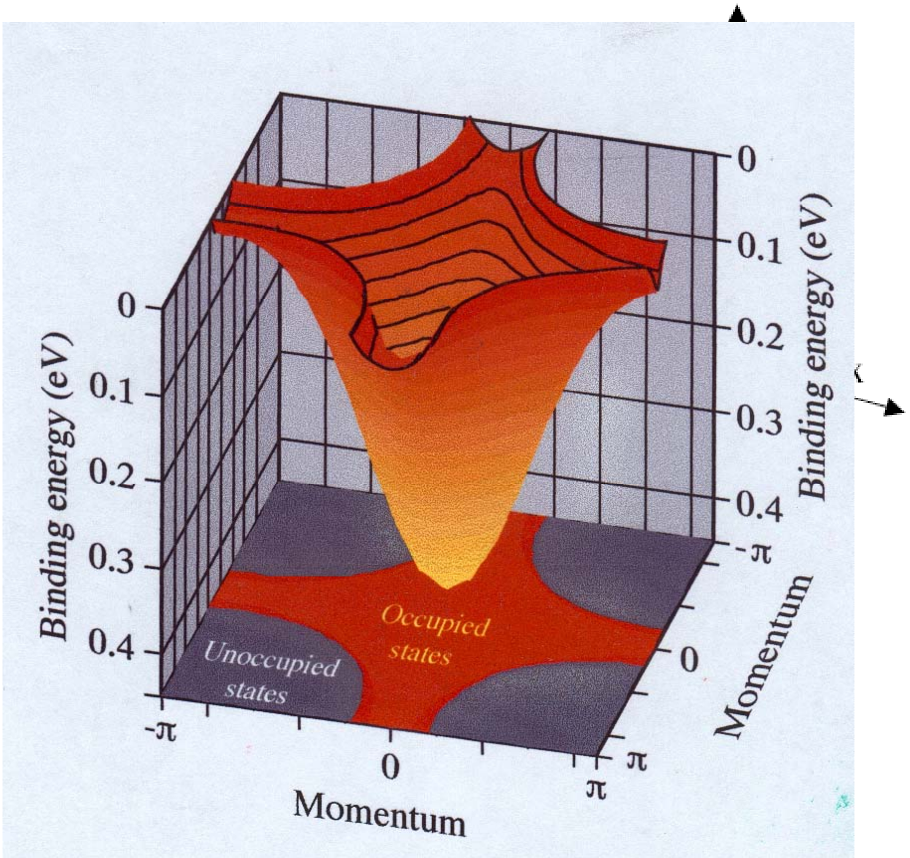
Angle Resolved Photoemission (ARPES)

$= \cos k_x - \cos k_y$

$= 1/2 \cos(2\phi)$

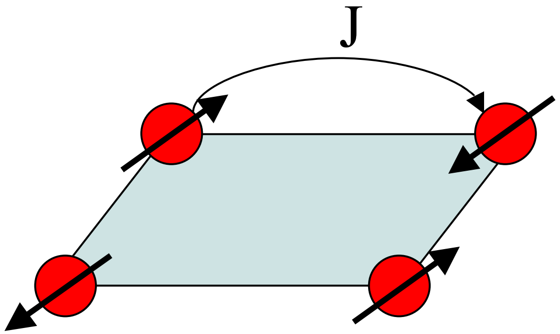




Can we demonstrate that in HTSC the glue binding the electrons/holes is of magnetic origin?

Magnetism: undoped HTSC

2D-square lattice ---> Heisenberg Hamiltonian $$H = \sum_{\langle ij \rangle} J\mathbf{S}_i \mathbf{S}_j$$



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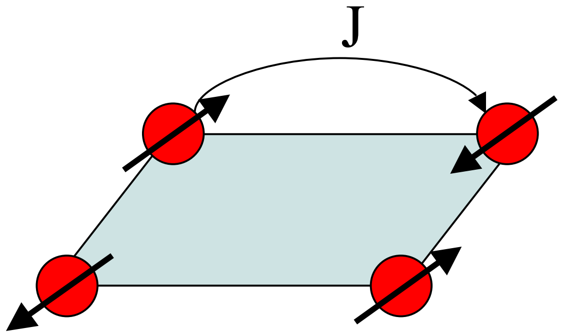

$$\hbar\omega(\mathbf{q}) = 2J\left[1 - \gamma^2(\mathbf{q})/4\right]^{1/2}$$

$$\gamma(\mathbf{q}) = \cos(q_x a) + \cos(q_y a)$$

 Kittel, Quantum Theory of Solids (Wiley, NY, 1963)

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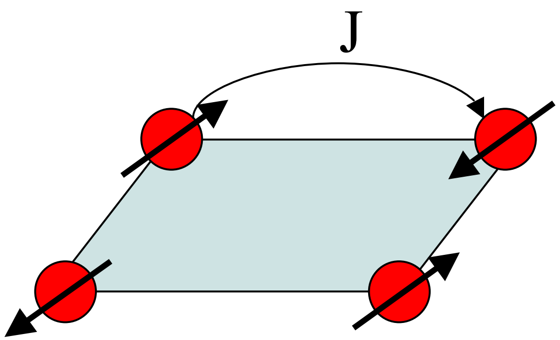

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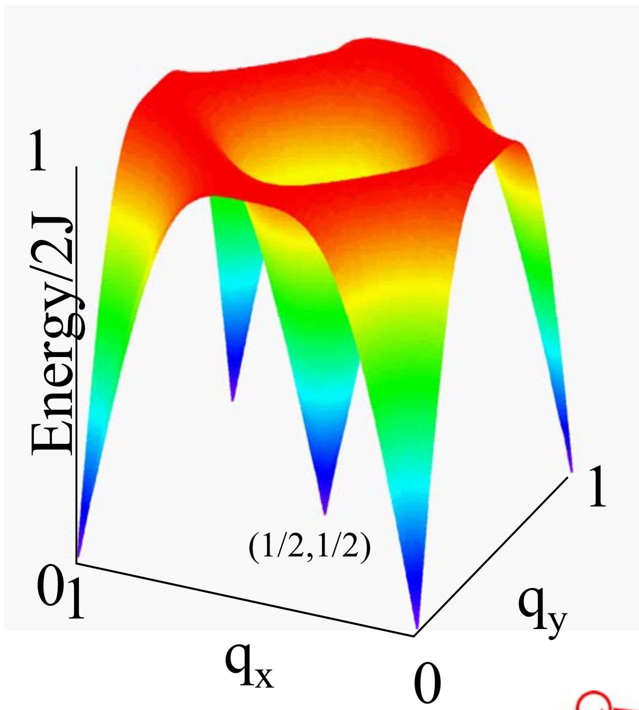
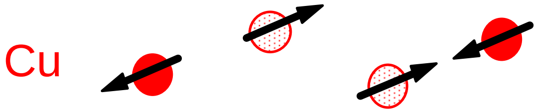


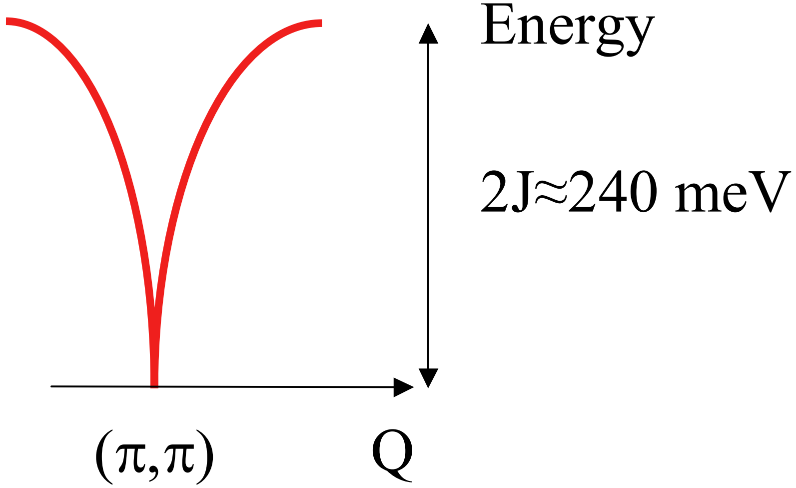
Kittel, Quantum Theory of Solids (Wiley, NY, 1963)

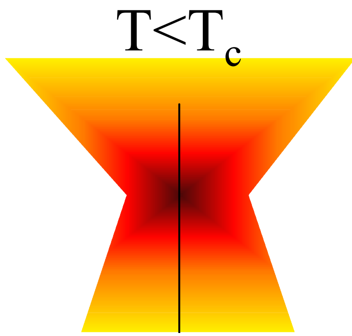
Doped Cuprates

<u>Undoped</u>



Ordre AF, **k**=(π, π)=(1/2,1/2)

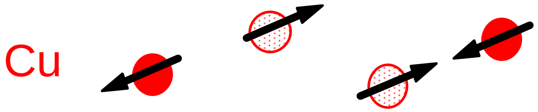




Fluctuations centered around (π,π)

Doped Cuprates

Undoped



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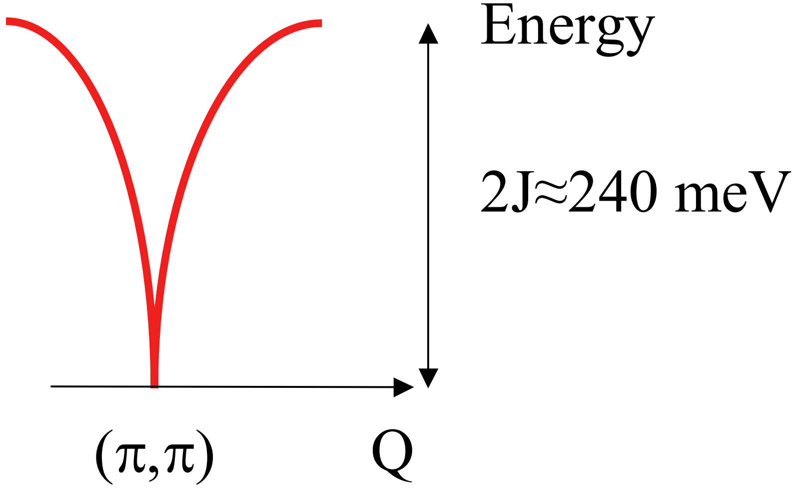

Doped

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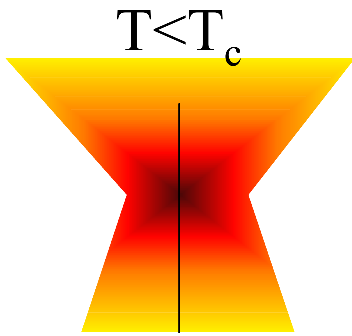

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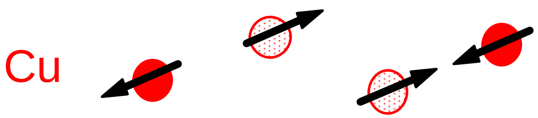
Doped Cuprates

Undoped



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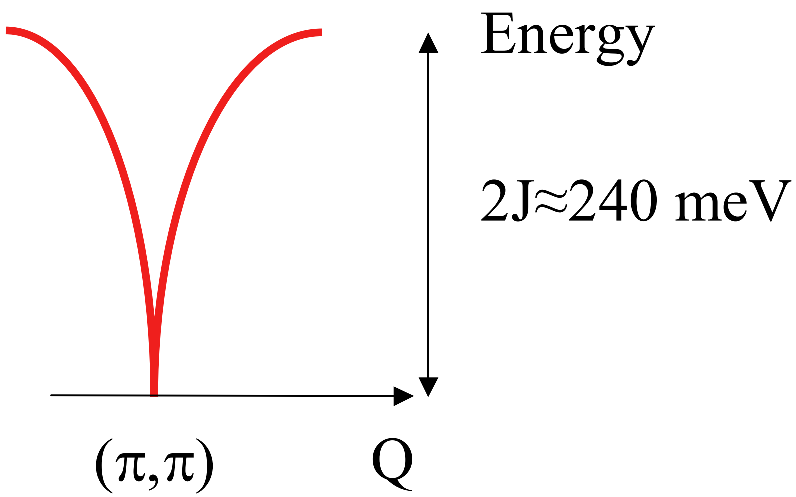
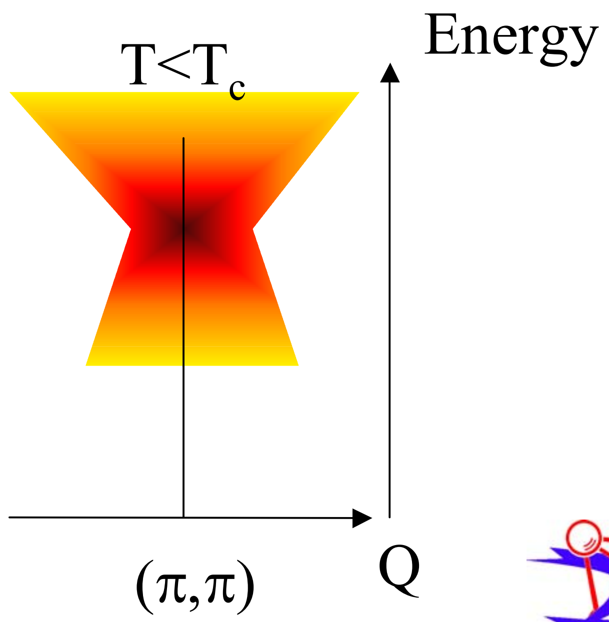


Doped

Fluctuations centered around (π,π)



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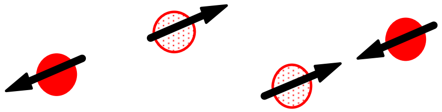
Doped Cuprates

<u>Undoped</u>

Cu



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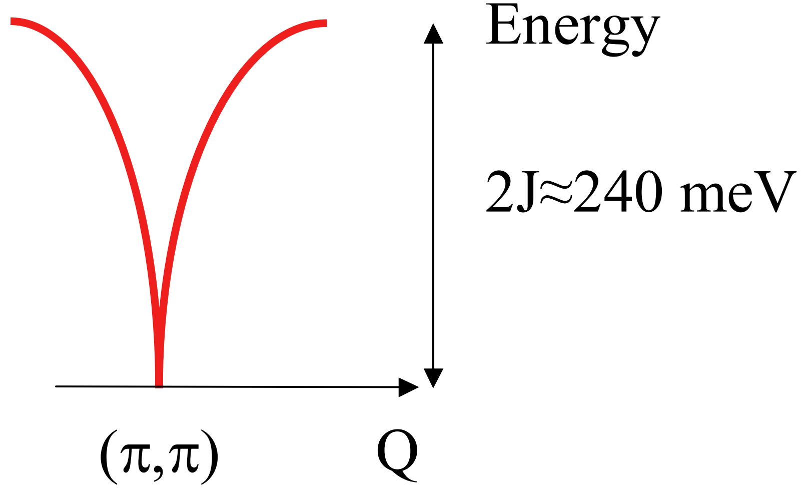
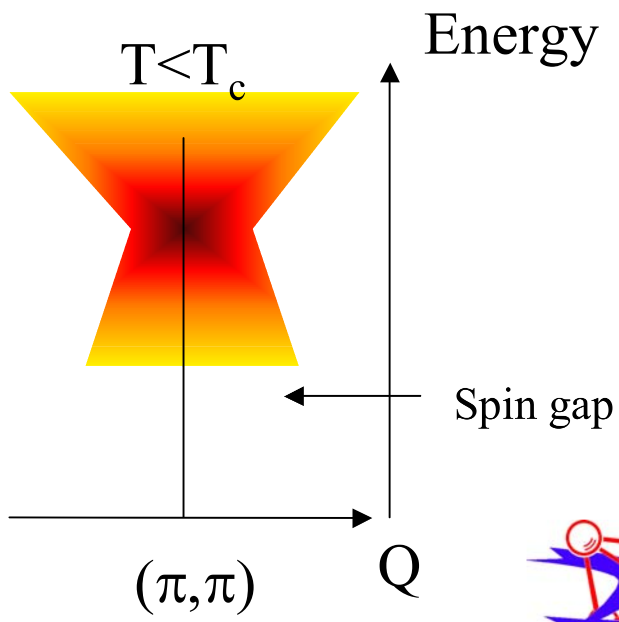
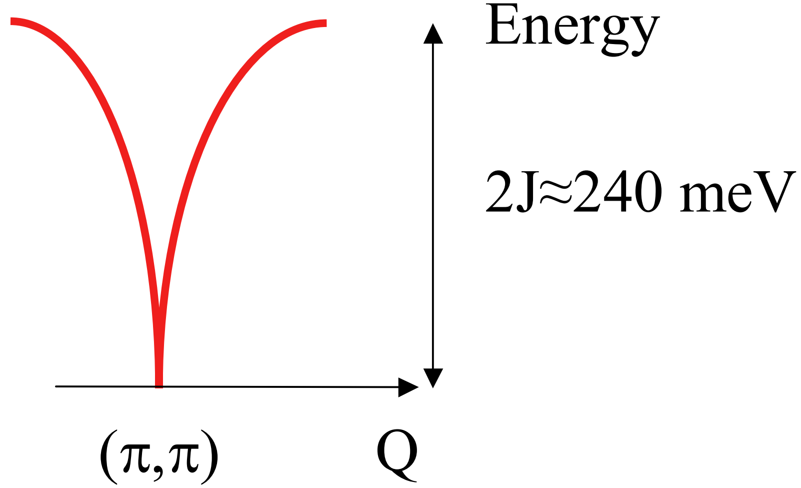


<u>Doped</u>

Fluctuations centered around (π,π)

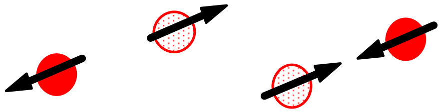


Doped Cuprates



Undoped

Cu



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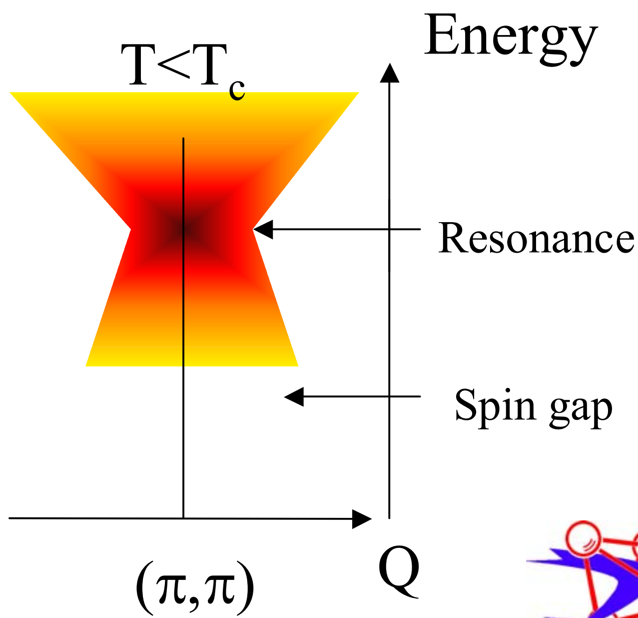

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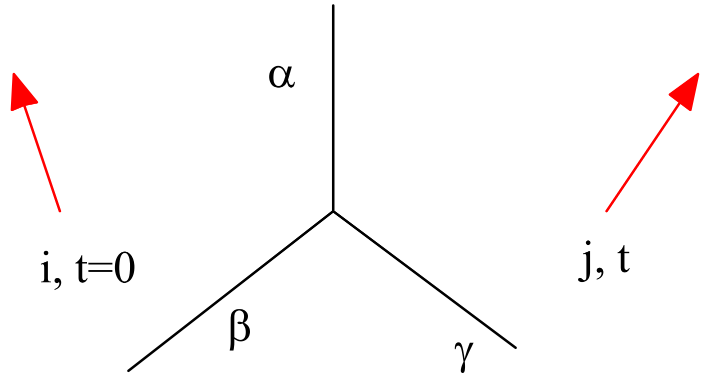
Inelastic Neutron Scattering: Magnetic Cross-Section

$$I(\mathbf{Q},\omega) = r_0^2 \frac{k_f}{k_i} f^2(\mathbf{Q}) e^{-2W(\mathbf{Q})} \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2} \right) \underline{S_{\alpha\beta}(\mathbf{Q},\omega)}$$

$$\underline{S_{\alpha\beta}(\mathbf{Q},\omega)} \propto \sum_{i,j} \int dt e^{i\left(\mathbf{Q}\mathbf{R}_{ij} - \omega t\right)} \left\langle S_i^\alpha S_j^\beta(t) \right\rangle$$

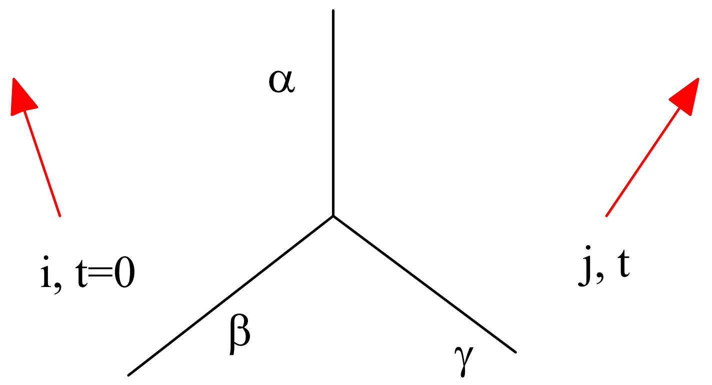


QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Inelastic Neutron Scattering: Magnetic Cross-Section

$$I(\mathbf{Q},\omega) = r_0^2 \frac{k_f}{k_i} f^2(\mathbf{Q}) e^{-2W(\mathbf{Q})} \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2} \right) \underline{S_{\alpha\beta}(\mathbf{Q},\omega)}$$

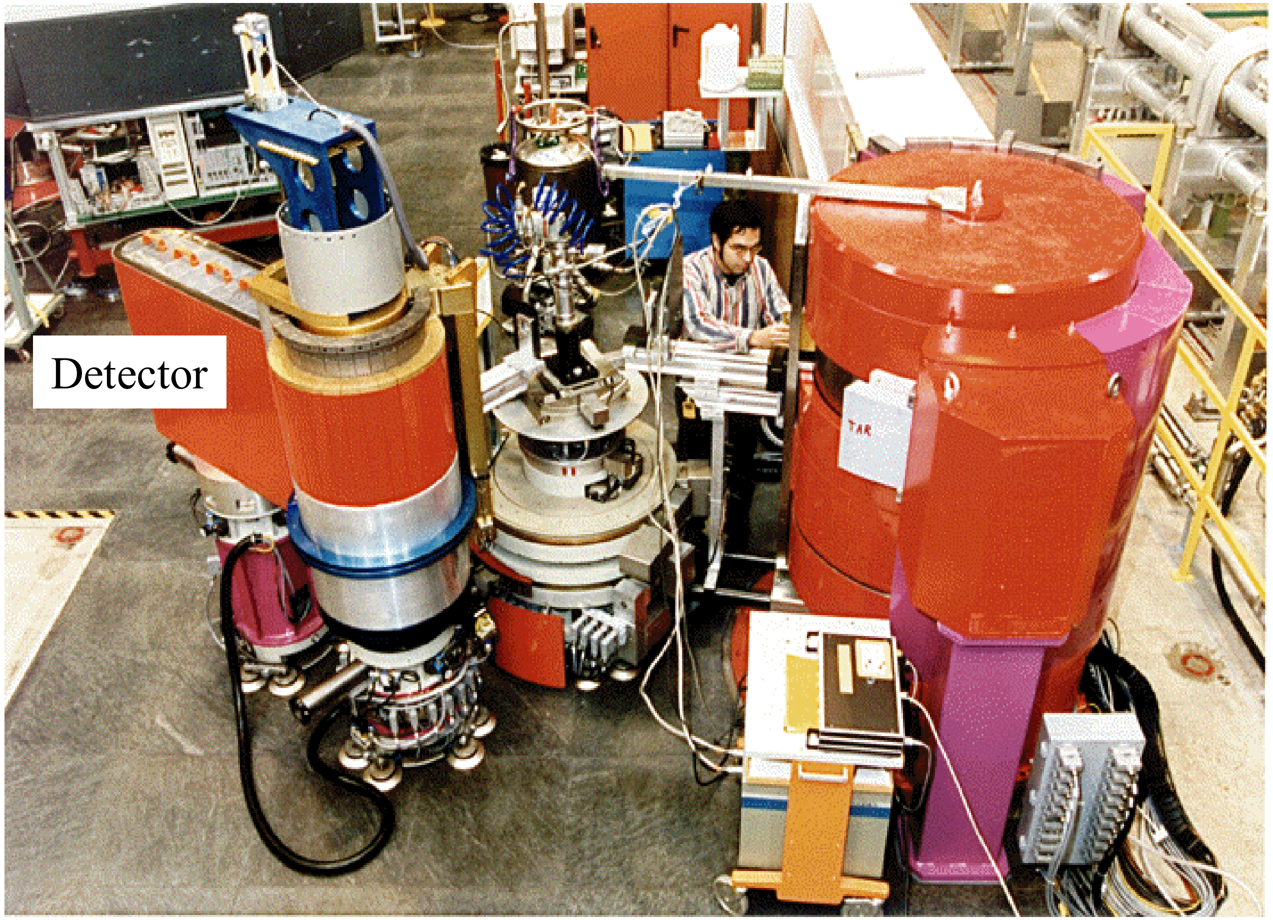
$$\underline{S_{\alpha\beta}(\mathbf{Q},\omega)} \propto \sum_{i,j} \int dt e^{i\left(\mathbf{QR}_{ij} - \omega t\right)} \left\langle S_i^\alpha S_j^\beta(t) \right\rangle$$



Fluctuation-dissipation theorem

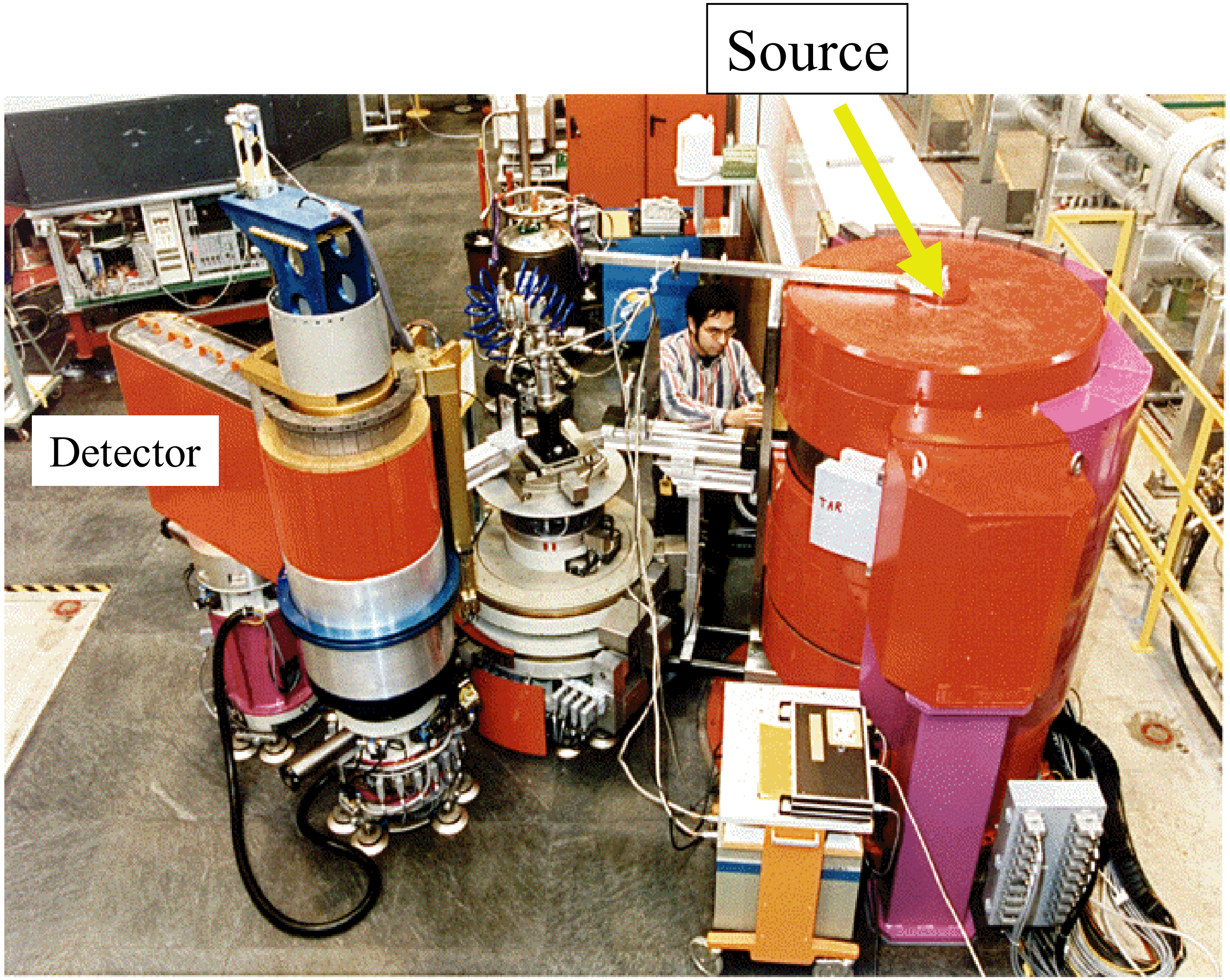
$$S_{\alpha\beta}(\mathbf{Q},\omega) = \frac{1 + n(\omega)}{\pi\left(\gamma\mu_B\right)^2} \chi''_{\alpha\beta}(\mathbf{Q},\omega)$$

