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**WITCH:  
WORLD INDUCED TECHNICAL CHANGE MODEL**

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# WITCH

## A World Induced Technical Change Hybrid Model

by

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### Abstract

The need for a better understanding of future energy and technological scenarios, of their compatibility with the objective of stabilizing greenhouse gas concentrations, and of their links with climate policy, calls for the development of hybrid models. Hybrid because both the technological detail typical of Bottom Up (BU) models and the long run dynamics typical of Top Down (TD) models are crucially necessary. We present WITCH –World Induced Technical Change Hybrid model– a neo-classical optimal growth model (TD) with a detailed energy input specification (BU) and endogenous technical change (ETC). In particular, the BU component includes both electric and non-electric energy use, with a total of six technologies for electricity generation and three for non electricity generation. In contrast to traditional, static BU model, the allocation of installed capacity among technologies is defined as an optimal intertemporal strategy. The model endogenously accounts for technological progress, both through learning curves affecting prices of new vintages of capital and through R&D investments. In addition, the model captures the main economic interrelationships between world regions and is designed to analyse the optimal economic and environment policies in each world region as the outcome of a dynamic game. This paper provides a detailed description of the WITCH model, of its baseline, and of the model calibration procedure.

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## 1. Introduction

Climate change is a long run global phenomenon. Its impacts are felt over a long time horizon, with different adverse geographical and sectoral effects. Climate change negatively affects welfare of present and future generations. It is an uncertain phenomenon and its control is likely to be difficult and costly. Because no one really believes or is ready to accept that the solution to the climate change problem is to reduce the pace of economic growth, policy analyses have often focused on changes in technology that could bring about the long sought de-coupling of economic growth from generation of polluting emissions. It is indeed widely recognized that without a drastic technological change, in particular in energy technologies, it will be difficult to control the dynamics of climate change and its impacts on ecosystems and economic systems.

However, the above is not an easy task. A model of technology development, adoption and diffusion should also take into account the long run dimension of the climate change problem, the interdependence of the needs of present and future generations, the linkages and externalities between different geographical regions and economic sectors, the dynamics of investments and population, and the uncertainty pervading the climate change phenomenon and its effects. The ideal model would feature all the above aspects and should be computationally manageable. Unfortunately, at present this ideal model does not exist. Existing classes of models stress or describe in detail some but not all above aspects. Generally speaking, economists pay more attention to the economic dimension of climate change within their top-down (TD) models, whereas system analysts or engineers deepen the technological dimension of the problem in their bottom-up (BU) models.

In this paper, we present a new model designed to at least partly fill the gap we have briefly outlined above. Given that our final objective is the analysis of optimal policy strategies within a game-theoretic analytical framework, the basic layout is a top-down model of optimal economic growth. As such, it is in the best position – at least in principle – for appropriately describing the dynamics of the relevant variables of the problem and for determining the optimal intertemporal policy decisions to control climate change. A model of this sort lends itself to incorporate uncertainty features and can be designed so as to disaggregate the world into different regions and, above all, endogenize the process of technological progress.

Relative to top-down computable general equilibrium (CGE) models, intertemporal optimization growth models are usually limited in terms of sectoral breakdown. Relative to bottom-up techno-economic models they are unable to account for a rich menu of alternative technologies among which to choose. Therefore, a better understanding of future energy and technological scenarios, of their compatibility with the objective of stabilizing greenhouse gas concentrations and of their links with climate policy, calls for the development of hybrid models. Hybrid because both the

technological detail typical of Bottom Up (BU) models and the long run dynamics typical of Top Down (TD) models are crucially necessary and are explicitly integrated within one another.

In this paper we present a new hybrid model called WITCH, World Induced Technical Change Hybrid, a top-down neo-classical optimal growth model with a detailed energy input specification proper of a bottom-up model. The WITCH model is a “hard-link” top-down-based hybrid model. Traditional hybrid models are in most cases formulated so that the link between the two cores -the energy and the economy systems- implies stand-alone optimization processes. Examples of such “soft-link” hybrid models are MESSAGE-MACRO (see Messner and Schrattenholzer, 2000, for a detailed description) and MERGE (see Manne and Richels, 1992). Conversely, the main novelty of WITCH is that the optimization process is completely integrated and all investment decisions in new technologies are taken strategically in all world regions. In particular, its main features can be summarised as follows.

The bottom-up component includes both electric and non-electric energy use, with a total of six technologies for electricity generation and three for non-electricity generation. In contrast to traditional, static bottom-up models, the allocation of installed capacity among technologies is defined as an optimal intertemporal strategy. Therefore, the dynamic profile of optimal investments in different technologies is one of the outcomes of the model. In addition, these investment strategies are optimally determined by taking into account both economic and environmental externalities. Therefore, the investment profile for each technology is the solution of an intertemporal game between the twelve regions in which world is disaggregated within the model. These regions are formed on the basis of geography and/or the structure of energy demand. More generally, these twelve regions behave strategically with respect to all decision variables by playing an open-loop Nash game. From a top-down perspective, this enables us to analyse both the geographical dimension (e.g. rich vs. poor regions) and the time dimension (e.g. present vs. future generations). All regions determine their optimal strategies by maximising social welfare, with climate damages taken into account. This is consistent with the concern for sustainability which calls for an objective function accounting for the welfare of future generation.

Following the recent research in climate modelling, WITCH incorporates a description of endogenous and induced technical change. If effective policies to mitigate CO<sub>2</sub> are to be implemented, technical change is going to play an increasingly crucial role. As such, integrated assessment and energy modellers, among others Goulder and Mathai (2000), Grubler, Nakicenovic and Victor (1999), have started endogenizing technical change in order to investigate the potential effect of technological change on the costs of climate protection. Results are various, but generally point to a reinforce of the bottom-up vs top-down divide (Clarke and Weyant (2002)).

BU models that are able to include a rich technology representation and account for Learning by Doing (LbD) effects usually find large effects of endogenous technological change (Manne and



Barreto, 2004). The experience process that is represented through LbD is supported by a large empirical body of research within several areas, and has received particular attention in the field of energy technologies.<sup>1</sup> Therefore, it is important to account for the learning effects when studying the potential causes and impacts of technological change.

On the other hand, TD models have traditionally looked at technical innovation from a wider perspective, focusing on the connection between climate policy and its macroeconomic effects. By modelling technical change through energy R&D investments, TD models have considered innovation as an outcome of deliberate investments and have accounted for its opportunity costs.

We believe both approaches deserve to be integrated, if technological change is to be modelled appropriately. For this reason, we developed a framework that allows us to reconcile these two distinct views, and to assess their combined impacts. In the bottom up part of the model we encompass the learning by doing effects by bringing in experience curves for all energy technologies, while, in the top down part, we account for the accumulation of knowledge (via R&D) and for its effects on energy intensity.

The structure of the paper is as follows. In the next section we start by presenting the structure of the model and its general features. Then in its subsections, we describe in detail how we model technology, fuel prices and endogenous technical change. In section 3, we describe the calibration procedure. Section 4 outlines the main features of our baseline scenario. A few concluding remarks are contained in Section 5.

## 2. Model Description

WITCH is a Ramsey-type neoclassical optimal growth model. The model is defined for twelve macro regions listed in Figure 1. Countries of the world are clustered on the basis of geography and/or the structure of energy demand. For each of the twelve macro regions a central planner chooses the optimal paths of the control variables – investments in different capital stocks and in fossil inputs – so as to maximize welfare, defined as the regional present value of per capita consumption.<sup>2</sup> WITCH is a truly dynamic model because at each time step forward looking agents decide simultaneously and strategically with respect to the other decision makers (given the dynamics of the stock variables). At each time step, WITCH does not simply take as given the stock of assets that result from previous times, but rather evaluates if the stock is adequate and optimally adjust it, backward and forward. Any

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<sup>1</sup> The IEA (2000) states that “experience curves demonstrate that investment in the deployment of emerging technologies could drive prices down so as to provide new competitive energy system for CO<sub>2</sub> stabilisation”, page 3.

<sup>2</sup> Population is currently exogenous in the model. The discount factor follows Nordhaus and Boyer (2000). The model equations not listed in the text can be found in the Appendix.

event in the future, such as environmental damage or a rise of fossil fuel prices, has immediate consequences on the present.

Consumption of the single final good is obtained via the economy budget constraint by subtracting total investment spending from net output. Specifically:

$$(1) \quad C(n,t) = Y(n,t) - I_C(n,t) - I_{R\&D}(n,t) - \sum_j I_j(n,t) - \sum_j O\&M_j(n,t),$$

where  $C$  is consumption,  $Y$  is net output and  $I$  denotes investment. The indexes  $n$  and  $t$  denote region  $n$  at time  $t$ . There are eight different types of investment: in capital for final good production,  $I_C$ , in R&D capital,  $I_{R\&D}$ , and in six electricity generation technologies indexed by  $j$ . In particular,  $j$  stands for coal, oil, gas, hydro, nuclear and renewables. We also model operation and maintenance costs ( $O\&M$ ) for electricity generation as an investment that fully depreciates every year. Capital for final good production and R&D capital cumulate following the usual perpetual inventory rule.

Gross output of final good is the result of the technological transformation of three inputs, capital, labor (taken to be equal to population) and energy services (see Figure 2). These are in turn obtained from various energy sources. Among them coal, oil and gas, whose consumption generates CO<sub>2</sub> emissions, computed by applying stoichiometric coefficients. Emissions are fed into a stylized three-boxes climate module (the dynamics is described in Nordhaus and Boyer, 2000) which returns the level of temperature increases relative to pre-industrial levels. Through a damage function the increase in temperature creates a wedge between output gross and net of climate change effects. The damage is nonlinear and can have either a positive or a negative impact on output, possibly switching from one region to the other as time goes by.

The model pays great attention to technological change. In addition to autonomous TFP growth in the final good production process, we allow for endogenous technical change in the form of LbD in electricity generation investment costs and of energy R&D affecting the energy efficiency of the different world regions. Prices of fossil fuels are endogenous and depend on cumulated and current world extraction.

The twelve regions in which we group world countries are not separate islands, but they strategically interact through four relevant channels, which are in turn strictly interdependent among each other. First, at each time period, the prices of oil, coal, gas and uranium depend on world cumulative extraction and on current extraction. Thus, investment decisions, consumption choices and R&D investment in any country at any time period indirectly affect all other countries choices. Since prices of fossil fuel have a strong impact on technology adoption and energy use this is a very important channel of interaction. Consider for example the impact of a massive reduction of oil consumption in the USA and in Europe alone; thanks to the strategic interaction among regions in WITCH, we can study the effect of a lower path of oil prices on the demand and technology adoption

in the rest of the world. We thus incorporate rebound effects not only at regional but also at world level.

