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Fundamentals of holographic data storage:
Diffraction of light by volume holographic gratings
LECTURE 1: The Physical Principles

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Fundamentals of holographic data storage: Diffraction of light by volume holographic gratings

LECTURE 1: "The Physical Principles"

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CONTENTS

- I. 1.- Objectives
- I. 2.- Some antecedents
- I. 3.- The electromagnetic framework
- I. 4.- The scalar approach
- I. 5.- Vector solution for an inhomogeneous optical medium with period dielectric permittivity
- I. 6.- The Kogelnik's solutions: The One-dimensional Coupled Wave Theory
 - I. 6.1.- First order approximation. Relevant parameters.
- I. 7.- Conclusions
- I. 8.- References

I.1.- OBJECTIVES

- We want to formulate the electromagnetic field originated by the interaction of light with a material medium.
- We study the particular conditions when the optical medium has a spatial periodic structure.
- We apply the possible solutions to the case of a holographic volume grating.
- Certain approximations simplify the formalism to be later applied to characterize holographic memories.

I.2.- Some antecedents

- The electromagnetic theory for the diffraction of light by a holographic volume grating is based upon the [Dynamical Diffraction of X-Rays by a perfect crystal](#).
- The original work was done in the early 20th century by: *C.G. Darwin* (1914), *P.P. Ewald* (1916), *M. von Laue* (1931): They introduced a vector solution in the form of a superposition of a number N of scattered fields following particular directions inside the crystal.
- The solutions yield the so-called *Bragg's law* for the experimental observation of diffraction of X-rays by a crystalline structure, (by *W.L. Bragg* in 1912).
- A review of these theories was done by *B.W. Batterman* and *H. Cole* in 1964.
- Volume holography was firstly experimentally developed by *Y. Denisyuk* (1962) as an extension of the pioneering experiments done by *D. Gabor* in 1947 (Nobel Prize, 1971) combined with the principles of color photography earlier obtained by *G. Lippmann* (1894).
- Separately, in a different context, *C.V. Raman* and *N.S.N. Nath* obtained evidence of volume grating behavior of an acoustic wave irradiated with light and studied the nature of the diffracted field in 1935.
- During the last decades in the past century an important amount of work was done to characterize volume holographic gratings under rigorous vector solutions and approximations: *H. Kogelnik*, *M.G. Moharam*, *T.K. Gaylord*, *L. Solymar*, *P. Cheben* and many others.
- The 21st century is called to be the century of information photonics in which holographic techniques appear to be among the most appealing ones.

I.2.- The electromagnetic framework

• Macroscopic Maxwell's equations for the propagation of a electromagnetic field in a finite dielectric medium, in the absence of conductivity

• **Main condition:**

We assume: The wavelength lies in the optical range:

$$400 \text{ nm} \leq \lambda \leq 750 \text{ nm}$$

And:

$$\Delta V \ll \lambda^3; \quad r_a^3 \ll \Delta V \ll \lambda^3; \quad \text{Thus: } r_a^3 \ll \Delta V \ll 0.064 \mu\text{m}^3$$

r_a : typical size of an atom. ΔV : infinitesimal volume element of the medium

• **Consequence:**

We consider ordinary optical phenomena. Optically dense media behave practically as continuous.

Macroscopic Maxwell's equations for a dielectric medium in the absence of conductivity

$$\begin{aligned} \nabla \times \bar{H} &= \frac{1}{c} \frac{\partial \bar{D}}{\partial t}; & \nabla \times \bar{E} &= -\frac{1}{c} \frac{\partial \bar{B}}{\partial t} \\ \nabla \cdot \bar{D} &= 0; & \nabla \cdot \bar{B} &= 0 \end{aligned}$$

Where:

$$\begin{aligned} \bar{H} &: \text{Magnetic field}; & \bar{E} &: \text{Electric field} \\ \bar{D} &: \text{Electric displacement}; & \bar{B} &: \text{Magnetic induction} \end{aligned}$$

All these macroscopic magnitudes are averaged over a volume ΔV , thus, with smooth variation with ΔV . In the case of: $\lambda \sim r_a$ the fields cannot be averaged for a high number of atoms. For example, the macroscopic formalism is not applicable to the interaction of X-Rays with atoms for which: $\lambda \sim 0.1 \text{ nm}$

The constitutive relations are:

$$\begin{aligned} \bar{D} &= \varepsilon_0 \bar{E} + \bar{P} = \varepsilon_0 \hat{\varepsilon} \bar{E}; & \bar{B} &= \mu_0 \bar{H} + \bar{M} = \mu_0 \hat{\mu} \bar{H}; \\ \text{and Ohm's law boils down to: } & \bar{j} &= 0. \\ \hat{\varepsilon} &: \text{Relative dielectric permittivity tensor} \\ \hat{\mu} &: \text{Relative magnetic permeability tensor} \end{aligned}$$

Linearity of the constitutive relations: Some discussions

- In the case where:

$$\hat{\varepsilon} = \hat{\varepsilon}(\overline{E}); \quad \text{or/and:} \quad \hat{\mu} = \hat{\mu}(\overline{H})$$

Then, the polarization and magnetization are as well functions of the electric and magnetic fields, respectively:

$$\overline{P} = \overline{P}(\overline{E}); \quad \text{or/and:} \quad \overline{M} = \overline{M}(\overline{H})$$

As a **consequence**, non linearity could appear in the range of low electromagnetic frequencies. For example, for low optical frequencies, second and third harmonic generation can take place. We need high power luminous density: $\sim 10^{13} W.cm^{-2}$

Comparable to electric internal fields responsible for maintaining the electrons bound in atoms and molecules: $\sim 10^{11} V.cm^{-1}$

In what follows, in this lecture, we will consider luminous power density: $\leq 1 W.cm^{-2}$

Differential equation for the electromagnetic field

•According to the previous conditions, by operating in Maxwell's equations and considering the constitutive relations for non magnetic and isotropic media, we obtain the vector wave equations for time-dependent fields:

$$\nabla \times \nabla \times \bar{E} + \frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} = -\frac{1}{c^2} \frac{\partial \bar{P}}{\partial t}$$
$$\nabla \times \nabla \times \bar{H} + \frac{1}{c^2} \frac{\partial^2 \bar{H}}{\partial t^2} = \frac{1}{c} \nabla \times \frac{\partial \bar{P}}{\partial t}$$