Neutron Scattering: TAS



QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Neutron Scattering: TAS



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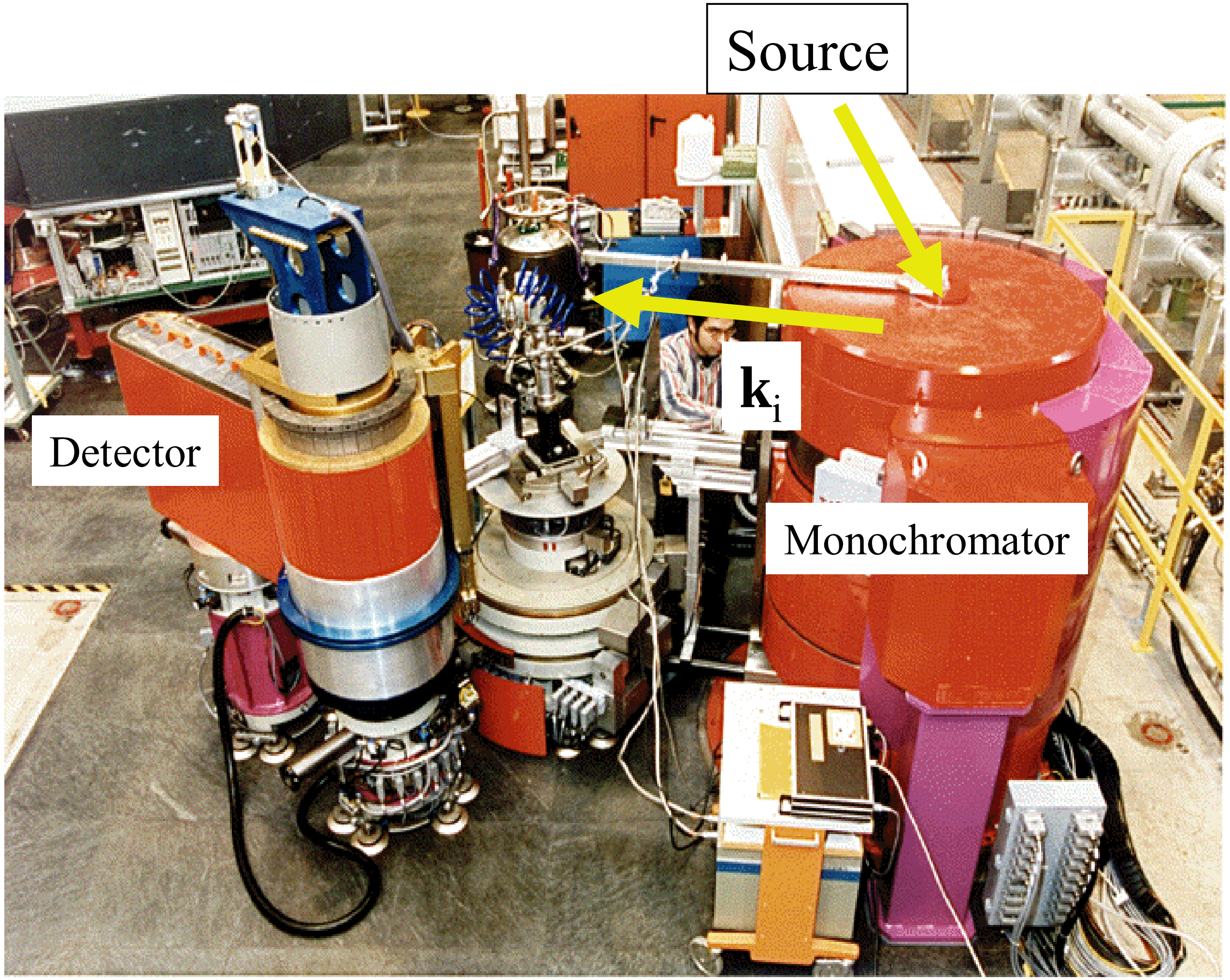
Neutron Scattering: TAS



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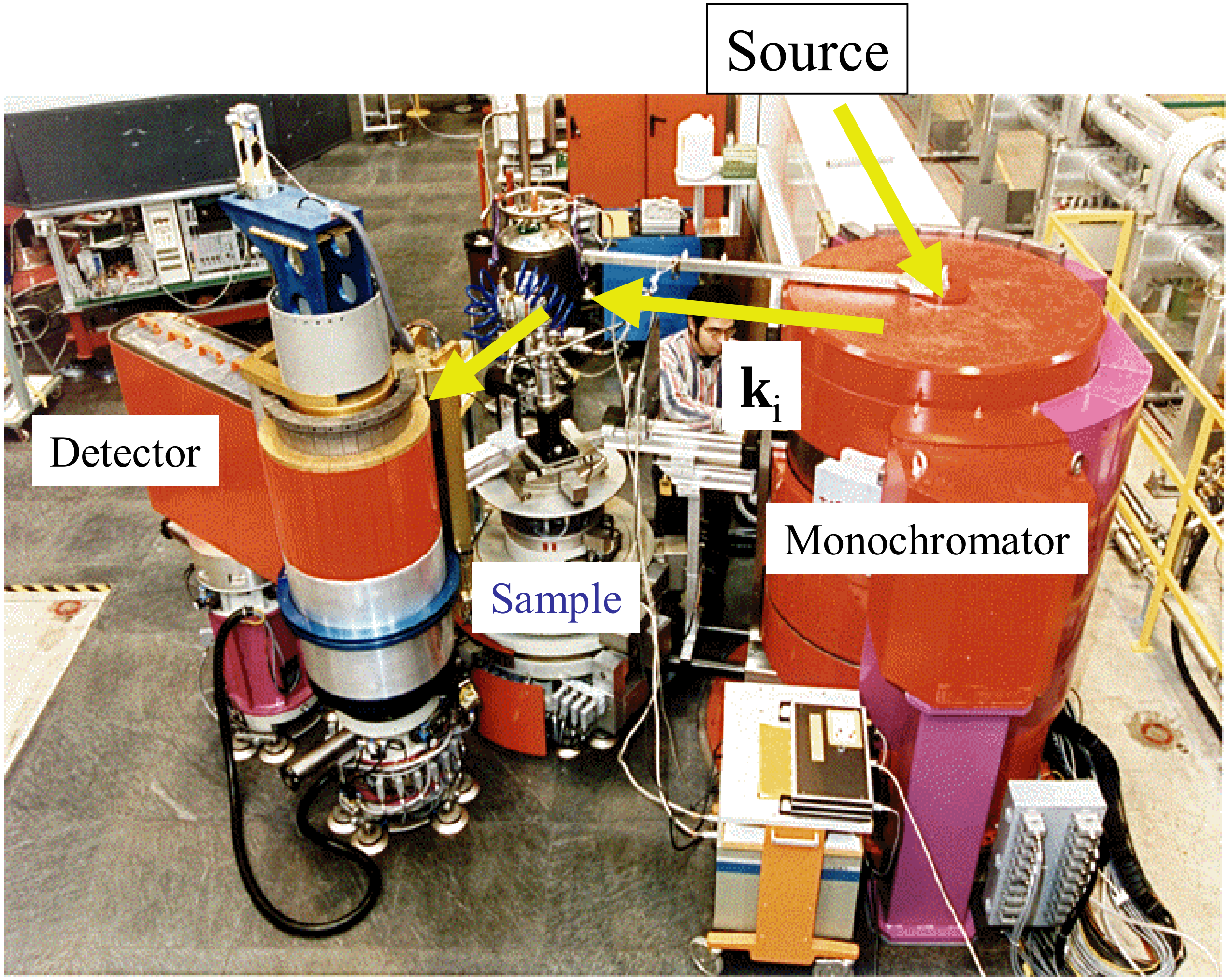
Neutron Scattering: TAS



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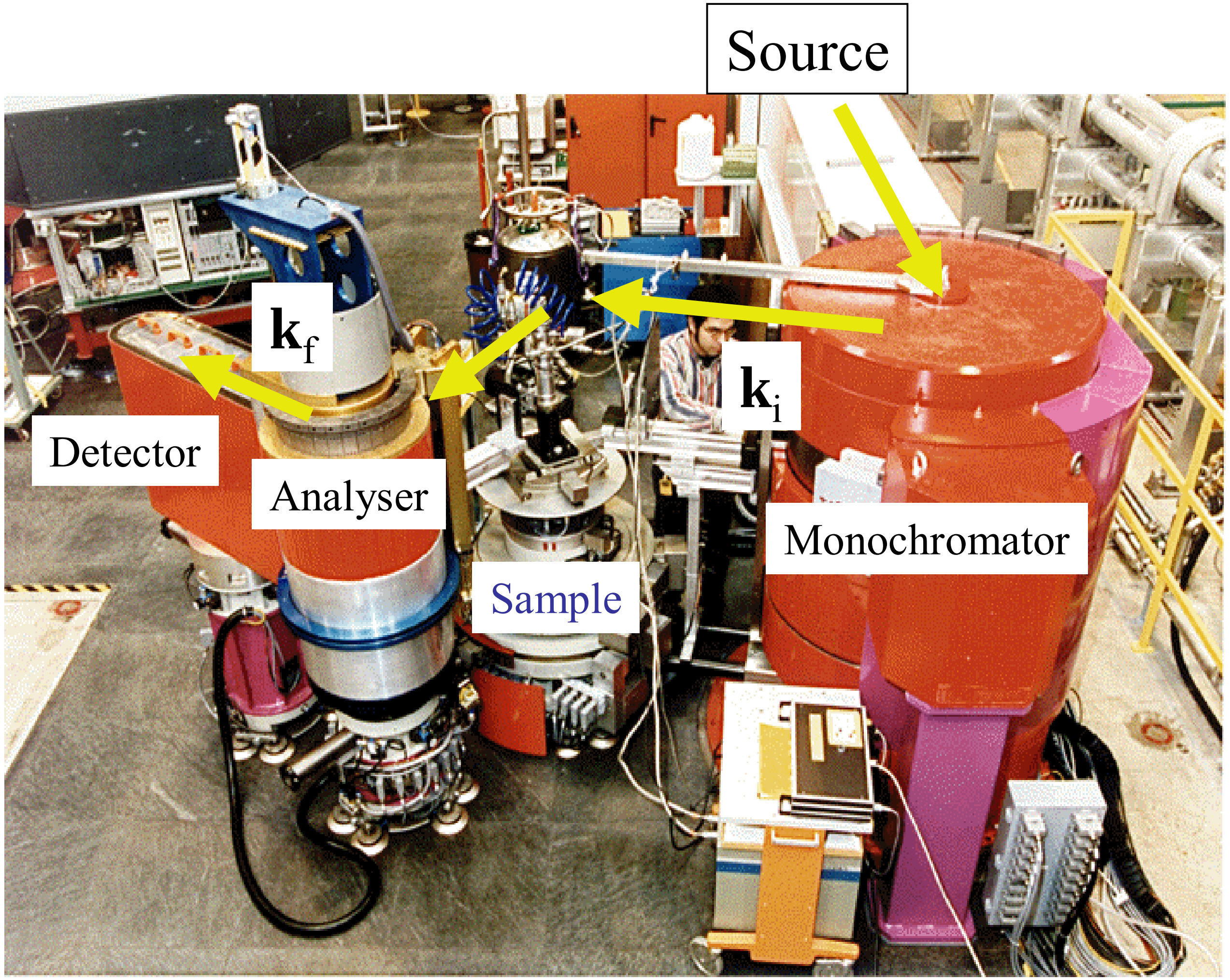
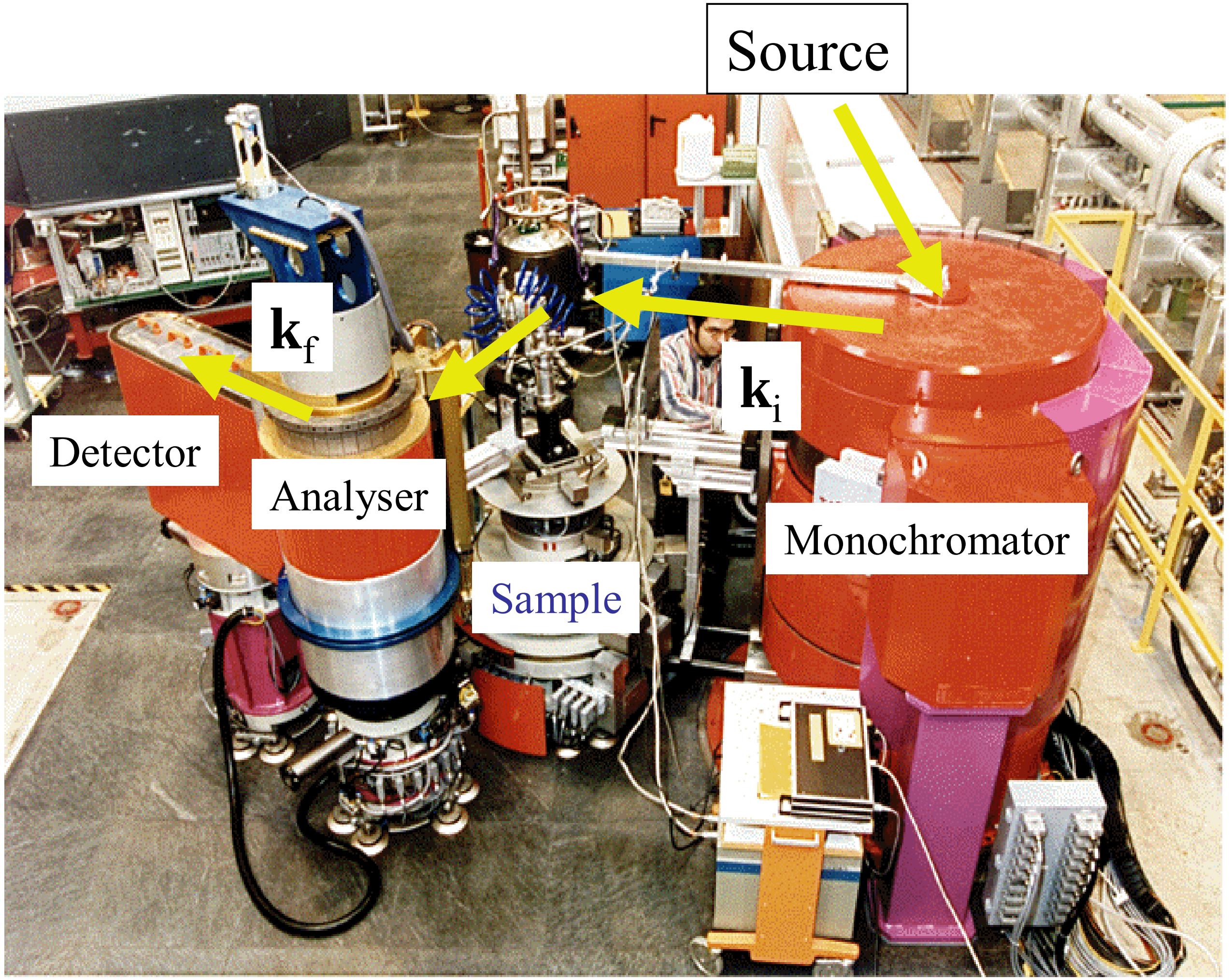
Neutron Scattering: TAS



Neutron Scattering: TAS



Sample

$\mathbf{Q}=-(\mathbf{k}_f-\mathbf{k}_i)$

$$\hbar\omega = -\left(E_f - E_i\right)$$

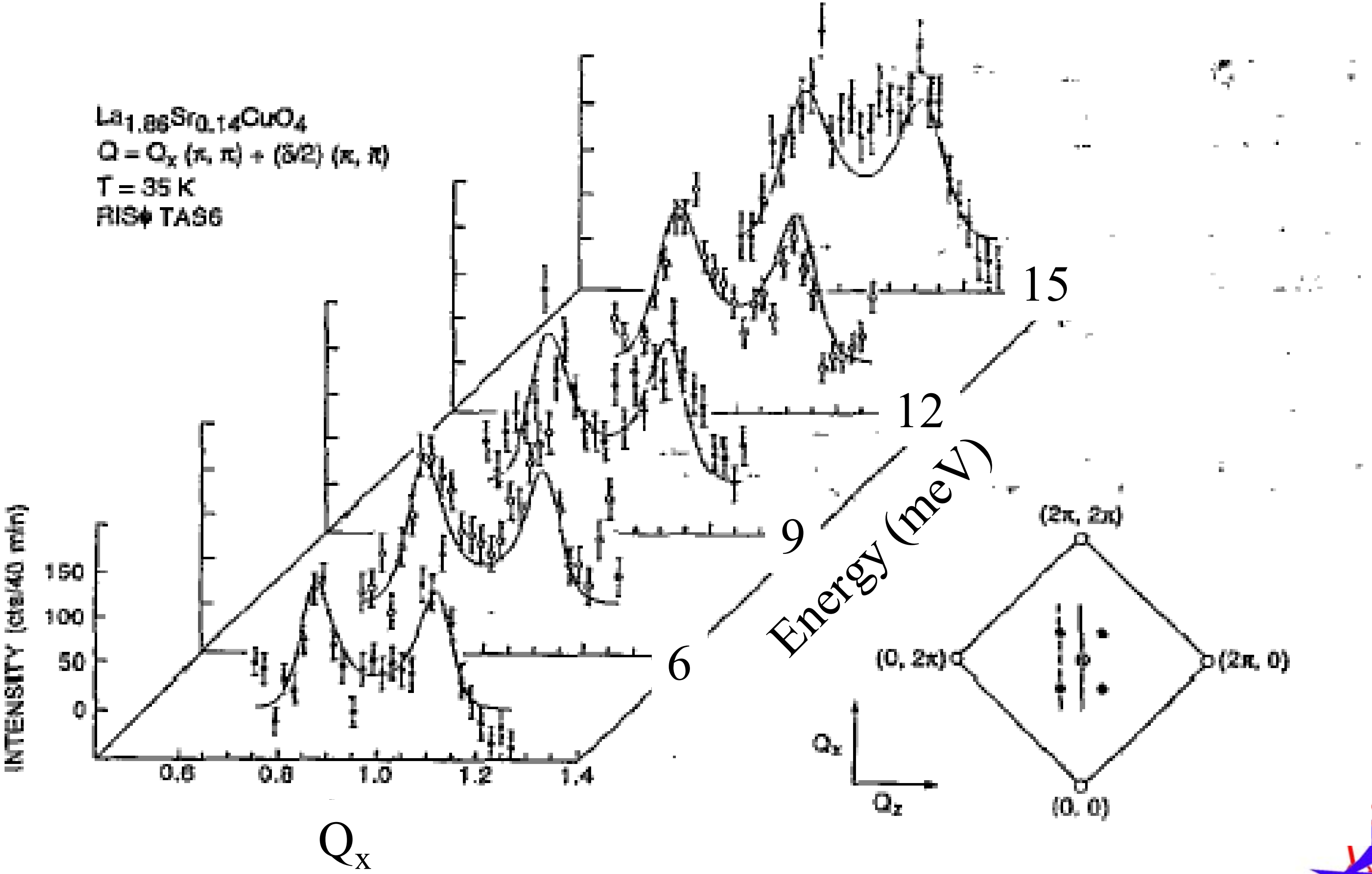
QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

QuickTime™ and a
Video decompressor
are needed to see this picture.

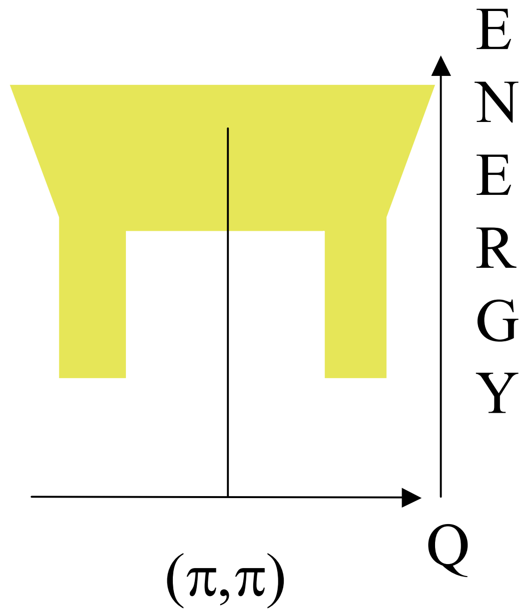
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

$La_{1.86}Sr_{0.14}CuO_4$ (normal state)

Mason et al., PRL 68 (1992) 1414

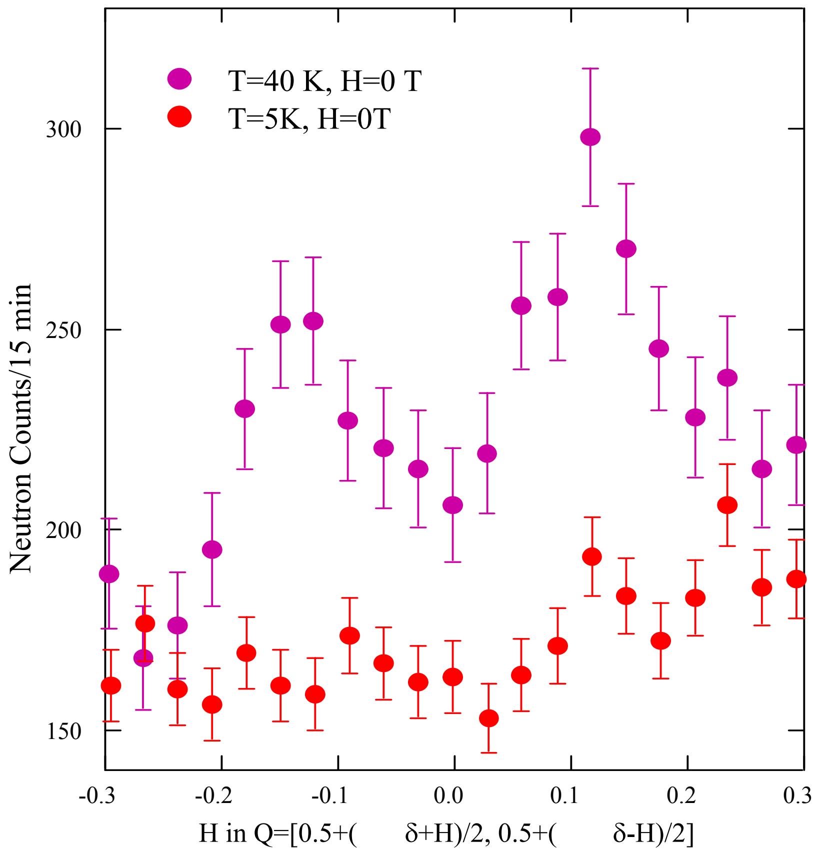


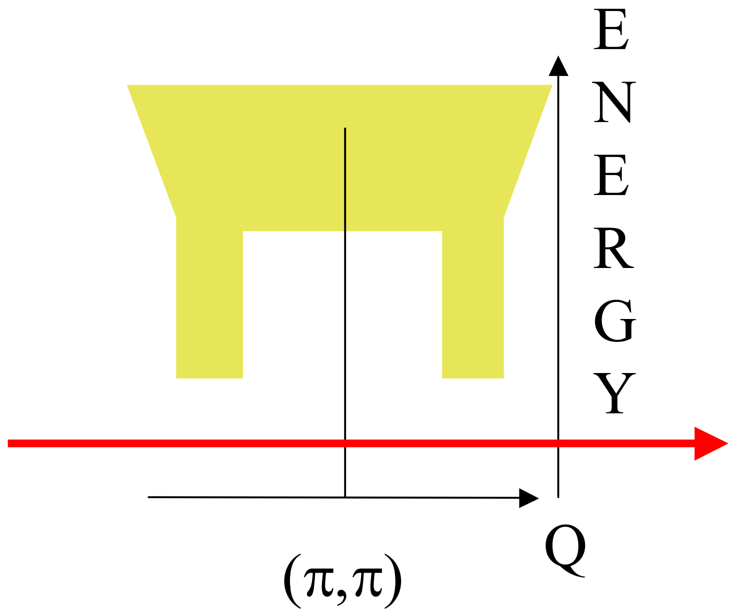
Overdoped LSCO (x=0.17, T_c=37 K)



Overdoped LSCO (x=0.17, T_c=37 K)

