The Abdus Salam
International Centre for Theoretical Physics

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WINTER COLLEGE

on

QUANTUM AND CLASSICAL ASPECTS

of

INFORMATION OPTICS

30 January - 10 February 2006

Optical Integral Transforms for Information Processing

Lecture 3: Applications for Information Processing

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Optical Integral Transforms for Information Processing

Lecture 3: Applications for Information Processing

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Winter College on Quantum and Classical Aspects of Information Optics
30 January - 10 February 2006, Miramare - Trieste, Italy

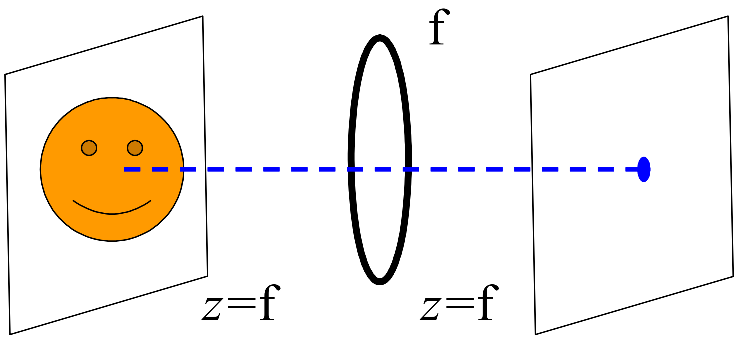
Outlines

- Convolution and some specific operations: image replication, differentiation, integration
- Filtering in different phase space domains
  - shift invariant and shift variant filtering
  - fractional convolution
- Image quality improvement, feature extraction
- Pattern recognition
- Security systems
- Multiresolution analysis: optical wavelet transform
- Conclusions

Integration in Fourier plane

- The value of the Fourier transform in the origin equals to the integral of the input image

$$\iint f(x,y)dxdy = F(0,0)$$



- It is always 0 for odd functions
- Insertion of the blocking mask in the input plane permits to vary the integration limits.
- Cylindrical lenses – integration on one coordinate

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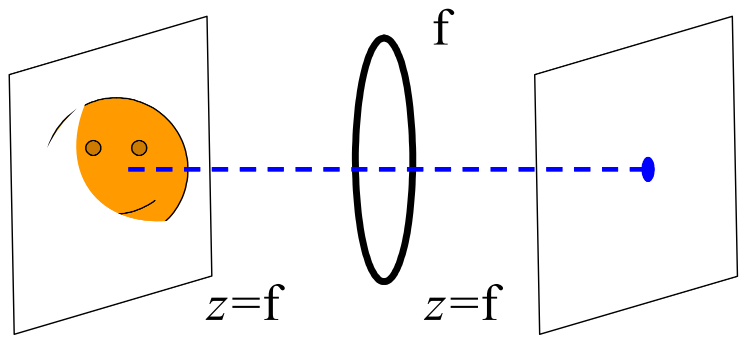

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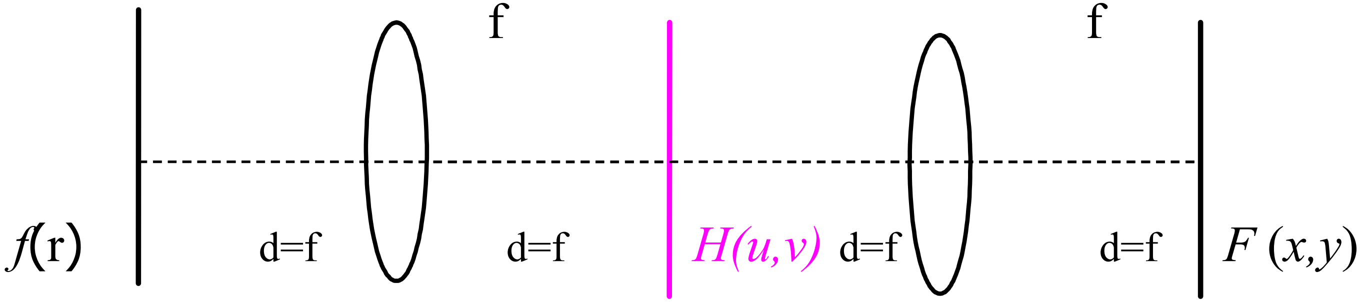
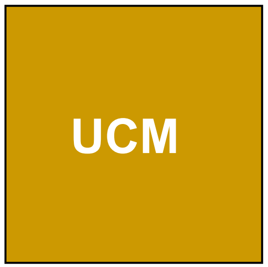
Periodic gratings

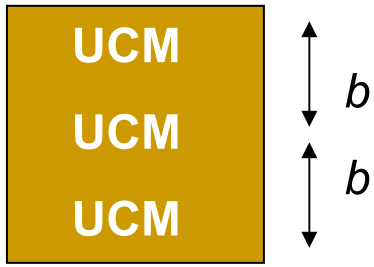
- periodic filter ⟹ object replication



$$H(u,v) = \tfrac{1}{2}\left\{1 + \exp\left[i(2\pi v b + \varphi)\right] + \exp\left[-i(2\pi v b + \varphi)\right]\right\}$$



triplication by *cosine* grating



$$F(x,y) = \tfrac{1}{2}\left\{f(x,y) + f(x,y-b)\exp(-i\varphi) + f(x,y+b)\exp(i\varphi)\right\}$$

- φ is related to the grating position with respect to the optical axis.

S. H. Lee (ed), *Topics in Applied Physics* 48: Optical information processing, Springer-Verlag, NY (1991)

Image addition and subtraction

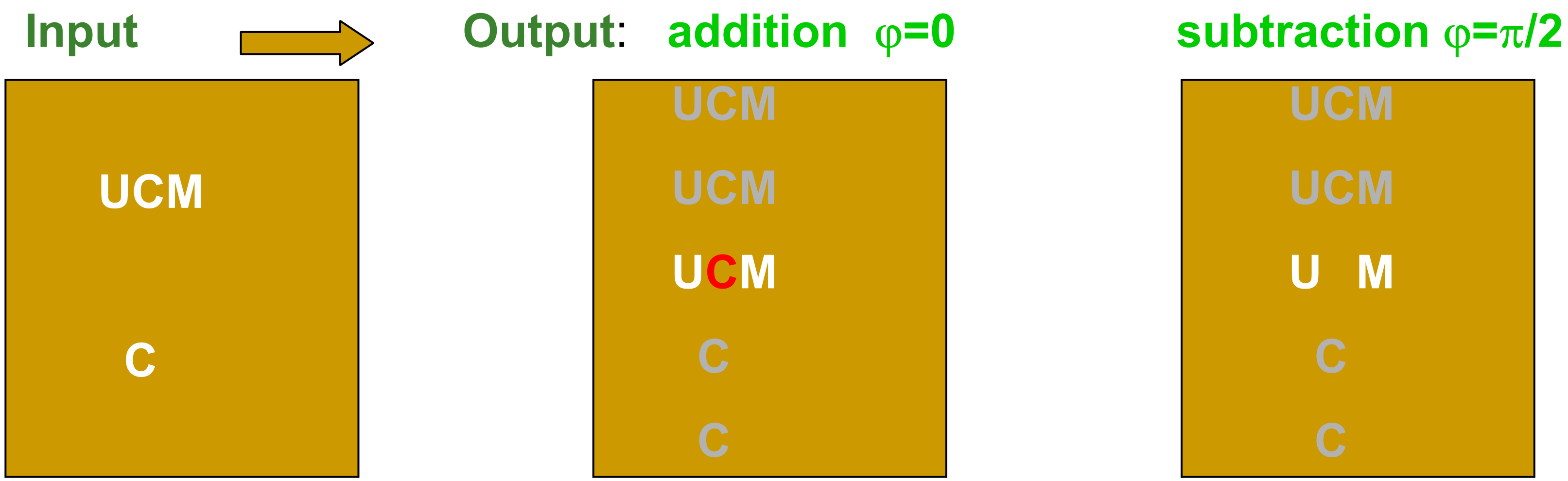
- If at the input there are two non-overlapping objects

$$f(\xi,\eta) = f_1(\xi,\eta - a) + f_2(\xi,\eta + a)$$

- and *a*=*b* (*b* is the frequency of the periodic *cosine* grating) then at the output

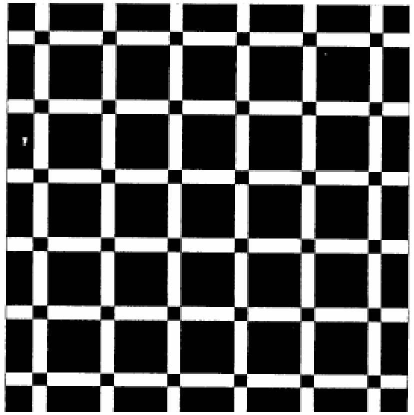
$$F(x,y) = \tfrac{1}{2}\left\{ f_1(x,y) + f_2(x,y)\exp(-i2\varphi)\right\}\exp(i\varphi)$$

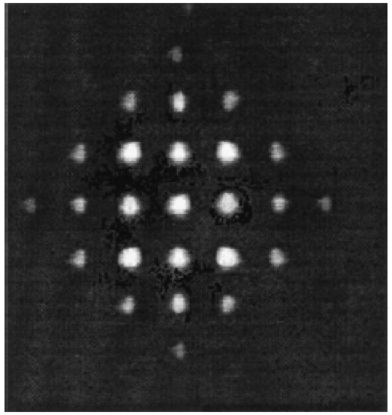
$$+\,4\,\text{other off-axis terms}$$



Dammann grating

- Binary phase gratings that have several diffraction orders of equal intensity are used to produce an array of images.
- Dammann grating produces an array of 3 x 3 diffraction orders of equal intensity





- 2-D array of 3 x 3 of replicated spectra of the input image (Dammann grating is placed just after the image and the FT is performed by lens)

I. Ouzieli and D. Mendlovic, *Appl. Opt.* 35, 5839 (1996)

Differentiation

- Fourier transform property

$$f(x,y) = TF^{-1}\{F(u,v)\} = \iint F(u,v)\exp\left[i2\pi(xu+yv)\right]dudv$$

$$\frac{\partial^{k+l} f(x,y)}{\partial x^k \partial^l y} = (i2\pi)^{k+l} \iint u^k v^l F(u,v)\exp\left[i2\pi(xu+yv)\right]dudv$$

- Differentiation of order *k, l* by filter $H(u,v) = (i2\pi)^{l+k} u^k v^l$

- This filter cannot be used at the FT plane origin (*u,v* near to 0); Negative values are realized by π phase step plate

J. K. T. Eu, C. Y. C. Liu, A. W. Lohmann, *Opt. Commun*. 9, 168 (1973); G. O. Reynolds, J. B. DeVelis, et al, *Tutorials in Fourier Optics*, SPIE Optical Engineering Press, Washington (1989)

Differentiation by composite gratings

- Composite grating: two *cosine* gratings with slightly different frequencies

$$H(u,v) = \lim_{\varepsilon \to 0} \left[\tfrac{1}{2} + \cos(bv) - \cos(bv + \varepsilon v) \right]$$