•We assume that the fields are **monochromatic**. The vectors: $\bar{E}(\bar{r}, t), \bar{H}(\bar{r}, t), \bar{P}(\bar{r}, t)$ oscillate harmonically with the time variable:

$$\nabla \times \nabla \times \bar{E} - k^2 \bar{E}(\bar{r}) = \bar{F}_e(\bar{r})$$
$$\nabla \times \nabla \times \bar{H} - k^2 \bar{H}(\bar{r}) = \bar{F}_m(\bar{r})$$

Both equations are vector Helmholtz equations. With: $k = \omega/c$. Also, the terms: $\bar{F}_e(\bar{r})$ and $\bar{F}_m(\bar{r})$ represent **sources** generating electromagnetic waves in the considered optical medium:

$$\bar{F}_e(\bar{r}) = k^2 \bar{P}(\bar{r})$$
$$\bar{F}_m(\bar{r}) = -ik \nabla \times \bar{P}(\bar{r})$$

I.3.- The Scalar Approach: The Concept of Optical Potential

If we operate in the vector Helmholtz equation for the electric field, and make use of the vectorial identity:

$$\nabla \times \nabla \times \bar{E} = \nabla (\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$$

We obtain the wave vector equation for the electric field:

$$\nabla^2 \bar{E} + k^2 \bar{E} = -k^2 \left[n^2(\bar{r}) - 1 \right] \bar{E} + \underbrace{\nabla (\nabla \cdot \bar{E})}_{\text{Term inducing polarization changes}}$$

Where: $\chi(\bar{r}) = n^2(\bar{r}) - 1$

And n is the refractive index of the inhomogeneous and isotropic medium.

An analogous formula can be obtained for the magnetic field.

The equation above indicates that there exist changes in the state of polarization of \bar{E} as a result of the interaction of the wave with the optical medium.

If the following condition for the refractive index of the medium holds: $\frac{\delta n^2}{\delta x} \gg \lambda$

The depolarization is negligible. The vector equation reduces to the **scalar representation**:

$$\nabla^2 U(\bar{r}) + k^2 U(\bar{r}) = F(\bar{r}) U(\bar{r})$$

Following the analogy with Quantum Mechanics:

$$F(\bar{r}) = -k^2 \left[n^2(\bar{r}) - 1 \right]$$

It is the **optical potential**

I.4.- Vector solution for a dielectric inhomogeneous medium with periodic dielectric permittivity

- We consider an optical medium with periodic dielectric permittivity.
- This case was earlier treated by *Von Laue* (1931), for thermal diffuse X-ray scattering by a large perfect crystal (including absorption properties): The **dynamical diffraction theory**, later revised by *Batterman and Cole* (1964).
- For simplicity we define the relative dielectric permittivity:

$$\varepsilon_r(\vec{r}) = \underbrace{\varepsilon_{r_0}}_{\text{average dielectric constant}} + \underbrace{\varepsilon_{r_1}}_{\text{phase modulation}} \cos(\vec{K} \cdot \vec{r})$$

- The product:

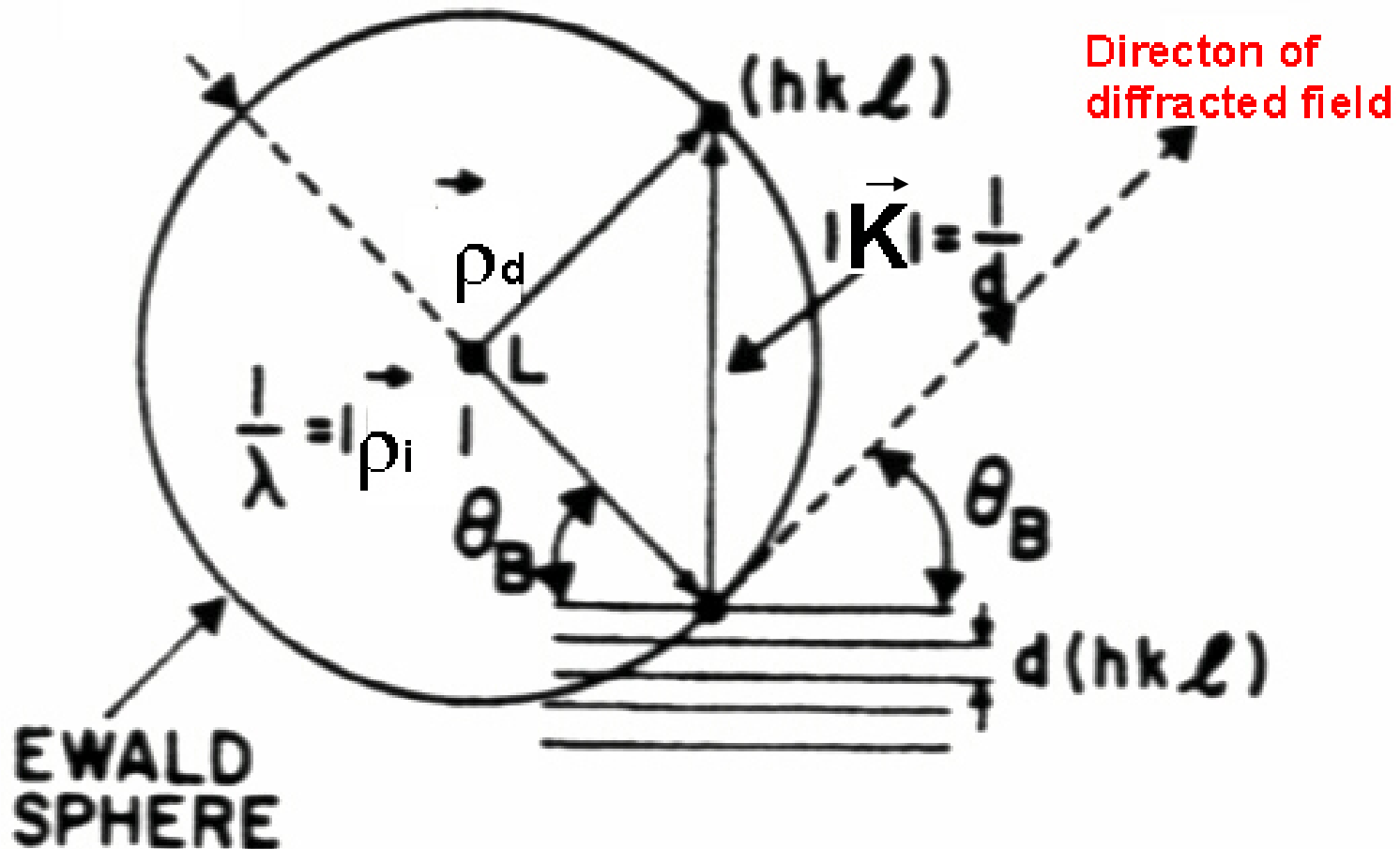
$$\vec{K} \cdot \vec{r} = xK_x + yK_y + zK_z = \text{constant}$$

Defines the condition for spatial location of constant values of the permittivity.

