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LINKING ENERGY SYSTEM AND MACROECONOMIC GROWTH MODELS - IS THE SUPPLY CURVE ENOUGH?

Nico BAUER

Paul-Scherrer Institute – PSI
Switzerland

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Linking Energy System and Macroeconomic Growth Models

Is the Supply Curve Enough?

Nico Bauer*

Energy Economics Group, General Energy Department

Paul Scherrer Institute (PSI), Switzerland

Ottmar Edenhofer

Department for Global Change and Social Systems

Potsdam Institute for Climate Impact Research (PIK), Germany

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Abstract

Analysing interactions of the energy sector and the overall economy are of interest to improve science and policy advice. Macroeconomic growth models (MGM) and energy system models (ESM) are available to represent these two sectors. These models can be linked by two approaches. The hard-link approach completely integrates the ESM into the MGM. In the soft-link approach both models are solved in isolation and information is exchanged between them in a iteration process. This is done by reducing the full-scale dynamic ESM into a reduced form model consisting of a set of static energy supply functions. The soft-link approach attracts interest because it could enable the analyst to use more detailed models of both sectors. However, the solution is at best an approximation of the hard-link approach. Although the

* *Corresponding author:* CH-5232 Villigen, Switzerland. Tel.: ++41 +56 310 4740. E-mail: nico.bauer@psi.ch.

hard-link leads to an economically consistent solution, it is worse in terms of computational efficiency: the level of detail in both models is more limited than in the soft-link approach. The aim of this study is to test the usefulness of the soft-link approach. We check it for internal consistency as well as for the goodness of approximation against the benchmark of the hard-link. It turns out that the use of energy supply functions in the soft-link leads to an energy market equilibrium, but it does not for the capital market, where the financial means are allocated to various investment opportunities. The reason is that the ESM does not consider the dynamics of capital scarcity determined in the macroeconomy through simple energy supply functions. Augmenting the soft-link by also considering a time variable interest rate in the ESM, which is a result of the MGM, does not improve the approach much. This leads to the conclusion that the soft-link is not a reasonable approximation to the hard-link because capital market equilibrium is not assured.

Keywords: Model coupling, computational economics, supply theory, capital theory, energy system model, growth model, transition dynamics.

1 Introduction

Analysing the interaction of the macroeconomy and the energy sector is essential for gaining insights into the mid to long term development of each of them. On the one hand energy is an essential production factor for the generation of economic values. This implies that increasing scarcity of energy is an impediment on economic growth and leads to a shift of income distribution in favour of energy at the expense of capital and labour. Moreover, as is found in econometric analysis energy prices affect the conflict between capital and labour with respect to their income shares. On the other hand the energy sector requires especially financial means to install the capacities as well as the related infrastructure with long pay-back times. Energy related investments are in competition with consumption, investments in other capital stocks and income redistribution within the living generation. The interplay between both parts induces investments into various energy technologies, which in turn generate particular primary and end-use energy mixes. Moreover, the energy sector is related to issues of local and global environmental problems, research and development, international trade and public finance.

The issues of mutual interdependency are the motive to integrate macroeconomic growth models (MGM) and energy system models (ESM). The MGM maximises an non-linear intertemporal social welfare function depending on the consumption time path by allocating the budget between consumption and investment. Investments add to the capital stock, which is needed in combination with labour to produce economic value. The ESM minimises the costs of the energy system subject to an exogenous energy demand path and characteristics of energy technologies by choosing capacity additions and their utilisation. The financial means of the expenditures are not constrained in an ESM.

Integration of both models requires – in a first step – that energy is considered as a production factor in the MGM and that the production of energy requires financial means that are accounted in the budget equation of the macroeconomy. In a second step one has to decide between two different approaches. First, the *hard-link* approach integrates the full ESM into the MGM as an additional set of functions and constraints and solves one very complex non-linear programming (NLP) problem. The *soft-link* approach integrates a reduced form model of the ESM into the MGM (call it $\mathrm{ESM_r}$), which results in a less complex model. The parameters of $\mathrm{ESM_r}$ are calibrated according to the optimal solution of the original ESM that gets the exogenous energy demands from the MGM. An iterative procedure adapts the parameters of the reduced form model until the changes of the energy demand paths get sufficiently small. The optimal solutions of the ESM and $\mathrm{MGESM}_{\mathrm{SL}}$ are the solution of the soft-link.

The hard-link approach leads to one highly complex optimisation problem. This limits the level of disaggregation and detail of both models because of the computational abilities of NLP solvers; see e.g. Zwaan et al. (2001) and Edenhofer et al. (2005). However, the solution obtained with a hard-linked model is consistent. The soft-link approach only approximates this solution because it relies on a reduced form model of the ESM. However, the ESM and the MGM could be highly complex and solvable while the optimisation problem of the hard-link approach of both models could not be solved at all.

The soft-link approach that is examined in this study has been introduced by Messner and Schrattenholzer (2000). This leads to the question of the consistency and the goodness of approximation of the soft-link approach. We use relatively simple MGM and ESM models and couple them using the hard as well as the soft-link approach. The reduced form model of the ESM in the soft-link uses supply curves for energy. We test the soft-link approach for internal consistency and assess its goodness of approximation using the hard-link solution as benchmark. The specific research questions will be outlined below (see Ch. 3.3).

Analysing the appearing differences between the two approaches will point to an inconsistency of the soft-link approach with respect to capital markets, which in turn has effects on the energy market. The reason is that the valuation of capital scarcity in the MGM is not considered in the ESM and therefore the supply curves are not consistent with the interest rate. It is a natural step to augment the soft-link approach by also considering the information of endogenously computed time-variable interest rate of the MGM in the ESM. It turns out that this augmentation does not improve the soft-link approach, since the distorting effects are not reduced, but only switch the sign of deviation. This paper can be seen as a contribution to the more general economic issue of supply function in macroeconomic growth models as well as capital theory.

The organisation of the paper is as follows. Sec. 2 presents the two models, while Sec. 3 presents the two coupling approaches in detail and develops the particular research questions on that basis. Sec. 4 presents the results and discusses the questions. The discussion leads to a natural augmentation of the soft-link approach that is explained and analysed in Sec. 4. Finally, conclusions are drawn and suggestions for further research are made in Sec. 6.

2 The Two Models

In this section the two models are introduced in detail, which will then be integrated following the two approaches in the next section. Sec. 2.1 will introduce the ESM and Sec. 2.2 the MGM.