Second, at any time period, emissions of CO<sub>2</sub> from each region affect the average world temperature and, through this channel, they impact final output in all other regions. Third, investment decisions in each electricity generation technology, in each country, at each time, affect other regions by changing cumulative world installed capacity which in turns affects investment costs via the Learning by Doing mechanism. Finally, the fourth channel of interaction derives from R&D spillovers that affect the price of the backstop technology. WITCH uses these four channels of interaction<sup>3</sup> to characterise the interdependencies of all countries' climate, energy and technology policies.

In term of the solution algorithm, we model the interactions among world regions as a non-cooperative Nash Game, which is solved recursively and yields an Open Loop Nash Equilibrium. The algorithm works as follows: at each iteration the social planner of every region takes as given the behaviour of other players -which in turn derives from the previous iteration- and sets the optimal value of all choice variables; then, we store the level of the variables to feed the next round of optimization. This continues until each region's behaviour converges in the sense that each region's choice is the best response to all other regions best responses to its behavior, which is a way of characterizing Nash equilibrium. We have found that convergence is rather fast and we tested the uniqueness of the solution by using alternative starting conditions. The algorithm proves to be well constructed because the solution is invariant to different regions orderings.<sup>4</sup>

## 2.1 The Final Good Sector

In each of the twelve regions of the model, net output available for consumption is defined as follows:

$$(2) \quad Y(n, t) = \left\{ TFP(n, t) K_C^{1-\alpha(n)-\beta(n)}(n, t) L^{\beta(n)}(n, t) ES(n, t)^{\alpha(n)} \right\} / \Omega(n, t) - \sum_j \sum_z P_j(n, t) X_{j,z}(n, t) - P_{CCS}(n, t) CCS(n, t)$$

where  $j$  stands for coal, oil, gas, hydro, nuclear and renewables and  $z$  stands for the electric and non-electric energy sectors. According to the standard three-input Cobb-Douglas production function in (2), output is produced by combining capital  $K_C$ , labor  $L$ , and energy services  $ES$ .  $TFP$  represents total factor productivity which evolves exogenously over time.  $\Omega$  is the damage, that is the feedback of climate onto output production. In calculating the net output we have to subtract the expenditure for fossil fuels: we consider it as a net loss for the economy, as if each region was importing fossil fuels

<sup>3</sup> A fifth channel will be operational when the model will be used to analyse the effects of some emission trading schemes. Indeed, when an emission permits market is open, regions interact via this channel which equalizes marginal abatement costs across regions, with all the necessary consequences of this result on R&D effort and investment choices.

<sup>4</sup> Unfortunately, this is not the case with Nordhaus and Yang (1996)'s algorithm.

from abroad, thus paying external factors of productions.  $X_{j,z}$  denotes the total consumption of fuel  $j$  in sector  $z$  with price  $P_j$ . We also model carbon capture and sequestration as a know-how that enables the economy to reduce emissions of CO<sub>2</sub> per unit of fossil fuel used in the electricity generation process.  $CCS$  thus stands for the amount of CO<sub>2</sub> captured from the atmosphere and  $P_{CCS}$  is the corresponding cost that the economy has to pay to the external supplier of  $CCS$  know-how.

## 2.2 The Energy Sector

The energy services factor of production  $ES$  is a combination of energy with cumulated energy R&D. As in Popp (2004a), an increase in R&D,  $HE$ , efforts improves the efficiency with which energy,  $EN$ , is translated into energy services,  $ES$ , e.g. more efficient car engines, trains, technical equipment or light bulbs. Again, we refer to Figure 2 for a diagrammatic description of the energy sector and its composing technologies. Energy  $EN$  is an aggregate of electric,  $EL$ , and non-electric energy,  $NEL$ . Contrary to what specified in other top-down growth models – such as DEMETER (Gerlagh and van der Zwaan, 2004a) and MIND (Edenhofer, Bauer, and Kriegler, 2005) – in WITCH the whole energy demand does not coincide with electricity. Indeed, other economic activities use petrol, coal or gas directly, besides electricity. So, for instance, to the extent that they are not equipped with electric engines, trucks use diesel to move, not electric sources. In our opinion this is a needed distinction as reducing emissions is traditionally more challenging in the non-electric sector, and its neglecting would seriously over-estimate the potential GHG control achievements. The distinction between electric and non-electric energy is clearly relevant if we consider that: (i) non-electric energy accounts for about 70% of total energy demand in industrialised countries and about 80% in Developing Countries, with China ranging in the middle at about 75%; (ii) non-electric energy includes the consumption of fossil fuels in the transport sector, which is the most fast growing source of energy demand, both in industrialised and developing economies.

Non-electric energy is obtained by combining in fixed proportions coal on the one hand and an oil-gas aggregate on the other. The use of coal in non-electric energy production is quite small and limited to a few world regions. The oil-gas aggregate in turn combines oil and natural gas sources according to a CES law. Oil is used mostly for transportation, natural gas mostly for heating purposes. Hence a modest substitutability is allowed between the two sources.

The equations representing the electric sector require a more detailed illustration. In WITCH we group electricity generation technologies into three big families: (i) fossil fuel-based generation, which includes thermoelectric plants using coal, oil and natural gas, to which we have added a backstop technology ( $FFB$ ); (ii) “traditional non fossil” generation, produced using nuclear and hydroelectric plants (to which we have added geothermal plants) ( $TNF$ ), and (iii) new carbon-free technologies, including wind turbines and photovoltaic panels ( $ELREN$ ).

In the production of electricity, substitution possibilities are contemplated among the three aforementioned aggregates, between the use of hydroelectric power (*ELHYDRO*) and of nuclear (*ELNUKE*) power in the generation of *TNF*, and in the combination of coal (*ELCOAL*), oil (*ELOIL*), and gas (*ELGAS*) to generate the fossil fuel aggregate (*FF*). For reasons explained below, we assume perfect substitutability between the backstop technology (*BACKSTOP*) and the fossil fuel aggregate (*FF*) when obtaining *FFB*.

For each technology  $j$  (renewables, hydroelectric, nuclear, coal, oil and gas), at time  $t$  and in each region  $n$ , electricity is obtained by combining in fixed proportions three factors: i) the installed power generation capacity ( $K$ ), ii) operation and maintenance equipment ( $O\&M$ ) and iii) fuel resources consumption ( $X$ ), when needed. The Leontief technology is then as follows:

$$(3) \quad EL_j(n, t) = \min\{\mu_{n,j} K_j(n, t); \tau_{n,j} O\&M_j(n, t); \zeta_j X_{j,EL}(n, t)\}.$$

The parameters governing the production function take into account the technical features of each power production technology.  $\mu$  translates power capacity (i.e. TW) into electricity generation (i.e. TWh) through the utilization rate (hours per year), which allows us to take into consideration the fact that some technologies, noticeably renewables such as wind power, are less continuous than others.  $\tau$  differentiates operation and maintenance costs among technologies, i.e. nuclear power is more expensive to run and maintain than natural gas combined cycle (NGCC). Finally, the parameter  $\zeta$  measures (the reciprocal of) power plants fuel efficiencies and returns us the quantity of fuel needed to produce a KWh of electricity. *ELHYDRO* and *ELREN* are assumed to have efficiency equal to one, as they do not consume any fuel, which reduces to assume a two factors Leontief production function.

It is important to stress the fact that the power generation capacity is not equivalent to the cumulated investment in that specific technology: different plants have different investment costs in terms of final output, i.e.:

$$(4) \quad K_j(n, t+1) = K_j(n, t)(1 - \delta_j) + \frac{I_j(n, t)}{SC_j(n, t)}.$$

where  $\delta_j$  is the rate of depreciation and  $SC_j$  is the price, in terms of final good, of installing power generation capacity of type  $j$ , which, as we will discuss more in-depth in the next section, is time and region specific. It is worth noting that the depreciation rates  $\delta_j$  are set consistently with the power plants lifetime, so that again we are able to incorporate the technical specifications of each different electricity production technology.

A crucial feature of WITCH is that the price of electricity generation is endogenously derived within the model. Let us explain how. In neoclassical optimal growth models, households supply labour and own assets of firms; in return they are paid a wage and a rental rate of capital at least equal to the interest rate that they could receive from bank deposits. Factors are paid their marginal product



and exhaust total gross output.<sup>5</sup> Households save up to the point at which the marginal utility of consumption in the present equals the discounted marginal utility of additional consumption made available by new investments in the future. Once investment decisions are made, efficiency dictates that they are allocated in such a way as to yield the same marginal product, i.e. the same return, in all sectors. Thus, the marginal benefit of devoting one more unit of investment to production of electricity through nuclear power plants must necessarily be equal to the marginal product of investment in final good capital. Otherwise investment would not occur. Keeping this in mind, it is possible to understand how prices of electricity are endogenously determined if we look at how we have constructed the energy (*EN*) input in WITCH.

We have modeled the energy sector as close as possible to an energy balance sheet where at successive stages we construct total energy supply in terms of thermal units. For this reason we express power plants capital in TW and the output of the Leontief function as electricity, measured in TWh. The result is that WITCH offers a detailed and intuitive view of how energy is supplied to the economy and the analysis gains in transparency. However, if we look at equation (4), which describes accumulation of electricity generation in power plants units, it is clear that we could equivalently represent the energy nest in terms of final good invested in electricity production. Energy is nothing else than capital cumulated over time in power plants and capital invested for fuels and O&M, which fully depreciates every period.<sup>6</sup>

Thus, WITCH calculates the cost of electricity generation, and of the energy input in general, as the sum of the shadow value of lost consumption for investing in plants, for paying O&M costs and fossil fuels. Since investment costs, O&M costs, fuel efficiency for each technology and fuel prices are region specific, we obtain a high degree of realism in constructing relative prices of different ways of producing electricity in the twelve regions considered. An important feature is that while the expenditure on fossil fuels, as well as O&M costs, are completely depleted each year, plants cumulate over time. Thus, even if in a single time period the price of one KWh of electricity from gas fired power plants and from wind turbines is the same,<sup>7</sup> WITCH, by optimizing intertemporally the allocation of resources, calculates a lower internal cost of generating electricity from renewable sources than from gas. The model tends to prefer capital intensive rather than fuel intensive electricity production.<sup>8</sup>

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<sup>5</sup> This is true if the production function is homogeneous of degree one in all inputs, which is our case; the property follows directly from the application of Euler's Theorem.