Q-scans, ΔE=4 meV





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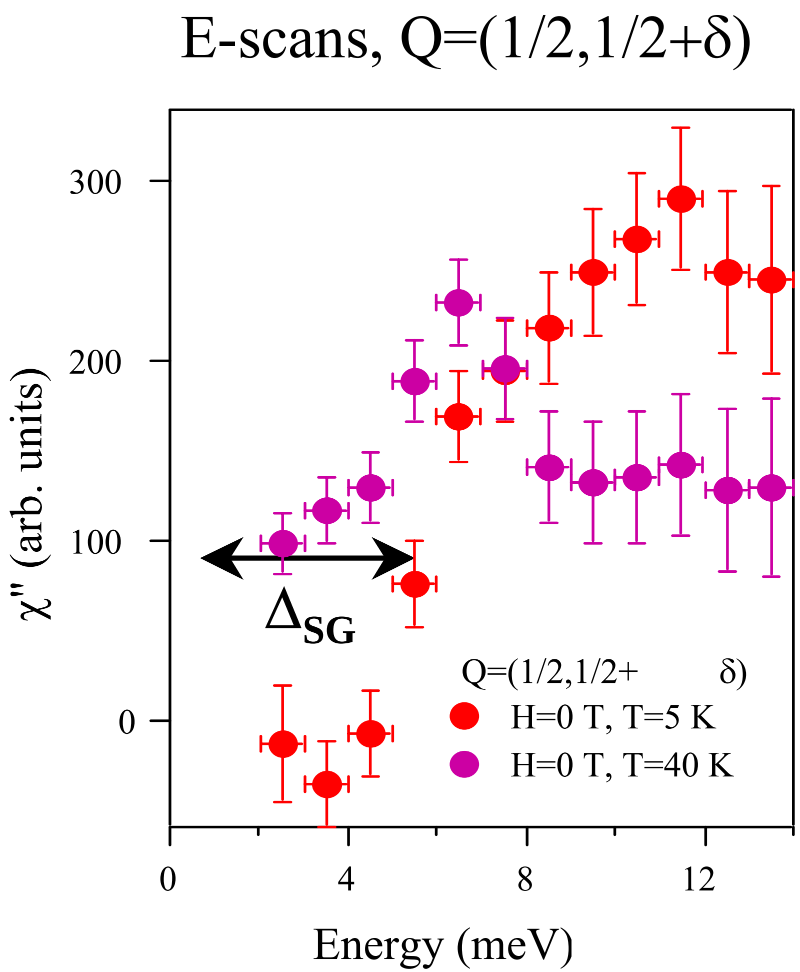
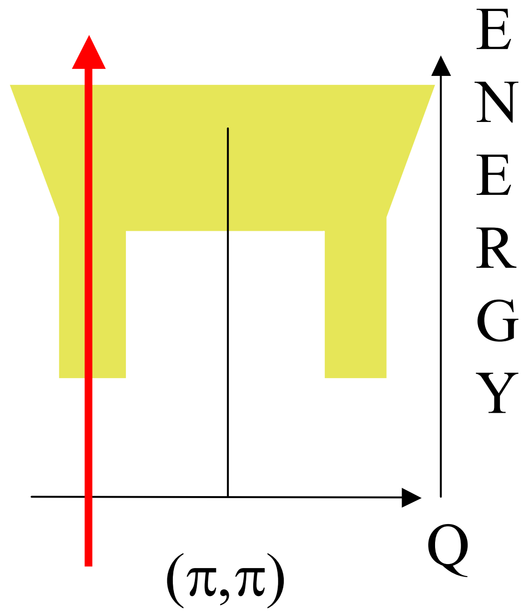
Overdoped LSCO (x=0.17, T_c=37 K)

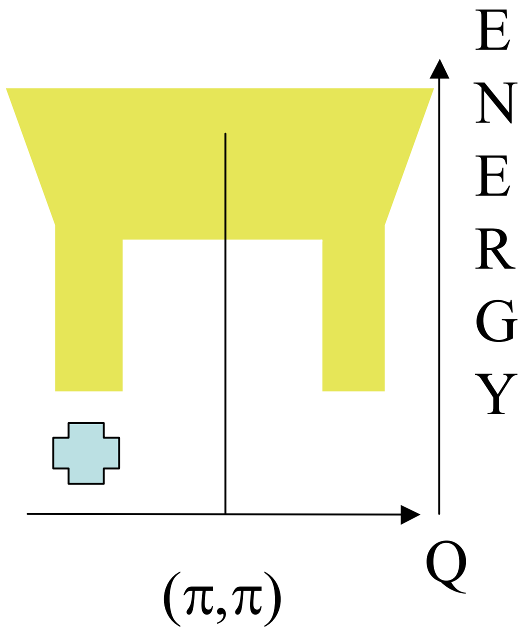




QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Spin gap: temperature scans at ΔE=2 meV

Overdoped (x=0.17, T_c=37 K)



QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Spin gap: temperature scans at ΔE=2 meV

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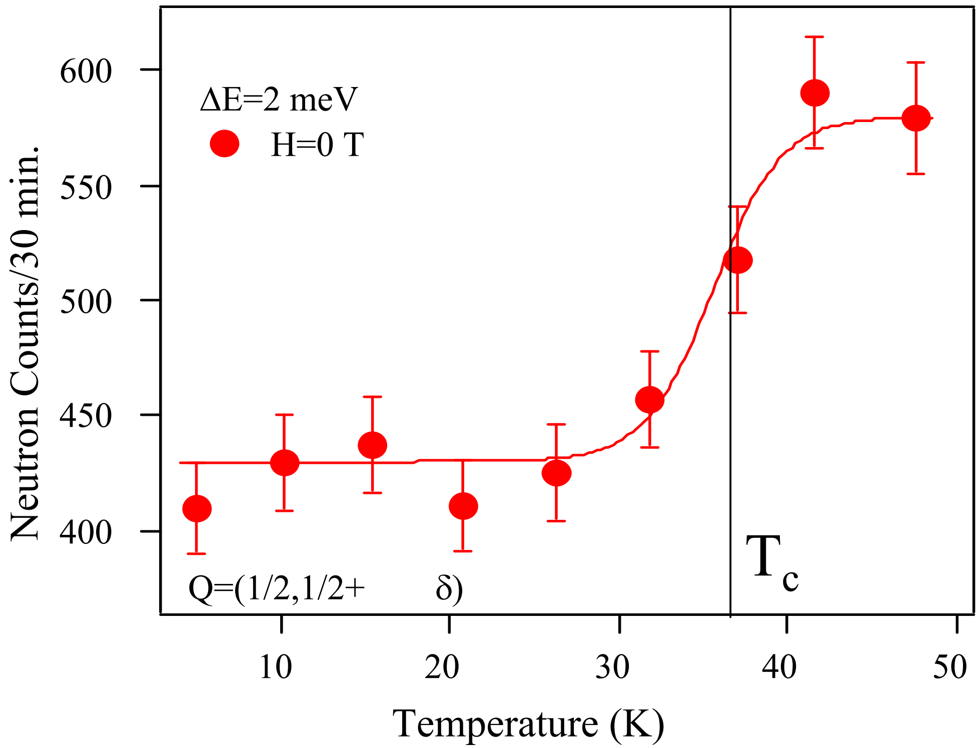
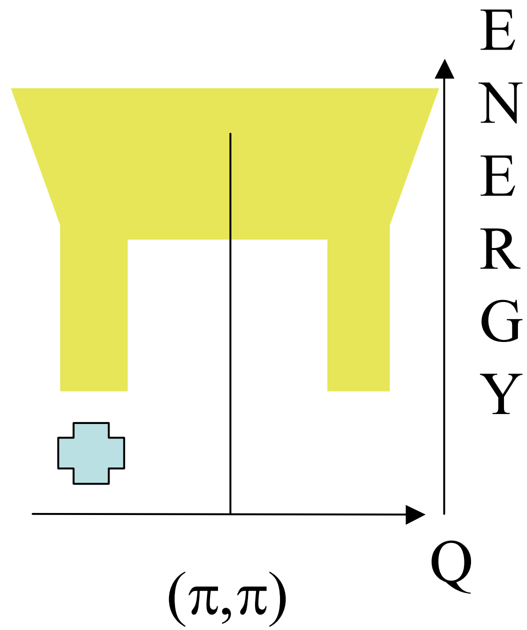
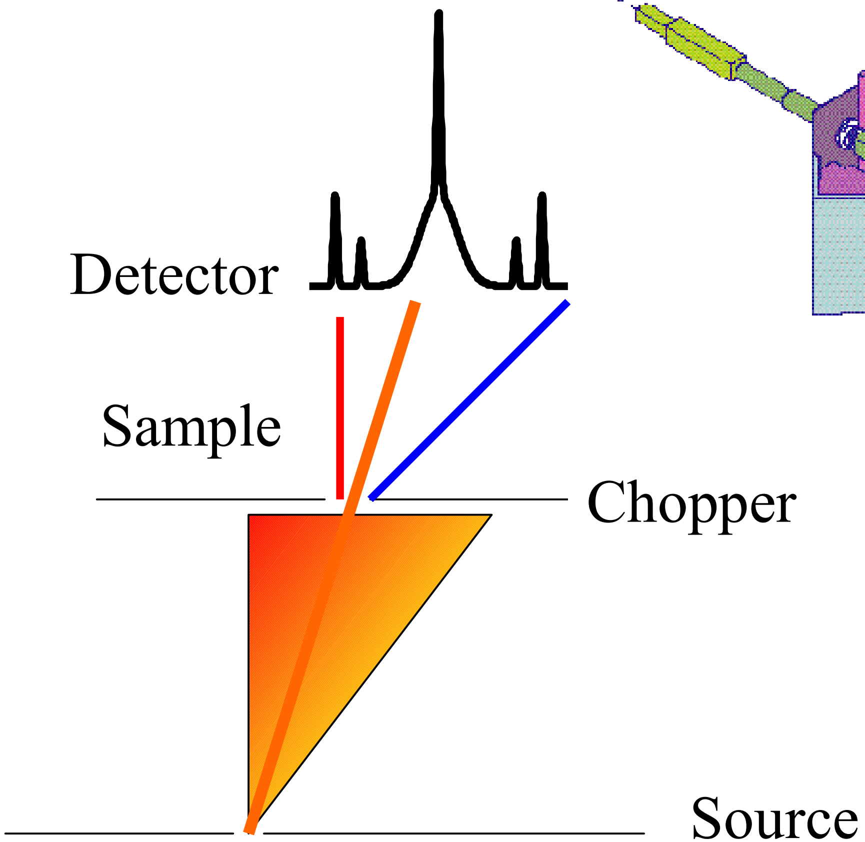
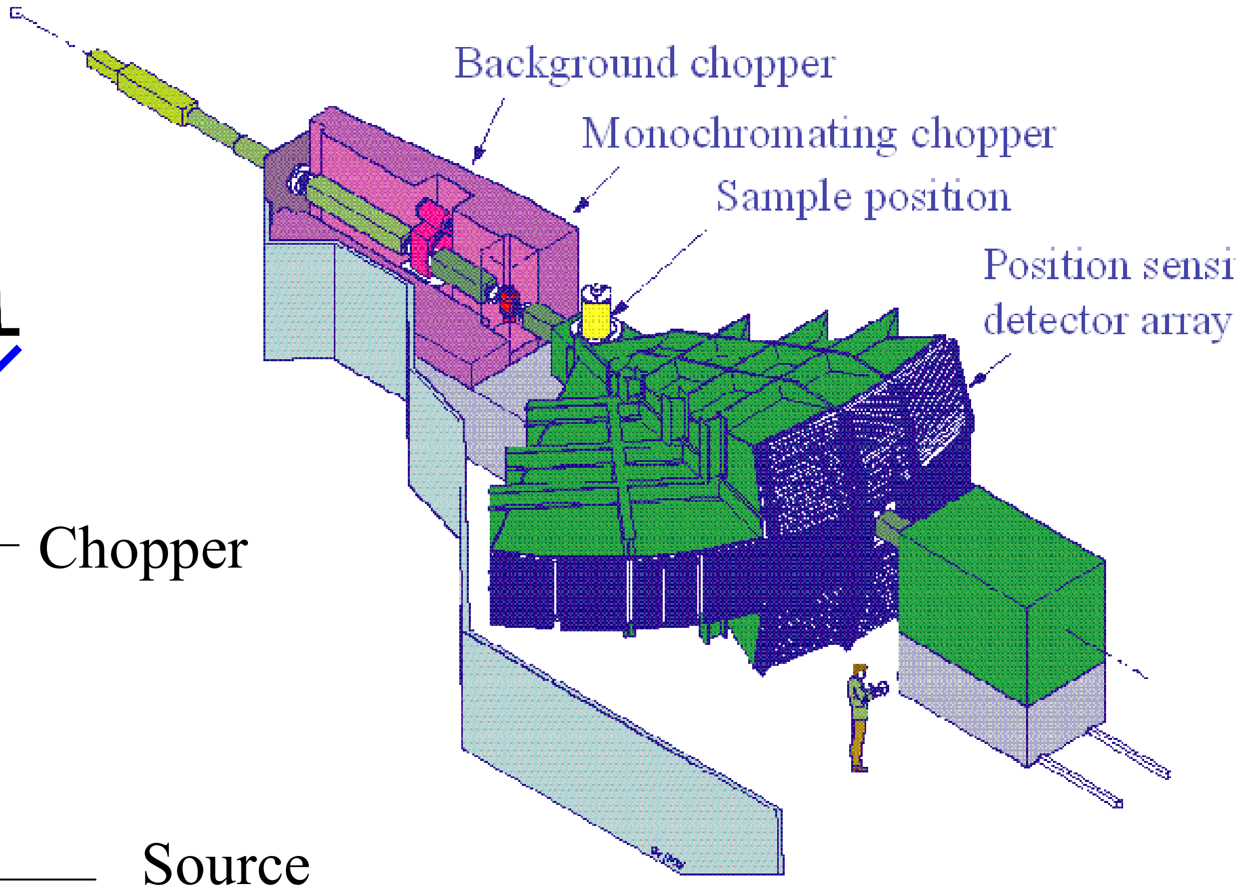




QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

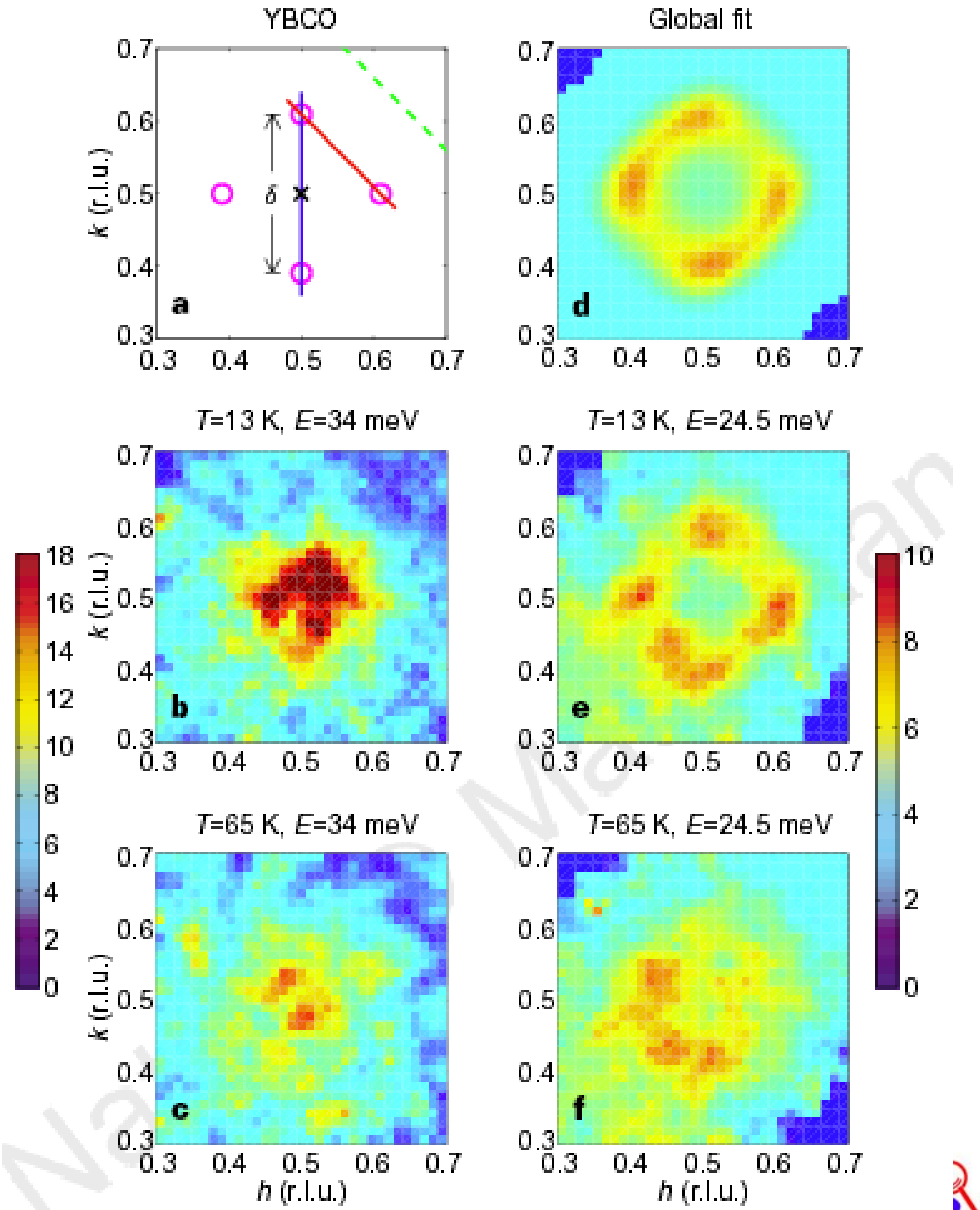
MAPS @ ISIS -RAL





Y123 x=6.6 / TOF

Mook *et al.*,
Nature **395** (1998) 580



QuickTime™ and a
TIFF (LZW) decompressor
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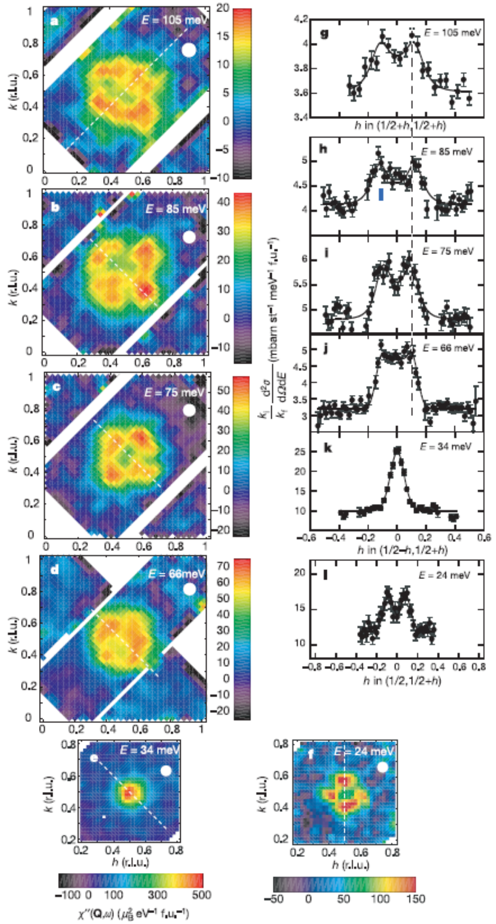
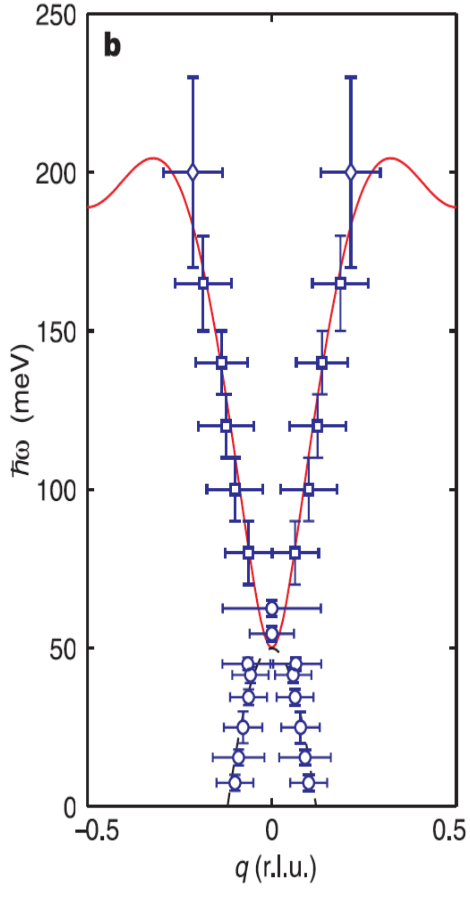
High-Energy (2004)

High-Energy (2004)

Underdoped YBCO non-SC LBCO



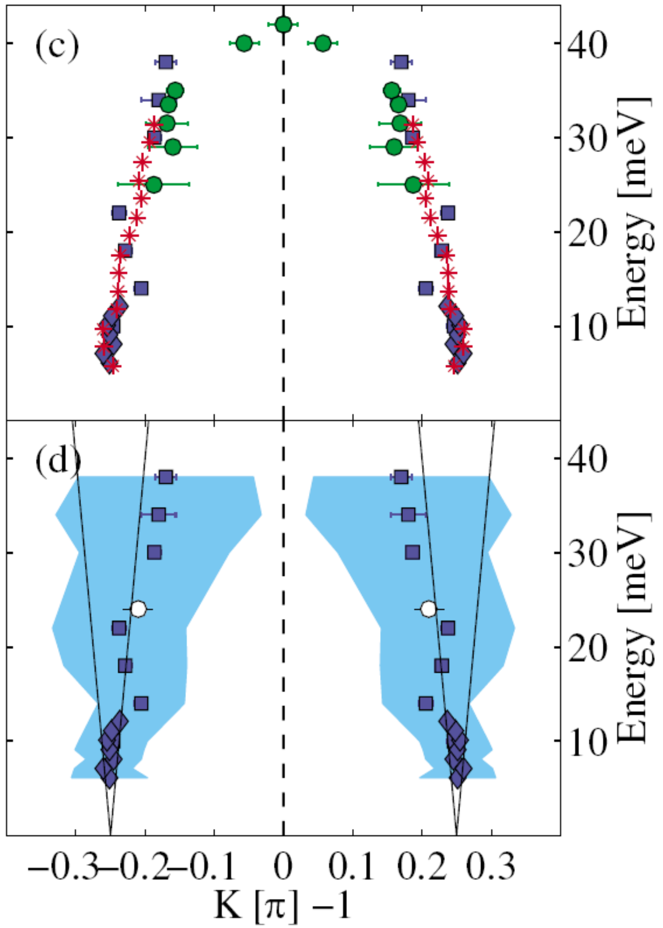


Hayden *et al.* Nature Tranquada *et al.* Nature

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

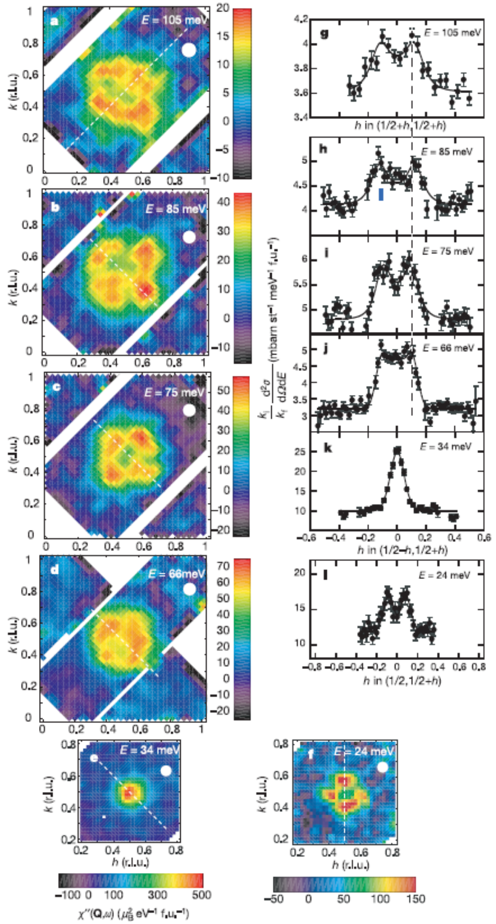
High-Energy (2004)

Opt LSCO /YBCO



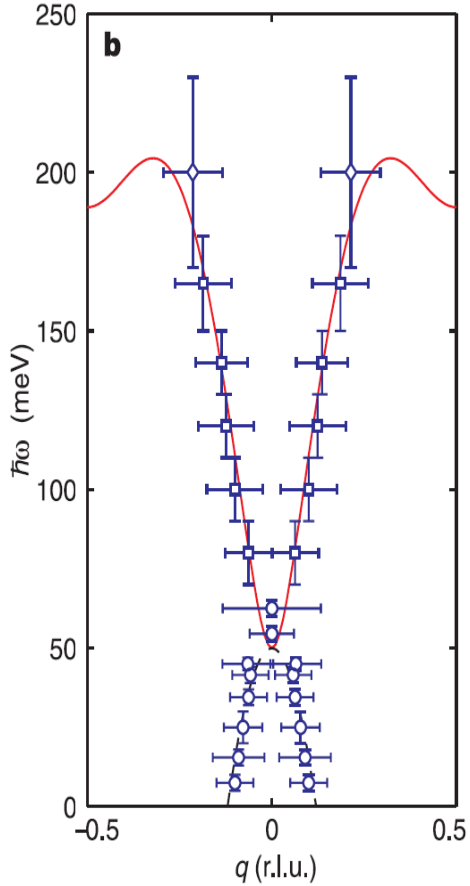
Christensen, *et al.* Phys. Rev. Lett.

Underdoped YBCO



Hayden *et al.* Nature

non-SC LBCO

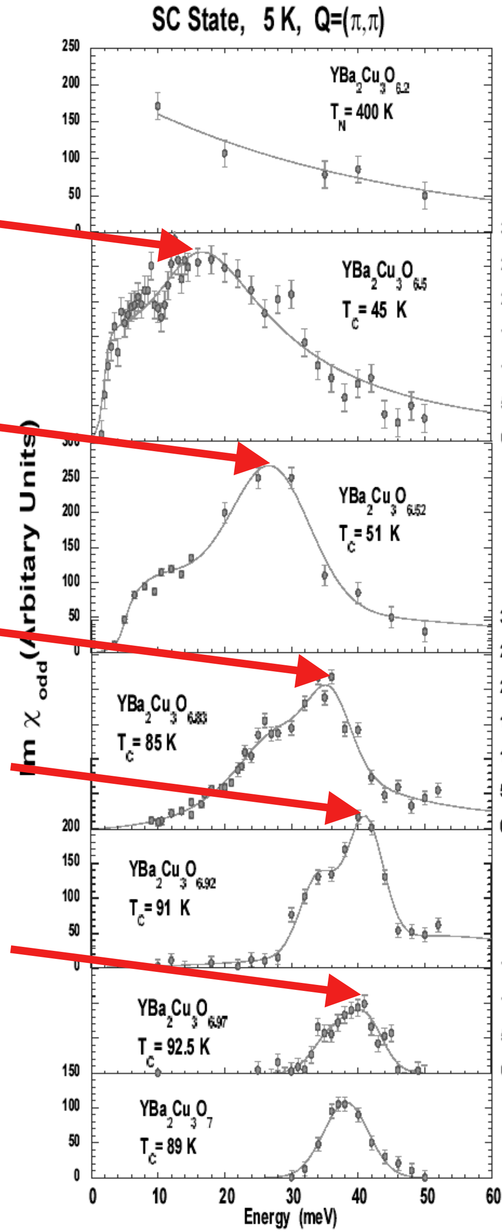


Tranquada *et al.* Nature

Resonance: doping dependence

Ph. Bourges cond/mat (9901333)

$YBa_2Cu_3O_{6+x}$



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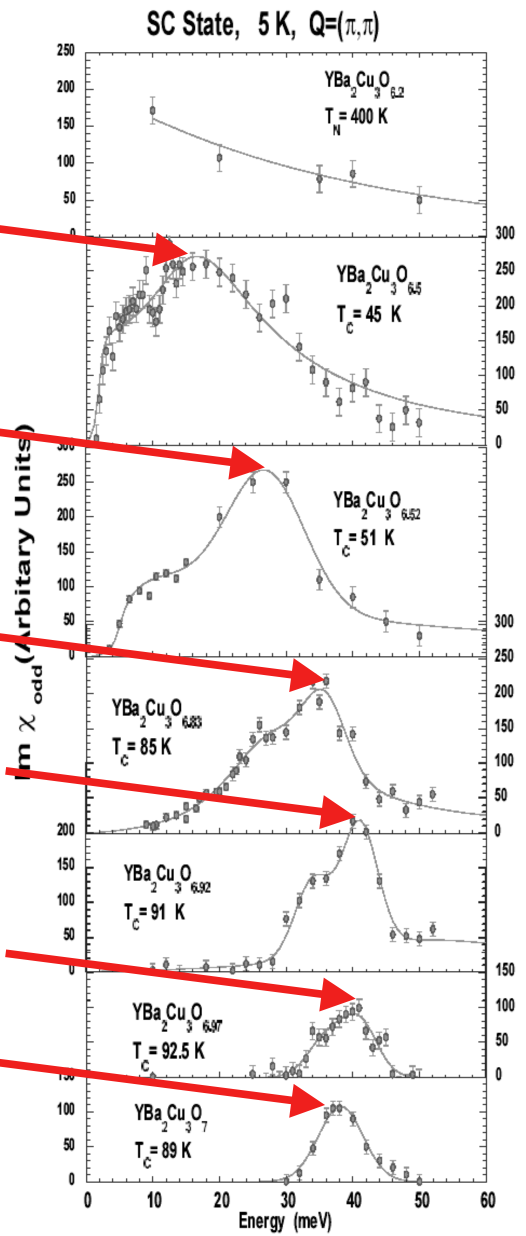


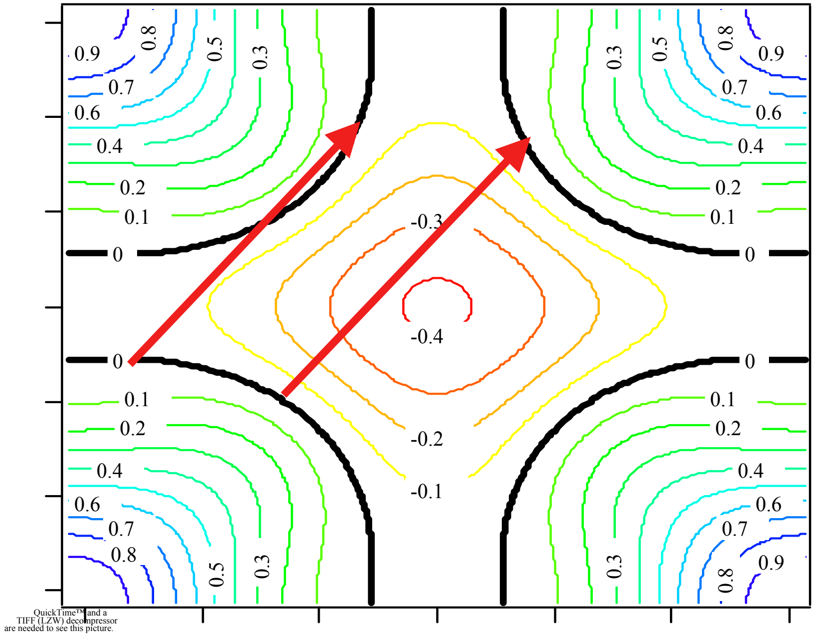
QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

What is the nature of the spin excitations?