2.1 The Energy System Model

An ESM computes a cost-minimal energy system by choosing additions of energy technology capacities and primary energy utilisation to satisfy a given end-use energy demand. The technologies are described technical characteristics like conversion efficiencies, emissions, etc. and economic characteristics like unit investment costs. Some energy technologies require primary energy carriers that are described in terms of availability, costs etc. The analyst is interested in issues like technology choice, energy mix and costs of policy constraints.

ESMs are partial fix-price models of the energy sector. The energy sector is seen as a relatively small one with respect to the overall economy, which justifies the assumption that changes in its factor demands do not change the factor prices. This assumption is necessary to justify the linearity of cost and production functions that are expressed in linear relationships with fixed coefficients. And this in turn implies constant prices.

The ESM used in this study is a very simple linear programming problem. The demands for energy E_j, with $j = 1, 2$ are exogenously given and each of the types of energy can be produced using either scarce fossil fuel conversion technologies or renewable energy sources that are in the following indexed with F and R, respectively. Therefore, we have the following balance equation:

$$E_j = \sum_{k=F,R} E_{jk}, \qquad \text{for } j = 1, 2. \tag{1}$$

The production of end-use energy requires capacities that have to be installed. The availability of capacities requires fixed operation and maintenance

(O&M) efforts. The production of energy by utilising the capacities requires variable O&M efforts and if $k = F$ fossil fuels according to the conversion efficiencies. Thus, there are three cost components of the total costs C_{tot} in each period: investment costs C_I, fixed and variable O&M costs C_{OM} and fossil fuel costs C_F:

$$C_{tot} = C_I + C_{OM} + C_F. \qquad (2)$$

The capacities K_{jk} are required for the production of useful energy. A capacity is available for an exogenously assumed time of a year ν_{jk}, which gives the energy production per year:

$$E_{jk} = \nu_{jk} K_{jk}, \qquad \text{for } j = 1,2 \text{ and } k = F, R. \qquad (3)$$

Addition of capacities leads to investment costs that are proportional according to the investment costs per unit ι_{jk}:

$$C_I = \sum_{j=1,2} \sum_{k=F,R} \iota_{jk} I_{jk}. \qquad (4)$$

The installed capacities decline exponentially according to depreciation rate δ_{jk}:

$$\dot{K}_{jk} = I_{jk} - \delta_{jk} K_{jk}, \qquad \text{for } j = 1,2 \text{ and } k = F, R. \qquad (5)$$

The fixed O&M costs are proportional to the capacities using the cost coefficient o^f_{jk} and the variable O&M costs are proportional to energy production using the cost coefficient o^v_{jk}:

$$C_{OM} = \sum_{j=1,2} \sum_{k=F,R} o^f_{jk} K_{jk} + o^v_{jk} E_{jk}. \qquad (6)$$

The fossil fuel costs C_F are determined by the fossil resource extraction R_{lF} and the price of fossil fuels p_{lF}:

$$C_F = \sum_{j=1,2} p_{lF} R_{lF}. \qquad (7)$$

The index l distinguishes the various deposits of fossil fuels, that are characterised by p_{lF} and the maximum available amount. The fuel extraction from all deposits has to cover the energy production that is influenced by the conversion efficiency η_j:

$$\sum_l R_{lF} = \sum_{j=1,2} \frac{1}{\eta_{jF}} E_{jF} \qquad (8)$$

The price p_{lF} increases with cumulative extraction in previous periods according to a stepwise linear function that are distinguished using the index l. This is implemented by defining a number of fossil fuel pools that are characterised by prices p_{lF} and maximum available amounts R_{lF}^{max}.

$$R_{lF}^{max} \geq \int_{\tau_1}^{\tau_2} R_{lF}(t)dt, \quad \forall\, l. \tag{9}$$

Finally, the use of fossil fuels and the production of fossil end-use energy have to be balanced:

$$R_{jF} = \sum_{j=1,2} \frac{1}{\eta_{jF}} E_{jF}. \tag{10}$$

The ESM can be run in a stand alone version, if the energy demands are given exogenously. The optimal control problem is to minimise the cumulative discounted costs of the energy system, where r is the interest rate used for discounting:

$$DESC = \int_{\tau_1}^{\tau_2} e^{-rt} C_{tot}(t)dt. \tag{11}$$

This objective function implicitly assumes that the energy sector can get as much financial means as it demands at a constant interest rate r.

2.2 The Macroeconomic Growth Model

A simple Ramsey-type MGM – without considering energy – computes a welfare optimal growth path of a total economy by allocating a finite economic income on investments and consumption. The economy is described by non-linear welfare and production functions, an accounting system of the entire economy and a capital motion equation with an initial capital stock. The analyst is interested in welfare optimal transition and steady-state paths of investment rates, interest rates, distribution of income and economic growth.

An MGM is a general flex-price model of a whole economy. The accounting system is consistent in such a way that all expenditures are incomes and all incomes are used for alternative purposes. Thus, the total economic output is distributed to the production factors and the consumption and investment expenditures can not exceed the the economic income. Balancing the consumption-investment decision in the light of limited income requires flexible prices considering the non-linear production and utility functions.

The MGM in this study is an extension of a simple Ramsey-model. It solves the consumption-investment-energy expenditure decision by maximising an intertemporal social welfare function given in the following equation:

$$W = \int_{\tau_1}^{\tau_2} e^{-\rho t} U\left(C(t)\right) dt, \qquad \text{with } W \in \mathbb{R} \text{ and } U(\cdot) \in \mathbb{C}^2; \tag{12}$$

$$U' > 0, U'' < 0; \lim_{c \to 0} U' \to \infty, \lim_{c \to \infty} U' \to 0.$$

W is the scalar welfare measure that is computed as the discounted sum over the time interval $[\tau_1, \tau_2]$ of the utility path; ρ is the discounting rate. Utility at a time is computed using the velocity function $U(\cdot)$, which is a concave function in consumption C.

