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WHAT ARE RELIABLE AND ECONOMICALLY EFFICIENT RISK-REDUCTION STRATEGIES IN THE FACE OF UNCERTAIN CLIMATE THRESHOLDS?

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What are reliable climate change strategies in the face of uncertain climate thresholds?

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ABSTRACT

Anthropogenic greenhouse gas emissions may trigger climate threshold responses, such as a collapse of the North Atlantic meridional overturning circulation (MOC). Climate threshold responses have been interpreted as an example of "dangerous anthropogenic interference with the climate system" in the sense of the United Nations Framework Convention on Climate Change (UNFCCC). The UNFCCC objective is to "prevent" such dangerous anthropogenic interference. The current uncertainty about important parameters of the coupled natural–human system implies, however, that the UNFCCC objective can only be achieved in a probabilistic sense. In other words, climate management can only reduce — but not entirely eliminate — the risk of crossing climate thresholds. Here we use an integrated assessment model of climate change to derive economically optimal risk-reduction strategies.

We implement a stochastic version of the DICE model and account for uncertainty about four parameters that have been previously identified as dominant drivers of the uncertain system response. The resulting model is, of course, just a crude approximation as it neglects, for example, some structural uncertainty and focuses on a single threshold, out of many potential climate responses. Subject to this caveat, our analysis suggests five main conclusions. First, reducing the numerical artifacts due to sub-sampling the parameter probability density functions to reasonable levels requires sample sizes exceeding 10^3. Conclusions of previous studies that are based on much smaller sample sizes may hence need to be revisited. Second, following a business-as-usual (BAU) scenario results in odds for an MOC collapse in the next 150 years exceeding 1 in 3 in this model. Third, an economically "optimal" strategy (that maximizes the expected utility of the decision-maker) reduces carbon dioxide (CO_2) emissions by approximately 25% at the end of this century. Perhaps surprisingly, this strategy leaves the odds of an MOC collapse virtually unchanged compared to a BAU strategy. Fourth, reducing the odds for an

MOC collapse to 1 in 10 would require an almost complete decarbonization of the economy within a few decades. Finally, further risk reductions (*e.g.,* to 1 in 100) are possible in the framework of the simple model, but would require faster and more expensive reductions in CO_2 emissions.

1 Introduction

Anthropogenic greenhouse gas emissions may trigger climate threshold responses [*Alley et al.*, 2003]. For example, some ocean models predict a collapse of the North Atlantic meridional overturning circulation (MOC) as a potential threshold response to anthropogenic carbon dioxide (CO_2) emissions [*Stocker and Schmittner*, 1997; *Cubasch and Meehl*, 2001]. An MOC collapse has the potential to cause widespread temperature and precipitation changes and nontrivial ecological and economic impacts [*Keller et al.*, 2000; *Vellinga and Wood*, 2002; *Link and Tol*, 2004]. The potential for adverse impacts due to anthropogenic greenhouse gas emissions raises the question of how — if at all — this risk should be reduced. An economically efficient strategy would find some optimal balance between the costs and benefits of reducing these impacts. Numerous studies have applied integrated assessment models (IAMs) of climate change to analyze risk management strategies in the face of uncertain climate thresholds (*e.g.*, *Nordhaus* [1992], *Toth et al.* [1997], *Kelly and Kolstad* [1999], *Keller et al.* [2000], *Zickfeld and Bruckner* [2003], *Keller et al.* [2004], *Bruckner and Zickfeld* [2004]). Here we use a simple IAM that accounts for uncertainty in key parameters and the potential for an MOC collapse to analyze two main questions: (i) What would be the odds of triggering an MOC collapse for a strategy with unabated CO_2 emissions? (ii) What would be economically optimal strategies to reduce the odds of triggering an MOC collapse to various levels? We address these questions using a simple integrated assessment model of climate change. This framework is, as any model, subject to numerous caveats (discussed at the end of this paper).

Our study improves on previous work by a more refined examination of the effects of parametric uncertainty and the effects of imposing a reliability constraint. Uncertain parameters are represented by sets of stratified Latin-hypercube samples. We find that sample sizes exceeding 10^3 are required to reduce the effects of sampling resolution to arguably reasonable levels. For a business-as-usual policy (*i.e.*, no CO_2 abatement) the odds for triggering an MOC collapse over the next 150 years exceed 1 in 3. Considering previously published estimates for climate change damages, we find that optimal carbon abatement increases steadily from more than 10% in 2005 to almost 30% over the next 150 years. However, this economically "optimal" policy does not considerably reduce the future odds of an MOC collapse compared to the BAU strategy. Constraining the odds of triggering an MOC collapse to below 1 in 10 requires CO_2 abatement exceeding 80% within the next 60 years. Reducing these odds further to 1 in 100 is possible in the model, but requires faster and more expensive abatement — close to 90% abatement over the next 40 years.