<sup>6</sup> Things get more complex as capital invested in a technology cumulates and LbD makes investment less expensive. Even if new plants are physically identical to the old ones, the new investment is more productive, in terms of electricity generation. As a consequence, the system will allocate more resources to the technologies with the fastest decreasing investment cost.

<sup>7</sup> We refer here to the price calculated as the sum of the internal cost of capital invested in plants, O&M and fuels.

<sup>8</sup> Together with the O&M costs which will follow and the projected life of the power plant.



Let us now look at another interesting feature of investment decisions that directly follows from the intertemporal optimal growth framework of WITCH. Once a power plant has been built, it works at full capacity for its entire life and depreciates at a constant rate  $\delta_j$ . Since the operations of generating electricity from a given capacity requires an annual expenditure in O&M costs and in fuels (when needed) and it cannot be reversed from one technology to the other, investment choices are forward looking. In particular, given the generally long life of power plants, future evolutions of investment costs and fossil fuels prices have a strong influence in determining present investment decision.

Four natural resources are employed in electricity generation: coal, crude oil, natural gas and uranium. They are non-renewable resources whose price responds to the short-term characteristics of supply and demand and obeys to a long-term trend that reflects their exhaustibility. We abstract from short term fluctuations and we model fossil fuel prices using a reduced-form cost function that allows for non-linearity in both the depletion effect and in the rate of extraction (see Hansen, Epple and Roberds, 1985). Namely:

$$(5) \quad c_j(n, t) = q_j \left( \chi_{j,n} + \gamma_j q_j(t) + \pi_j \left[ \frac{Q_j(t-1)}{\bar{Q}_j} \right]^{\psi_j} \right),$$

where  $c$  is the cost,  $q$  is the current extraction rate,  $Q$  is cumulative extraction,  $\gamma$  measures economies of scale in current extraction and  $\chi_n$  is a region specific markup. Index  $j$  stands for coal, oil, gas, hydro, nuclear and renewables refers to coal, oil, and gas.  $\bar{Q}$  is the point at which scarcity starts rocketing the price of fossil fuel.<sup>9</sup> Thus, with the assumption of competitive markets, price  $P_j$  is equal to marginal cost:

$$(6) \quad \begin{aligned} P_j(n, t) &= \chi_{j,n} + 2\gamma_j q_j(t) + \pi_j \left[ \frac{Q_j(t-1)}{\bar{Q}_j} \right]^{\psi_j}, \\ Q_j(t-1) &= Q_{0j} + \sum_{s=0}^{t-1} \sum_n X_j(n, s) \end{aligned}$$

where the second expression represents cumulative extraction.

As noted above, we also allow for a backstop technology in carbon-free energy production. This is a technology whose price declines as R&D is performed. It enters the carbon-free nest linearly. Therefore, if its price becomes lower than the other fossil fuel prices, the backstop technology substitutes out all other sources of electricity generation from fossil fuels. Notice that in our model the backstop technology does not coincide with solar or wind electricity generation nor with advanced nuclear, since these technologies already have dedicated channels to enter the production function, but rather with something close to nuclear fusion or to some other major innovation still yet to come. The price of electricity produced using the backstop technology  $P_B$  evolves according to a rule which we adapt from Popp (2004b):

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<sup>9</sup> WITCH therefore exhibits a higher degree of realism and transparency than previous optimal growth models also thanks to a familiar price system of natural resources: USD per barrel of crude oil, USD per metric tonne of coal, USD per boe (barrel of oil equivalent) of natural gas and USD per Kg of uranium.

$$(7) \quad P_B(t) = \frac{P_B^0}{v^{\vartheta} \sum_n K_{R\&D}(n, t)^{\vartheta}}.$$

Finally, WITCH offers the rare possibility, at least in optimal growth models, of tracing with precision consumption of different fossil fuels. Thus, GHGs emissions from combustion of fossil fuels can easily be derived by applying the appropriate stochiometric coefficients to the total amount of fossil fuels burnt each year. Even though we presently use a climate module that is reactive only to CO<sub>2</sub> emissions, a multi-gas climate module can easily be incorporated in WITCH. In addition, we can introduce gas-specific emissions ceilings and study how the system reacts to them. For each region  $n$  CO<sub>2</sub> emissions are derived as follows:

$$(8) \quad CO_2(n, t) = \sum_z \sum_i \omega_{i, CO_2} X_{i,z}(n, t) - CCS(n, t),$$

where  $\omega_{i, CO_2}$  are the stochiometric coefficients and  $CCS$  stands for the amount of CO<sub>2</sub> captured and sequestered from the atmosphere.

### 2.3 Endogenous Technical Change

In the Introduction we stressed the importance of incorporating endogenous technical change, and we have noticed how bottom-up and top-down approaches differently portray it, thus leading to different impacts. WITCH, thanks to its hybrid nature, allows us to bring together these two distinct views and to account for both Learning by Doing (LbD) and R&D investment effects.

First, we incorporate LbD effects in electricity production, and are thus able to reproduce the observed empirical relation for which the investment cost of a given technology decreases with accumulation of knowledge represented by cumulated installed capacity. This representation has proved a strong explanatory power in areas such as the renewable energy sector, where, for example, the installation costs of wind turbines have steadily declined at a constant rate. Learning rates depend on a variety of factors – not least of public nature – and vary considerably across countries. In our framework we resorted to use world learning curves, where investment costs decline with the world installed capacity. That is, we assume perfect technology spillovers and constant learning rates across countries, which is fairly reasonable considering that any time step in the model corresponds to five years.

In the learning curves, the cumulative (installed) world capacity is used as a proxy for the accrual of knowledge that affects the investment cost of a given technology,  $j$ :

$$(9) \quad SC_j(n, t) = B_j(n) \sum_i \sum_n K_j(t-1)^{-\log_2 PR_j} + \xi_n,$$

where  $\xi_n$  is a regional markup and  $PR$  is the progress ratio that defines the speed of learning. With every doubling of cumulative capacity the ratio of the new investment cost to its original value is constant and equal to  $PR$ , until a fixed floor level is reached. The decline in investment cost subsequently translates into an increase in capital productivity.

By having several electricity production technologies, the model is given the flexibility to change the power production mix and invest in the more appropriate technology for each given climate policy, thus creating the conditions to foster the learning by doing effects for the clean but yet too pricey electricity production techniques.<sup>10</sup>

Second, we introduce energy R&D as a device for increasing energy efficiency. Following Popp (2004a) technological advances are captured by a stock of knowledge that aggregates with energy in a constant elasticity of substitution (CES) function, and thus stimulates energy efficiency improvements:

$$(10) \quad E(n, t) = \left[ \alpha_H HE(n, t)^\rho + \alpha_{EN} EN(n, t)^\rho \right]^{1/\rho}.$$

The stock of knowledge  $HE(n, t)$  derives from energy R&D investments through an innovation possibility frontier that models diminishing returns to research at any given time and across time periods, and depreciates similarly to a physical stock :

$$(11) \quad HE(n, t+1) = a I_{R\&D}(n, t)^b HE(n, t)^c + HE(n, t)(1 - \delta_{R\&D}),$$

$\delta_{R\&D}$  being the depreciation rate of knowledge.

As social returns are found to be higher than private one in the case of R&D, the positive externality of knowledge creation is accounted for by assuming that the return on energy R&D investment is four times higher than the one in physical capital. At the same time, the opportunity cost of crowding out other forms of R&D is obtained by subtracting four dollars of private investment from the physical capital stock for each dollar of R&D crowded out by energy R&D,  $\psi_{R\&D}$ , so that the net capital stock for final good production becomes:

$$(12) \quad K_C(n, t+1) = K_C(n, t)(1 - \delta_C) + (I_C(n, t) - 4\psi_{R\&D} I_{R\&D}(n, t)),$$

where  $\delta_C$  is the depreciation rate of the physical capital stock.

### 3. Calibration

Model complexity comes at the cost of increased calibration efforts. The high number of regions and the detailed structure of the energy sector require particular attention in setting all the parameters.

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<sup>10</sup> In a future extension of the model, the learning curves will be extended into two-factor learning curves, in which both learning by researching and learning by doing are taken into account.

To identify their values in the absence of statistical data to perform econometric analyses, we relied on existing literature (where available) and on expertise otherwise.

We follow Nordhaus and Boyer (2000) to calibrate the general structure of the model and of the climate module. The model is calibrated on economic and energy data for the base year 2002. All energy data are obtained from the ENERDATA (2005) and IEA databases. Output and population assumptions are adapted respectively from the World Bank (2004) and the Common POLES IMAGE (CPI) baseline (van Vuuren *et al.* (2004)).

The main production function is a Cobb-Douglas, where we have set the factor share of labour to 0.7 in accordance to standard literature. The elasticities of substitution of the energy sector are the most uncertain parameters to be set. Figures are reported in Figure 2 in the Appendix. Electric and non-electric energies are aggregated using a moderate elasticity of 0.4. Although possibilities of switching from direct energy to electricity exist in many sectors –a good example is the housing heating and cooking systems– lock-in in past investments and large up-front costs make the two forms of energy only slightly substitutable.

For electricity production, we use an elasticity of 3. This way the electricity produced via different technologies is assumed to be substitutable, although imperfectly. In doing so, we are able to take into account the different characteristics of the various power production technologies (flexibility, load) and allow for niche markets. At the bottom of the electricity sector, we need to determine the amount of capital, fuels and O&M costs that are necessary for electricity production. The Leontief functions serve this scope by aggregating inputs in fixed proportions, which are in turn determined by plants operating hours, fuel efficiencies and O&M costs. Data for power plant technical specifications are taken from NEA/IEA (1998 and 2005), and are region specific though are held constant through time. The same Leontief functional form is used in aggregating the coal and oil-gas nests in the non-electric energy tree.

All other parameters of the production function and of the energy sector are calibrated to reproduce the base year data, to respect the energy balances and to equate marginal products to their respective prices. As we mentioned above, electricity is ultimately produced by investment in the energy sector; the price of electricity can thus be derived as the sum of the remuneration of capital employed in its production. We have assumed that all capital invested in plants for electricity production has the same productivity (which is equal to say that the cost of all plants using the same technology is the same, as well as the efficiency and O&M costs) and we have calculated the rental rate of capital as the sum of the depreciation rate plus the interest rate from bank deposits, which is assumed to be region specific.

Costs for new investments and maintenance in power generation are adopted from NEA/IEA (1998 and 2005). Investment costs decline with cumulated installed capacity at the rate set by the learning curve progress ratios. For the technology specification currently represented in the model, we have assumed that learning occurs in the renewable electricity production only, at the progress ratio of 0.87.