C is the residual of economic income Y that is alternatively allocated to capital investment I or to energy related expenditures EE:

$$Y = C + I + EE. \tag{13}$$

The investments are allocated to two different capital stocks K_A and K_B that are subject to depreciation and the initial condition of historically given capital stocks K_i^0:

$$\dot{K}_i = I_i - \delta_i K_i, \qquad K_i(t = \tau_1) = K_i^0, \quad \text{for } i = A, B. \tag{14}$$

The capital stocks are required for the production of economic values. Each capital stock is combined with two types of energy E_1 and E_2 in order to form sectoral composites S_A and S_B. The two composites are again combined to an aggregate capital-energy composite S_{KE} that in turn is combined with exogenously assumed labour L in order to compute the produced economic value Y. The general form of this production structure is:

$$Y = F\left\{A \cdot L, S_{KE}\left[S_A(K_A, E_{A1}, E_{A2}), S_B(K_B, E_{B1}, E_{B2})\right]\right\}. \tag{15}$$

Applying the concept of constant elasticity of substitution (CES) production functions results in the CES-nesting structure shown in Fig. 1 and more

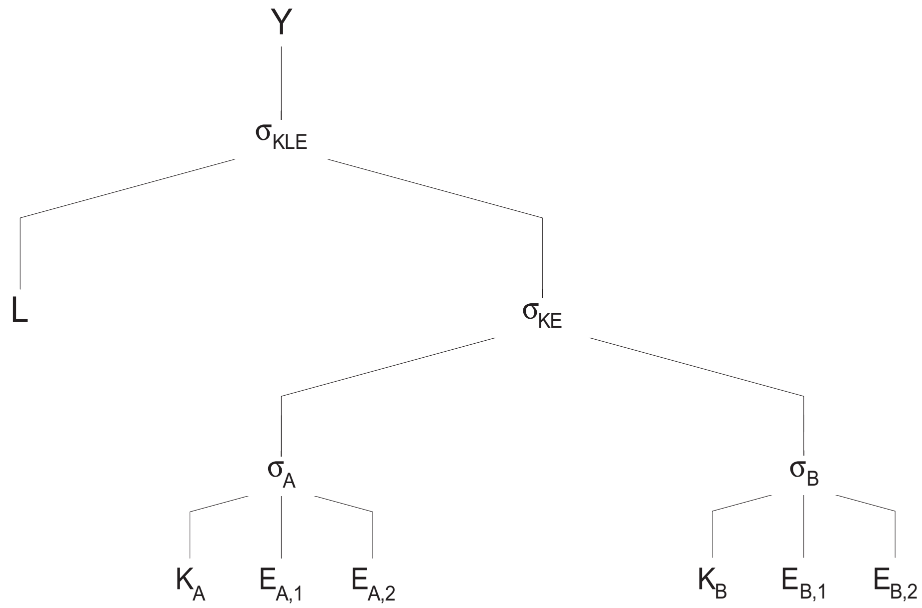


Figure 1: Production structure of the nested CES given in Eq. 16 – Eq. 18

formally to the following equations:

$$Y = F(A \cdot L, S_{KE}) = \phi_{S_{KLE}} \left[\tilde{\xi}_L (A \cdot L)^{\tilde{\sigma}_{S_{KLE}}} + \tilde{\xi}_{S_{KE}} S_{KE}^{\tilde{\sigma}_{S_{KLE}}} \right]^{\frac{1}{\tilde{\sigma}_{S_{KLE}}}}; \quad (16)$$

$$\text{with } S_{KE}(S_A, S_B) = \phi_{S_{KE}} \left[\sum_{i=A,B} \tilde{\xi}_{S_{KE},i} S_i^{\tilde{\sigma}_{S_{KE}}} \right]^{\frac{1}{\tilde{\sigma}_{S_{KE}}}}; \quad (17)$$

$$\text{with } S_i(K_i, E_j) = \phi_{S_i} \left[\tilde{\xi}_{K,i} K_i^{\tilde{\sigma}_{S_i}} + \sum_{j=1,2} \tilde{\xi}_{E,ij} E_{ij}^{\tilde{\sigma}_{S_i}} \right]^{\frac{1}{\tilde{\sigma}_{S_i}}}, \text{ for } i = A, B. \quad (18)$$

In these equations ϕ, ξ and σ denote the parameters for scaling, distribution and substitution, respectively, as they are common in CES production functions; see e.g. Arrow et al. (1961). The parameter A is a factor that represents the efficiency level of labour and it changes with time.

Energy of both types is delivered from the energy sector as will be described in the next section. For both types of energy balance equations have to be full-filled:

$$E_j = \sum_{i=A,B} E_{ij}, \qquad \text{for } j = 1, 2. \quad (19)$$

The model is not complete, yet. It lacks of the relationship that maps time paths of EE into time paths of energy E_j, with $j = 1, 2$. This relationship depends on the coupling approach applied to link it with the ESM. This will be explained in the next section.

3 Two Approaches

In this section the two approaches for coupling MGM and ESM are introduced in detail. Sec. 3.1 introduces the hard-link approach and Sec. 3.2 deals with the soft-link approach. In Sec. 3.3 we add some remarks and motivate the research questions of this study.

3.1 The Hard-Link Approach

The hard-link approach integrates the full scale ESM into the MGM; call the resulting model MGESM_{HL}. This means that the Eq. 1 – 10 become additional constraints in the MGM and the energy related expenditures EE equal the total energy system costs C_{tot} of Eq. 2. Therefore, the budget equation in the hard-linked model is:

$$Y = C + I + C_{tot}. \tag{20}$$

The objective function in Eq. 11 is obsolete in the hard-link approach, since the one and only optimisation problem to be solved maximises W of the MGM from Eq. 12. For the sake of clarity, the optimisation problem of MGESM_{HL} is to maximise W subject to the ESM equations Eq. 1 – 10 and MGM equations Eq. 14 – 20.

The hard-link approach optimises the overall system set up by the MGM with the fully integrated ESM in one optimisation problem. Thus, to study the appropriateness of the soft-link approach the solution of MGESM_{HL} is set as a benchmark for the goodness of approximation that the soft-link approach could approach.

3.2 The Soft-Link Approach

Formalising the soft-link approach using the two models of the previous section requires four steps:

1. ***Reduced form model*** has to be assumed that should represent the full scale ESM.

2. ***Parameter determination*** of $\mathrm{ESM_r}$ from the solution of the full scale ESM.
3. ***Information flow*** between the models has to be defined.
4. ***Stopping criterion*** has to be defined in order to stop the iteration.

The first step in formulating the soft-link approach is to build a reduced form model $\mathrm{ESM_r}$ of the ESM, which is then integrated into the MGM. This model is then called MGESM_{SL}. A commonly used ESM_r is to apply the supply curve concept. The energy related expenditures EE in Eq. 13 depend on the amount of the two types of energy delivered to the macroeconomic production function:

$$EE(t) = a(t) + \sum_{j=1,2} b_j(t) E_j(t)^c. \tag{21}$$

In the second step the time paths of the parameters a and $b = (b_1, b_2)$ are determined by the optimal solution of ESM for a particular set of exogenously given energy demand paths $E = (E_1, E_2)$ in Eq. 1. The parameter c is usually assumed to equal two or three. Hence, EE is convex in E, which presupposes a production function with diminishing returns to scale.