2 The integrated assessment model

We adopt the Dynamic Integrated model of Climate and the Economy, DICE [*Nordhaus*, 1994], as a simple and transparent framework. This model links an economic growth model, as pioneered by *Ramsey* [1928], to a description of the CO_2 induced climate change and the resulting economic impacts. Here we describe the general structure and key parameters of the model. An excellent and very thorough description is given in *Nordhaus* [1994].

The objective of the decision-problem is to determine abatement and investment policies that maximize the discounted sum of utility across the considered states of the world

$$U^* = \sum_{s=1}^{N} \sum_{t=t_0}^{t^*} U(s,t)(1+\rho)^{-t}\,, \tag{1}$$

where t_0 is an initial point (the year 1965), t^* is the considered time horizon, ρ is the pure rate of social time preference, and s denotes a particular state of the world (SOW) out of N considered discrete samples (discussed below). Well-being in the

DICE model is represented by a flow of utility

$$U(t) = L(t) \ln c(t) , \tag{2}$$

where L is population, specified exogenously, and c is per capita consumption. (The used symbols are summarized and defined in the Appendix.)

Consumption is the difference between economic output and the sum of investment in capital stocks, damages caused by climate change and investment in reducing CO_2 emissions. The DICE model describes economic output using a constant-returns-to-scale Cobb Douglass production function with parameters for labor, capital, and level of technology. Labor inputs are proportional to population.

Economic activity has the side effect of CO_2 production with an exogenously evolving ratio between economic output — measured as gross world product (GWP) — and CO_2 production. A linear carbon cycle model links CO_2 emissions to atmospheric CO_2 concentrations. Radiative forcings by greenhouse gases besides CO_2 and through aerosols are prescribed exogenously.

The climate system in DICE is represented by a simple model consisting of an atmosphere, a surface ocean, and a deep ocean [*Schneider and Thompson*, 1981]. Higher greenhouse gas levels enhance radiative forcing, warming the atmosphere and (over time) the deep ocean. The climate sensitivity is an important parameter in this model and represents the equilibrium temperature increase for a hypothetical doubling of atmospheric CO_2. Economic damages due to climate change are approximated by a polynomial fit to previous studies, as described in *Nordhaus* [1994]. The original DICE model is modified to consider economic damages caused by an MOC collapse. Following previous studies, we approximate the model results of *Stocker and Schmittner* [1997] by a critical equivalent CO_2 concentration, beyond which the MOC collapses [*Keller et al.*, 2004]. The cost of carbon emissions abatement is expressed as a function of the fractional reduction in CO_2 emissions, $\mu(t)$, relative to uncontrolled emissions.

3 Representation of parametric uncertainty

Previous work identified four parameters as key drivers of the optimal policy under uncertainty [*Nordhaus and Popp*, 1997; *Keller et al.*, 2004]. We improve on these studies by considering (i) a more refined resolution, (ii) the combined effects of multiple parameter uncertainty, and (iii) the effects of a "reliability constraint" on optimal policies (discussed below). The considered parameters are: (i) the damage associated with an MOC collapse (θ_3); (ii) the climate sensitivity ($\lambda^\star$); (iii) the decline in population growth rate (δ_L); and (iv) the initial growth of the CO_2–GDP ratio, (g_σ). The specific choices of parameter distributions, given in Table 1, are discussed below.

Following *Keller et al.* [2004], we adopt a uniform distribution for the damages due to an MOC collapse (θ_3) between 0% and 3% GWP. For the climate sensitivity ($\lambda^\star$) we adopt the empirical distribution reported by *Andronova and Schlesinger* [2001] with an expected value of 3.4°C and 95% confidence limits of 0.9°C and 12.7°C. Population growth in the DICE model declines at a constant rate, δ_L. We fit a Weibull distribution to the reported discrete samples given in *Nordhaus and Popp* [1997] and constrain samples to positive values. The ratio between CO_2 production and economic output initially grows at a rate of g_σ. We fit a negative Weibull distribution to the discrete samples given in *Nordhaus and Popp* [1997], constraining all samples to negative values. The probability density functions for the four uncertain parameters are displayed in Figure 1.