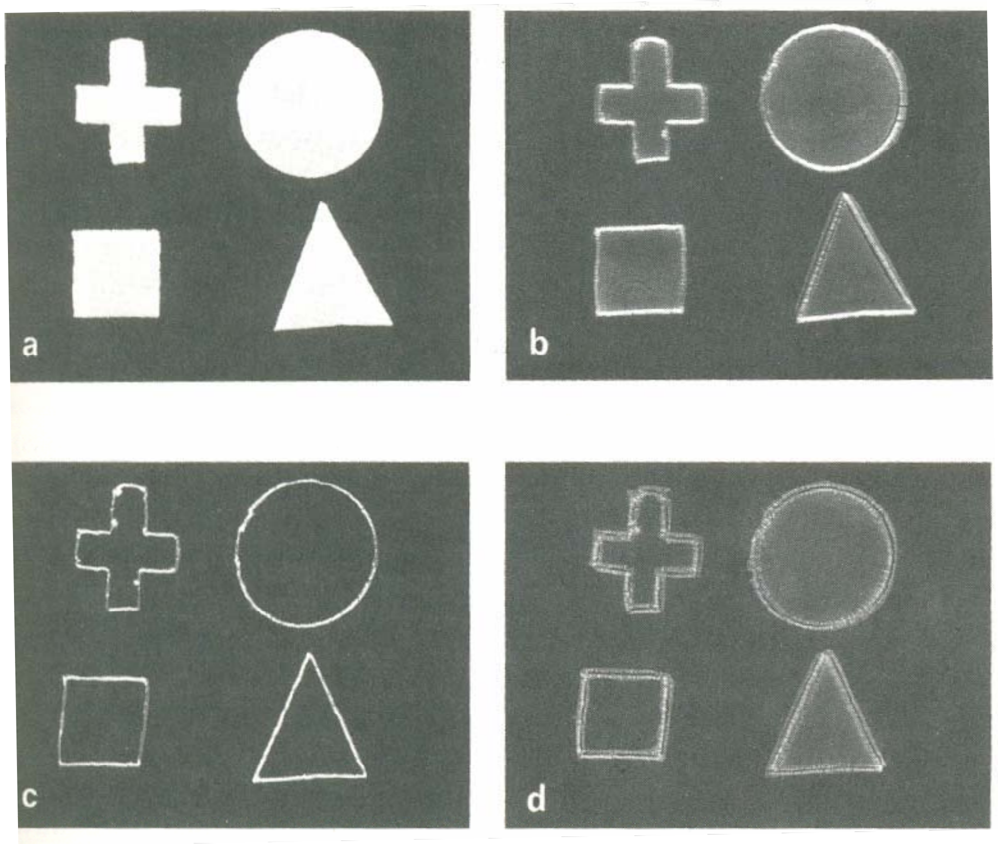
- This filter has an impulse response

$$h(x,y) = \lim_{\varepsilon \to 0} \tfrac{1}{2} \left[\delta(x,y) + \delta(x, y \pm b) - \delta(x, y \pm (b + \varepsilon)) \right]$$

with off-axis component can perform differentiation operation

$$\frac{\partial}{\partial y} f(x,y) = -\lim_{\varepsilon \to 0} \tfrac{1}{\varepsilon} \left[f(x, y+b) - f(x, y+b+\varepsilon) \right]$$

Optical differentiation: examples



Optical differentiation by composite gratings

a - original pattern $f(x,y)$

b - $df(x,y)/dy$ (horizontal edge enhancement)

c - $df(x,y)/dx + df(x,y)/dy$

d - $d^2f(x,y)/dx^2 + d^2f(x,y)/dy^2$

S. H. Lee (ed), *Topics in Applied Physics* 48: Optical information processing, Springer-Verlag, NY (1991)

Outlines

- Convolution and some specific operations: image replication, differentiation, integration
- **Filtering in different phase space domains**
  - **shift invariant and shift variant filtering**
  - **fractional convolution**
- **Image quality improvement, feature extraction**
- Pattern recognition
- Security systems
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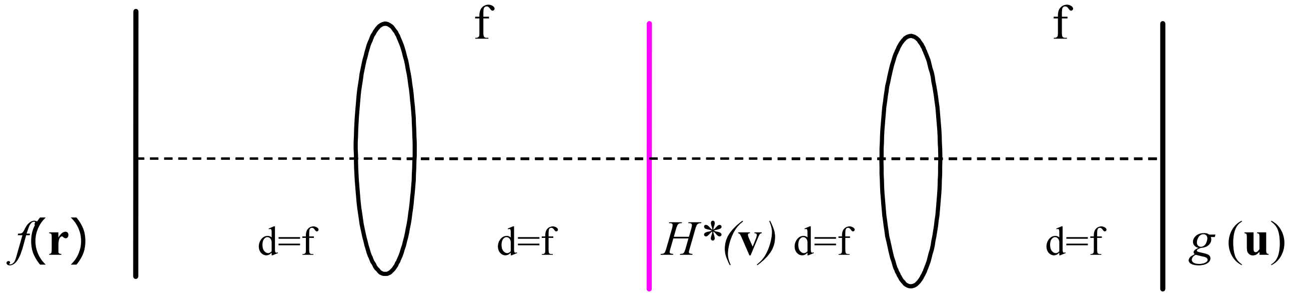
- Conclusions

Filtering in different domains

- Filtering in space domain (example: cutting a part of image)
- Filtering in Fourier domain affects the entire image: low (high) frequencies – big (small) size image details



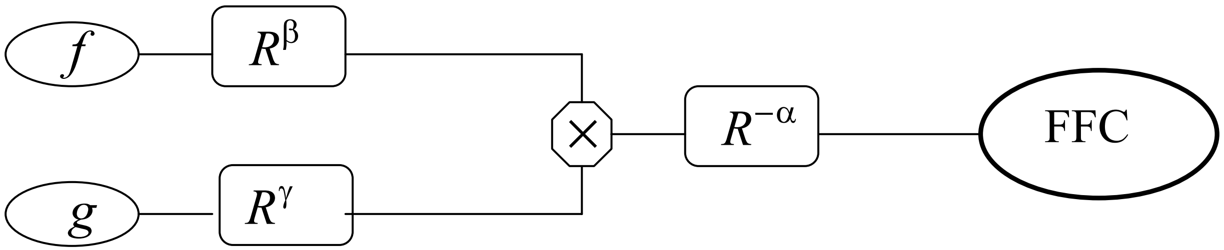
$$g(\mathbf{u}) = FT\left\{F(\mathbf{v})H^*(\mathbf{v})\right\}$$

- Filtering in the fractional FT domain (space-frequency filtering)

Fractional Fourier Convolution

- Ordinary convolution $G_{f,g}(\mathbf{r}) = FT^{-1}\left\{FT\left[f(\mathbf{r})\right]FT\left[g(\mathbf{r})\right]\right\}$
- Fractional FT convolution

$$H_{f,g}(\mathbf{r},\alpha,\beta,\gamma) = R^{-\alpha}\left\{R^{\beta}\left[f(\mathbf{r})\right]R^{\gamma}\left[g(\mathbf{r})\right]\right\}$$



- What parameters α, β, and γ are useful? It depends on the applications:
  - Pattern recognition (shift variant): $\beta = -\gamma$; $\alpha = \pi/2$
  - Filtering (denoising, encryption, watermarking, etc): various angles

O. Akay and G. F. Boudreaux-Bartels, *IEEE Trans. Sign. Proc.* 49, 979(2001); T. Alieva, M. J. Bastiaans, M. L. Calvo, *EURASIP J. Appl. Sign. Proc.* 2005, 1498 (2005).

Amplitude filtering in FT domain

- Low pass frequency filter $H=\exp(-a\mathbf{u}^2)$

  ⇩

  image smoothing

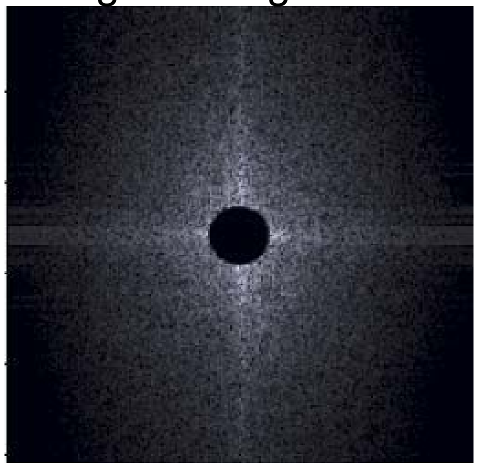
- High pass frequency filter

  ⇩

  edge enhancement

Original image

After Gaussian smooth filtering

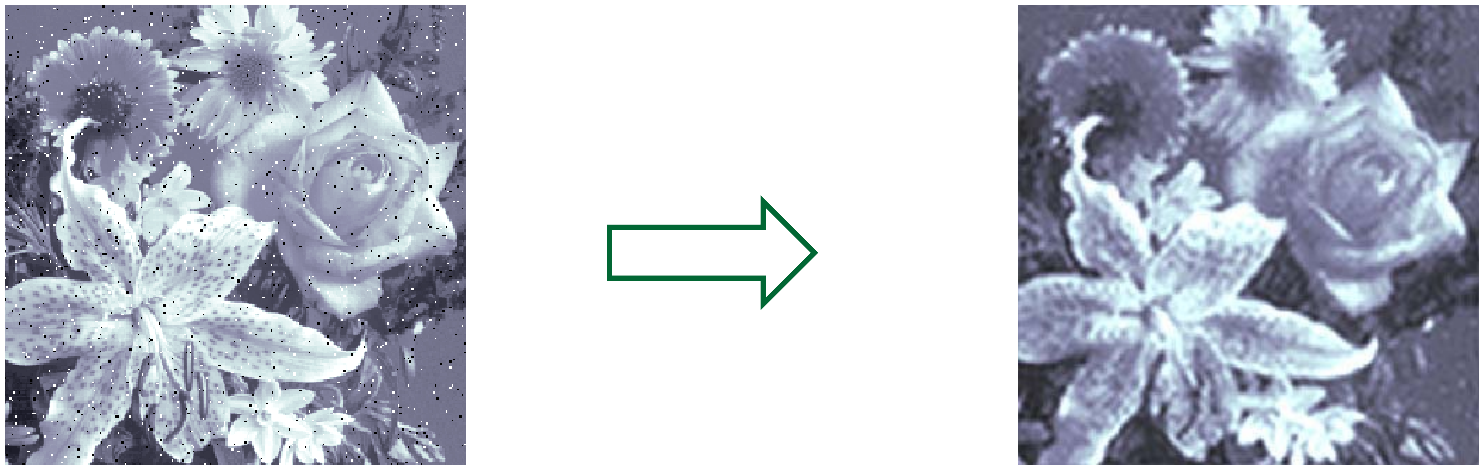


Low frequency blocking of FT

Reconstruction

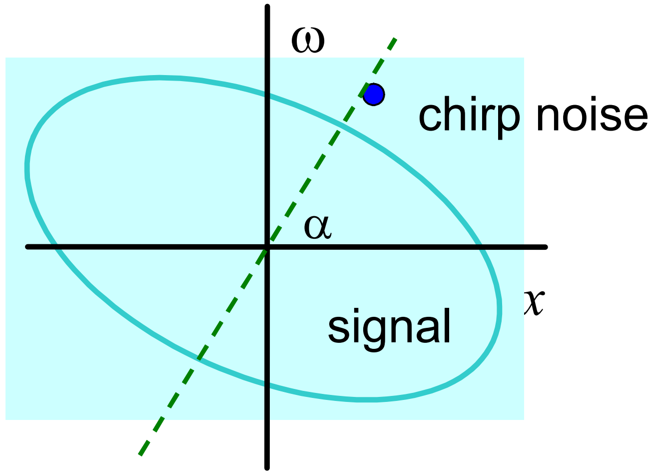
Noise reduction in FT domain

- Salt and pepper noise
  - has only high-frequency components
  - is distributed over all the image
- Low pass filtering in FT domain