The parameters a and b are determined by the following equation that is derived from the partial derivative of Eq. 21 with respect to E_j. The partial derivative is assumed to equal the price of energy, which is assumed to be the shadow price of energy type j denoted $\mathcal{P}_j$, which is the dual variable of the energy balance equation Eq. 1 of the optimal solution of the ESM in each time step:

$$b_j = \frac{\mathcal{P}_j}{c \cdot E_j^{c-1}}. \tag{22}$$

The parameter a can be interpreted as a technically necessary correction parameter. The variable energy costs $\sum_j b_j E_j^c$ might not equal the total energy system costs C_{tot} computed with the ESM. Thus, the fixed energy costs a are introduced to correct for this difference:

$$a = C_{tot} - \sum_j b_j E_j^c. \tag{23}$$

Defining in the third step the information flow between the models the ESM can be interpreted as a function $\mathcal{E}(\cdot)$ that maps time paths of E into time paths of a and b using optimisation procedures:

$$\binom{a}{b} = \mathcal{E}(E). \tag{24}$$

In turn, MGESM_{SL} can be interpreted as a function $\mathcal{M}(\cdot)$ that maps the supply curve parameters a and b into energy demand paths E using optimisation procedures:

$$E = \mathcal{M}(a, b). \tag{25}$$

The mappings Eq. 24 – 25 are sufficient to define an iteration procedure, since the input of one model is the output the other. As a stopping criterion one can ask in the final step for less than ϵ-changes of E from one to the next iteration step. The approximated solution is found as soon as the iteration stops due to this convergence criterion.

3.3 Remarks

Some remarks have to be made with respect to the two approaches. Three remarks are directly related to the research questions and treated first. First, the iteration that approximates the solution in the soft-link approach stops, if the changes of the energy paths E become sufficiently small. The algorithm does not look for energy market equilibria nor is it implicitly guaranteed. This leads to the first question: are the supply and demand prices equal?

Second, the parameter a in Eq. 23 closes the gap between the variable costs of the energy system and the actual energy costs. Since there is delivery of energy to the household sector the parameter a could also be interpreted as a lump-sum tax transfer to the energy sector. This leads to the second question: Is the lump-sum transfer sufficiently small compared to the total energy system costs?

Third, any approximation procedure is subject to the question, whether the iterative solution is sufficiently close to the original solution. We assume the solution of the hard-link approach as the benchmark and ask: What is the difference of energy prices and quantities of the soft-link solution relative to the hard-link solution?

The next two remarks are with respect to the static nature of (energy) supply functions. Any supply function assumes that all rewards and expenses are realised during the same period in which the production takes place. The ESM is essentially different in this respect, since the expansion of supply in one period induces the expansion of supply in future periods because the capacities are installed. To put it differently: The expansion of supply requires financial means for investments that have a payback period that is much longer than a period for which the supply increase is considered. The supply curve concept assumes full flexibility of quantity choices between periods.

From this observation a problem arises in determining the parameters of the supply function. One should note, that the supply curve functions are calibrated as convex function (quadratic or cubic) using only one single price-quantity point to determine it. An alternative procedure is to compute several price-quantity combinations using the ESM by varying the time paths of E at all time steps. But since a perturbation in t has consequences in all $t + \tilde{t}$, with $\tilde{t} \geq 1$, the supply curve in each period can not be properly determined. The fundamental reason is that the supply curve concept simply aims at summarising an essentially dynamic formal structure into static structure.

4 Results

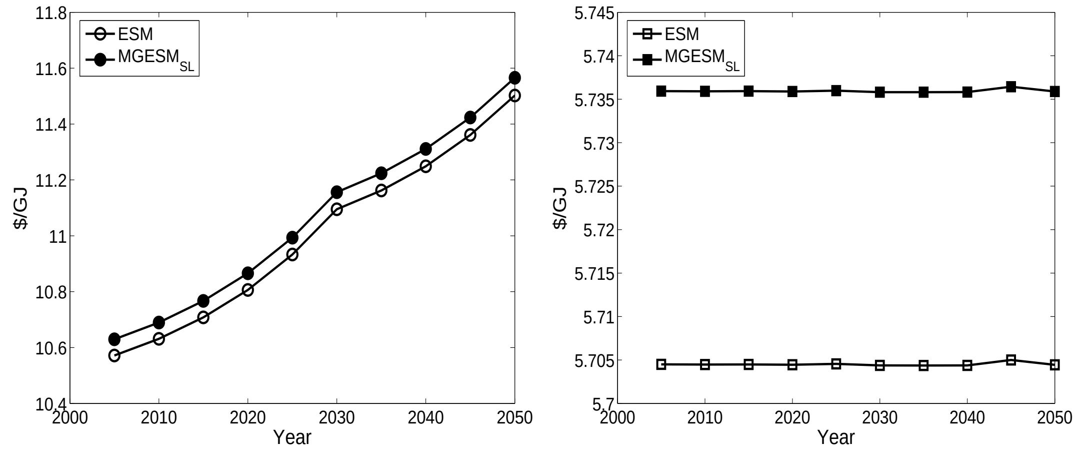
The computations in the following are performed using arbitrary numbers for the calibration. It is not intended to use the computations to show the effect for any particular country or region. Sec. 4.1 asks for the internal consistency of the soft-link approach by examining the equality of supply and demand prices by focusing on the accounting of the energy costs. Sec. 4.2 we address the relationship between the soft and the hard-link approach by assessing the goodness of approximation of the soft-link approach, which suggests a more detailed examination of the way both approaches deal with the scarcity of capital.

4.1 Consistency of the Soft-Link Approach

The soft-link approach does not guarantee that the supply and demand prices equalise. The demand and supply quantities are equal, since the energy demand of the MGM is the boundary condition for the ESM. The question related to the energy prices does only touch the consistency of the soft-link approach.

Fig. 2 compares the demand and supply prices of energy that are computed with the soft-link. The supply price of a type of energy is determined by the slope of the energy supply curve of ESM_r. The demand price equals the shadow price of both energy types $E_j, j = 1, 2$, using Eq. 19 of the MGM.