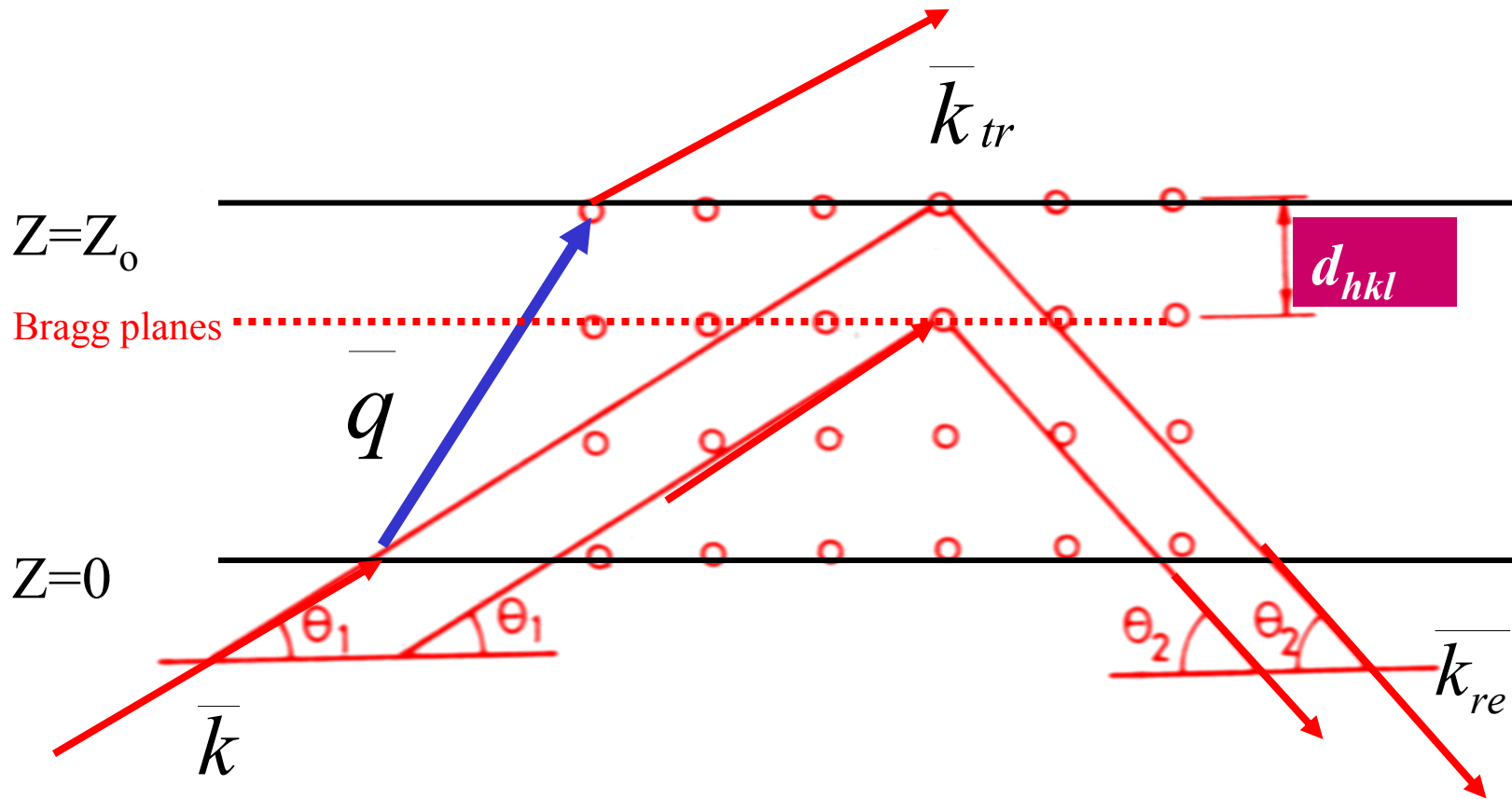
- We will analyze the type of periodic solution for the electric field representing the scattering wave originated by the interaction of the electromagnetic field with the so represented periodic medium.
- The vector \vec{K} accounts on the periodicity of the optical medium.
- **MAIN CONDITION:** We need a **geometry** for describing the spatial distribution of the scattered amplitude.

EWALD'S GEOMETRICAL CONSTRUCTION: The Ewald's sphere: Diffraction occurs in a sphere of radius $1/\lambda$. Having its center back along the incident beam direction. It is constructed in the reciprocal lattice. Vector \mathbf{K} denotes an allowed solution.

Direction of incidence
of the incoming field



DIFFRACTION BY A CRYSTAL STRUCTURE. THE BRAGG REGIME



The angle of incidence satisfies the Bragg-condition: $2 \sin \theta_B = \frac{n\lambda}{d_{hkl}}; n = \pm 1, \pm 2, \dots$

h, k, l : Miller indices

Type of vector solution

•MAIN ASSUMPTIONS:

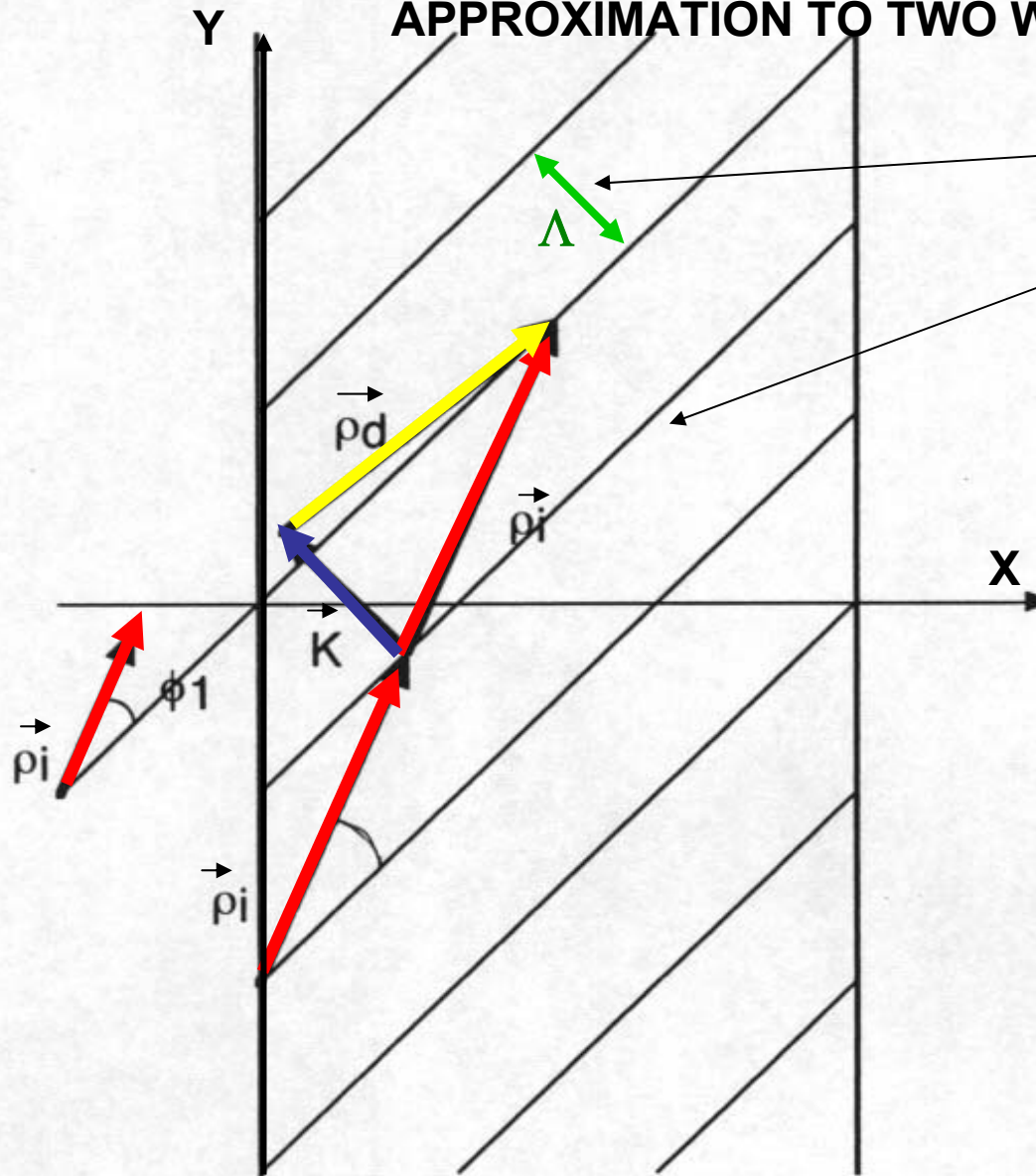
1.- The wavefield inside the periodic medium satisfies Bragg's law and Maxwell's equations.

2.- The solution is taken as a sum of N plane waves, each one associated to a particular vector in the reciprocal lattice:

$$\bar{E}(\bar{r}, t) = \left[\sum_N \bar{E}_N \exp(-2\pi i \bar{K} \cdot \bar{r}) \right] \exp(-2\pi i \bar{\rho}_i \cdot \bar{r}) \exp(2\pi i \nu t)$$

Defined as the so-called *Bloch's* function. It is a wave of wavevector $\bar{\rho}_i$ and amplitude expressed as a Fourier series.

REPRESENTATION OF THE PERIODICITY OF THE MEDIUM IN THE PLANE: APPROXIMATION TO TWO WAVES SOLUTION



$$2\pi/\Lambda = |\vec{K}| = |\vec{\rho}_d + \vec{\rho}_i|$$

Planes of constant irradiance:
(after material processing: planes of constant phase)

$$\vec{K} \cdot \vec{r} = \text{constant}$$

We consider that **only two waves** are of appreciable amplitude.

IMPORTANT: Physically all waves exist in the periodic medium

The incident wave with wave vector: $\vec{\rho}_i$ is scattered by the periodic structure.

The wave vector of the scattered wave is: $\vec{\rho}_d = \vec{K} + \vec{\rho}_i$

Considerations on the polarization states

- The field vectors: $\bar{\mathbf{E}}_i$ and $\bar{\mathbf{E}}_d$, each have two components in the plane of incidence:
- **Longitudinal component**: Along their respective wavevectors
- **Transverse component**: Orthogonal to their respective wavevectors
- But the longitudinal components can be seen to be negligible. This is consistent with the condition:

$$\nabla \cdot \bar{\mathbf{D}}(\bar{\mathbf{r}}) = \nabla \cdot [\varepsilon(\bar{\mathbf{r}}) \bar{\mathbf{E}}(\bar{\mathbf{r}})] = \varepsilon(\bar{\mathbf{r}}) \nabla \cdot \bar{\mathbf{E}}(\bar{\mathbf{r}}) + \bar{\mathbf{E}}(\bar{\mathbf{r}}) \cdot \nabla \varepsilon(\bar{\mathbf{r}}) = 0$$
$$\nabla \cdot \bar{\mathbf{E}}(\bar{\mathbf{r}}) = -\bar{\mathbf{E}}(\bar{\mathbf{r}}) \cdot \frac{\nabla \varepsilon(\bar{\mathbf{r}})}{\varepsilon(\bar{\mathbf{r}})} = -\bar{\mathbf{E}}(\bar{\mathbf{r}}) \cdot \nabla \ln \varepsilon(\bar{\mathbf{r}})$$

- The remaining transverse components lie in the plane of incidence and are orthogonal to their respective wavevectors.
- We can consider a simplified situation: To work with non coupled internally diffracted fields: single mode condition.

Since: $\bar{\rho}_i \in XY$; $\bar{K} \in XY$

And: $\nabla \varepsilon(\bar{r}) \parallel \bar{K}$; and $|\nabla \varepsilon(\bar{r})| = \varepsilon_{r1} |\bar{K}|$

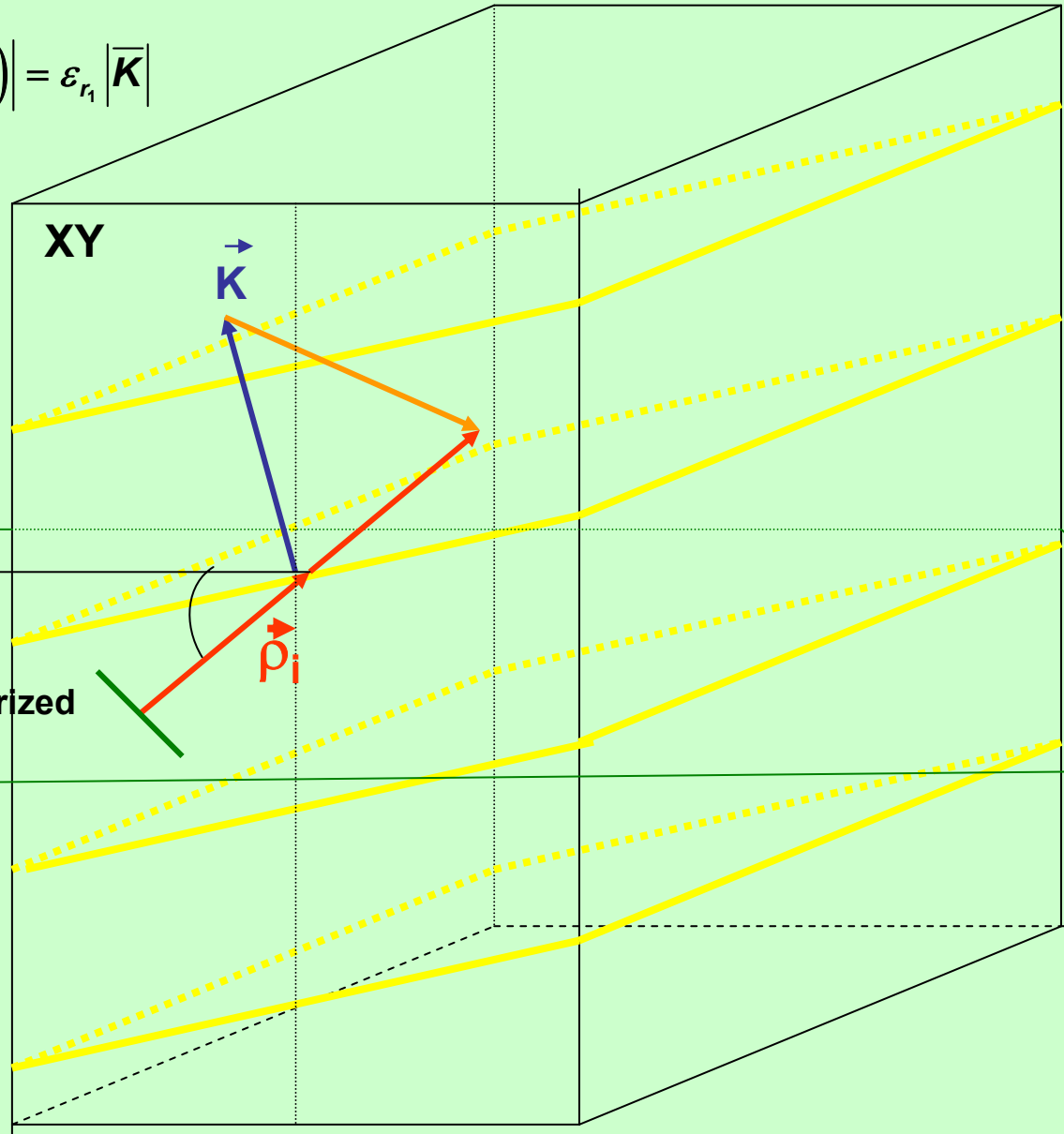
We can choose the electric field \bar{E} orthogonal to the plane of incidence (TE polarization)

XZ

Plane-polarized wave

There will be no polarization conversion, and:

$$\nabla \bar{E}(\bar{r}) = -\bar{E}(\bar{r}) \cdot \frac{\nabla \varepsilon(\bar{r})}{\varepsilon(\bar{r})} = 0$$



I. 5.- The Kogelnik's solution: The One-dimensional Coupled Wave Theory

- In 1969 *H. Kogelnik* obtained an approximate solution for the case of two uncoupled vector fields solutions.
- The vector solution reduces to the scalar approximation:

$$\nabla^2 E(\bar{r}) + k^2 E(\bar{r}) = 0$$

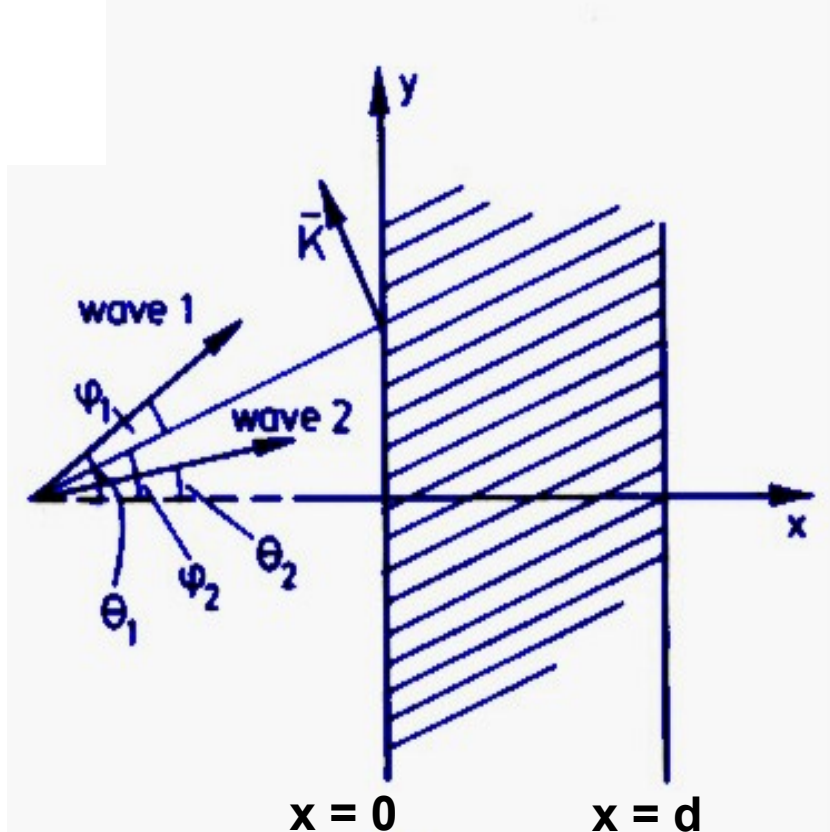
with:

$$k^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_{r_0} \left(1 + \frac{\varepsilon_{r_1}}{\varepsilon_{r_0}} \cos(\bar{K} \cdot \bar{r}) \right)$$

•Description of the recording of the volume holographic grating:

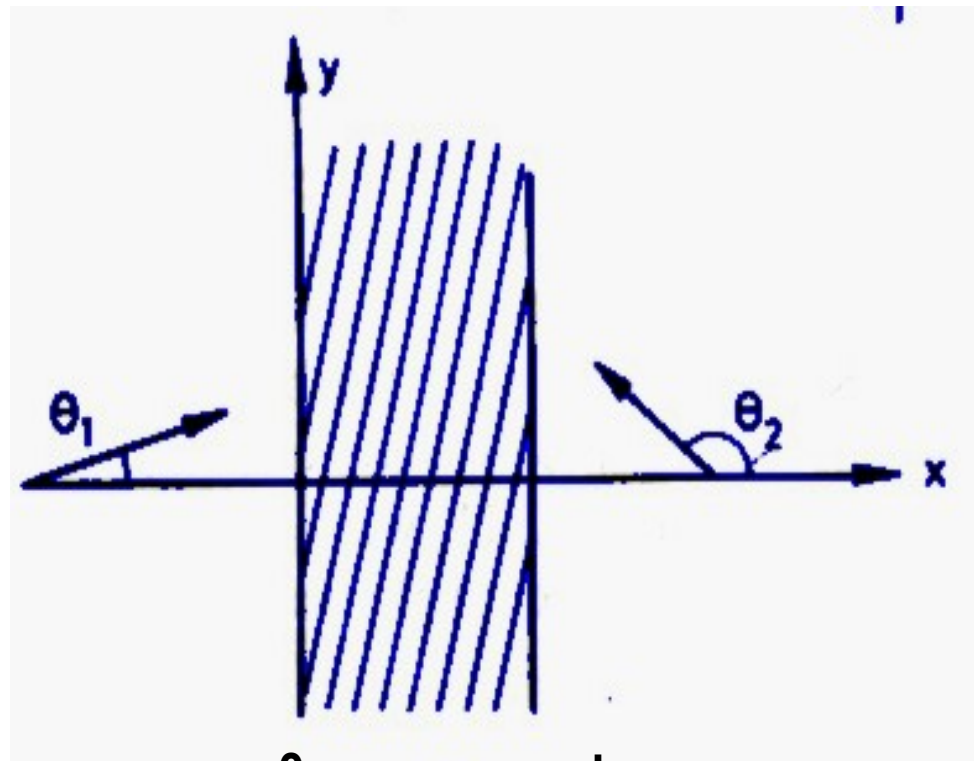
Let us describe the process for recording the volume holographic grating.

It can be recorded by illuminating the holographic material by two waves. The simplest case is the **recording with two plane waves**.



$x = 0$ $x = d$
 $R(0)=1$ $S(0)=0$

Recording of a transmission holographic grating. The two waves impinge the photomaterial from the same side.

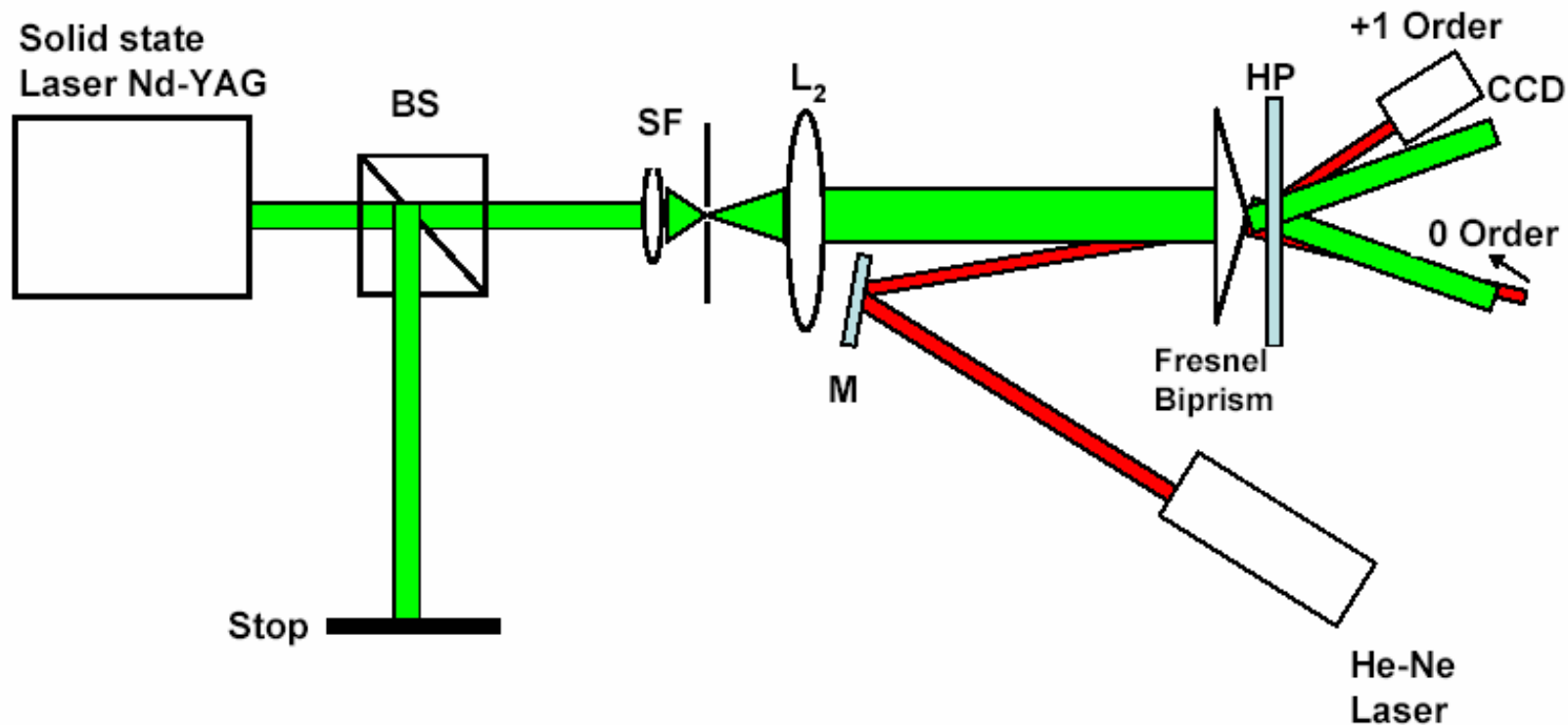


$x = 0$ $x = d$
 $R(0)=1$ $S(d)=0$

Recording of a reflection holographic grating. The two waves impinge the photomaterial from opposite sides.

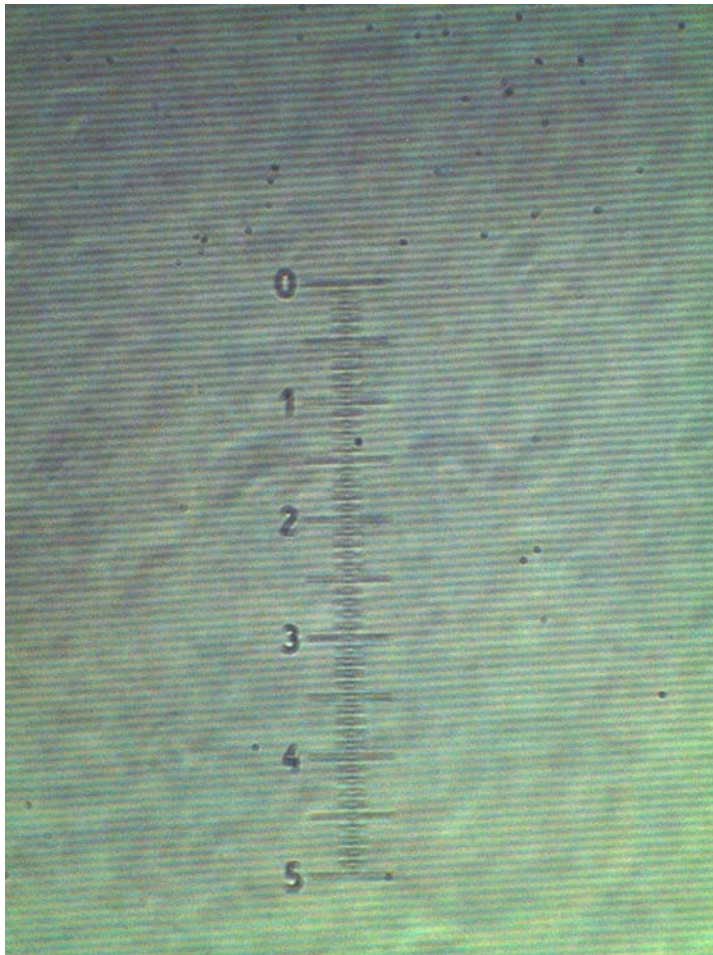
In both cases the angles of wavevectors with X-axis are not coinciding. This situation produces a slanted grating (planes of constant phase are not parallel to X-axis). Opposite, while coinciding produces an unslanted grating.

Experimental arrangement for the recording of a holographic grating. Recording beam at 550 nm and reading beam at 632 nm. BS: Cube beam splitter. SF: Spatial filter. L: convergent lens. M: First surface mirror. HP: Holographic photomaterial. CCD: Camera for diffraction analysis.



Scheme for the writing (green beam : 532 nm) and the reading (red beam: 633 nm) of a holographic grating

SPECIFICATIONS OF THE HOLOGRAPHIC GRATING



Relevant parameters:

- Spatial frequency: **500 lines/mm**
- Periodicity: $\Lambda = 2 \mu\text{m}$
- Modulus of grating wavevector:
 $K = 3 \mu\text{m}^{-1}$
- Thickness: $d = 80 \mu\text{m}$
- Diffraction regime:

$$Q = \frac{K^2 d}{\beta}; \quad \beta = \omega \sqrt{\mu \varepsilon_0 \varepsilon_{r_0}}$$

$Q = 42$; $Q \gg 10$: Bragg regime

- The scale for measurement is a standard one: **2,25mm**
- Ocular lens: **10X**

I.5.1.- First order approximation

- We consider a simple one-dimensional scalar model.
- The holographic grating presents no absorption: pure phase grating.
- The two waves are polarized orthogonally to the plane of incidence.
- Both waves are striking the photomaterial with angles fulfilling Bragg's condition.
- Only two waves propagate inside the optical medium: the incident and the first diffracted ones, respectively:

$$\Psi_i = R(x) \cdot \exp[-i\beta(x \cos \theta_1 + y \sin \theta_1)]$$

$$\Psi_d = S(x) \cdot \exp[-i\beta(x \cos \theta_2 + y \sin \theta_2)]$$

- With:
$$\beta = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_{r0}} = \frac{2\pi}{\lambda_0}$$

- It is easy to calculate the total amplitude as a sum of both waves.