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

What is the nature of the spin excitations?

Fermi nesting



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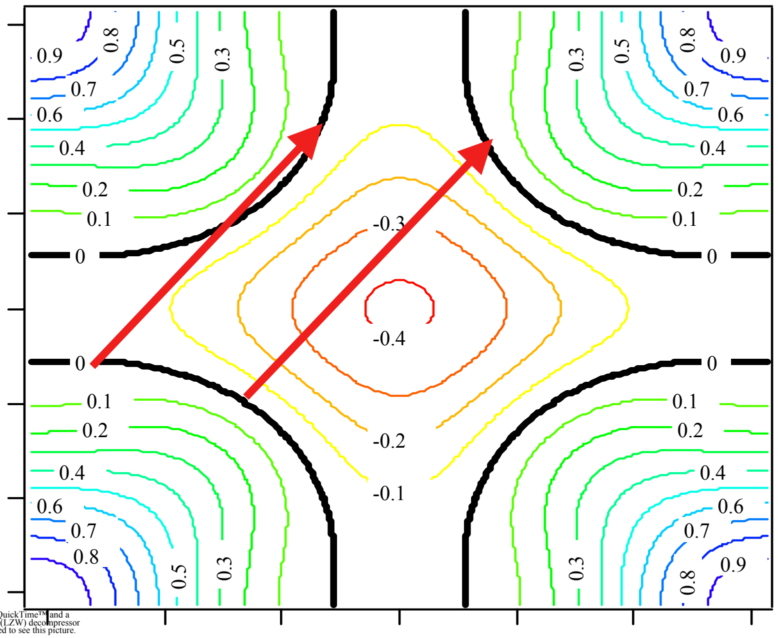
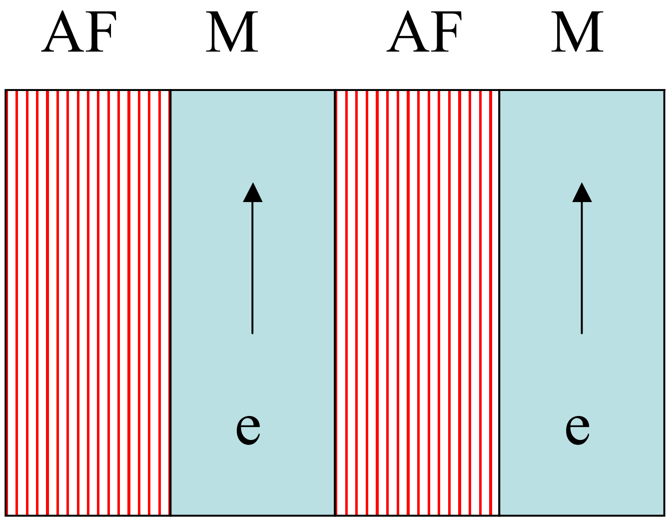


Stripes



Fermi Nesting Scenario

$$\chi_0(q,\omega) = \sum_k \left(1 - \frac{\varepsilon_k \varepsilon_{k+q} + \Delta_k \Delta_{k+q}}{E_k E_{k+q}}\right)\left(\frac{f\left(E_{k+q}\right) + f\left(E_k\right) - 1}{\omega - \left(E_{k+q} + E_k\right) + i\delta}\right)$$

$$E_k = \sqrt{\varepsilon_k^2 + \Delta_k^2}$$

QuickTime™ and a
TIFF (LZW) decompressor
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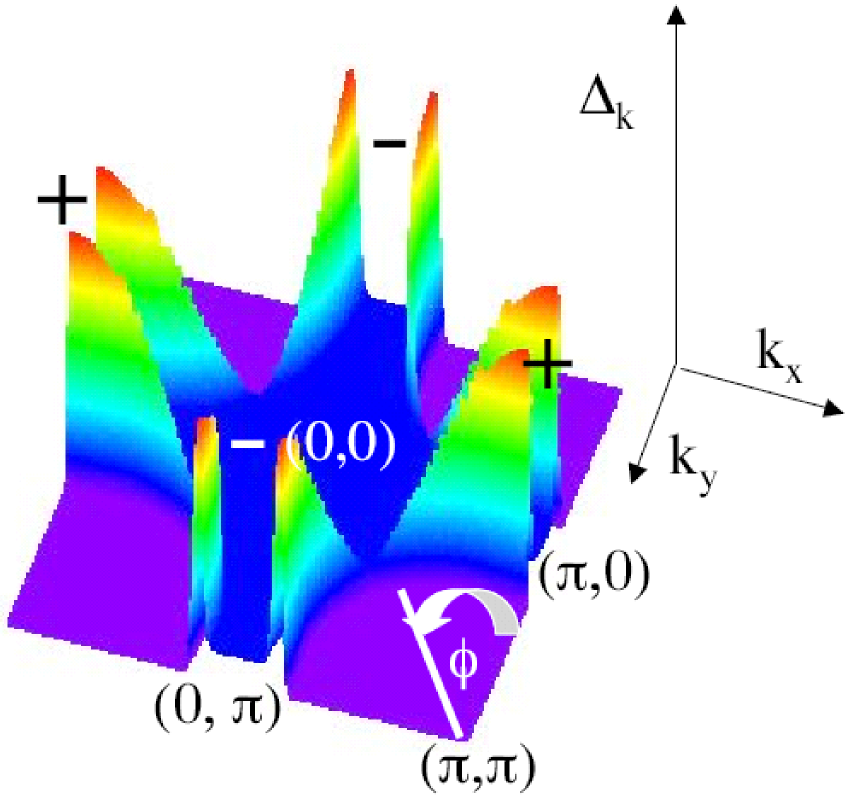

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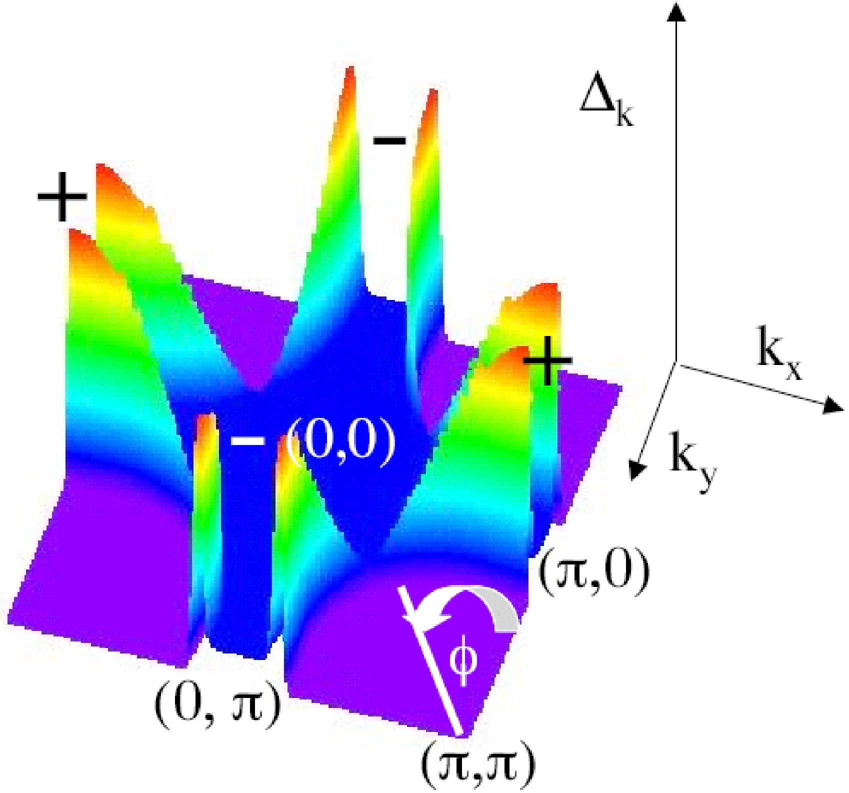
$\varepsilon_k=0$ --> 1-sign(Δ_k) sign(Δ_{k+q})



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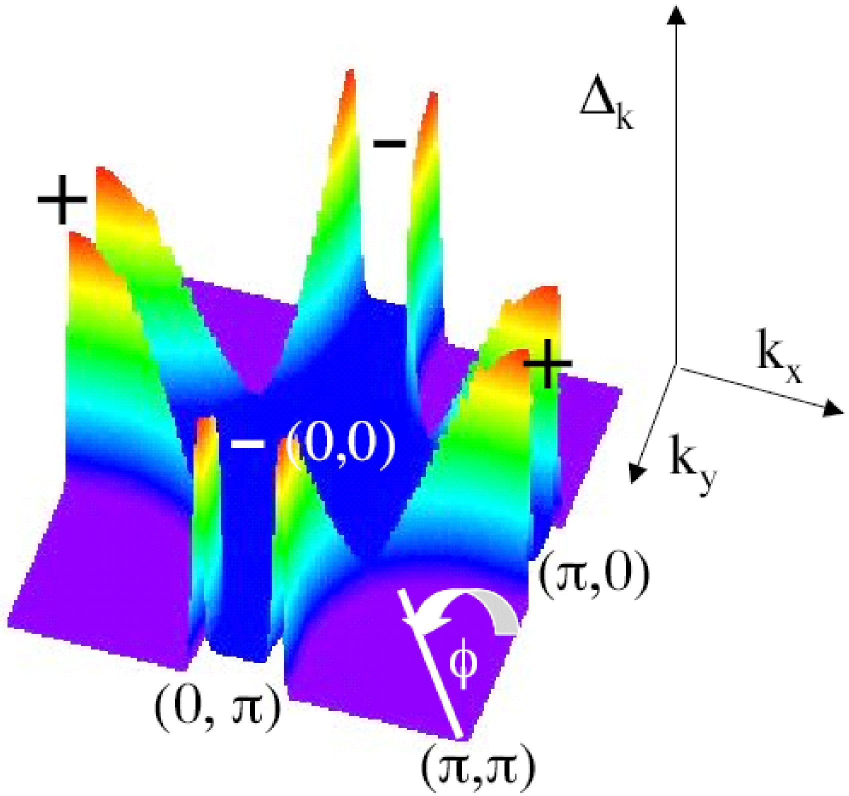
q=(π,π)-->	0	if Δ_k even (s)
	2	if Δ_k odd (d)



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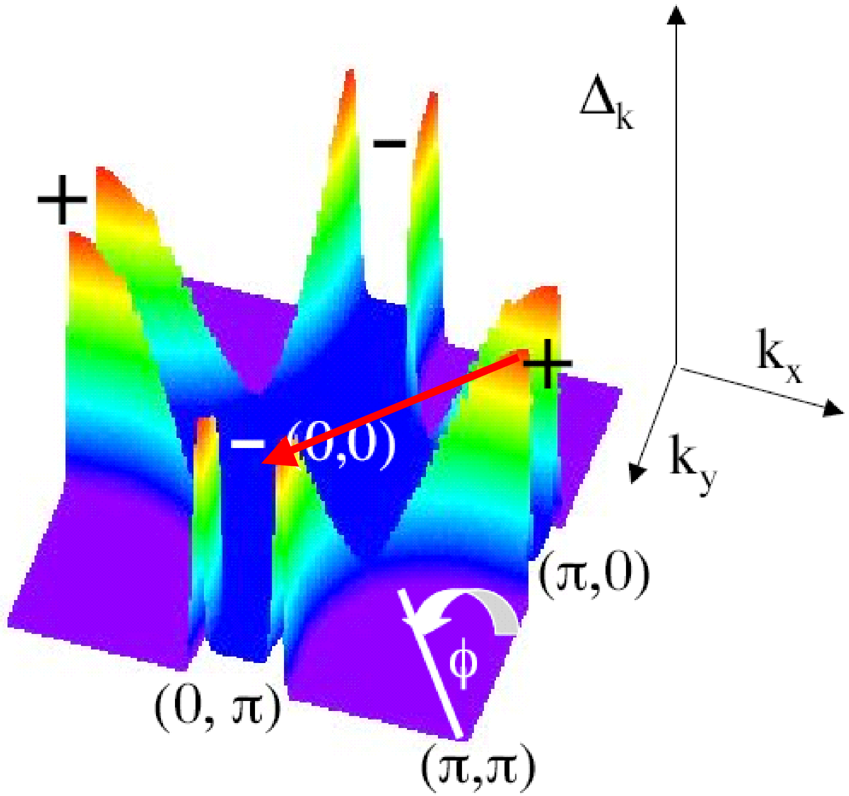

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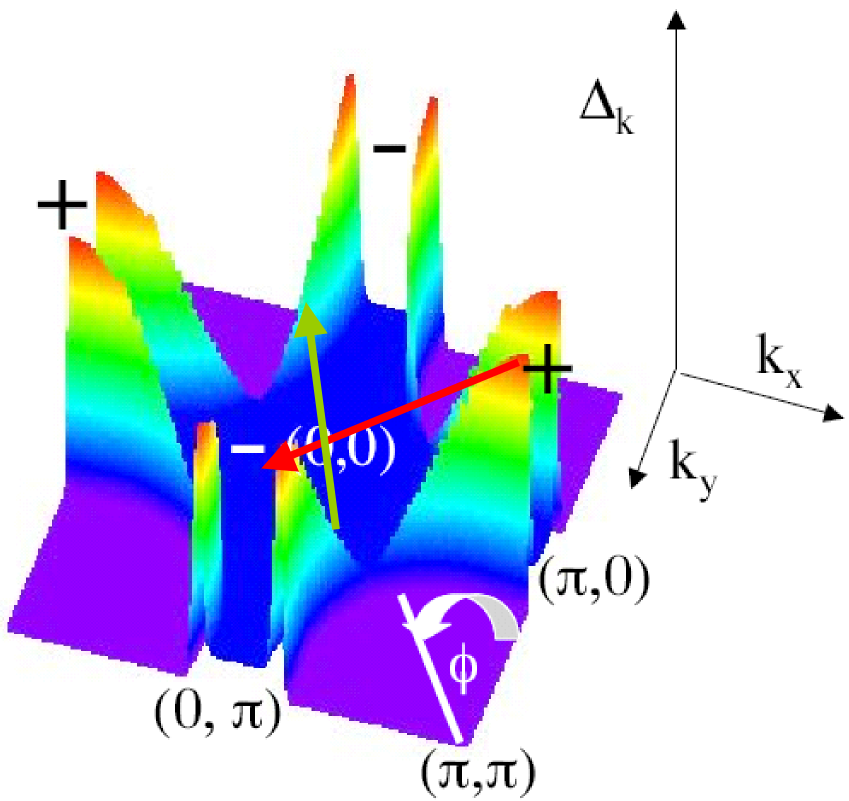

$q=(\pi,\pi)$ ---> $q=(\pi+\delta,\pi)$

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Renormalized Susceptibility

M. Lavagna PRB **49** (94) 4235, D. Z. Liu, PRL **75** (95) 4130, N. Bulut, PRB **53** (96) 5149, J. Brinckmann, PRL **82** (99) 2915, Norman, PRB **61** (00) 14751

+interactions (RPA)

$$\chi(q,\omega) = \frac{\chi_0(q,\omega)}{1 - J(q)\,\chi_0(q,\omega)}$$

$$J(q) = J\,(\cos(q_x a) + \cos(q_y a))\,/2$$

QuickTime™ and a
TIFF (LZW) decompressor
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Renormalized Susceptibility

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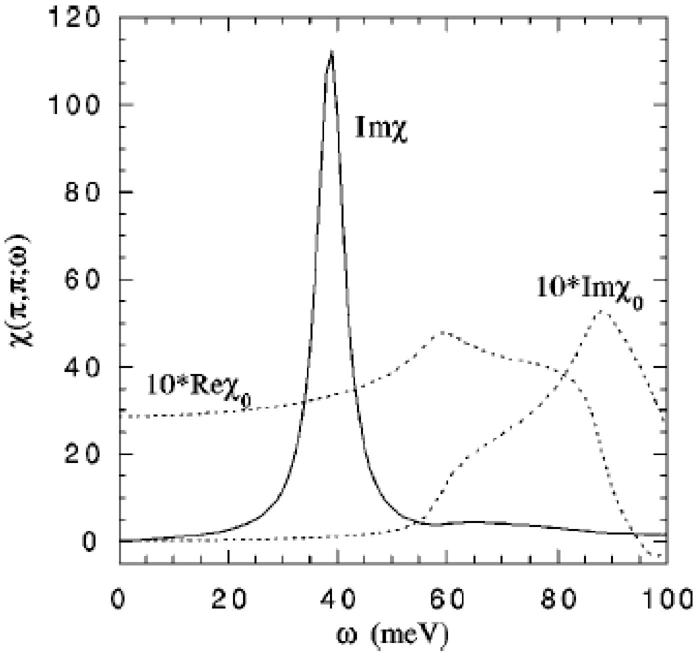

Norman, PRB **61** (00) 14751

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QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Fermi nesting approach:

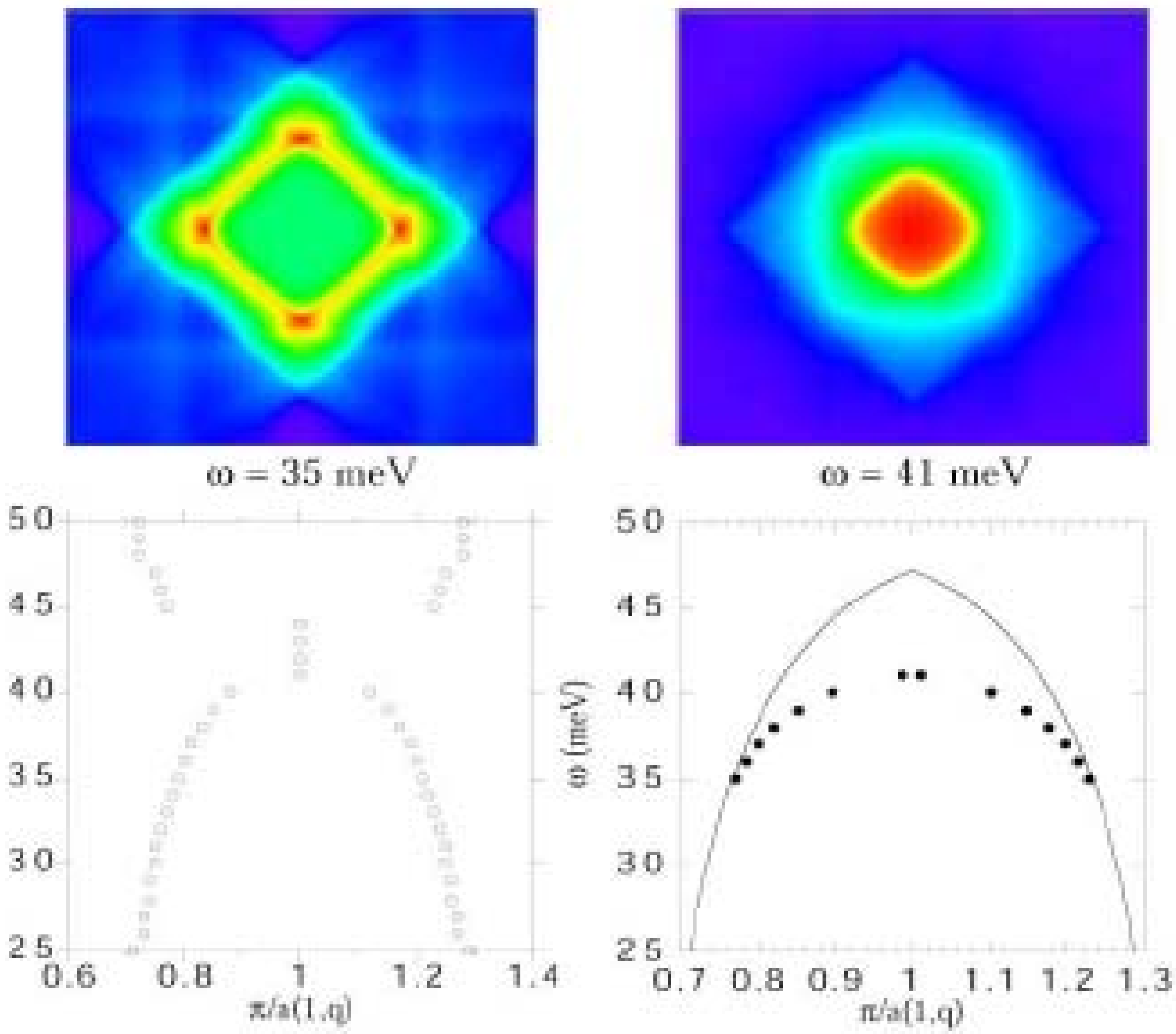
YBCO

Norman, Pépin

Cond-mat/0302347

Schneider et al.

Phys. Rev. B. 2004

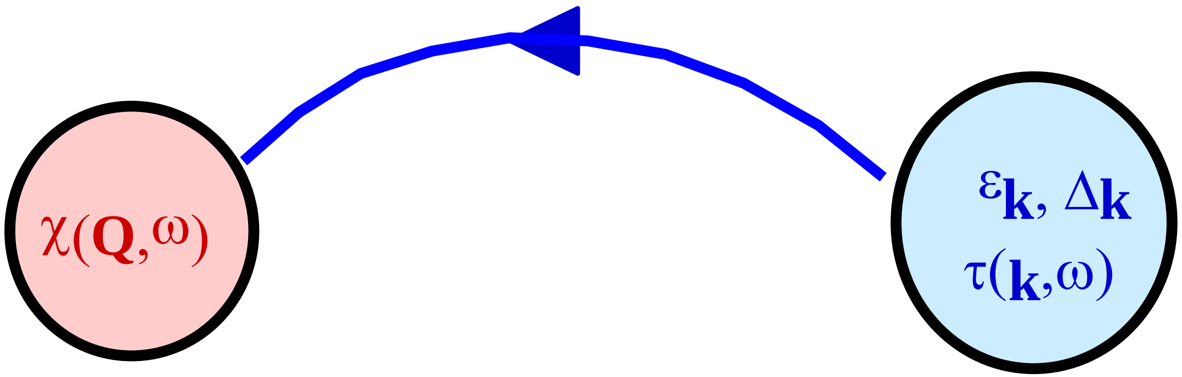


QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Is magnetism relevant for HTSC?

Magnetism

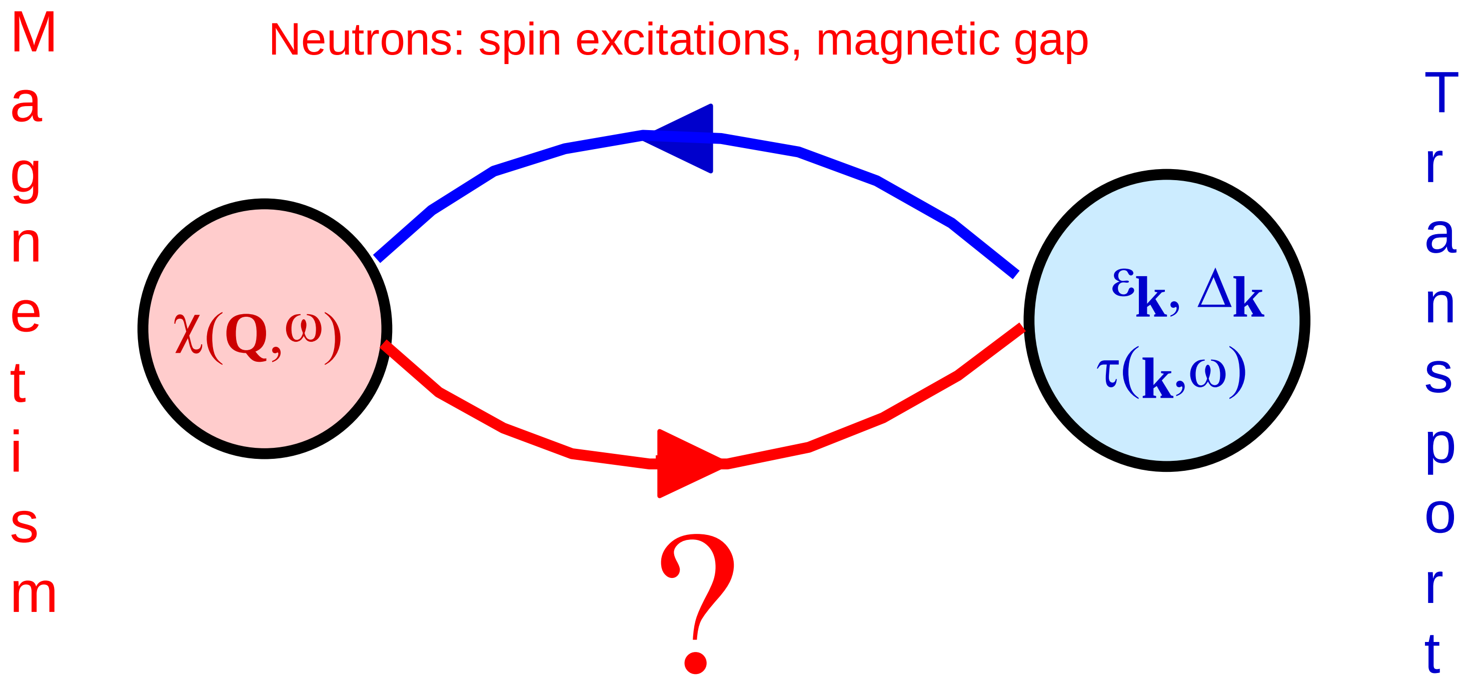
Neutrons: spin excitations, magnetic gap



Transport

QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Is magnetism relevant for HTSC?



