The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

International Atomic Energy Agency

SMR.1738 - 8

**WINTER COLLEGE**

**on**

**QUANTUM AND CLASSICAL ASPECTS**

**of**

**INFORMATION OPTICS**

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# The Origins of Light's angular Momentum

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*Trieste Feb 2006*

# The Origins of Light's angular Momentum

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UNIVERSITY of GLASGOW

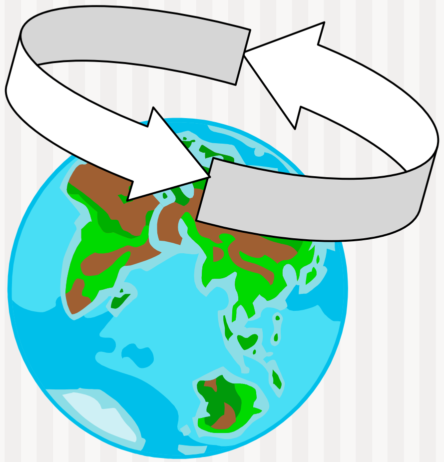
# How does optical angular momentum arise?

Allen *et al.* Phys. Rev. A 1992

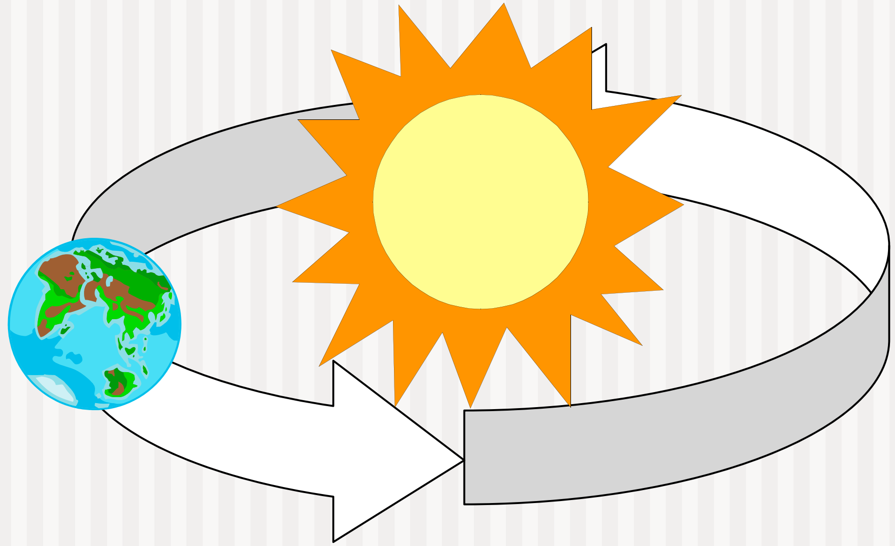
Allen *et al.* Prog. in Optics 1999

# Angular momentum

Spin angular momentum
(SAM)

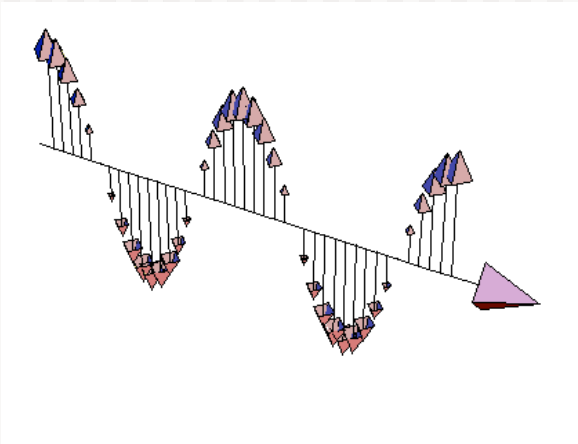


Orbital angular momentum
(OAM)

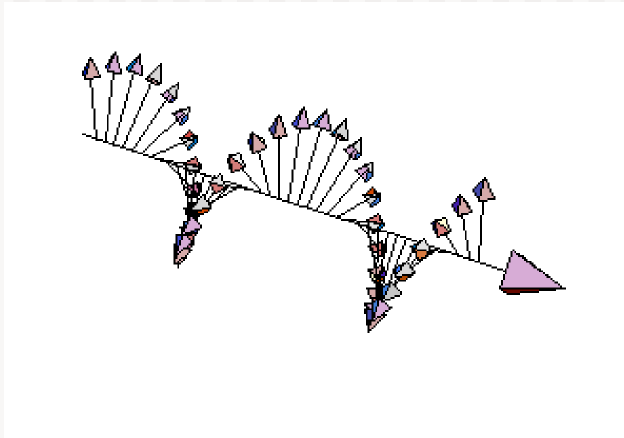


# Light beams with angular momentum (spin)

**Plane polarisation**



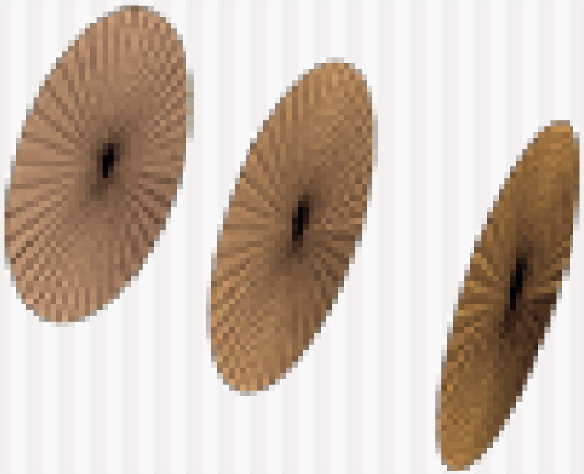
**Circular polarisation**



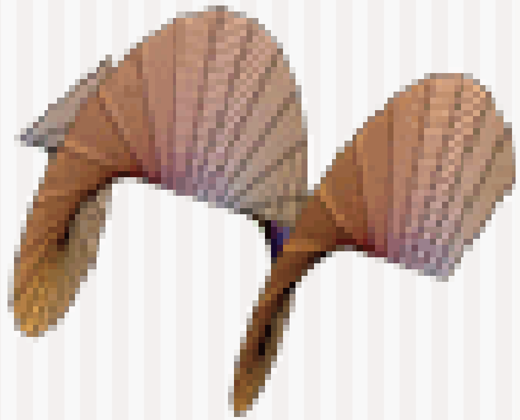
Gives spin angular momentum

# Light beams with angular momentum (orbital)

**Plane phasefronts**



**Helical phasefronts**



Gives Orbital angular momentum

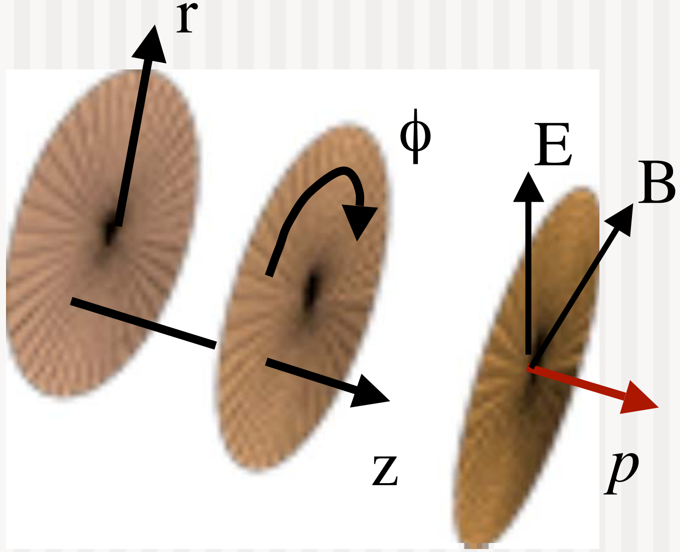
No restriction on polarisation

# Linear-momentum of light

- Energy and momentum flow within a light beam given by Poynting vector
- Linear Momentum

$$p = \frac{\varepsilon_0}{2}\left(E^* \times B + E \times B^*\right)$$

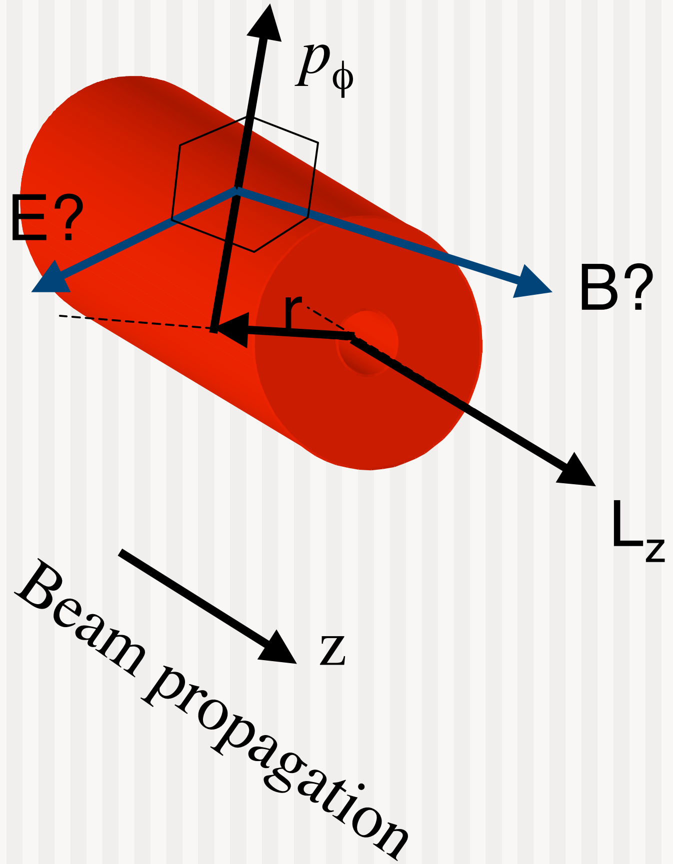
  - $p \,\alpha\, \mathbf{D} \times \mathbf{B}$
- Can evaluate a local value of momentum at every position within a light beam




Direction of energy and momentum flow within a plane wave

# Angular-momentum of light

- In the “classical world” all effects can be explained by the electro-magnetic field
  - Angular momentum z-direction requires linear momentum in $\phi$-direction
    - i.e. $L_z = r\, p_\phi$
  - Linear momentum in $\phi$-direction needs component of E or B in z-direction
- Angular momentum requires field component in direction of propagation



# Calculate AM from EM field

Depends upon phase structure of beam

Depends upon polarisation state & intensity gradient of beam

$$p = \frac{\varepsilon_0}{2}\left(E^* \times B + E \times B^*\right) = i\omega \frac{\varepsilon_0}{2}\left(u^* \nabla u - u \nabla u^*\right) + \omega k \varepsilon_0 |u|^2 z + \omega\sigma \frac{\varepsilon_0}{2} \frac{\partial |u|^2}{\partial r} \Phi$$

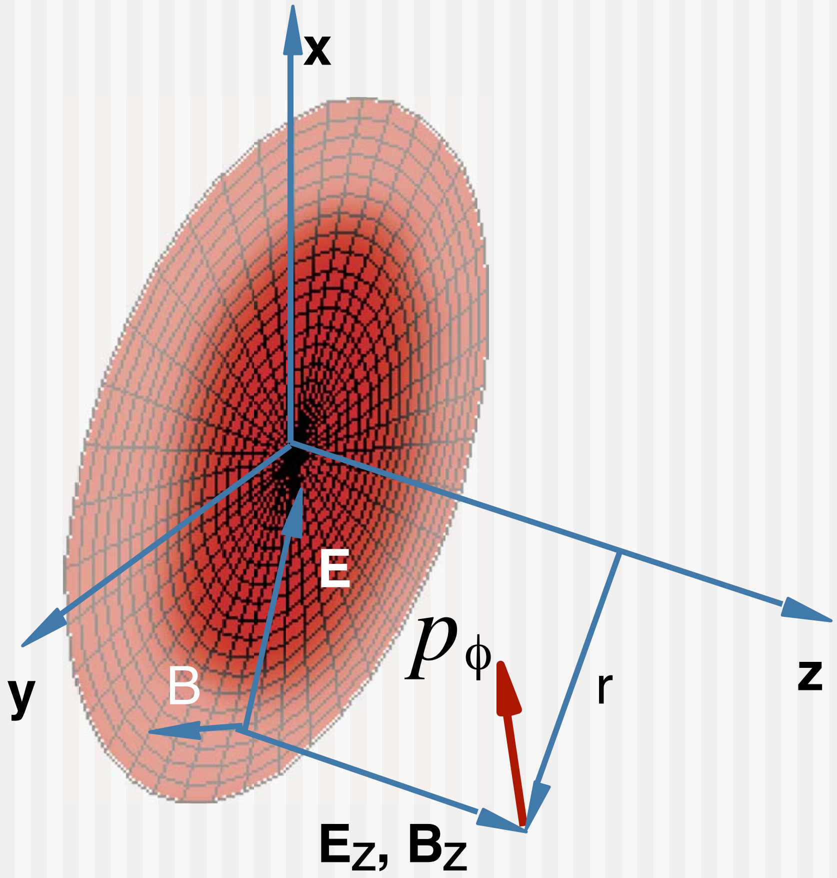
$\phi$- component gives OAM

$\phi$- component gives SAM

u ≈ the local amplitude of the beam (proportional to E)
Orbital terms arises from phase gradient
Spin term arises from intensity gradient

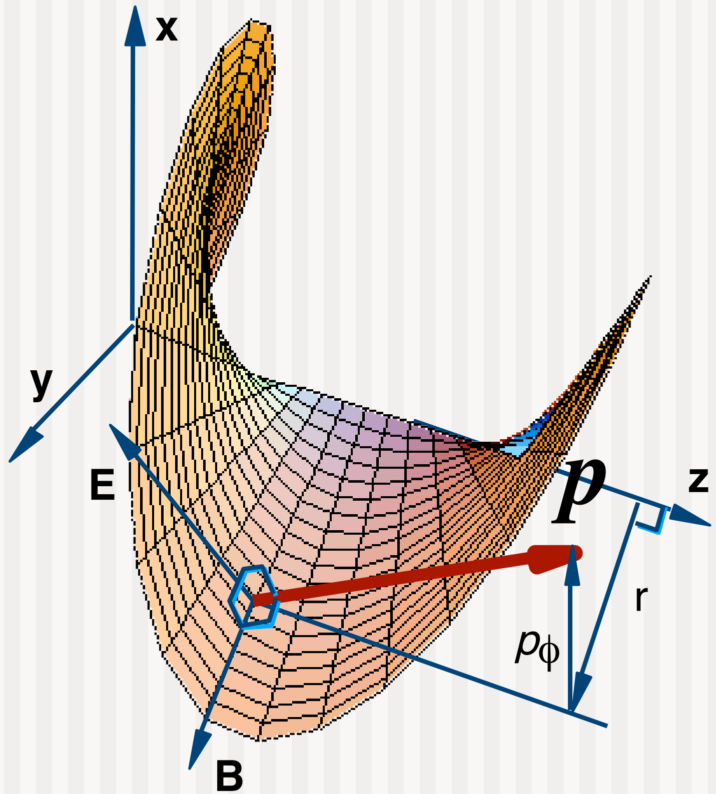
# Spin AM (more complicated!)

- SAM requires both circular polarisation & an intensity gradient!
  - B $\alpha$ Curl E
  - e.g. if $\frac{dE_y}{dx} \neq 0$ & $\sigma \neq 0$
  - $B_z \neq 0$
- Intensity gradient approach gives right answer to
  - Transfer of SAM to particles



# Orbital angular momentum

- OAM arises from helical phasefronts
  - $E_z$ & $H_z \neq 0$
  - $p_\phi \neq 0$
  - $L_z \neq 0$
- OAM arises from "skew rays"
- Skew rays give the right answer to
  - Transfer of OAM to particles
  - Generation of OAM
  - Frequency shift

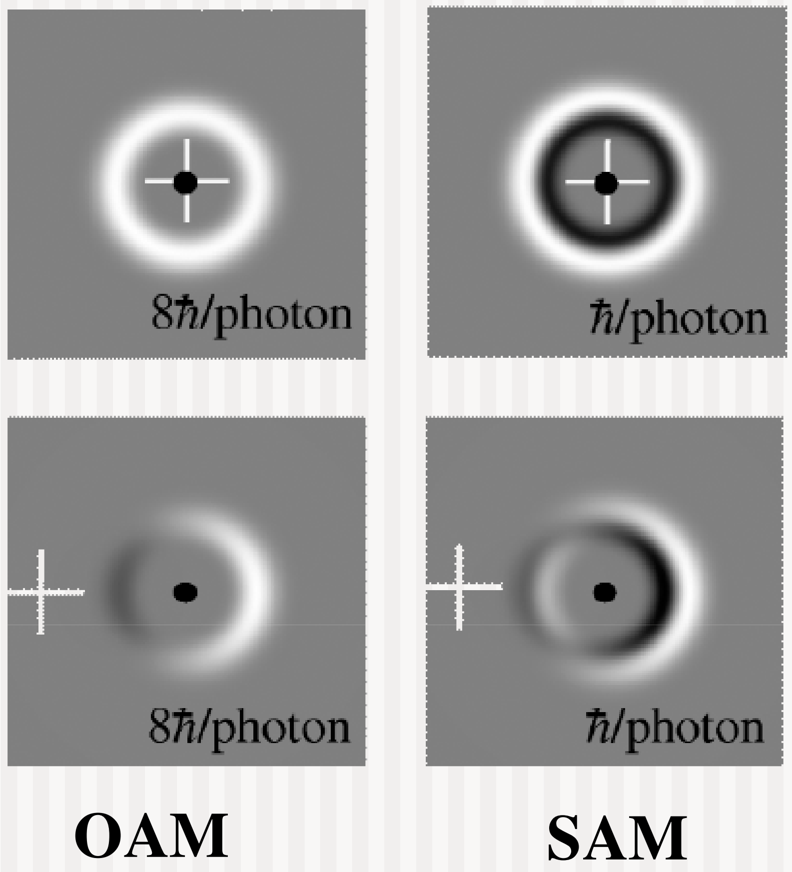



Simmons and Guttmann (1970)

# Numerical calculation of AM density

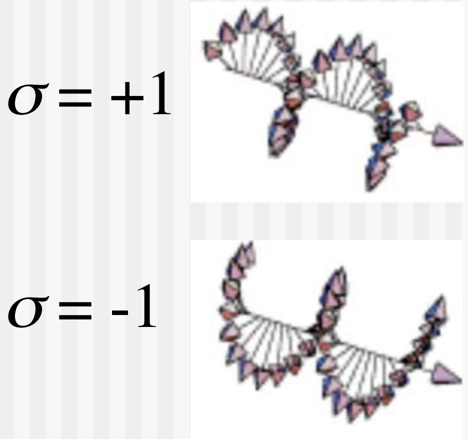
- OAM arises from phase gradients
- SAM arises from circular polarisation & intensity gradient
  - Intensity gradient set by “edge” of observation
- Numerical evaluation of fields gives correct answer

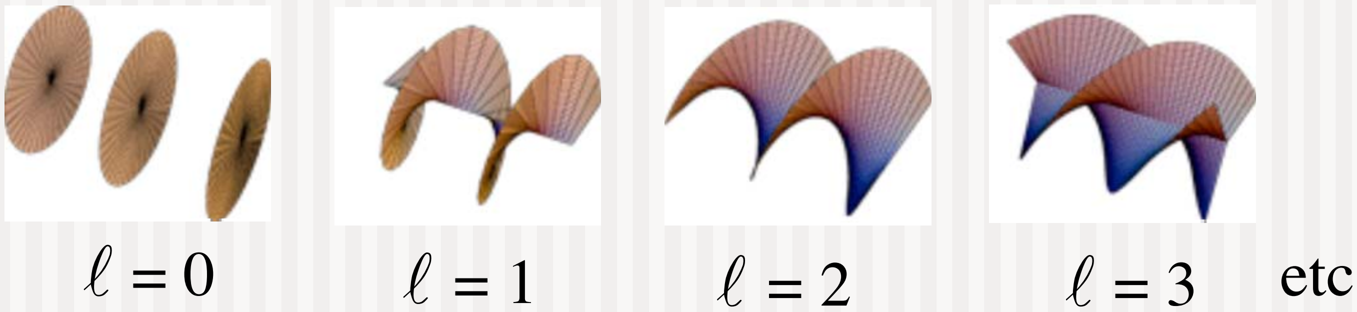
LG beam, $\ell$=8, σ=1



# Angular momentum in terms of photons

- Spin angular momentum
  - Circular polarisation
  - $\sigma\hbar$ per photon
- Orbital angular momentum
  - Helical phasefronts
  - $\ell\hbar$ per photon



# Ray-optics to model OAM

Padgett and Allen, Opt Commun 1995

# Poynting vector

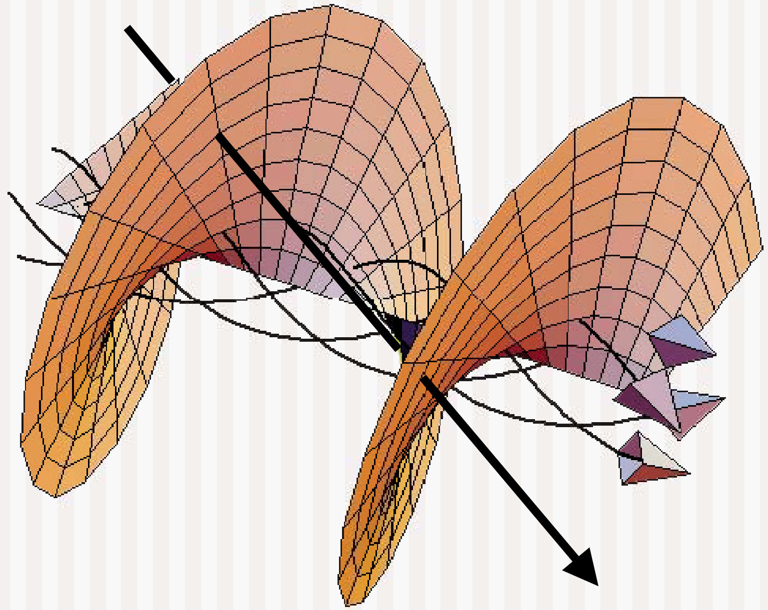
- Poynting vector (paraxial)

$$S\alpha\left(\frac{zr}{z_r^2+z^2}\hat{r}+\frac{\ell}{kr}\hat{\phi}+\hat{z}\right)$$

- Radius of max intensity (p=0)

$$r(@I_{\max})=\sqrt{\frac{\ell}{2}}w(z)$$

- Direction of Poynting vector at r (@$I_{max}$)
  - Straight line!
  - Skew angle $\theta = \ell / kr$



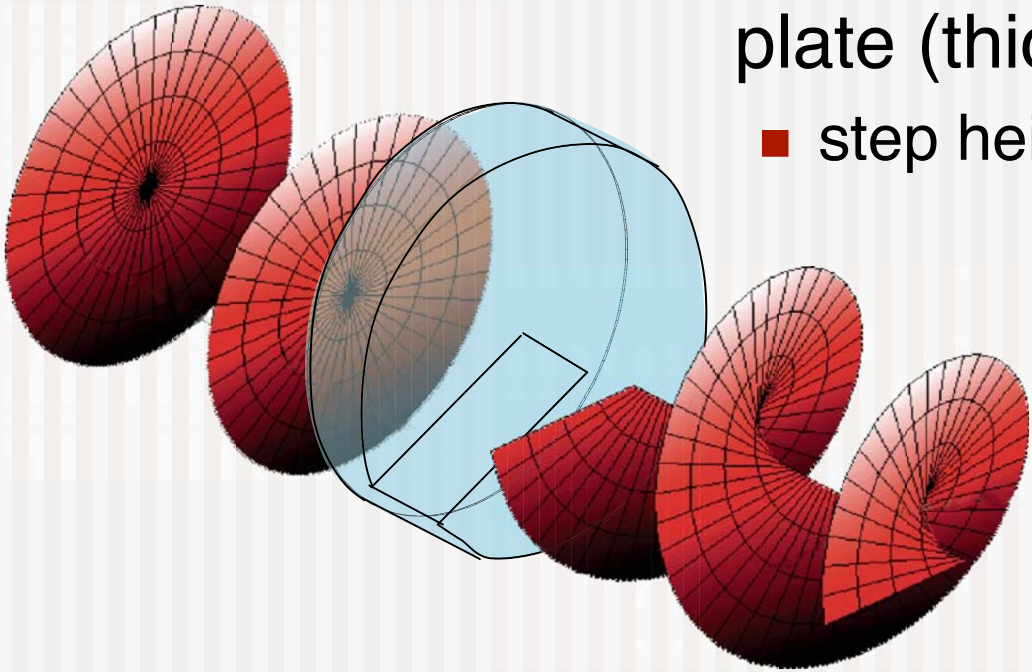
Momentum and energy flow perpendicular to phasefronts

# Generation of Beams with Orbital Angular Momentum

Beijersbergen *et al.* Opt Commun.1992

Bazhenov, *et al.* JEPT Lett. 1990

# Making helical phasefronts



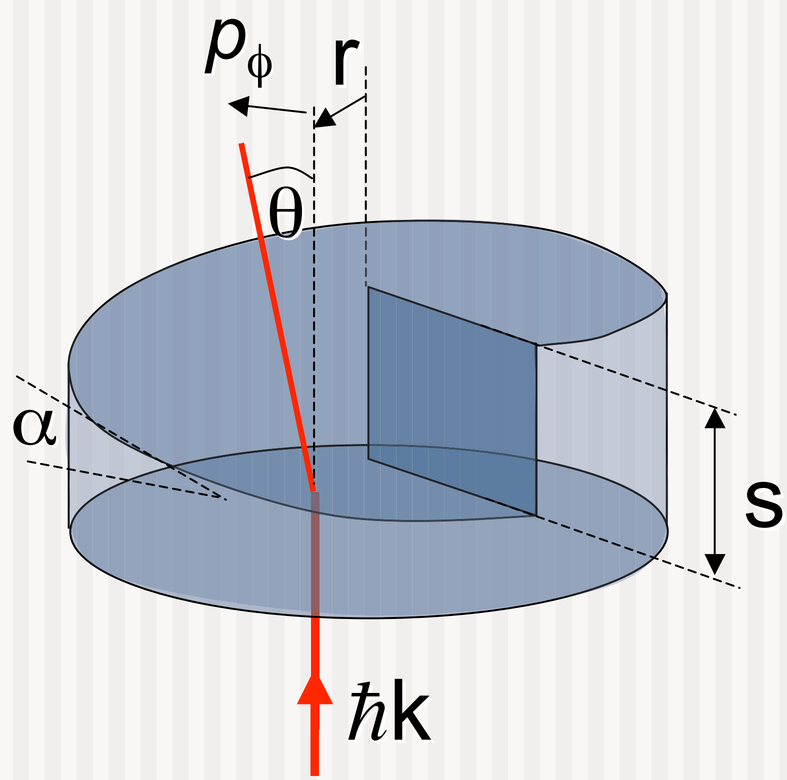
- Pass plane-wave through a spiral-phase plate (thickness $\alpha\ \phi$)
  - step height= $\ell\lambda/(n-1)$

# Ray-optics to model OAM

Turnbull *et al.* Opt. Commun. 1996

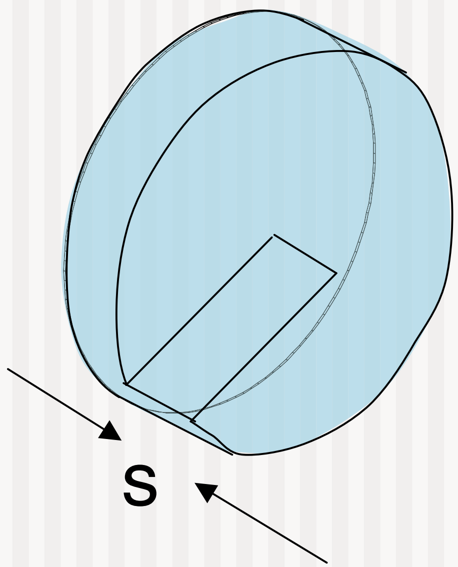
# Spiral Phase-plate

- Step height constraint
  - $s = \ell\lambda/(n-1)$
- Local slope of plate
  - $\alpha = s/2\pi r$
- Refraction at surface
  - $\theta = (n-1)s/2\pi r$
  - $p_\phi = \hbar k(n-1)s/2\pi r$
- Angular momentum
  - $L_z = p_\phi r = \ell\hbar$ per photon




Ray-Optics gives the right answer

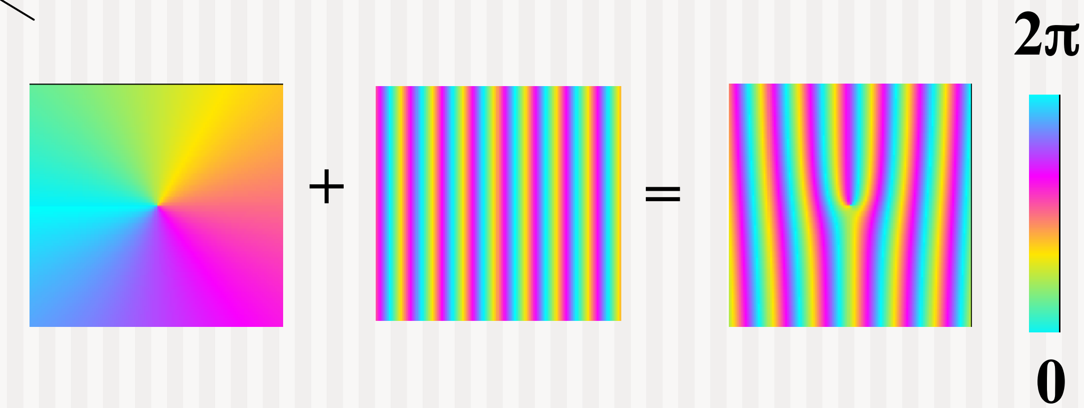
# Designing helical phase hologram




- Spiral Phase-plate
  s= $\ell\lambda/(n-1)$

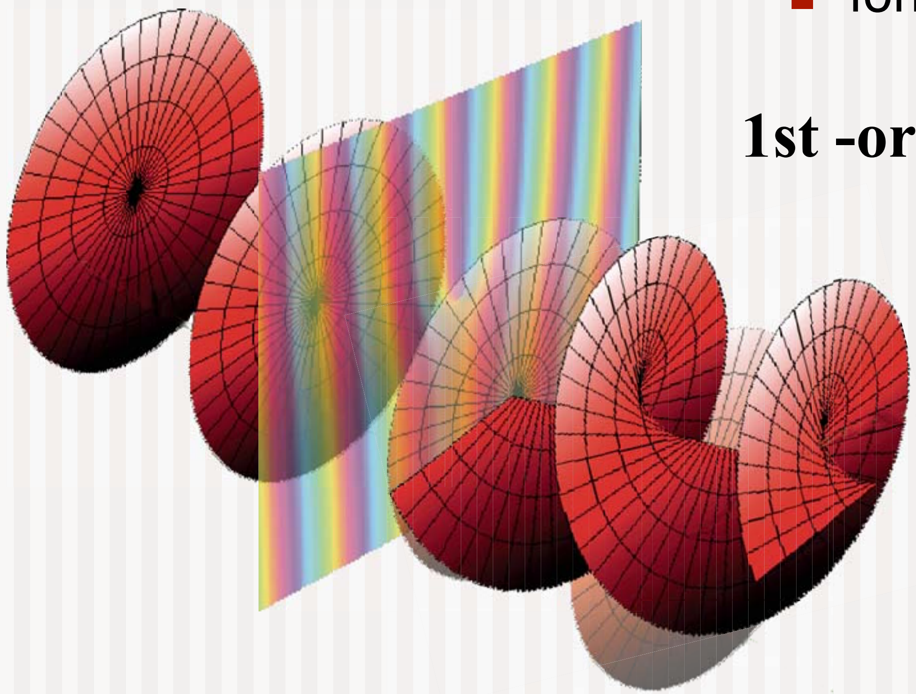
- Holographically

  e.g. $\ell = 1$ ▷



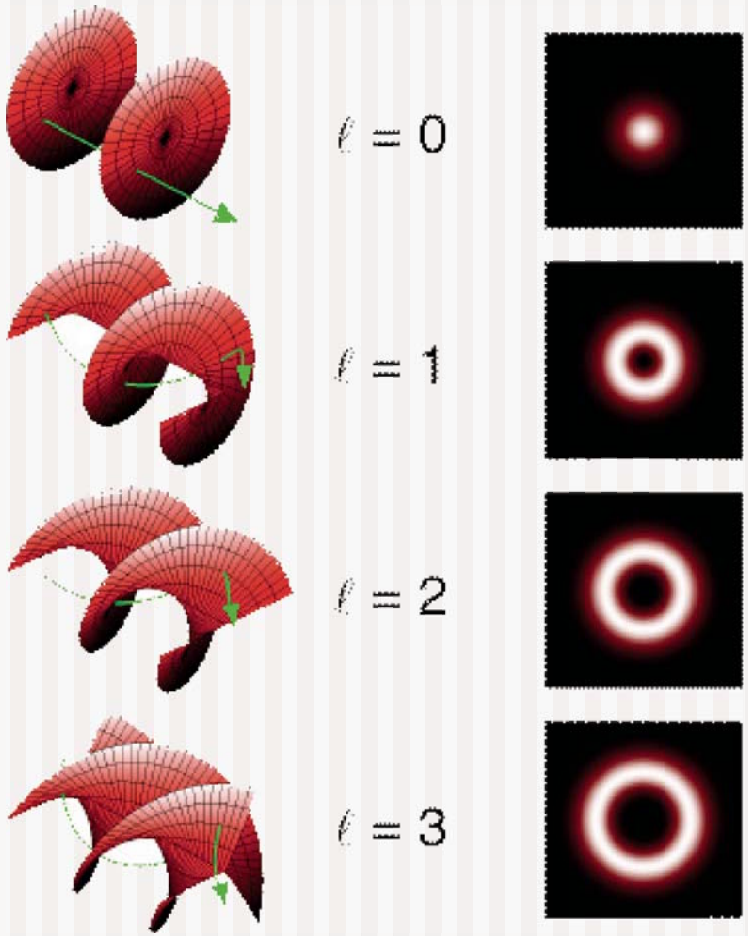
# Making helical phasefronts

- Holographic equivalent (diffraction grating)
  - fork no. = $\ell$

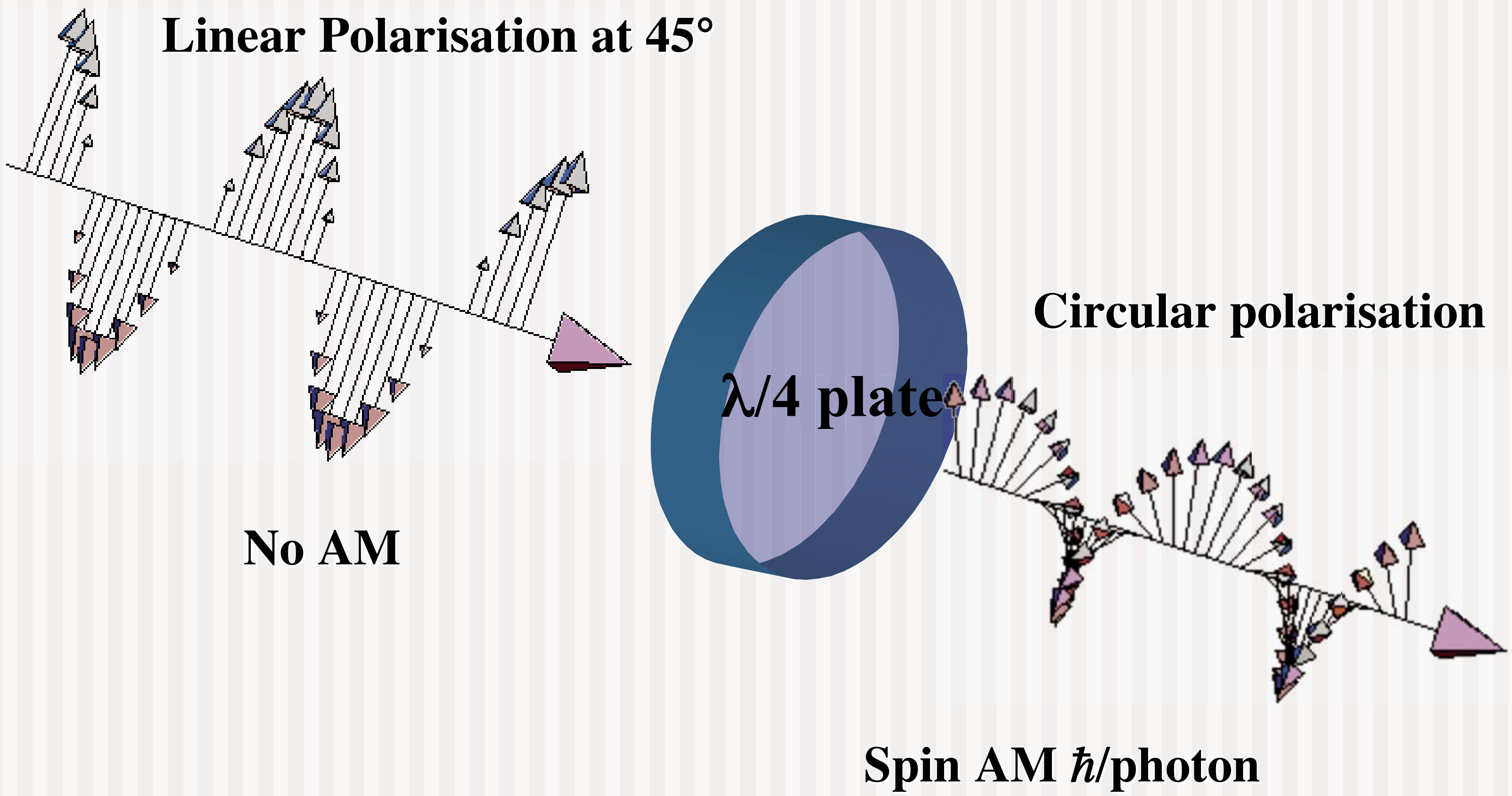


# What do these beams look like?

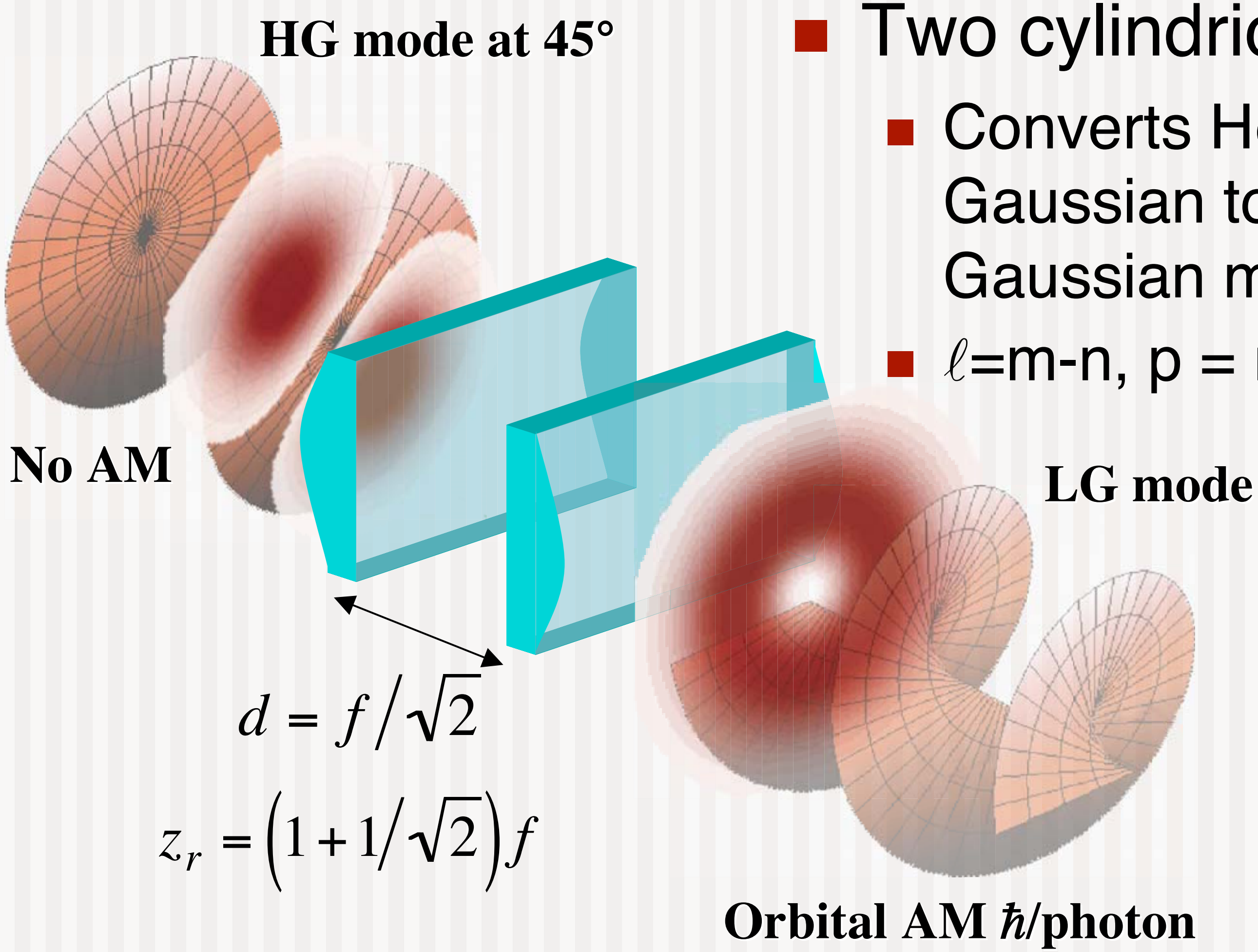
- Any beam with “skewed” phase-fronts carries OAM
- The “pure” OAM states have $\ell$-intertwined helical phasefronts
  - $u(\phi) = \exp(i\ell\phi)$
  - OAM $=\ell\hbar$ per photon
- Example OAM beams
  - Laguerre-Gaussian modes
  - High-order Bessel beams



# Making circularly polarised light



# Making helical phasefronts




$$d = f/\sqrt{2}$$

$$z_r = \left(1 + 1/\sqrt{2}\right) f$$

- Two cylindrical lenses
  - Converts Hermite-Gaussian to Laguerre-Gaussian modes
  - $\ell$=m-n, p = min(m,n)

# Orthogonal states for spin and orbital angular momentum

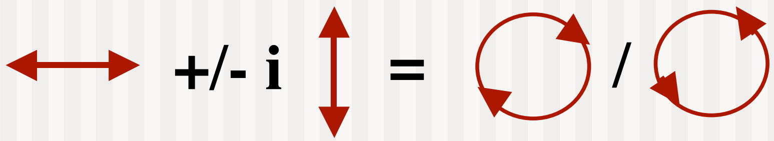
Beijersbergen *et al*. Opt Commun. 1992

Padgett and Courtial, Opt Lett.1999

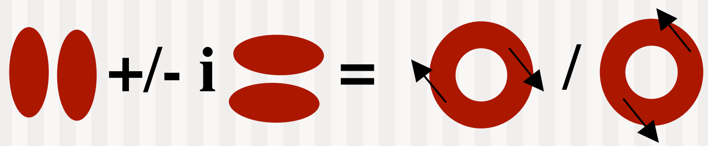
Allen, *et al.* Phys Rev E. 1999

# Transformations between basis sets

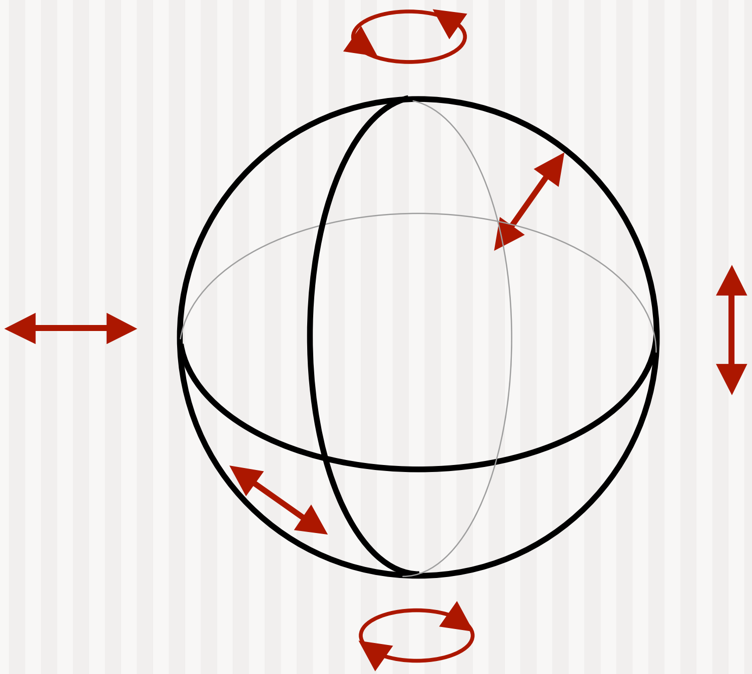
- Polarisation (spin angular momentum)
  - Linear cf circular polarisation
  - Transformation using waveplates



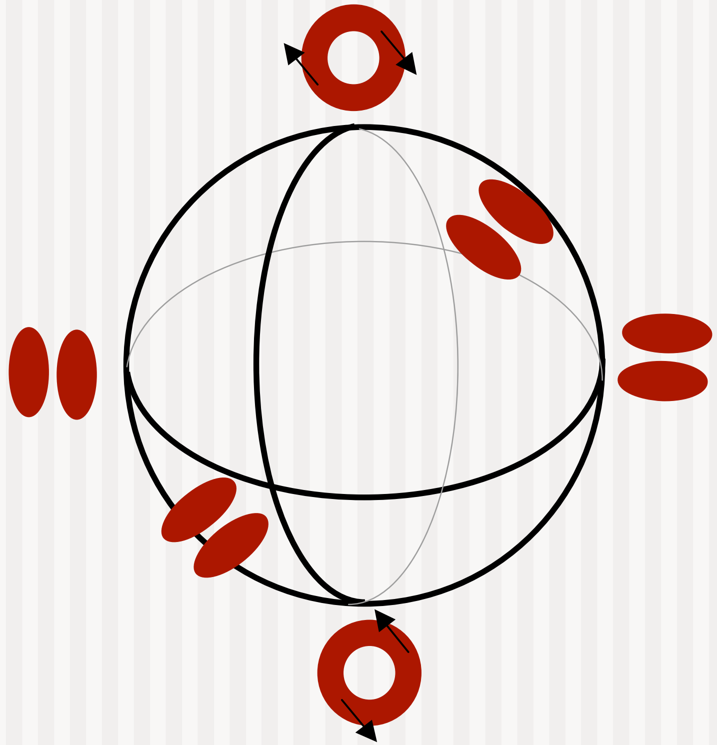

- Mode structure (orbital angular momentum)
  - Hermite-Gaussian cf Laguerre-Gaussian modes
  - Transformation ?



# Spin and orbital AM



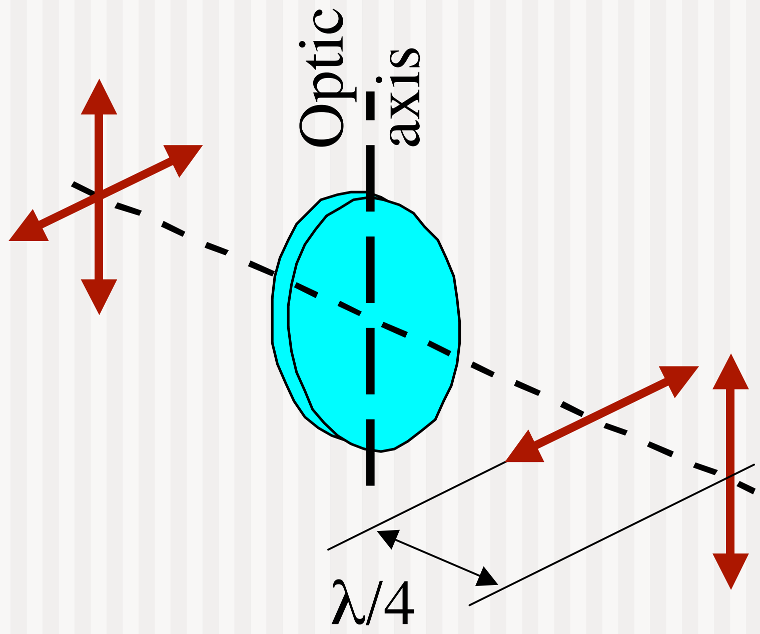
Poincaré Sphere



Poincaré Sphere for OAM

# How to change phase of orthogonal SAM states

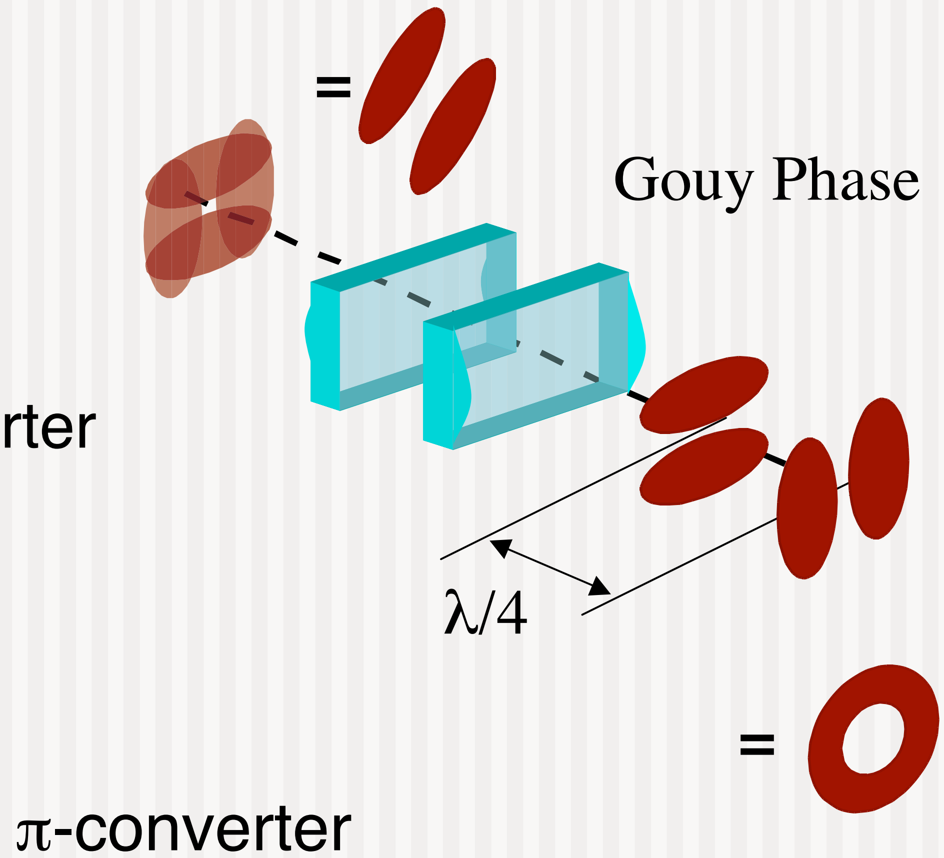
- Change the relative phase of linear polarisation states using a waveplate
  - 180° use half-wave plate
  - 90° use quarter-wave plate




Quarter-waveplate

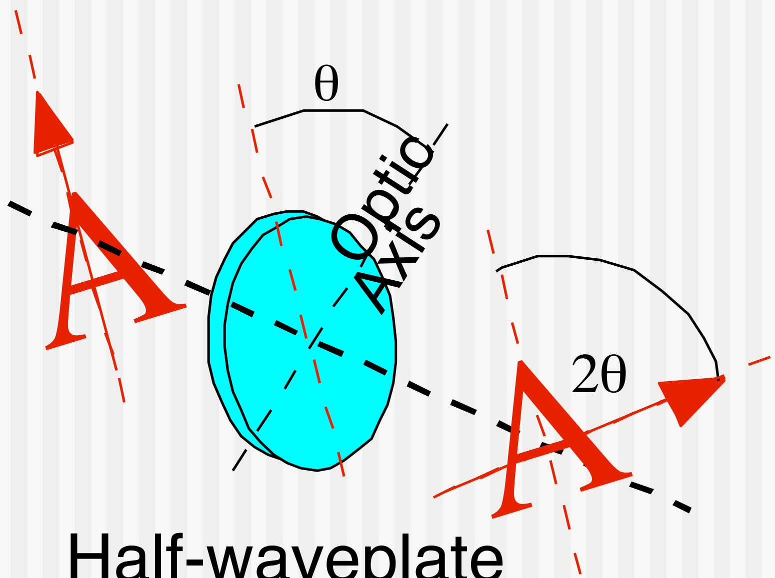
# How to change phase of orthogonal OAM states

- Change the relative phase of linear polarisation states using a cylindrical lenses
  - 180° use π -converter
  - 90° use π/2 -converter



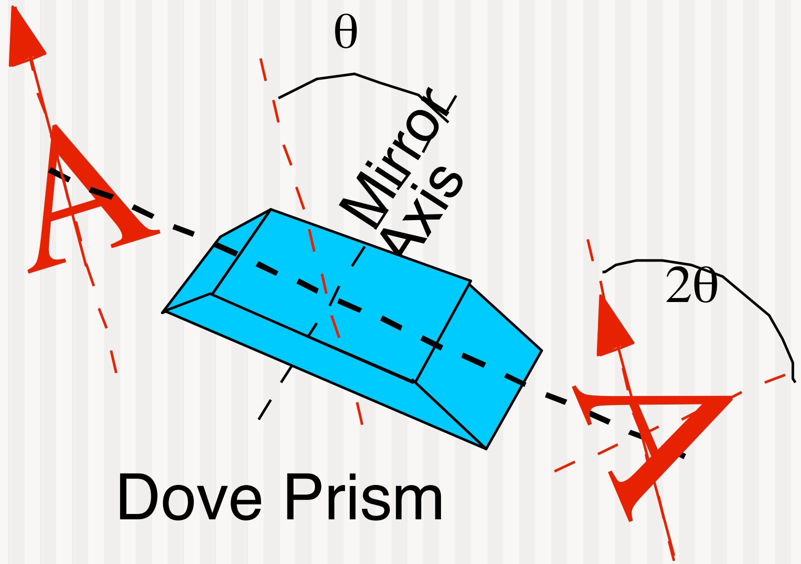
# How to rotate a light beam (or at least the polarisation)

- Rotate polarisation by rotating half-waveplate (but does not rotate image)
- Phase change
  - $\Delta\psi = 2\theta\sigma$



# How to rotate an image (phase of a light beam)

- Rotate phase structure with Dove prism (but does not rotate polarisation)
- Phase change
  - $\Delta\psi = 2\theta\ell$



# Spin and orbital equivalence

| Spin | Vs | Orbital |
|---|---|---|
| Birefringence | Equiv. | Astigmatism |
| Optical Activity | Equiv. | Image rotation |

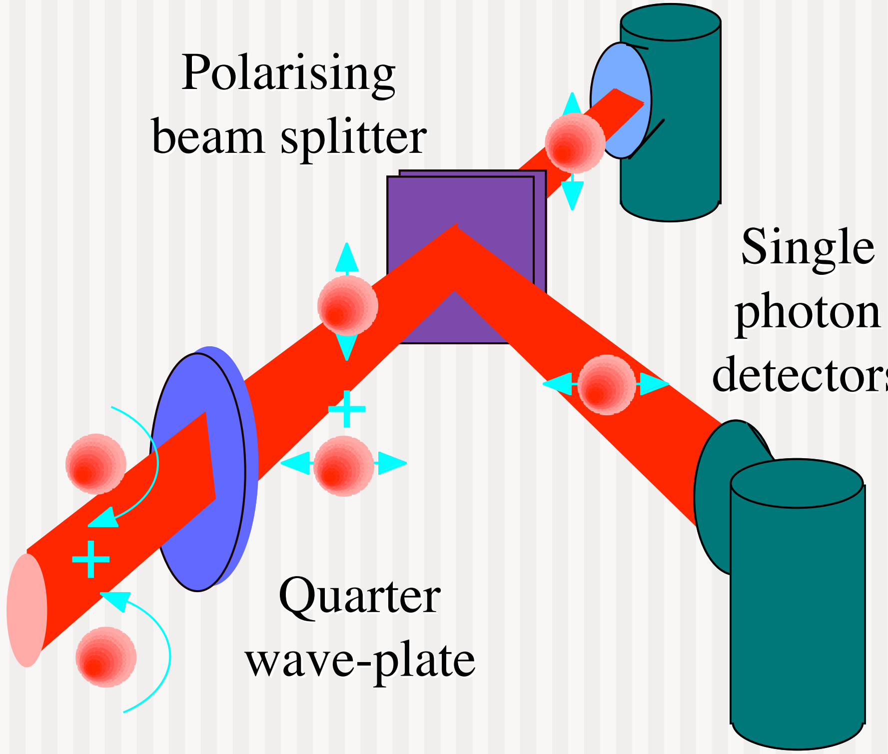
# Measuring OAM

Mair *et al.* Nature 2001.

Leach *et al.* Phys. Rev. Lett. 2002

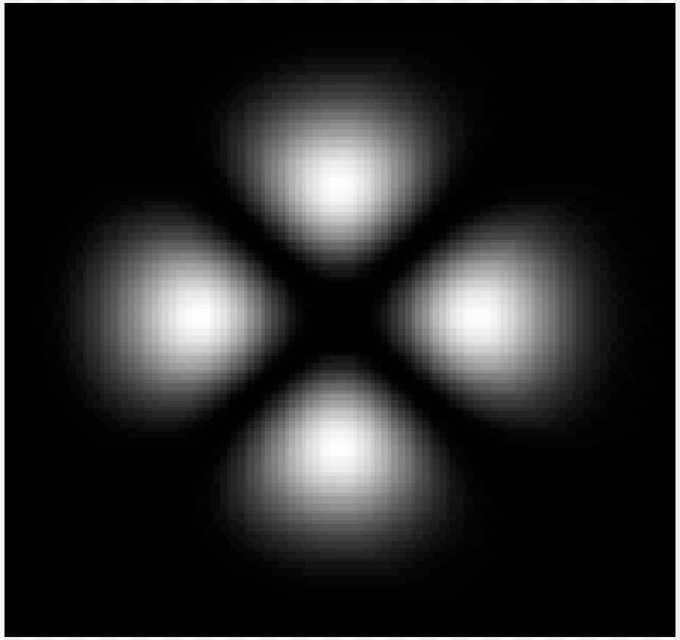
# Measuring SAM of single photons

- Measuring SAM is the same as measuring polarisaion
- Quarter- waveplate converts circular to linear polarisation
- Polarising beam splitter separates states
- Polarisation measured to be in one of two states

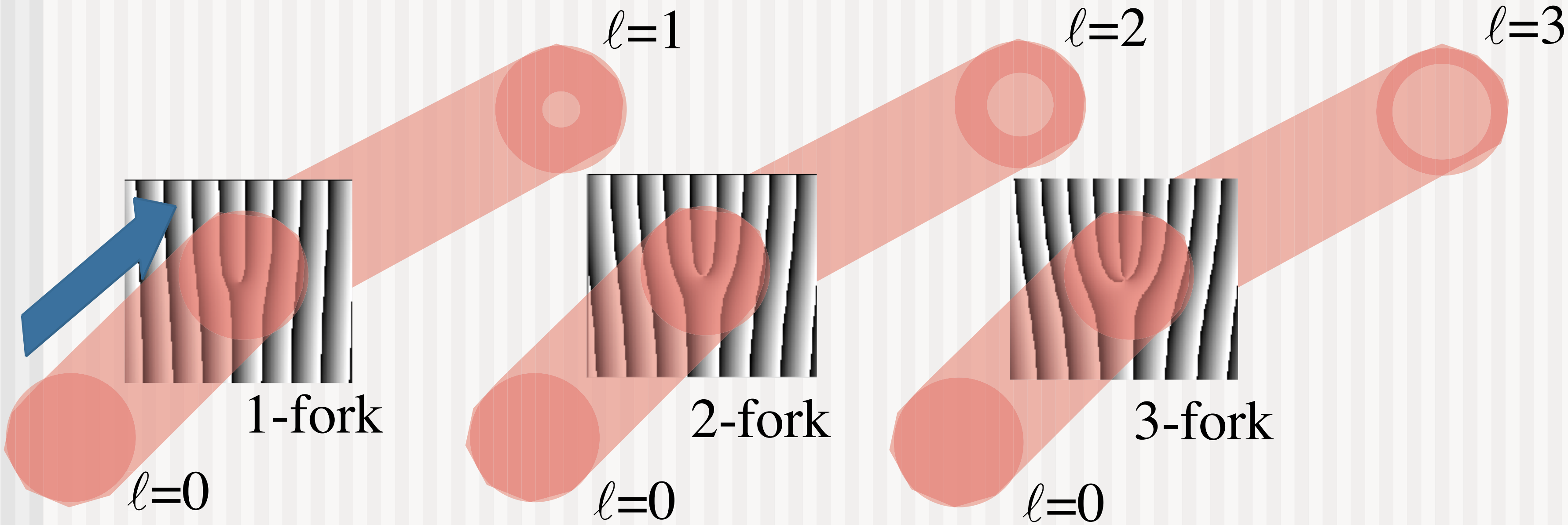


# Measuring OAM of beams

- Interfere a helical beam with a plane wave
  - gives $\ell$ spokes
  - or $\ell$ -fork dislocations
- Requires many photons in the same state