Each uncertain parameter is sampled from the appropriate distribution. All other parameters are considered as certain and their numerical values are adopted from the DICE model [*Nordhaus*, 1994]. We refer to a combination of four samples (one specific sample from each probability density function of the four uncertain parameters) as a single state of the world (SOW). Taking N_S samples from each distribution yields $N = {N_S}^4$ SOWs. We adopt the Latin-hypercube sampling technique outlined in *Nordhaus and Popp* [1997] to define the SOW sets. Probability density functions of the parameters are divided into N_S intervals with equal probability mass. The expected value of the distribution over each interval provides the N_S samples with each SOW being equally likely. The results of this sampling method are illustrated in Figure 1 for $N_S = 7$, corresponding to $N = 2,401$ SOWs. This sampling technique ensures that the mean value of the samples is equal to the mean value of the distribution, but it neglects potential correlations between the parameters.

4 The solution method

We determine a single policy that maximizes the expected utility defined in Equation (1). We apply a numerical optimization method, since analytic solutions are not available for this problem. The presence of a climate threshold in the DICE model can introduce non-smooth gradients and local optima in the objective function [*Keller et al.*, 2004]. As a result, gradient based methods can result in severely biased results and global optimization methods are required. Based on previous benchmark results [*Moles et al.*, 2004] we implement the SRES algorithm [*Runarsson and Yao*, 2000] to solve this non-convex and nonlinear global optimization problem. SRES is a genetic optimization algorithm that uses a sequence of mutation and selection to improve an objective function. The algorithm starts with an initial random population of trial vectors that are evaluated by the objective function. Some fraction of well-performing members of this population are selected and used to generate new trial vectors (with random perturbation, akin to the effects of mutation in the evolution of natural populations). The repeated application of selection and mutation results in a generally improving objective function without the typical pitfalls of gradient based methods [*Goldberg*, 1989]. We consider a time horizon of 470 years to ensure that the finite time horizon has virtually no influence on the reported results over the next two centuries.

Finding the global optimum for this non-convex optimization problem is non-trivial. As with all global optimization problems without a known analytical solution, one cannot mathematically prove that the precise global optimum has been identified in a finite computation time. However, we implement an algorithm for detecting misconvergence that ensures a close approximation to the optimal solution. Specifically, we run the SRES algorithm with two sets of initial conditions and compare optimal strategies at the end of each iteration. When the decision variables for each optimal strategy differ by a root mean square error of less than 0.5%, the two solutions are for practical purposes identical and likely a good approximation to the global maximum. In case this approach does not lead to convergence, we switch to another global optimization algorithm [*Runarsson and Yao*, 2005]. Solutions that still have not converged with the second algorithm are discarded and the process is reinitialized with a new random initial population. This procedure is continued until the convergence criteria is satisfied. The model code is available upon request.

5 Analyzed strategies

We compare three strategies: (i) business-as-usual (BAU), (ii) unconstrained optimal, and (iii) reliability constrained optimal. For BAU, carbon emissions remain unabated. This is implemented by setting abatement (μ) to zero and optimizing utility (Equation 1) as a function of relative investment (I/Q) only. For the unconstrained optimal policy, we maximize the objective function by choosing μ and I/Q. Abatement is set to zero between 1965 and 1995, to reflect past policies, and BAU investment values are used for these times. Reliability constrained optimal policies maximize the expected utility with the additional constraint to keep the odds of triggering an MOC collapse below a specified level. This is implemented by introducing a penalty to the objective function when the odds of an MOC collapse exceed the specified reliability.

6 Results and discussion

6.1 Effects of sampling resolution

Previous studies analyzing related decision problems use relatively low resolutions for the SOW sampling, ranging on the order of 10^0 to 10^2 samples (*e.g., Chao* [1995], *Yohe* [1996], *Zickfeld and Bruckner* [2003], *Bruckner and Zickfeld* [2004], *Keller et al.* [2004]). For a nonlinear system, such low resolutions can introduce considerable biases [*Tol*, 2003]. Here we evaluate these biases for a subset of the uncertain parameters and model structure. To this end, we compare optimal strategies estimated at different resolutions to identify when further resolution increases have negligible effects. In each case, N_S samples of each uncertain parameter are taken, leading to ${N_S}^4$ SOWs. Figure 2 illustrates the unconstrained optimal abatements when one (circles), seven (squares), and 15 (stars) samples of each uncertain parameter are taken. Note that the linear increase in samples per model parameter (N_S) results in a geometric growth of the number of SOWs (following $N = {N_S}^4$) and the approximate computational requirements. Solving the optimal control problem for $N_S = 15$ implies $N = 50,625$ SOWs and requires approximately 16 hours on a high performance computer cluster (8 nodes with dual 2.4 GHz processors).