QuickTime™ and a
TIFF (LZW) decompressor
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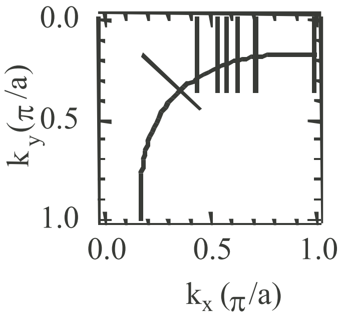
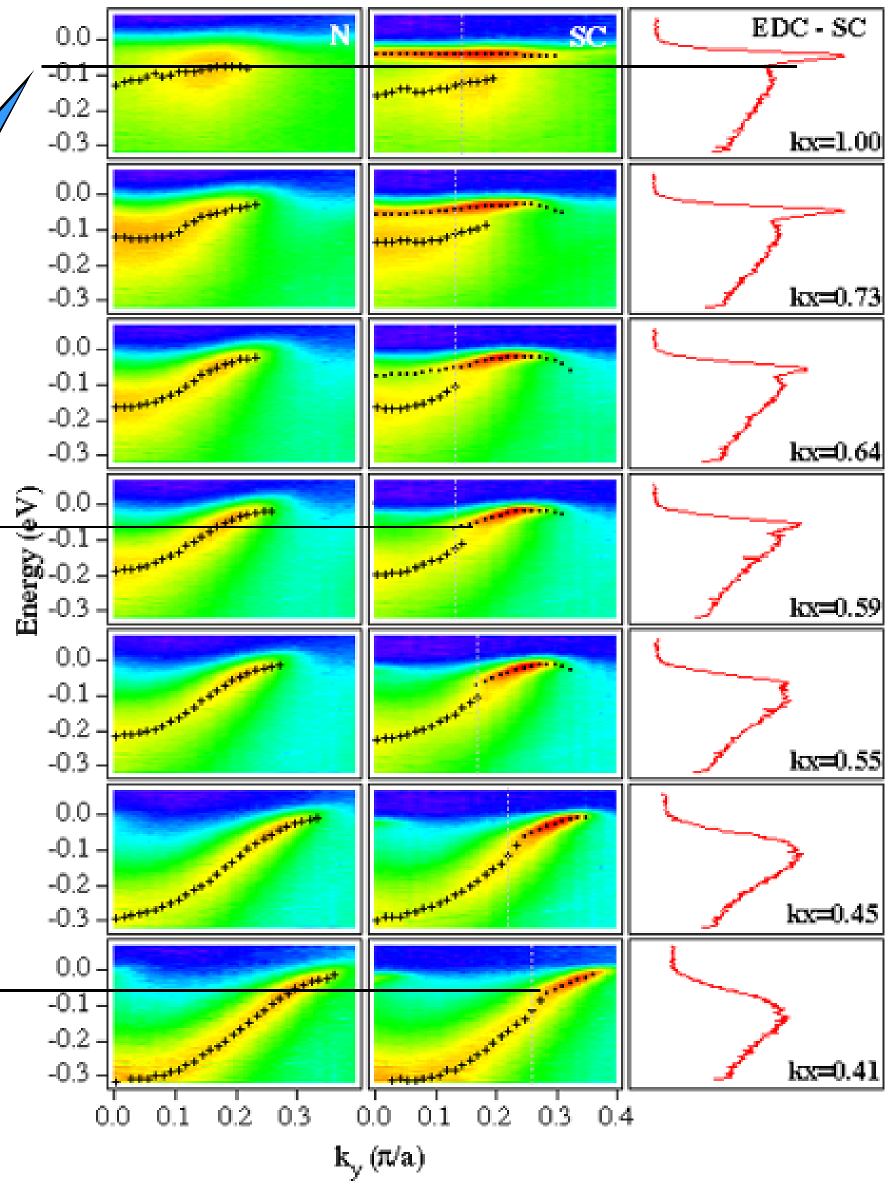
Electronic renormalization (ARPES)?



The effect is present at all k-points

Bi2212 Opt



QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

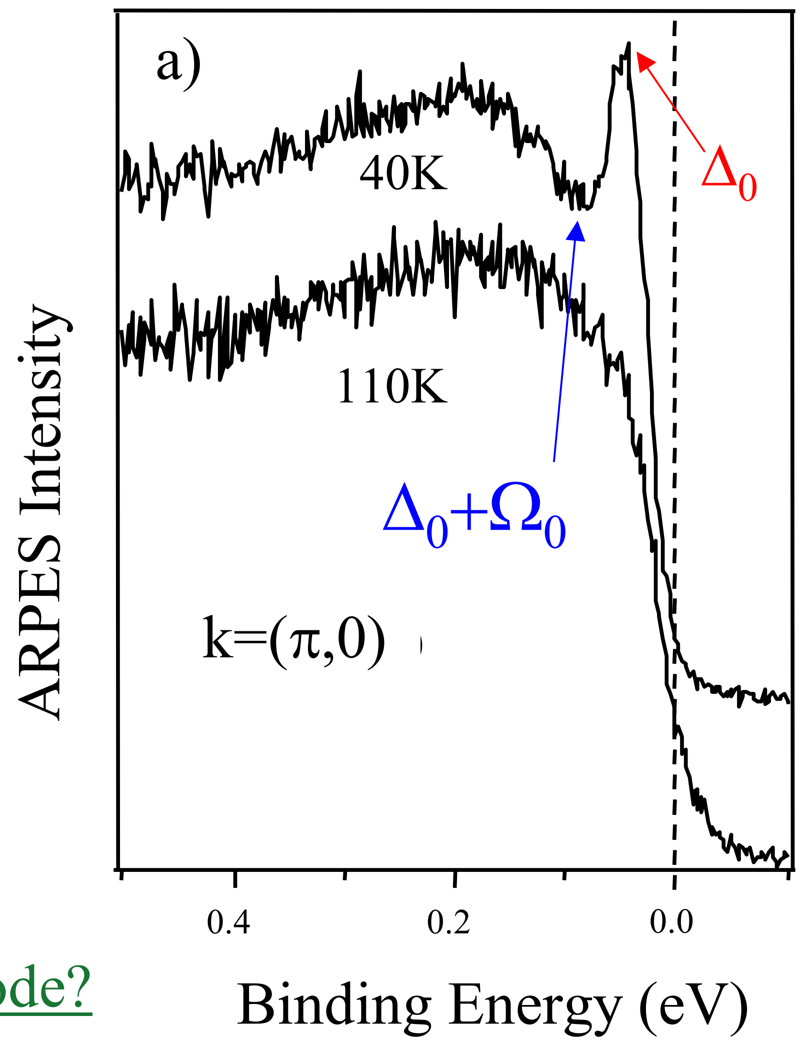
Origin of the electronic renormalization?

Interaction of the electrons with a collective mode of energy $\Omega_0 < 2\,\Delta$

-Norman *et al.* PRL 79 (1997) 3506
-Abanov *et al.* PRL 83 (1999) 1652
-Dahm *et al.* PRB 58 (1999) 12454

$$A(\mathbf{k},\omega) = \frac{\operatorname{Im}\Sigma(\mathbf{k},\omega)}{|\omega - E_{\mathbf{k}} - \operatorname{Re}\Sigma(\mathbf{k},\omega)|^2 + |\operatorname{Im}\Sigma(\mathbf{k},\omega)|^2}$$

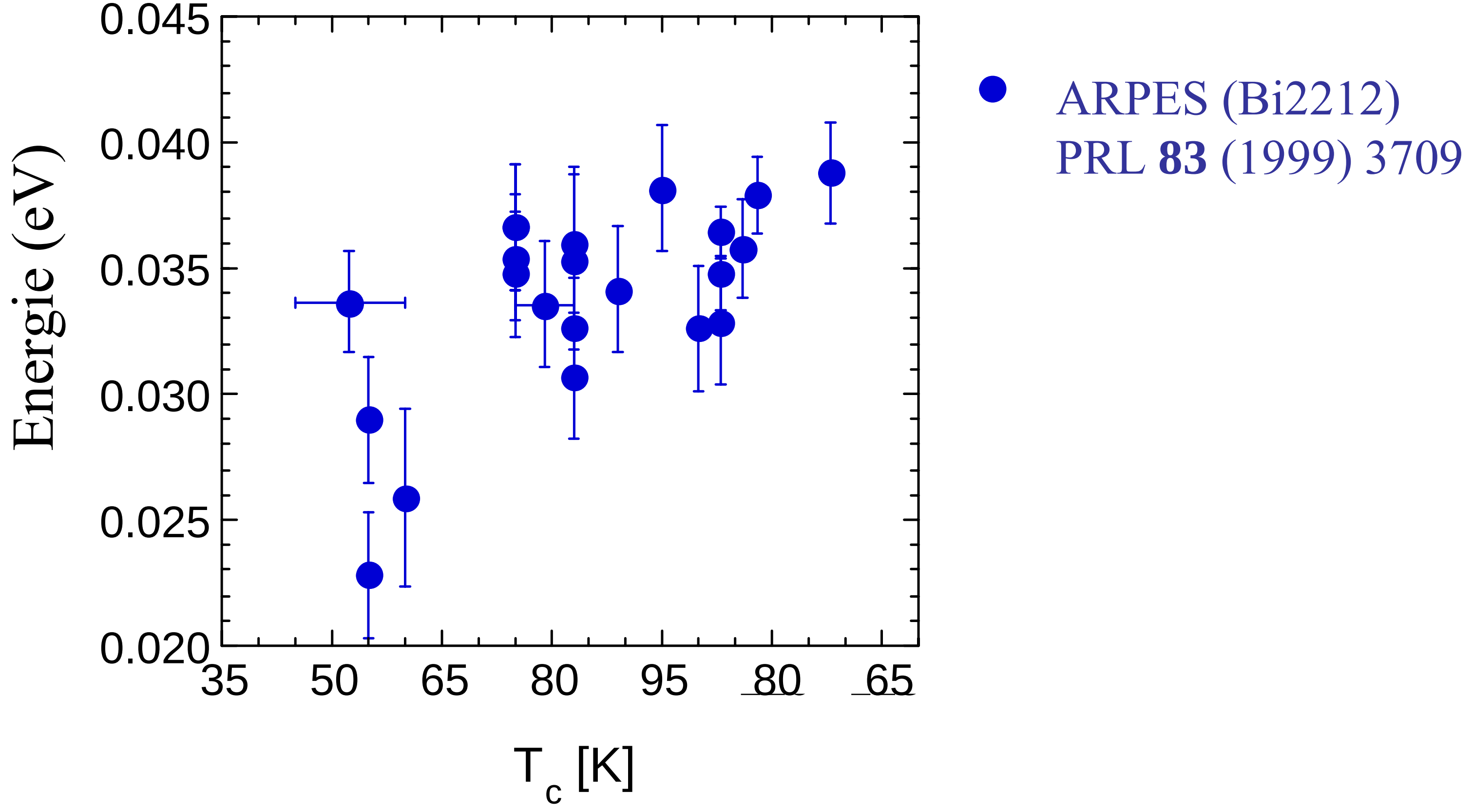
$$(\text{at } E_F) \rightarrow \frac{1}{|\operatorname{Im}\Sigma(\mathbf{k},\omega)|} \approx \tau\,(\text{life time})$$



What is the nature of the collective mode?

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Collective mode = resonance ?



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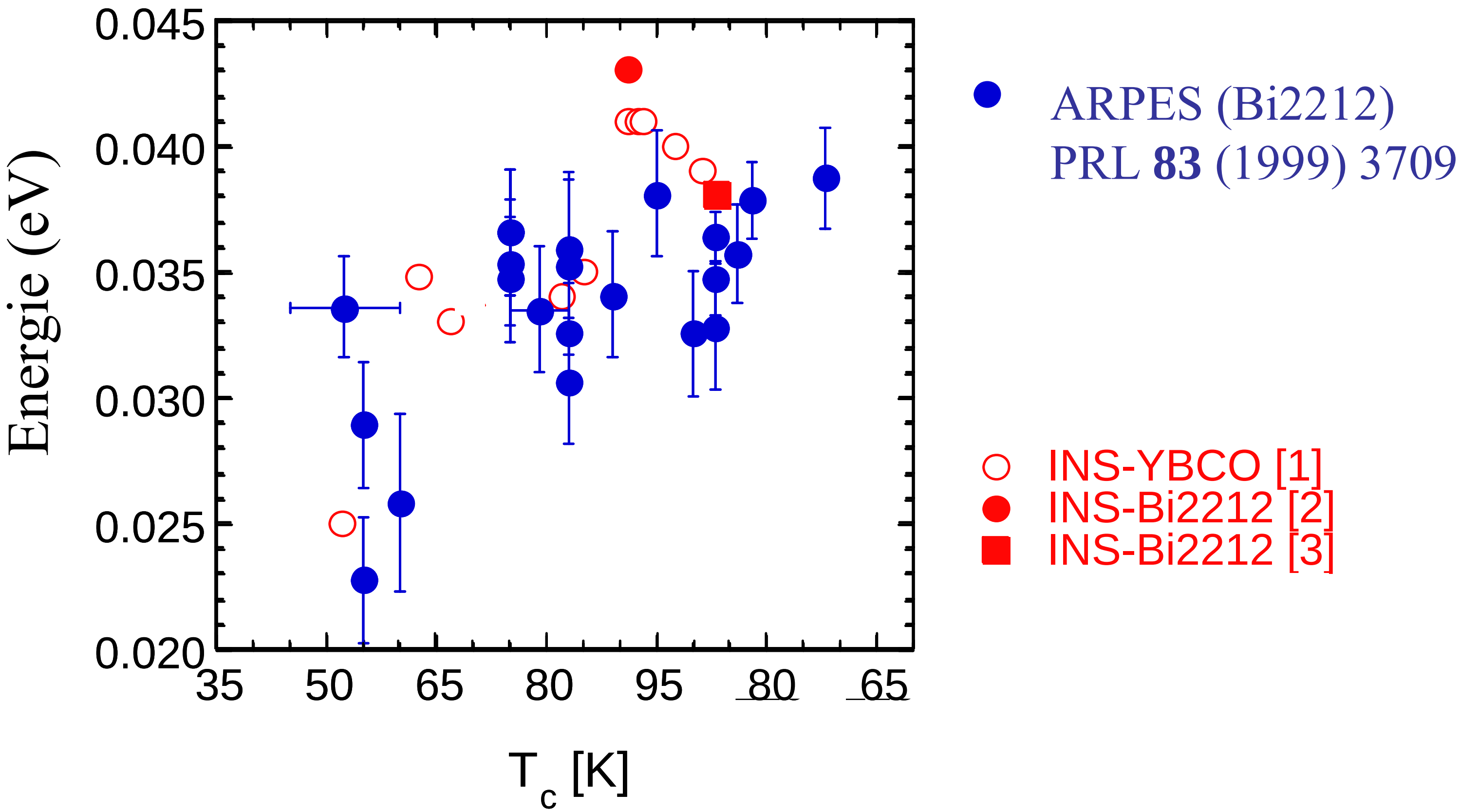


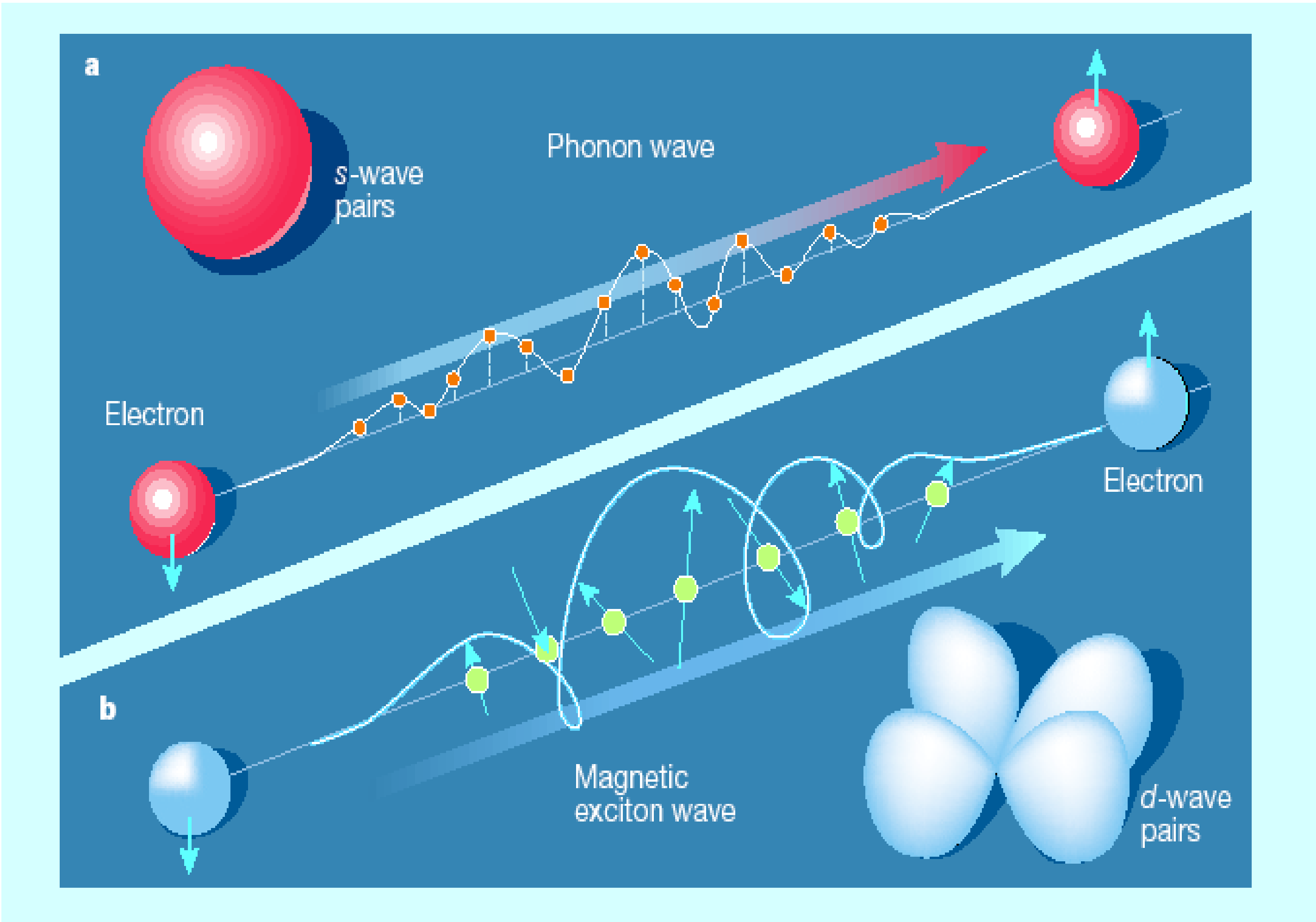
QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

[1] Ph. Bourges, cond-mat/9901333
[2] H. Fong, Nature **398** (99) 588
[3] H. He, cond-mat/0002013

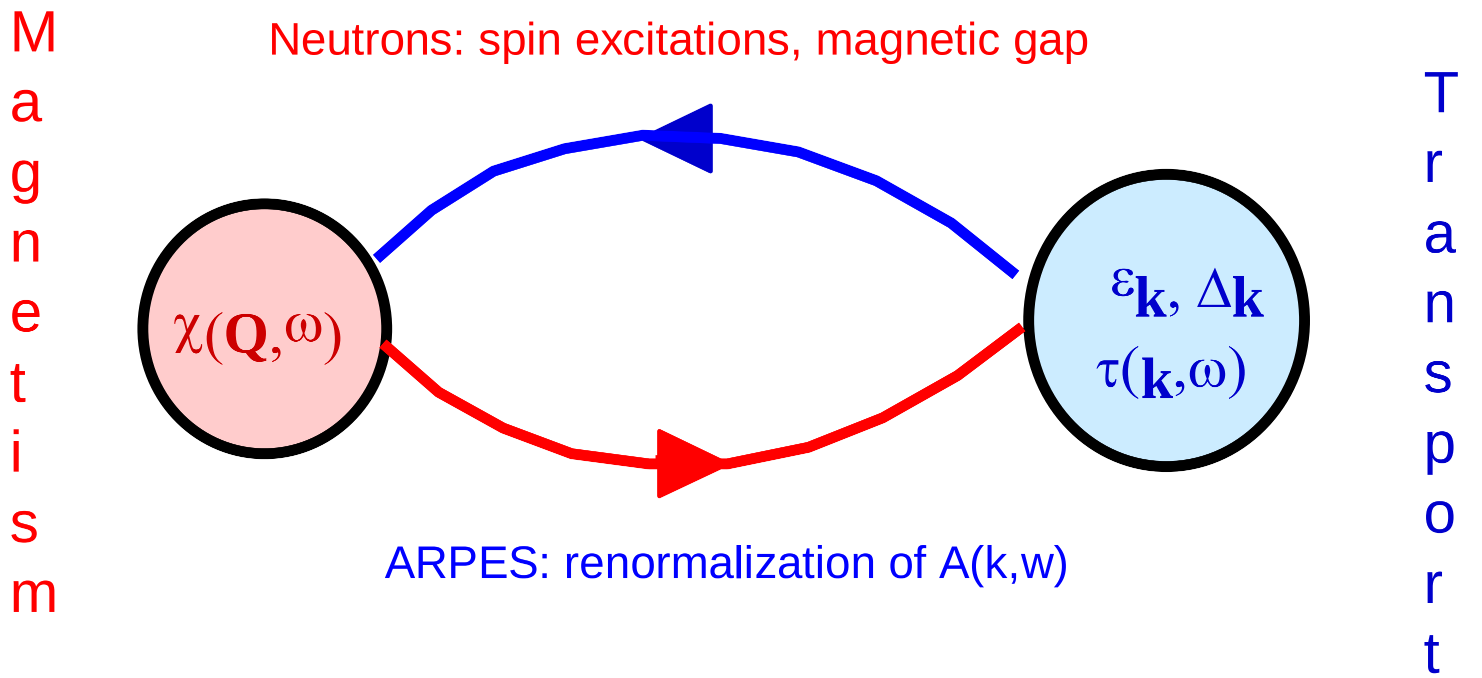
Low- vs high-T_c Superconductors

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Low- vs high-T_c Superconductors

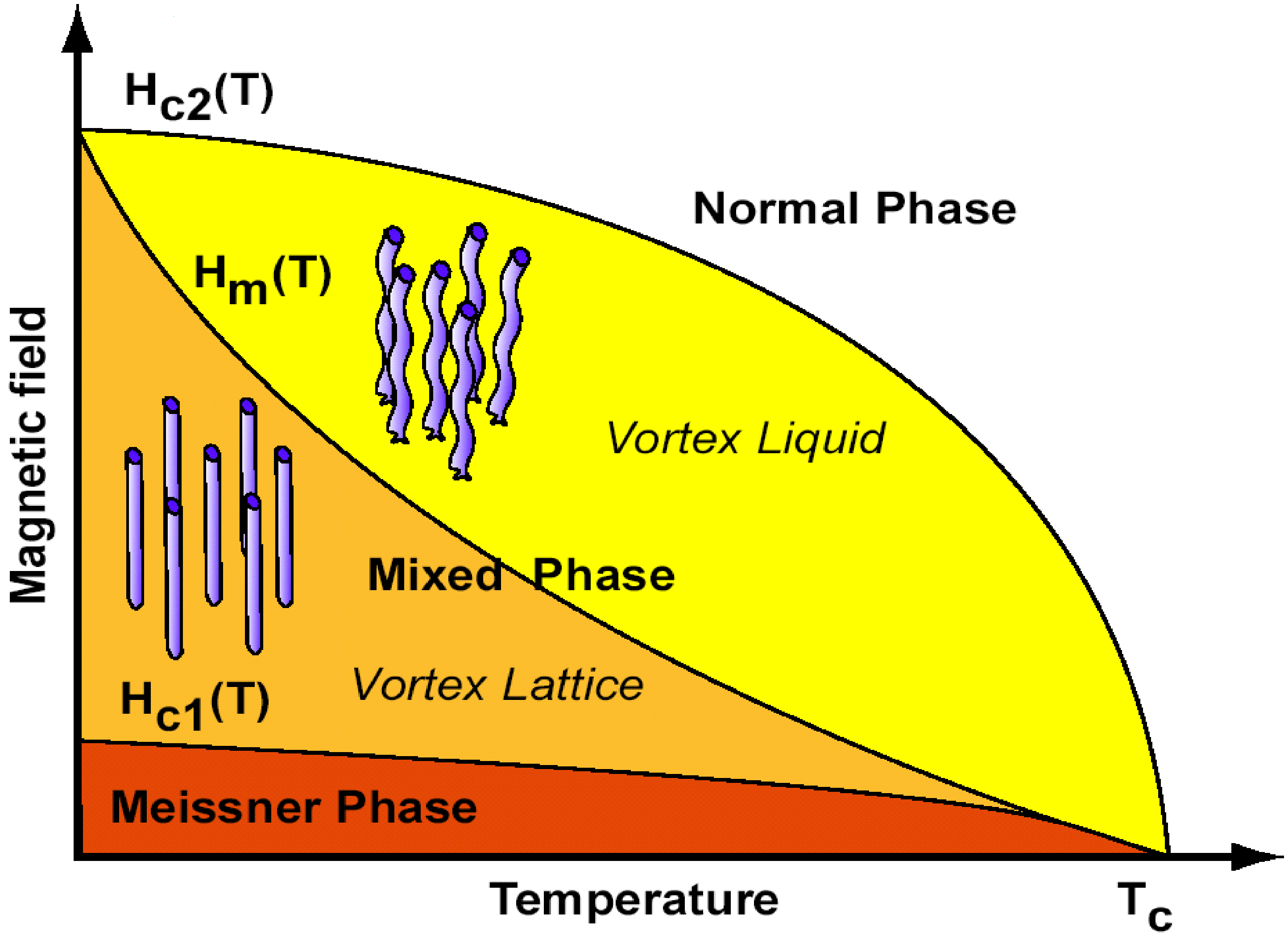


A complex and fascinating problem!



QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

Magnetic Phase Diagram



Meissner effect (magnetic)

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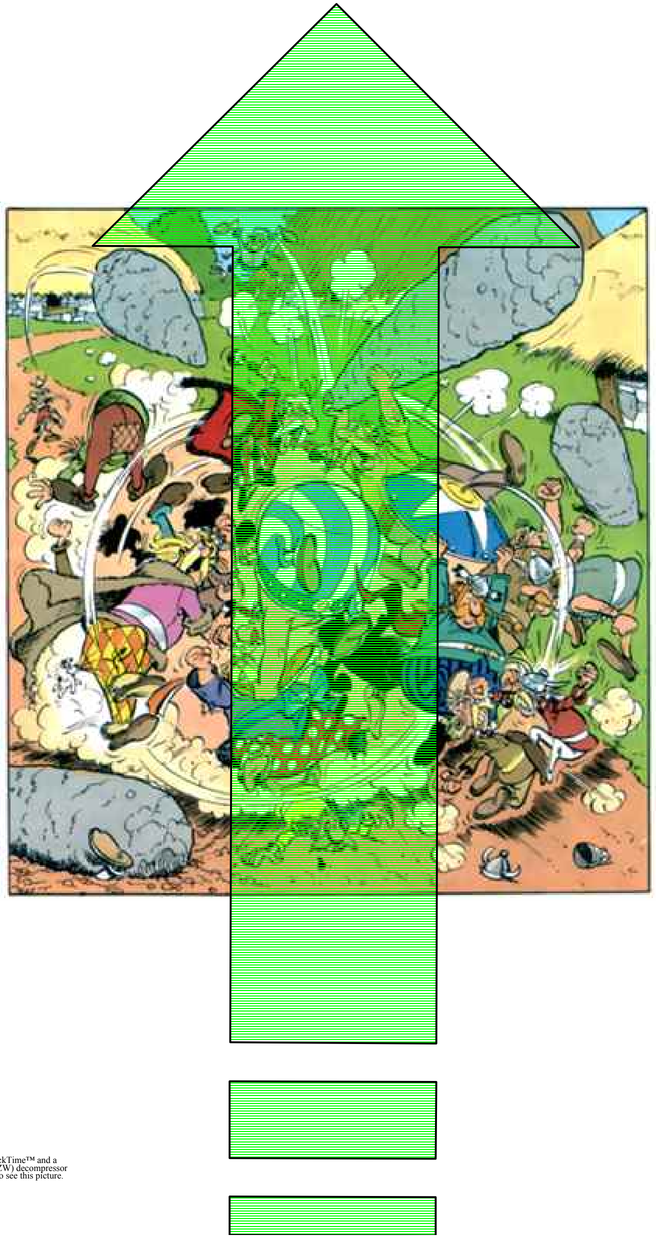

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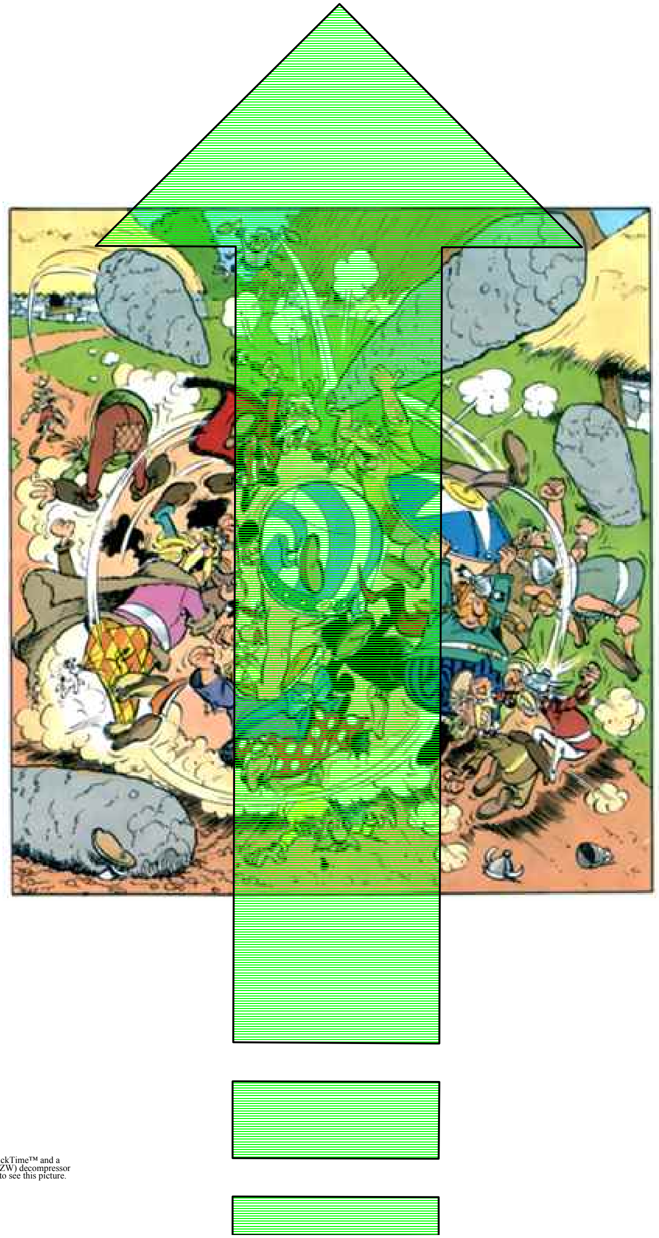

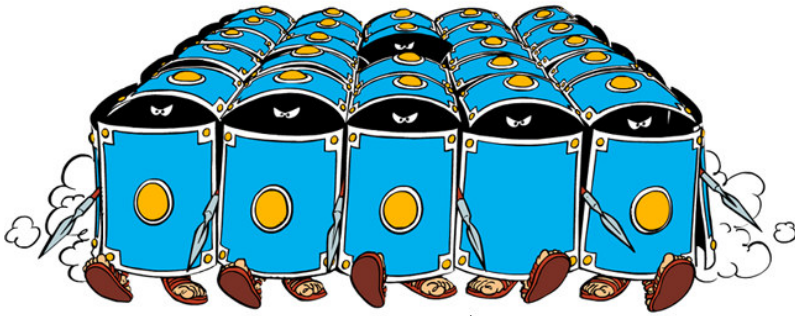

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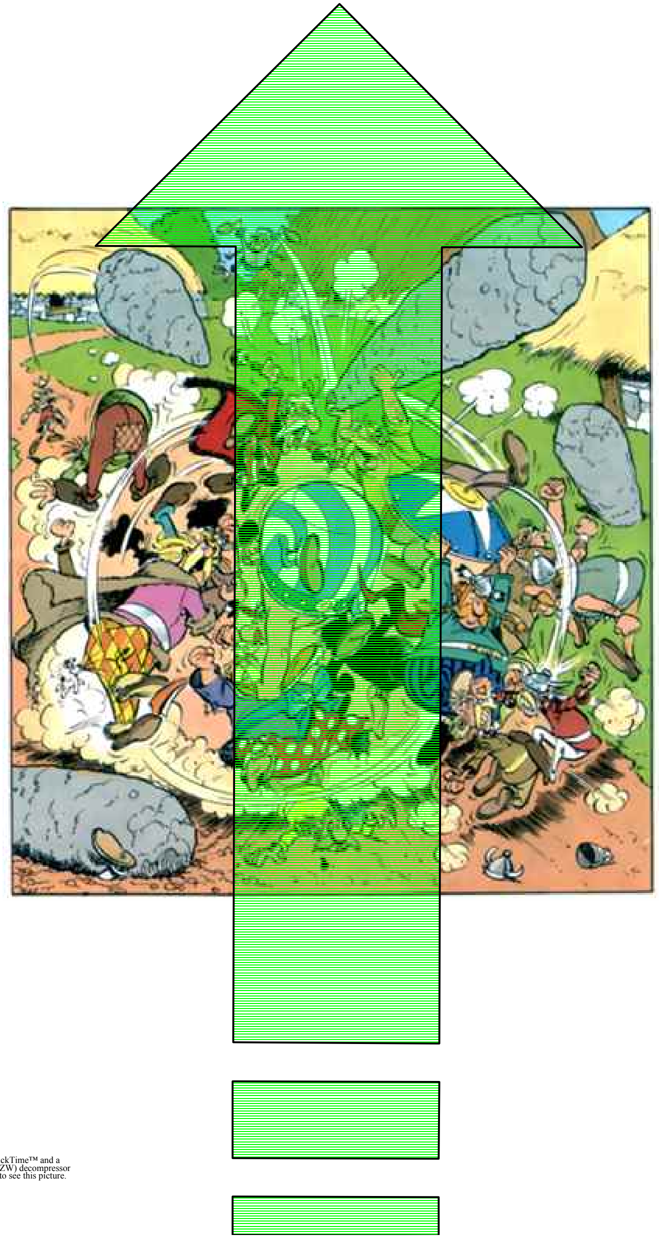

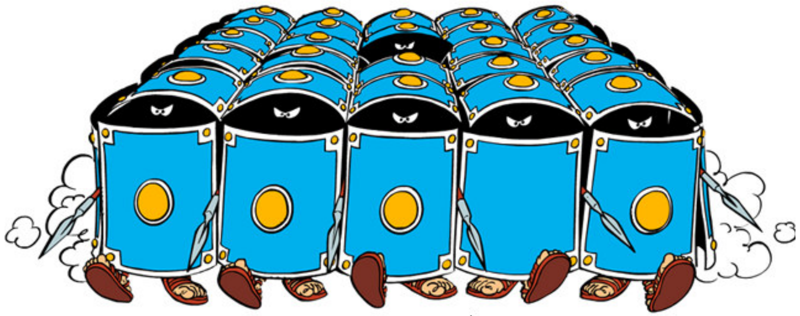

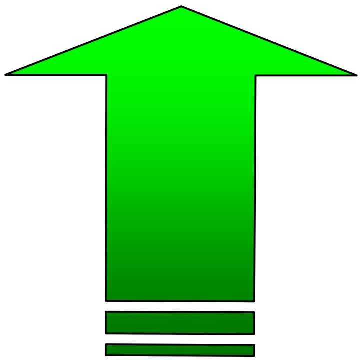

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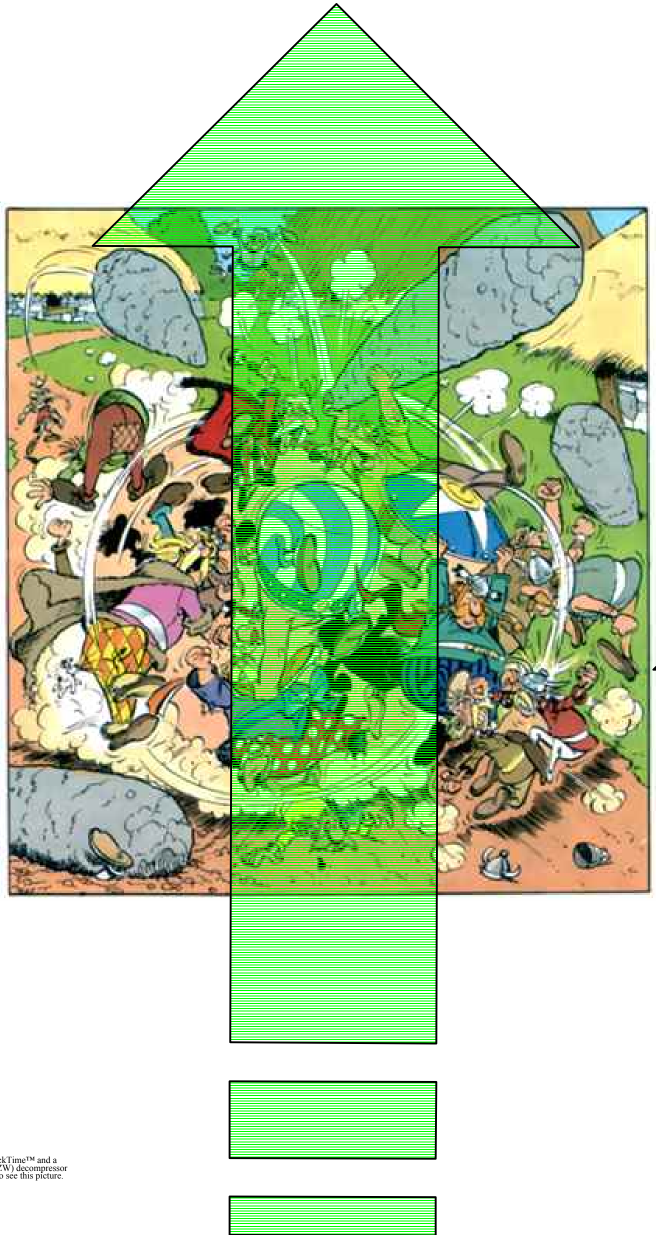

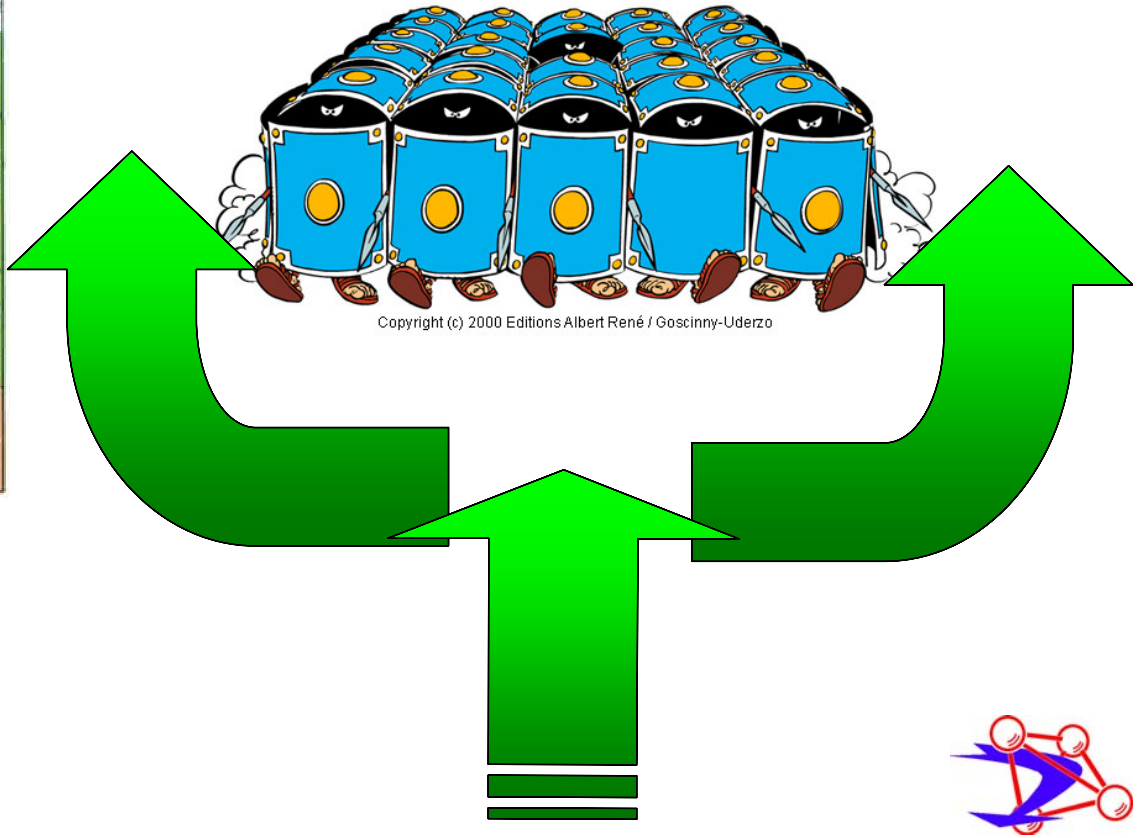

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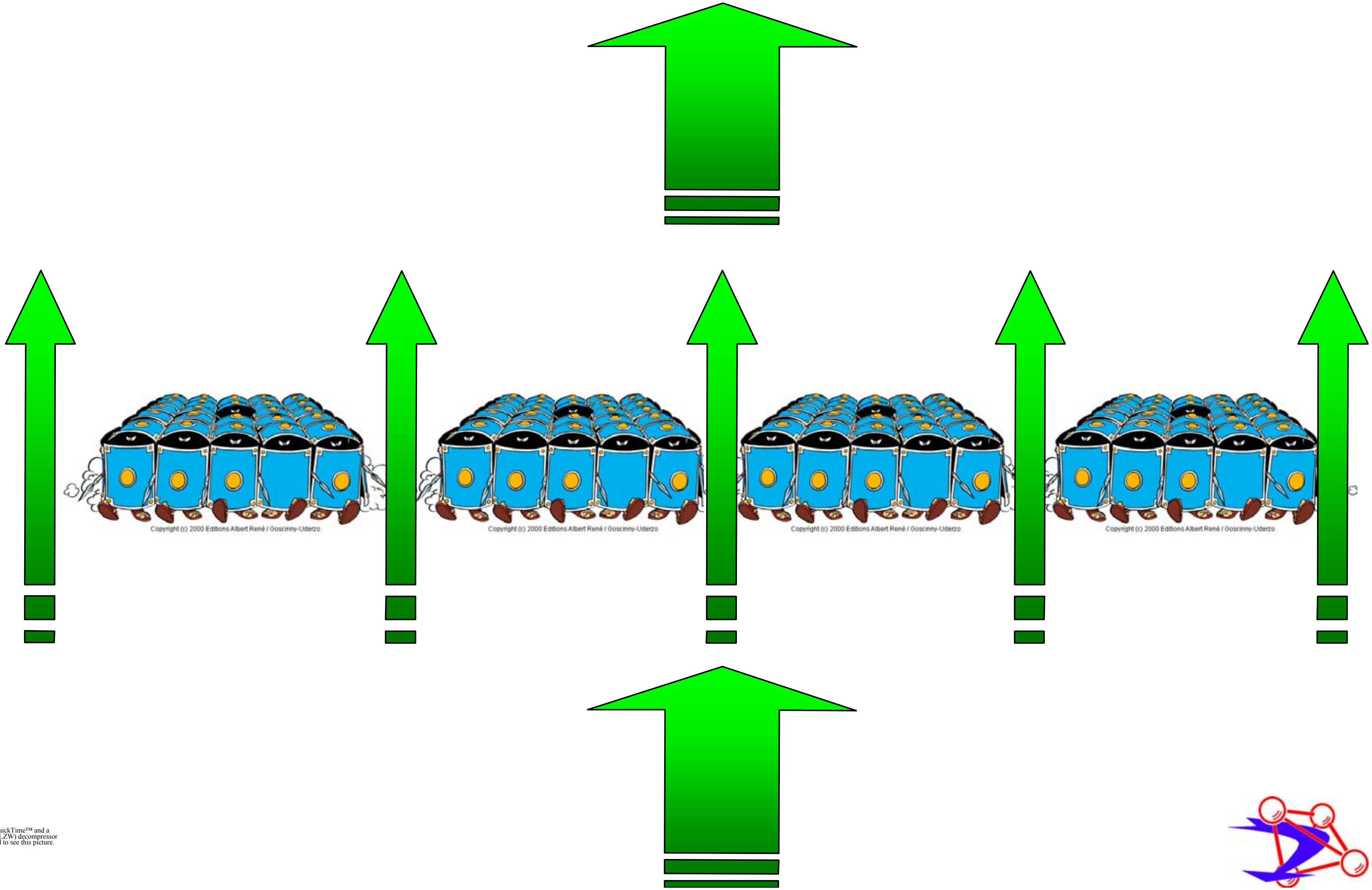
Flux lines (magnetic)



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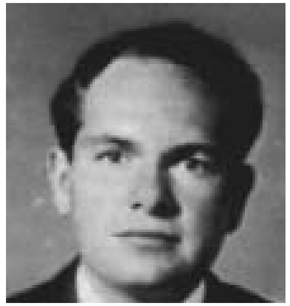
Abrikosov Lattice



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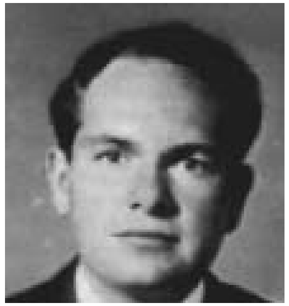

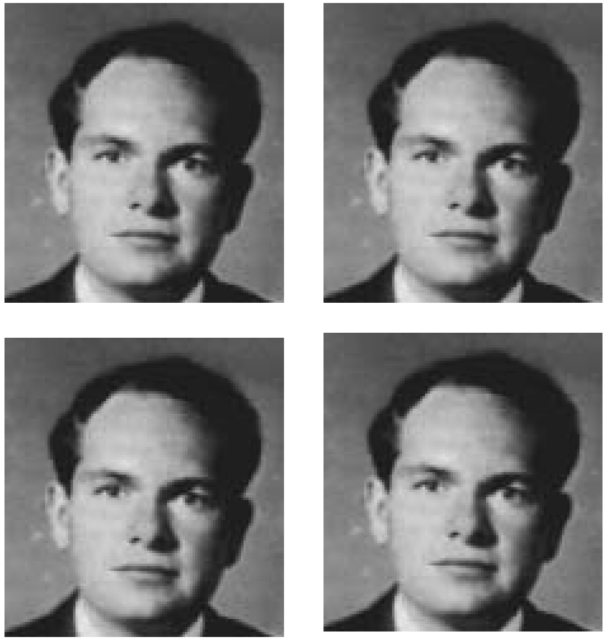

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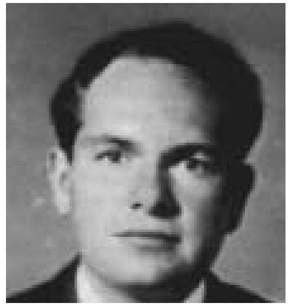

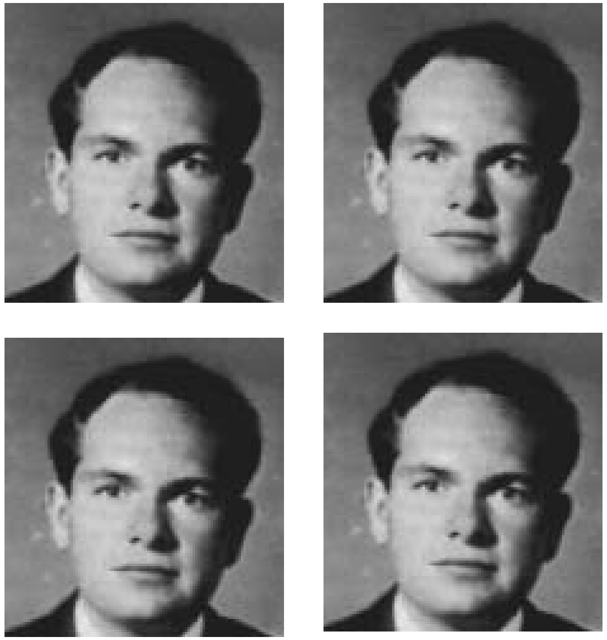

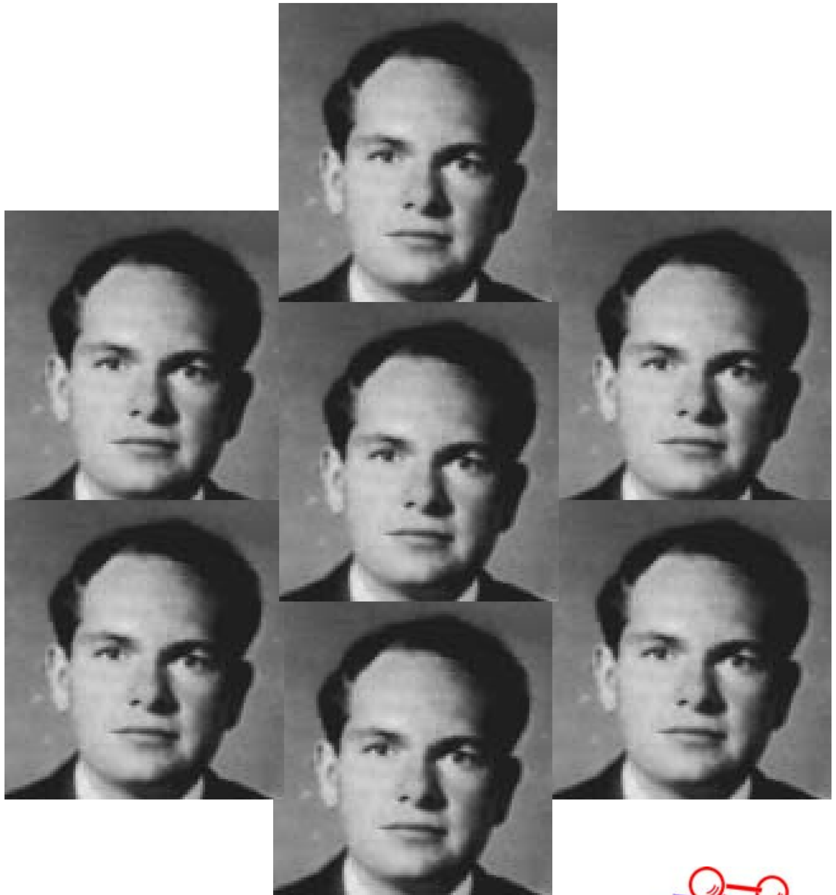

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Small Angle Neutron Scattering (SANS)

Diffraction of neutrons from flux lines ----> Bragg law: $\lambda = 2d \sin(\Theta)$

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$$d = \alpha\sqrt{\frac{\Phi_0}{B}}$$

square: $\alpha=1.000$

hexagonal: $\alpha=1.075$

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B=1 Tesla -----> d=455 Å

λ=10 Å -----> Θ=0.63 degrees

SMALL ANGLES!!!