Fig. 2(a) shows the time path of prices for E_1. There is a constant absolute wedge between both with slightly higher supply price. The increase of the



(a) Price of E_1 (b) Price of E_2

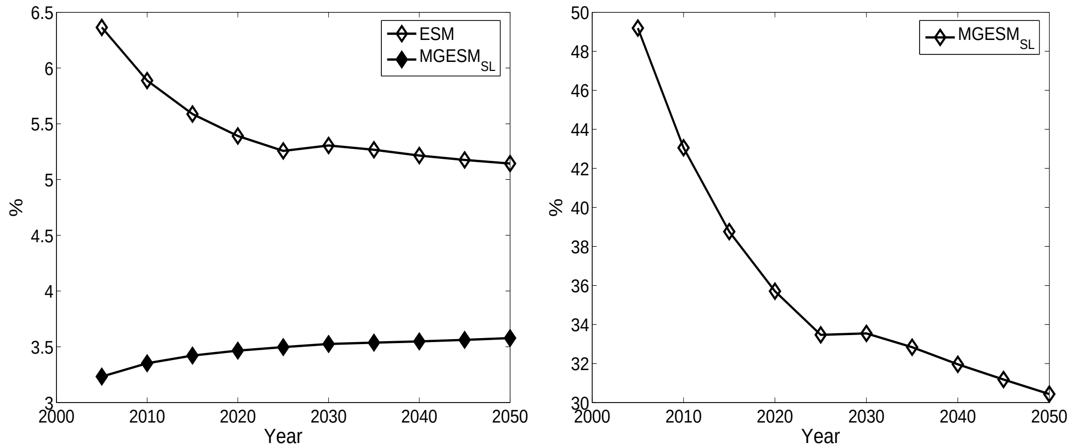
Figure 2: Time path of energy prices for E_j in the soft-link approach for ESM and MGESM_{SL}.

price is due to the fact that the production of E_1 relies on the use of the scarce fossil energy sources. There is also a small, constant price wedge observed for E_2 in Fig. 2(b); the constancy of the prices is implied by the use of infinite renewable energy sources.

The differences are – from our point of view – sufficiently small, so that we conclude that the soft-link approach performs well with respect to prices on the energy market. The result is all but trivial, especially because the approach requires less than ten iterations to find such a solution. Algorithms based on finding equilibrium prices might perform quite worse.

We turn to the second research question, which addresses the consistency of expenditures to the energy sector. Although prices and quantities are equivalent for both approaches, there is no guarantee that the energy expenditures in each period computed with the ESM are the same as those computed with MGESM_{SL} using the supply curve. A possible gap is closed by introducing the technical correction factor a and the question is whether this fixed cost share is significant or negligible.

Fig. 3 illustrates the model behaviour. Fig. 3(a) shows the share of the energy system costs computed by the ESM and the MGESM_{SL} relative to the macroeconomic output. The computation for the ESM used the total undiscounted energy system costs C_{tot} of Eq. 2; for the MGESM_{SL} used $EE - a$ appearing in Eq. 21. Both shares rely on the same macroeconomic output computed with MGESM_{SL}. The time paths show qualitative different and



(a) Share of energy system costs

(b) Fixed cost share

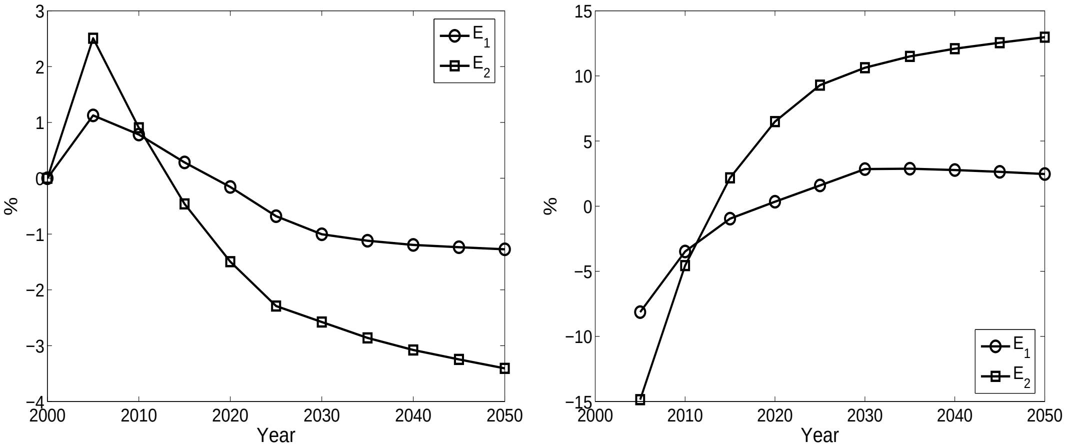
Figure 3: Time path of the energy system cost computed with the ESM and the $\mathrm{MGESM}_{\mathrm{SL}}$.

quantitative significant differences. Initially, the share of the ESM is decreasing while that of $\mathrm{MGESM}_{\mathrm{SL}}$ is slightly increasing. In the long-run there is a difference of about 1.5%-points of macroeconomic income, while the absolute share of $\mathrm{MGESM}_{\mathrm{SL}}$ is only slightly higher than 3.5%.

Fig. 3(b) shows the share of the fixed costs a relative to the total energy system costs EE of MGESM_{SL}. In the initial period nearly half of the energy system costs computed with the supply curve are financed by the lump-sum transfer. This share decreases, but remains between 30 and 35%.

The reason for this model behaviour is related to the particular form of the energy supply functions. The quadratic form of Eq. 21 used in this study implies that the first unit of energy is produced at costs equal to zero. However, the energy production costs of any unit of energy computed with the ESM are higher than zero. The resulting gap in energy costs is closed by the introduction of a.

In summary, we conclude that the soft-link approach leads to an internally consistent solution with respect to prices and quantities on the energy market. However, in the soft-link approach energy costs justified by economic exchange substantially deviate. The deviation is addressed by introducing the correction factor a that allows the interpretation of a lump-sum income transfer. Therefore, the soft-link approach is useful, if one is interested in a solution assuring energy market equilibrium and no other requirements should be fulfilled.



(a) Quantities (b) Prices

Figure 4: Differences of the soft-link relative to the hard-link approach for energy quantities and prices.

So far, the analysis does not reveal whether the soft-link approach leads to systematic biases with respect to energy prices and quantities compared to the hard-link approach. This question is addressed in the next sub-section and it turns out that the deviations are substantial.

4.2 Goodness of Approximation

In the following we assess the goodness of approximation of the soft-link approach by comparing it with the hard-link. The latter is the benchmark that should be approximated by the former.