Carbon abatement exceeds 10% in 2005 and rises to more than 20% over the subsequent 80 years, regardless of the reliability constraint. For $N_S = 1$ abatement increases sharply immediately, reaching approximately 60% in 2155. After 2155, abatement levels drop to around 20%. In this case an MOC collapse is delayed, but not avoided. Increasing the numerical resolution changes the results considerably. For $N_S = 7$ and $N_S = 15$, abatement increases steadily to around 30% over the next 150 years. The optimal abatement levels for $N_S = 7$ are practically indistinguishable from the more computationally involved calculations using $N_S = 15$.

The biases introduced by the discreet sampling of the underlying probability density functions decrease almost monotonically, with small changes beyond $N_S = 7$ (Figure 3). Beyond $N_S = 7$ the solution has, for practical purposes, converged. This suggests that $N_S = 7$ provides a reasonable approximation for the specific parametric uncertainty in the chosen model structure. We adopt this value for the more detailed analysis below. It is important to note that this study focuses on uncertainty about a subset of the model parameters. As a result, the true uncertainty is likely larger than the one reported here.

6.2 What are economically optimal risk reduction strategies?

The analysis, so far, adopts the expected utility maximization approach of previous studies [*Nordhaus*, 1992]. Expected utility maximization may be a useful framework for cases with well known costs and benefits and where the decision-makers actually adopt this decision criteria. Expected utility maximization can, however, be a poor description of decision making when the negative impacts are not easily quantified or the underlying probability functions are deeply uncertain [*Bradford*, 1999; *Lempert*, 2002]. Decision problems that arguably fall into these categories include the design of dikes, water reservoirs and nuclear power stations. Such problems are often solved by imposing a reliability constraint on the accepted odds of failure [*Ouarda and Labadie*, 2001; *van Manen and Brinkhaus*, 2005]. Here we analyze how a reliability constraint on the odds of avoiding an MOC collapse affects the timing and extent of CO_2 abatement.

The odds for an MOC collapse over the next 150 years exceed 1 in 3 for the

BAU policy (Figure 4 b). Optimal reductions of CO_2 emissions without a reliability constraint do not considerably decrease these odds. The fact that the optimal policy in this model considering published estimates of economic damages of climate change (including damages due to an MOC collapse) results in considerable odds of an MOC collapse is in contrast to previous results and warrants some discussion. *Nordhaus* [1994, p.115] considers the effects of a climate catastrophe (with economic impacts in the tens of percent of GWP) on optimal climate policy for a single SOW. In this case, the optimal policy without parameter uncertainty is to avoid the catastrophe. *Keller et al.* [2004] show that much smaller damages (less than 2% of GWP) results in an optimal policy that avoids an MOC collapse for a single SOW. Considering parametric uncertainty in the model of *Keller et al.* [2004] results in lower optimal abatement levels. The current study considers a more thorough representation of parametric uncertainty. Hence, the current results are generally consistent with the previously observed effects of parameter uncertainty in this decision problem [*Keller et al.*, 2004]. We hypothesize that improving some of the known shortcomings of the DICE model (*e.g.*, the static representation of the technology for reducing CO_2 emissions [*Keller et al.*, 2003] and the use of a relatively high rate of social time preference that decreases the importance of future climate change impacts [*Nordhaus*, 1994]) would result in higher CO_2 abatement and lower odds of triggering an MOC collapse in the optimal policy.

One might ask how much CO_2 abatement is required to significantly reduce the odds for an MOC collapse? Decreasing the odds of an MOC collapse to 1 in 10 increases the (constrained) optimal abatement to approximately 80% in the next 60 years (Figure 4 a). Decreasing these odds further to 1 in 20 or 1 in 100 is possible (in the framework of the model), but requires higher and more expensive abatement. A reliability constraint of 1 in 100 implies almost 90% abatement in the next 40 years. Whether such a rapid and almost complete decarbonization of the global economy is feasible is an open question.

Parametric uncertainty translates into predictive uncertainty. This is illustrated in Figure 5 for BAU, the unconstrained optimal policy and the reliability constrained optimal policy with odds for an MOC collapse of 1 in 10. Emissions increase substantially between 2055 and 2105 for the BAU and optimal policies (Figure 5 a,b); the mean value of emissions increases from 24 $\mathrm{Gt\,C\,a^{-1}}$ to 38 $\mathrm{Gt\,C\,a^{-1}}$ for BAU and from 19 $\mathrm{Gt\,C\,a^{-1}}$ to 28 $\mathrm{Gt\,C\,a^{-1}}$ for the optimal policy. In contrast, the optimal reliable policy results in mean CO_2 emissions of 5.7 $\mathrm{Gt\,C\,a^{-1}}$ in 2055 and 2.8 $\mathrm{Gt\,C\,a^{-1}}$ in 2105. The differences in CO_2 emissions drive differences in atmospheric CO_2 concentrations (Figure 5 c,d). Between 2055 and 2105, mean CO_2 concentrations increase from 560 ppm to 850 ppm for the BAU policy, and from 520 ppm to 725 ppm for the optimal policy. The optimal reliable policy, in contrast, has a reduced mean CO_2 concentration of 460 ppm in 2055 and 450 ppm in 2105. The reduced CO_2 concentrations for the optimal reliable policy result in decreased global warming (Figure 5 e,f). For the BAU and optimal policies, the mean temperature increases to 2.3°C and 2.1°C respectively. This increase is 1.9°C for the optimal reliable policy. The effect of different strategies on temperatures becomes more noticeable over time. In 2105, the mean temperature increase for the BAU, optimal and reliability constrained optimal case are 3.6°C, 3.3°C and 2.2°C respectively.