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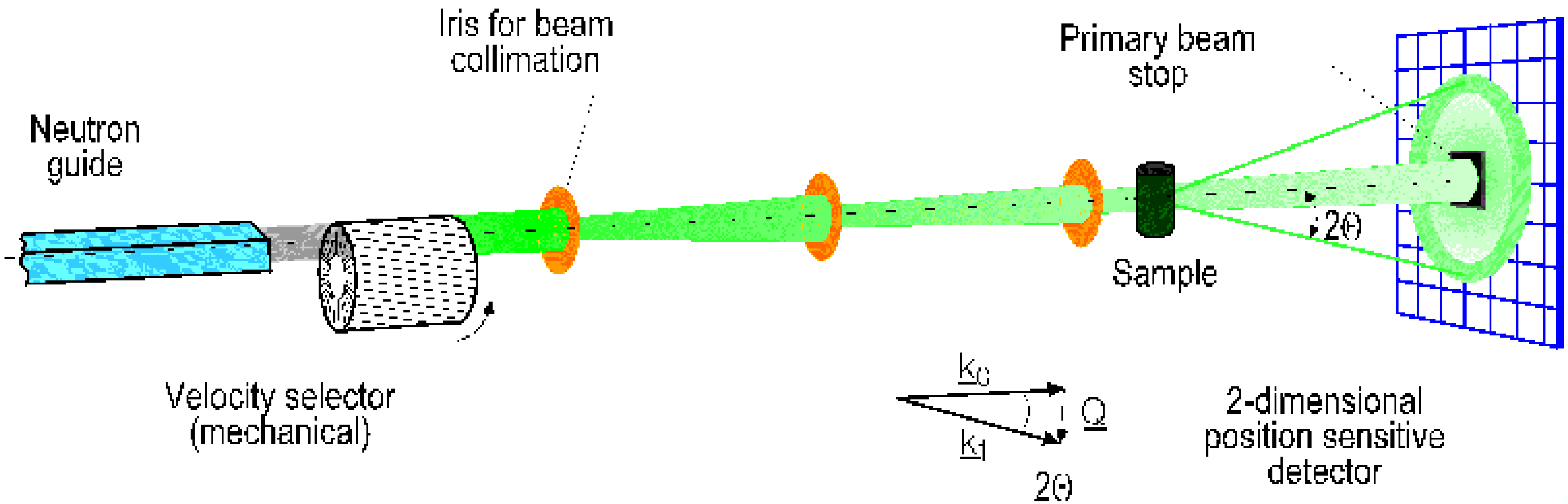

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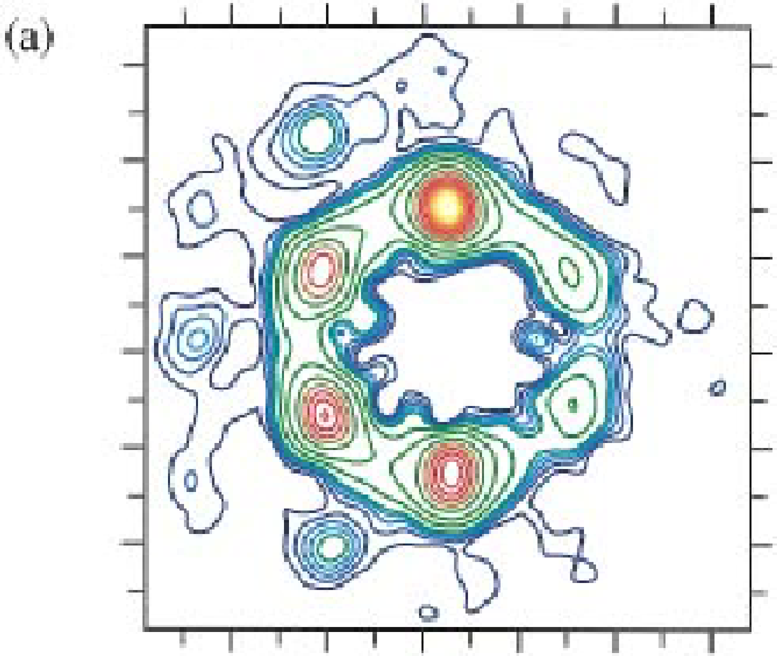
11-Tesla Magnet

HTSC low fields

$YBa_2Cu_3O_7$ (B=0.2 T)



Johnson *et al*. PRL **82** (1999) 2792

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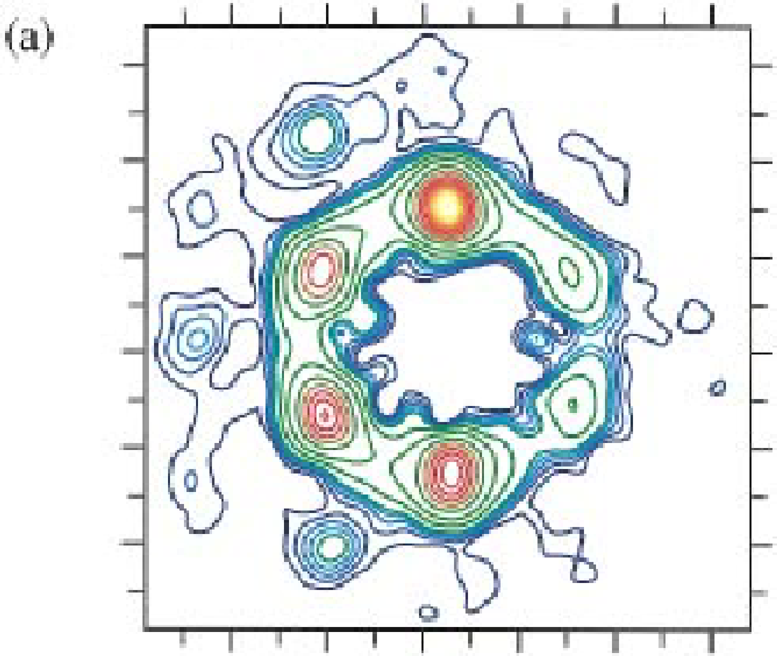
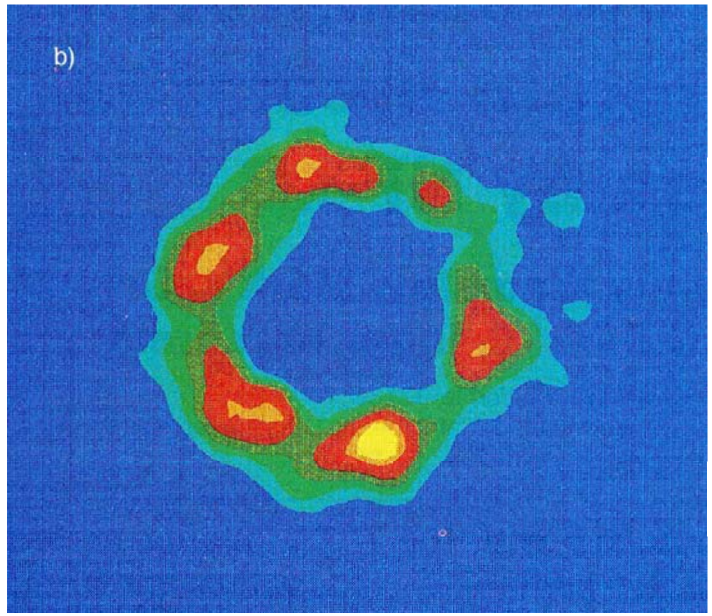


Johnson *et al*. PRL **82** (1999) 2792

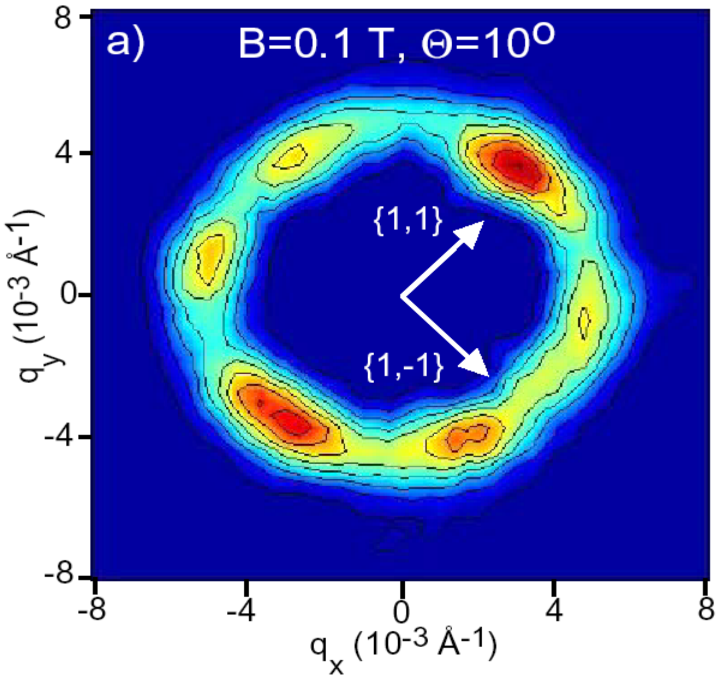
$Bi_2Sr_2CaCu_2O_8$ (B=0.05 T)



Cubitt *et al*., Nature **365** (1993) 407

$La_{2-x}Sr_xCuO_4$ (x=0.17)

{1,1}=(Cu-O-Cu)



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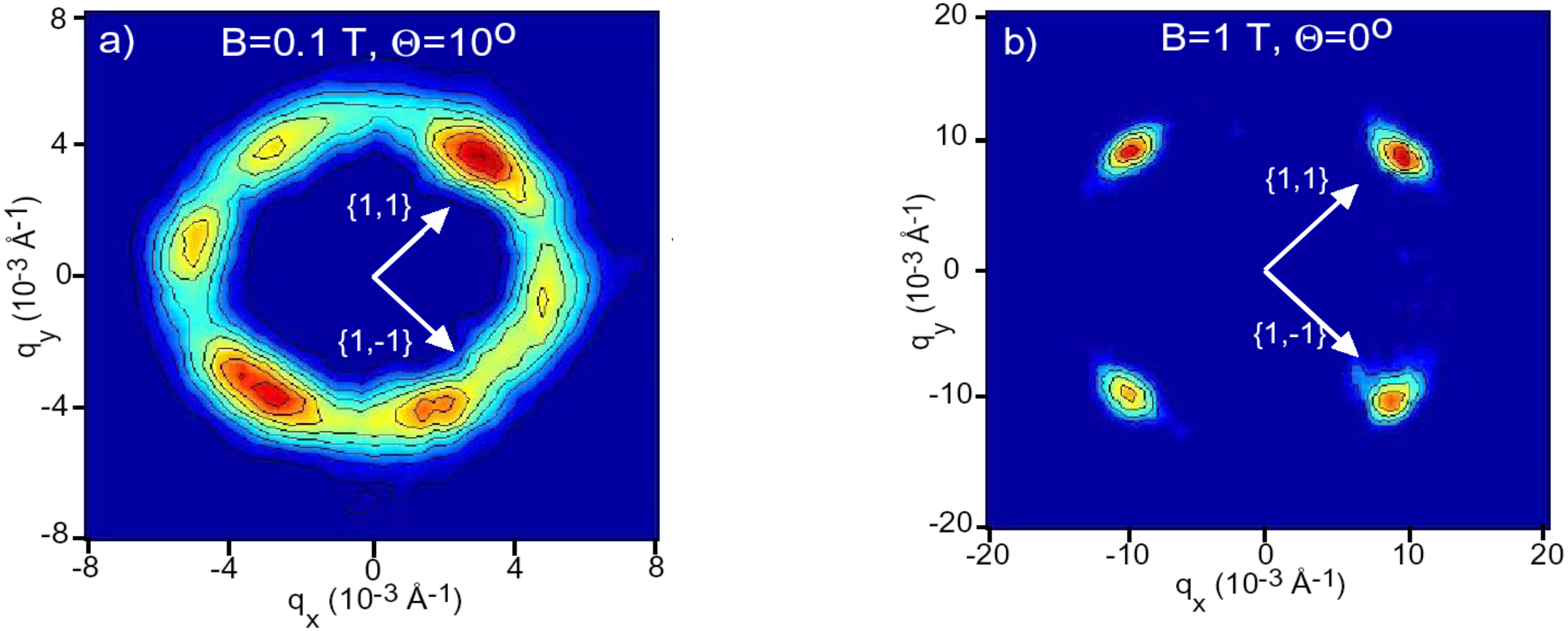

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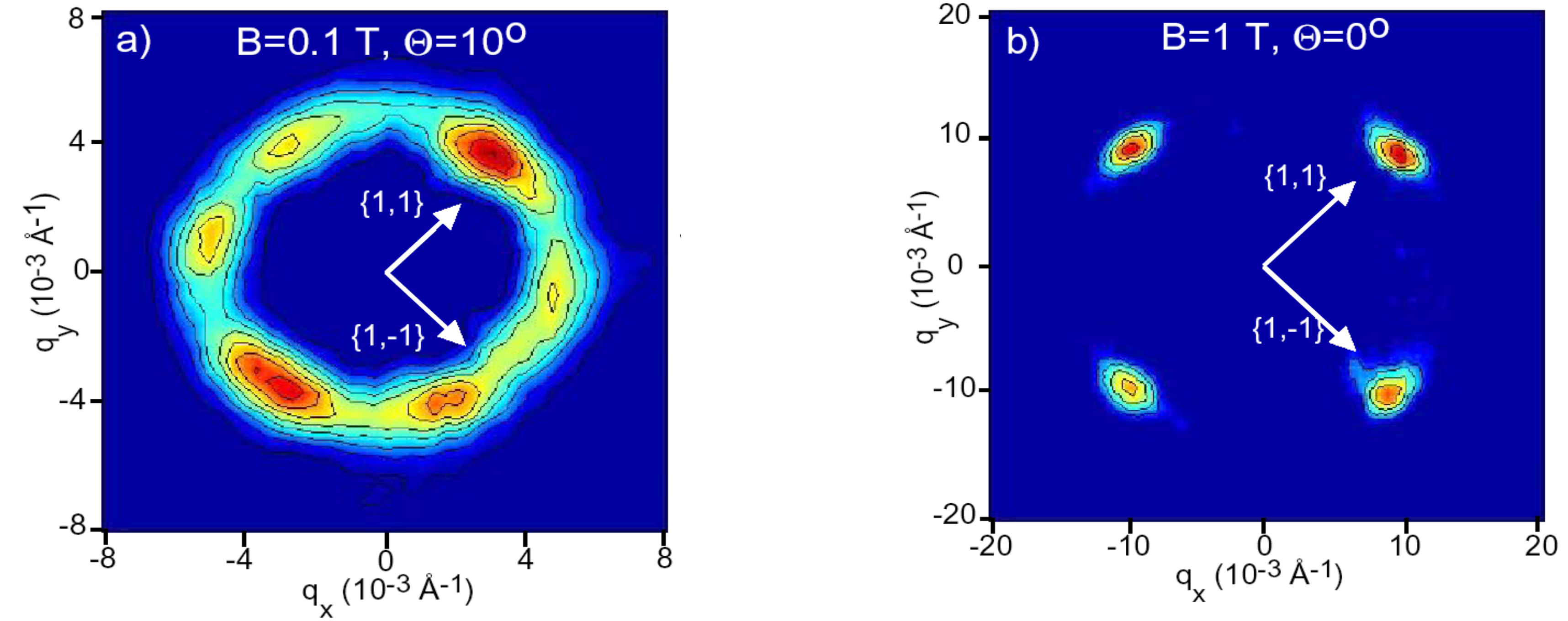
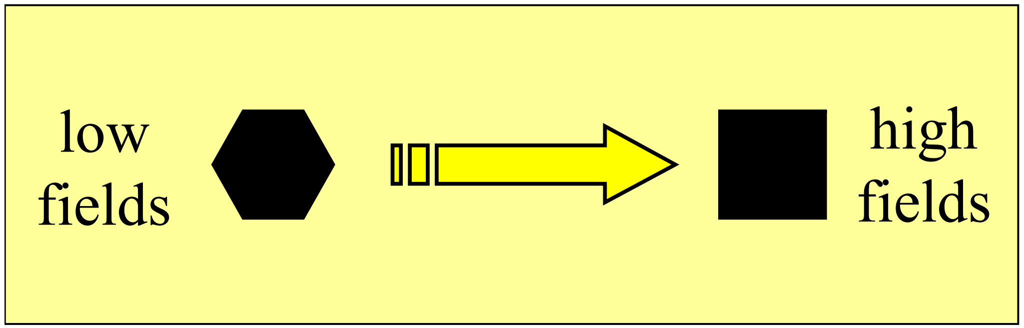
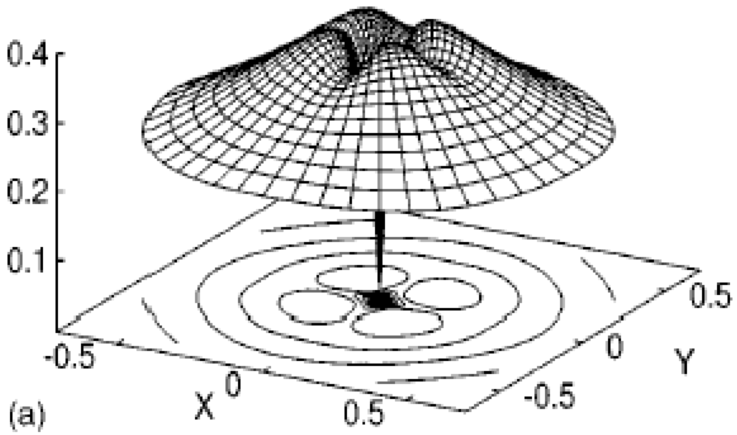


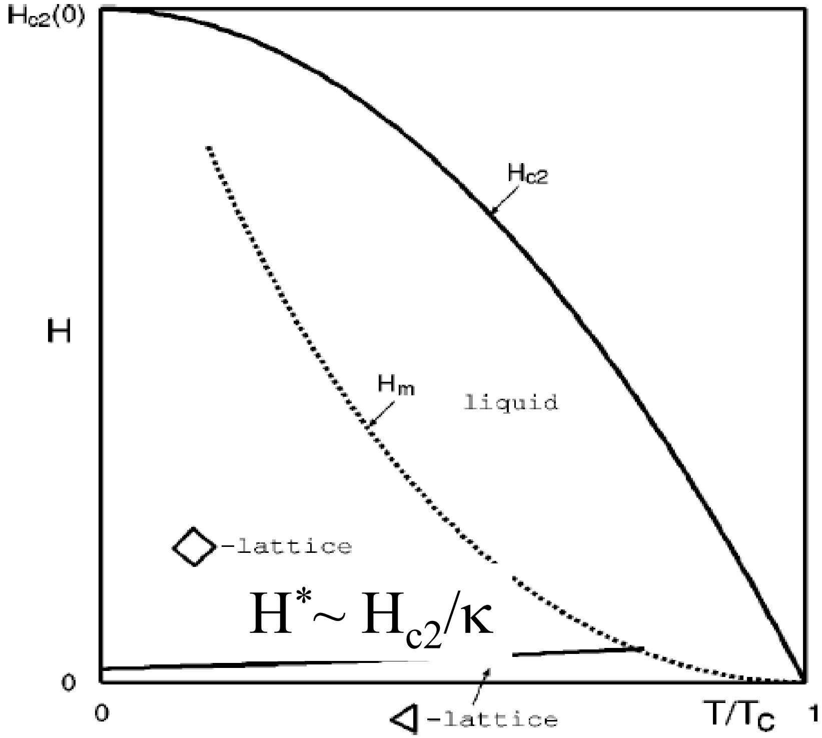


R. Gilardi *et al.*,
PRL **88** (2002) 217003

Vortex structure in d-wave superconductors

M. Ichioka et al., Phys. Rev. B **53** (1996) 15316





J. Shiraishi et al., PRB ***59*** *(1999) 4497*

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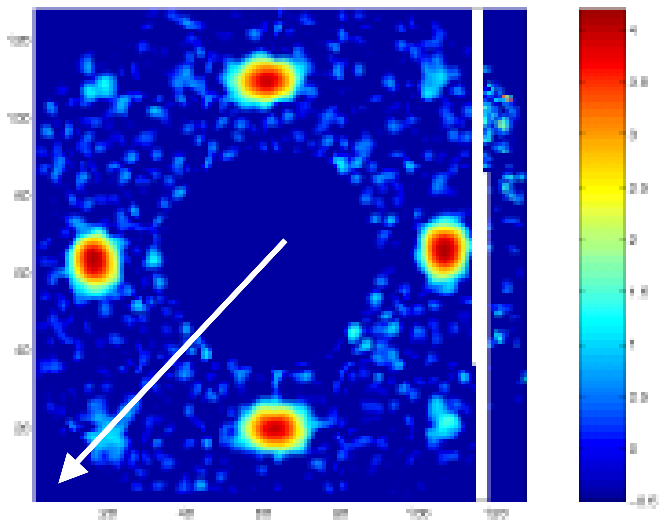
Crossover hexagonal to square in YBCO at higher fields

B=9 Tesla



(Cu-O-Cu)

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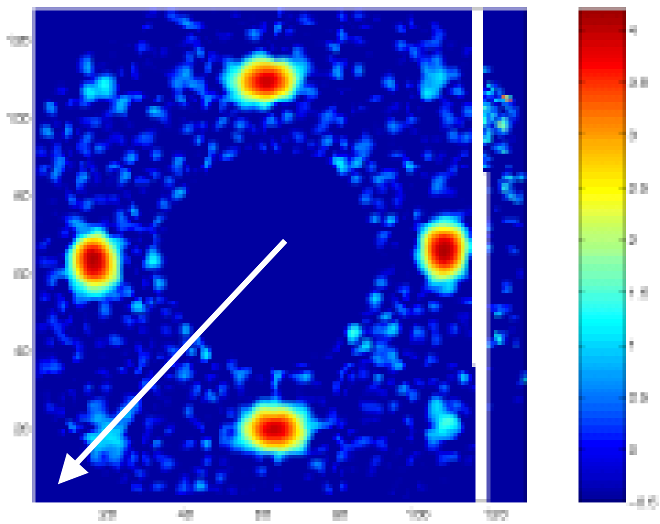

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$$\frac{H_{cross}(YBCO)}{H_{cross}(LSCO)} \approx 22.5$$

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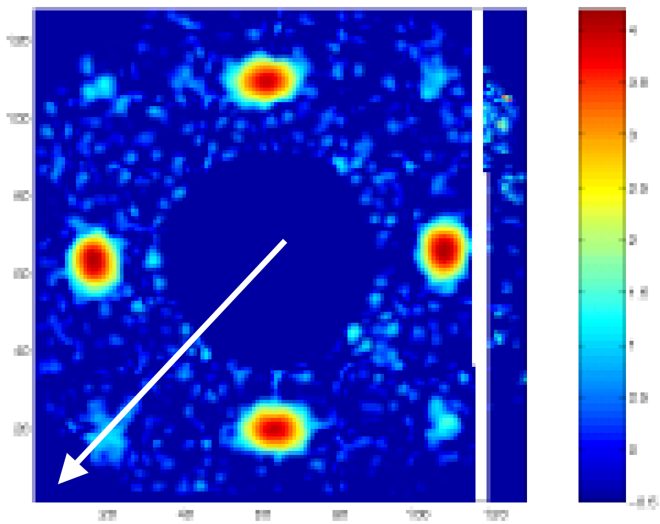

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$$\frac{H_{cross}(YBCO)}{H_{cross}(LSCO)} \approx 22.5$$

Lattice orientation

YBCO	LSCO
Nodal	**Anti-nodal**

- S. Brown, T. Forgan, unpublished
- Keimer et al., PRL **75** (1994) 3459

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Origin of the transition?

Predictions from theoretical works:

A.J. Berlinsky et al., Phys. Rev. Lett. ***73*** *(1995) 2200*

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-Presence of stripes?

Vortex structure in d-wave superconductors

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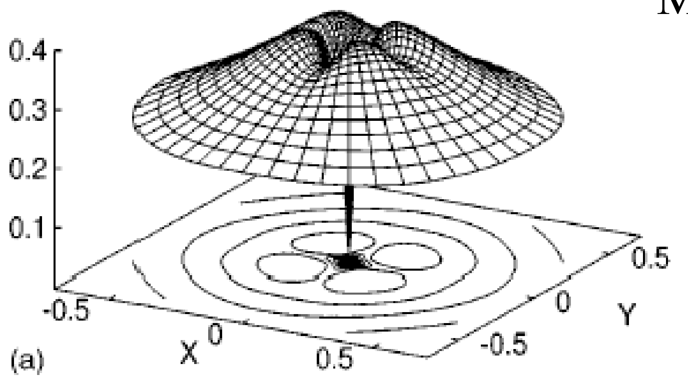

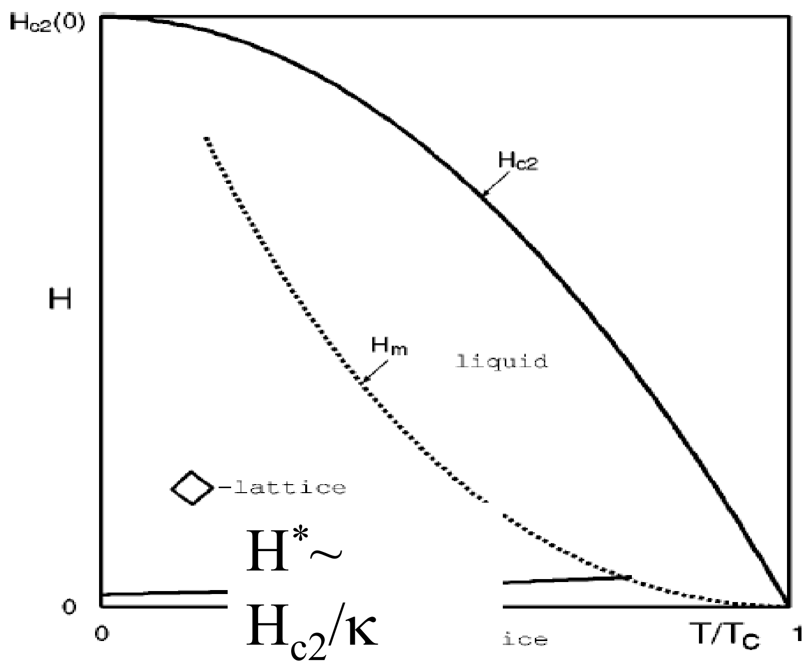

$H^{*} \sim H_{c2}/\kappa$

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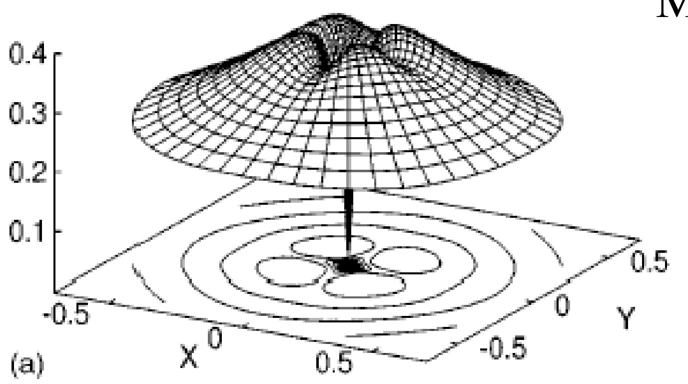
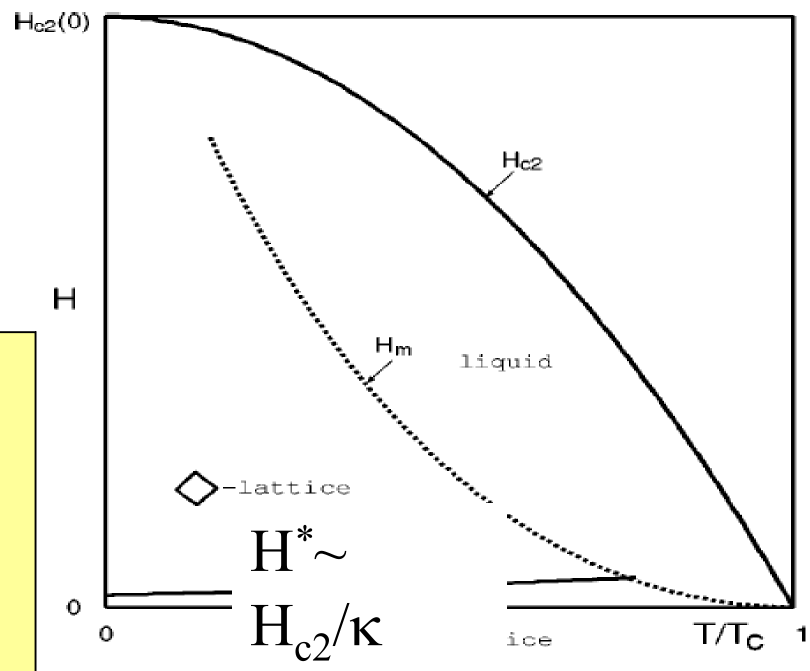




$H^* \sim H_{c2}/\kappa$

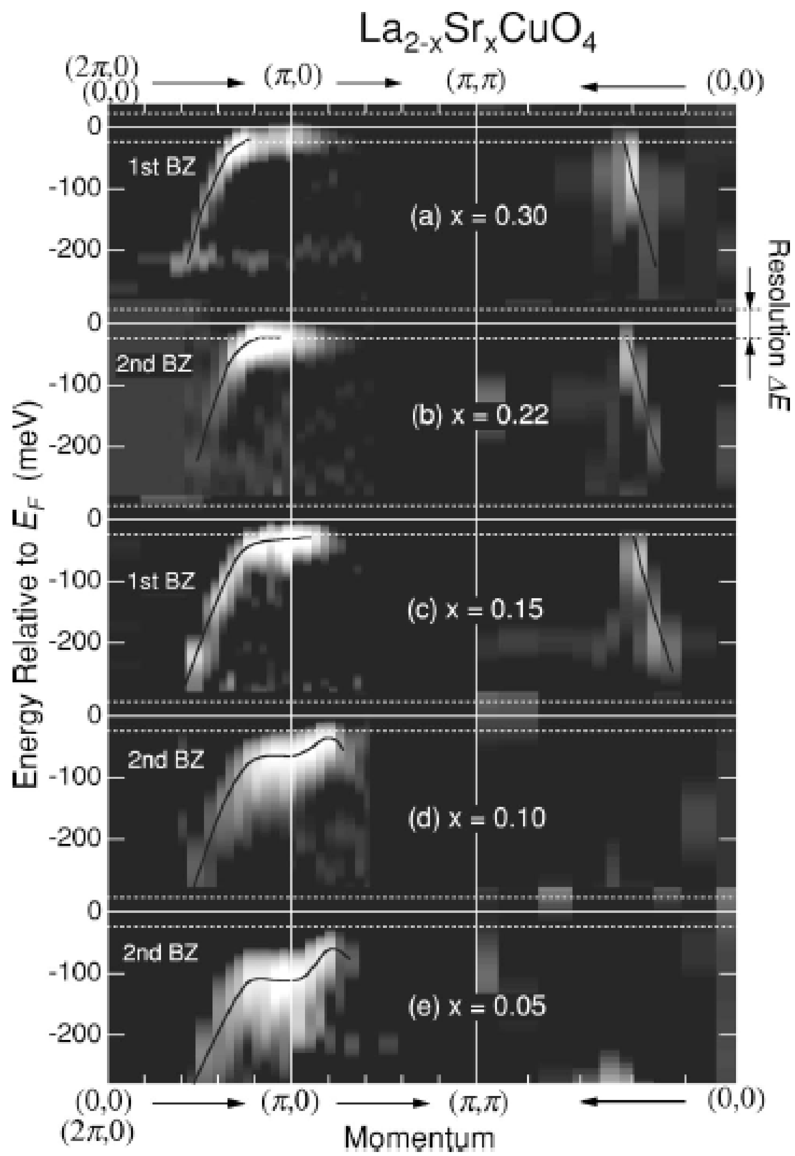
*J. Shiraishi et al., PRB **59** (1999) 449?*