The subsection contains two steps. In a first step the goodness of approximation can be assessed by looking at the solutions for the prices and quantities of energy of the soft and the hard-link. Explaining the remarkable differences of the results leads us in the second step to a closer examination of transition dynamics of a growing economy. The explanation for the differences requires to consider the scarcity of capital and the intertemporal valuation of investments and consumption that are treated differently with respect to energy related capital in the two approaches. This leads us in Sec. 5 to augment the soft-link approach so that the scarcity of capital is also considered in the ESM of the soft-link.

The differences of the amounts of traded energy and the corresponding prices for the soft-link relative to the hard-link are shown in Fig. 4. For both types of energy the traded quantity shows a clear pattern in Fig. 4(a). For the first periods there is remarkably higher energy production in the soft-link approach, which inverts into constantly lower energy production in later periods. This pattern is mirrored in the time paths of the relative differences of the prices as shown in Fig. 4(b). The energy prices in the soft-link are initially lower because of the higher energy production, then the difference is reversed because of the lower energy production in the soft-link. Please note that the relative price differences (-15 – 15%) are more pronounced than the relative differences of quantities (-4 – 2%) due to the assumed poor possibilities to substitute energy by capital.

We observe that the time paths of the relative quantity differences are not monotonous in time. In the beginning the soft-link approach shows a greater flexibility in increasing the supply of energy, but in the longer term the energy production capacities are not expanded as much as in the hard-link approach. This non-monotonous differences point into the direction that the soft and the hard-link approach have different dynamic properties related to the accumulation of capacities in the energy sector. This issue is in turn related to the scarcity of capital and the intertemporal valuation of consumption and investment. Hence, the soft-link approach using the static supply curve deals in a different way with the scarcity of capital than the hard-link model that integrates the overall dynamics of the energy sector.

An analysis of the scarcity of capital requires to look at the own rates of return (ORR) of capital and capacity stocks, which are the indicators for the amount and allocation of investments of the economy; see e.g. Dorfman et al. (1958, Ch. 12). ORR are specific to distinct investment opportunities $r_{i'}$; in our case the index i' distinguishes investments that increase the capital and capacity stocks. The various $r_{i'}$ are related to the economy's interest rate r^*.

The ORR of a technology is the return in monetary terms that could be paid by the investor to the financier for lending the financial means; i.e. credits, stocks, etc. The investors know their ORR and ask for credits at the capital market. If the ORR of a project is higher than r^* the investors will demand as much credits as the investment opportunity pays off. If $r_{i'}$ of a technology is less than r^*, no investor will demand credits to carry out such investment. Therefore, in capital market equilibrium one of the following two conditions for all i' hold:

$$r_{i'} \leq \quad r^* \qquad \Leftrightarrow \qquad I_{i'} = 0; \tag{26}$$

$$r_{i'} = r^* > 0 \qquad \Leftrightarrow \qquad I_{i'} > 0. \tag{27}$$

Both conditions can be summarised in a single equation:

$$(r^* - r_{i'})I_{i'} = 0; \qquad \forall\, i'. \tag{28}$$

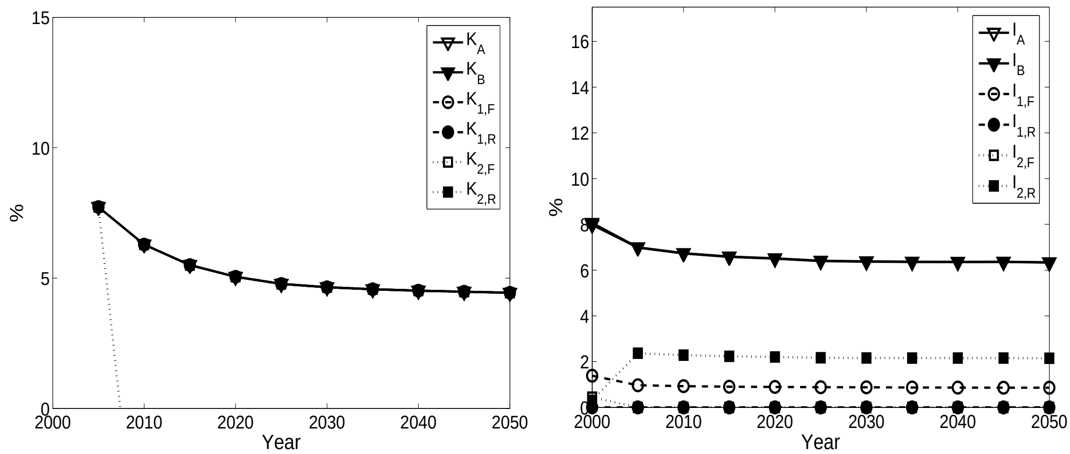
In capital market equilibrium at every point in time all ORRs of all technologies to which investments are allocated are equal and positive. Investments in each investment opportunity are increased up to the point where $r_{i'}$ would fall below r^*. The market interest rate in turn is the result of all investment opportunities, their profitability and the preferences of the households. ORRs of all other technologies that do not attract credits are lower than r^* and could be negative. This implies that for two alternative investment projects producing the same good three outcomes are possible: investment in both alternatives, investment in one alternative or no investment at all; see e.g. Pitchford (1979).

Thus ORR are directly related to the investment rates (IR) of the corresponding capital and capacity stocks, because investments into a capital or capacity stock require a minimum ORR. Fig. 5 illustrates ORR and IR for $\mathrm{MGESM_{HL}}$; i.e. the hard-link approach.

Fig. 5(a) shows the ORR for all six capital and capacity stocks in the model. The ORR of the macroeconomic capital stocks are computed by dividing the shadow price of the capital motion equation Eq. 14 by the shadow price of the budget equation Eq. 13, which results in ORR present values for each period. The ORR of the energy capacities also require that this value is divided by the investment costs $\iota_{i,j}$.

The model calibration for the initial period indicates that capital is relatively scarce. The ORRs are equal for all capital stock in which financial means are invested. As can be seen in Fig. 5(b) investments $I_{2,F}$ are positive and become zero and therefore the ORR becomes lower relative to other investment opportunities. There are no investments at all in $I_{1,R}$

The initial scarcity of capital is reduced by relatively high investment rates that are reduced in subsequent periods. Decreasing capital scarcity leads to lower ORRs. After about 25 years the constant levels are approached, which is indicated by the constancy of ORRs and IRs. The process towards the steady-state is known as transition dynamics. Note that ORR of the energy related



(a) Own rate of return (b) Investment rates

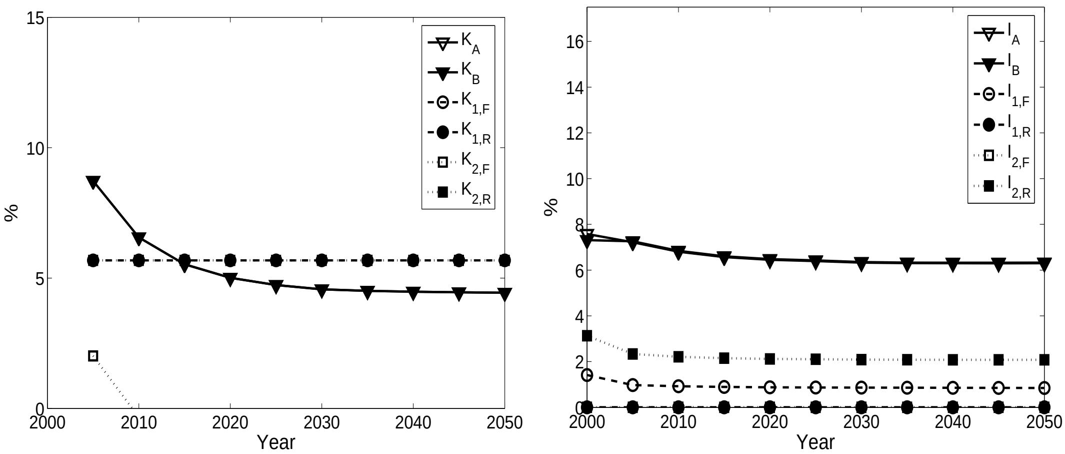
Figure 5: Own rate of interest and investment rates for the capital and capacity stocks of MGESM_{HL}. Triangles ($\bigtriangledown$) denote the macroeconomic capital stocks, circles ($\bigcirc$) capital stocks related to energy type 1, squares ($\square$) capital stocks related to energy type 2 and F and R denote fossil and renewable based energy technologies, respectively.

capacity stocks is in correspondence with the ORR of the macroeconomic capital stocks. This means that all ORR of profitable investment opportunities follow along the same time path.

Fig. 6 shows the corresponding results for the soft-link approach. Fig. 6(a) presents the ORR of the capital and capacity stocks. It that the ORR of the two macro-economic capital stocks exhibit the same shape as in the hard-link, but those of $K_{1,F}$ and $K_{2,R}$ immediately switch to constant levels.[1] These two capacity stocks are those to which energy related investments are allocated as can be seen in Fig. 6(b). The immediate switch of the ORR below the level of the macro-economic capital stocks requires larger energy related investments than in the hard-link approach. Thus, the differences in investments arise from a capital market dis-equilibrium in the soft-link approach that is due to the energy supply functions, which do not consider the scarcity of capital.

A comparison of energy capacity related investment rates in early periods reveals that the soft-link computes higher investment rates than the hard-link. This is in correspondence with the higher energy production and lower energy prices observed in Fig. 4.

[1]The constant ORR level is at about 5.7%, which is slightly higher than the exogenously assumed discount rate r that equals 5%. The reason for this difference is not clear.



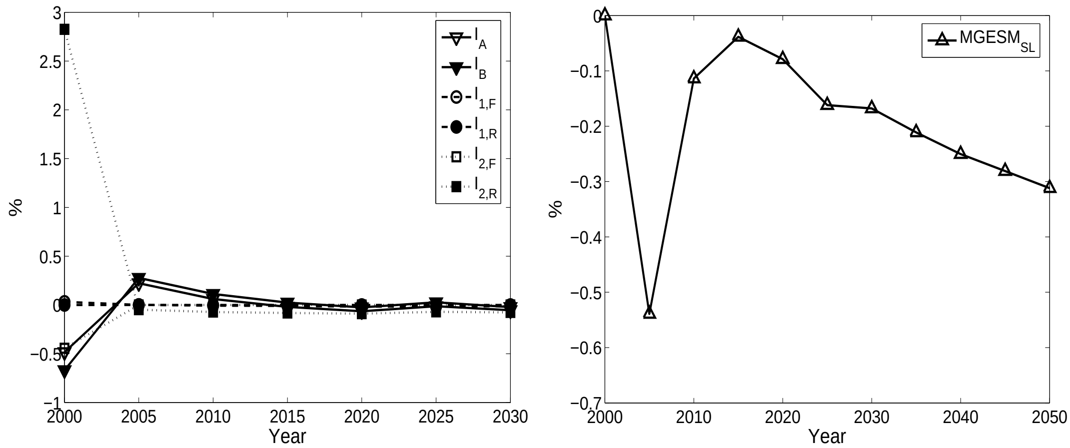
(a) Own rate of return (b) Investment rates

Figure 6: Own rate of interest and investment rates for the capital and capacity stocks of the soft-link approach.

A closer look at the investment dynamics is given in Fig. 7(a). This graph shows the differences of the IRs by subtracting the IR of the hard-link from the soft-link's IR. It shows that in the initial period $I_{2,R}$ is 2.8% higher than in the hard-link. These higher investments are partially financed by lower investments in macroeconomic capital and energy capacities. The total initial investments of the soft-link are higher than in the hard-link, which is financed by reduced consumption. The lower initial investments in macroeconomic capital in the soft-link have to make up leeway in subsequent periods, which is the reason for the positive difference in the following two periods. However, in the long-run the IRs for all capital and capacity stocks are not as high as in the hard-link approach.

Fig. 7(b) shows the overall effect on economic income as the difference of the soft-link relative to the hard-link. It turns out that the long run economic income is about 0.3% lower than in the hard-link approach.

In summary, the energy prices and quantities in both approaches behave quite differently in initial periods. The reason is that both approaches deal differently with the intertemporal valuation of capital scarcity, which is most pronounced in early periods. The soft-link approach using the energy supply functions leads to an inconsistency at the capital market, which induces a different allocation of investments. The overall economic effect of this misallocation of investment is permanently negative. Therefore, the analysis of the



(a) Investment rates

(b) Income

Figure 7: Relative differences of investment rates and economic income of soft-link relative to hard-link.

goodness of approximation pointed to an inconsistency of the soft-link with respect to the capital market that has an effect on the energy market through the biased allocation of investments.