7 Caveats

This study improves on previous studies by considering the effects of reliability constraints and more refined numerical resolution, but is still silent on potentially important questions. For example, we analyze only the effects of a single threshold out of many possible climate responses [*Keller et al.*, 2005], consider only uncertainty about a subset of the model parameters, and neglect structural uncertainty, stochastic variability and the distinct possibility that we may learn in the future [*Kelly and Kolstad*, 1999].

The issue of structural uncertainty can be illustrated by the adopted choice of a single MOC model [*Stocker and Schmittner*, 1997]. As discussed in *Cubasch and Meehl* [2001], the MOC varies widely across model implementations. Several models (*e.g.* *Latif et al.* [2000]) suggest a stabilizing feedback that results in a basically insensitive MOC. Which MOC model is a more appropriate description of reality is currently deeply uncertain [*Cubasch and Meehl*, 2001]. This structural uncertainty could be represented by a Bayesian Model Averaging approach [*Hoeting et al.*, 1999]. Assuming a binary mixture of the adopted sensitive MOC model [*Stocker and Schmittner*, 1997] and a member of the insensitive MOC models [*Latif et al.*, 2000] with ignorant priors ($p = 0.5$ for both) would reduce the reported odds of triggering an MOC collapse by a factor of two. As discussed in *Keller et al.* [2005], future observations have a strong potential to reduce this uncertainty.

8 Conclusion

We examine the effect of parametric uncertainty and the potential for an MOC collapse in an economic optimal growth model. We expand on previous studies by considering (i) the effects of a reliability constraint, (ii) a larger number of uncertain parameters, and (iii) a more realistic numerical resolution.

A business-as-usual strategy results in considerable odds of an MOC collapse (exceeding 1 in 3 in the next 150 years in this simple model). An unconstrained optimal policy does not reduce these odds considerably. Reducing these odds to 1 in 10 requires an almost complete decarbonization (greater than 80%) over the next 60 years. A further reduction to 1 in 100 reduces this decarbonization time scale to

40 years.

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Appendix 1: List of symbols

symbol	definition
BAU	business-as-usual
CO_2	carbon dioxide
c	per capita consumption per year
δ_L	decline in population growth rate
GDP	gross domestic product
GWP	gross world product
g_σ	initial growth in CO_2–GDP ratio
I	gross investment
L	population
$\lambda^\star$	climate sensitivity
μ	CO_2 abatement relative to the BAU scenario
N	number of SOWs
N_S	number of samples of each uncertain parameter
Q	GWP
ρ	pure rate of social time preference
SOW	state of the world
s	SOW index
t	time
t_0	start of considered time horizon
t^*	end of considered time horizon
θ_3	threshold specific fractional economic damage
U	flow of utility
U^*	discounted sum of utility

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Table Legends

Table 1: Summary of the considered uncertain parameters. MOC refers to the North Atlantic meridional overturning circulation.

Figure Legends

Figure 1: Probability density functions (PDFs) for the considered uncertain parameters. Shown are: (i) the damage associated with an MOC collapse collapse (θ_3), (ii) the climate sensitivity ($\lambda^{\star}$), (iii) the decline in population growth rate (δ_L), and (iv) the initial growth in CO_2 to GDP ratio (g_σ). The triangles denote the locations of the stratified Latin-hypercube samples for $N_S = 7$. See text for details on the sampling procedure.

Figure 2: Optimal abatement levels for $N_S = 1$ (circles), $N_S = 7$ (stars) and $N_S = 15$ (squares). These sampling densities correspond to 1, 2,401 and 50,625 states of the world respectively.

Figure 3: Effects of increasing sampling resolution on approximation error. Shown are the mean difference (calculated from 2005 to 2155) between optimal abatement as a function of the numbers of states of the world (SOWs) compared with the optimal solution for 50,625 SOWs. The number of samples of each uncertain parameter are marked on the plot. The solution has practically converged beyond $7^4 =$ 2,401 SOWs.

