The Abdus Salam
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WINTER COLLEGE

on

QUANTUM AND CLASSICAL ASPECTS

of

INFORMATION OPTICS

30 January - 10 February 2006

Classical and Quantum Imaging

Peter KNIGHT
Imperial College of Science, Technology and Medicine
the Blackett Laboratory - Optics Section
Prince Consort Road
SW7 2BW London
United Kingdom

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

Classical and Quantum Imaging

Peter Knight
Imperial College London

ICTP Trieste

Winter College on

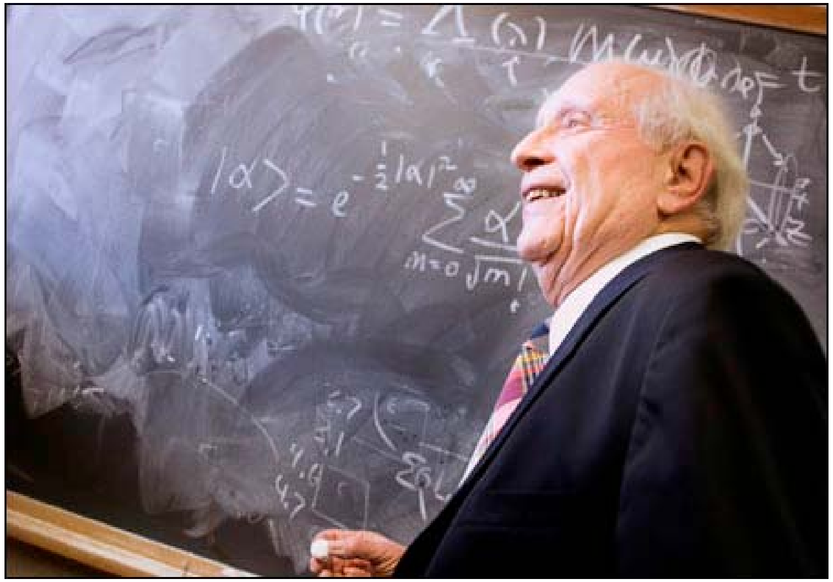
Quantum & Classical Aspects of Information Optics

menu

- What will I cover?
- What will the School cover?
- *Grateful to my friends Gigi Lugiato, Martin Plenio, Mike Raymer and Antonm Zeilinger for figures*

Quantum coherence?

- Define field modes
- Apply sho quantization to each mode
- Excitation of a normal mode is a photon
- Fock states - no coherence
- Superpositions and minimum uncertainty
- Wigner correlations
- Two mode correlations and information



$$|\alpha\rangle \equiv \exp\left(-\tfrac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

FIELD QUANTIZATION

Expand field in normal modes with SHO quantization

$$\underline{\hat{E}}^{(+)}(\underline{r},t) = i\sum_j \sqrt{\frac{\hbar\omega_j}{2\varepsilon_0}}\ \hat{b}_j\ \underline{u}_j(\underline{r})\exp(-i\omega_j t) \qquad (\omega_j > 0)$$

monochromatic plane-wave modes:

$$\underline{u}_j(\underline{r}) = V^{-1/2}\varepsilon_j \exp(i\underline{k}_j \cdot \underline{r})$$

Polarization unit vector

photon annihilation and creation operators: $\hat{b}_j,\ \hat{b}_k^{\dagger}$

See eg R Loudon, Quantum Theory of Light, OUP (2000)

commutator:

$$[\hat{b}_j, \hat{b}_k^{\dagger}] = \delta_{jk}$$

one-photon state:

$$|1_\omega\rangle = \hat{b}_\omega^{\dagger}|vac\rangle$$

n-photon state:

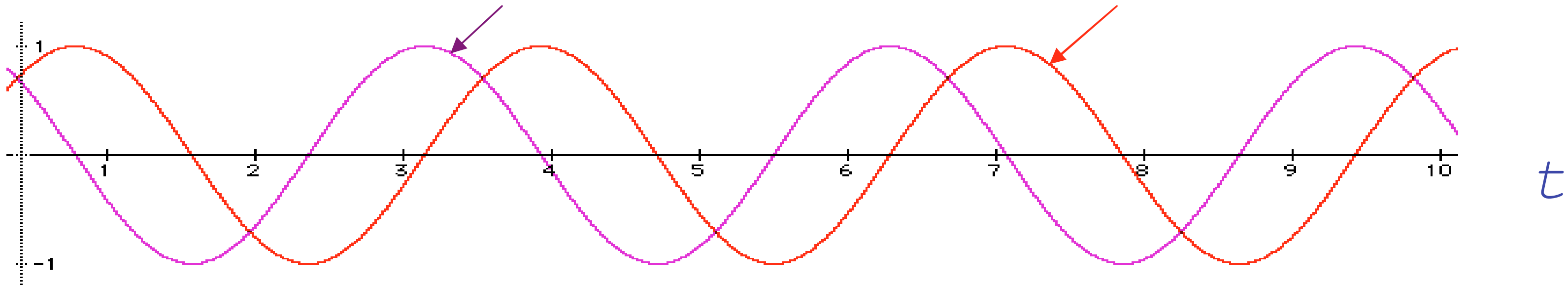
$$|n_\omega\rangle = (\hat{b}_\omega^{\dagger})^n|vac\rangle$$

Field Uncertainty, quadrature operators and squeezing

Single monochromatic mode: $$\underline{\hat{E}}^{(+)} = i\sqrt{\frac{\hbar\omega_j}{2\varepsilon_0}}\;\hat{b}\;\underline{u}_0(z)\exp(-i\omega_0\,t)$$

Hermitian operators:

$$\hat{q} = (\hat{b} + \hat{b}^\dagger)\,/\,2^{1/2} \qquad \hat{p} = (\hat{b} - \hat{b}^\dagger)\,/\,i2^{1/2}$$

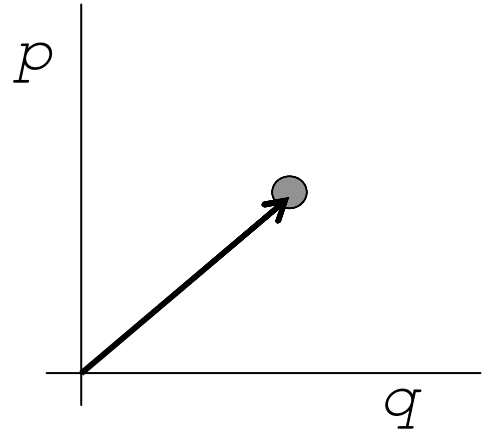


$$\hat{E}^{(+)}(z,t) \propto \underline{\hat{q}}\cos(\omega_0 t - k_0 z) + \underline{\hat{p}}\sin(\omega_0 t - k_0 z)$$

q, *p* = quadrature operators which obey:

Uncertainty relation: $[\hat{q},\,\hat{p}] = i$

$$\Delta(q)\;\Delta(p)\;\geq\;1/2$$



Coherent states

- Definition $$|\alpha\rangle \equiv \exp\left(-\tfrac{1}{2}\,|\alpha|^2 \right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle$$
- Right e-state of a $$a\,|\alpha\rangle = \exp\left(-\tfrac{1}{2}\,|\alpha|^2 \right) \sum_{n=1}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\sqrt{n}\,|n-1\rangle = \alpha\,|\alpha\rangle$$
- Overcomplete $$\left|\,\langle\alpha|\beta\rangle\,\right|^2 = \exp\left(-|\alpha-\beta|^2 \right)$$
- Poisson number distribut $$P_m = \left|\,\langle m|\alpha\rangle\,\right|^2 = \exp\left(-|\alpha|^2 \right) \frac{|\alpha|^{2m}}{m!} = \exp\left(-\langle n\rangle_\alpha \right) \frac{\langle n\rangle_\alpha^m}{m!}$$

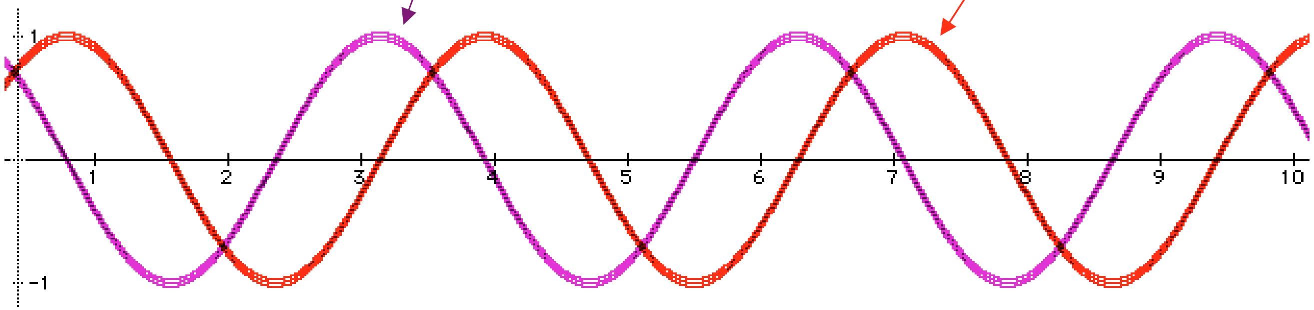
$$\langle n\rangle_\alpha = \langle\alpha|\,a^\dagger a\,|\alpha\rangle = \|\,a\,|\alpha\rangle\,\|^2 = |\alpha|^2$$

Coherent State - ideal laser output:

Size of field fluctuations

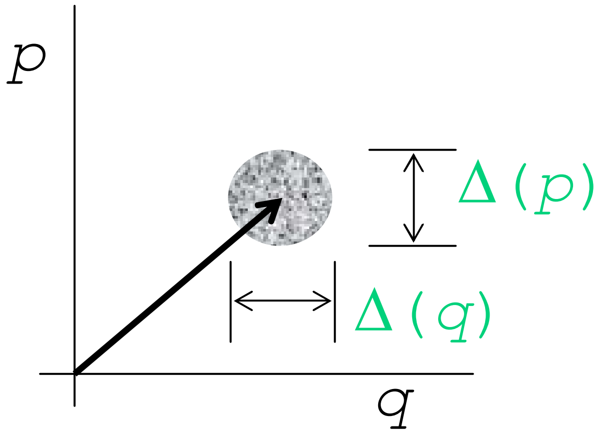
$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

quadrature operators: $\hat{q} = (\hat{b} + \hat{b}^\dagger)/2^{1/2}$ $\hat{p} = (\hat{b} - \hat{b}^\dagger)/i2^{1/2}$



Equal Uncertainties:

$$\Delta(q) = \Delta(p) = 1/\sqrt{2}$$



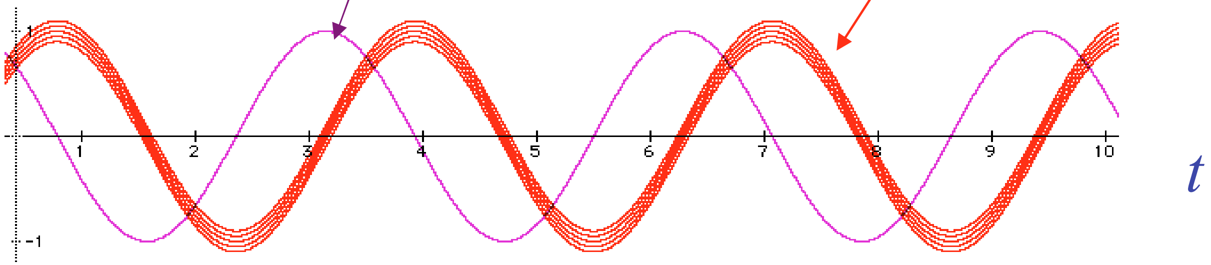
Field Uncertainty and Squeezing

See eg R Loudon and P L Knight, J Mod Opt 34, 709 (1987) and references therein

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

quadrature-squeezed light:

$$\hat{q} = (\hat{b} + \hat{b}^\dagger)/2^{1/2} \qquad \hat{p} = (\hat{b} - \hat{b}^\dagger)/i2^{1/2}$$

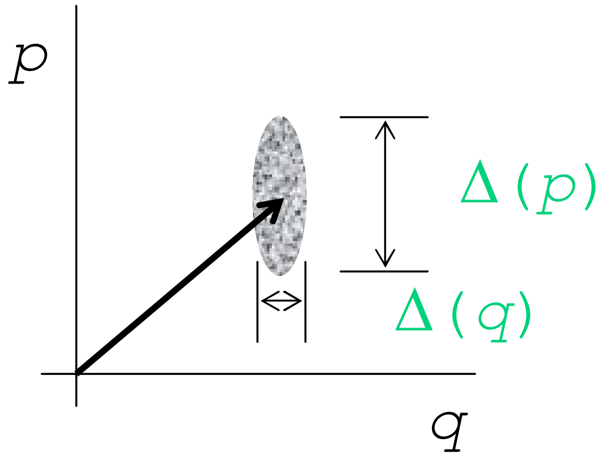


q noise reduced

p noise increased

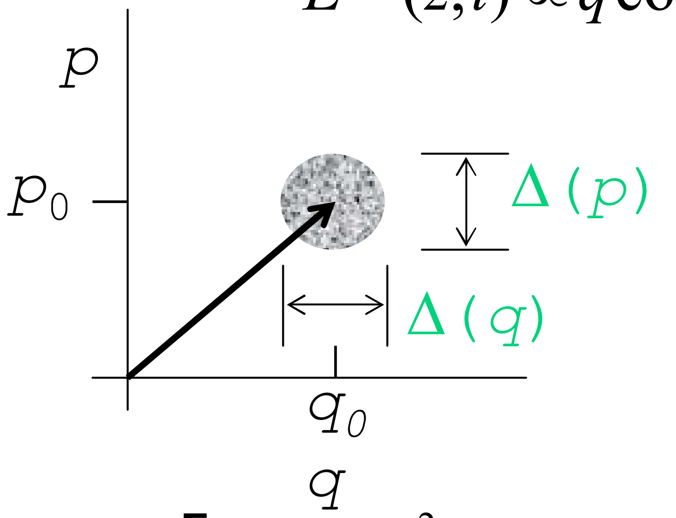
Measured by homodyne interference

Major nonclassical resource in QIP



Coherent and Squeezed States

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

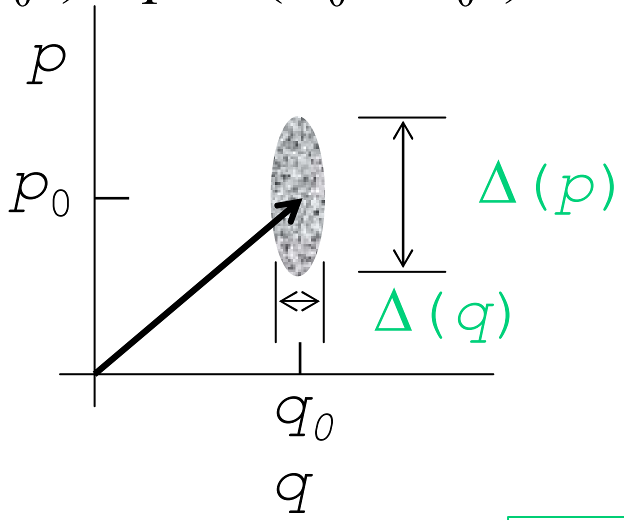


$$\psi(q) = \exp\left[-(q-q_0)^2/2 \; - \; i\,p_0\,q\right]$$

photon number probability:
Poisson

$$p(n) = \left|\langle n|\psi\rangle\right|^2 = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$$

$$\bar{n} = |\alpha|^2 \; , \; \alpha = \frac{q_0 + ip_0}{\sqrt{2}}$$



$$\beta^2 = (1/2)e^{-2s}$$

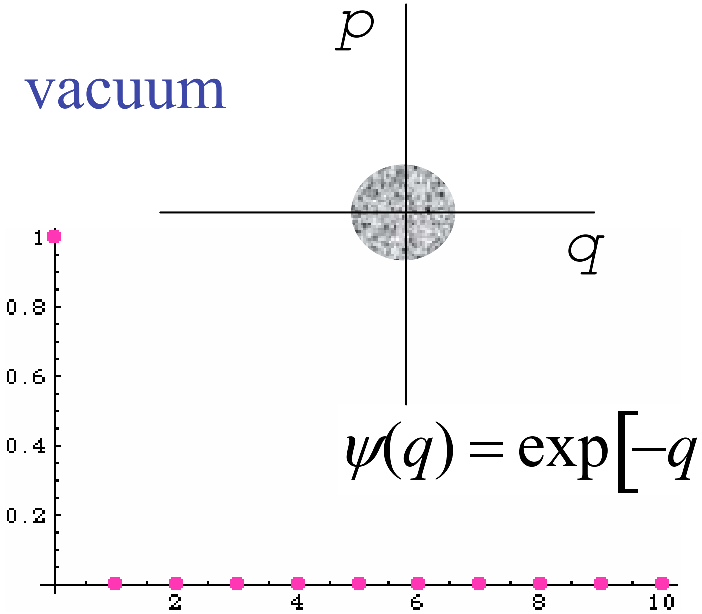
$$\psi(q) = \exp\left[-(q-q_0)^2/(2\beta^2) \; - \; i\,p_0\,q\right]$$

photon number probability?

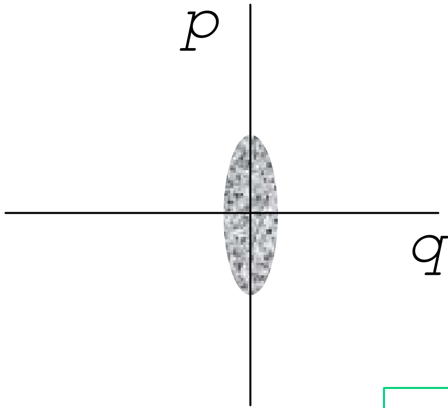
Quadrature-Squeezed Vacuum States

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

vacuum

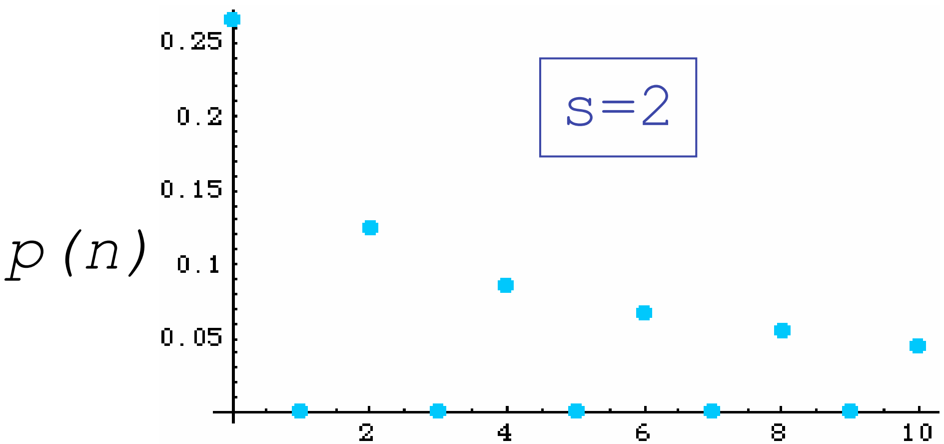


$$\psi(q) = \exp\left[-q^2/2\right]$$



$$\beta^2 = (1/2)e^{-2s}$$

$$\psi(q) = \exp\left[-q^2/(2\beta^2)\right]$$

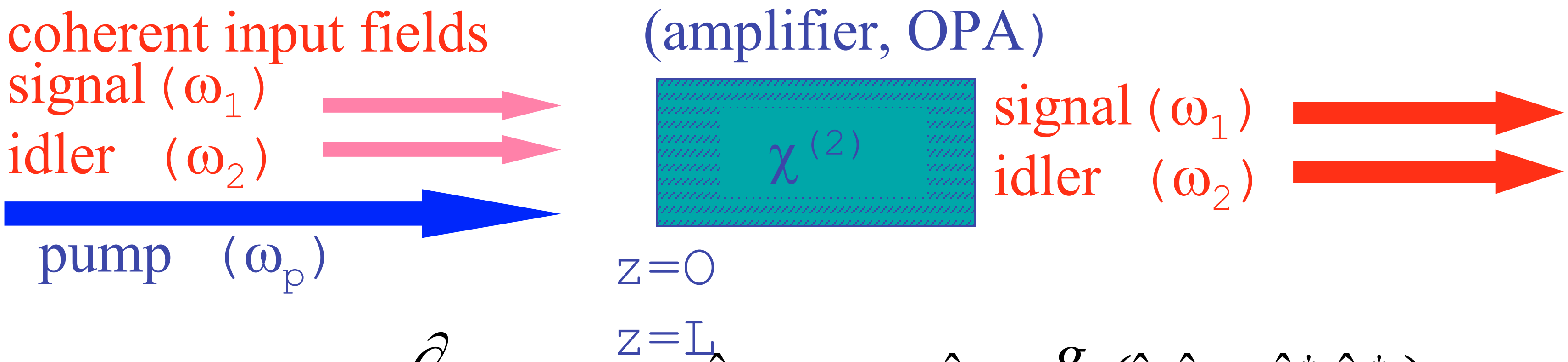


$p_{odd}(n) = 0$ (pair creation)

$$p_{even}(n) = \left|\langle n|\psi\rangle\right|^2 = \binom{n}{n/2}\frac{1}{\cosh(s)}\left(\frac{1}{2}\tanh(s)\right)^n$$

Two-Mode Squeezed States by Second-order Nonlinearity: Optical Parametric Amplification:

pump (ω_p) --> signal (ω_1) + idler (ω_2)



$$\frac{\partial}{\partial z}|\psi\rangle = -i\hat{H}\,|\psi\rangle \ , \quad \hat{H} = i\frac{g}{2}\left(\hat{b}_1\hat{b}_2 - \hat{b}_1^{\dagger}\hat{b}_2^{\dagger}\right)$$

photon difference number N_D is a constant of the motion:

$$\hat{N}_D = (\hat{n}_1 - \hat{n}_2) = \left(\hat{b}_1^{\dagger}\hat{b}_1 - \hat{b}_2^{\dagger}\hat{b}_2\right)$$

$$\left[\hat{N}_D, \hat{H}\right] = 0$$

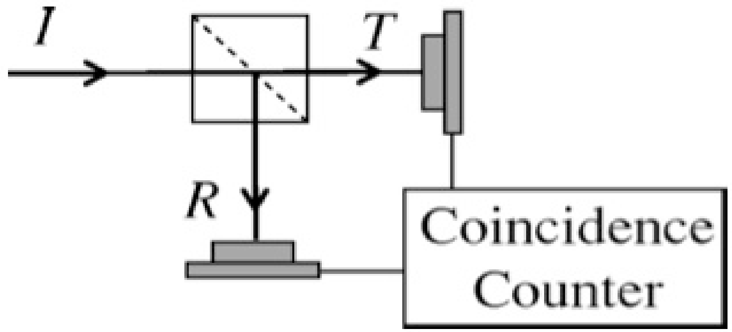
Burnham & Weinberg (1970)

Identical photon numbers in signal and idler beams: used in metrology etc

Intensity correlations

- Mode operators

$$\hat{a}_R = \frac{1}{\sqrt{2}}(\hat{a}_I + \hat{a}_V), \quad \hat{a}_T = \frac{1}{\sqrt{2}}(\hat{a}_I - \hat{a}_V)$$



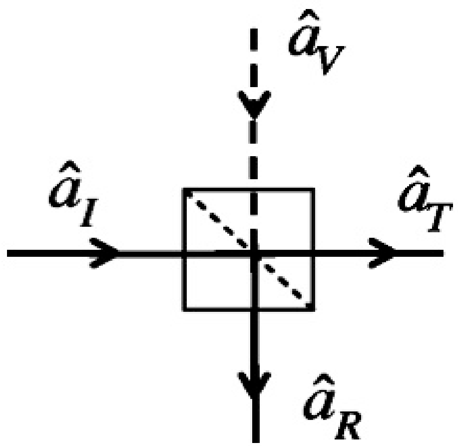
- Intensity correlation

$$g^{(2)}_{T,R}(\tau) = \frac{\langle I_T(t+\tau) I_R(t)\rangle}{\langle I_T(t+\tau)\rangle\langle I_R(t)\rangle}$$

$$g^{(2)}_{T,R}(0) = \frac{\langle :\hat{I}_T \hat{I}_R :\rangle}{\langle \hat{I}_T\rangle\langle \hat{I}_R\rangle} \qquad g^{(2)}_{T,R}(0) = g^{(2)}(0) \geqslant 1 \quad \text{(classical fields)}$$

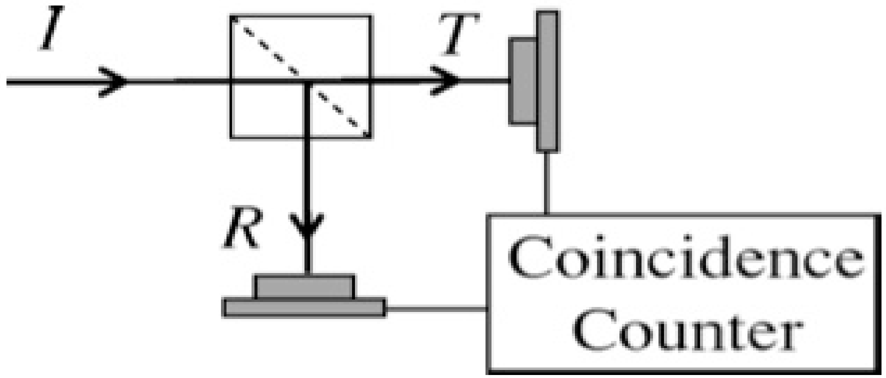
$$g^{(2)}_{T,R}(0) = \frac{\langle :\hat{I}_T \hat{I}_R :\rangle}{\langle \hat{I}_T\rangle\langle \hat{I}_R\rangle} \qquad g^{(2)}_{T,R}(0) = \frac{\langle :\hat{n}_T \hat{n}_R :\rangle}{\langle \hat{n}_T\rangle\langle \hat{n}_R\rangle} = \frac{\langle \hat{a}_T^\dagger \hat{a}_R^\dagger \hat{a}_R \hat{a}_T\rangle}{\langle \hat{a}_T^\dagger \hat{a}_T\rangle\langle \hat{a}_R^\dagger \hat{a}_R\rangle}$$

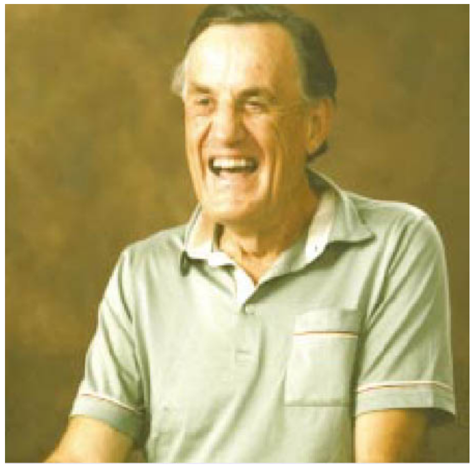
$$g^{(2)}_{T,R}(0) = \frac{\langle \hat{n}_I(\hat{n}_I - 1)\rangle}{\langle \hat{n}_I\rangle^2} = g^{(2)}_{I,I}(0) = g^{(2)}(0)$$



Hanbury Brown & Twiss

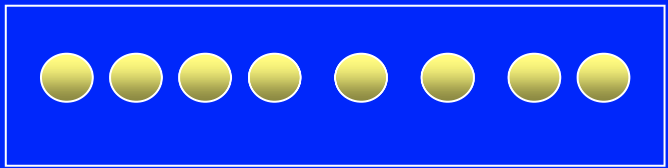
- Second order correlations
- Can you beam-split a photon?
- Early work in quantum optics by G I Taylor
- Photon bunching
- Photon antibunching





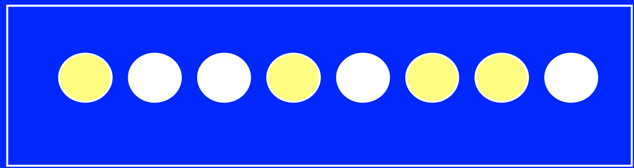
Diversion on mixed states and classicality

- What is a mixed state?



pure states

$$\frac{1}{\sqrt{2}}\left(|y\rangle + |b\rangle\right)$$



mixed states

$$\frac{1}{2}|y\rangle\langle y| + \frac{1}{2}|b\rangle\langle b|$$

- Why mixed states? As soon as a quantum state is embedded in an environment, the pure state becomes mixed.

- M B Plenio and V Vitelli, Contemp Phys 42, 25 (2001) and refs therein

Parametric Down Conversion:

from A Migdall

Physics Today 1999

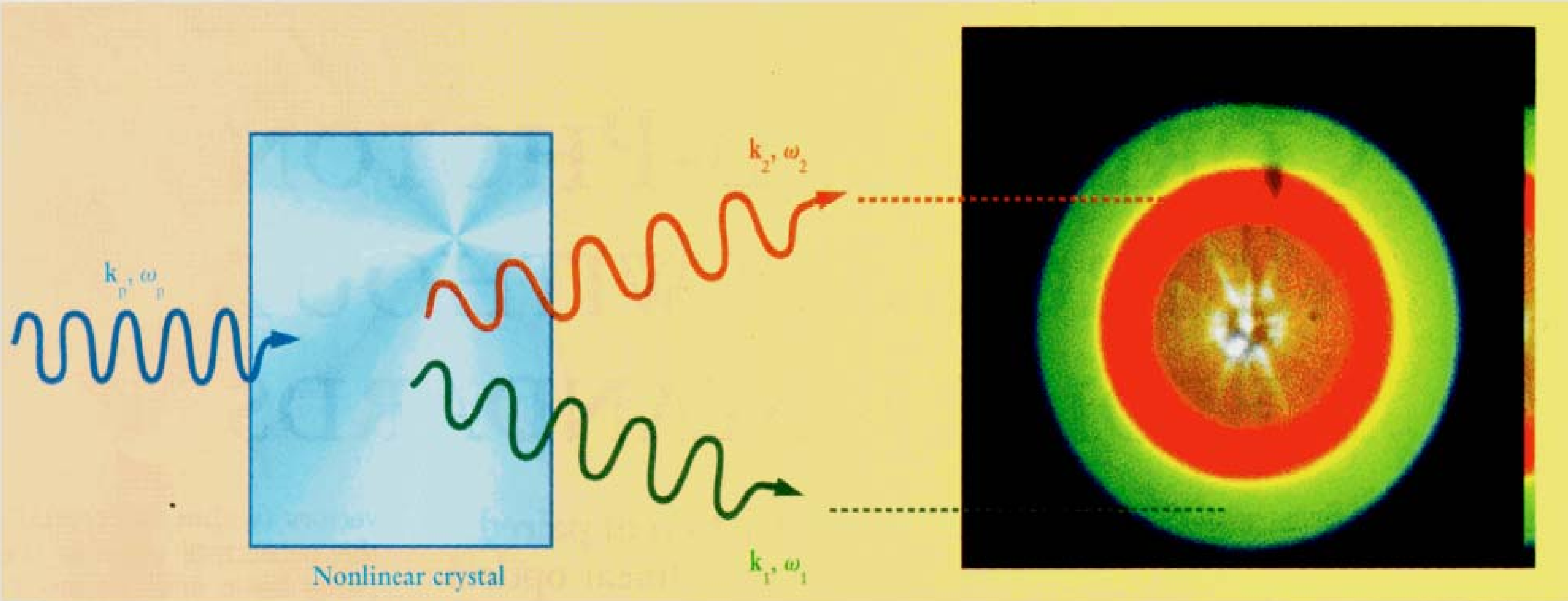
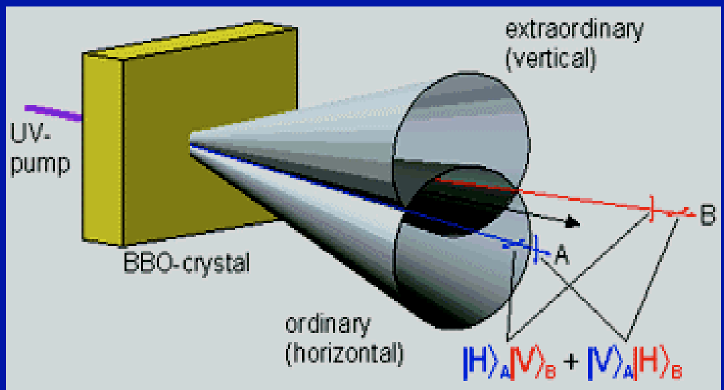


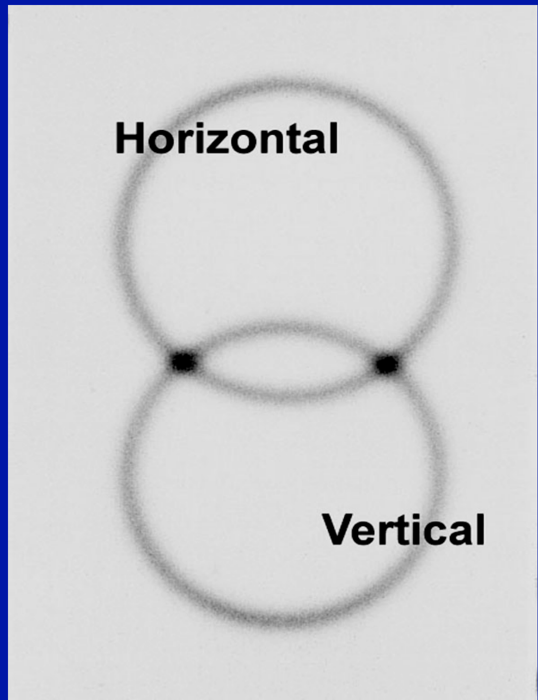
FIGURE 1. PARAMETRIC DOWN-CONVERSION, turning a single photon entering an optically nonlinear crystal into two photons coming out, is essentially the inverse of sum-frequency generation. The energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$ of the incident photon equals the sums of the outgoing energies and momenta. The concentric circles of output light in different colors (at right), azimuthally symmetric about the monochromatic pump-beam axis, indicate the broad spectral range of the down-converted light. At their center, one sees some of the pump light leaking around a beam stop.

Type-II, CW parametric downconversion

Source of polarization-entangled photon pairs

(Kwiat et al PRL 75, 4337 (1995))





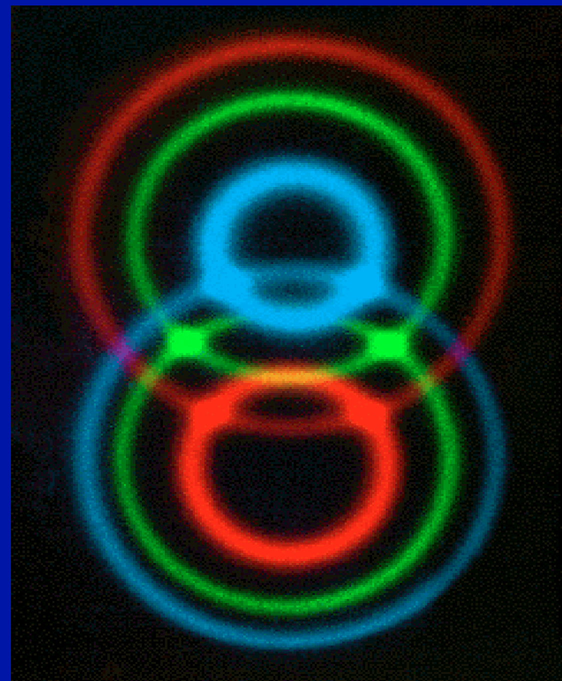
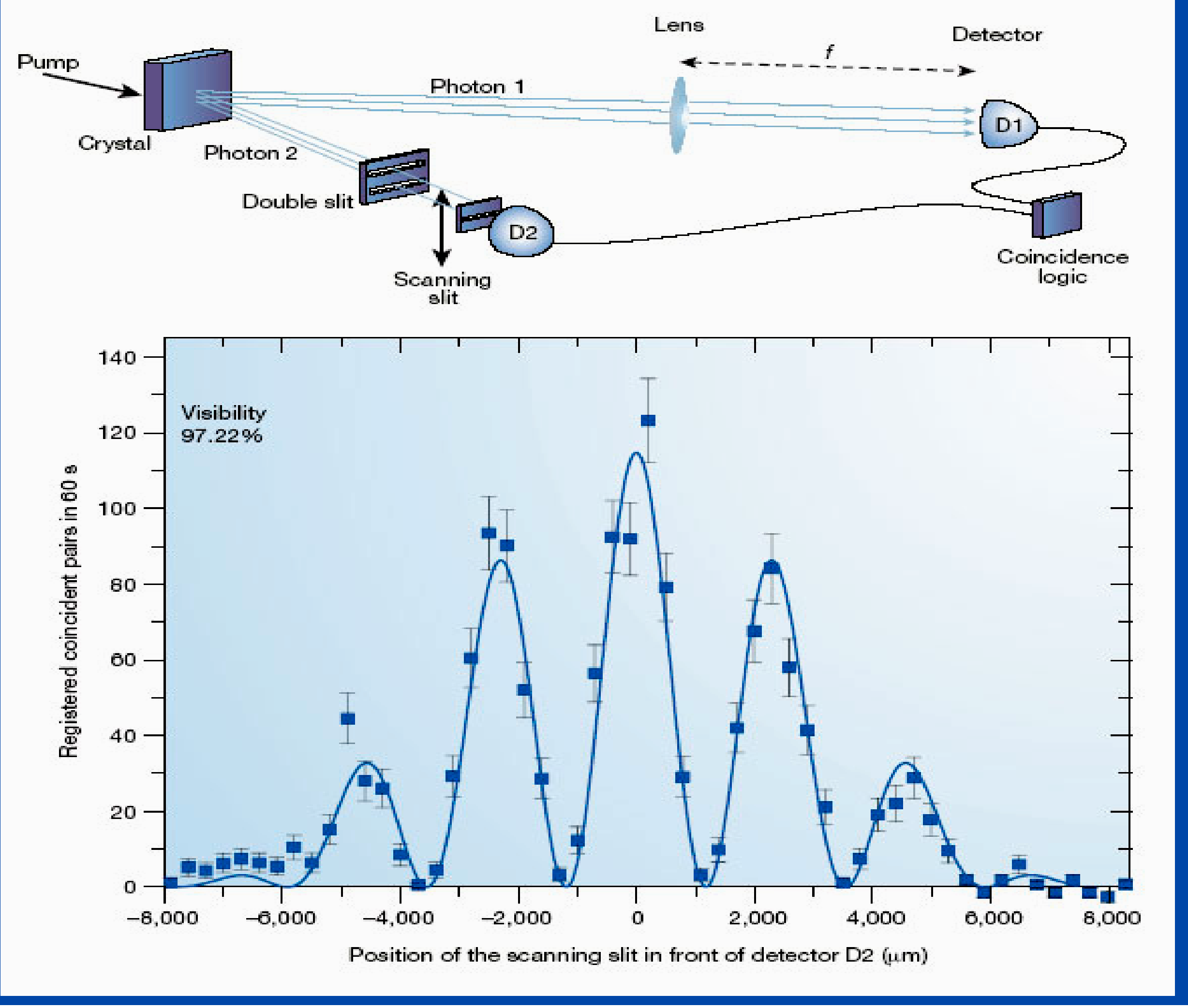


Figure 2 Single-photon double-slit interference. A pair of momentum-entangled photons is created by type-I parametric down-conversion. Photon 2 enters a double-slit assembly and photon 1 is registered by a detector D1 placed at distance *f* in the focal plane of the lens. This projects the state of photon 2 into a momentum eigenstate which cannot reveal any positional information and, hence supplies no information about slit passage. Therefore, in coincidence with a registration of photon 1 in the focal plane, photon 2 exhibits the interference pattern shown. On the other hand, when the detector is placed in the imaging plane, it does reveal the path photon 2 takes through the slit assembly, which therefore does not show the interference pattern. The observed count rate of at most two photons per second implies that the average spatial distance between photons registered would be of the order of 100,000 km or more. Therefore, most of the time the apparatus is empty (from refs 11 and 12). The error bars (s.d.) show the statistical errors of photon counting.

Zeilinger et al
Nature 433
230 (2005)



Beam splitting a photon?
Clauser (1972), Mandel & Kimble (1977)

- Can you beam split a photon?
- Heralded photons
- Antibunching and violation of Cauchy Schwartz inequality

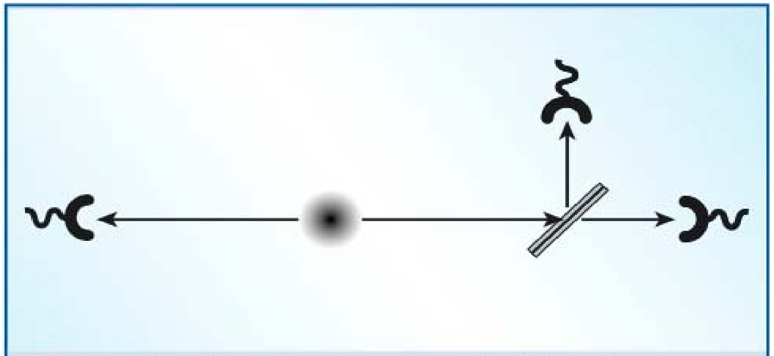
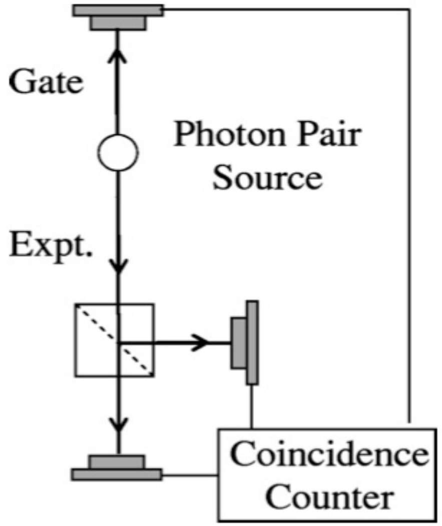


Figure 1 Principle of Clauser's experiment with correlated pairs of photons (simplified). The source emits two photons. Registration of a photon on the left detector provides the information that one and only one photon at the right side encounters a 50/50 beam splitter where it is either reflected or transmitted. The fact that only one of the two detectors behind the beam splitter registers and never both



2 photon interference (from Zeilinger)

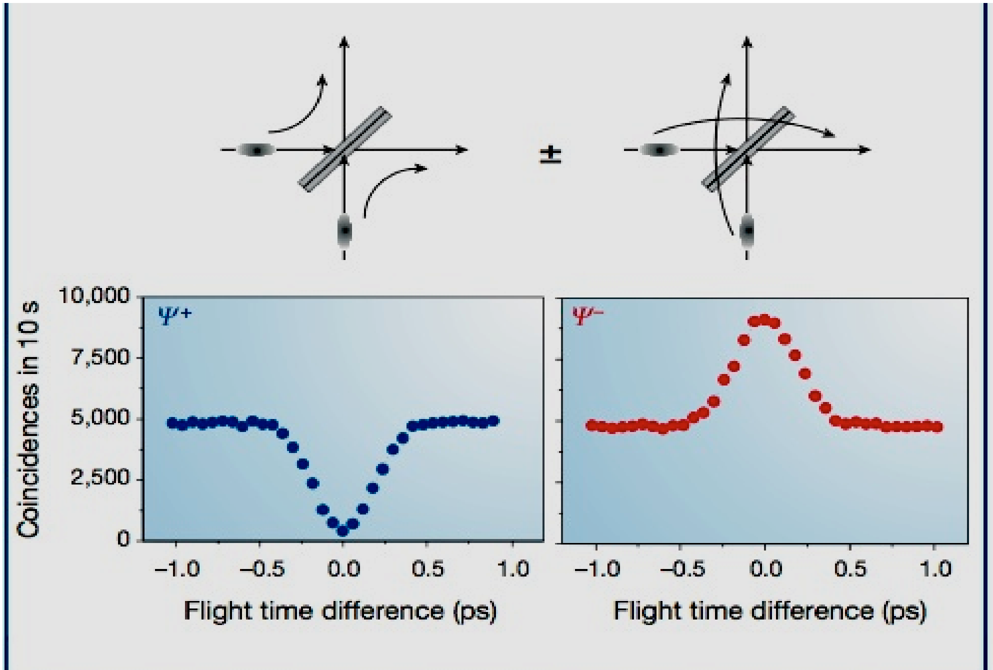


Figure 3 Bunching (left) or antibunching (right) behaviour of photon pairs. One photon each is incident from each input of a 50/50 beam splitter. Coincidences between detectors in the two output beams are registered as a function of the flight time difference of the incident photons. No coincidences are observed for zero flight time difference (left) for the usual symmetric spatial state of the two photons. This is because the probability amplitudes for the transmission of both photons and for the reflection of both photons destructively interfere: the latter one picks up a minus sign owing to the phase shift of the photons upon reflection. Interestingly, the two incident photons can also be in an antisymmetric spatial state (which occurs if the two-photon spin state is also antisymmetric). In this case, the two amplitudes interfere constructively. This results in the two photons always exiting in separate beams for zero flight time difference. The observed coincidence peak (right) confirms this expected antibunching.

Rarity: metrology using correlated photons

(from Migdall Physics Today)

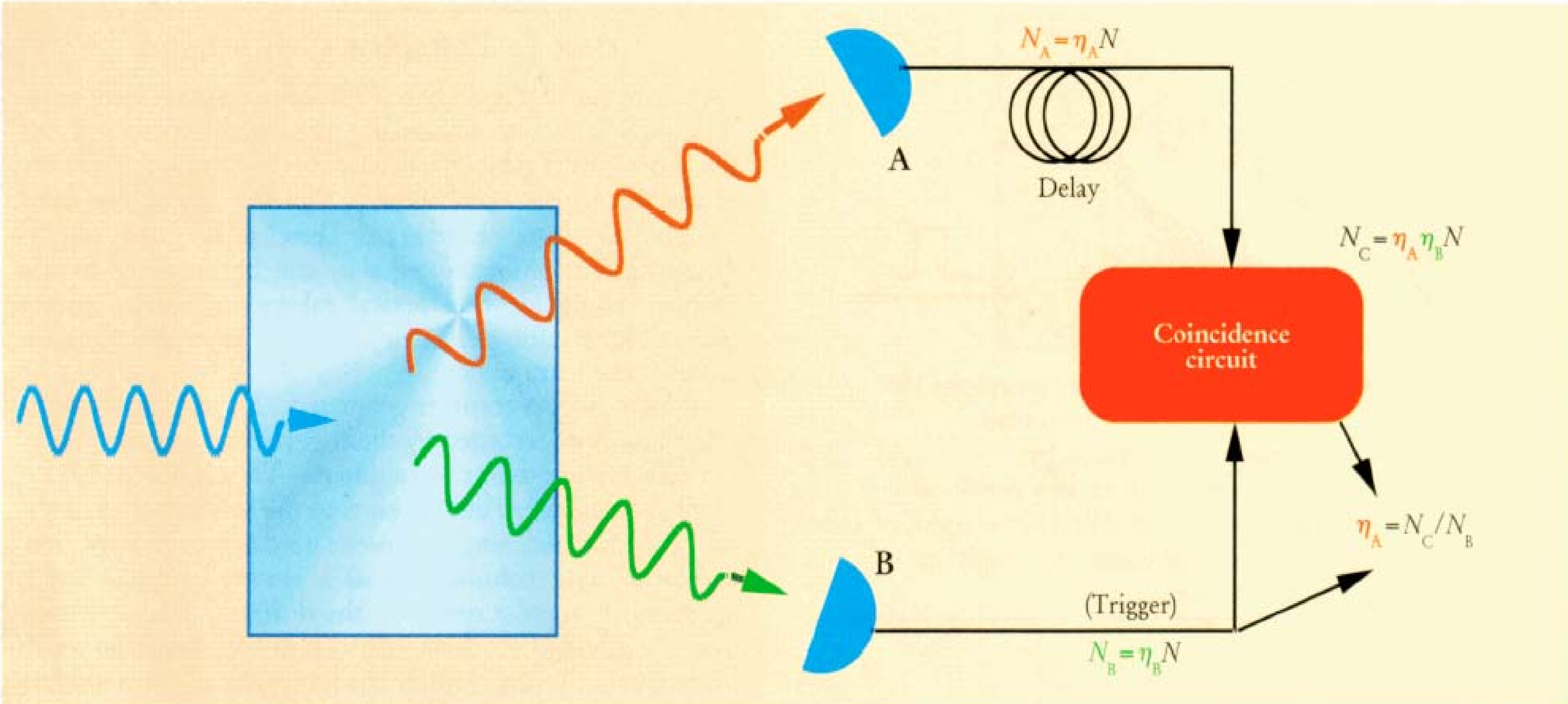


FIGURE 2. ABSOLUTE QUANTUM-EFFICIENCY DETERMINATION. N is the true number of correlated photon pairs produced in the down-conversion crystal, and N_A and N_B are the tallies of photons recorded individually by detectors A and B, with respective unknown efficiencies η_A and η_B. The number of expected coincidence counts N_C being N times the product of these two efficiencies, one arrives at the efficiency of A, the detector to be calibrated, without having to know the efficiency of B, the trigger detector.

Single photon source
(from Migdall PT)

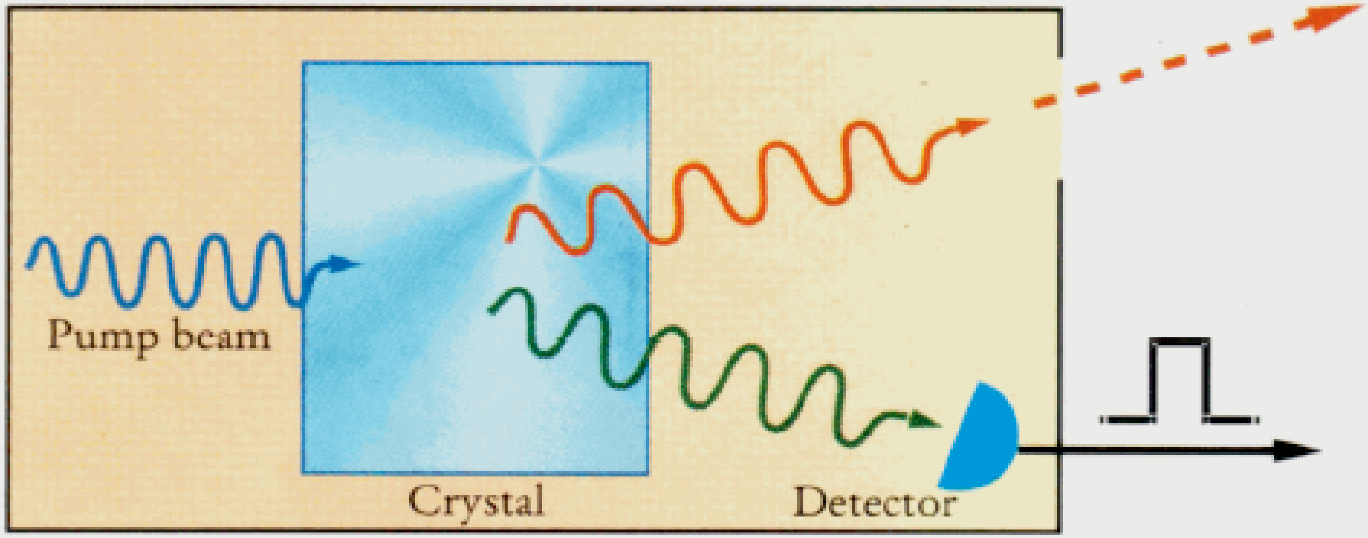


FIGURE 4. NEW ABSOLUTE PHOTON SOURCE exploits the constraints of simultaneity and energy–momentum conservation on pairs of photons produced by down-conversion of pump-beam photons in a nonlinear crystal to specify not only the direction and wavelength of an output photon, but also its time of emission, simply by recording its partner in a detector.

Non-classical features: Wigner Functions

- What is a non-classical state?
- you could say negative Wigner functions

$$W(q,p) \equiv \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\left\langle q+\frac{1}{2}x\middle|\hat{\rho}\middle|q-\frac{1}{2}x\right\rangle e^{ipx/\hbar}dx$$

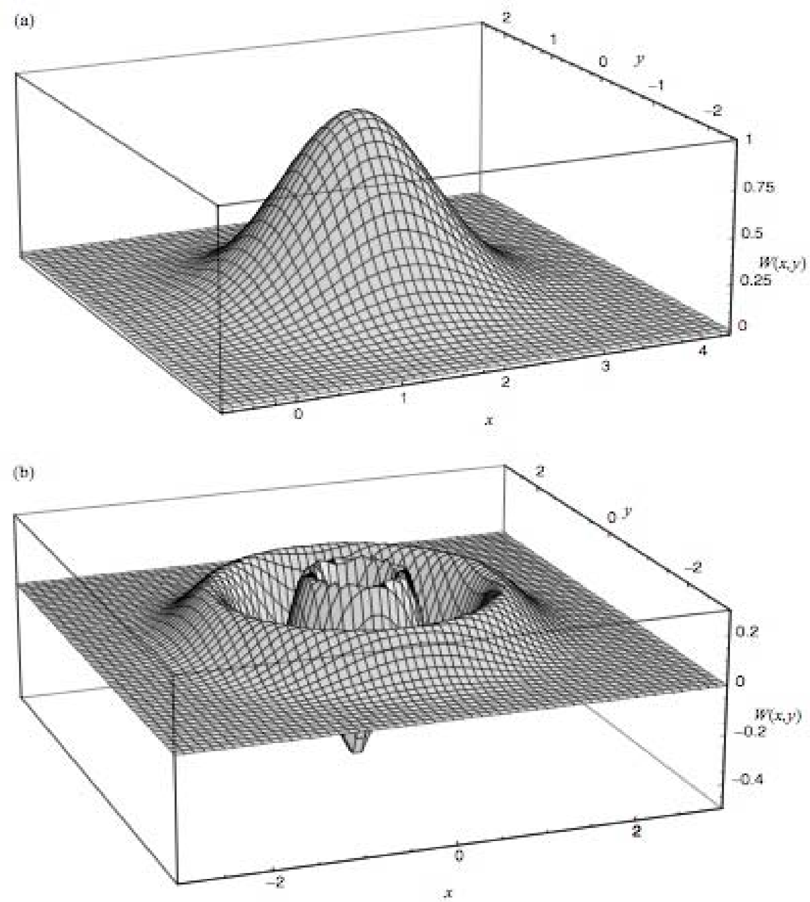


Fig. 3.10. Wigner function for (a) a coherent state with $\bar{n} = 10$, (b) a number state with $n = 3$.

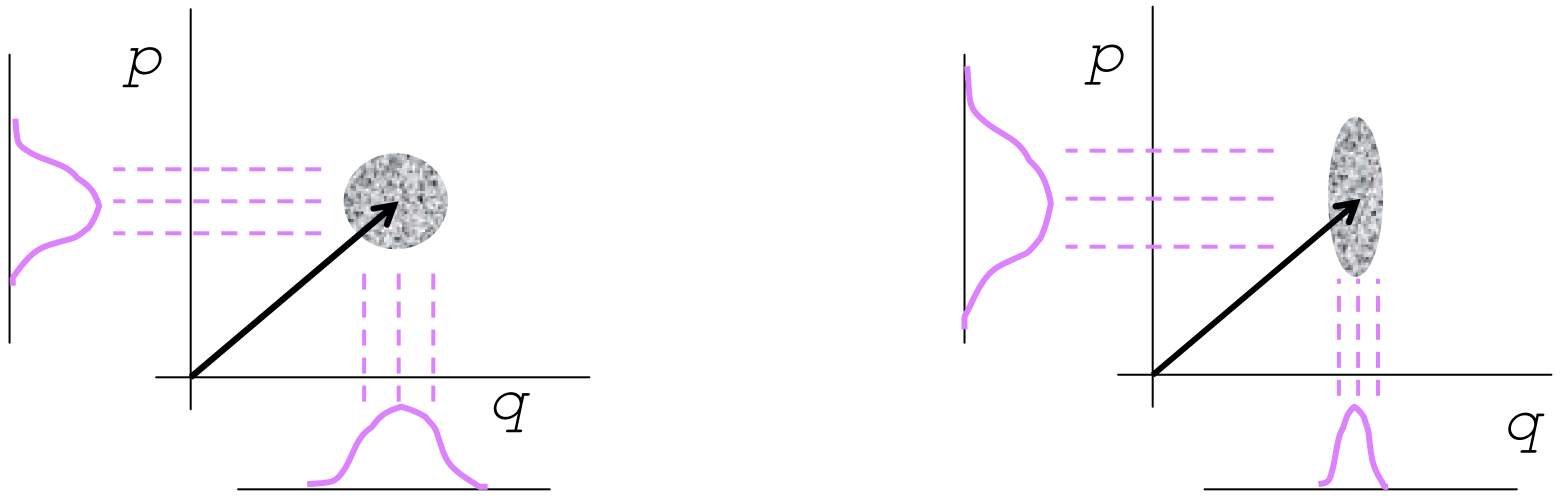
"In 1936 came a shock ... Princeton dismissed me ... they never explained why ... I could not help feeling angry." He seemed to forgive them as he came back as a Professor within a couple of years

WIGNER DISTRIBUTION

Representation of state of a single mode in phase space.

$$\hat{E}^{(+)}(z,t) \propto \hat{q}\cos(\omega_0 t - k_0 z) + \hat{p}\sin(\omega_0 t - k_0 z)$$

projected distributions: *Pr(q)*, *Pr(p)*



Underlying Joint Distribution?

$$W(q,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} \psi(q + x/2)\, \psi^*(q - x/2) \exp(-ipx)\, dx$$

WIGNER DISTRIBUTION
in phase space: marginal distributions.

$$W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(q+q'/2)\ \psi^*(q-q'/2)\ \exp(-i\,q'\,p)\ dq'$$

$$Pr(q) = \int_{-\infty}^{\infty} W(q,p)\ dp \qquad , \qquad Pr(p) = \int_{-\infty}^{\infty} W(q,p)\ dq$$

W*(q,p)* acts like a joint probability distribution.

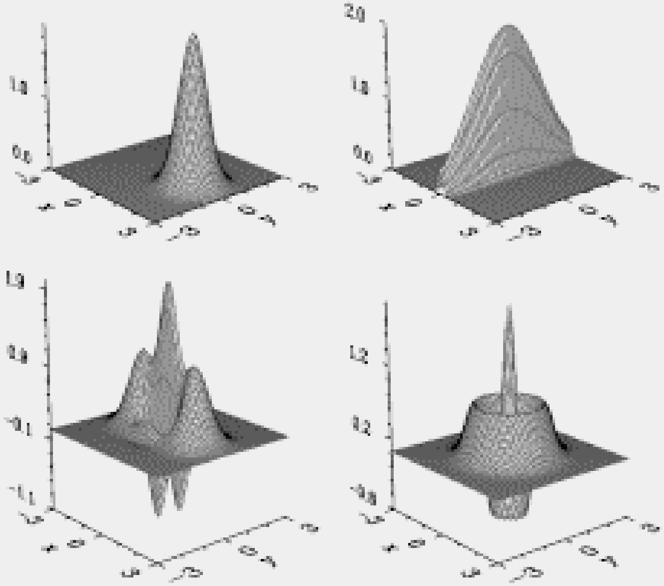
But it can be negative. See eg M G Raymer, Contemp Phys 38, 343 (1997)

Wigner Functions of Light States

$$W(q,p) = \frac{1}{2\pi\hbar}\int C(q',p')\exp\left[-\frac{i(qp'-pq')}{\hbar}\right]dq'dp'$$

characteristic function $C_{\hat{\rho}}^{(W)}(q,p) = \mathrm{Tr}\left[\hat{\rho}\hat{D}(q,p)\right]$

displacement operator $\hat{D}(q,p) = \exp\left[\frac{i}{\hbar}(\hat{q}p - \hat{p}q)\right]$



Marginal distributions

$$P_{\hat{\rho}}(q) \equiv \frac{1}{\sqrt{2\pi\hbar}}\int dp\, W_{\hat{\rho}}(q,p) = \sqrt{2\pi\hbar}\langle q|\hat{\rho}|q\rangle$$

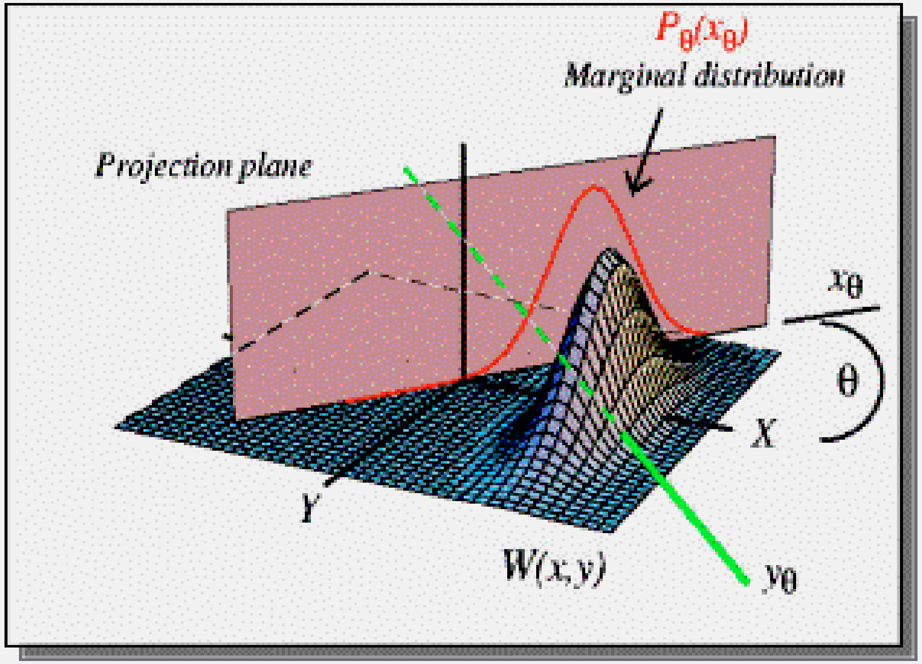
M.Hillery, R.F.O'Connell, M.O.Scully, and E.P.Wigner, *Phys. Rep.* **106**, 121 (1984)

Quantum Tomography

- rotated quadratures

$$\hat{x}_\theta = \sqrt{\frac{\hbar}{2}}\left[\hat{a}\mathrm{e}^{-i\theta} + \hat{a}^\dagger \mathrm{e}^{i\theta}\right] \qquad \hat{x}_{\theta+\pi/2} = \frac{\sqrt{\hbar}}{i\sqrt{2}}\left[\hat{a}\mathrm{e}^{-i\theta} - \hat{a}^\dagger \mathrm{e}^{i\theta}\right]$$

- marginal distribution for $P_\theta(x_\theta)$



K.Vogel and H.Risken, *Phys. Rev. A* **40**, 2847 (1987);

U.Leonhardt: *Measuring the quantum state of light* (Cambridge University Press, Cambridge, 1997).

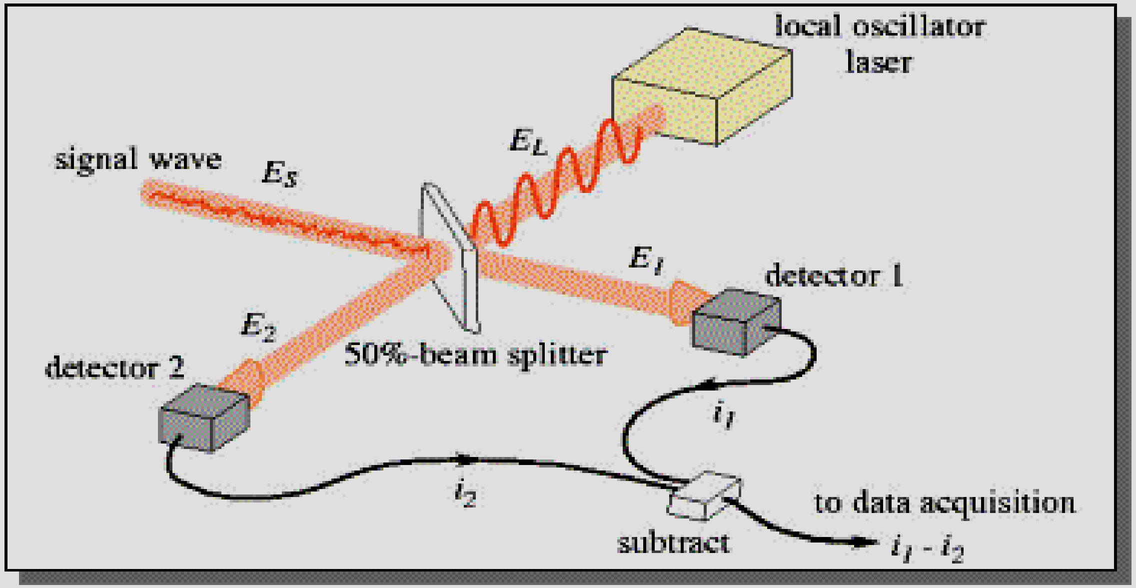
Can you measure -ve Wigner?

- Yes by homodyne

Inverse Transformations

$$P_\theta(x_\theta)\forall\{-\infty \leq x_\theta \leq \infty; 0 \leq \theta \leq \pi\} \longrightarrow W(q,p)$$

- Radon transformation
- Transformation via sampling functions $\rho_{mn} = \int_0^\pi \int_{-\infty}^{\infty} P_\theta(x_\theta) F_{mn}(x_\theta,\theta)\, dx_\theta\, d\theta$



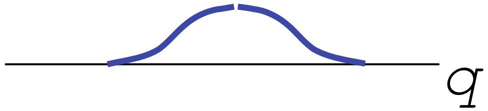
K.Vogel and H.Risken, *Phys. Rev. A* **40**, 2847 (1987);

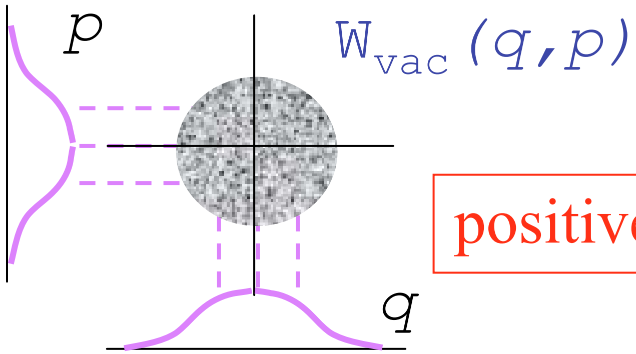
Th.Richter, *Phys. Lett. A* **211**, 327 (1996);

G.M.D'Ariano, C.Machiavelo, and M.G.A.Paris, *Phys. Rev. A* **50**, 4298 (1994).

WIGNER DISTRIBUTION
for vacuum state $|0\rangle$

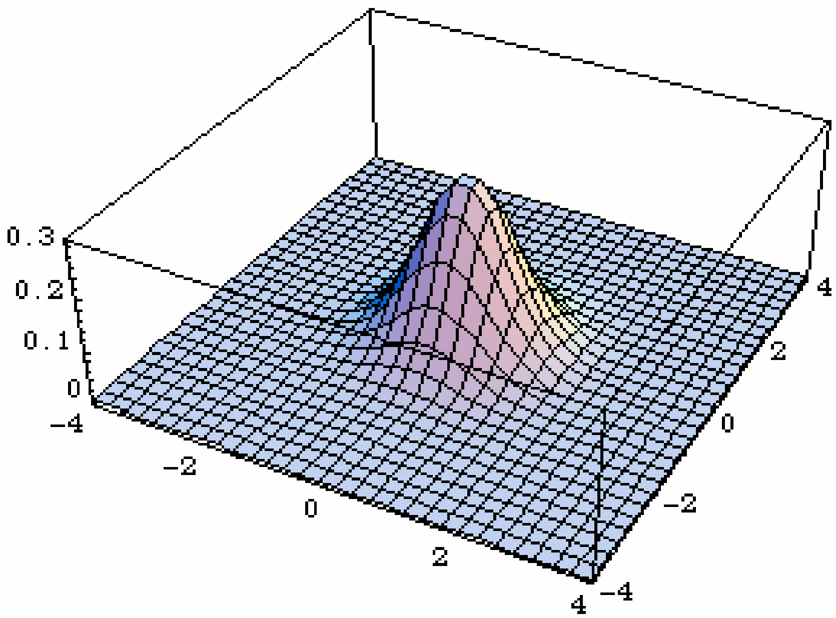
$$\psi_{vac}(q) = \exp\left[-q^2/2\right]$$





positive

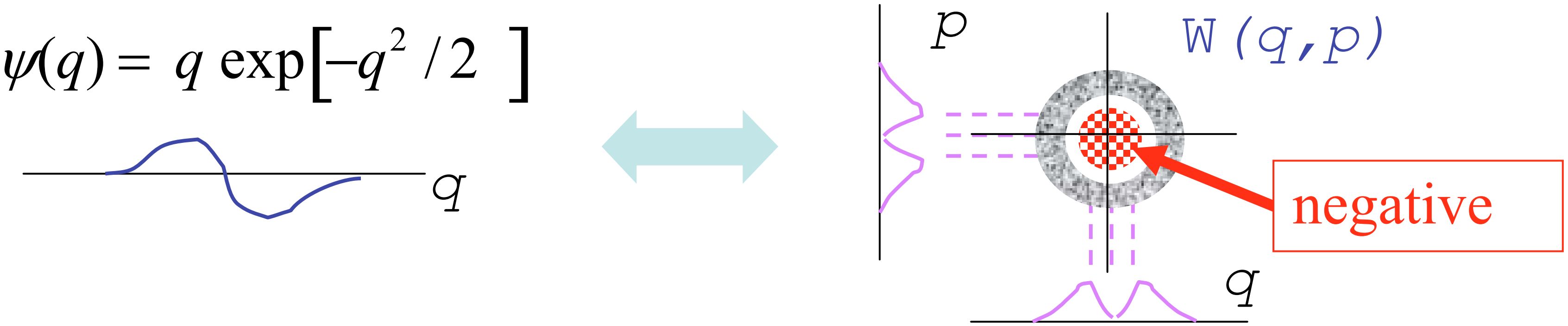
$$W(q,p) = \frac{1}{\pi}\exp(-q^2 - p^2)$$



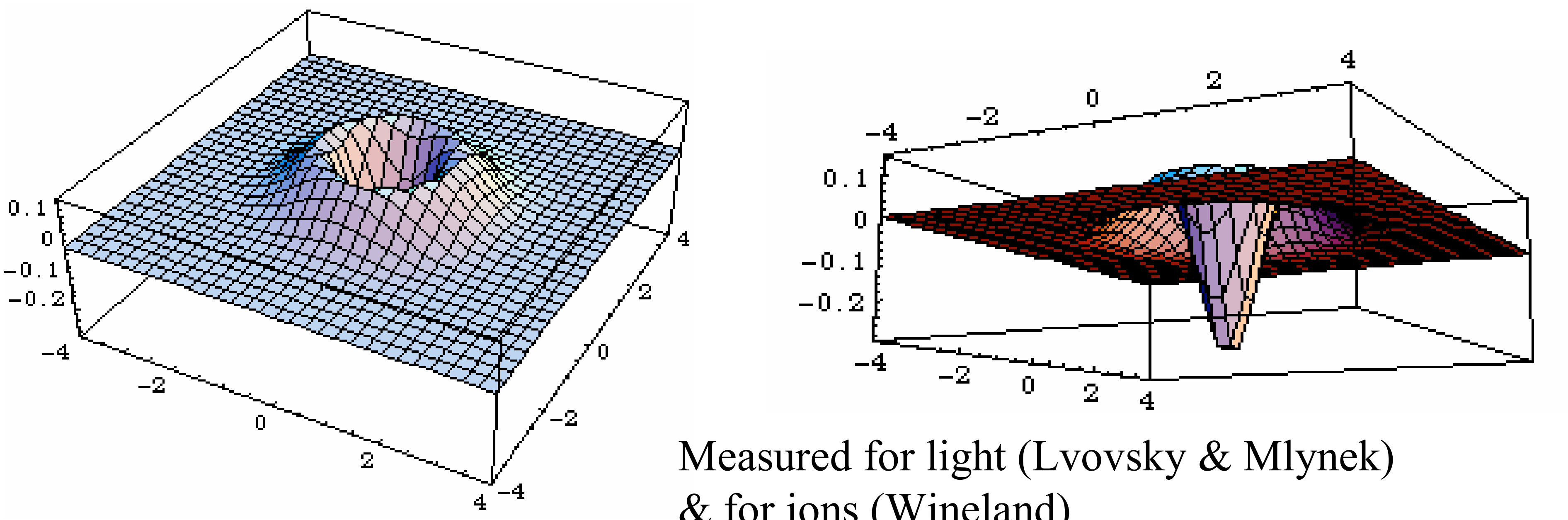
Gaussian vacuum state: the only pure state Wigner function which is positive everywhere (the Hudson-Piquet theorem)

WIGNER DISTRIBUTION
for one-photon state | 1 >

$$\psi(q) = q\exp\left[-q^2/2\right]$$



$$W(q,p) = \frac{2q^2 + 2p^2 - 1}{\pi}\exp(-q^2 - p^2)$$



Measured for light (Lvovsky & Mlynek)
& for ions (Wineland)

Multimode Fields

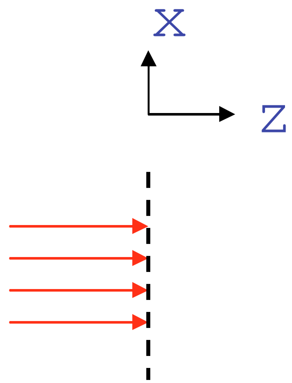
Photon-flux amplitude operator:

$$\hat{\underline{\Phi}}^{(+)}(\underline{r},t) = i\sqrt{c}\sum_j \hat{b}_j\ \underline{u}_j(\underline{r})\ \exp(-i\omega_j t)$$

monochromatic plane-wave modes:

$$\underline{u}_j(\underline{r}) = V^{-1/2}\varepsilon_j \exp(i\,\underline{k}_j\cdot\underline{r})$$

Photon flux through a plane at z=O:



$$\hat{I}(t) = \int_{Det} d^2x\ \hat{\underline{\Phi}}^{(-)}(\underline{x},0,t)\cdot\hat{\underline{\Phi}}^{(+)}(\underline{x},0,t)$$

Integrated photon number in time T:

$$\hat{N} = \int_0^T \hat{I}(t)\ dt$$

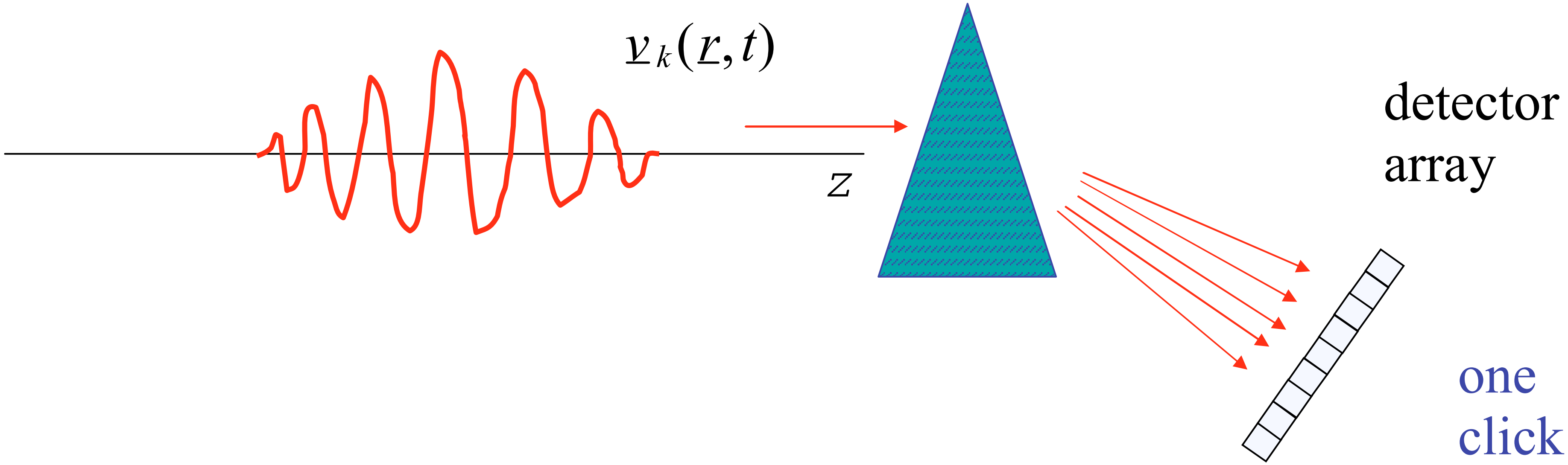
Non-monochromatic Wave-Packet Modes

$$\hat{\Phi}^{(+)}(\underline{r},t) = i\sqrt{c}\sum_k \hat{a}_k \; \underline{v}_k(\underline{r},t)$$

$\hat{a}_k{}^\dagger$ operator creates <u>one</u> photon in the wave packet

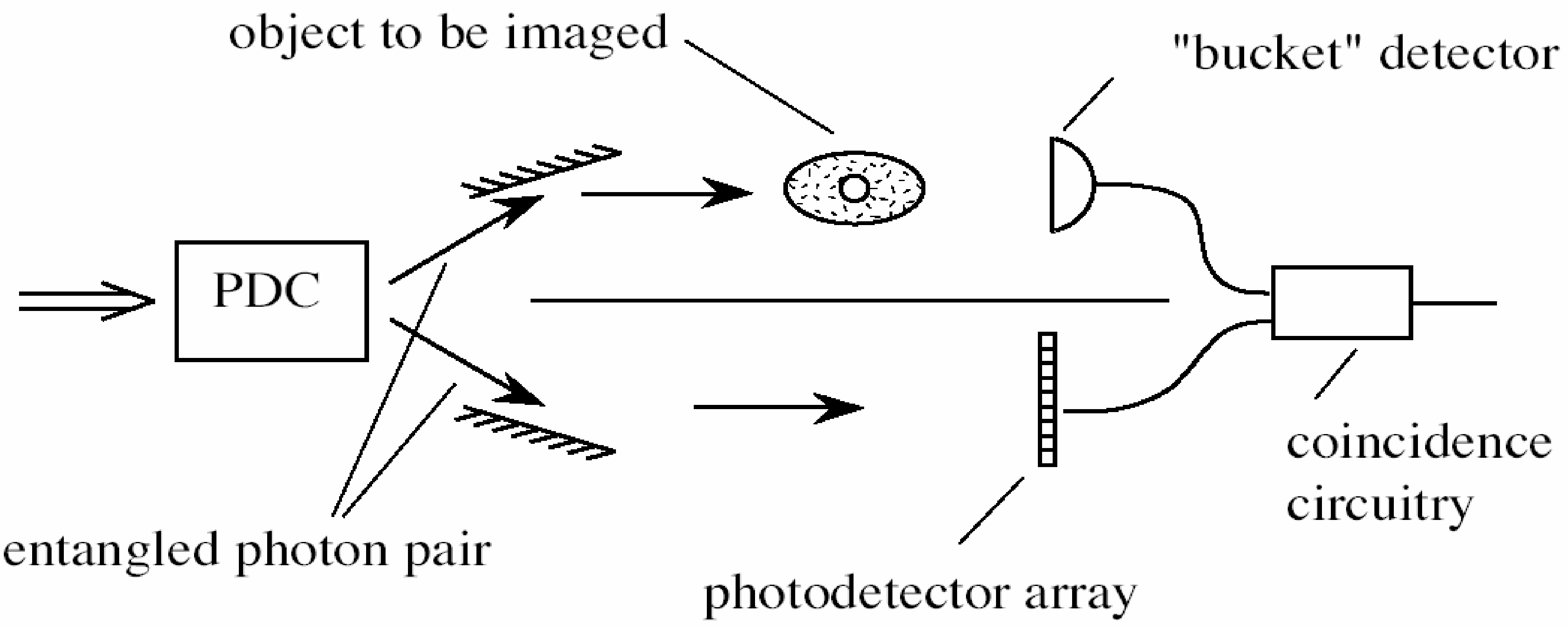
$$\hat{a}_k{}^\dagger |vac\rangle = |0, 0, \ldots 1_k, 0, 0 \ldots\rangle$$

$\underline{v}_k(\underline{r},t)$



Twin photons and Ghost imaging

Ghost (Coincidence) Imaging



Obvious applicability to remote sensing!

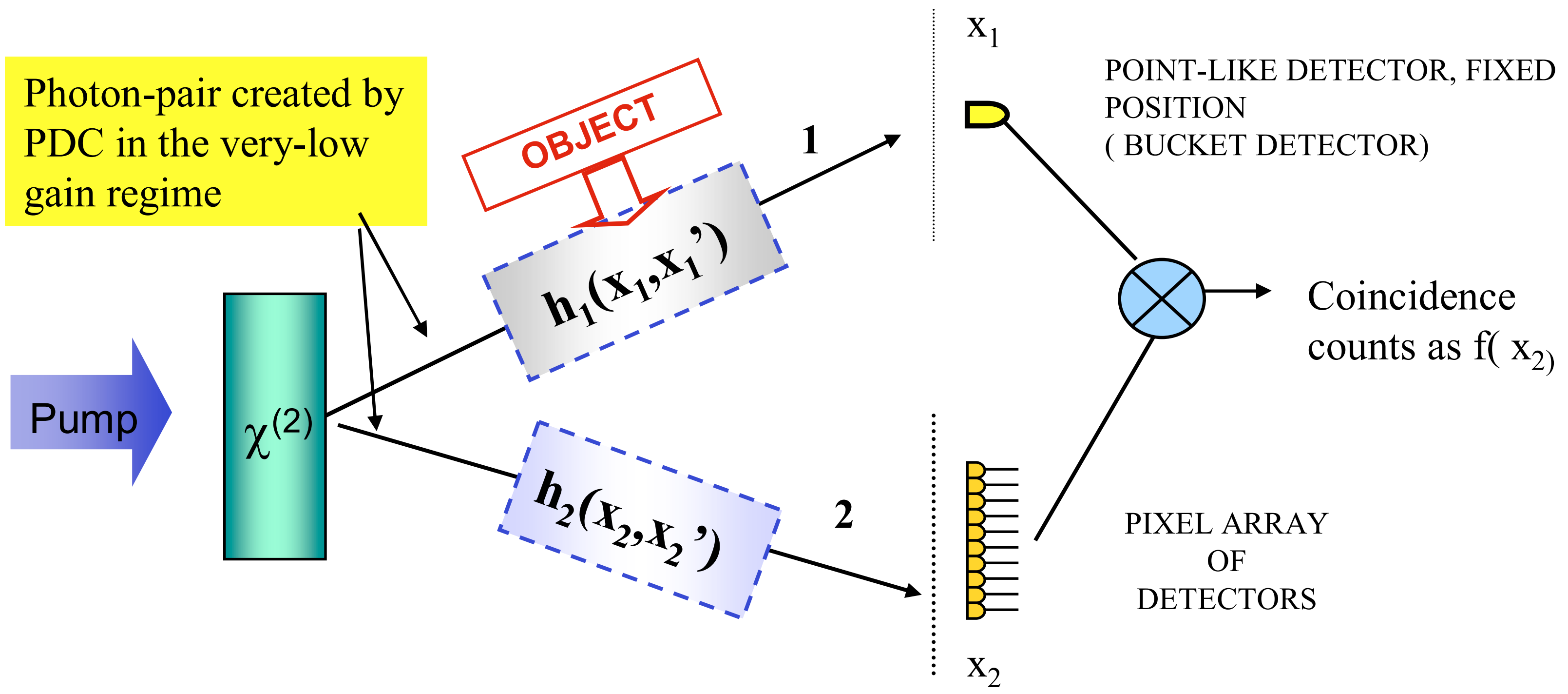
Ghost imaging using two-photon quantum entanglement

Belinsky and Klyshko, Sov. Phys JETP 78, 259 (1994)

•Pittman et al, PRA 52, R3429 (1995) GHOST IMAGE EXP

•Abouraddy et al, Phys.Rev.Lett. **87**, 123602 (2001) THEORY

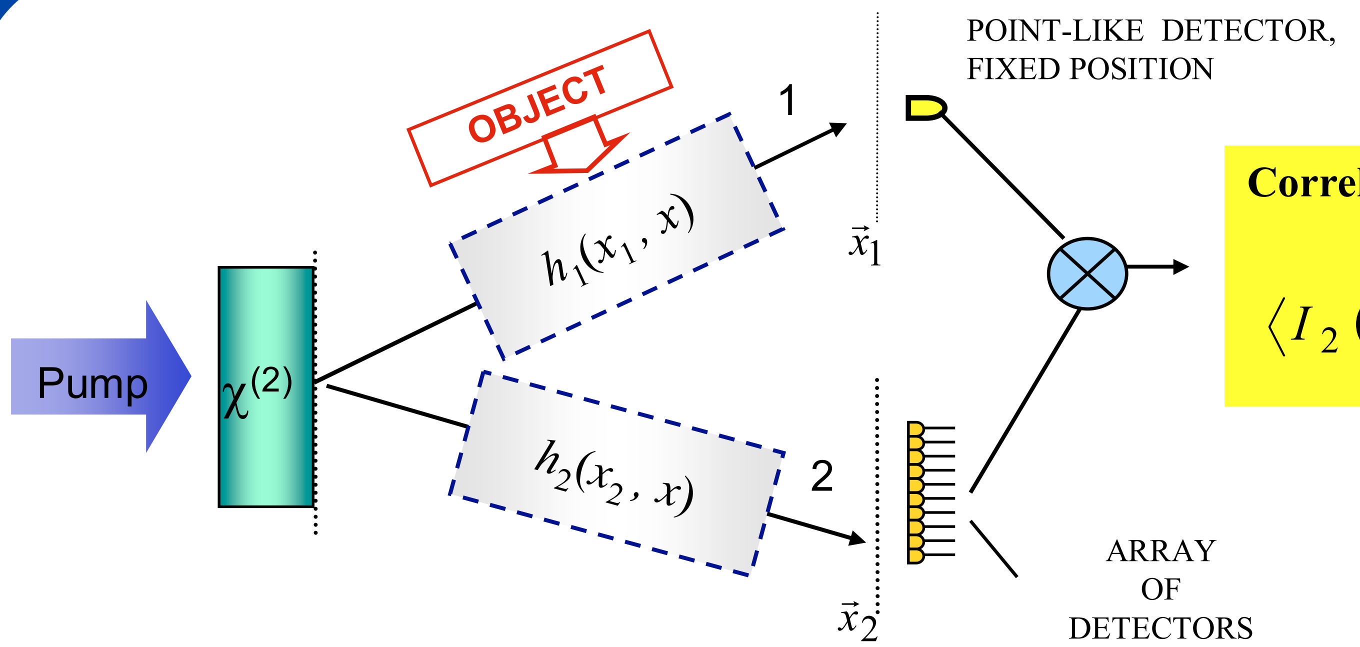
} GHOST DIFFRACTION EXP



Imaging information is extracted from the coincidence counts as a function of the position of the reference photon 2

signal-idler intensity correlation function

[Gatti, et al, PRL 90, 133603 (2003)]



Correlation function of intensities

$$\langle I_2(\vec{x}_2) I_1(\vec{x}_1)\rangle$$

IMAGING INFORMATION IS CONTAINED IN THE INTENSITY FLUCTUATION CORRELATION FUNCTION $\delta I_i = I_i - \langle I_i \rangle$

$$\langle I_2(\vec{x}_2) I_1(\vec{x}_1)\rangle = \langle I_2(\vec{x}_2)\rangle\langle I_1(\vec{x}_1)\rangle + \langle \delta I_2(\vec{x}_2)\delta I_1(\vec{x}_1)\rangle$$

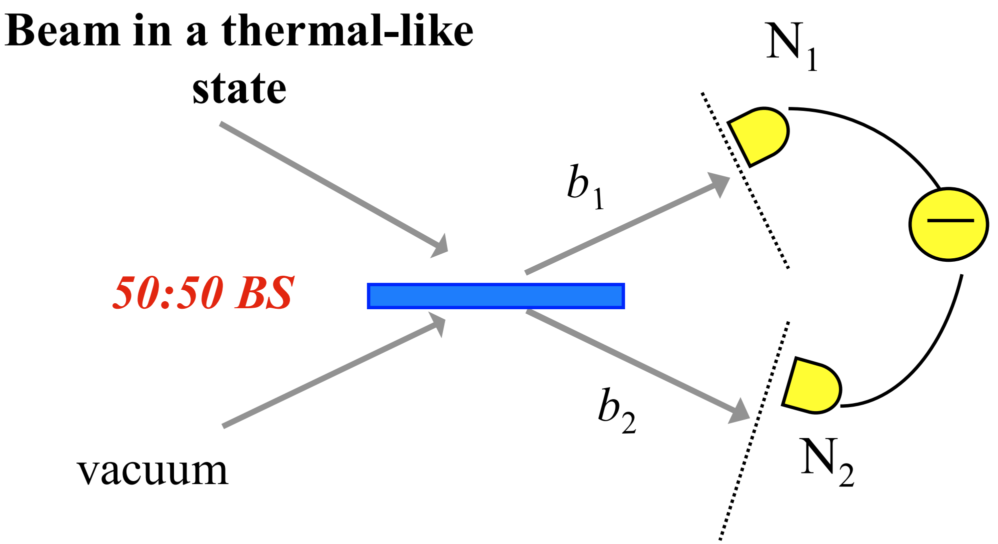
no information, background

Imaging information

Gatti et al, PRL **93**, 093602 (2004), Phys. Rev. A **70**, 013802 (2004),

Is entanglement of the two beams necessary for ghost imaging? NO!

A spatially incoherent thermal-like beam divided at a beam splitter generates two spatially correlated beams that can be used for ghost imaging exactly in the same way as the entangled beams, except with limited visibility.

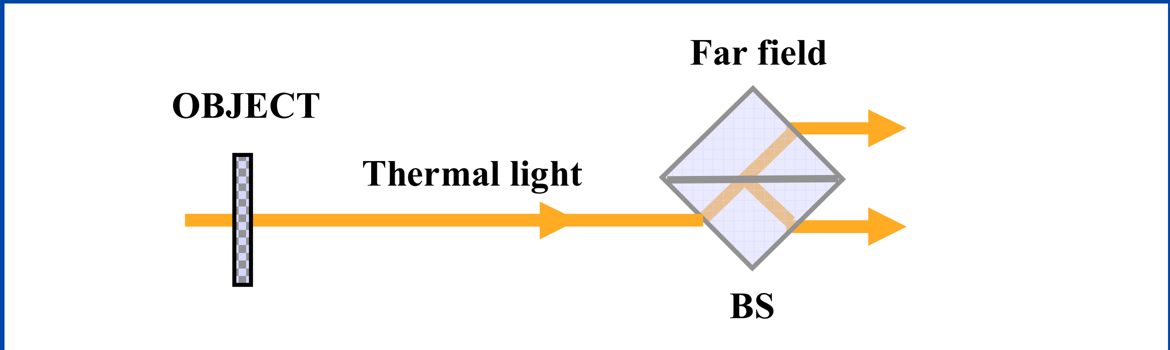


$$N_- = N_1 - N_2$$

$$\langle \delta N_-^2 \rangle = \langle N_1 \rangle + \langle N_2 \rangle$$

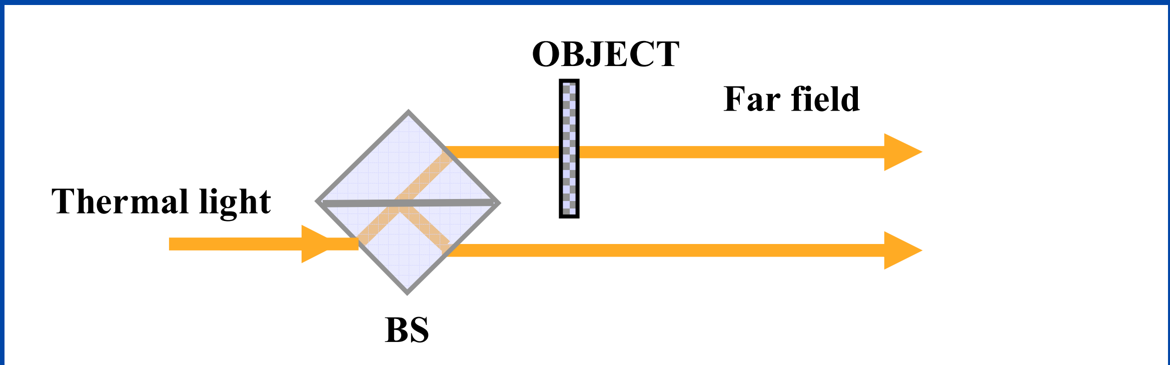
SHOT - NOISE LEVEL

HBT TECHNIQUE



Auto-correlation

GHOST IMAGING TECHNIQUE



Cross- correlation

In this case, one obtains the Fourier transform of the object even in the presence of phase modulation. Hence this is truly coherent imaging with incoherent light.

Key papers on ghost imaging:

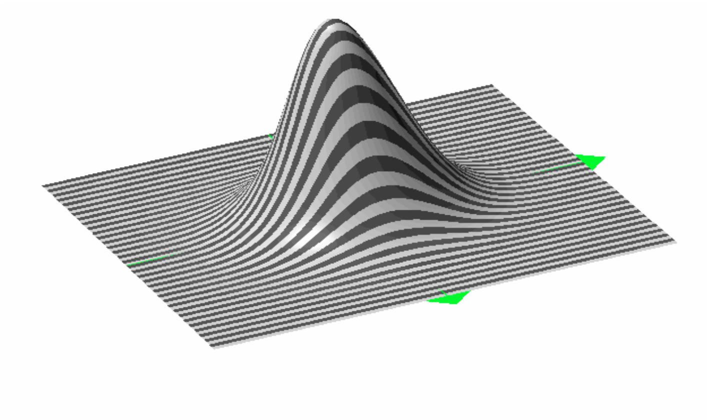
-Abouraddy, Saleh, Sergienko, Teich, Phys. Rev. Lett. 87, 123602 (2001)
-Bennink, Bentley, Boyd, Phys. Rev. Lett. 89, 113601 (2002)
-Gatti, Brambilla, Lugiato, Phys. Rev. Lett. 90, 133603 (2003)
-Gatti, Brambilla, Lugiato, quant-ph/0307187 (2003) $\rightarrow$ Phys. Rev. Lett. 93, 093602 (2004); Phys. Rev. A 70, 013802 (2004)
-Bennink, Bentley, Boyd, Howell, Phys. Rev. Lett. 92, 033601 (2004)
- Cheng, Han, Phys. Rev. Lett. 92, 093903 (2004)
- Valencia, Scarcelli, D'Angelo, Shih, Phys. Rev. Lett. 94, 063601 (2005)
- Wang, Cao, Phys. Rev. A 70, 041801R (2004)
- Cai, Zhu, Opt. Lett. 29, 2716 (2004)
- Ferri, Magatti, Gatti, Bache, Brambilla, Lugiato, Phys. Rev. Lett. 94, 183602 (2005)
- Zheng, Zhai, Chen, Wu, Opt. Lett. 30, 2354 (2005)

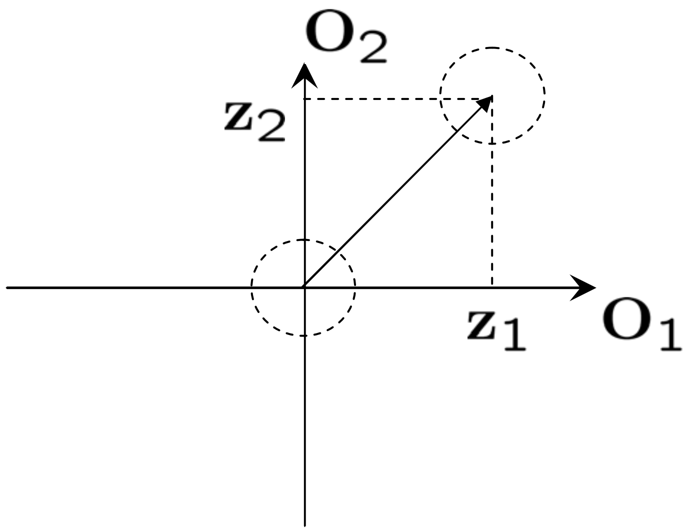
Continous variables

- Use nonclassical resourc?
- squeezing

Characteristic function

- **Characteristic function** (Fourier transform of Wigner function)

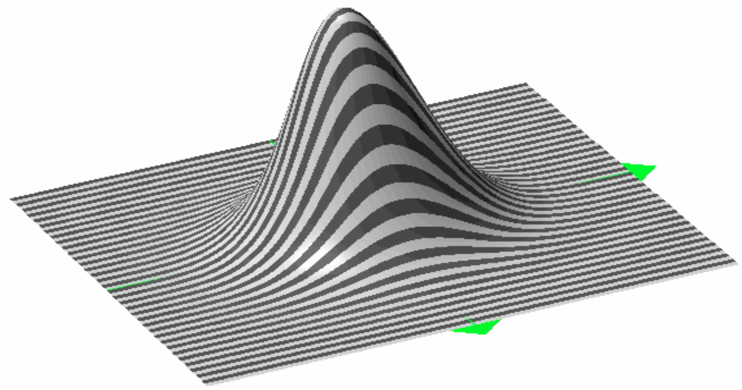




General CV states too general: Restrict to Gaussian states

- A state is called **Gaussian**, iff its characteristic function (or its Wigner function) is a Gaussian

$$\chi(x,p) \propto \exp\left(-x^2\right)\exp\left(-p^2\right)$$



- Gaussian states are completely determined by their first and second moments

- Are the states that can be made experimentally with current technology

- coherent states
- squeezed states (one and two modes)
- thermal states

non-classical features

- What is a non-classical state?
  - State without a positive well-defined P function

$$\rho = \int P(\alpha) |\alpha\rangle\langle\alpha| d^2\alpha$$

- Or you could say negative Wigner functions
- But discreteness of photons in principle observable (JCM and Haroche…)

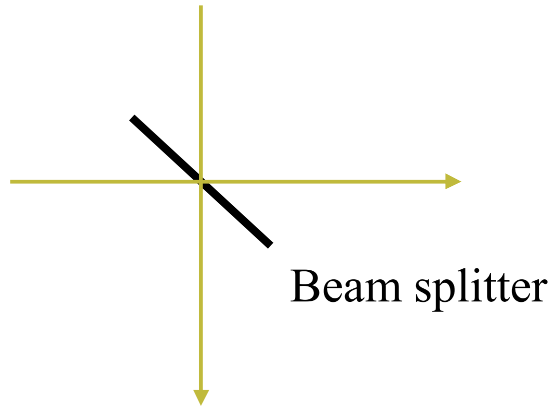
Entanglement

- If a system of two (or more) particles is not represented by a weighted sum of product states, the particles are said to be entangled: $\rho \neq \sum_{\mathrm{i}} \mathrm{p}_{\mathrm{i}} \rho_a(i) \otimes \rho_b(i)$
- Peres criterion [***A. Peres, Phys.Rev.Lett. 77, 1413* (1996)**]
  If the partial transposition of its density matrix has a negative eigenvalue, the state is said to be entangled.
- For example, $\frac{1}{\sqrt{2}}\left(|1\rangle|0\rangle + |0\rangle|1\rangle\right)$
- Two important ingredients: Random & deterministic

Beam splitter as an entangler

- Two Fock state inputs always result in an entangled output.
- Well known result

$$|1\rangle|1\rangle \xrightarrow{bs} \frac{1}{\sqrt{2}}\left(|2\rangle|0\rangle + |0\rangle|2\rangle\right)$$



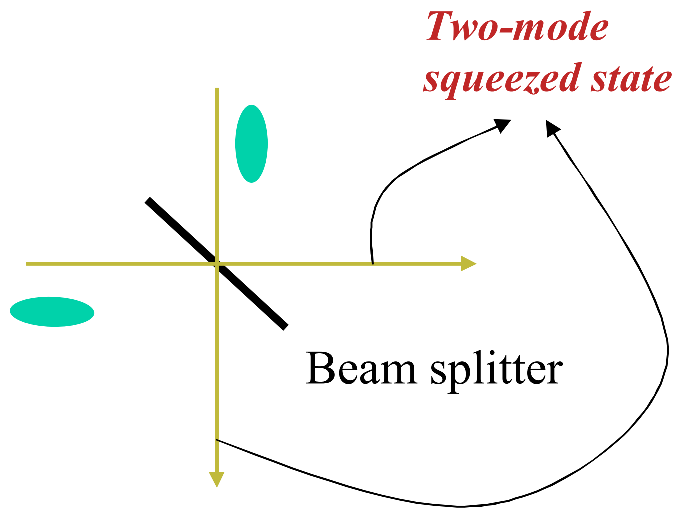
- Two single-mode squeezed input fields & Two-mode squeezed output

Single-mode squeezed state: $\sum C(n)|2n\rangle$

Two-mode squeezed state: $\sum B(n)|n\rangle|n\rangle$

Two single-mode squeezed state:

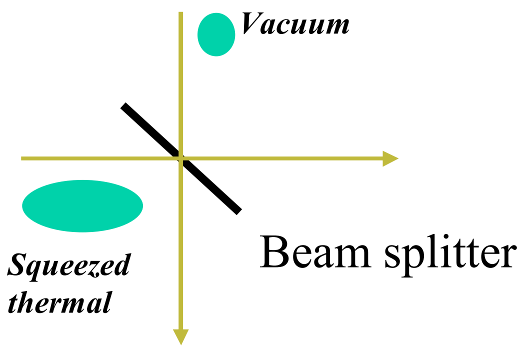
$a_1|0\rangle|0\rangle + a_2|0\rangle|2\rangle + a_2|2\rangle|0\rangle + \ldots..$



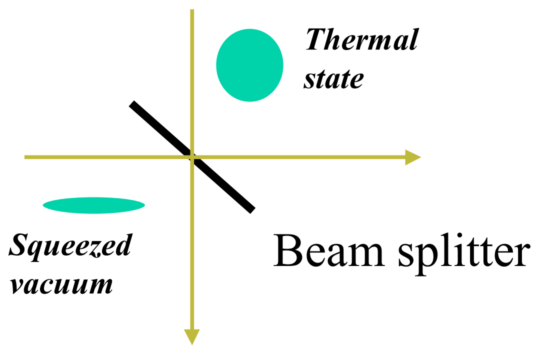
Non-classicality vs. Entangled output

Kim, Son, Buzek & Knight, quant-ph/0106136

Straightforward to prove that two classical input fields do not result in an entangled output.



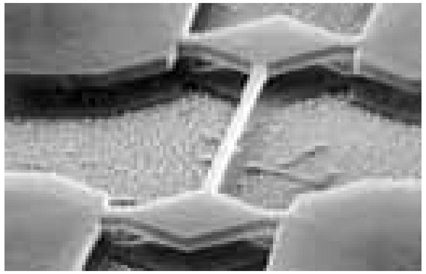
Output fields are entangled iff the squeezed thermal state becomes non-classical.

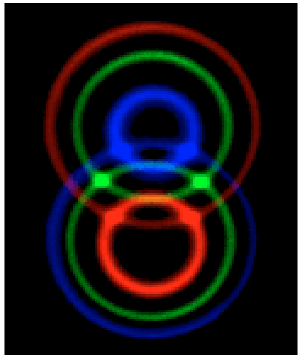


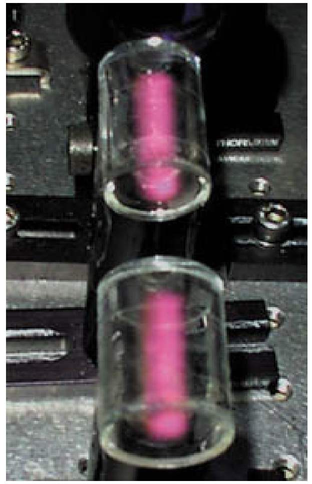
Output fields are entangled iff a reduced output field becomes non-classical.

Quantum Continuous Variable Systems

- Harmonic oscillators, light modes or cold atom gases.







- canonical variables with commutation relations

$$(\mathbf{O}_1, \mathbf{O}_2, \ldots, \mathbf{O}_{2n-1}, \mathbf{O}_{2n}) = (X_1, P_1, \ldots, X_n, P_n)$$

Further reading: textbooks

- M O Scully and M S Zubairy, Quantum Optics, CUP (1997)
- C C Gerry and P L Knight, Introductory Quantum Optics, CUP (2005)
- L Mandel and E Wolf, Optical Coherence and Quantum Optics, CUP (1995)