We calibrate energy R&D as in Popp (2004a). Parameters of the CES function between energy and knowledge and of the innovation possibility frontier are chosen so to be consistent with historical levels, to reproduce the elasticity of energy R&D with energy prices and to achieve a return 4 times the one of physical capital, in order to account for the positive externality of knowledge creation.

Similarly, the effectiveness of investments in the carbon free backstop technology follows closely that in Popp (2004b).

Concerning CCS technologies, we adopt the rule of thumb assumption that costs are quadratic in the level of effort with a linear component of 10 USD/tC and marginal costs increasing up to 400 USD, as proposed by Gerlagh and van der Zwaan (2004b). We take 10 GtC per year as the maximum volume of CCS (corresponding to the marginal cost of 400 USD/tC). Besides, no leakages or auto-consumption of energy are assumed for CCS.

The climate module is adopted from Nordhaus and Boyer (2000). Figures have been adjusted for the different time step length (5 years in our formulation) and initial base year.

Capital invested in output production and R&D depreciates at a rate of 10% per year. Depreciation of investments in electricity production is set in agreement with plant lifetimes. The depreciation rate for the natural resources and O&M is assumed to be equal to 1, as if consumption corresponded to one period complete depreciation.

Intertemporal discount is set equal to 3%. Interest rates are 5% for industrialized regions (USA, OLDEURO, NEWEURO, KOSAU, CAJANZ) countries and 7% for the others.

As in Nordhaus and Boyer (2000), the total factor productivity is assumed to exogenously grow over time to reflect technological progress. The exponential trend is calibrated to fit the output projection underlying the Common POLES IMAGE (CPI) baseline (van Vuuren *et al.* (2004)).

We calibrate endogenous extraction cost functions for oil and coal. We set coefficients on current extraction ( $\gamma$ ) equal to zero because in the construction of the baseline we were more interested on long run dynamics of fuels extraction rather than on short term frictions in the markets.

As for oil, we use data on total ultimately recoverable resources of oil from IEA (2004) and we set them equal to 3345 billion barrels;  $\bar{Q}_{OIL}$  is equal to  $\frac{3}{4}$  of total ultimately recoverable resources.  $\bar{Q}_{OIL}$  grows at an exogenous constant rate of 1% per year. By allowing total resources not to be finite we stabilize prices of oil in the long run. The cumulative extraction component is assumed quadratic. The marginal extraction cost  $\chi_{j,n}$  is set equal to 14.3 US\$ per barrel, plus a regional markup to take

care of transportation and other factors that affect the price of oil. By using a base year international oil price of 21 US1995\$ per barrel we computed the value of  $\pi_{OIL}$ .

Coal extraction cost function is calibrated similarly; we compute total ultimately recoverable resources using data from IEA (2004) and ENERDATA (2005). The cumulative extraction component is quadratic and scarcity becomes relevant when  $\frac{3}{4}$  of resources have been depleted; resources grow at an exogenous rate of 0.1 % per year. We use a base year international price of 35 US1995\$ per tonne of coal, to which we added regional mark-ups.

Gas prices are derived by applying additive regional mark ups to prices of oil, thus assuming a perfect correlation of oil to gas prices. Our choice incorporates the analysis of gas markets contained in IEA (2004).

Uranium extraction cost grows at an initial exogenous rate of 5% per (5 year) period which declines in later periods. Uranium reserves are indeed limited, but we expect the emergence of advanced nuclear reactors characterised by a fuel cycle that reclaims energy contained in spent fuel - thus greatly extending the availability of nuclear fuel.

#### 4. WITCH Business as Usual Scenario

Given the model parameters and dynamic path of the exogenous variables described above, WITCH has been used to maximise welfare in all world regions. As consequence, the optimal intertemporal values of all control variables have been computed by solving a dynamic open loop Nash game. Let us recall that the control variables are investments in all energy technologies, in physical capital and in R&D. The time horizon of the optimisation runs is 2100. Optimal investments have been computed for all control variables (9), all regions (12), and all periods (20). The algorithm was briefly described in section 3.<sup>11</sup>

Given the optimal values of the control variables, the model, i.e. the dynamic equations that constitute the structure of WITCH, yield the optimal time path of a large set of endogenous variables, which include economic, technological and climate variables. These time paths define our business-as-usual (BAU) scenario.

Let us start by describing the macroeconomic features of our BAU scenario. Figure 3 (see the Appendix) shows the dynamics of GDP in the twelve world regions. World output is 34.6 trillions in 2002 and grows to 74 trillions in 2030 to reach 193 trillions in 2100, almost a six fold increase; the average annual world output growth rate is 3.5 in 2002, 1.9% in 2030 and 1.2% in 2100. USA, OLDEURO, CAJANZ have mature economies that grow at a decreasing rate and approach their

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<sup>11</sup> As previously mentioned, numerical optimisation runs have also been used to calibrate the parameters for which we could not have statistical information or experts' judgements.



steady state level. Their share of world GDP decreases from 75% at 2002 to 60% in 2030 and finally reaches 31% in 2100. Fast growth is registered by all developing economies except for Sub Saharan Africa whose share of world output remains negligible. The reason for this result is that Sub Saharan Africa economy is still prevalently based on non market activities that are not well accounted for in the classical optimal growth framework we adopted for WITCH. While output expansion for the first 30 years is in line with projections of other institutions, it is possible that we underestimate market based output in the very long run due to the difficulties of modelling the structural break that instead already became visible in other developing economies.

Figure 4 shows the dynamics of total investments (left hand scale) and energy R&D investments (right hand scale). In the BAU, energy R&D investments are between 0% and 0.3% of total investments (the smallest value is in region SSA, the highest in USA). They represent between 0% and 0.1% of total GDP (again, in the same regions). This information is important because in WITCH technical change is endogenous and reacts to climate policy. Therefore, one of the applications of the model would be to analyse the changes in energy R&D investments and in the adoption of new technologies induced by the implementation of a given climate policy (e.g. a stabilisation target or a permit market).

Another related important information on the properties of our BAU is revealed by Figure 5, which shows the future amount of electricity produced using the different energy technologies modelled in WITCH. In our baseline scenario, increasing shares of coal and renewables will be used to produce electricity. The share of renewable energy is expected to become 4.7% in 2100, whereas the share of coal will be 51% (from 38.1% in 2002). Nuclear energy (the optimal amount of nuclear energy) is also going to increase slightly (18% from 16.9% in 2002).

There are several factors that explain these results. First the model is forward looking. Therefore, when making investment decisions, investors in different countries take into account the expected prices of energy sources in future periods. As a consequence, they take into account the future high costs of oil and gas and the present and future high costs (including social and political ones) of nuclear energy.

The rapid increase in the use of coal based technologies can be explained both by the aforementioned future cost differentials and by two environmental factors: (i) the absence in the BAU of any climate policy, and (ii) the low impact that future changes in climate impacts have on the present and discount values of GDP and consumption. Therefore, in our baseline, decision makers have little incentive to internalise the externalities produced by coal consumption.

Notwithstanding this non environment-friendly projected evolution of the energy-mix, energy policy in all countries has some positive environmental features. Indeed, investors take into account the increasing costs of energy sources and therefore reduce the amount of energy per unit of output over time. Consider, for example, Figure 6 (see the Appendix again), which shows aggregate energy

intensity in all world regions. The dynamics of energy intensity clearly suggest a strong future reduction of energy per unit of output and, most importantly, a convergence of all world regions to very similar values of energy intensity.

A more detailed information on the energy sector in WITCH is provided by Figure 7 and 8, which display the regional disaggregation of electricity technologies. The two figures show, among other things, that gas will no longer be used to produce electricity in developed countries in 2100, whereas in these countries nuclear energy will expand. In “old” European countries there will be a considerable effort on renewables that the model does not predict for the other world regions. Coal will mainly be used in the US, in the developing countries and in the “new” European countries.

Some information on the environmental features of our BAU is provided by Figures 9 and 10. Here results from WITCH are very similar to those from other models. Total CO<sub>2</sub> emissions will increase in the next century and the increase will be large in developing countries above all. The developed countries’ efforts to control CO<sub>2</sub> emissions will be offset by the emission increase in developing countries. Given the aforementioned dynamics of the energy-mix, emission increase in the BAU is mainly explained by the increased use of coal to produce electricity, again in developing countries above all.

Let us stress two important features of our BAU that are behind the results just described. First, in the BAU, no carbon capture and sequestration (CCS) technology is adopted in the different world regions. The reason is that, at current and expected prices of energy sources and given the investment costs, the present and future costs of CCS make it non-profitable in the absence of any climate policy.<sup>12</sup> Second, and for the same reasons, no backstop technology becomes profitable in the BAU scenario in the absence of any climate policy. Therefore, there is limited scope for emission reductions in our BAU. The only incentives to reduce emissions come from the increased price of energy. This explains the fall of energy intensity. However, there are little incentives (i) to modify the fuel-mix, and (ii) to foster investments in new technologies and in climate-friendly R&D.

Finally, let us consider the dynamics of technical change the BAU scenario. From Figure 11, we can say that investments in new energy technologies will increase over time and that this increase is larger in the BAU scenario (BSL in the figure) than in the scenarios in which no endogenous technical change is considered (in Figure 11, the NO ITC scenario, where no R&D investments and no LbD are allowed for). Notice that the presence of LbD and of international learning spillovers reduce the incentives to invest in new energy technologies. This is due to the several channels of interaction across regions that the model considers and to the free-riding incentives that characterise the game. Indeed, given that a given player benefits from the other players’ investments in new technologies,

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<sup>12</sup> Let us recall that we use the cost parameter proposed by Gerlagh and van der Zwaan (2004b).

his/her incentives to pay for the investment costs become lower the larger the spillovers and the LbD effects.

## 5. Conclusions

This paper has presented the main characteristics and properties of a new model designed to be used for climate policy analysis. This model, called WITCH (World Induced Technical Change Hybrid), is a bottom-up energy model integrated with a top-down macro model. The model possesses some interesting features. It contains a detailed specification of energy investments. These investments are the outcome of a dynamic open loop Nash game with perfect foresight. Investments depend on the dynamics of technical change, which is its self endogenous and depends on investment paths as well as on prices and other economic and climatic variables (including climate policy). Investment decisions in one country depend on those in the other countries, given the several interdependency channels specified in the model.

The model has been calibrated using the available information on the model parameters (taken from the existing literature or from expert judgement). The remaining parameters, for which no or little information was available, have been calibrated to yield a baseline which replicates the expected dynamics of the main macro variables of the model (e.g. GDP and investments).