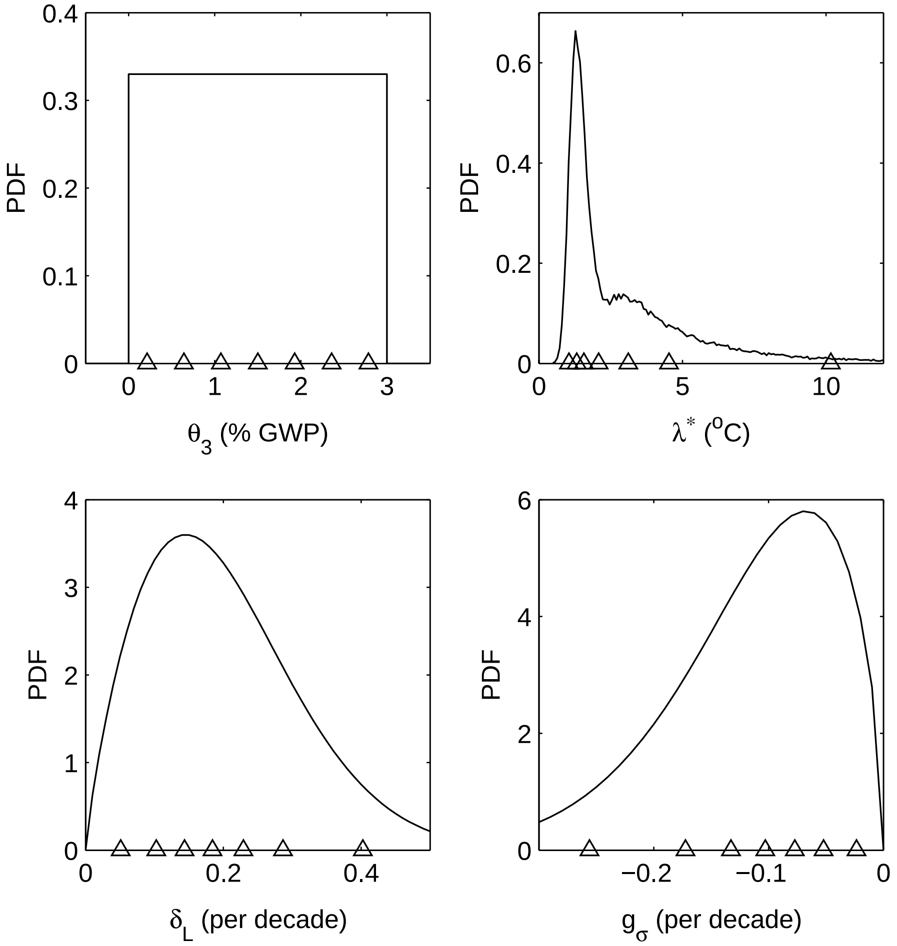
Figure 4: (a) Abatement strategies and (b) the potential odds of an MOC collapse for the business-as-usual policy (BAU), the unconstrained optimal policy and the optimal policies with odds for an MOC collapse constrained below 1 in 10, 1 in 20 and 1 in 100.

Figure 5: Cumulative density functions (CDFs) of carbon dioxide (CO_2) emissions in (a) 2055 and (b) 2105; atmospheric CO_2 concentration in (c) 2055 and (d) 2105; and temperature increase in (e) 2055 and (f) 2105. The business-as-usual strategy is represented by the dotted lines, the unconstrained optimal strategy by the dashed

lines and the optimal strategy with odds of an MOC collapse constrained below 1 in 10 by the solid lines.

Table 1:

Parameter	Units	Symbol	Mean	95% confidence limits	Distribution type	Reference
MOC specific damages	% GWP	θ_3	1.5	$[0.075, 2.925]$	Uniform	*[Keller et al., 2004]*
Climate sensitivity	$^\circ$C	$\lambda^\star$	3.4	$[0.9, 12.7]$	Empirical	*[Andronova and Schlesinger, 2001]*
Decline in population growth	Per decade	δ_L	0.2	$[0.03, 0.46]$	Weibull	*[Nordhaus and Popp, 1997]*
Initial decline of CO_2 to GDP ratio	Per decade	g_σ	-0.1168	$[-0.30, 0.01]$	Weibull	*[Nordhaus and Popp, 1997]*