$\kappa \approx 100$

$H_{c2} \approx 60$ T

$H^* \sim 0.6$ Tesla ✓

But: VL expected to be rotated by 45°
(expected along nodal direction) ✗



Ino et al., PRB (2002) 094504

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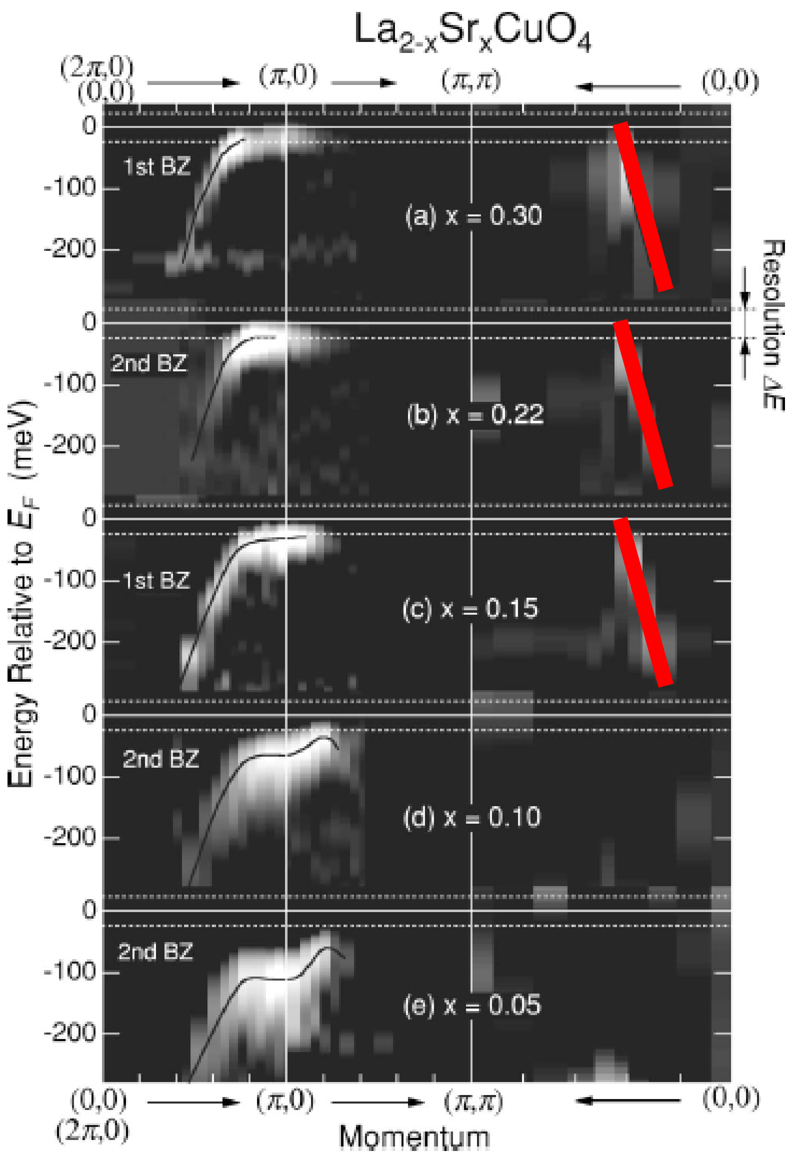

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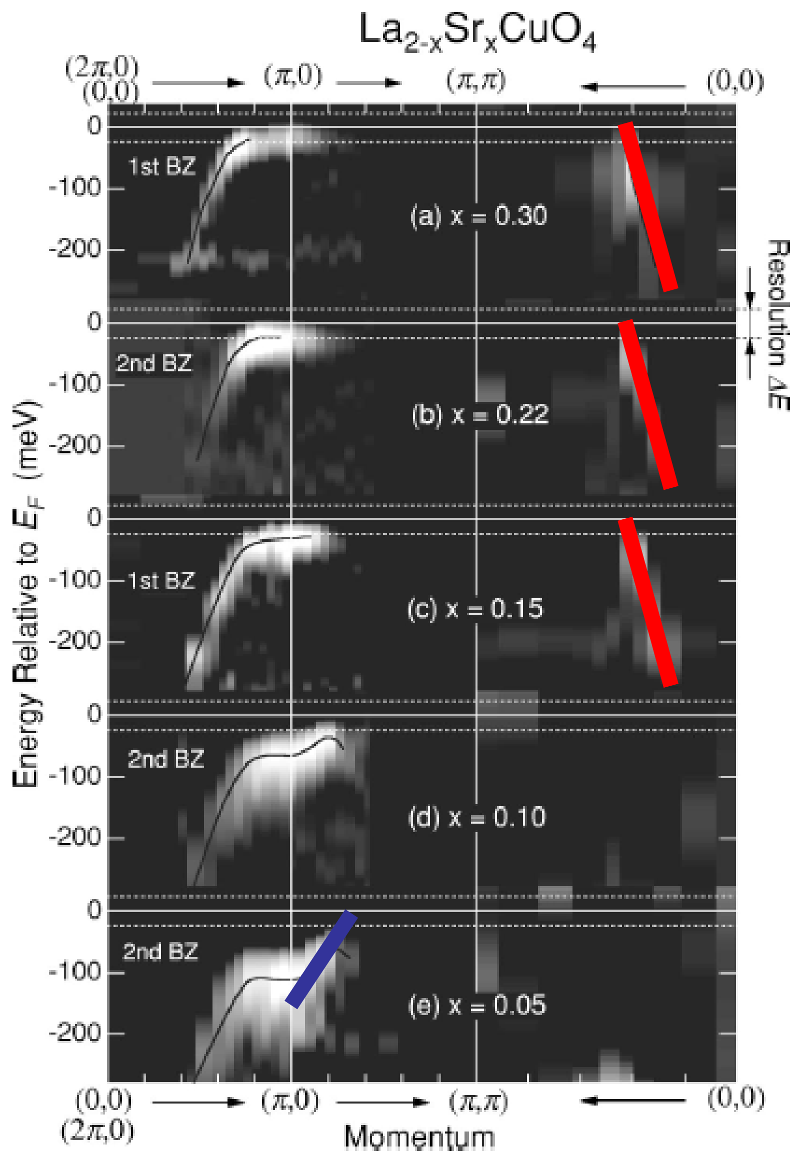

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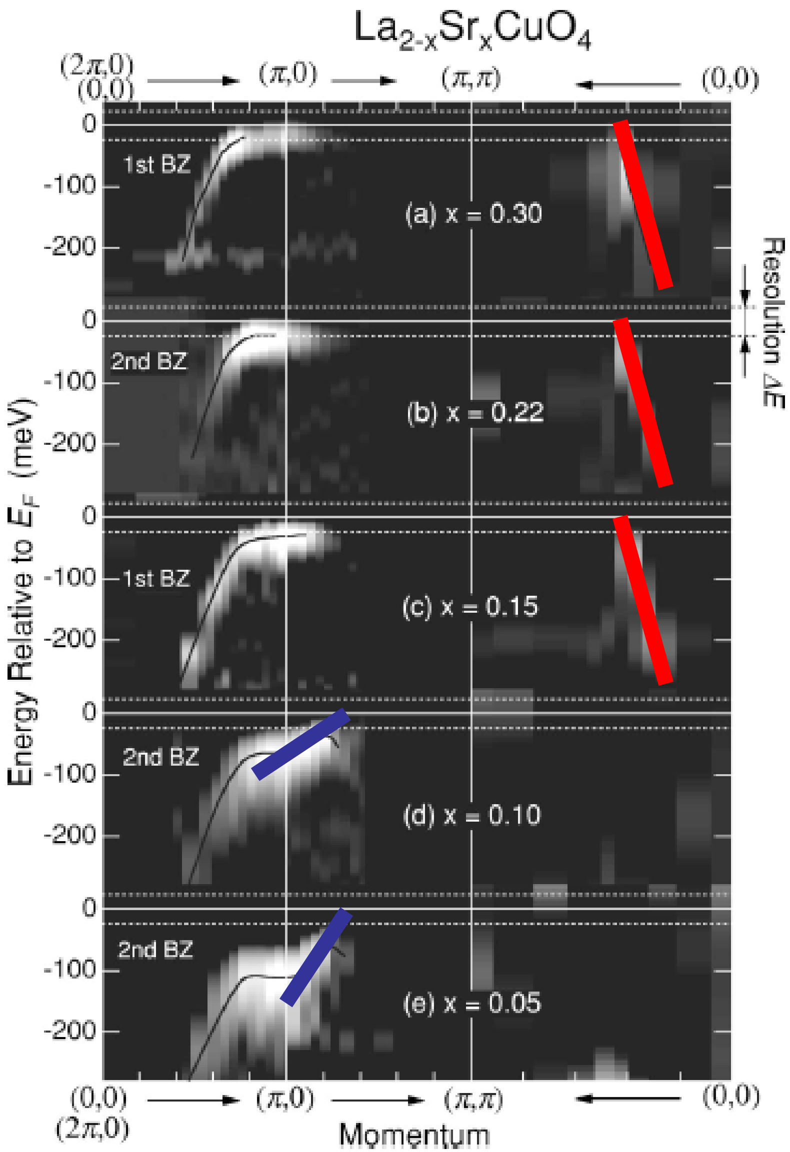

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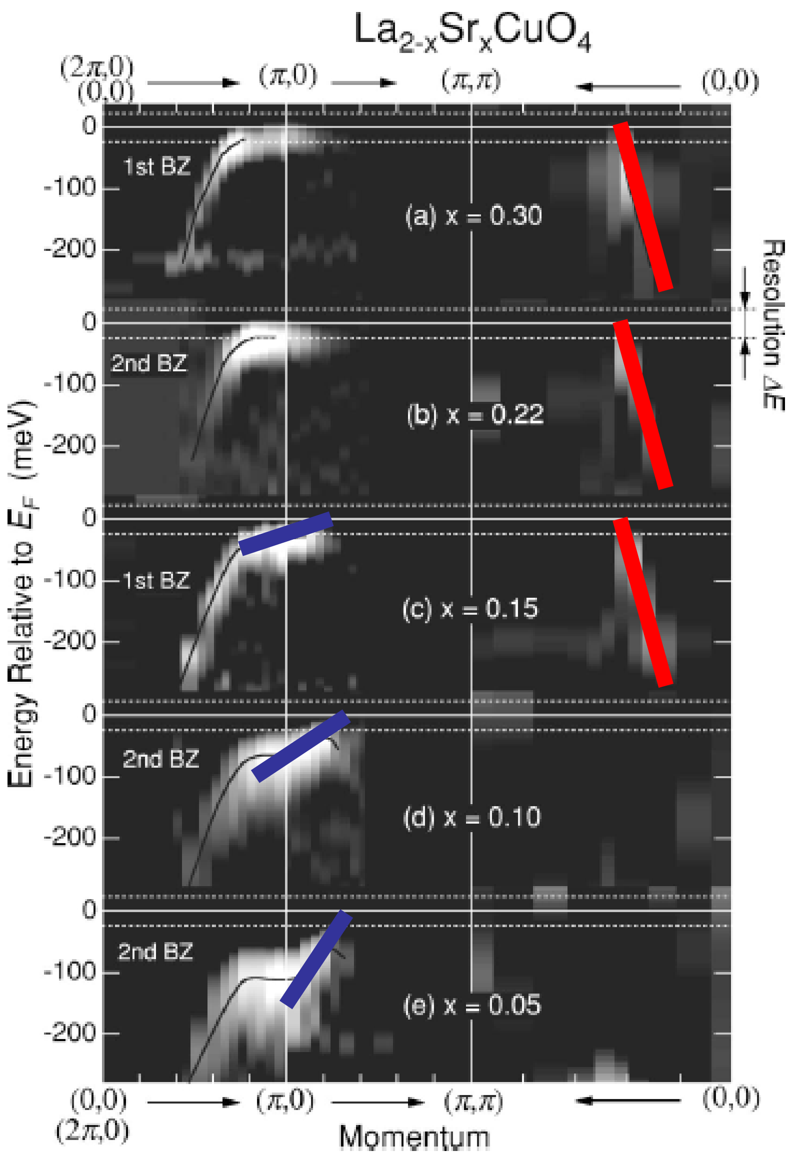

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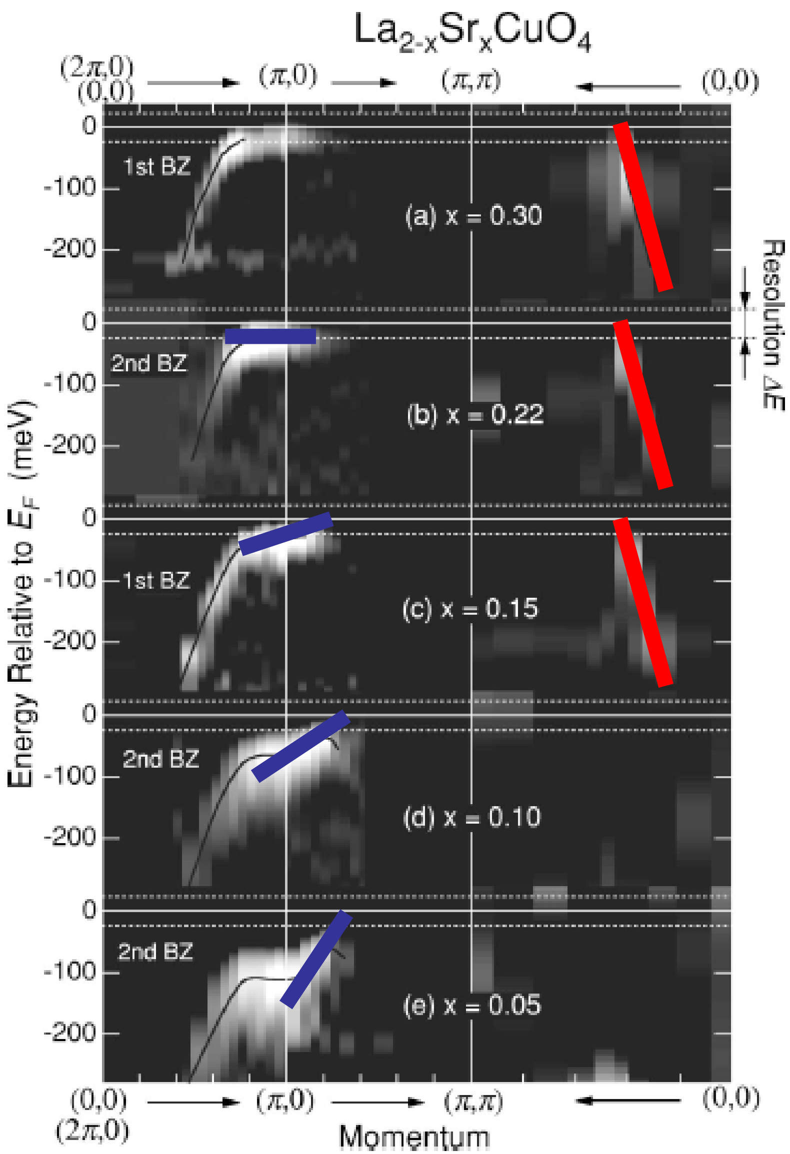

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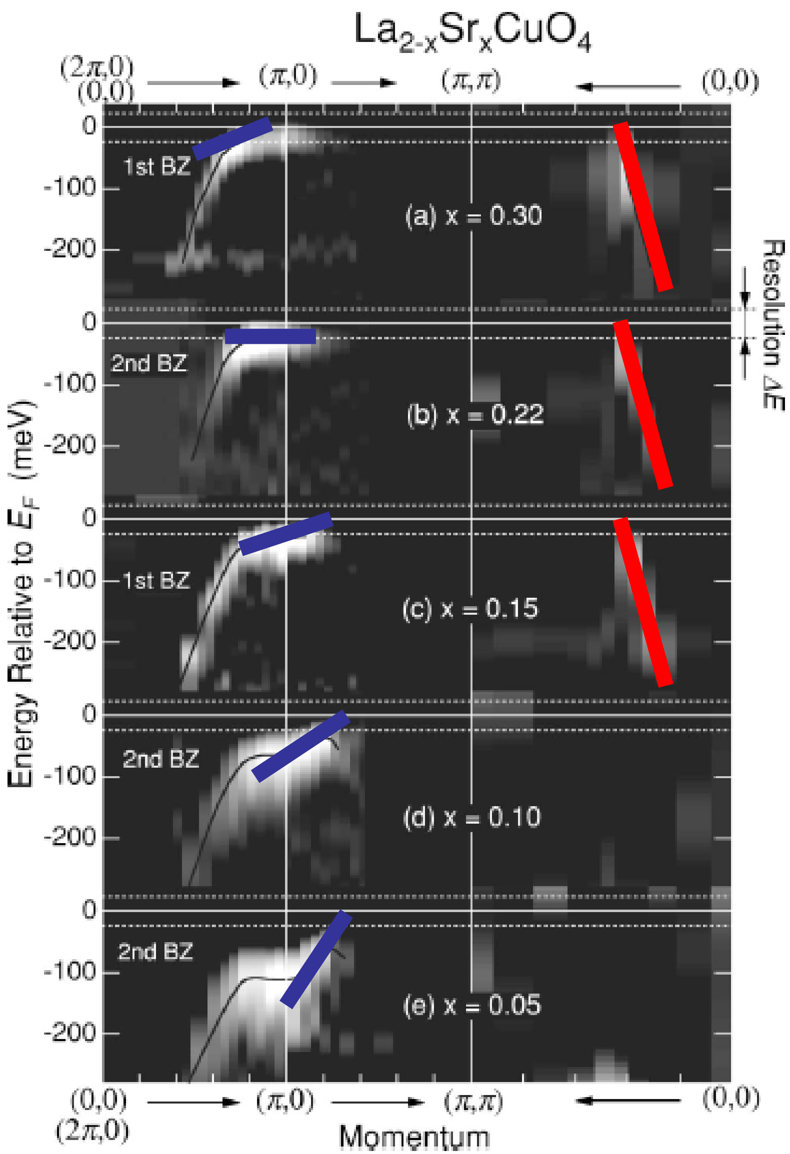

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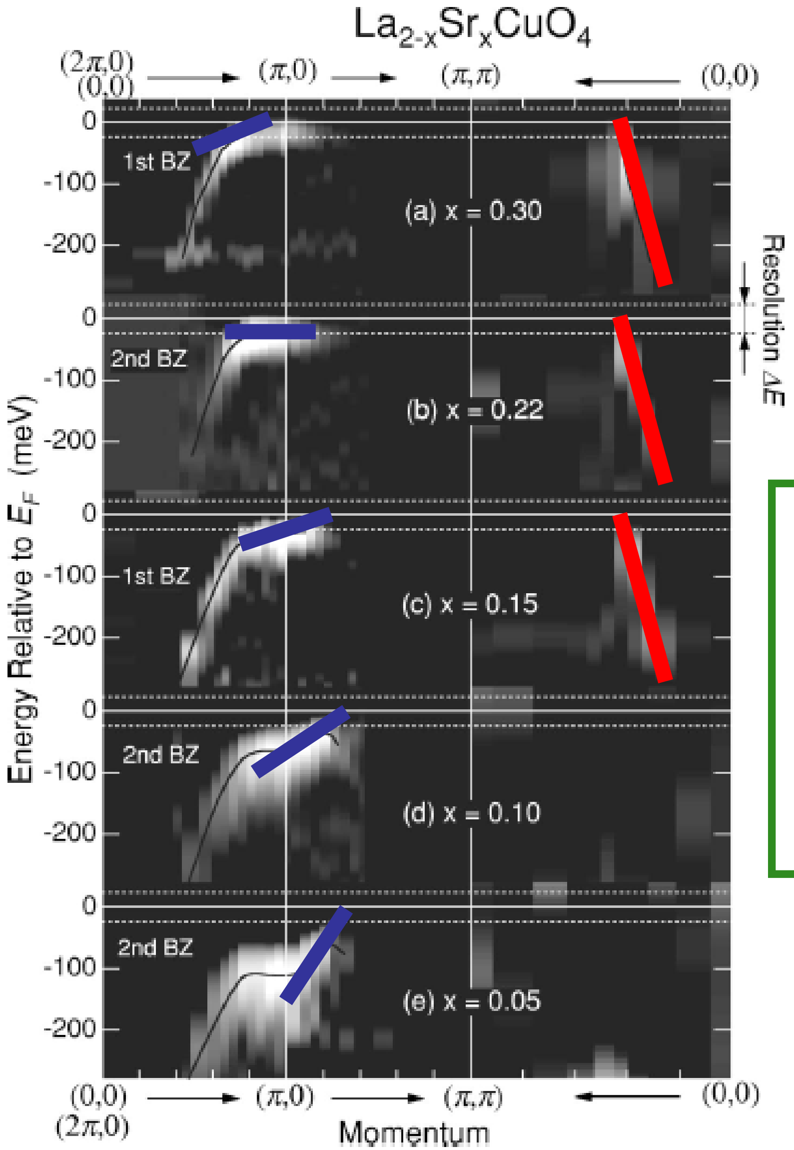


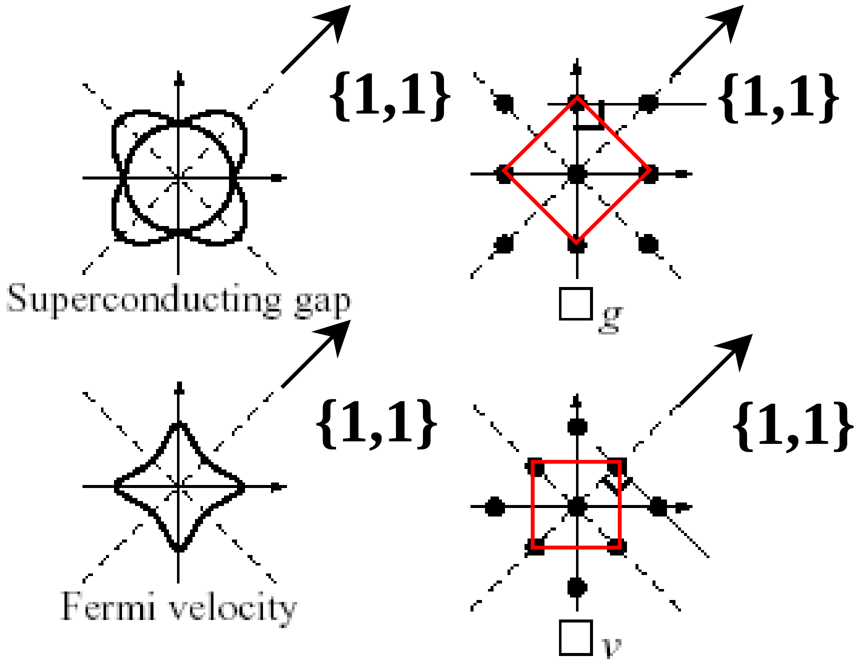
Ino et al., PRB (2002) 094504

V_F(nodal) > V_F(anti-nodal)

+

V_F(anti-nodal) doping dependent !

Interplay/Competition between Fermi velocity anisotropy & gap anisotropy



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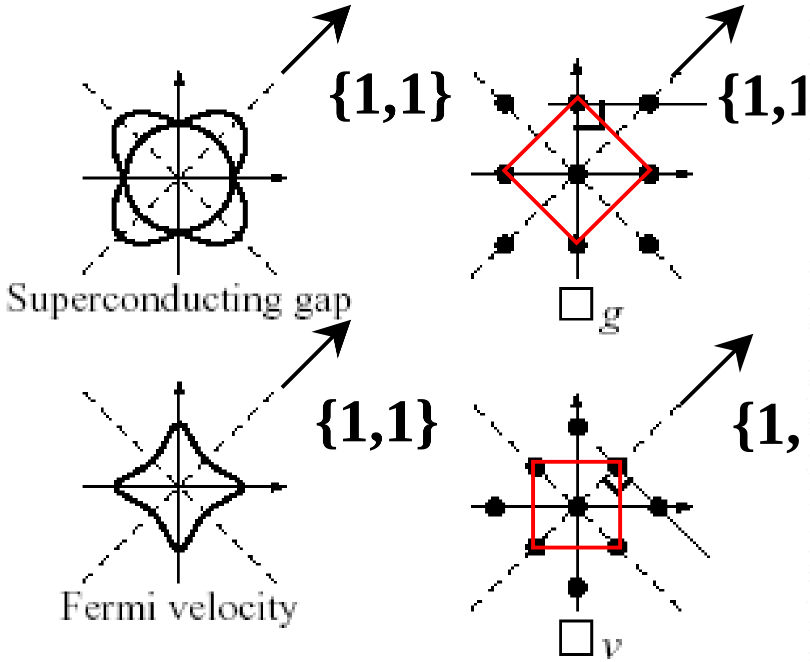

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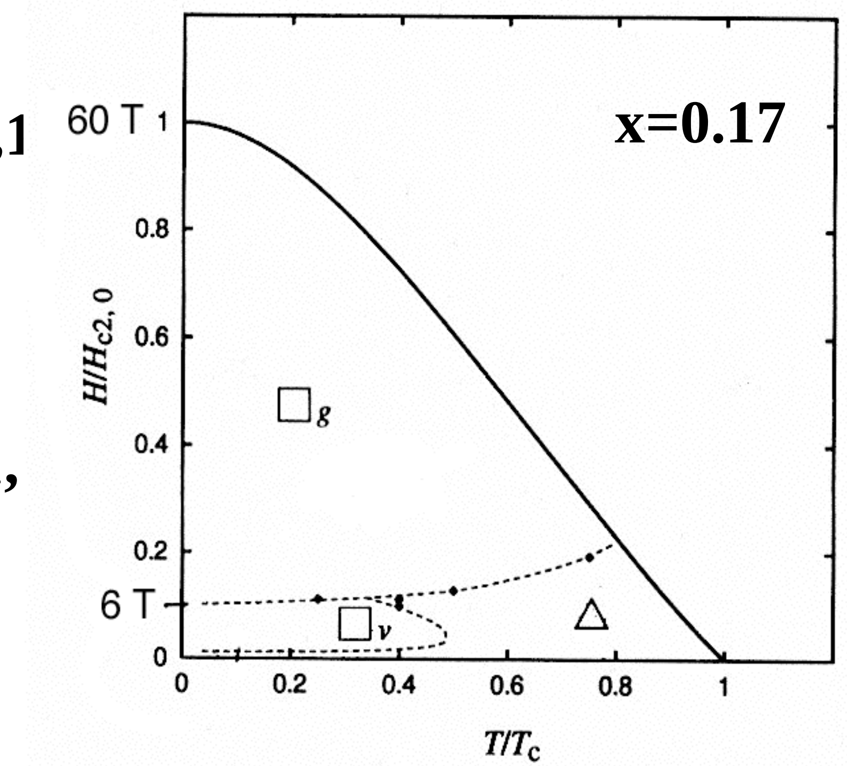

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