The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR1735

Advanced School and Conference on Representation Theory and Related Topics

(9 - 27 January 2006)

ABSTRACTS

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

REALIZATION OF SIMPLE LIE ALGEBRAS VIA HALL ALGEBRAS OF DOMESTIC CANONICAL ALGEBRAS

HIDETO ASASHIBA

We realize simple complex Lie algebras $\mathfrak{g}(\Delta)$ of Dynkin type $\Delta = A, D, E$ as quotient algebras $L(A)_1^{\mathbb{C}}/I(A)$ of complex degenerate composition Lie algebras $L(A)_1^{\mathbb{C}}$ by some ideal $I(A)$ defined via Hall algebras of domestic canonical algebras A. The preprint in 2004 contained an error and a gap. Namely, the definition of $I(A)$ given there was wrong and a proof of the injectivity of a map $\phi\colon \mathfrak{g}(\Delta) \to L(A)_1^{\mathbb{C}}/I(A)$ defined there had a gap. In fact, under the wrong definition we checked that $L(A)_1^{\mathbb{C}}/I(A) = 0$ in some cases. In this talk we will present a right definition of $I(A)$ and give an outline of the proof of the injectivity of ϕ. In addition, using a notion of Gabriel-Roiter submodules we will give a simpler proof of the fact that for each exceptional A-module X the symbol $u_{[X]}$ corresponding to X is contained in $L(A)_1^{\mathbb{Q}}$.

ON THE VANISHING OF EXT AND $\varprojlim^1$

S. BAZZONI

We consider the relations between the vanishing of Ext functors and the derived functors of the inverse limit.

In particular, we focus on the Mittag-Leffler condition on countable inverse limits and we illustrate how this condition can be used to solve a problem on tilting classes of modules and on the class of Baer modules.

Moreover, we discuss how to relate the vanishing of $\varprojlim^n$ with an open problem on Σ-cotorsion modules, namely the problem of determining if pure submodules of Σ-cotorsion modules are cotorsion.

Dipartimento di Matematica Pura e Applicata, Università di Padova, Via Belzoni 7, 35131 Padova

E-mail address: bazzoni@@math.unipd.it

TILTING THEORY IN ABELIAN CATEGORIES

APOSTOLOS BELIGIANNIS

Abstract. Tilting theory has been established as a fundamental tool in representation theory. Its main aspect is that it gives an effective comparison, via suitable equivalences or dualities at various levels, of usually large parts of the categories which we are interested in. There are several notions of tilting in various settings (mainly for module categories) depending on suitable finiteness conditions. Our aim in this talk is to present a unified treatment of tilting theory in general abelian categories, in some cases without necessarily enough projective and/or injective objects, and to give in this setting the connections with (co)torsion pairs, derived equivalences and, times permits, related homotopical structures.

Department of Mathematics, University of the Aegean, 83200 Karlovassi, Samos, Greece
E-mail address: `abeligia@aegean.gr`, `abeligia@mail.otenet.gr`

CLUSTER-TILTED ALGEBRAS

ASLAK BAKKE BUAN

Let Γ be a cluster-tilted algebra, that is $\Gamma = \mathrm{End}_{\mathcal{C}}(T)$ for some tilting object T in a cluster category $\mathcal{C}$. An important property of cluster-tilted algebras is that $\Gamma/\Gamma e\Gamma$ is also cluster-tilted, for any idempotenet e in Γ. We sketch a proof for this fact, and indicate some of the applications of this in the theory of cluster algebras/cluster-tilted algebras.

This is based on joint work with Robert Marsh and Idun Reiten.

Institutt for matematiske fag, Norges teknisk-naturvitenskapelige universitet, N-7491 Trondheim, Norway

E-mail address: aslakb@math.ntnu.no

Noncommutative Desingularisation of the Generic Determinant

Ragnar-Olaf Buchweitz

In this joint work with Graham Leuschke and Michel van den Bergh we show that the generic determinant admits a noncommutative crepant desingularization by a 'Quiverized Clifford Algebra'.

The talk will explain these terms and show how this result relates to very concrete questions such as the following posed (and mainly answered) by George Bergman: If X is an $n \times n$- matrix with indeterminate entries and $adj(X)$ is its classical adjoint, can one factor $adj(X) = UV$ with noninvertible $n \times n$-matrices U, V ?