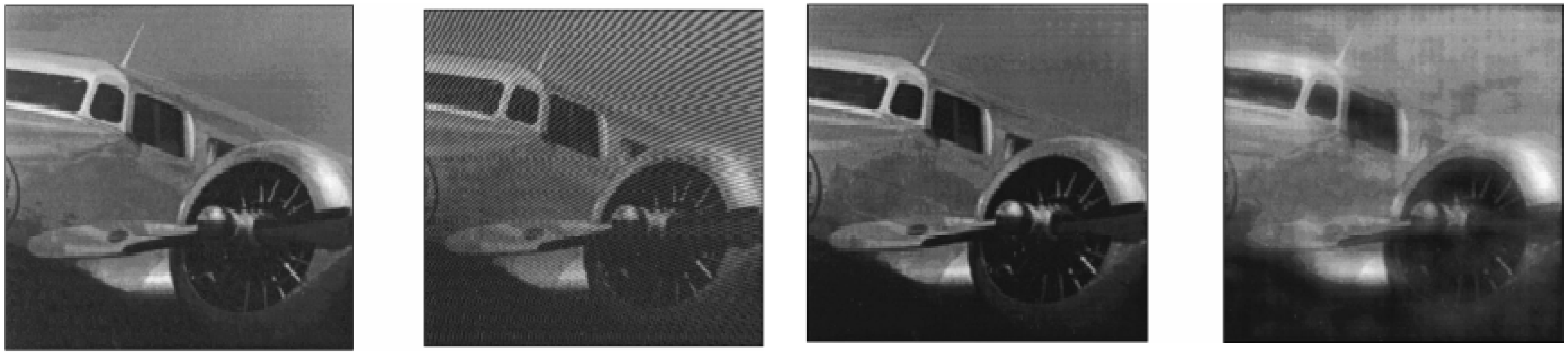
Noise reduction in fractional FT domain

- Chirp location in phase plane
- Fractional FT of a linear frequency modulated signal
  $x(t) = \exp(i\pi p t^{2} + i2\pi q t)$
  is a delta pulse located
  at position $q \sin\alpha$
  for the angle $\alpha = \pi/2 + \arctan p$



Noise filtering in fractional FT domain

- Filtering in fractional Fourier domains is effective in eliminating chirp like noise



Original image | noise corrupted image | filtering in optimum fractional FT domain | filtering in FT domain

- = fractional convolution $H_{f,g}(\mathbf{r},\alpha,\alpha,0) = R^{-\alpha}\left\{R^{\alpha}\left[f(\mathbf{r})\right]R^{0}\left[g(\mathbf{r})\right]\right\}$

α is determined by chirp location; g(**r**) is blocking point mask

M. Alper Kutay and H. M. Ozaktas, *J. Opt. Soc. Am. A* 15, 825 (1998)

Hilbert transform

- Hilbert transform of real function f(x)

$$H[f(x)](u) = H(u) = \pi^{-1} \int_{-\infty}^{\infty} \frac{f(x)}{u-x} dx$$

- Inverse transform

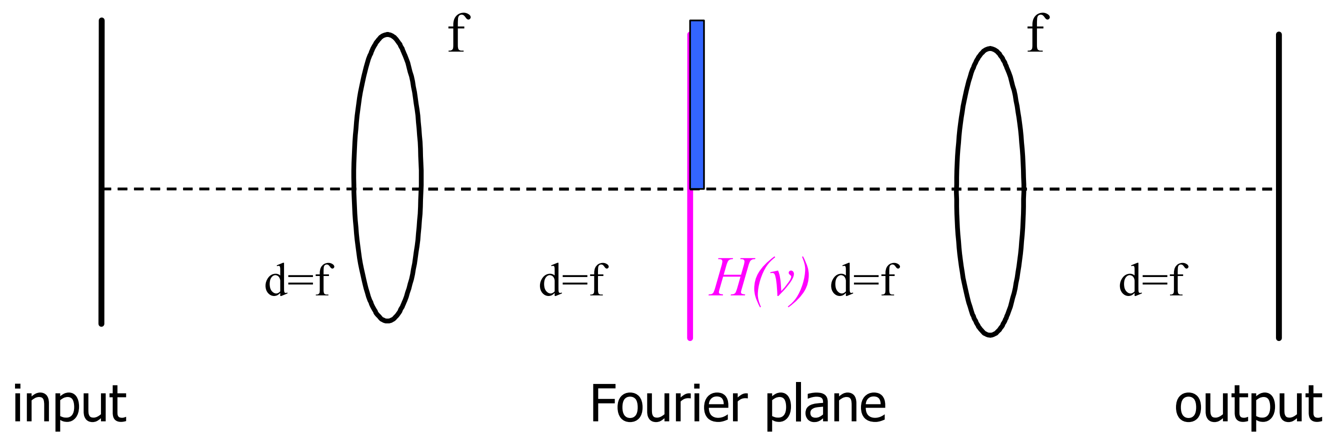
$$f(x) = -\pi^{-1} \int_{-\infty}^{\infty} \frac{H(u)}{u-x} du$$

- It can be generated by multiplying its frequency spectrum of f(x) by $-i\,\mathrm{sgn}(\omega)$ and then performing the inverse FT

$$\mathrm{sgn}(\omega) = \begin{cases} 1, & \omega > 0 \\ 0, & \omega = 0 \\ -1, & \omega < 0 \end{cases}$$

Optical Hilbert transform

- Optical spatial filter consists of a π–phase shifting plate covered half plane



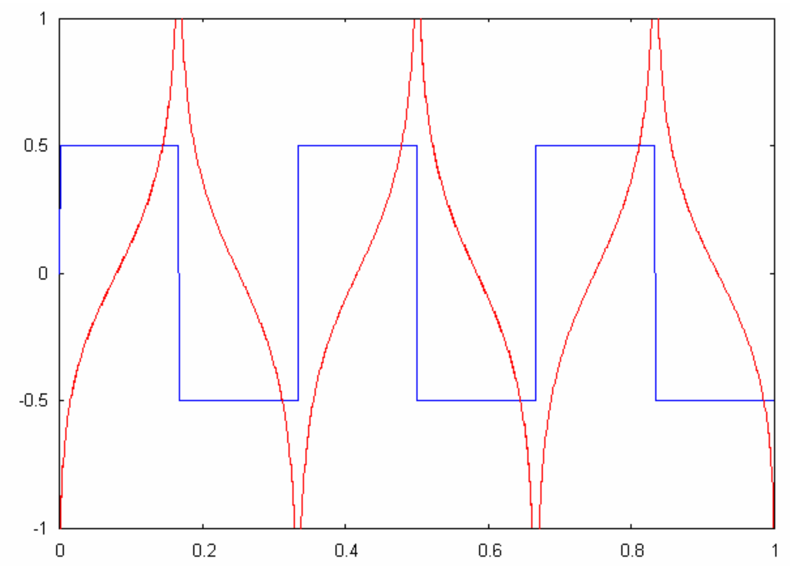
A. Kastler, *Rev. Opt*. 29, 308 (1950);
S. Lowenthal and Y. Belvaux, *Appl. Phys. Lett.* 11, 49 (1967)

Hilbert transform for edge detection

- Hilbert transform as a convolution

$$H(x) = f(x) * \frac{1}{\pi x}$$

- The Hilbert transform, in red, of a square wave, in blue



http://en.wikipedia.org/wiki/Hilbert_transform

Fractional Hilbert transform

- There are different fractional Hilbert transforms
- One of them is applied for selective edge enhancement

$$H_p(u) = \int_{-\infty}^{\infty} K(p,x,u) f(x) dx$$

$$K(p,x,u) = \delta(x-u)\cos[\tfrac{1}{2}\pi p] + (\pi(u-x))^{-1}\sin[\tfrac{1}{2}\pi p]$$

- It can be performed by the same optical set up as the Hilbert transform but with filter function

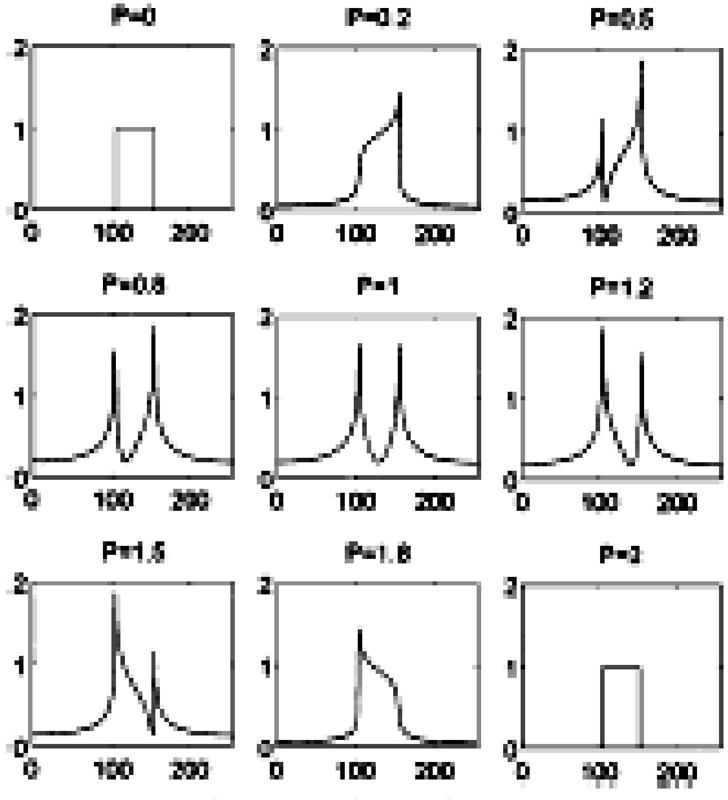
$$H(v) = \cos[\tfrac{1}{2}\pi p] + i\,\mathrm{sgn}(v)\sin[\tfrac{1}{2}\pi p]$$

A. W. Lohmann, D. Mendlovic, and Z. Zalevsky, *Opt. Lett*. 21, 281 (1996);
T. Alieva and M. L. Calvo, *J. Opt. Soc. Am. A* 17**,** 2330 (2000).

Fractional Hilbert transform for edge enhancement

- The fractional Hilbert transform enhances differently the negative slope and positive slope edges.

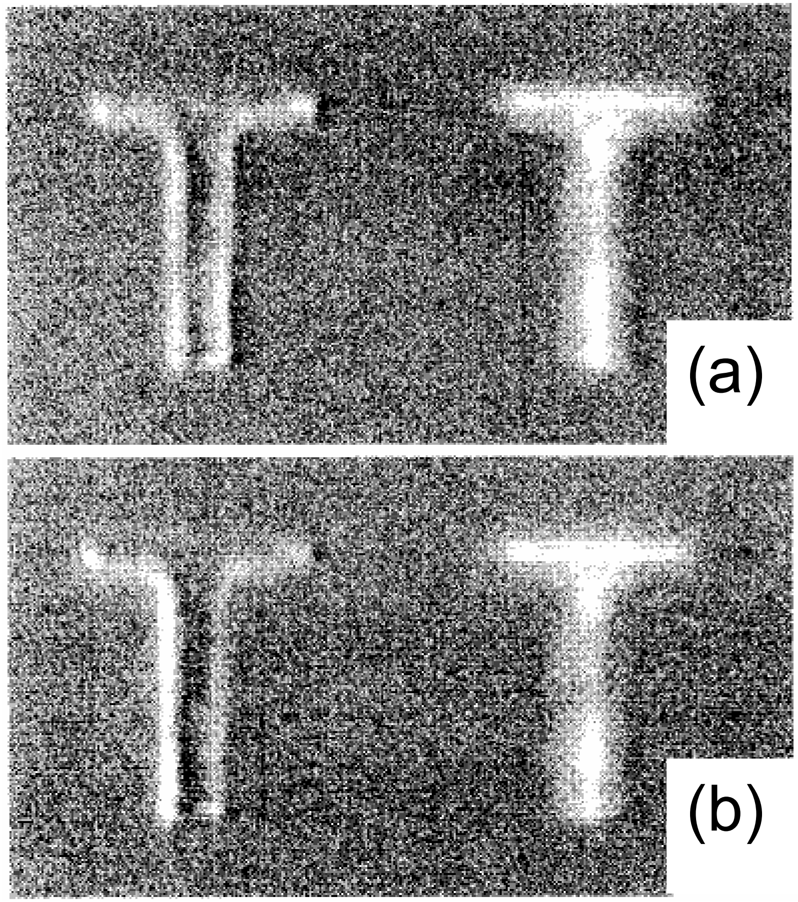
- For fractional orders *p*∈[0,1] the negative slope edges are emphasized; for *p*∈[1,2] - the positive slope edges.



A. Lohmann, et al, *Opt. Lett.* 21, 281 (1996); A. Lohmann, et al, *Appl. Opt.* 36, 6620 (1997); J. Davis, et al, *Appl. Opt.* 37, 6911 (1998)

Selective edge detection

- (a) p=1 both edges are emphasized,
- (b) p= 1.4 the left edge is emphasized



J. A. Davis and M. D. Nowak *Appl. Opt.* 41, 4835 (2002)

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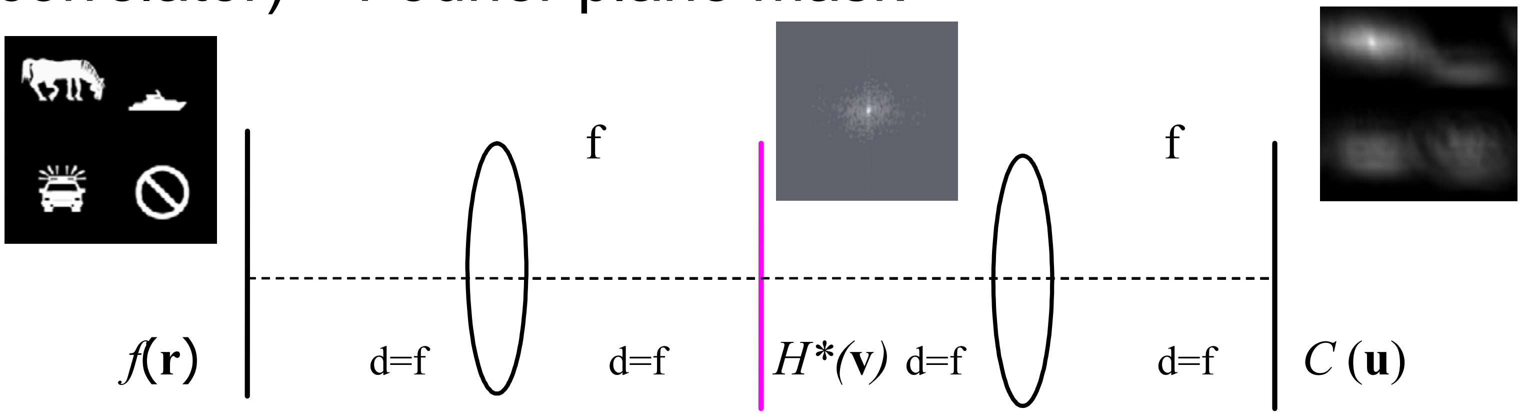
Pattern recognition

- 4-f Van der Lugt optical processor (frequency plane correlator) – Fourier plane mask



$$C(\mathbf{u}) = FT\left\{F(\mathbf{v})H^{*}(\mathbf{v})\right\} = Cor(-\mathbf{u})$$

- To detect pattern *g* at scene *f* we can use matched filter *H**=*G**, which provides the maximum output signal-to-noise ratio (SNR), defined as the ratio of the average output peak value to its standard deviation

A. Van der Lugt, *IEEE Trans. Inf. Theory* IT-10, 139 (1964); A. Van der Lugt, *Optical Signal Processing*, John Wiley, NY (1992)

How to create matched filter?

Matched filter is a complex function

$$G^{*}(\mathbf{v}) = \left|G(\mathbf{v})\right| \exp(-i\phi(\mathbf{v}))$$

- Hologram
- Computer generated hologram
- Real time implementation – application of the Spatial Light Modulator (SLM)
- Practical difficulties of matched filter:
  - Sensitive to small changes of the reference signal
  - Light inefficient
  - SLMs cannot accommodate the full complex frequency response needed for matched filter

Phase-only filter (POF)

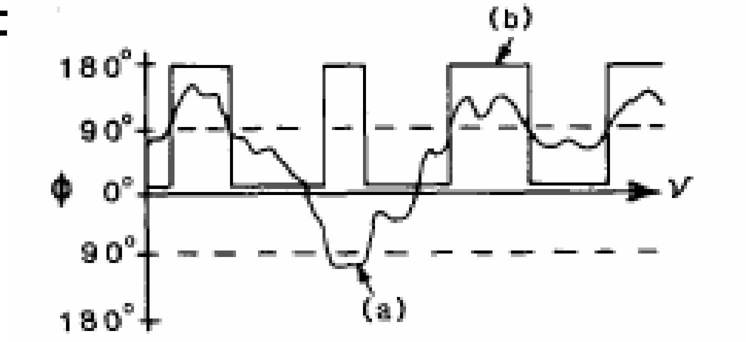
- It does not absorb light - higher optical efficiency
  - Important for low power optical correlators
  - Produces higher correlation peak
  - Better performance for noisy input
- Reduction of stored data
- Most SLMs cannot encode fully complex functions
- POF examples: Glasses, lenses, prisms

- POF simplification: Binary POF

  Symmetric input – real FT:

  1 for positive values

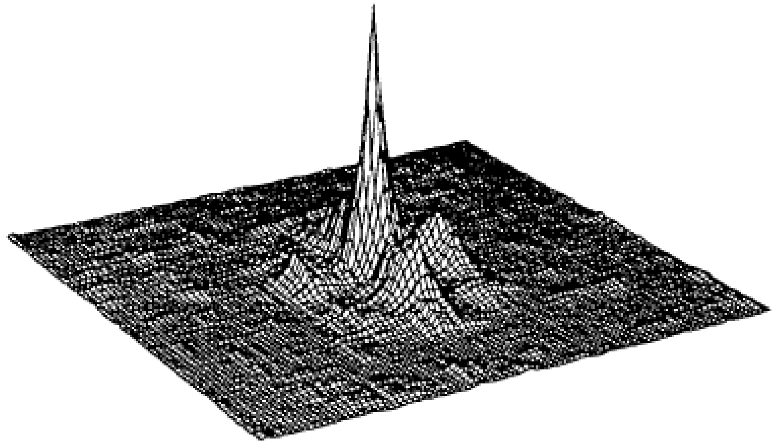
  -1 for negative values



J. L. Horner and P. D. Gianino, *Appl. Opt.* 23, 812 (1984); D. Psaltis, E. Paek, and S. Venkatesh, *Opt. Eng.* 23, 698 (1984)

POF versus matched filters

- Matched filter: Intensity of the autocorrelation peak



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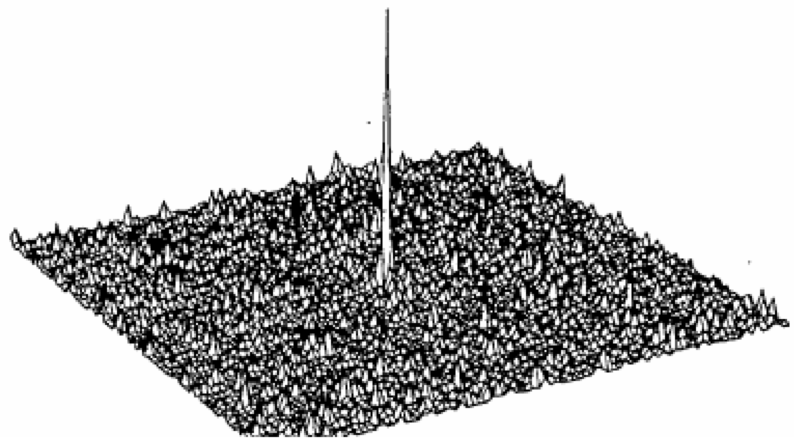
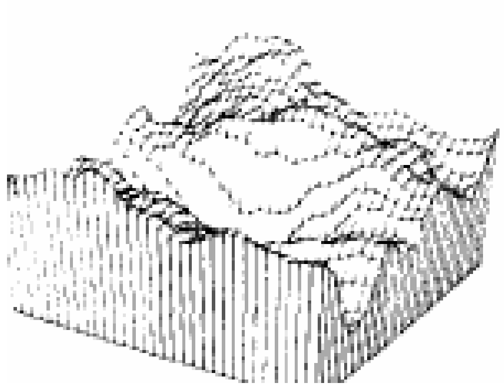


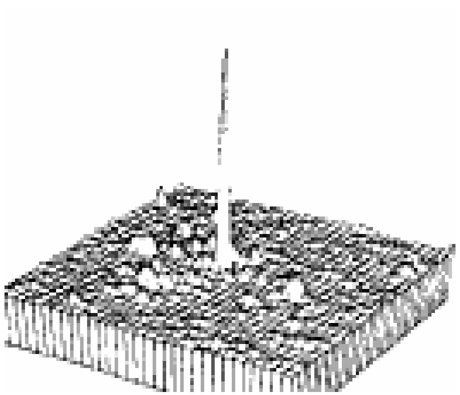
J. L. Horner and P. D. Gianino, *Appl. Opt.* 23, 812 (1984)