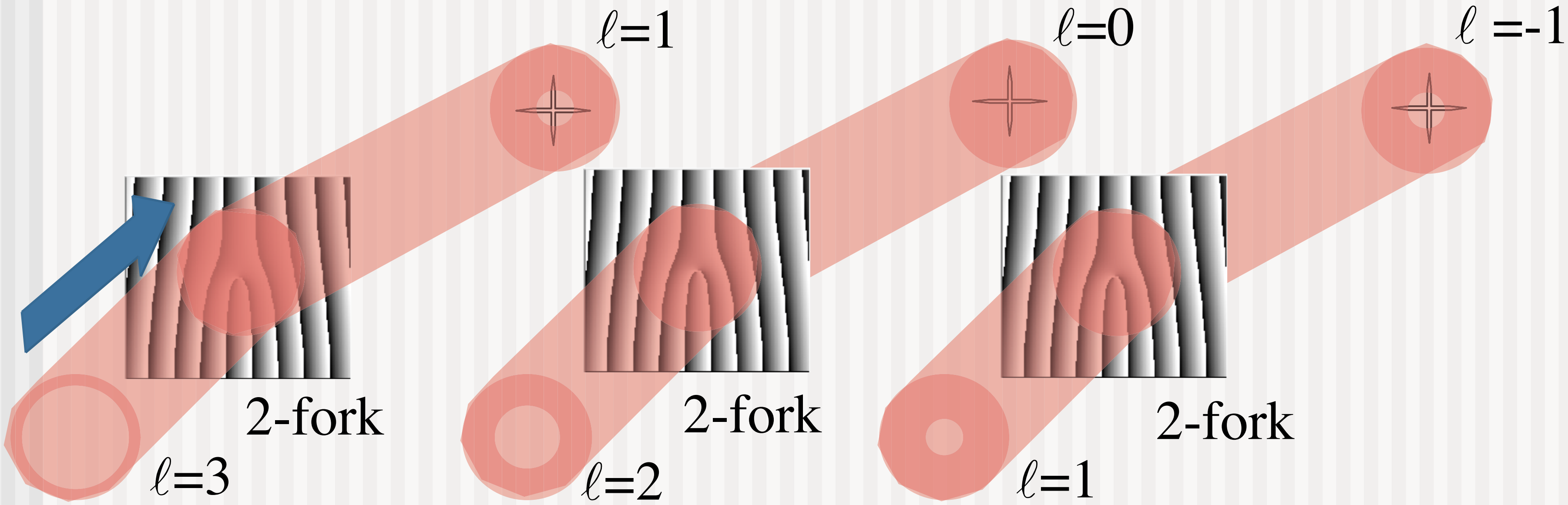


# Generating LG beams




- Diffractive optical components (computer generated holograms)
- Incident Gaussian beam converted to LG mode
- Number of dislocations sets "*l*" of first-order diffracted beam

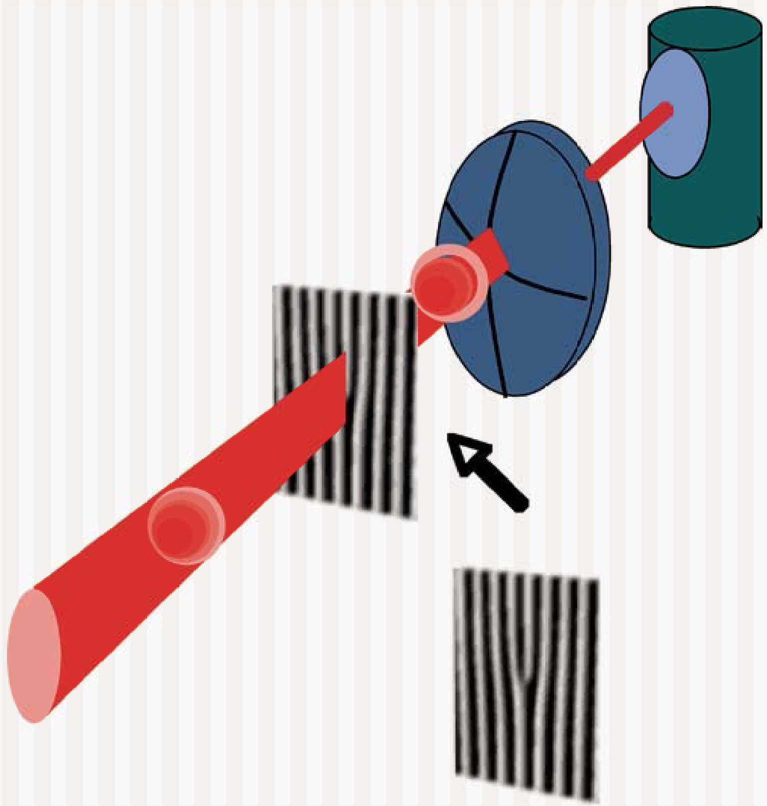
# Measuring LG beams




- Incident beam ℓ changed by fork number
- Measurement of on-axis intensity gives ℓ=2 component
- Inherent efficiency 1/number of test channels
- Make interactive by using SLM as diffractive optic

# Confirming OAM of single photons

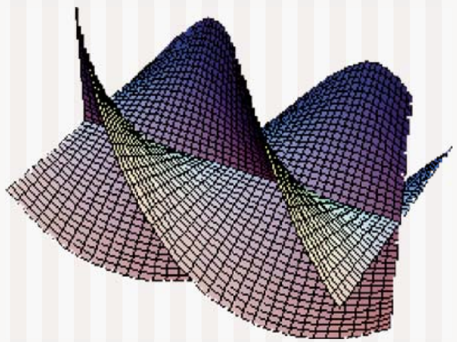
- Infinite number of discrete OAM states
- Can use hologram to CONFIRM particular $\ell$-value,
- e.g. $\ell$=1 ? yes or no
- But cannot make a general measurement of OAM



# Measuring OAM - using rotational symmetry

- rotate the mode through 180°

even *l*



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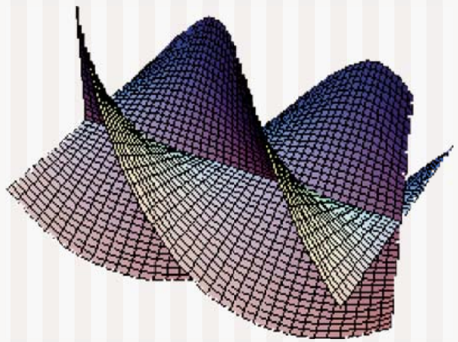
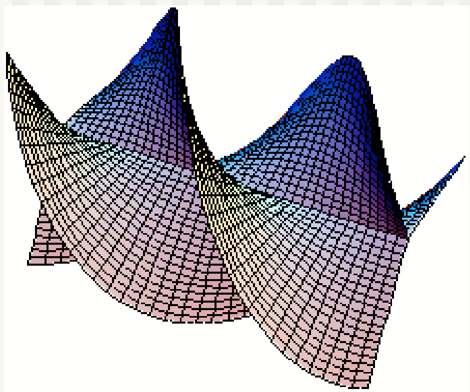


constructive interference

odd *l*



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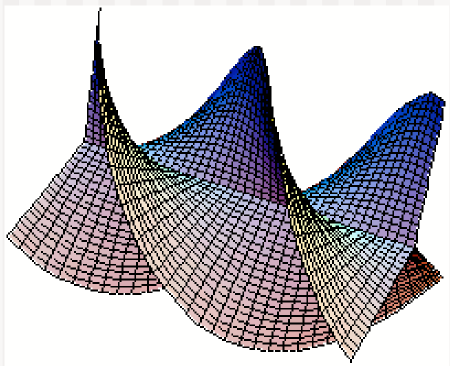
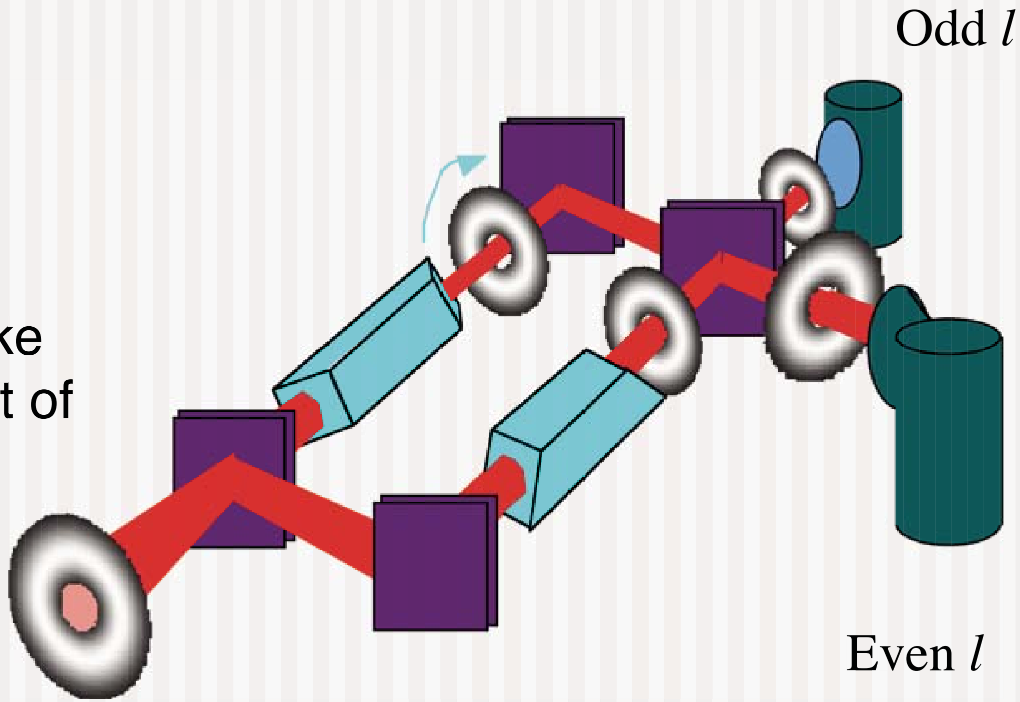


destructive interference

# Measuring OAM of single photons

- Use "beam rotating" interferometer to sort modes by their rotational symmetry
- Employ additional stages to give further sorting
- Can (in principle) make general measurement of OAM



# Sorting odd and even $l$-values

- Change $\ell$-values of input mode
- Image both interferometer outputs - see what happens
- Can now use interferometer to sort odd and even $\ell$-values