A Hall algebra approach to cluster algebras

Philippe Caldero

This is joint work with Bernhard Keller, [CK1], [CK2]. The cluster category is a triangulated category which enjoys a nice symmetry of Calabi-Yau type. We show how this symmetry property provides morphisms among varieties of triangles of the category. We then define Hall algebra type multiplication rules between objects of the cluster category, whose coefficients are Euler characteristics of varieties of triangles. On an opposite side, cluster algebras are commutative algebras defined inductively by generators and relations. We show how the multiplication rule connects explicitly the cluster category with an associated cluster algebra. This Hall algebra approach of cluster algebras enables to solve conjectures of Fomin and Zelevinsky, [?].

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Anticyclic operads and Auslander-Reiten translation

Frédéric Chapoton

arXive:math.QA/0502065

We will show that two different constructions lead to the same actions of cyclic groups on some Abelian groups. The first of these constructions lives in the framework of the theory of operads, and more precisely revolves around the notion of anticyclic operad. The other construction is provided by the Coxeter transformation, which is the action induced by the Auslander-Reiten functor on the Grothendieck group of a finite-dimensional algebra.

There are only two examples so far for this relationship. The first one is between the Diassociative operad and the sequence of hereditary algebras of the A_n quivers. This is of course a very classical setting. The other one is between the Dendriform operad and the sequence of incidence algebras of the Tamari lattices. This is related to some more recent developments, such as the theory of cluster algebras.

MINIMAL NON-TILTED ALGEBRAS

FLÁVIO U. COELHO, JOSE A. DE LA PEÑA, AND SONIA TREPODE

Introduced in the early 80's by Happel-Ringel, the class of tilted algebras has played an important role in the development of the representation theory of algebras. However, it is not always easy to identify a tilted algebra looking, for instance, at its ordinary quiver. On the other hand, the tilted algebras behave nicely with respect to their semi-convex subcategories, that is, if A is a tilted algebra and B is a semi-convex subcategory of A, then also B is tilted. Recall that, given an algebra A, a subcategory B of A is *semi-convex* provided there is a sequence $B = B_s, \cdots, B_0 = A$ such that $B_i = C_i[M_i']$ (respectively, $B_i = [M_i']C_i$) is a one-point (co-)extension of a convex subcategory C_i of $B_{i-1} = C_i[M_i' \oplus M_i'']$ (respectively $B_{i-1} = [M_i' \oplus M_i'']C_i$) by a C_i-module M_i', possibly $M_i'' = 0$. Clearly, convex subcategories are semi-convex.

By looking at the minimal non-tilted algebras, we hope to get some insight of how the tilted algebras are built up. By *minimal non-tilted* we mean a triangular algebra which is not tilted but any of its semi-convex subcategory is tilted.

In this talk, we will discuss this problem and give several results of classification of classes of minimal non-tilted algebras.

Departamento de Matemática-IME, Universidade de São Paulo, CP 66281, São Paulo, SP, 05315-970, Brazil
E-mail address: fucoelho@ime.usp.br

Instituto de Matemáticas, UNAM, Ciudad Universitaria, Mexico 04510, D.F, Mexico
E-mail address: jap@penelope.matem.unam.mx

Depto de Matemáticas, FCEyN, Universidad de Mar del Plata, 7600, Mar del Plata, Argentina
E-mail address: strepode@mdp.edu.ar

On two classes of modules of the second kind for galois coverings

Piotr Dowbor (UMK, Toruń)

Galois covering technique is an important tool of modern representation theory of algebras. In many cases it was effectively used to reduce a studied problem for modules over an algebra to analogous one, but simpler, for its cover. Nevertheless, the basic problem of understanding and description of the structure, for category of the second kind modules for a Galois covering, in general case reminds still open.

The main aim of this talk is to define two classes of indecomposables of the second kind, the so-called non-orbicular and non-regularly orbicular modules, and discuss the existence problem for them. We present constructions, which in some situations give possibilities of forming large, usually wild, families of such modules. We also discuss properties of maximal Cohen-Macaulay modules (in some specific sense) over certain skew group algebras. We show that this class can be treated as an useful platform for studying modules of the second kind for Galois coverings.

$\mathrm{G_1T}$-MODULES, AR-COMPONENTS, AND GOOD FILTRATIONS

ROLF FARNSTEINER

Let $(\mathfrak{g}, [p])$ be a finite dimensional restricted Lie algebra, defined over an algebraically closed field k of characteristic $\mathrm{char}(k) = p > 0$. In comparison with representations of complex Lie algebras, one of the main technical problems in the study of $\mathfrak{g}$-modules resides in the lack of information provided by the weight space decomposition

$$M = \bigoplus_{\lambda \in X(\mathfrak{h})} M_\lambda$$

of a $\mathfrak{g}$-module M relative to a Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$. While the above provides a $\mathbb{Z}^n$-grading in the classical context, one obtains a gradation by a p-elementary abelian group in the modular setting.

In 1979 J. Jantzen transferred the well-known results by Bernstein-Gel'fand-Gel'fand concerning modules belonging to the category $\mathcal{O}$ of complex semi-simple Lie algebras to Lie algebras $\mathfrak{g} := \mathrm{Lie}(G)$, associated to (smooth) reductive groups G. His main tool was the highest weight category $\mathrm{mod}\, G_1T$ of G_1T-modules, defined by the first Frobenius kernel G_1 of G and a maximal torus $T \subset G$. Roughly speaking, the objects are finite dimensional $\mathfrak{g}$-modules which admit a grading by a free group (the character group of T) that is compatible with the weight space decomposition relative to the Cartan subalgebra $\mathfrak{t} := \mathrm{Lie}(T)$ of $\mathfrak{g}$.

Using results by Gordon and Green, we show that the Frobenius category $\mathrm{mod}\, G_1T$ has almost split sequences. Rank varieties are employed to establish the analogue of Webb's Theorem and to investigate components of the stable AR-quiver of $\mathrm{mod}\, G_1T$ containing modules affording a good filtration.

Fakultät für Mathematik, Universität Bielefeld, Postfach 10 01 31, 33501 Bielefeld, Germany.
E-mail address: rolf@mathematik.uni-bielefeld.de

Ext and noetherianity for Koszul algebras

Edward L. Green, Virginia Tech, USA

In this talk I will discuss joint work with N. Snashall, O. Solberg and D. Zacharia. I will present recent results relating the condition that a Koszul algebra is noetherian to conditions on both the category of graded modules and on the category of graded modules of the Koszul dual (the Ext-algebra).

Koszul algebras and distributive triples

Lutz Hille

Let A be a $\mathbb{Z}$–graded quadratic algebra. Then we can define the quadratic dual algebra $A^!$ and the graded dual B of $A^!$. The Koszul complex $B \otimes A$ (with its canonical differential and a natural grading) is a complex of left projective A–modules. The algebra A is called Koszul, if the Koszul complex is a resolution of the semisimple A–module $A/A_{\geq 1}$. There is a result relating the the Koszul complex to certain triples of vector spaces associated to the algebra A. We formulate this result and derive several consequences. In particular, we obtain a new characterisation of Koszul algebras in terms of distributive triples of vector spaces.

Weighted locally gentle quivers and Cartan matrices

Thorsten Holm
(joint work with C. Bessenrodt; arXiv:math.RT/0511610)

We study weighted locally gentle quivers. This naturally extends gentle quivers and gentle algebras, which have been intensively studied in the representation theory of finite-dimensional algebras, to a wider class of potentially infinite-dimensional algebras. Weights on the arrows of these quivers lead to gradings on the corresponding algebras. For the natural grading by path lengths, any locally gentle algebra is a Koszul algebra.

Our main result is a general combinatorial formula for the determinant of the weighted Cartan matrix of a weighted locally gentle quiver. This determinant is invariant under graded derived equivalences of the corresponding algebras. We show that this weighted Cartan determinant is a rational function which is completely determined by the combinatorics of the quiver, more precisely by the number and the weight of certain oriented cycles. This leads to combinatorial invariants of the graded derived categories of graded locally gentle algebras.

Tilting modules over Calabi-Yau algebras

Osamu Iyama

This is a report on a joint work with I. Reiten.

Let R be a complete regular local ring of dimension d and Λ a module-finite R-algebra. We denote by $\mathrm{flmod}\Lambda$ the category of Λ-modules of finite length.

Definition For an integer n, Λ is called *Calabi-Yau-n* if there exists a functorial isomorphism

$$\mathrm{Hom}(X, Y) \simeq \mathrm{Hom}(Y, X[n])^*$$

for any $X, Y \in D^b(\mathrm{flmod}\Lambda)$.

We call Λ an *R-order* if it is a projective R-module. We call Λ a *symmetric* R-algebra if $\mathrm{Hom}_R(\Lambda, R)$ is isomorphic to Λ as a (Λ, Λ)-module.

One of our results is the following.

Theorem *Assume that Λ is a faithful R-module. Then Λ is Calabi-Yau-n for some n if and only if Λ is a symmetric R-order with* $\mathrm{gl.dim}\Lambda = d$. *In this case, $n = d$ holds.*

Then we will study tilting modules on Calabi-Yau algebras. Especially, we study Fomin-Zelevinsky mutation on tilting modules over Calabi-Yau-3 algebras, and a relationship with non-commutative crepant resolution of singularities introduced by Van den Bergh.

On the question: is tame open?
Stanisław Kasjan

Let T, W, SSC denote the subsets of the variety of d-dimensional algebras over a fixed algebraically closed field K, consisting of the tame (resp. wild, strongly simply connected) algebras. Using the characterization of tame strongly simply connected algebras due to Brüstle and Skowroński we prove that the sets $T \cap SSC$ and SSC are Zariski-open. Moreover, the sets are defined by polynomials with integral coefficients chosen independently on the field K.

The (still open) question if T is open is an objective of the article of Yang Han [*J. Algebra*, 284, 801-810 (2005)], where so called rank of a wild algebra is introduced and "Wild-Rank Conjecture" (implying that T is open) is formulated. Note that the methods of Han show that $T \cap SSC$ is open in SSC.

In what follows we use a slightly modified definition of rank, but the modification does not change the most important features of the concept. Consider a one-parameter regular family A_t, $t \in K$, of d-dimensional algebras. We prove that there is a function $\beta : \mathbb{N} \to \mathbb{N}$ such that if A_t is wild of rank less than or equal r for at least $\beta(r)$ values of t, then A_t is wild for any $t \in K$. The function β can be explicitly calculated and depends only on d.

CONSTRUCTING TILTING MODULES

OTTO KERNER AND JAN TRLIFAJ

Let A be an Artin algebra and T be a right A-module. Then T is a *tilting module* provided that (T1) p.dim$T \leq 1$, (T2) $\mathrm{Ext}^1_R(T, T^{(I)}) = 0$ for any set I, and (T3) there is a short exact sequence $0 \to A \to T_0 \to T_1 \to 0$ where T_0 and T_1 are direct summands in a direct sum of (possibly infinitely many) copies of T.

If T is a tilting A-module, $T^\perp$ denotes the full subcategory

$$T^\perp = \{M \in \text{Mod-}A \mid \mathrm{Ext}^1_A(T, M) = 0\},$$

which is a torsion class in Mod-A, the category of all A-modules. If T' is another tilting module then T is said to be *equivalent* to T' if $\{T\}^\perp = \{T'\}^\perp$.

Conbining a recent Theorem of Bazzoni and Herbera with a result of Kerner and Trlifaj, the assignment $T \mapsto T^\perp \cap \text{mod-}A$ defines a bijection between the set of torsion classes in mod-A, containing all injective modules in mod-A, and the set of equivalence classes of tilting A-modules.

This fact motivates the study of A-modules X, which can be completed to a tilting module $T = X \oplus Y$. Restricting to hereditary Artin algebras, on can show:

Theorem. *Let A be a connected hereditary Artin algebra, and X be an H-module, finitely generated over its endomorphism ring and with $\mathrm{Ext}^1_H(X, X^{(I)}) = 0$, for any set I. Then there exists an H-module Y, such that $X \oplus Y$ is a tilting H-module.*

The proof is done on two steps: (a) X is faithful, (b) X is not faithful.

As a consequence one gets for example for hereditary algebras: If X is a *stone*, which means that X is endo-finite, indecomposable and without self-extensions, then X is a direct summand of a tilting module.

Generalising an almost relatively true statement

Steffen König

The derived categories of diagram algebras, such as Brauer algebras or partition algebras, usually are stratified by derived categories of group algebras of symmetric groups. By a result of Hemmer and Nakano, Specht modules of symmetric groups usually form standard systems in the sense of Dlab and Ringel.

This is a report of joint work with Robert Hartmann, Anne Henke and Rowena Paget.

Preprojective algebras and cluster algebras

Bernard Leclerc

I will review recent results on the relation between cluster algebras and preprojective algebras. This is a joint work with Christof Geiss and Jan Schröer.

Serre functors, category $\mathcal{O}$, and symmetric algebra

Volodymyr Mazorchuk

This is a joint work with Catharina Stroppel.

Let $\Bbbk$ be a field. A Serre functor on a $\Bbbk$-linear category, $\mathcal{C}$, with finite-dimensional homomorphism spaces is an auto-equivalence, F, of $\mathcal{C}$ which gives isomorphisms

$$\mathrm{Hom}_{\mathcal{C}}(X, FY) \cong \mathrm{Hom}_{\mathcal{C}}(Y, X)^*$$

natural in both X and Y. If A is a finite dimensional algebra, then it is well-known that the bounded derived category $\mathcal{D}^b(A)$ has a Serre functor if and only if gl.dim.$A < \infty$, and if the latter is the case, the Serre functor is just the left derived of the Nakayama functor $A^* \otimes_A -$. In particular, it follows that there is a Serre functor for all blocks of the BGG category $\mathcal{O}$, associated with a semi-simple complex finite-dimensional Lie algebra. However, since in this case the algebra A is not explicitly given, the Serre functor is not easy to compute. One of our results is the following explicit description of the Serre functor on $\mathcal{O}$ (the geometric counterpart of this result was recently obtained by Beilinson, Bezrukavnkov and Mirkovic):

Theorem. Let T_{w_0} be the global Arkhipov's twisting functor on the regular block of the category $\mathcal{O}$. Then $\mathcal{L}\mathrm{T}_{w_0}^2$ is the Serre functor on this block.

Using the recent results of Khomenko on functors, naturally commuting with translation functors, the above theorem allows us to explicitly describe Serre functors on the regular blocks of the parabolic category $\mathcal{O}$ introduced by Rocha-Caridi. Using the connection between the Serre functors and symmetric algebras one obtains that the endomorphism algebra of the basic projective- injective module in the parabolic block is symmetric. This confirms a conjecture of Khovanov.

Department of Mathematics, Uppsala University, Box 480, 75106, Uppsala, Sweden, e-mail: `mazor@math.uu.se`, web: http://www.math.uu.se/˜mazor/

OMNIPRESENT EXCEPTIONAL MODULES FOR HYPERELLIPTIC ALGEBRAS

HAGEN MELTZER

A hyperelliptic algebra is a canonical algebra in the sense of Ringel of type $(2, 2, \ldots, 2)$, t entries, $t \geq 5$.

Whereas for domestic canonical algebras explicit descriptions for all indecomposable modules are known and for tubular canonical algebras we have results concerning the coefficients of the matrices of the exceptional modules the situation is much more complicated in the wild case. However a nice class of exceptional modules can be described explicitly by vector spaces and matrices.

It is well known that for a canonical algebra Λ the module category $\operatorname{mod}(\Lambda)$ admits a trisection $(\operatorname{mod}_+(\Lambda), \operatorname{mod}_0(\Lambda), \operatorname{mod}_-(\Lambda))$, where $\operatorname{mod}_0(\Lambda)$ is a separating family of stable tubes. We call a module M from $\operatorname{mod}_+(\Lambda)$ omnipresent if for each $S \in \operatorname{mod}_0(\Lambda)$ there is a non-zero homomorphism from M to S.

Theorem 1 *Let Λ be a canonical algebra of type $(p_1, p_2, \ldots, p_t)$ and M an omnipresent exceptional Λ-module from $\operatorname{mod}_+(\Lambda)$. Then $\operatorname{rk}(M) \geq t - 1$.*

Theorem 2 *Let Λ be a hyperelliptic algebra of type $(2, 2, \ldots, 2)$, t entries. Then, up to duality and "up to shift", there exists a unique omnipresent exceptional module of rank $t - 1$ in $\operatorname{mod}_+(\Lambda)$.*

In the hyperelliptic case we will give explicit matrices for all omnipresent exceptional modules of minimal rank $t - 1$.

The method in the proof is based on the study of universal extensions of line bundles on the corresponding weighted projective line and is useful also in other situations. In particular in the tubular case these techniques provide an algorithmic way for an explicit description of all exceptional modules. In joint work with P. Dowbor we are developing a computer program concerning this problem.

Characterisations of supported algebras.

María Inés Platzeck

We give several equivalent characterisations of left (and hence, by duality, also of right) supported algebras. These characterisations are in terms of properties of the left and the right parts of the module category, or in terms of the classes $\mathcal{L}_0$ and $\mathcal{R}_0$ which consist respectively of the predecessors of the projective modules, and of the successors of the injective modules.

This is a report of joint work with I. Assem, J. A. Cappa, and S. Trepode.

Tilted algebras and cluster tilted algebras

C. M. Ringel (Bielefeld)

Let B be a tilted algebra, say $B = \mathrm{End}_A(T)$, where A is a hereditary artin algebra and T is a tilting module. Buan, Marsh and Reiten have studied the corresponding cluster tilted algebra $\widetilde{B} = \mathrm{End}_C(T)$, where $C = C_A$ is the cluster category for A, as defined by Buan, Marsh, Reineke, Reiten, Todorov. The aim of the lecture is to outline a direct procedure for studying the category of $\widetilde{B}$-modules in terms of A-modules, bypassing the cluster category C itself. In addition, we will provide an elementary reformulation of the description of the complex of cluster tilting objects in C, as given by Marsh, Reineke, Zelevinsky.

A classification of torsion torsionfree triples in module categories

Manolo Saorín

In 1965 Jans ([2]) introduced the concept of **torsion torsionfree (TTF) triples** in an abelian category. They are triples $(\mathcal{C}, \mathcal{T}, \mathcal{F})$ of full subcategories such that $(\mathcal{C}, \mathcal{T})$ and $(\mathcal{T}, \mathcal{F})$ are both torsion theories. In case the ambient category is the module category $ModA$ over an associative ring A with unit, he gave a bijection between the set of those triples and the set of (two-sided) idempotent ideals of A. A TTF triple as above is called **centrally split** when both constituent torsion theories are split. Jans also proved that the above bijection restricted to another one between centrally split TTF triples of $ModA$ and (ideals generated by) central idempotents of A. On the other hand, the existence of **one-sided split** TTF-triples (i.e. such that only one of $(\mathcal{C}, \mathcal{T})$ and $(\mathcal{T}, \mathcal{F})$ is a split torsion theory) has been known for a long time (cf. [4]). However, the idempotent ideals of A associated to them by Jans' correspondence had not been identified.

In this joint work with Pedro Nicolás (see [3]), we identify those idempotent ideals, thus providing a full classification of one-sided split TTF- triples in module categories. In the particular case when A is an Artin algebra the bijections obtained (for left and right split TTF-triples, respectively) can be obtained one from each other by duality and were considered in an earlier work with Assem ([1]).

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TOP-STABLE AND LAYER-STABLE HOM-ORDER AND DEGENERATIONS

S. O. SMALØ (AND A. VALENTA)

This talk is a report on joint work with Anita Valenta.

Using geometrical methods, Birge Huisgen-Zimmermann has shown that if M is a module with simple top, then M has no proper degeneration $M <_{\deg} N$ such that $\mathfrak{r}^t M/\mathfrak{r}^{t+1} M \simeq \mathfrak{r}^t N/\mathfrak{r}^{t+1} N$ for all t. Given a module M with square-free top and a projective cover P, she shows also that $\dim_k \operatorname{Hom}(M, M) = \dim_k \operatorname{Hom}(P, M)$ if and only if M has no proper degeneration $M <_{\deg} N$ where $M/\mathfrak{r} M \simeq N/\mathfrak{r} N$. In this talk I will give an alternative approach to these results by using the more general hom-order instead of the degeneration-order and then obtain a direct algebraic proof of a more general statements.

Norwegian University of Science and Technology, Department of Mathematical Sciences, N-7491 Trondheim, Norway.

E-mail address: sverresm@math.ntnu.no

2000 *Mathematics Subject Classification* 14L30, 16D10, 16E30, 16G10, 16G20.

SUPPORT VARIETIES OVER COMPLETE INTERSECTIONS

ØYVIND SOLBERG

Let $(R, \mathfrak{m})$ be a (complete) local complete intersection with maximal ideal $\mathfrak{m}$. The support of a perfect complex over R is defined in terms of the prime ideal spectrum of R. The support of a finitely generated R-module is defined in terms of the prime spectrum of the ring $R/\mathfrak{m}[\chi_1, ..., \chi_t]$ (here $t = \operatorname{codim} R$). The aim of this talk is to show that these two support theories can both be derived from a theory of support varieties for the derived category of bounded complexes of finitely generated R-modules. Along the way we indicate how a theory of support varieties for bounded complexes is obtained as a special case of support varieties of a triangulated category admitting an action from a tensor triangulated category.

This talk is based on joint work with: Aslak B. Buan, Henning Krause and Nicole Snashall.

Institutt for matematiske fag, NTNU, N–7491 Trondheim, Norway
E-mail address: oyvinso@math.ntnu.no

QUOTIENT TRIANGULATED CATEGORIES ARISING IN REPRESENTATION THEORY

PU ZHANG

Department of Mathematics, Shanghai Jiao Tong University
Shanghai 200240, P. R. China, pzhang@sjtu.edu.cn

This is a joint work with Xiao-Wu Chen at USTC.

Several quotient triangulated categories arising naturally from the representations of algebras are studied.

1. For a self-orthogonal A-module T, the quotient triangulated category $D^b(A)/K^b(\mathrm{add}T)$.

(It is a funny trianulated category; if T is generalized tilting (resp. cotilting) then it is exactly the singularity category $\mathcal{D}_P(A) := D^b(A)/K^b(A\text{-proj})$ (resp. $\mathcal{D}_I(A) := D^b(A)/K^b(A\text{-inj})$); when A is Gorenstein then this generalizes a beautiful result of Happel, and a recent work of Orlov; this naturally related to a work of Auslander-Reiten, and a work of Ringel; one may expect to get some information on modules from this quotient triangulated category.)

Theorem 0.1. *Let T be a self-orthogonal module, $M \in \mathcal{X}_T$ and $N \in T^{\perp}$. Then there is a natural isomorphism of vector spaces* $\mathrm{Hom}_A(M,N)/T(M,N) \simeq \mathrm{Hom}_{D^b(A)/K^b(\mathrm{add}T)}(M,N)$, *where $T(M,N)$ is the subspace of A-maps which factor through* $\mathrm{add}T$.

In particular, the natural functor $\mathcal{X}_T \cap T^{\perp} \longrightarrow D^b(A)/K^b(\mathrm{add}T)$ induces a fully-faithful functor $\overline{\mathcal{X}_T \cap T^{\perp}} \longrightarrow D^b(A)/K^b(\mathrm{add}T)$, where $\overline{\mathcal{X}_T \cap T^{\perp}}$ is the stable category of $\mathcal{X}_T \cap T^{\perp}$ modulo $\mathrm{add}T$.

Corollary 0.2. *Let A be a Gorenstein algebra and T an A-module. Then T is generalized cotilting if and only if T is generalized tilting.*

Theorem 0.3. *Assume that* $\mathrm{inj.dim}\, {}_AA < \infty$. *Let T be a generalized cotilting A-module. Then the natural functor ${}^{\perp}T \longrightarrow D^b(A)/K^b(\mathrm{add}T) = \mathcal{D}_I(A)$ is dense.*

Moreover, if in addition A is Gorenstein, then the natural functor ${}^{\perp}T \cap T^{\perp} \longrightarrow D^b(A)/K^b(\mathrm{add}T)$ is dense.

Theorem 0.4. *Assume that* $\mathrm{inj.dim}\, {}_AA < \infty$. *Let T be a generalized tilting module. Then the natural functor ${}^{\perp}T \cap T^{\perp} \longrightarrow \mathcal{D}_I(A)$ is dense.*

In particular, the natural functor ${}^{\perp}A \longrightarrow \mathcal{D}_I(A)$ is dense.

Theorem 0.5. *Let A be Gorenstein, and T be a generalized cotilting module (= a generalized tilting module). Then the natural functors induce an equivalences of categories*

$$\underline{{}^{\perp}T \ \cap T^{\perp}} \simeq \mathcal{D}_I(A) = \mathcal{D}_P(A).$$

Corollary 0.6. *The following are equivalent*

(i) $\mathrm{gl.dimA} < \infty$*;*

(ii) $\mathrm{inj.dim}\, {}_AA < \infty$, *and ${}^{\perp}T \cap T^{\perp} = \mathrm{add}T$ for any generalized tilting module;*

(ii)′ proj.dim ${}_AD(A_A) < \infty$, *and* ${}^{\perp}T \cap T^{\perp} = \mathrm{add}T$ *for any generalized cotilting module.*

2. Let $T(A) := A \oplus D(A)$ be the the trivial extension algebra of A. It is $\mathbb{Z}$-graded with $\mathrm{degA} = 0$ and $\mathrm{degD(A)} = 1$. Let $T(A)^{\mathbb{Z}}$-mod be the category of finite-dimensional $\mathbb{Z}$-graded $T(A)$-modules with morphisms of degree 0. A theorem of Happel says that there exists a fully-faithful, exact functor $F : D^b(A) \longrightarrow T(A)^{\mathbb{Z}}\text{-}\underline{\mathrm{mod}}$; and F is dense if and only if $\mathrm{gl.dimA} < \infty$.

Theorem 0.7. *Let A be a Gorenstein algebra. Then under Happel's functor $F : D^b(A) \longrightarrow T(A)^{\mathbb{Z}}\text{-}\underline{\mathrm{mod}}$ we have*

$$D^b(A) \simeq \mathcal{N} := \{\oplus_{n\in\mathbb{Z}} M_n \in T(A)^{\mathbb{Z}}\text{-}\underline{\mathrm{mod}} \mid \mathrm{proj.dim}\ {}_AM_n < \infty,\ \forall\ n \neq 0\}$$

and

$$K^b(A\text{-}proj) \simeq \mathcal{M}_P := \{\oplus_{n\in\mathbb{Z}} M_n \in T(A)^{\mathbb{Z}}\text{-}\underline{\mathrm{mod}} \mid \mathrm{proj.dim}\ {}_AM_n < \infty,\ \forall\ n \in \mathbb{Z}\}.$$

3. The stable category $\underline{\mathfrak{a}(T)}$ of the Frobenius exact category $\mathfrak{a}(T) := \mathcal{X}_T \cap {}_T\mathcal{X}$.

Theorem 0.8. *Let T be self-orthogonal such that $\overbrace{\mathrm{add}T} \subseteq T^{\perp}$ and $\underbrace{\mathrm{add}T} \subseteq {}^{\perp}T$. Then there is an equivalence of triangulated categories $K^{ac}(T) \simeq \underline{\mathfrak{a}(T)}$, where $K^{ac}(T)$ is the full subcategory of $K(A)$ consisting of acyclic complexes with components in* $\mathrm{add}T$.

The notations of ${}^{\perp}T$, $T^{\perp}$, $\underbrace{\mathrm{add}T}$, $\overbrace{\mathrm{add}T}$, $\mathcal{X}_T$, ${}_T\mathcal{X}$, $\underset{\vee}{\mathrm{add}}T$, $\widehat{\mathrm{add}T}$ are same as in [AR].

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Codimension two singularities of orbit closures for representations of tame quivers

Grzegorz Zwara

Let k be an algebraically closed field, $Q = (Q_0, Q_1, s, e)$ be a quiver and $\mathbf{d} = (d_i) \in \mathbb{N}^{Q_0}$ be a dimension vector. The representations $M = (M_i, M_\alpha)_{i \in Q_0, \alpha \in Q_1}$ of Q with fixed vector spaces $M_i = k^{d_i}$, $i \in Q_0$, form an affine space denoted by $\mathrm{rep}_Q(\mathbf{d})$. The group $\mathrm{GL}(\mathbf{d}) = \prod_{i \in Q_0} \mathrm{GL}_{d_i}(k)$ acts on $\mathrm{rep}_Q(\mathbf{d})$ by

$$(g_i)_{i \in Q_0} \star (M_\alpha)_{\alpha \in Q_1} = (g_{e(\alpha)} \cdot M_\alpha \cdot g_{s(\alpha)}^{-1})_{\alpha \in Q_1}.$$

Then the orbits correspond to isomorphism classes of representations. Let M be a representation in $\mathrm{rep}_Q(\mathbf{d})$ and $\mathcal{X} = \overline{\mathrm{GL}(\mathbf{d}) \star M}$ be the (Zariski) closure of the orbit $\mathrm{GL}(\mathbf{d}) \star M$. The aim of this talk is to present some results concerning codimension two singularities occurring in $\mathcal{X}$, especially if Q is a Dynkin or Euclidean quiver.

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