PDF
0.4
0.3
0.2
0.1
0
0
1
2
3
θ3 (% GWP)
PDF
0.6
0.4
0.2
0
0
5
10
λ* (oC)
PDF
4
3
2
1
0
0
0.2
0.4
δL (per decade)
PDF
6
4
2
0
−0.2
−0.1
0
gσ (per decade)

Figure 1:

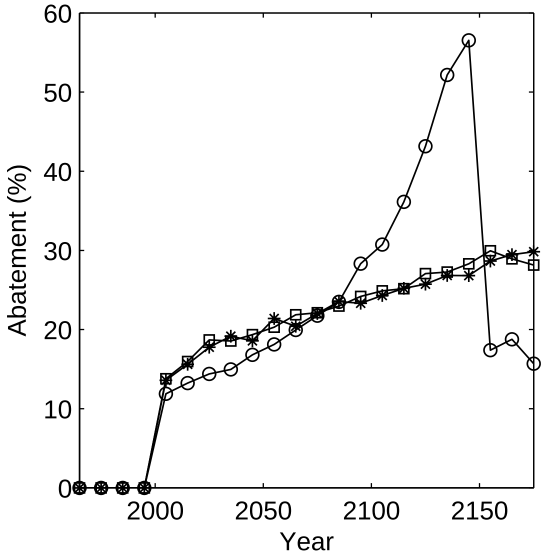
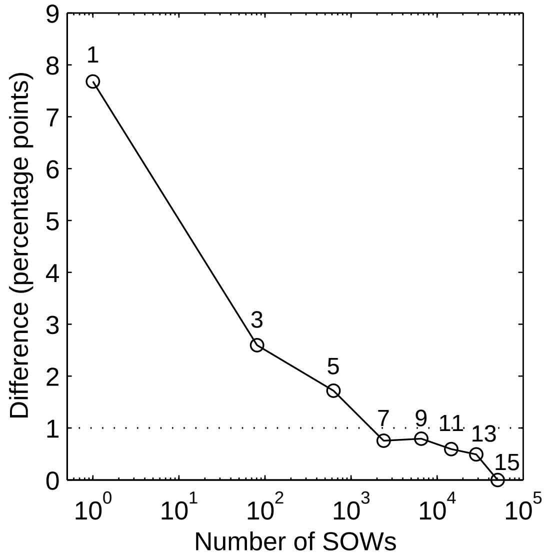
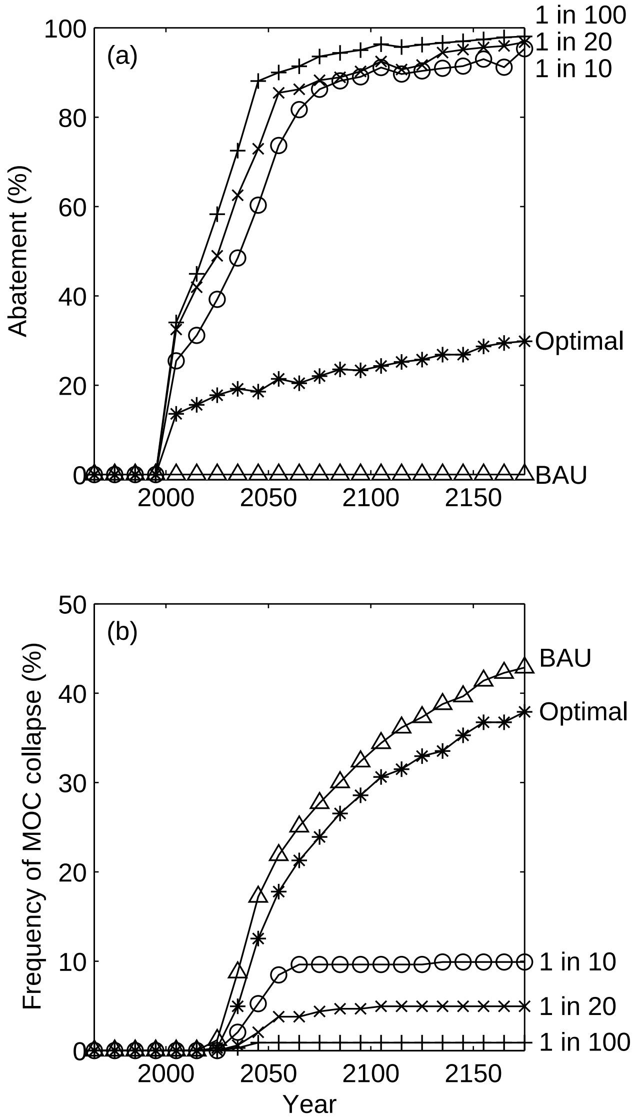


Figure 2:



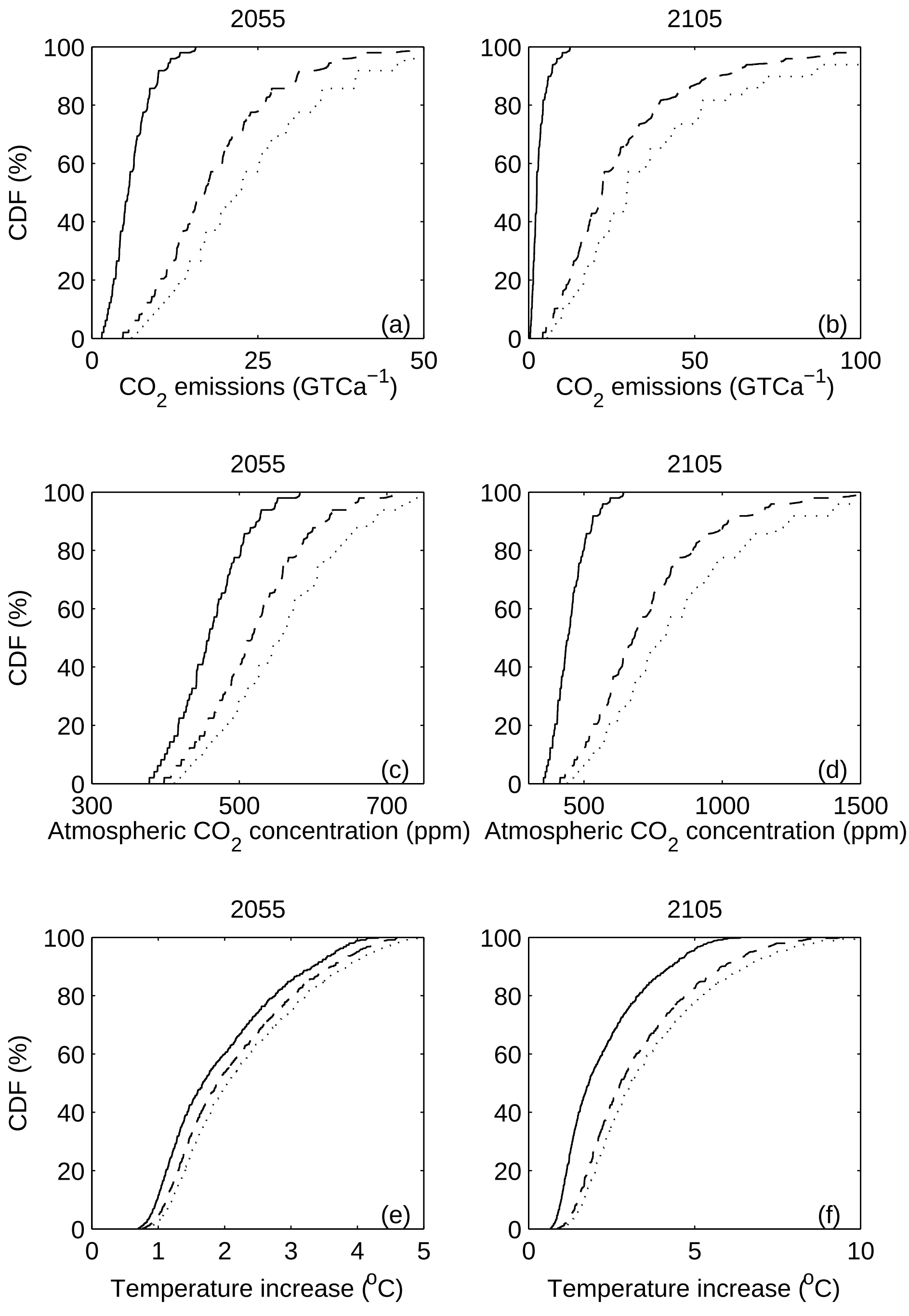
Difference (percentage points)
Number of SOWs
9
8
7
6
5
4
3
2
1
0
10^0
10^1
10^2
10^3
10^4
10^5
1
3
5
7
9
11
13
15

Figure 3:



(a)
Abatement (%)
100
80
60
40
20
0
2000
2050
2100
2150
1 in 100
1 in 20
1 in 10
Optimal
BAU
(b)
Frequency of MOC collapse (%)
50
40
30
20
10
0
2000
2050
2100
2150
Year
BAU
Optimal
1 in 10
1 in 20
1 in 100

Figure 4:



2055
2105
CDF (%)
(a)
(b)
CO_2 emissions ($GTCa^{-1}$)
(c)
(d)
Atmospheric CO_2 concentration (ppm)
(e)
(f)
Temperature increase (^oC)

Figure 5: