The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1738 - 9

WINTER COLLEGE

on

QUANTUM AND CLASSICAL ASPECTS

of

INFORMATION OPTICS

30 January - 10 February 2006

Spin cf. Orbital Angular Momentum

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Spin cf. Orbital Angular Momentum

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of
GLASGOW

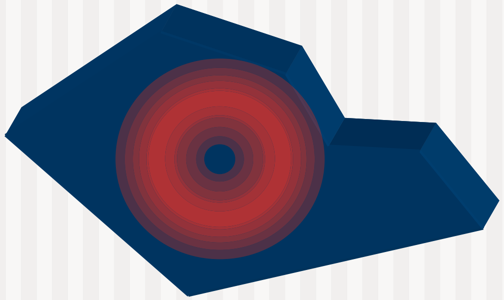
Transfer of AM to micro-objects

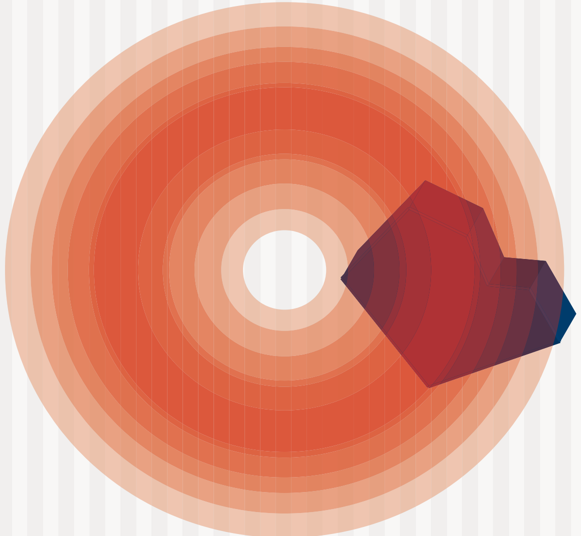
He *et al.* Phys Rev. Lett. 1995

Simpson *et al*. Opt. Lett. 1997

O'Neil *et al.* Phys. Rev. Lett. 2002

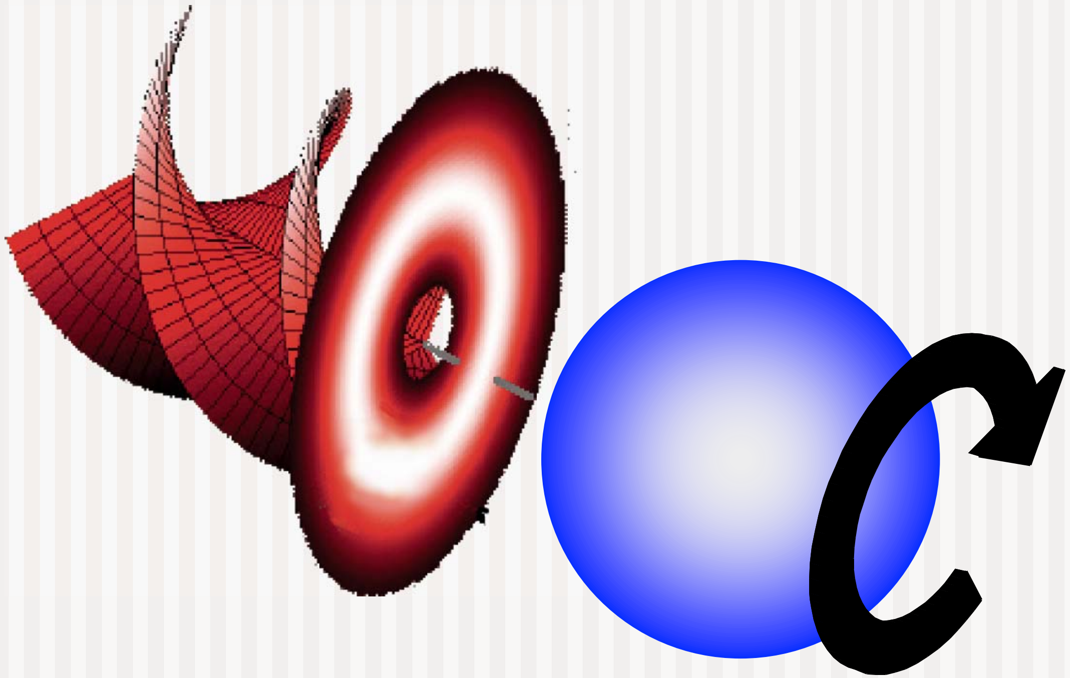
Angular momentum interactions with particles





- Object larger than beam
  - Spin AM = Orbital AM (for absorption)
- Beam larger than object
  - Spin AM ≠ Orbital AM

On-axis Spin and Orbital transfer



SAM &or OAM

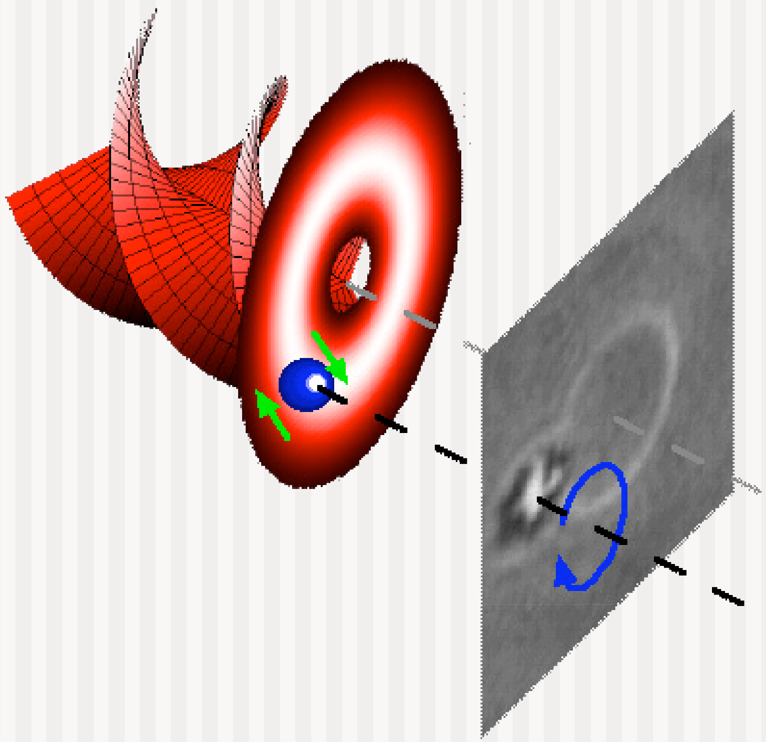
Particle spins on beam axis

OAM / SAM transfer to particle held in optical tweezers

OAM ($\hbar$) +/-SAM ($\hbar$)

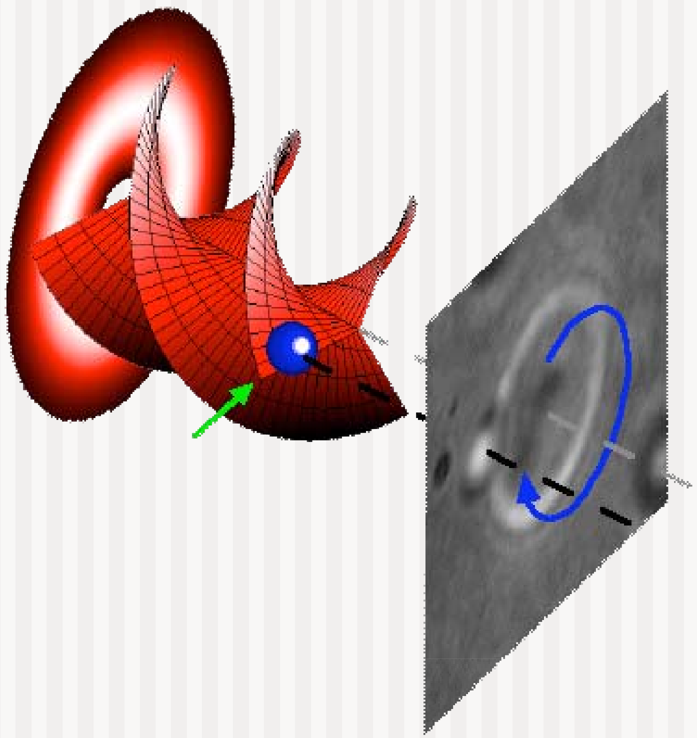
Particle spins and stops

Off-axis Spin and Orbital transfer



SAM

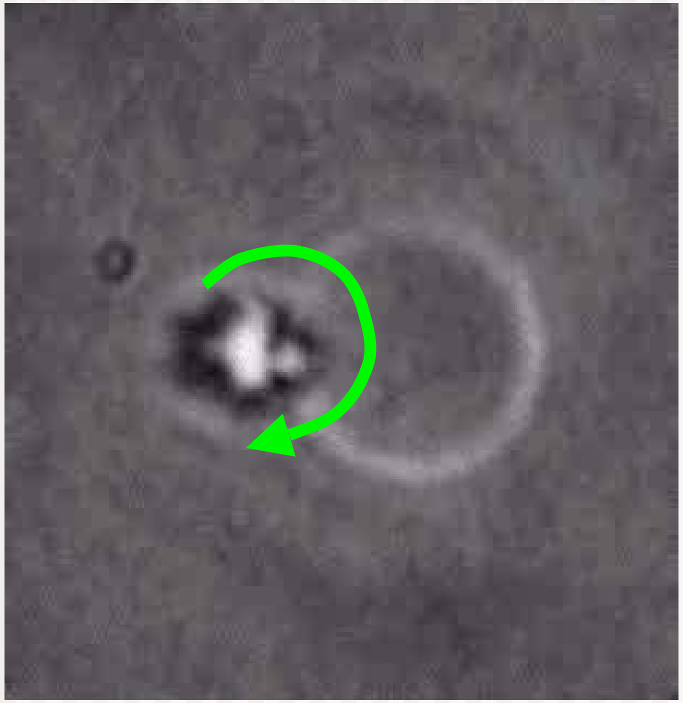
Particle spins on its own axis



OAM

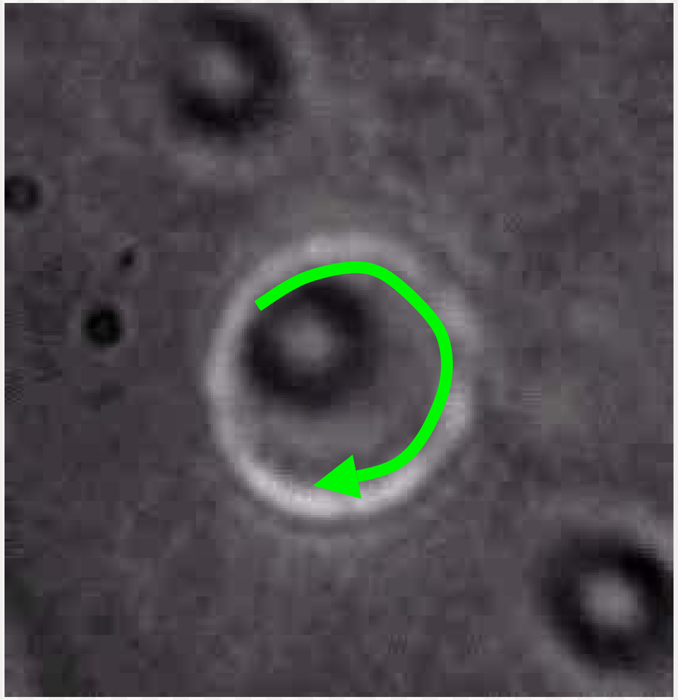
Particle orbits the beam axis

OAM / SAM transfer to particle held in optical tweezers



SAM

Particle spins on its own axis



OAM

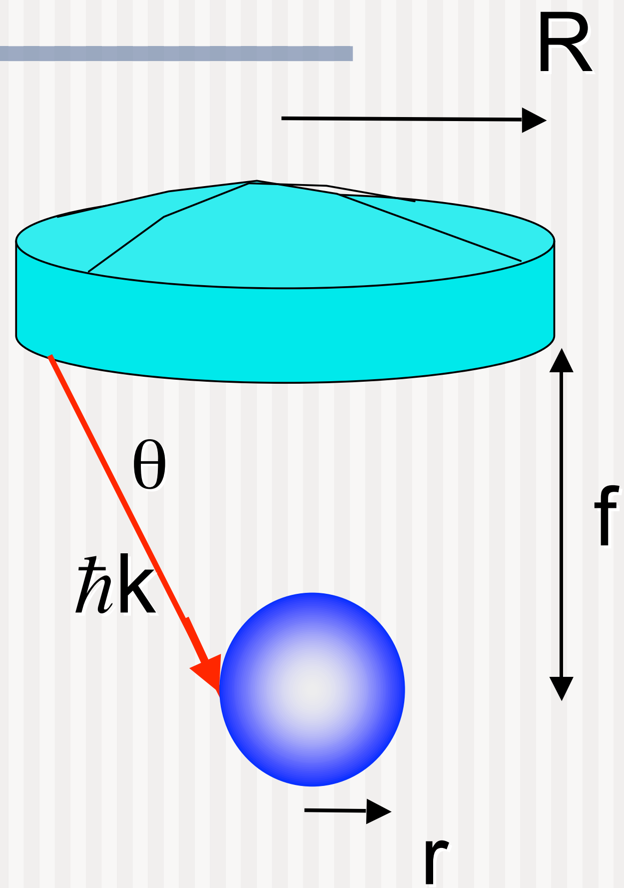
Particle orbits the beam axis

Ray-optics to model OAM

Courtial and Padgett Opt. Commun. 2000

Transfer of angular momentum

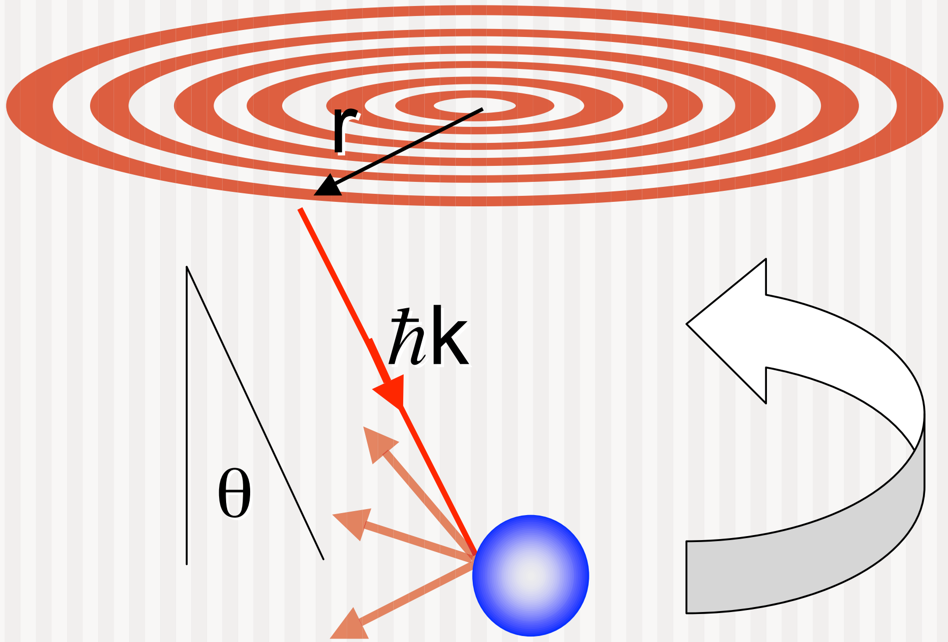
- Angular momentum arises from skew rays
  - $\theta = \ell / kr$
- The skew angle sets the azimuthal component to the momentum density
  - $p_\phi = \hbar k\, \theta = \ell\hbar / r$
  - $L = p_\phi\, r = \ell\hbar$
  - $L_{max} = \hbar k\, r\, (R/f)$



Ray-Optics gives the right answer

Transfer of orbital AM (e.g. from Bessel beam)

- Local intensity (Bessel Beam)
  - $I \propto 1/r$
- Angular momentum arises from skew rays
  - $\theta = \ell / kr$
  - i.e. $\theta \propto 1/r$
- Circumference of ring
  - $\propto r$
- (orbital) rotation rate
  - $\Omega \propto 1/r^3$



Ray-Optics gives the right answer

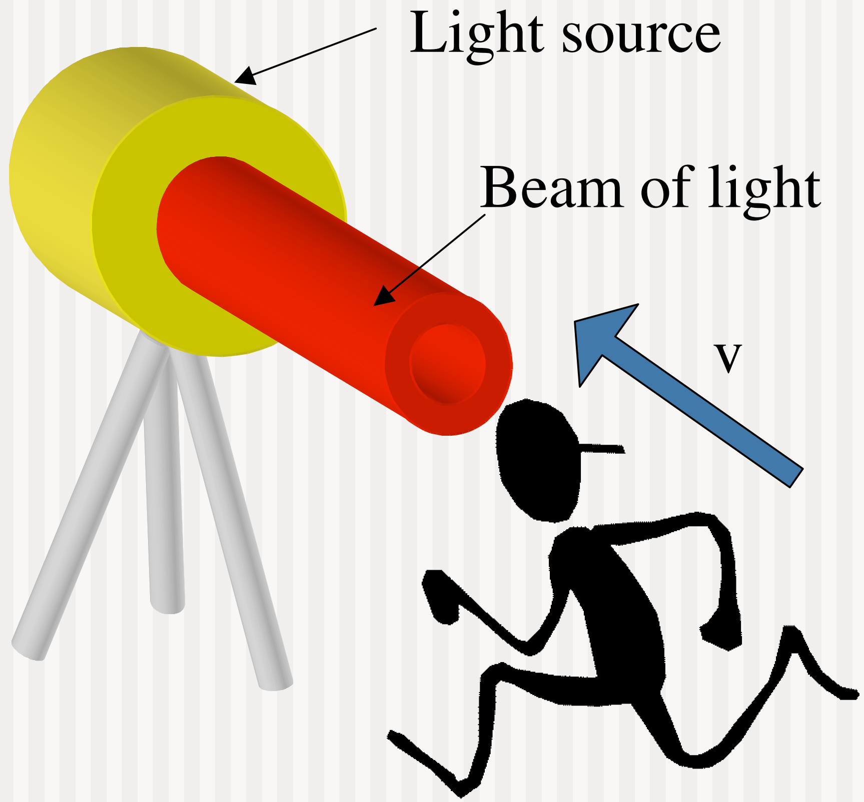
Rotational frequency shifts

Garetz and Arnold Opt. Commun. 1979

Courtial *et al.* Phys. Rev. Lett. 1998

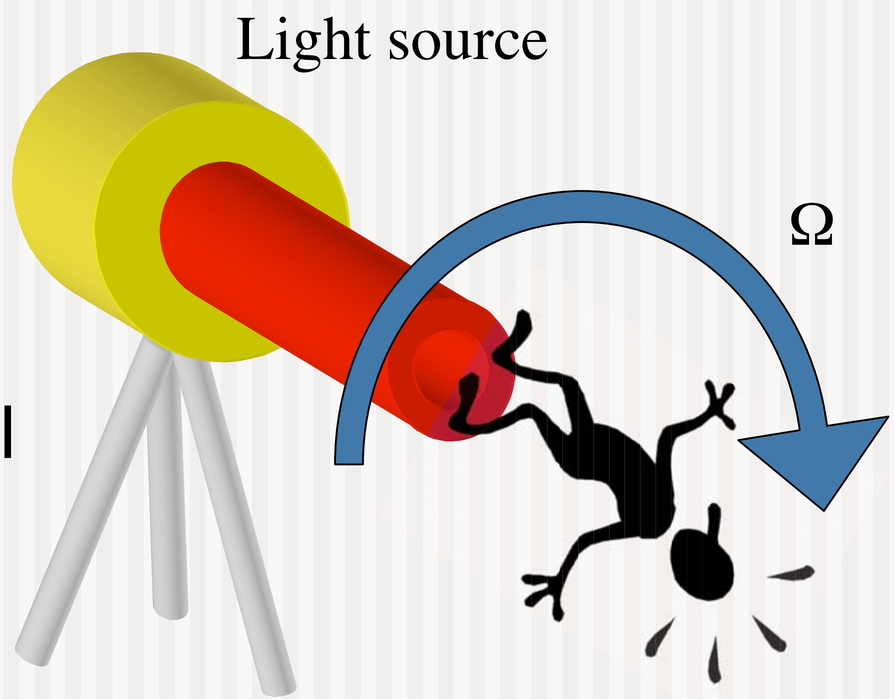
The linear Doppler shift

- Light source moves towards or away from detector giving Doppler shift
  - $\Delta\omega = \omega_0 \times v/c$
- Re-express in terms of linear momentum per photon, p
  - $\Delta\omega = v \times p/\hbar$



The annular Doppler shift

- Light source rotates with respect to detector giving Doppler shift
  - $\Delta\omega = \Omega \times (\ell + \sigma)$
- Also called rotational frequency shift



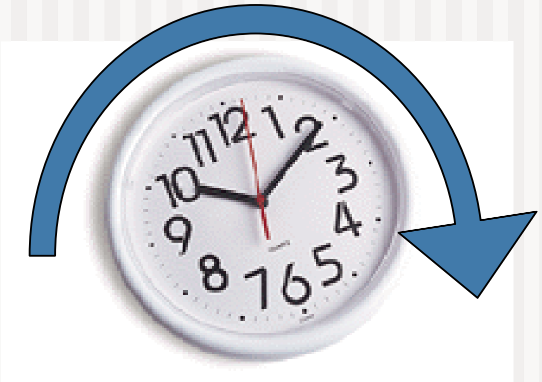
Annular Doppler shift for circularly polarised light

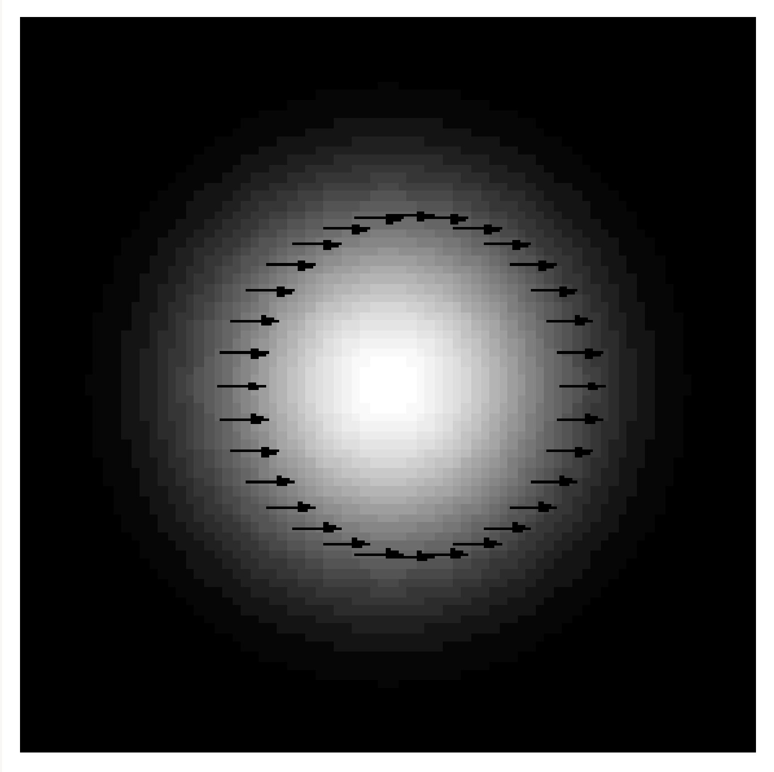
- Additional rotation of polarisation (at W) shifts frequency

  $\Delta\omega = \Omega$

  $= \sigma\Omega \quad (\sigma=\pm 1)$

- c.f. time speeds up if you rotate a clock!





Annular Doppler for helically phased circ. polarised light -1

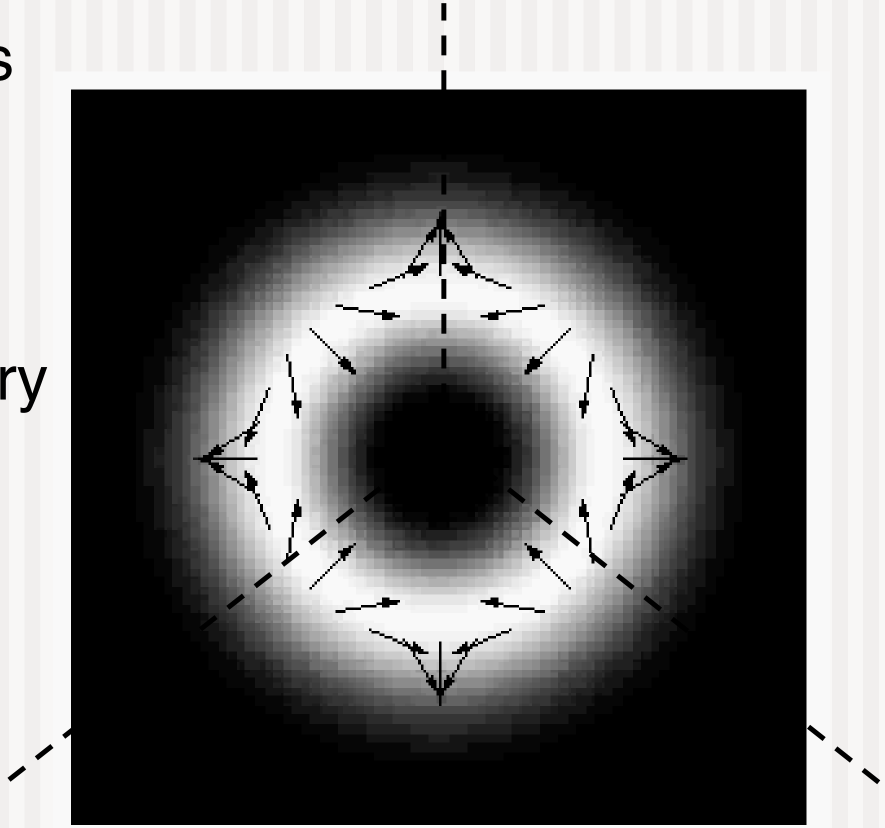
- Such a beam contains both SAM and OAM
- Example 1

  $\ell = 3$, $\sigma = +1$
- Four fold rot. Symmetry
- Rotate beam at Ω

  $\Delta\omega = (\ell + \sigma)\Omega$

  $= J\Omega$

  $= 4\Omega$



Rot. Doppler for helically phased, circ. polarised light -2

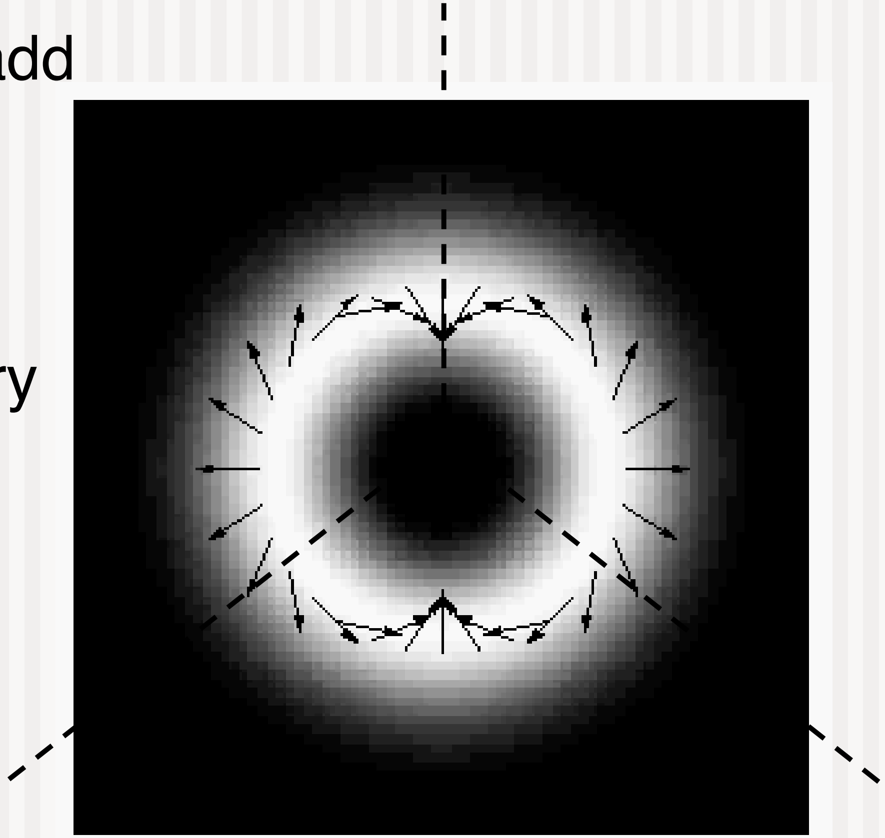
- The SAM and OAM add or subtract
- Example 2

  ℓ = -3, σ =+1
- Two fold rot. Symmetry
- Rotate beam at Ω

  $\Delta\omega = (\ell+\sigma)\Omega$

  $= J\Omega$

  $= 2\,\Omega$

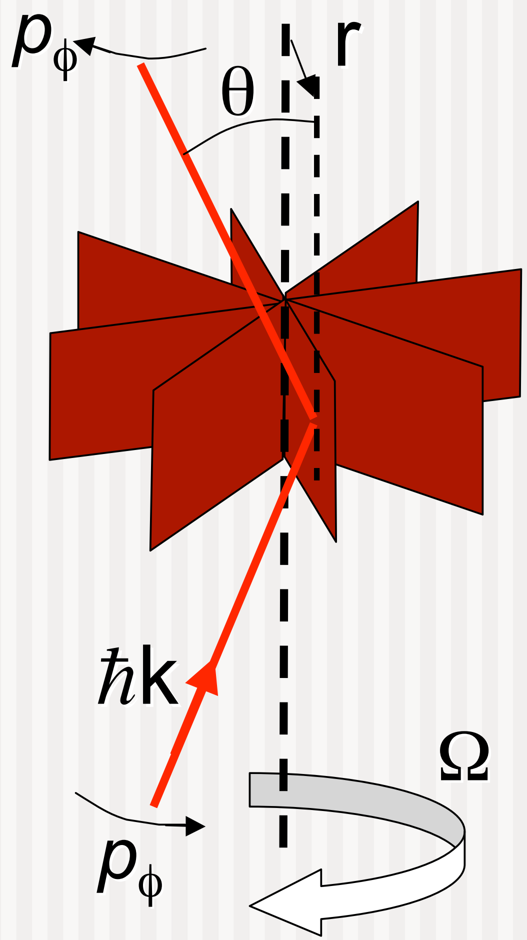


Ray-optics to model OAM

Padgett, J Opt A 2004

Rotationally induced frequency shifts

- Waveplate reverses p_ϕ
- Exerts force
  - $F = 2\hbar k \sin\theta$
- Skew angle of ray
  - $\theta = \ell / kr$
- Work done (per photon)
  - $W = Fv = 2\ell\hbar\Omega$
- Frequency shift
  - $\Delta\omega = 2\ell\Omega$



Ray-Optics/Photon Pressure gives the right answer

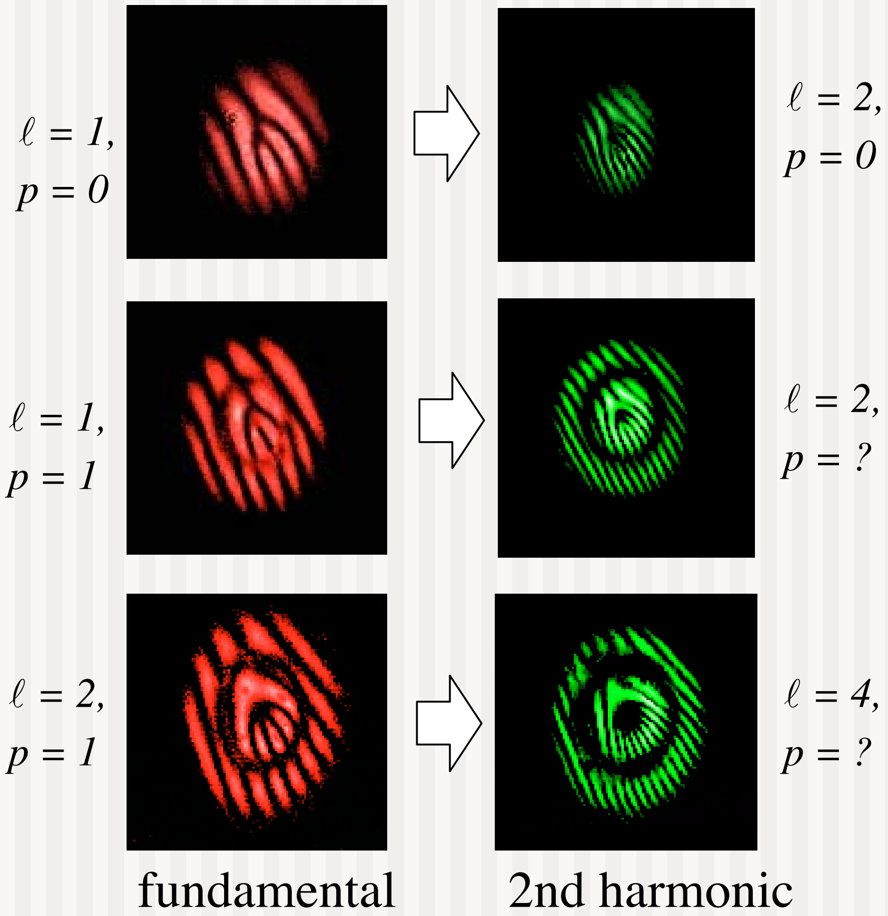
Non-linear optics

Courtial *et al.* Phys. Rev. A 1997

Mair *et al.* Nature 2001

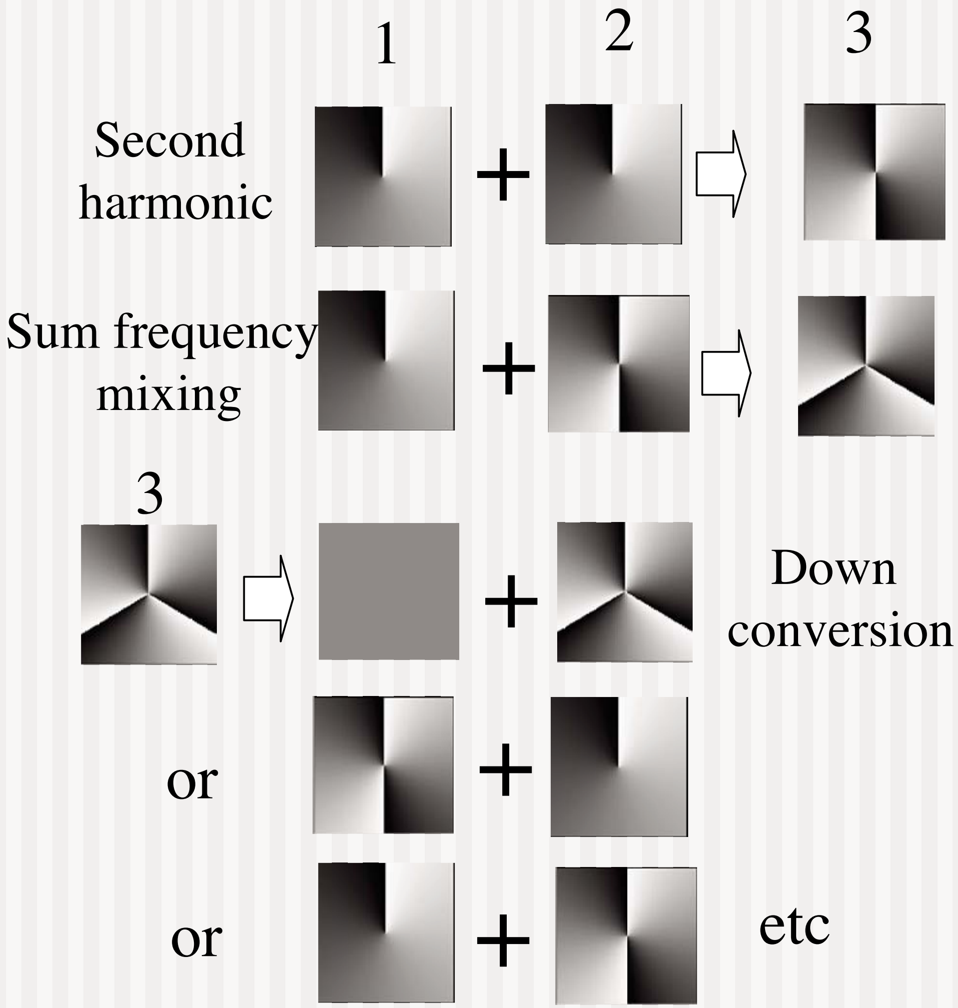
OAM conserved in SHG

- OAM conserved in the light beam
- c.f. SAM in which OAM is not conserved
- But, down conversion is more complicated!



OAM in three-wave interactions

- Fixed phase relationship between three fields
  - $\psi_1 + \psi_2 + \psi_3 = \pm\pi/2$
- Azimuthal phase terms are linked to each other, giving
  - $\ell_1 + \ell_2 = \ell_3$

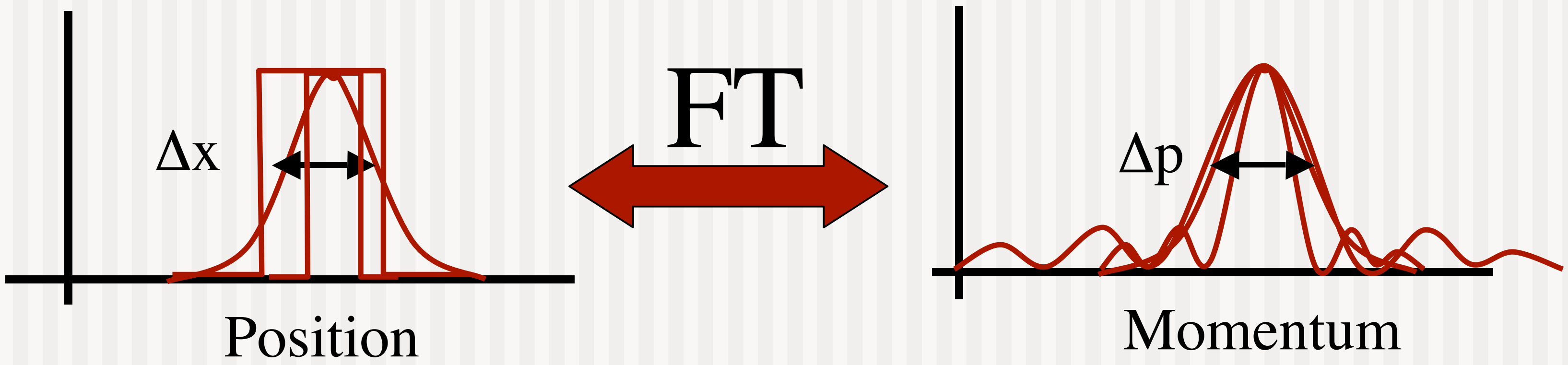


The (angular) uncertainty principle

Franke-Arnold *et al.* NJP 2004

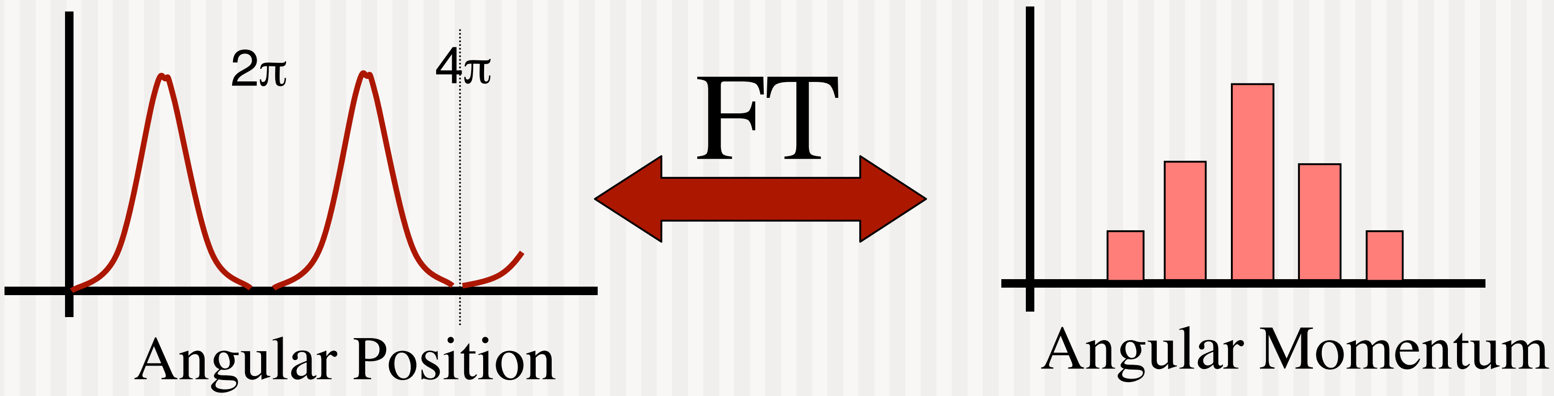
EPSRC 2004-2006

Uncertainty relationships



- Heisenberg's Uncertainty principle
  - $\Delta x \Delta p \geq \hbar/2$
- For Gaussian distribution
  - $\Delta x \Delta p = \hbar/2$ (Gaussian gives minimum uncertainty state product)
- What about angular momentum?

Uncertainty in Angular Momentum

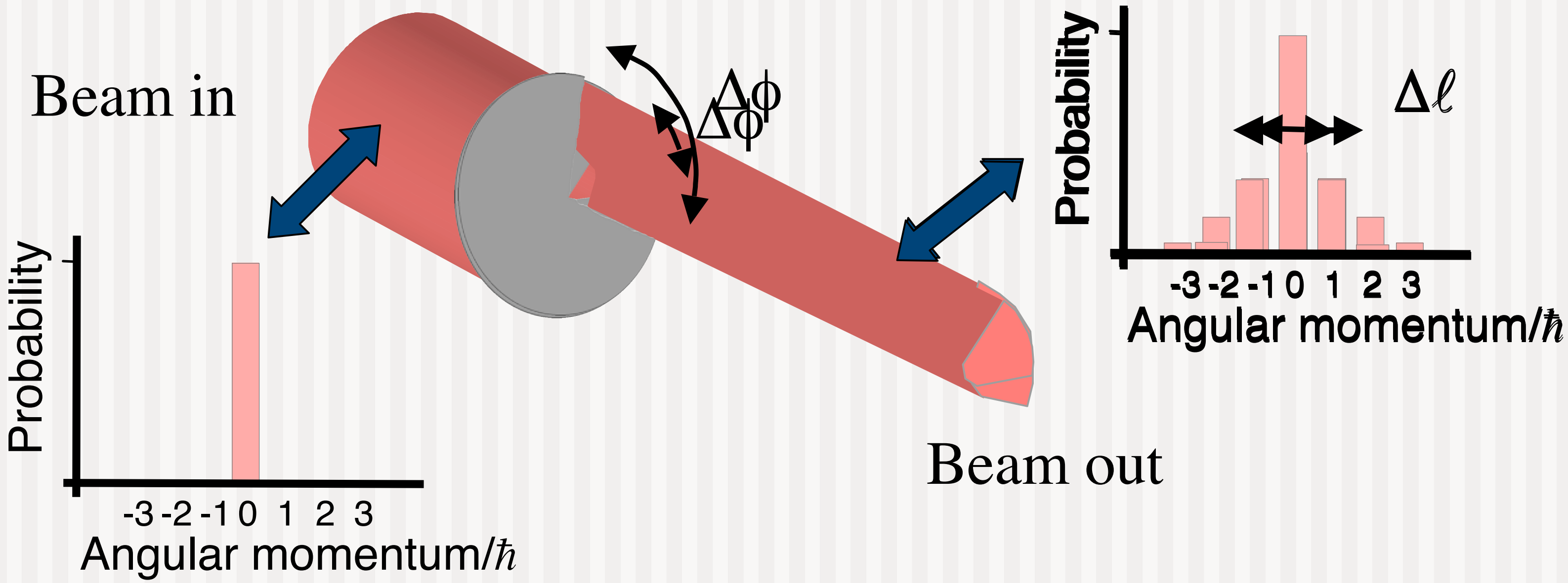


- Angular position repeats modulo 2π,
  - FT of repeating position gives discrete angular momentum values
  - $\Delta\phi\Delta L \geq \hbar/2$?
- For no restriction on ϕ, $\Delta\phi$ still finite, but L can be measured exactly
  - With no restriction, $\Delta\phi=\pi/\sqrt{3}$, but $\Delta L=0$, i.e. $\Delta\phi\Delta L = 0$
- What are the minimum uncertainty states?

Angular position
Angular momentum

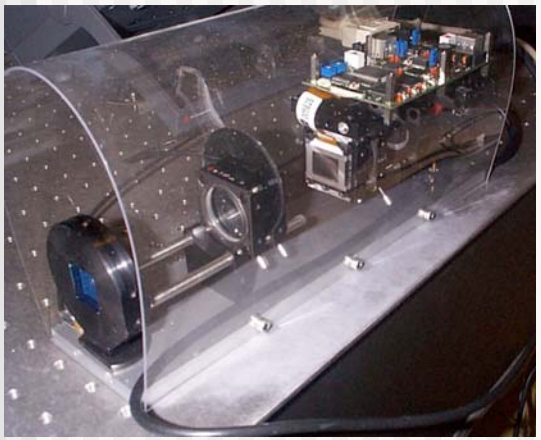
FT

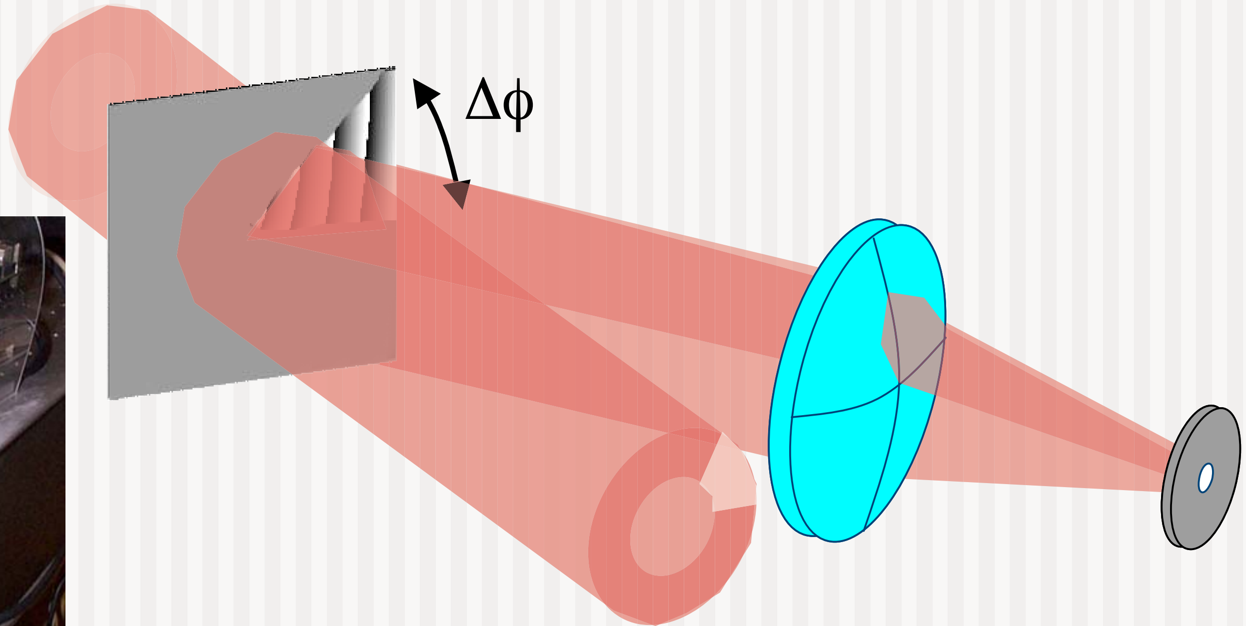
- A “cake-slice” aperture placed in a light beam restricts the angular position (≈ of the photon)



Doing the experiment

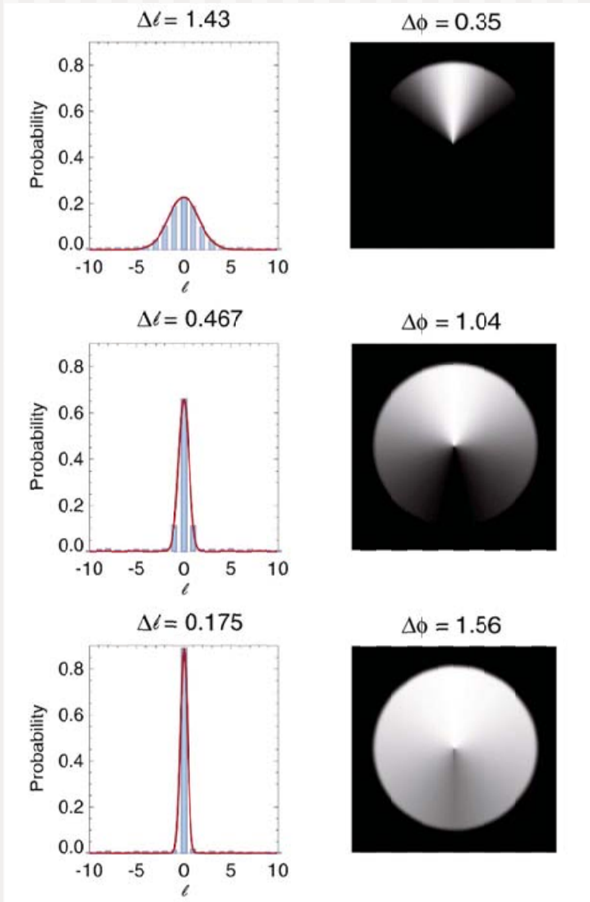
- SLM used to measure ℓ
- Same SLM used to impose aperture
- Transmission through aperture gives ℓ components





The uncertainty relationship for angular momentum

- For small “slices” we find an uncertainty relationship
  - $\Delta L \Delta \phi \geq \hbar/2$
  - i.e. $\Delta \ell \Delta \phi \geq 1/2$
- For no aperture
  - $\Delta L = 0$
- More complicated for large “slices”…..



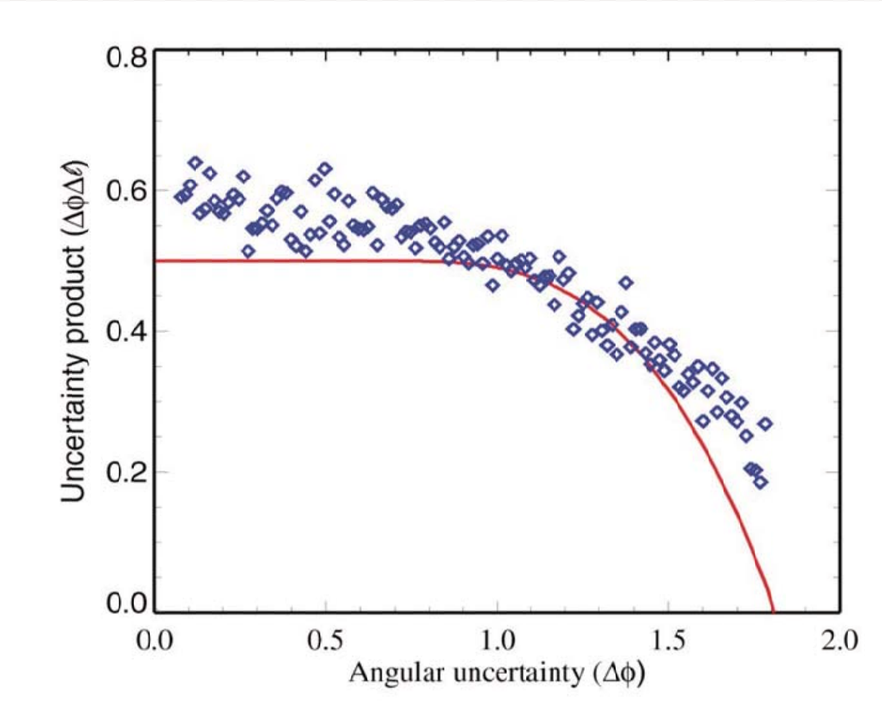
$\Delta \ell \Delta \phi = 0.501$

$\Delta \ell \Delta \phi = 0.486$

$\Delta \ell \Delta \phi = 0.273$

The minimum uncertainty states

- More complicated for large “slices”…..
  - $\Delta\ell\Delta\phi \geq 1/2(1-2\pi(P_{edge})$?
- Minimum uncertainty position state
  - Gaussian symmetrically truncated by $-\pi$ to $+\pi$?
- Results inconclusive?



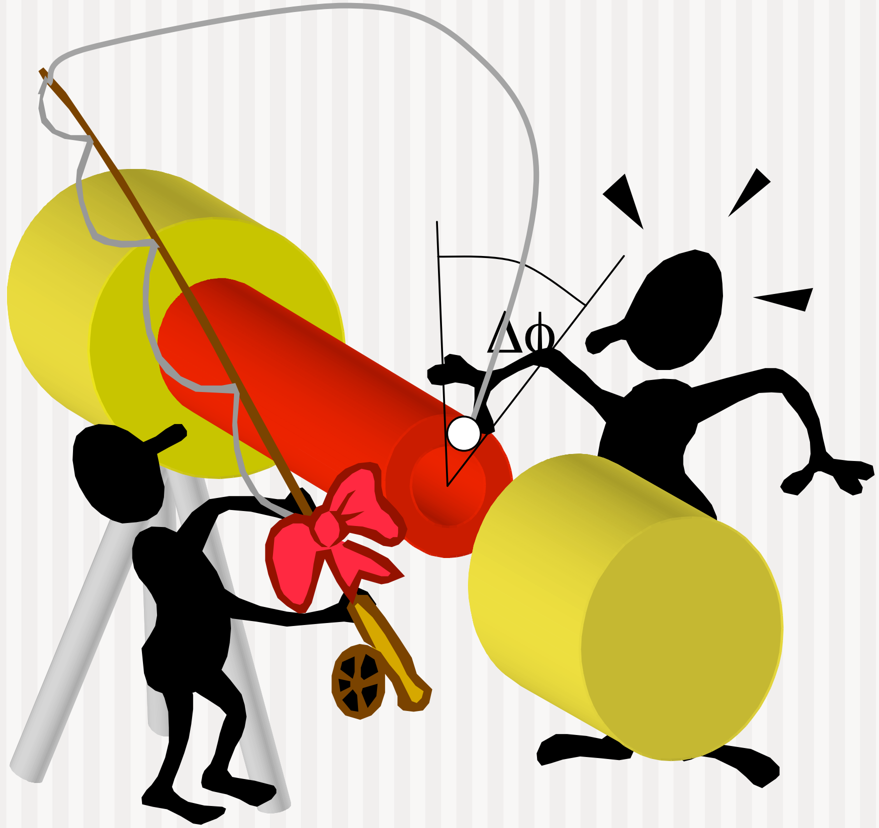
The OAM communicator

Gibson *et al.*

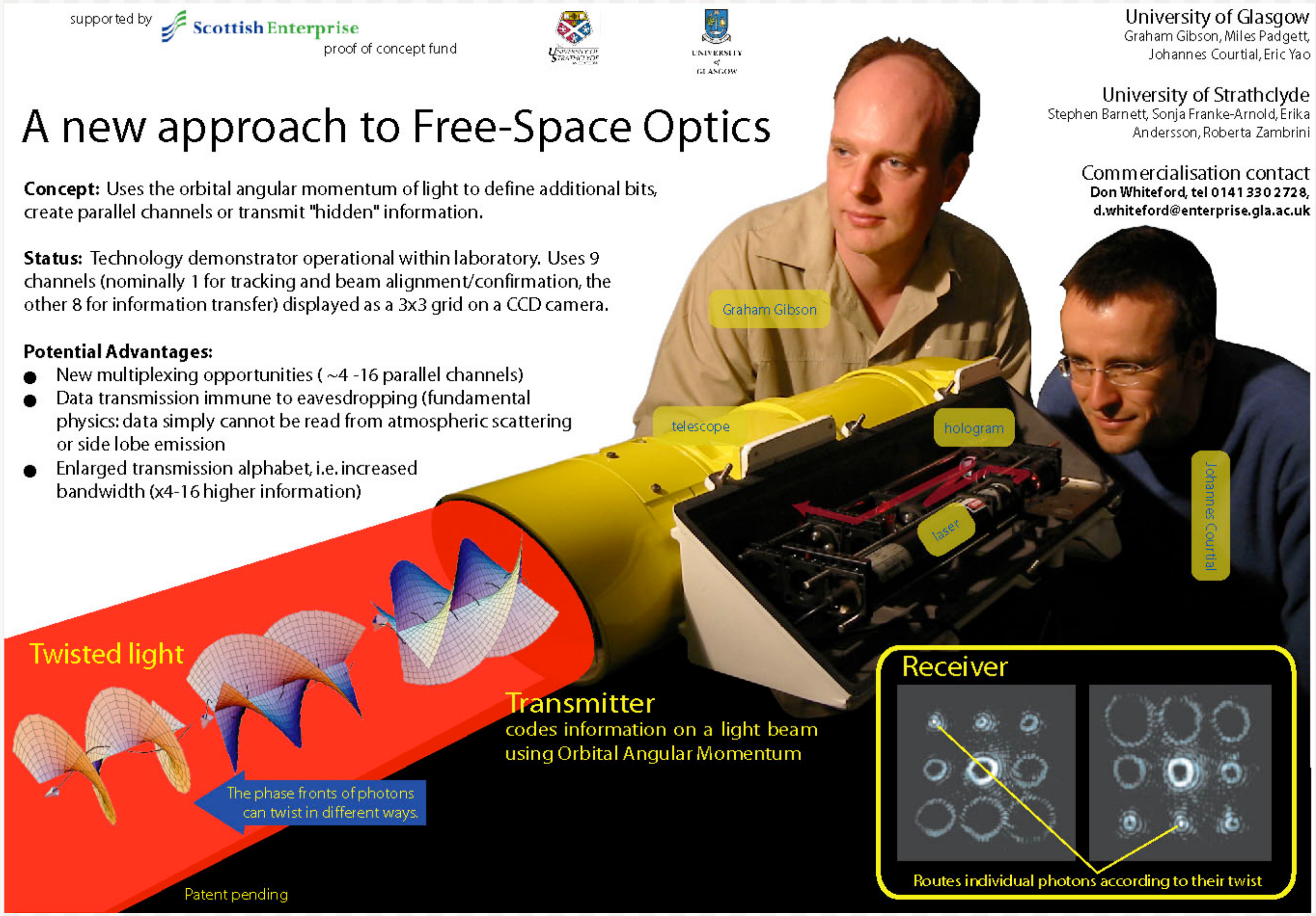
Opt Express **12,** 5448 (2004)

Measuring OAM gives secure(ish) communication

- Use OAM (i.e. ℓ-index) to encode data on a light beam
  - Each photon can take ANY value of ℓ hence increases data capacity
- $\Delta\ell\Delta\phi \geq 1/2$ gives security
  - Can't measure ℓ from only part of beam
  - e.g. Can't measure ℓ from scattered light
  - e.g. Can't measure ℓ from side lobe