5 Augmenting the Soft-Link Approach

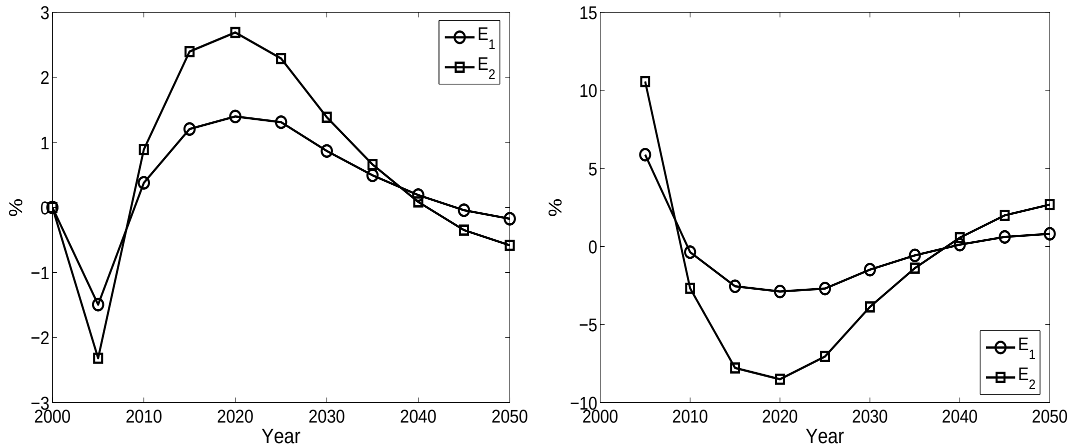
The poor performance of the soft-link in approaximating the hard-link result has been attributed to the bias on the capital market due to the fixed interest rate in ESM. Since the MGM computes the interest rate endogenously in a consistent way, it is a natural extension of the soft-link approach to pass this information to the ESM. Therefore, we augment the mappings Eq. 24 – 25 for the iteration:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \mathcal{E}(E, r); \tag{29}$$

$$\begin{pmatrix} E \\ r \end{pmatrix} = \mathcal{M}(a, b). \tag{30}$$

This means that the ESM considers the time varying interest rate r in Eq. 11 from the MGM and summarises this additional information in the parameters a and b. The question is whether this augmented soft-link approach improves the goodness of approximation.[2]

[2]The advantages and shortcomings discussed in Sec. 4.1 remain unchanged.



(a) Quantities (b) Prices

Figure 8: Differences of the augmented soft-link relative to the hard-link approach for energy quantities and prices.

Fig. 8 shows the relative differences of energy quantities and prices of the augmented soft-link approach relative to the hard-link approach. The relative differences of energy quantities in Fig. 8(a) shows the converse result as did the original soft-link approach. Now, the near-term energy production is lower in the soft-link approach, which leads to higher energy prices shown in Fig. 8(b). The lower long-term energy production comes with higher energy prices. This suggests that the interest rate does not correctly translates into the supply function parameters and therefore the price signals in the MGM remain biased, even though the direction is converse.

From this we can conclude that the attempt to repair the defect of the soft-link has not been successful. Only the hard-link model which considers all opportunities and constraints in a single optimisation problem leads to a consistent and efficient result. The result could not be approximated using a *static* reduced form model of the *dynamic* full scale model.

6 Discussion and Conclusions

In this paper we analysed two common approaches linking MGM and ESM. The hard-link integrates all constraints of the ESM into the MGM and the soft-link approach integrates energy supply functions into the MGM that are derived from the optimal solution of the ESM. The hard-link produces a consistent solution, which might be approximated using the soft-link approach.

The motive for this study stems from the need to analyse energy related policies using large-scale ESM and MGM. The hard-link limits the level of technological detail represented in the ESM because of limitations of available NLP solvers. The soft-link differentiates the total problem in two subproblems that are solved via an iterative procedure using energy supply functions. The study asks for the economic consistency of the soft-link approach and its performance in approximating the result of the hard-link.

In a first step it turned out that the soft-link produces an equilibrium at the energy market. Although the efficiency conditions are fulfilled, the energy costs using the variable part of the energy supply functions is different from the energy system costs obtained from ESM. This is due to the particular form of the variable part of the non-linear energy supply functions compared to the assumptions of the linear ESM. The difference is taken into account by the fixed part of the energy supply functions.

In a second step we tested the goodness of approximation of the soft-link compared to the hard-link solution. The complex pattern of differences on the energy market is due to an inconsistency at the capital market. The ESM assumes that infinite amounts of capital can be received from the capital market at a constant interest rate. However, the MGM computes a time path of the interest rate according to preferences and production technologies. Hence, the ESM ignores endogenous valuation of capital that is the main purpose of the MGM. Moreover, the interest rate is time variable, if the economies initial condition does not coincide with the steady-state. We conclude that the supply curve concept is not appropriate in an economy not starting in the steady-state.

A natural augmentation of the soft-link approach is to consider the time path of the MGM's interest rate in the ESM in order to take account of capital scarcity. However, the soft-link was not improved. This means that there must be other reasons leading to this bias. We have not found any following from our economic intuition. It can not be excluded that the deviation is due to numerical reasons.

In this study we started with analysing approaches of coupling models of the macroeconomy and the energy sector. The discussion of problems lead us to fundamental issues of production and supply theory as well as capital market equilibrium in growing economies. The analysis shows that during transition phases towards the steady-state the time variable interest rate leads to short run biases in investments. The bias in energy production is greater the greater is the distance of the economy from the steady state. As is analysed in Bauer (2005, Ch. 4) and Bauer and Edenhofer (Submitted) the imposition of climate mitigation policies can have impacts on the interest rate, which is reduced by the availability of climate friendly technologies. A reduction of the interest rate improves the competitiveness of investment opportunities with a high cost share of capital, like renewable energy technologies. The significance of interest rate changes on the choice of technologies is greater the greater are the differences of capital cost shares of various technologies.

The study showed that the shape of energy supply functions does not only depend on technologies, resource availability and cost coefficients, but also on the macroeconomic interest rate. An issue that is not considered in this study is the impact of the wage rate development on the growth of the energy sector and technology choice therein. The experience gathered from economic time series suggests for most developing countries an increasing wage rate. The growth of energy production will depend on labour productivity growth in the overall economy and in the energy sector as well as in the corresponding investment good sectors. Moreover, since the ratio of wage and interest rate as well as technological progress varies across sectors and technologies, the relative competitiveness of investment opportunities across all sectors is permanently reevaluated and hence technology choice is adjusted in accordance.

From the present study we conclude that sound coupling of ESM and MGM requires the hard-link approach. The soft-link approach does not guarantee simultaneous equilibrium at the energy and the capital market. However, this strategic choice of modelling limits the level of detail and complexity of a particular ESM. Nonetheless, also the hard-link approach is questionable since all cost coefficients in an ESM are independent of the macroeconomic development that interacts with these coefficients. This critique is mainly due to the complexity of market equilibrium and economic growth that can not be solved with available numerical algorithms.

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