POF and binary POF for image recognition

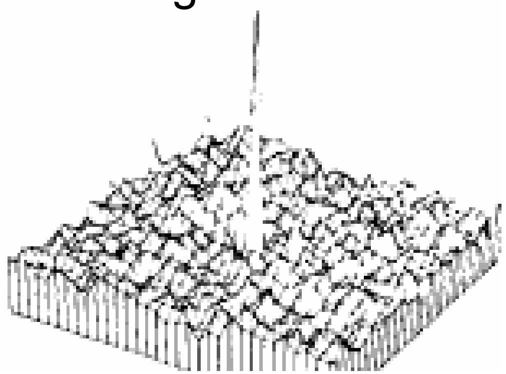
Input image



Correlation with matched filter made from the tank without background



Correlation with POF

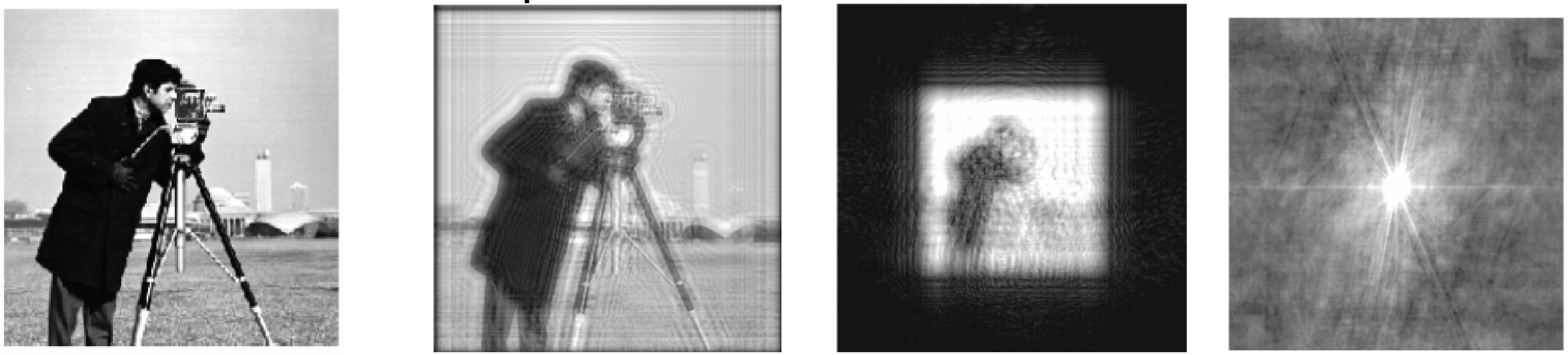


Correlation with binary POF

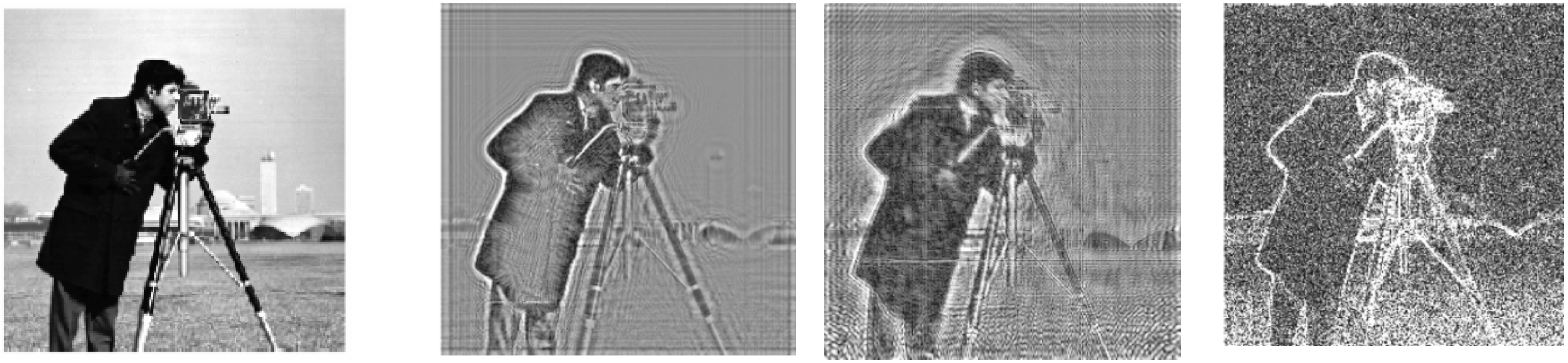
D. L. Flannery and J. L. Horner, *Proc. IEEE* 77, 1511 (1989)

Can we use POF in fractional FT domain?

- Signal restoration from only the amplitude or from only the phase of its fractional FT by applying the inverse fractional FT
- Reconstruction from amplitude information $\alpha=\pi/20$ $\alpha=\pi/4$ $\alpha=\pi/2$



- Reconstruction from phase information $\alpha=\pi/20$ $\alpha=\pi/4$ $\alpha=\pi/2$

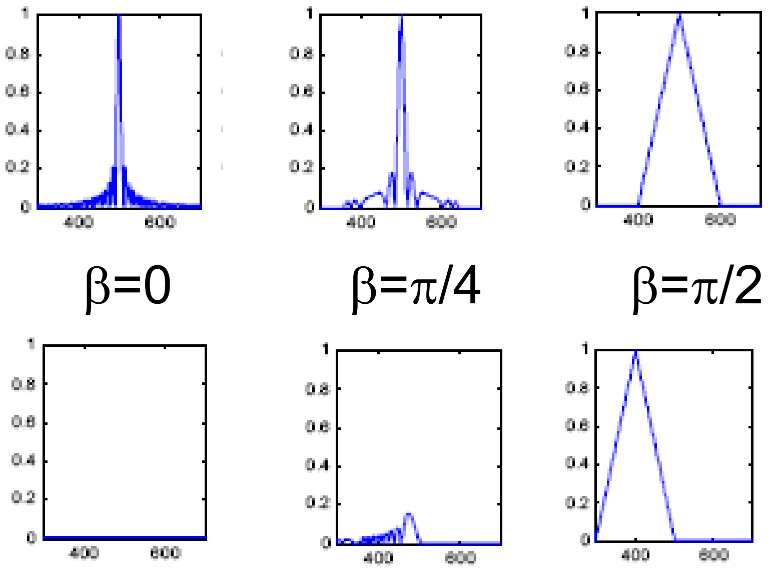


T. Alieva and M. L. Calvo, *J. Opt. Soc. Am. A* 20, 533 (2003)

Space variant recognition in fractional FT domain

- Fractional convolution is space variant

$$H_{f,g}(\mathbf{r},\pi/2,\beta,-\beta)=R^{-\pi/2}\left\{R^{\beta}\left[f(\mathbf{r})\right]R^{-\beta}\left[g(\mathbf{r})\right]\right\}$$



fractional autocorrelations of rectangular signal of size σ

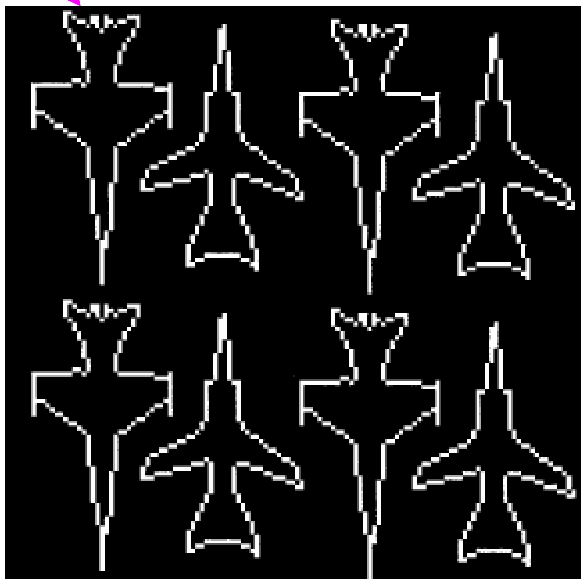
fractional correlations between rectangular signal and its shifted replica (shift s=σ)

O. Akay and G. Faye Boudreaux-Bartels, *IEEE Trans. Sign. Proc.* 49, 979 (2001)

Space-variant detection of several objects

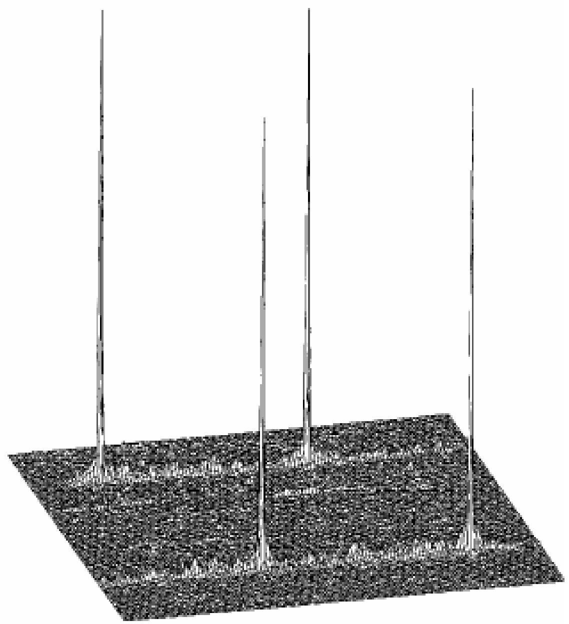
Input image

F18

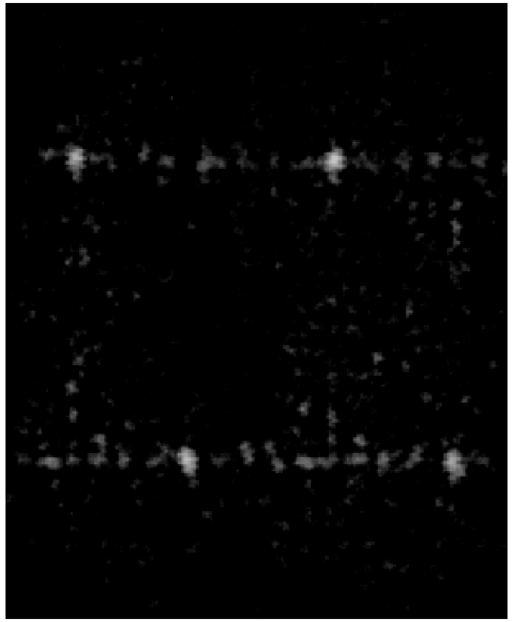


Tornado

Correlation peaks (simulations)



Correlation peaks (experiment)



$\beta=(\pi/2;\ \pi/4)$: x- invariant, y- variant detection

Shift tolerance condition $\pi\ \sigma s \cot\beta<1$ (s - signal shift, σ - signal width; dimensionless variables)

J. Garcia, D. Mendlovic, Z. Zalevsky, and A. Lohmann, *Appl. Opt.* 35, 3945 (1996);
D. Sazdon, Z. Zalevsky, E. Rivlin, and D. Mendlovic, *Pattern Recog.* 35, 2993 (2002)

Problems in pattern recognition

- Usually a recognition system has to be invariant to the input changes:
  - Position (for position dependent recognition - fractional or canonical correlations)
  - Rotation
  - Scale
  - Projection (perspective)
  - Distortion
- Algorithms handling rotation, scale, tilt (one dimensional scale) distortions:
  - Linear mapping algorithms (composite filter, synthetic discrimination functions, the least squares technique)
  - Eigenvector analysis

Filters matching to only part of input information

B. V. K. Vijaya Kumar, *Appl. Opt.* 31, 4773 (1992)

Scale invariant pattern recognition

- Object is decomposed into orthogonal set of Mellin radial harmonics (MRH): each harmonic exhibits scale invariance
- Two dimensional object expressed in polar coordinates is decomposed as

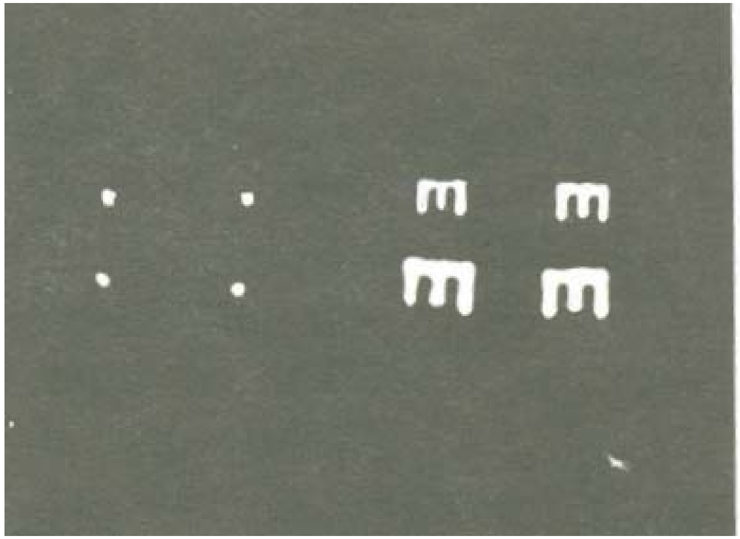
$$f(r,\theta) = \sum_{M=-\infty}^{\infty} f_M(\theta) r^{i2\pi M-1} = \sum_{M=-\infty}^{\infty} h_M(r,\theta)$$

$$f_M(\theta) = \int_{r_0}^{R} f(r,\theta) r^{-i2\pi M-1} r dr$$

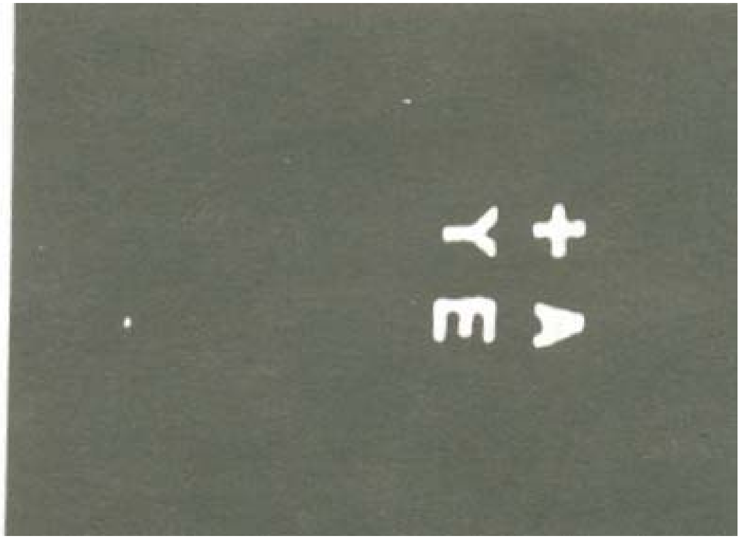
where *R* is finite size of the pattern, r_0 is the smallest radius used in the expansion