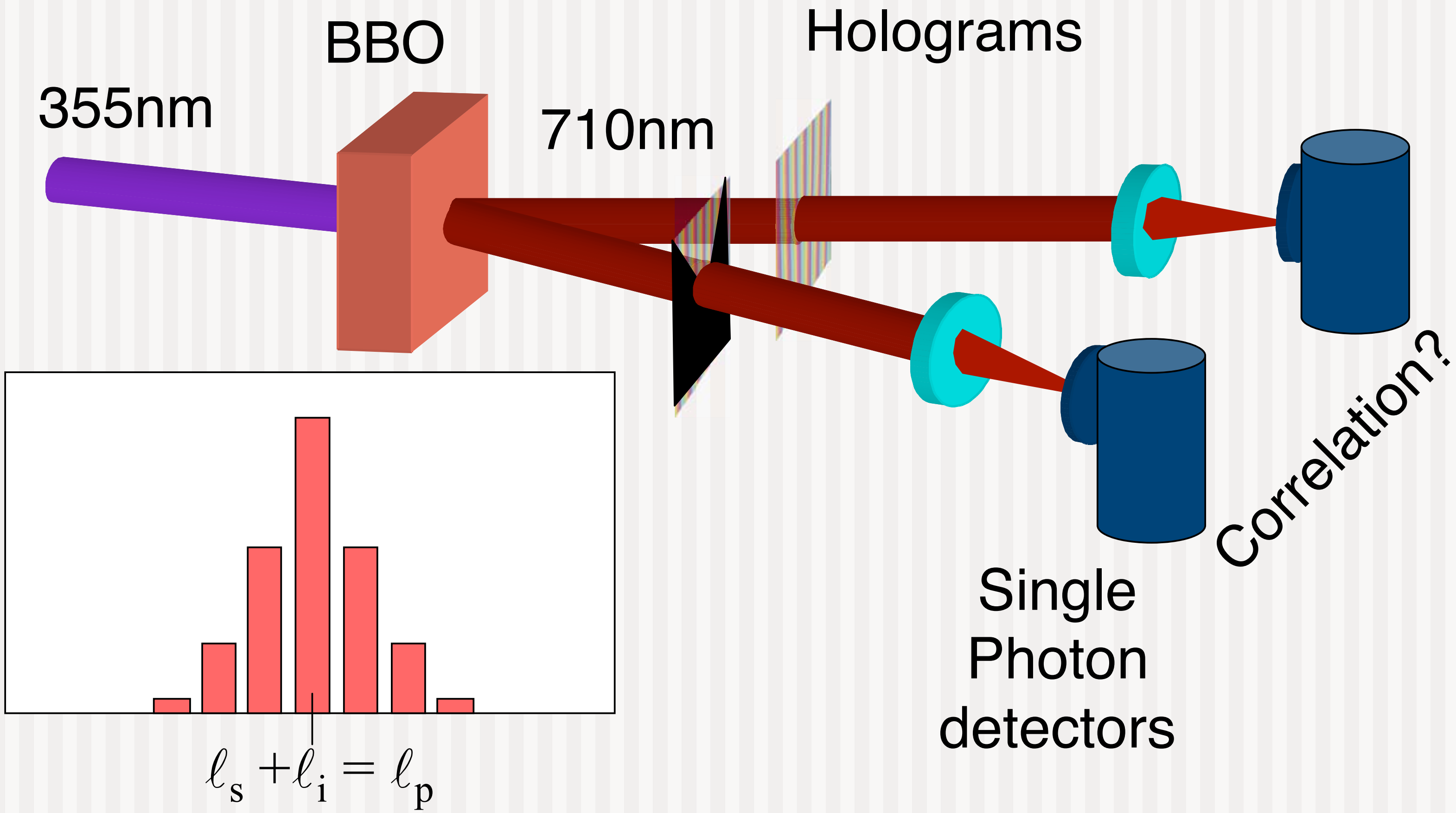
Free-space comms



Entanglement of angular position and angular momentum

EPSRC 2004-2006

Measuring angle and angular momentum



The (angular) momentum paradox

Brevik Phys Rep 1972

Loudon Phys Rev. A 2003

Padgett *et al.* J. Mod Opt 2003

Mansuripur Opt. Exp 2005

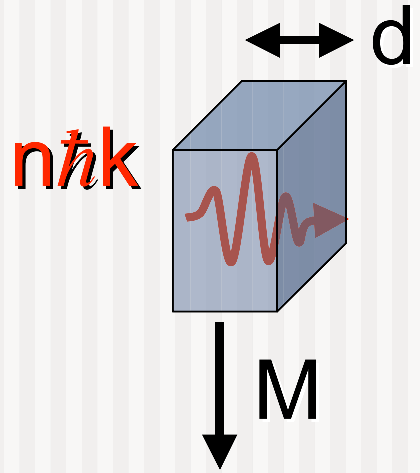
EPSRC 2005-2008

The momentum of light in a dielectric

- What is the momentum of light (a photon) inside a dielectric (refractive index n)?
  - $p = n\hbar k$ (Minkowski) {equiv. $\boldsymbol{p} = \mathbf{D} \times \mathbf{B}$}
  - $p = \hbar k/n$ (Abraham) {equiv. $\boldsymbol{p} = \mathbf{E} \times \mathbf{H}/c^2$}
- What is the angular momentum of light (a photon) inside a dielectric (refractive index n)?
  - $L = (l + \sigma)\,\hbar$ (Minkowski)
  - $L = (l + \sigma)\,\hbar/n^2$ (Abraham)

Minkowski

- A short, single-photon pulse traverses a block
- During transit of block, momentum of photon is increased
- Block moves towards source



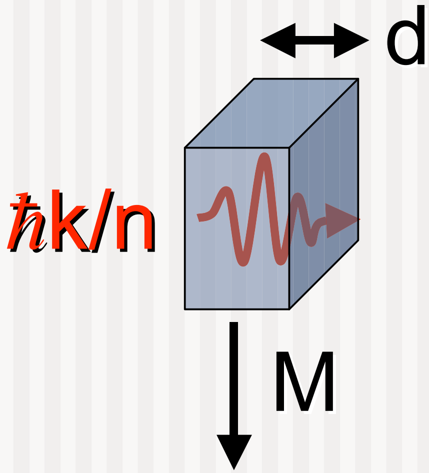
$$p_{block} = (1-n)\hbar k$$

block moves

$$\Delta z = (n-n^2)\hbar k\, d/cM$$

Abraham

- A short, single-photon pulse traverses a block
- During transit of block, momentum of photon is decreased
- Block moves away from source



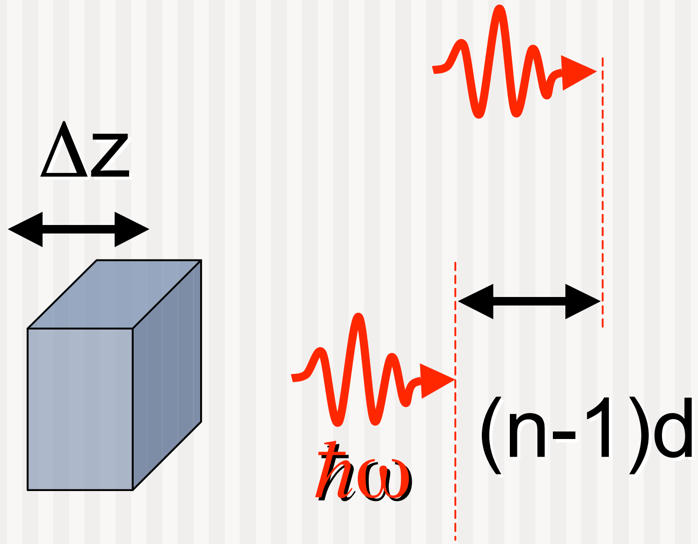
$p_{block} = (1-1/n)\hbar k$

block moves

$\Delta z = (n-1)\hbar k\, d/cM$

Einstein Box -> Abraham

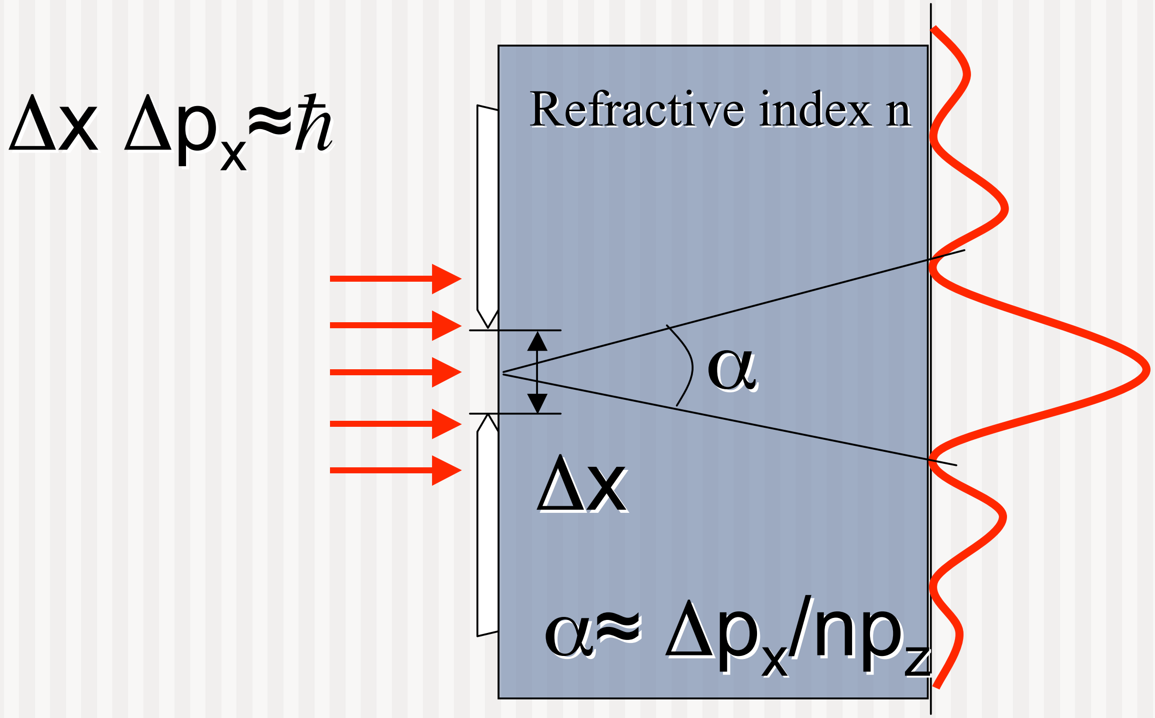
- Alternative argument based only on displacement of the centre of mass-energy
- Delay of photon energy equated to energy displacement of block
- Agrees with Abraham formulation



$$\Delta z\, Mc^2 = (n-1)d\, \hbar\omega$$

$$\Delta z = (n-1)\hbar k\, d/cM$$

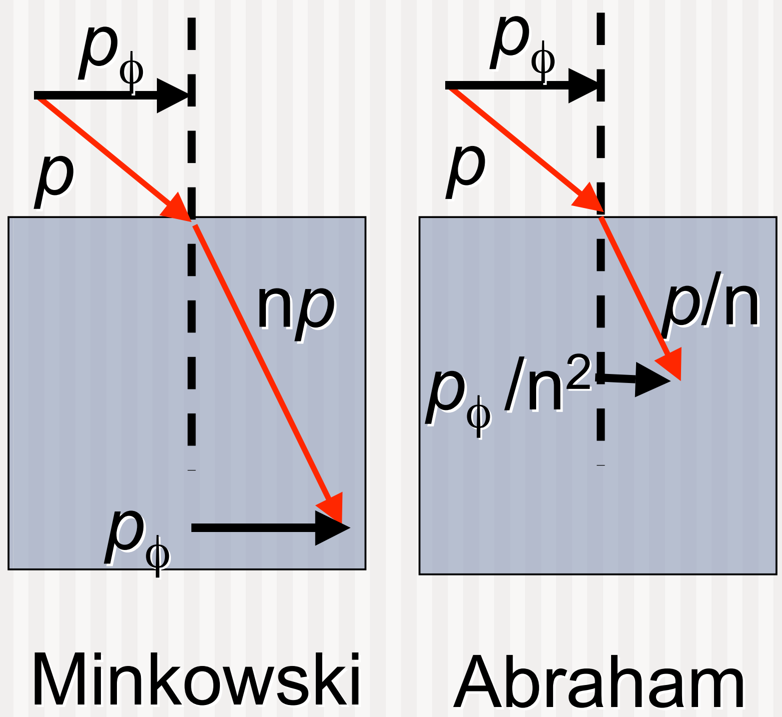
Diffraction -> Minkowski



- Diffraction arises from Uncertainty Principle
- Shrinking pattern implies Minkowski formulation