This paper briefly describes the properties of the baseline scenario produced by WITCH. It must be stressed that in our BAU scenario, free-riding incentives characterise the development and adoption of new climate-friendly technologies (e.g. the use of renewables for electricity production, carbon sequestration or clean energy from fossil fuels). Even though the model explicitly allows for the possible use of carbon sequestration and/or of a backstop technology, in the absence of any climate policy, all investors in all world regions do not find it convenient to adopt them. For the same reasons, the increase of climate friendly R&D investment is limited. As a consequence, the fuel-mix remains fairly stable over the time (the only significant change is an increase of coal use in electricity production).

Therefore, the BAU produced by WITCH is fairly conservative. The many options that WITCH allows for to reduce emissions are not switched on in our BAU. Other incentives, and in particular climate policy, would be necessary.<sup>13</sup> It is thus crucial to analyse what would be the impacts of different climate policies (e.g. stabilisation targets or emission trading) in WITCH. Given the many channels of transmission of climate policy into the economic system (from forward looking investments to learning by doing, from energy R&D expenditure to technological spillovers, etc.), climate policy is likely to have an important impact of the dynamics of the main economic variables in

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<sup>13</sup> Another reason which explains this result is that WITCH, despite the important reduction in energy intensity in the BAU, does not over estimate energy saving in the transport sector.

WITCH. Under what conditions can climate policy achieve the goal of stabilising GHG concentrations? What are the features of an optimal climate policy? How much would it be technology-based? All these above are issues and questions that WITCH can easily address and that will be the subject of future applications of the model.

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## Appendix

### Model Equations:

In this appendix we reproduce the main equations of the model. The list of variables is reported at the end of the Appendix. In each region, indexed by  $n$ , a social planner maximizes the following utility function

$$(A1) \quad W(n) = \sum_t U[C(n,t), L(n,t)]R(t) = \sum_t L(n,t) \{\log[c(n,t)]\}R(t),$$

where  $t$  are 5-years time spans and the pure time preference discount factor is given by:

$$(A2) \quad R(t) = \prod_{v=0}^t [1 + \rho(v)]^{-5},$$

whereas the pure rate of time preference  $\rho(v)$  is assumed to decline over time. Moreover,  $c(n,t) = \frac{C(n,t)}{L(n,t)}$  is per capita consumption.

### Economic module

The budget constraint defines consumption as net output minus investments:

$$(A3) \quad C(n,t) = Y(n,t) - I_C(n,t) - I_{R\&D}(n,t) - \sum_j I_j(n,t) - \sum_j O\&M_j(n,t).$$

Net output is produced through the Cobb-Douglas function to which we subtract the cost of the natural resources and CCS ( $j$  indexes technologies,  $z$  electric and non-electric energy):

$$(A4) \quad Y(n,t) = \left\{ TFP(n,t) K_C^{1-\alpha(n)-\beta(n)}(n,t) L^{\beta(n)}(n,t) ES(n,t)^{\alpha(n)} \right\} / \Omega(n,t) - \sum_j \sum_z P_j(n,t) X_{j,z}(n,t) - P_{CCS}(n,t) CCS(n,t)$$

$TFP(n,t)$  evolves exogenously with time. Final good capital accumulates as:

$$(A5) \quad K_C(n,t+1) = K_C(n,t)(1 - \delta_C) + (I_C(n,t) - \psi_{R\&D} I_{R\&D}(n,t)).$$

Labour is assumed to be equal to population and evolves exogenously. Energy services is an aggregate of energy and a stock of knowledge through a CES function:

$$(A6) \quad ES(n, t) = \left[ \alpha_H HE(n, t)^{\rho_{ES}} + \alpha_{EN} EN(n, t)^{\rho_{ES}} \right]^{\frac{1}{\rho_{ES}}}.$$

The stock of knowledge  $HE(n, t)$  derives from energy R&D investments:

$$(A7) \quad HE(n, t+1) = a I_{R\&D}(n, t)^b HE(n, t)^c + HE(n, t)(1 - \delta_{R\&D}).$$

Energy is a combination of electric and non-electric energy:

$$(A8) \quad EN(n, t) = \left[ \alpha_{EL} EL(n, t)^{\rho_{EN}} + \alpha_{NEL} NEL(n, t)^{\rho_{EN}} \right]^{\frac{1}{\rho_{EN}}}.$$

Each factor is further decomposed into several sub-components. Figure 2 shows a graphical illustration of the energy sector. Factors are aggregated using CES, linear and Leontief production functions. For example the backstop technology enters the carbon-free nest linearly, thus substituting out all other sources of electricity generation from fossil fuels when its price is the lowest one. Its price evolves as:

$$(A9) \quad P_B(t) = \frac{P_B^0}{\vartheta \sum_n HE(n, t)^\eta}.$$

For illustrative purposes, let us show how electricity is produced via capital, operation and maintenance and resource use through a zero-elasticity Leontief aggregate:

$$(A10) \quad EL_j(n, t) = \min \{ \mu_{n,j} K_j(n, t); \tau_{n,j} O\&M_j(n, t); \varsigma_j X_{j,EL}(n, t) \}.$$

Capital for electricity production technology accumulates in the usual way:

$$(A11) \quad K_j(n, t+1) = K_j(n, t)(1 - \delta_j) + \frac{I_j(n, t)}{SC_j(n, t)},$$

where the new capital investment cost  $SC(n, t)$  decrease with the world cumulated installed capacity by means of learning by doing:

$$(A12) \quad SC_j(n, t) = B_j(n) \left[ \sum_t \sum_n K_j(n, t) \right]^{-\log_2 PR_j} + \xi_n.$$

Operation and maintenance is treated like an investment that fully depreciates every year. The resources employed in electricity production are subtracted from output in equation (A4). Their prices are calculated endogenously using a reduced-form cost function that allows for non-linearity in both the depletion effect and in the rate of extraction:

$$(A13) \quad P_j(n, t) = \chi_{j,n} + 2\gamma_j q_j(t) + \pi_j [Q_j(t-1)/\bar{Q}_j]^{\psi_j},$$

where  $q$  is the extraction rate and  $Q$  the cumulative extraction:

$$(A14) \quad Q_j(t-1) = Q_{0j} + \sum_{s=0}^{t-1} \sum_n X_j(n, s).$$

*Climate Module:*

GHGs emissions from combustion of fossil fuels are derived by applying the stoichiometric coefficients to the total amount of fossil fuels utilized minus the amount of CO<sub>2</sub> sequestered:

$$(A15) \quad CO_2(n, t) = \sum_i \sum_n \omega_{i,CO_2} X_{i,n}(n, t) - CCS(n, t).$$

The damage function impacting output varies with global temperature:

$$(A16) \quad \Omega(n, t) = \frac{1}{1 + (\theta_{1,n} T(t) + \theta_{2,n} T(t)^2)}.$$

Temperature increases through augmented radiating forcing  $F(t)$ :

$$(A17) \quad T(t+1) = T(t) + \sigma_1 \{F(t+1) - \lambda T(t) - \sigma_2 [T(t) - T_{LO}(t)]\}.$$

It depends on CO<sub>2</sub> concentrations:

$$(A18) \quad F(t) = \eta \left\{ \log \left[ M_{AT}(t) / M_{AT}^{PI} \right] - \log(2) \right\} + O(t),$$

caused by emissions from fuel combustion and land use and change:

$$(A19) M_{AT}(t+1) = \sum_n [CO_2(n, t) + LU_j(t)] + \phi_{11}M_{AT}(t) + \phi_{21}M_{UP}(t),$$

$$(A20) M_{UP}(t+1) = \phi_{22}M_{UP}(t) + \phi_{12}M_{AT}(t) + \phi_{32}M_{LO}(t),$$

$$(A21) M_{LO}(t+1) = \phi_{33}M_{LO}(t) + \phi_{23}M_{UP}(t).$$

**List of variables:**

$W$  = welfare

$U$  = instantaneous utility

$C$  = consumption

$c$  = per-capita consumption

$L$  = population

$R$  = discount factor

$Y$  = production

$I_c$  = investment in final good

$I_{R\&D}$  = investment in energy R&D

$I_j$  = investment in technology  $j$

$O\&M$  = investment in operation and maintenance

$TFP$  = total factor productivity

$K_c$  = final good stock of capital

$ES$  = energy services

$\Omega$  = damage

$P_j$  = fossil fuel prices

$X_j$  = fuel resources

$P_{CCS}$  = price of CCS

$CCS$  = CO2 sequestred

$HE$  = energy knowledge

$EN$  = energy

$EL$  = electric energy

$NEL$  = non-electric energy

$K_j$  = stock of capital of technology j

$SC_j$  = investment cost

$CO_2$  = emissions from combustion of fossil fuels

$M_{AT}$  = atmospheric  $CO_2$  concentrations

$LU$  = land-use carbon emissions

$M_{UP}$  = upper oceans/biosphere  $CO_2$  concentrations

$M_{LO}$  = lower oceans  $CO_2$  concentrations

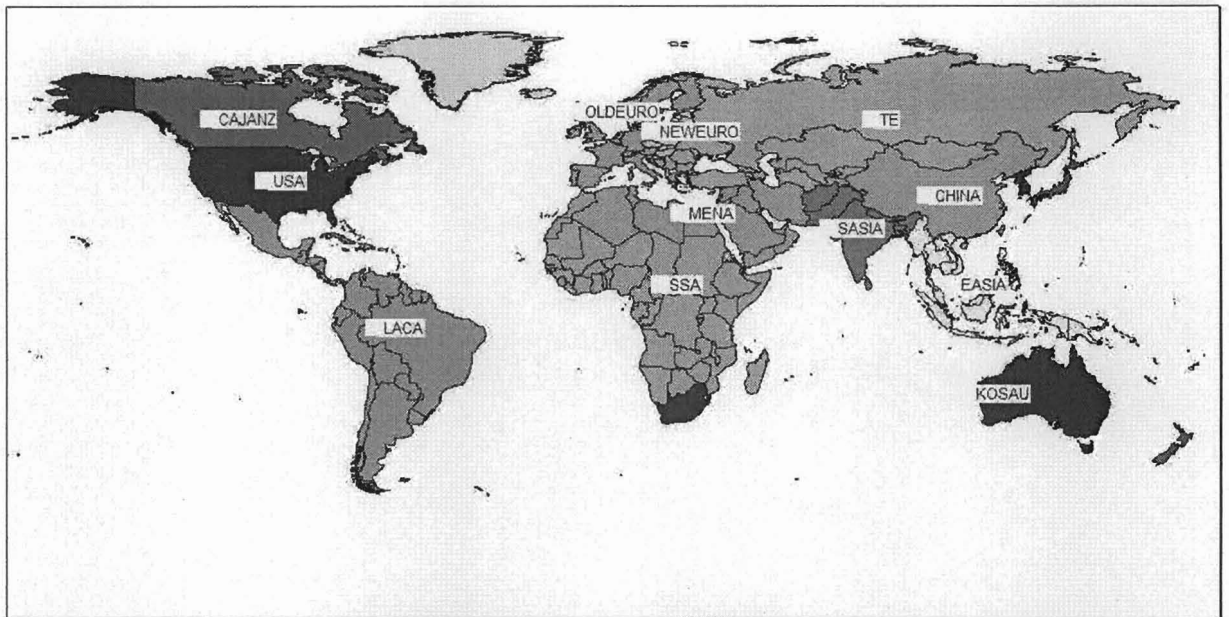
$F$  = radiative forcing

$T$  = temperature level

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**Figures and Tables:**

**Figure 1: Regions of the WITCH Model**

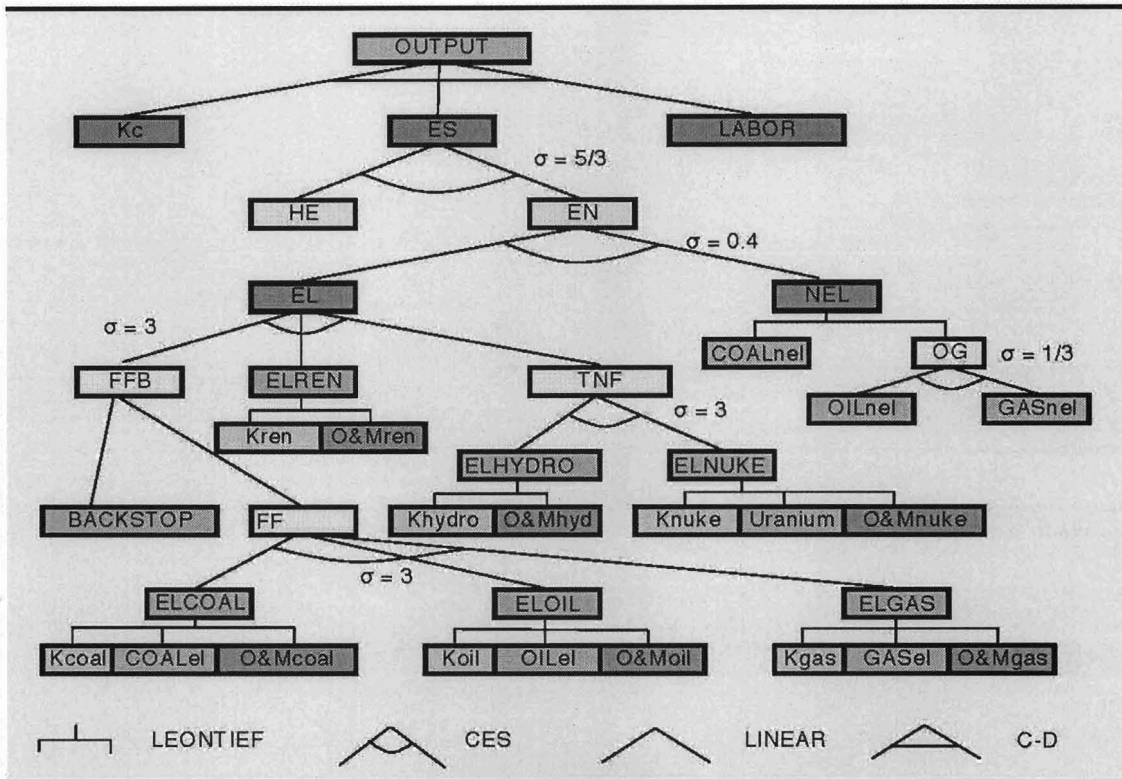


**Regions:**

- 1) CAJANZ (Canada, Japan, New Zealand)
- 2) USA
- 3) LACA (Latin America, Mexico and Caribbean)
- 4) OLDEURO (Old Europe)
- 5) NEWEURO (New Europe)
- 6) MENA (Middle East and North Africa)
- 7) SSA (Sub-Saharan Africa excl. South Africa)
- 8) TE (Transition Economies)
- 9) SASIA (South Asia)
- 10) CHINA (including Taiwan)
- 11) EASIA (South East Asia)
- 12) KOSAU (Korea, South Africa, and Australia)



Figure 2: Energy Technologies



Kc = capital invested in production of final good, expressed in 1995 US\$

HE = Energy R&D capital, measured 1995 US\$

ES = Energy services, expressed in energy measure (TWh)

EN = Energy, expressed in energy measure (TWh)

EL = Electricity, expressed in energy measure (TWh)

NEL = Non-electric energy use, expressed in energy measure (TWh)

FFB = Fossil Fuel and Backstop nest, aggregation is linear, expressed in energy measure (TWh)

TNF = Traditional non-fossil fuel electricity, CES nest, expressed in energy measure (TWh)

FF = Fossil fuel nest, CES nest, expressed in energy measure (TWh)

Backstop = Electricity produced using the Backstop technology, expressed in energy measure (TWh)

ELj = Electricity generated with the technology j, expressed in energy measure (TWh)

O&Mj = Operation and Maintenance costs for production of electricity with technology j, expressed in 1995 US\$

Gasz = natural gas used in sector z, expressed in energy measure (TWh)

Note: REN stand for Renewables;  $\sigma$  denotes the elasticity of input substitution

Figure 3. GDP level by region

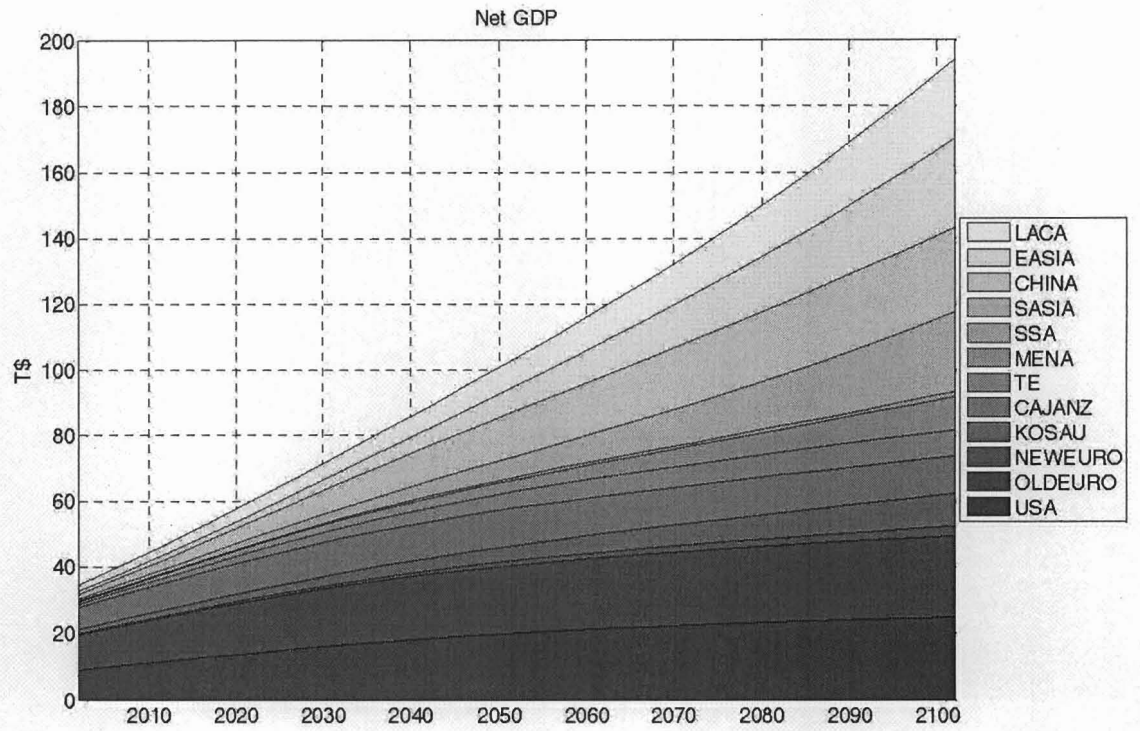


Figure 4. Total and energy R&D investments

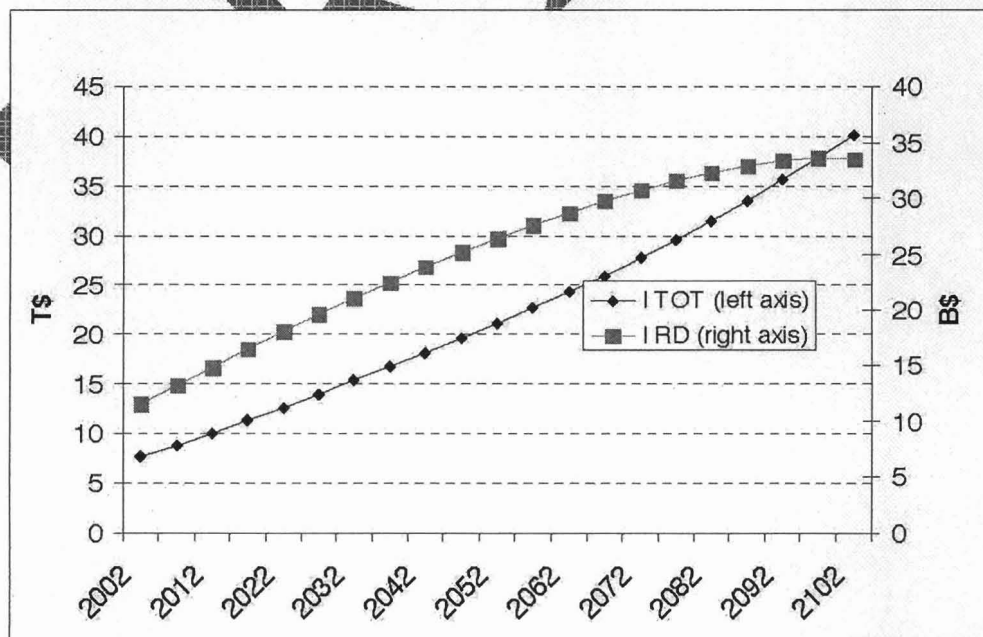


Figure 5. World Electricity Generation Fuel Mix

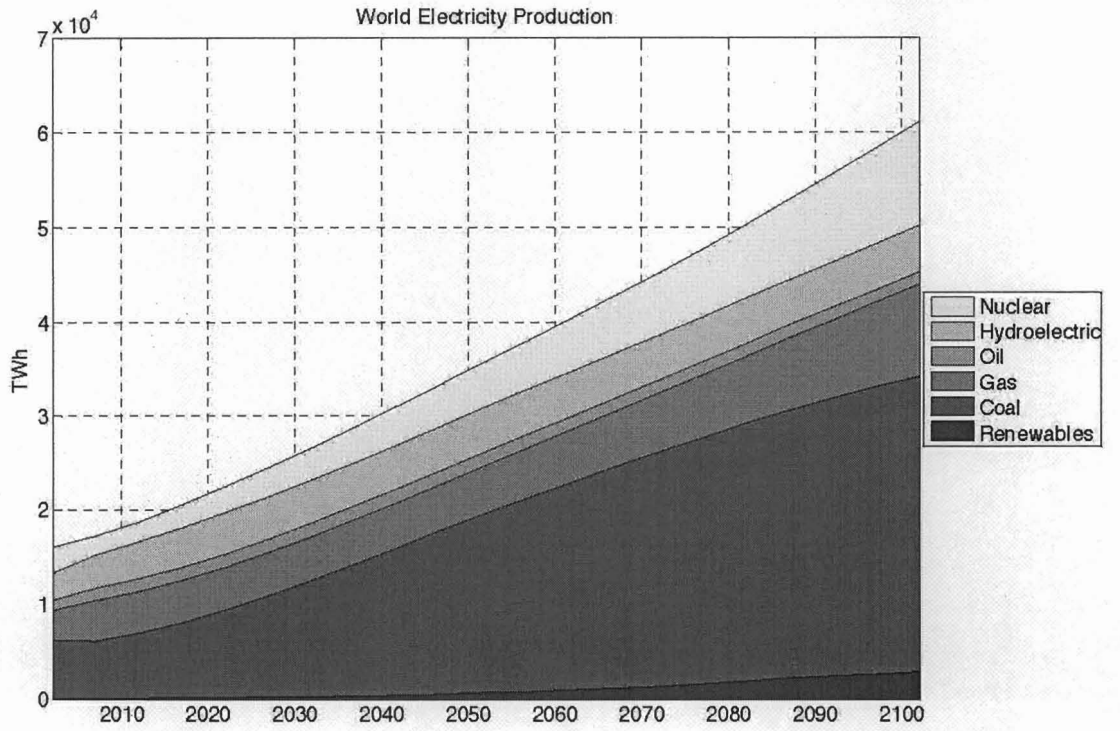


Figure 6. Energy intensities in all world regions

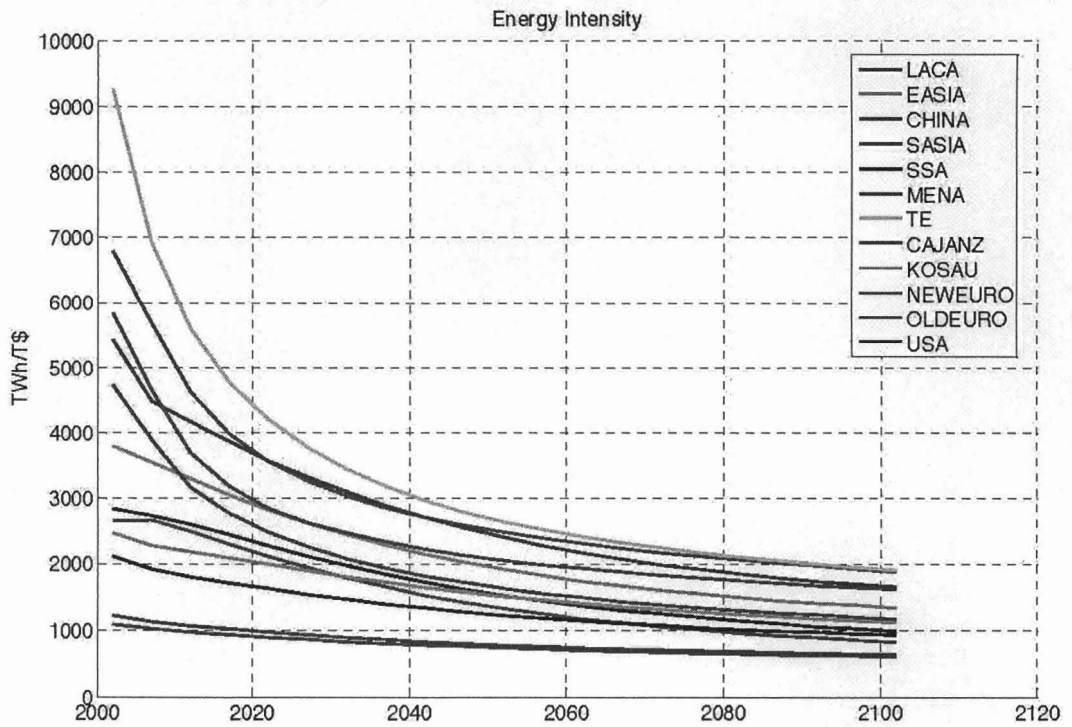


Figure 7. Electricity Generation by Technology in Individual WITCH Regions/1

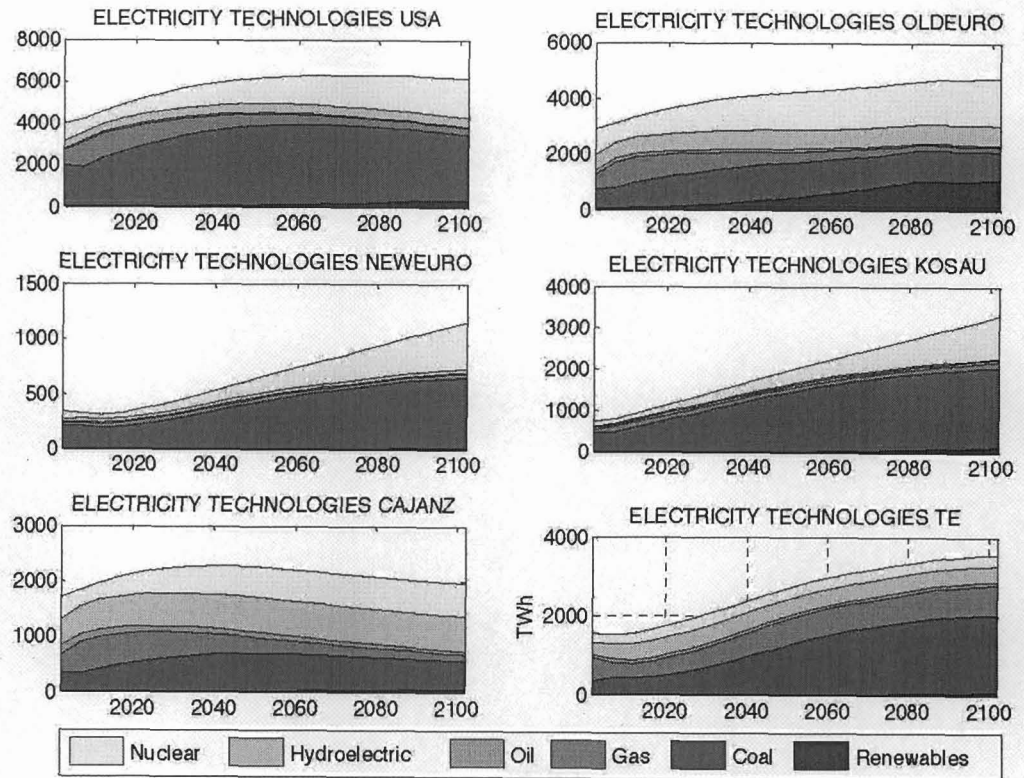


Figure 8. Electricity Generation by Technology in Individual WITCH Regions/2

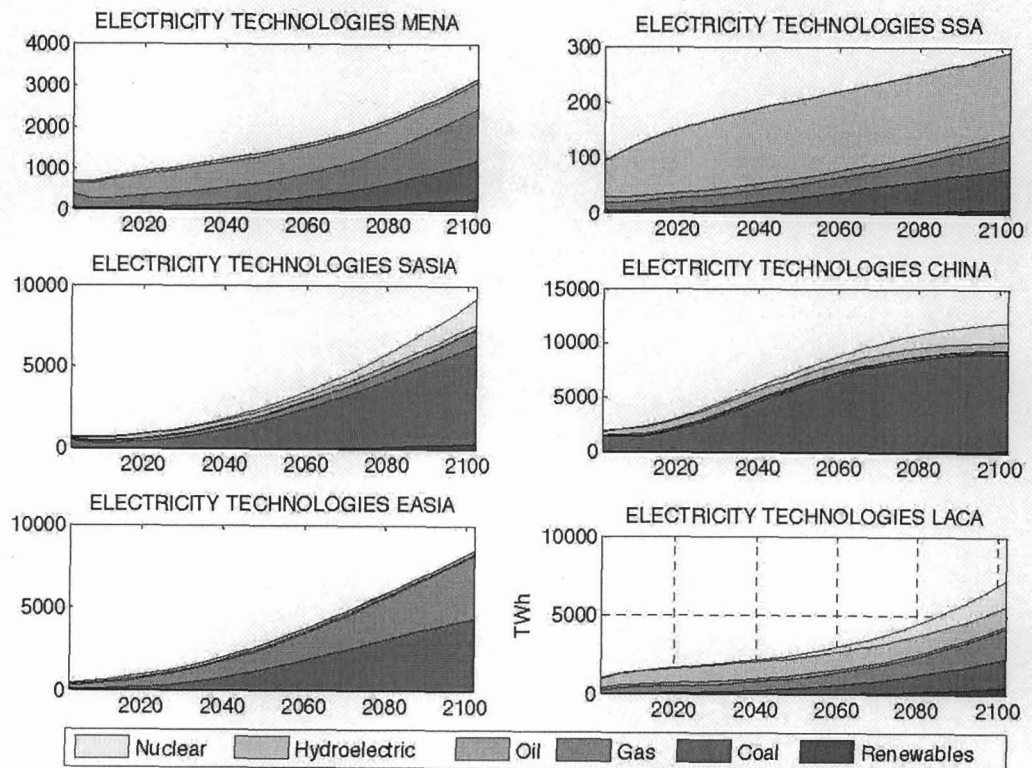




Figure 9. Carbon emissions (regional disaggregation)

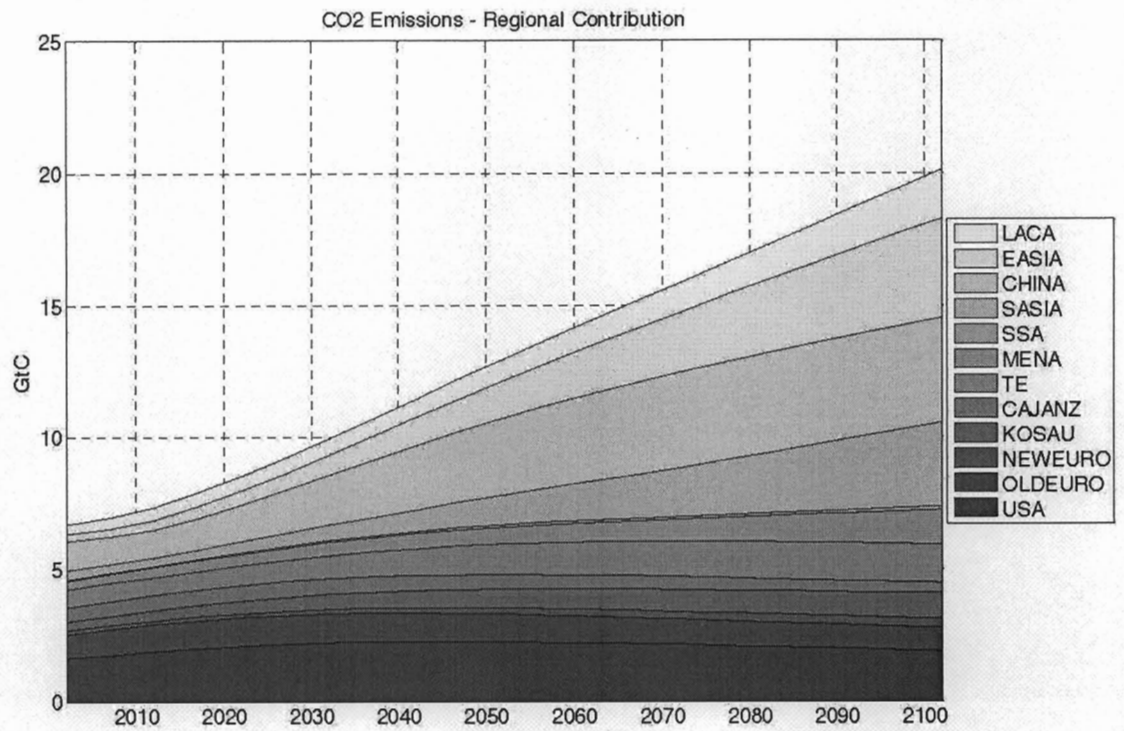


Figure 10. Total Primary Fossil Fuel Supply

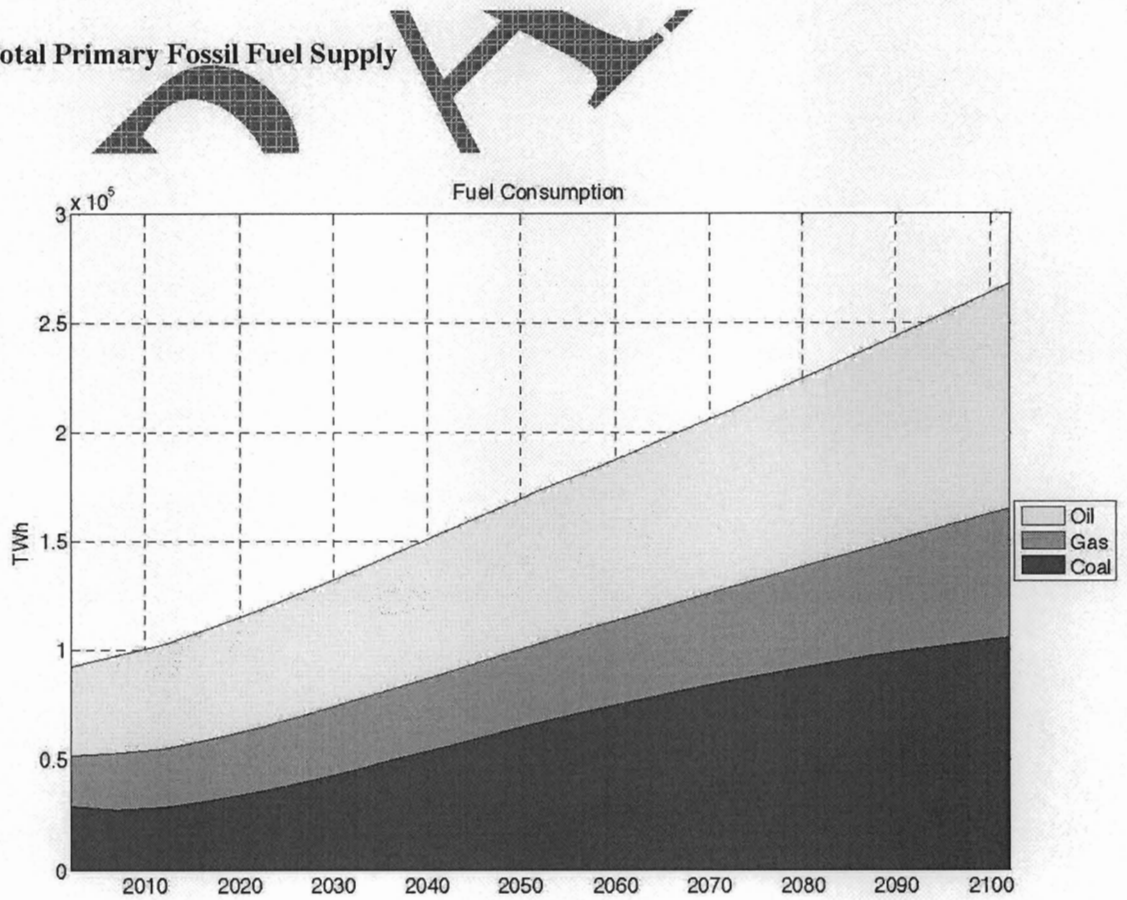


Figure 11. Learning by Doing in New Technologies

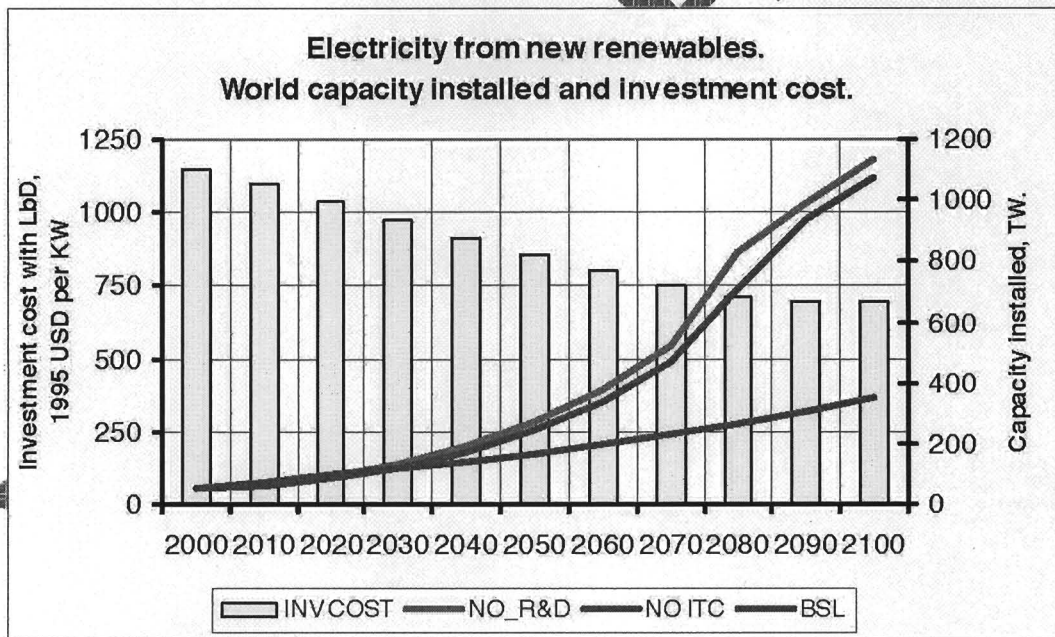
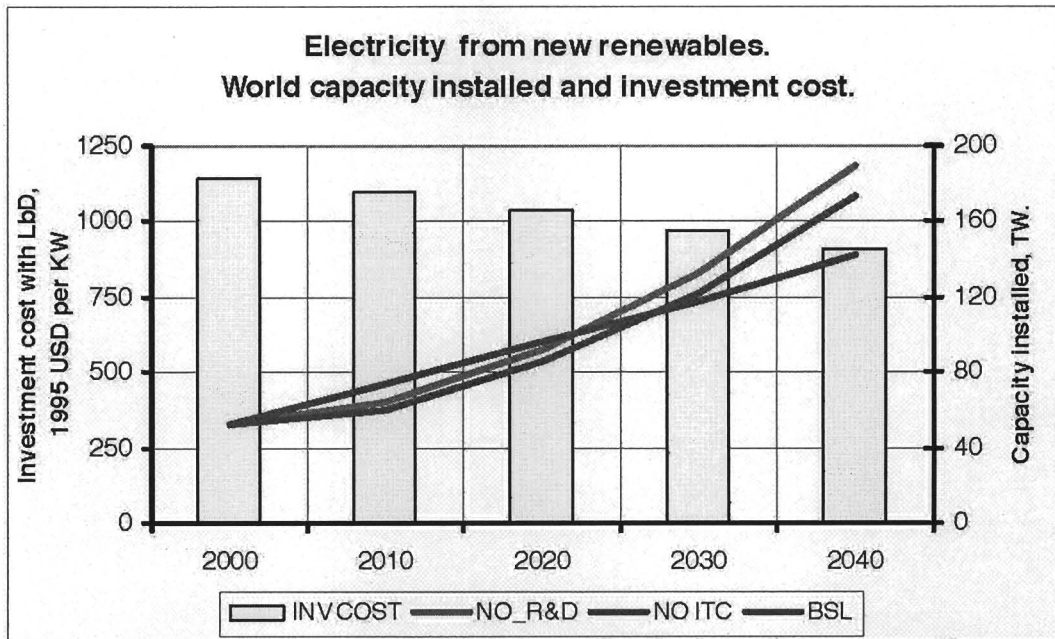


Table 1. Share of different electricity sources from 2002 to 2102.

	COAL	OIL	GAS	NUCLEAR	HYDRO	RENEWABLES
2002	38.13%	7.31%	19.50%	16.91%	17.82%	0.33%
2032	46.19%	5.37%	17.27%	12.92%	17.26%	1.00%
2102	51.21%	2.21%	16.02%	17.99%	7.89%	4.67%