Shift and scale invariant correlation based on Mellin radial harmonic

- A single MRH of letter E is used as a matched filter



Scale invariant recognition of E



Discrimination capacity of the filter

D. Mendlovic, E. Marom, and N. Konforti, *Opt. Commun.* 67, 172 (1988)

Rotation invariant pattern recognition

- Circular harmonic decomposition

$$f(r,\theta) = \sum_{m=-\infty}^{\infty} f_m(r)\exp(im\theta) = \sum_{m=-\infty}^{\infty} h_m(r,\theta)$$

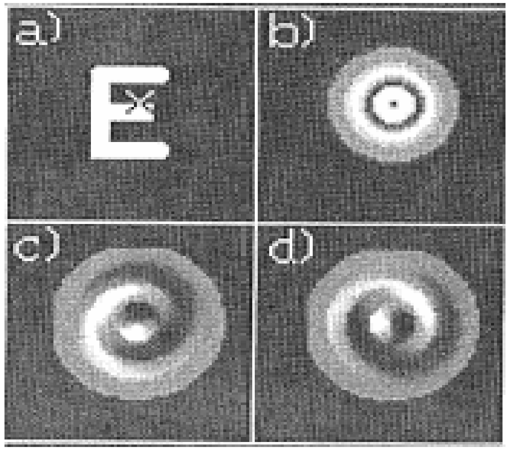
$$f_m(r) = \frac{1}{2\pi}\int_0^{2\pi} f(r,\theta)\exp(-im\theta)d\theta$$

- Single component $h_m(r,\theta)$ is used as a matched filter $\Rightarrow$ a correlation peak maximum value intensity is invariant to rotation of the object

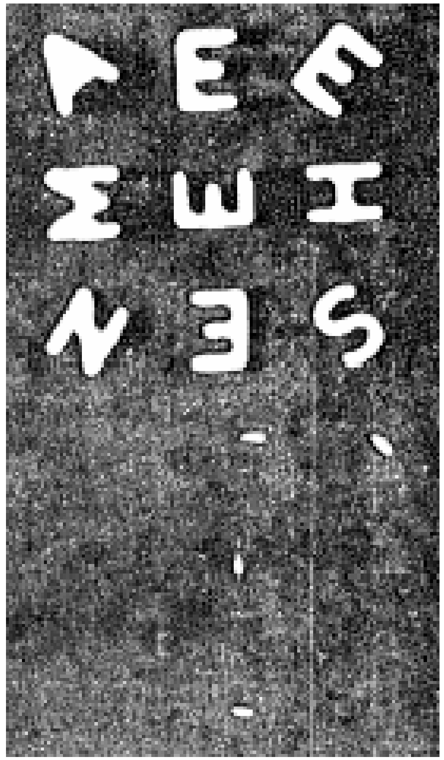
- Combination of several harmonics $\Rightarrow$ limited invariance to in-plane rotation

Y. N. Hsu and H. H. Arsenault, *Appl. Opt.* 21, 4016 (1982)

Rotation invariant recognition: example



First-order circular harmonic component: (a) target with X denoting the proper center; (b) amplitude of the circular harmonic component (m = 1); (c) real part of the circular harmonic component; (d) imaginary part of the circular harmonic component.



Thresholded output

Problem – low light efficiency: most of the target energy is thrown away

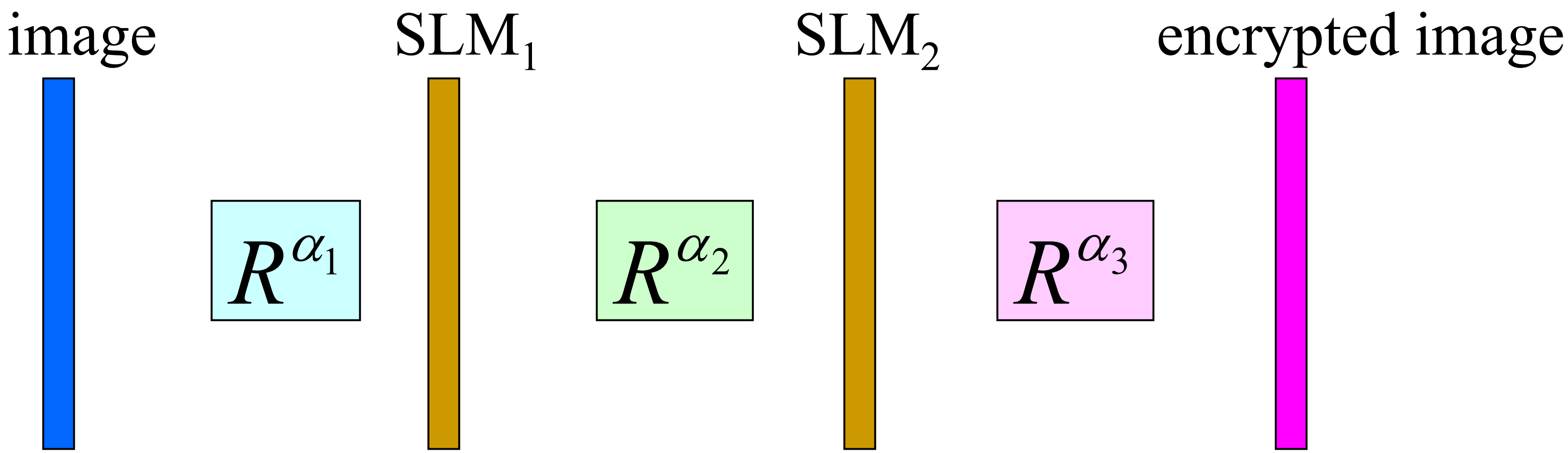
Security systems

- Optical systems are used for
  - Pattern recognition
  - Encryption/decryption of information
  - Watermarking
- Methods for encryption:
  - Random phase masks at different fractional FT domains
  - Holography
- Methods for watermarking:
  - Introduction of invisible in image domain marks which are difficult to take away since they are distributed over the entire image (chirp like signals, holographic images out of principal plane)

Special issue on optical security, *Opt. Eng.* 35 (1996); P. Refregier and B. Javidi, *Opt. Lett.* 20, 767 (1995); B. Javidi and T. Nomura, *Opt. Lett.* 25, 28 (2000)

Encryption in the fractional FT domain

The random masks and angles of the fractional FT domains are the encryption parameters

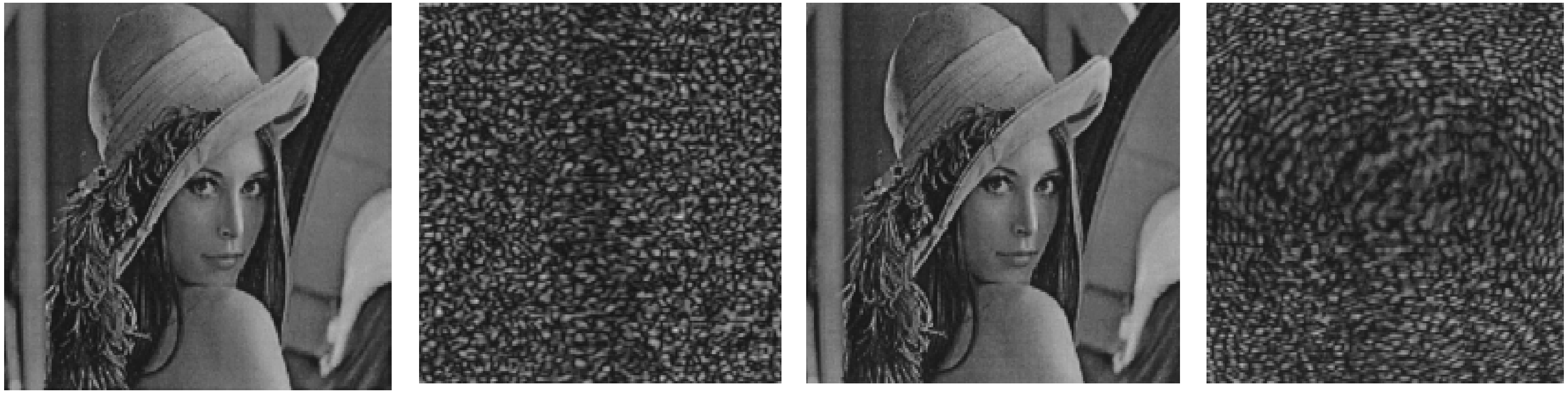


SLM – spatial light modulator: $\exp[-i\varphi(x,y)]$, where $\varphi(x,y)$ is a random function, $\varphi(x,y) \in [0,2\pi]$

S. Liu and L. Yu, B. Zhu, *Optics Commun*. 187, 57 (2001)

Encryption - decryption operations in the fractional Fourier domains

Original image | Encrypted image | Reconstructed with right masks and keys | Reconstructed with right masks but wrong keys



Fractional orders of fractional FT convolution: (0.83,0.56; 0.34,0.48; -0.78,-0.92)

Fractional orders are used as encryption keys

B. Zhu and S. Liu, *Optics Commun.* 195, 371 (2001)

Outlines

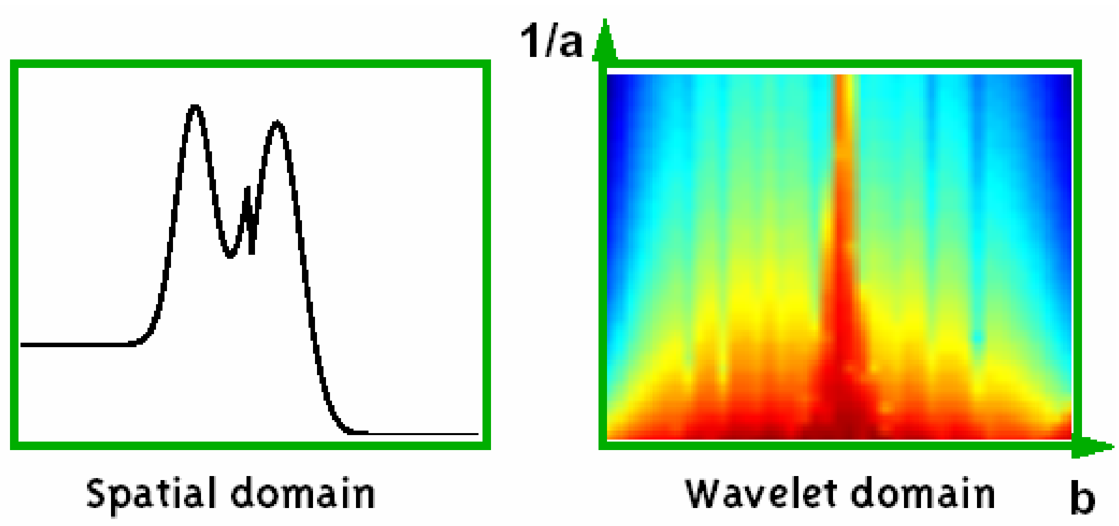
- Convolution and some specific operations: image replication, differentiation, integration
- Filtering in the different phase space domains
  - shift invariant and shift variant filtering
  - fractional convolution
- Image quality improvement, feature extraction
- Pattern recognition
- Security systems
- **Multiresolution analysis: optical wavelet transform**
- Conclusions

Wavelet transform

- Wavelet transform (WT) with wavelet ψ

$$WT\left[f(x)\right](a,b) = \int f(x)\frac{1}{\sqrt{a}}\psi^*\left(\frac{x-b}{a}\right)dx$$

- Low position resolution at low frequencies and high position resolution at high ones.



- Localization of signal singularities: find a scale *a* at position *b*

WT as a filter bank

- Alternative formula for WT

$$WT\left[f(x)\right](a,b) = \sqrt{a}\int F(u)\Psi^*(au)\exp\left(ibu\right)du$$

where $F(u)$ and $\Psi(u)$ are the FT of analyzed function $f(x)$ and a mother wavelet $\psi(x)$

- If the mother wavelet satisfies the admissibility condition

$$C_\psi = \int\left|\Psi(\omega)\right|^2\left|\omega\right|^{-1}d\omega < \infty$$

  - wavelets have a band-pass like spectrum
  - there exists inverse wavelet transform

$$f(x) = C_\psi^{-1}\int_0^\infty \frac{da}{a^2}\int_{-\infty}^{\infty} WT(a,b)\psi\left(\frac{x-b}{a}\right)db$$

Different wavelets

- Mexican hat mother wavelet and its FT (satisfies admissibility condition)

$$\psi(x) = (1 - x^2)\exp(-x^2/2)$$

$$\Psi(u) = 4\pi^2 u^2 \exp(-2\pi^2 u^2)$$

- Morlet (Gabor) wavelet and its FT (does not satisfy admissibility condition)

$$\psi(x) = \exp(2\pi i k x)\exp(-x^2/2)$$

$$\Psi(u) = \sqrt{2\pi}\exp(-2\pi^2 (u-k)^2)$$

Fresnel diffraction as a wavelet transform

- Fresnel diffraction of *f*(*x*) at distance *z*

$$F(z,u)=\frac{C}{\sqrt{\lambda z}}\int f(x)\exp\left(i\pi\frac{(x-u)^2}{\lambda z}\right)dx$$

  as wavelet transform with chirp mother wavelet

$$F(z,u)\rightarrow WT[f(x)](a,b)$$

$$\exp\left(-i\pi x^2\right)\rightarrow\psi(x)$$

$$a=\sqrt{\lambda z}\qquad b=u$$

- The chirp does not satisfy the admissibility condition, but the Fresnel transform is invertible

L. Onural, *Opt. Lett.* 18, 846 (1993)

Optical wavelet transform in 2D

WT of 2-D signal is four-dimensional

$$WT\left[f(x,y)\right](a_x,a_y,b_x,b_y)=\int f(x,y)\frac{1}{\sqrt{a_x a_y}}\psi^*\left(\frac{x-b_x}{a_x},\frac{y-b_y}{a_y}\right)dxdy$$

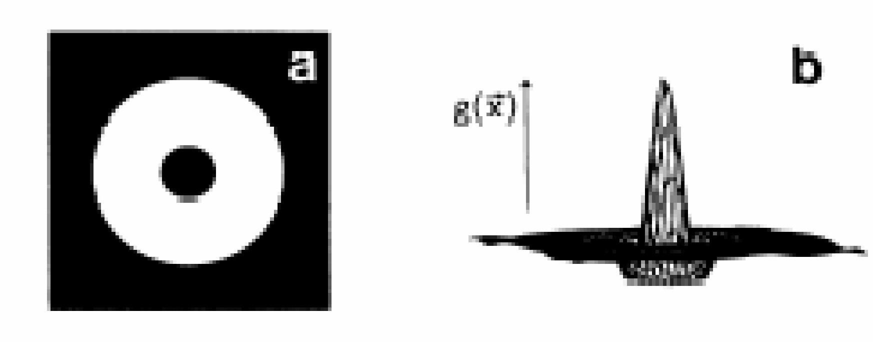
space multiplexing

time multiplexing

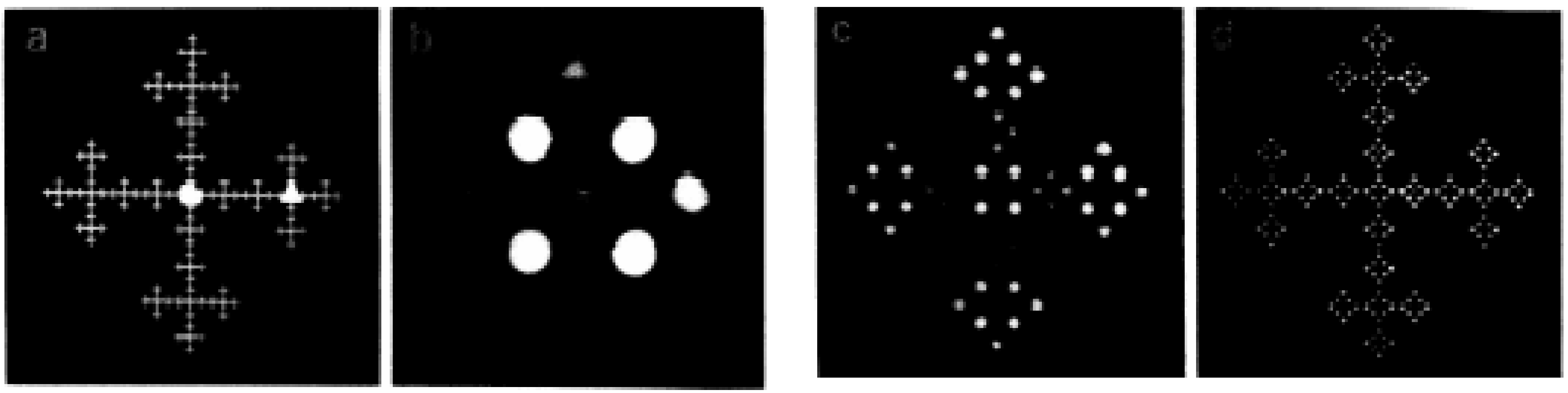
Y. Li, H. H. Szu, Y. Sheng, and H. J. Caulfield, *Proc. IEEE* 84, 720 (1996)

Temporal multiplexing

- Temporal replacing of the filter in the optical correlator
- Filters – transparent rings



which select spatial frequencies $sq_1<q<sq_2$. It is a binary approximation of the radial Mexican hat (scale $s=3^n$)

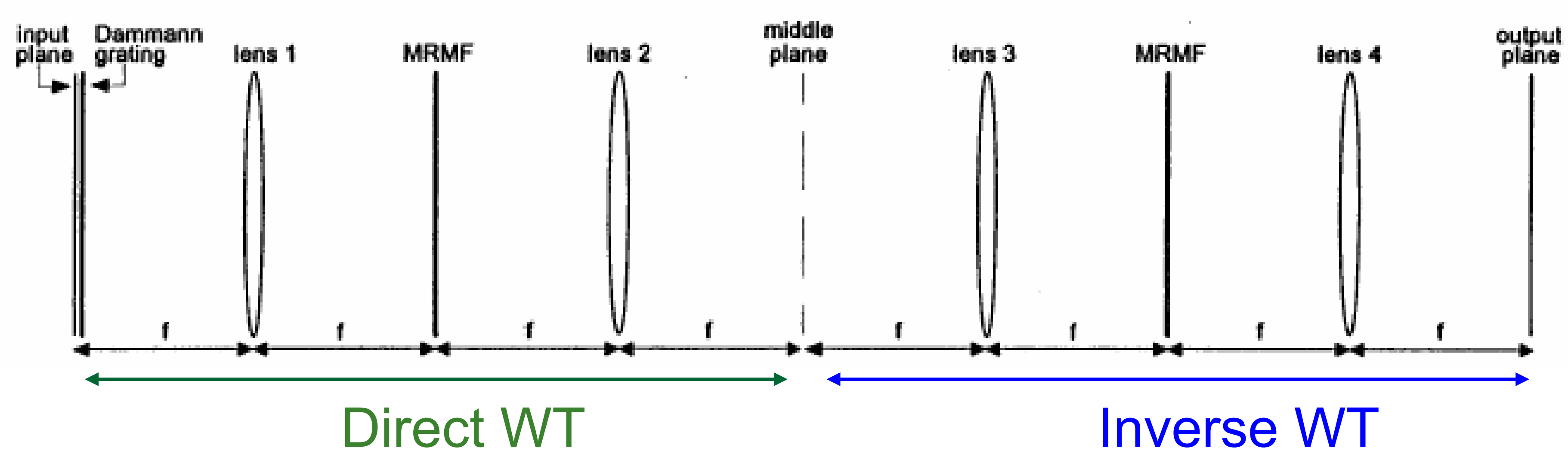


Original image   Wavelet transform for s =3 , 3^2, 3^3

E. Freysz, B. Pouligny, F. Argoul, and A. Arneodo, *Phys. Rev. Lett.* 64, 745 (1990)

Spatial multiplexing: multichannel correlator

- Multichannel correlator: generation of an array of image FTs + matched WT filter bank
- Dammann grating is used to produce array of image FTs
- Multireference matched filter (MRMF): each daughter wavelet at different location is encoded with a different reference beam



I. Ouzieli and D. Mendlovic, *Appl. Opt.* 35, 5839 (1996)

Optical inverse WT

- The zero order zone at the output of the first correlator is eliminated
- The inverse WT is a weighted sum of the correlations between a daughter wavelet and the WTs obtained from the same daughter wavelet
- The same multireference-matched filter, but with normalization $1/a^2$ of each daughter wavelet, is used

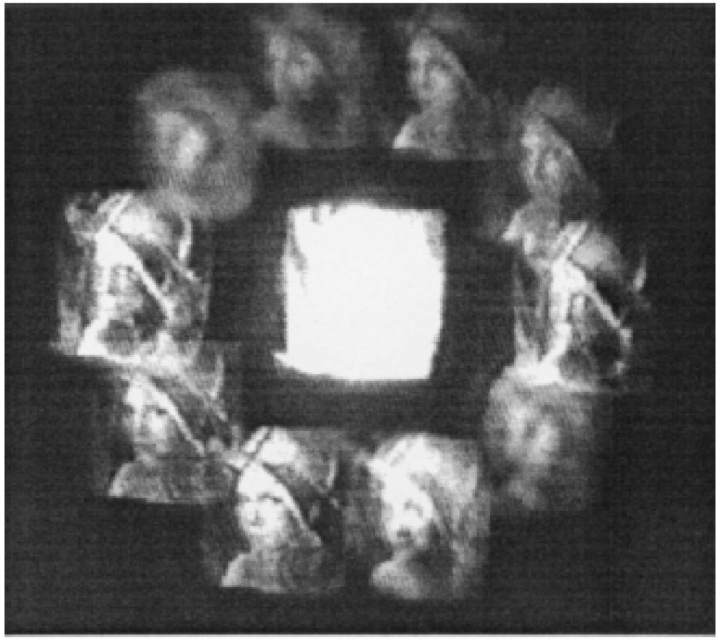
$$f(x) = C_{\psi}^{-1} \int_{0}^{\infty} \frac{da}{a^2} \boxed{\int_{-\infty}^{\infty} WT(a,b)\psi\left(\frac{x-b}{a}\right) db}$$

- The inverse WT is achieved in the zero diffraction order

Optical WT: example

- 5 Mexican hat wavelets with magnification *a*= 1,2,4,8,16 are obtained

Reconstructed inverse WT in zero diffraction order



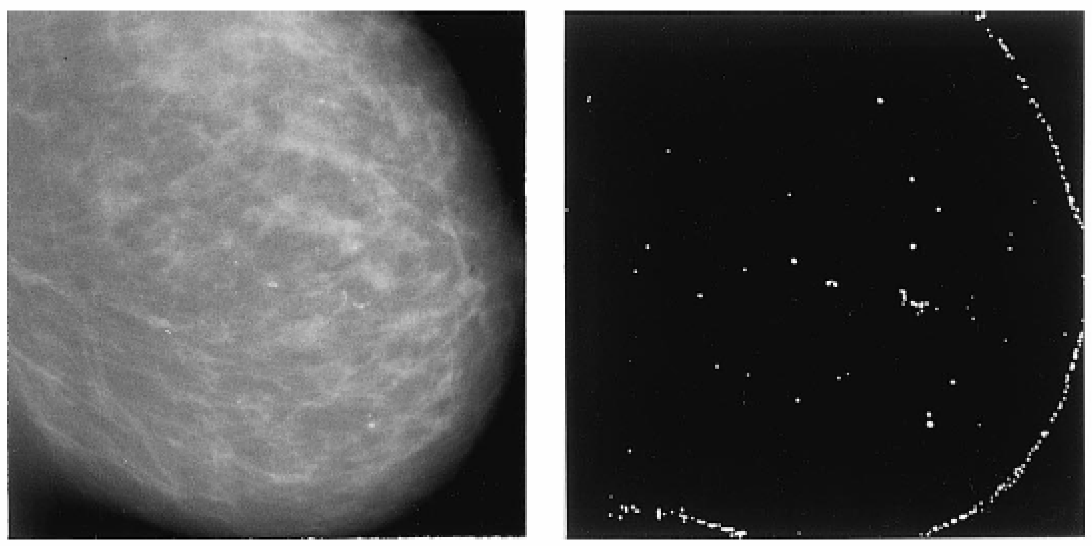
I. Ouzieli and D. Mendlovic, *Appl. Opt.* 35, 5839 (1996)

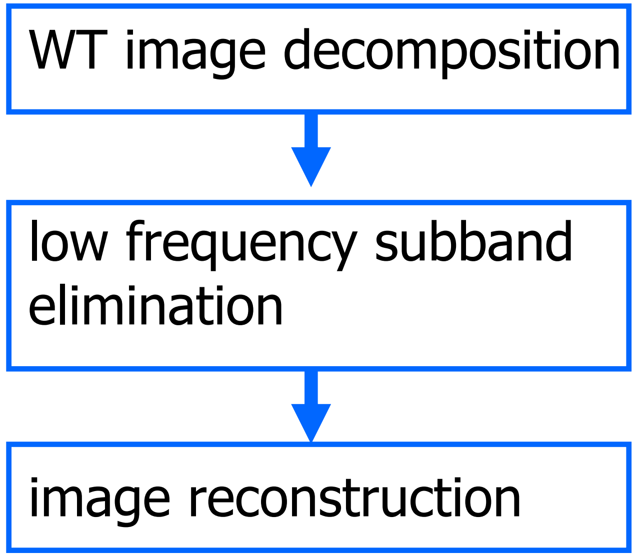
Wavelet techniques in biomedicine

- noise reduction, edge enhancement of biomedical images
- analysis of the bio-acoustical signals (heart, lung, blood flow sounds), electrocardiograms, electroencephalograms
- image compression
- object reconstruction in CAT and MRI, functional image (PET) analysis
- detection of microcalcifications in mammograms
- texture analysis and image classification
- image fusion

M. Unser and A. Aldroubi, “A Review of wavelets in biomedical applications,” *Proc. IEEE* 84, 626 (1996)

Detection of microcalcifications using WT

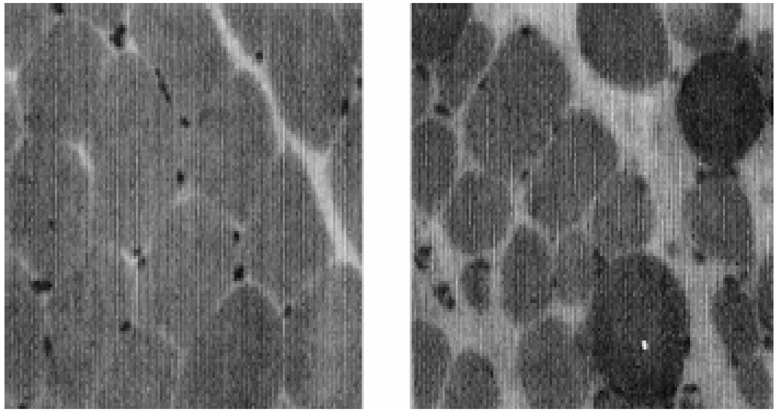




Primary signs of breast cancer:
granular microcalcifications of 0.05-1mm

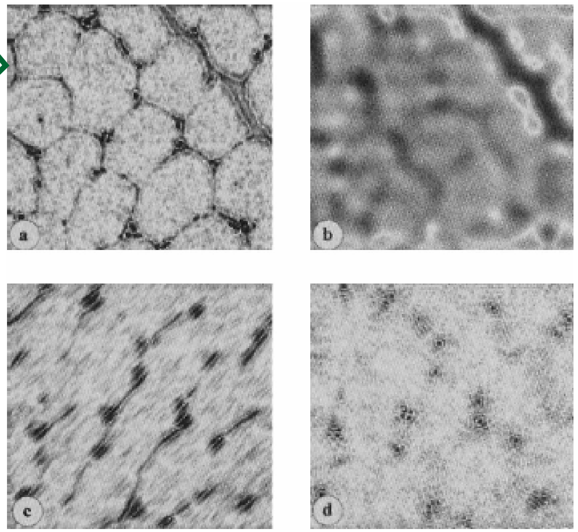
T. C. Wang and N. B. Karayiannis, *IEEE Trans. Med. Imag.* 17, 498 (1998)

Optical image classification



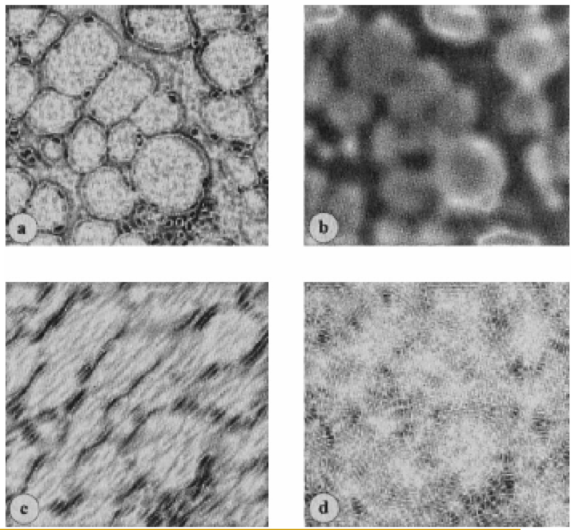
healthy and **diseased**
samples of biopsied skeletal muscle

main features:
- regularity of muscle membranes
- space between the membranes
- distribution of dark spots between the muscle fibers



2 isotropic Mexican-hat ▶ membranes' shape, rough bright areas

- separable Mexican-hat ▶ directional membrane features,
- Morlet wavelet ▶ spots

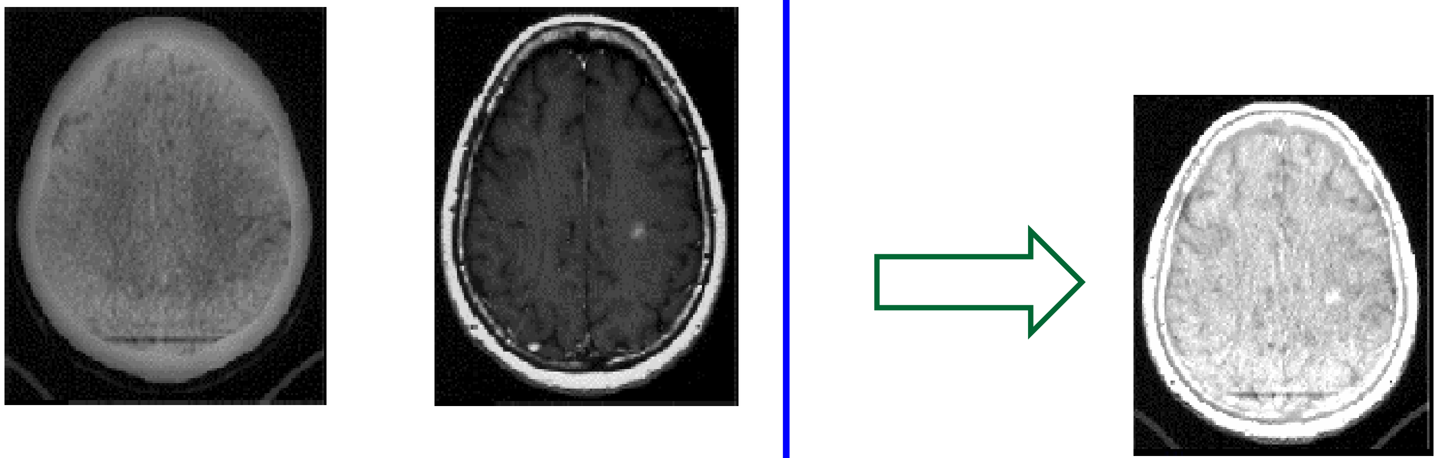


A. Stollfuss, S. Teiwes, and F. Wyrowski, *Appl. Opt*. 34, 1579 (1995)

Image fusion by WT

CAT ▶ denser tissue; MRI ▶ soft tissue;

PET ▶ flood activity with low space resolution



CAT + MRI

Fusion scheme:

to keep the max of WT maxima modulus of both images at different levels and apply inverse WT

G. Qu, D. Zhang and P. Yan, *Opt. Exp.* 9, 184 (2001)

Conclusions

- Linear canonical integral transforms are the basis for the generation of a long list of other integral transforms and bilinear distributions

- They are used for: filtering, phase retrieval, beam characterization and manipulation, edge enhancement, pattern recognition, signal analysis and synthesis, encryption, watermarking, motion analysis, neural networks construction, etc.

- Areas of applications: machine vision, robotics, automation, security, medicine, defense

Concluding remarks on optical information processing

- The main information processing tools can be implemented optically
- Not optical but hybrid optoelectronical information processing
- Optical information processing: synthesis, analysis, classification, etc. of optical signals
- Optical information processing is beneficial when
  - Data are obtained by optical modalities
  - Similar treatment of huge amount of information
- Main problems: low flexibility, data input/output
- Perspectives: new generation of SLM, CCD,CMOS, new holographic materials

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