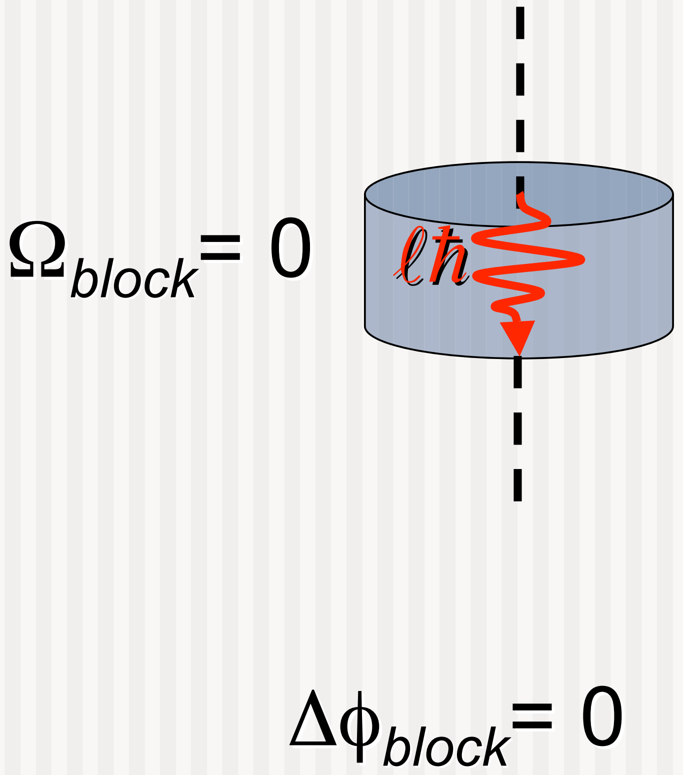
Angular momentum

- Angular momentum arises from ϕ comp. of *p*
- At interface, ϕ comp. reduced by 1/n (Snell's Law)
- True for both Abraham and Minkowski
- Implications re AM
  - Minkowski $L = \ell\hbar$
  - Abraham $L = \ell\hbar/n^2$



Minkowski equiv for AM

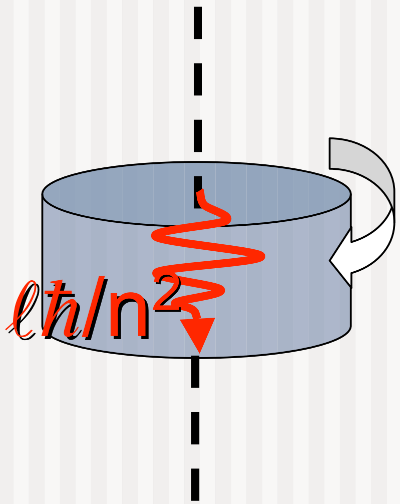
- Single photon pulse carrying angular momentum
- During transit of block, angular momentum is unchanged
- Block does not rotate



Abraham equiv for AM

- Single photon pulse carrying angular momentum
- During transit of block, angular momentum is changed
- Block does rotate

$$\Omega_{block}=(1-1/n^2)\ell\hbar/I$$

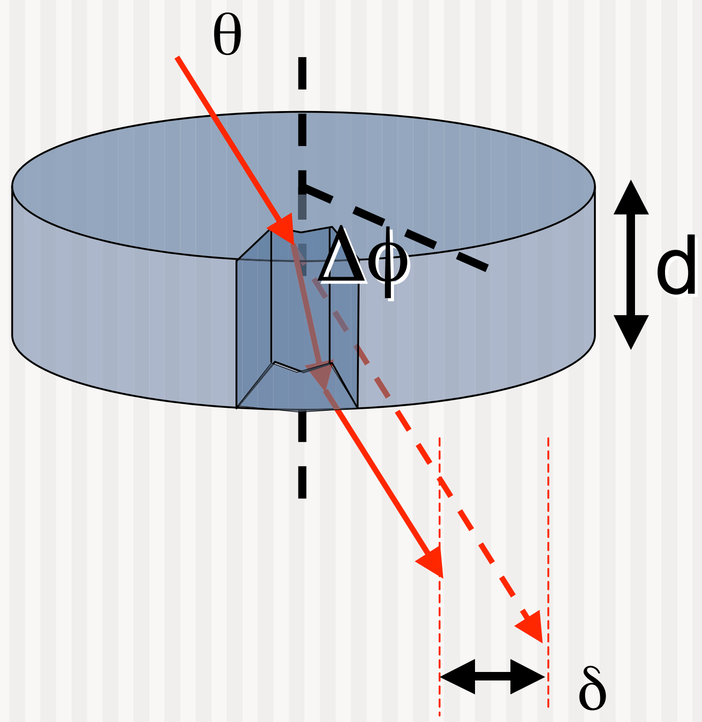


$$\Delta\phi_{block}= (n-1/n)\ell\hbar\ d/cI$$

Einstein Box for Angular Momentum

- Equate the lateral delay of the photon energy to the mass energy displacement of the disc element
- Sub. in for skew angle ($\theta = \ell / kr$) and integrate over disc
  - $\Delta\phi_{block} = (n-1/n)\ell\hbar\ d/cI$

    (I = moment of inertia)
- The Abraham result!
- Also true for spin AM?
  - $\Delta\phi_{block} = (n-1/n)(\ell+\sigma)\hbar\ d/cI$



$$\Delta\phi_{block} r\ Mc^2 = \delta\hbar\omega = \hbar\omega(n-1/n)d\theta$$

The Mechanical rotation (Faraday) Effects

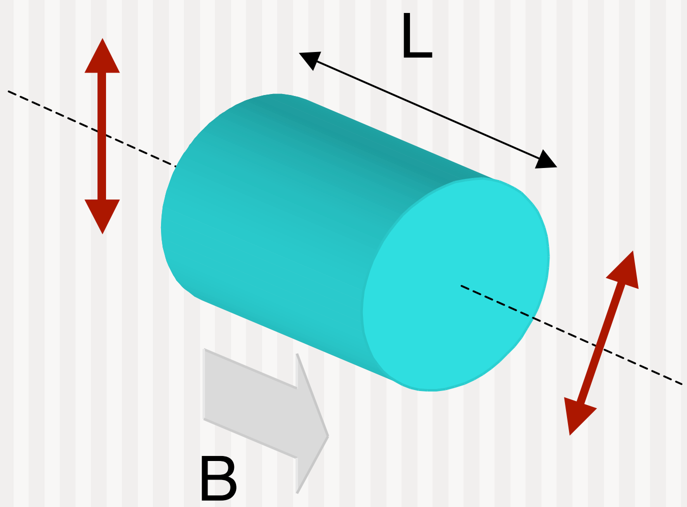
Jones *et al.* J. Proc. Roy Soc. A 1976

Nienhuis *et al.* Phys Rev A 1992

SUPA 2005-2006

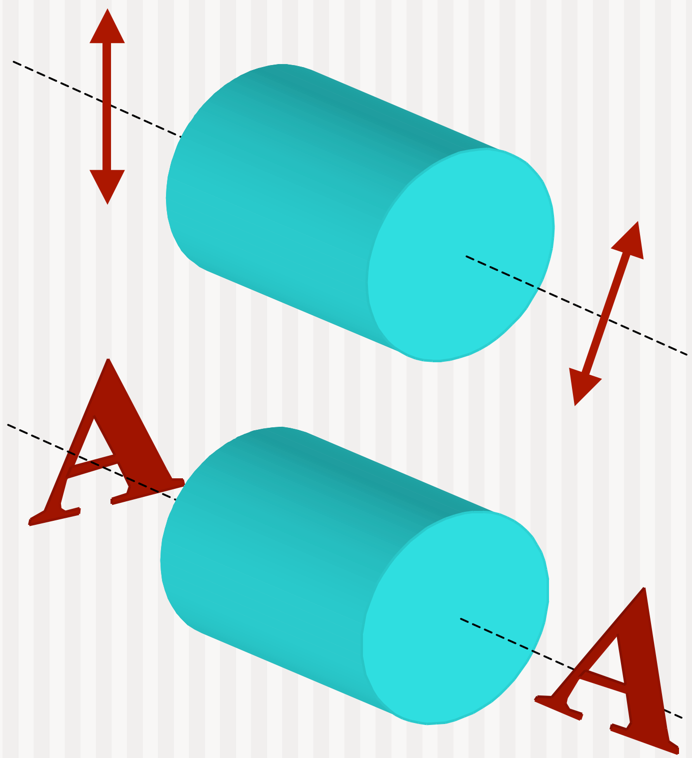
Magnetic Faraday effect

- Rotation of plane polarised light
  - $\Delta\theta = BLV$
    - V Verdet constant
- OR treat as phase delay of circularly polarised light
  - $\Delta\phi = \sigma BLV$
- Are SAM and OAM equivalent?



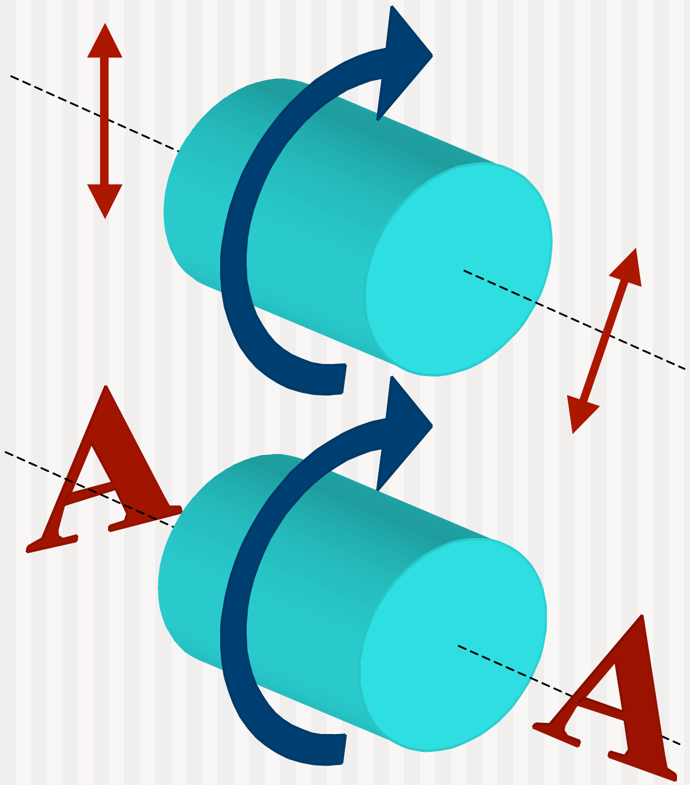
Rotations of Polarisation and Image

- SAM -> Polarisation rotation
- OAM-> Image rotation
- Look through a Faraday isolator ($\Delta\theta \approx 45°$), is the "world" rotated - NO
  - SAM and OAM are not equivalent in the Magnetic Faraday effect



Mechanical Faraday effect

- Photon drag, gives Polarisation rotation
  - $\Delta\theta = \Omega(n-1/n)L/c$
- Phase delay equiv.
  - $\Delta\phi = \sigma\Omega(n-1/n)L/c$
- Does photon drag give image rotation?
  - $\Delta\theta = \Omega(n-1/n)L/c$



Photon drag

- Transverse photon drag
  - $u_x=(1-1/n^2)v$
- For transit time Ln/c, gives displacement
  - $\Delta x=L(1-1/n^2)nv/c$
- In cylindrical frame
  - $\Delta\theta r=L(1-1/n^2)nr\Omega/c$
  - $\Delta\theta=L(n-1/n)\Omega/c$
- OAM equiv. SAM?