- We only need to obtain the solutions of the differential wave equation:

$$\nabla^2 E + \beta^2 \left[1 + \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \cos \beta(p_1 - p_2) \right] E = 0$$

- With: $\bar{\rho}_i \cdot \bar{r} = \beta(x \cos \theta_1 + y \sin \theta_1) = \beta p_1$; $\bar{\rho}_d \cdot \bar{r} = \beta(x \cos \theta_2 + y \sin \theta_2) = \beta p_2$

- The so-called first-order approximation implies that we neglect phase terms:

$$\exp(-2i\beta p_1) \approx \exp(-2i\beta p_2) \approx 1$$

- Also, the amplitudes $R(x)$ and $S(x)$ are smoothly varying. This implies that the interchange of energy inside the optical medium is very weak. Therefore, we neglect the second derivative terms: eikonal approach

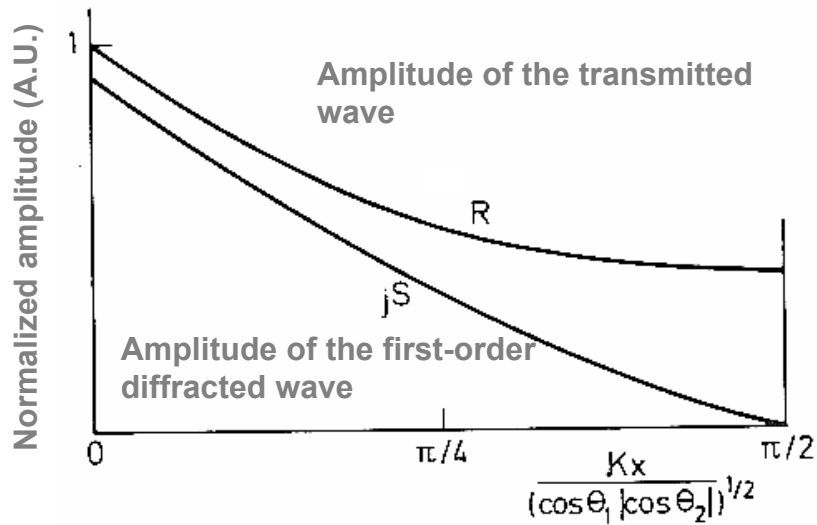
- Partial differential equation system :**

$$\left\{ \begin{array}{l} \frac{dR(x)}{dx} + i\kappa \sec \theta_1 S(x) = 0 \\ \frac{dS(x)}{dx} + \kappa \sec \theta_2 R(x) = 0 \end{array} \right.$$

- And Kogelnik's coupling coefficient:

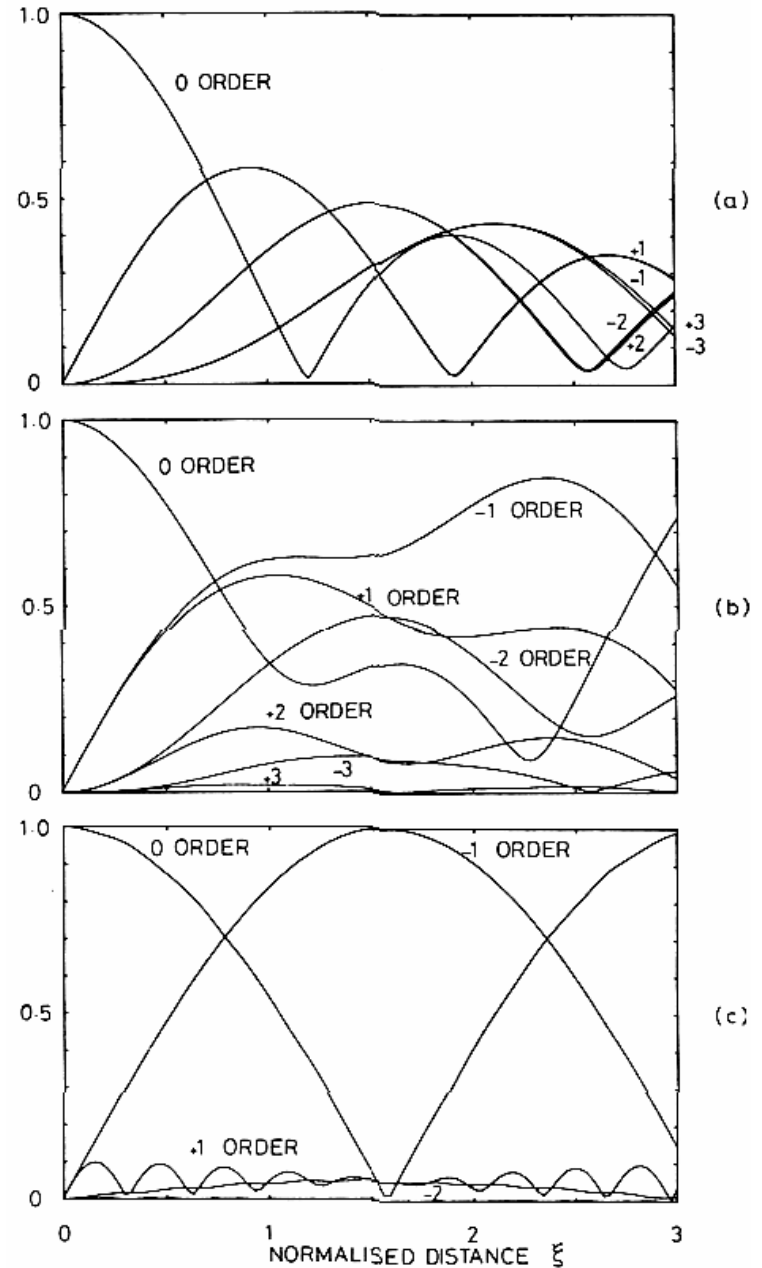
$$\kappa = \frac{\beta \varepsilon_{r1}}{4 \varepsilon_{r0}} \ll 1$$

EXAMPLES OF NUMERICAL SOLUTIONS



Reflection volume holographic grating under Bragg condition and first order approximation (Kogelnik)

Transmission holographic grating:
 (a) Raman-Nath regime (Thin hologram)
 (b) Transition regime
 (c) Bragg regime (Thick hologram)



Power conservation

- According to the linear behavior assumed in the diffraction process, there must be a balance of power.
- The power of the transmitted and diffracted waves flows in the direction of vectors $\bar{\rho}_i$ and $\bar{\rho}_d$, respectively.

- The total flux of energy in the X-direction is:

$$P = \cos \theta_1 |R|^2 + \cos \theta_2 |S|^2$$

That is invariant along the X-direction where there are no absorption losses. So, that:

$$\nabla \cdot \langle \bar{S}(\bar{r}) \rangle = 0$$

- This result is a consequence of the conservation of Poynting's vector as formulated by the dynamical diffraction theory. There are no energy “sinks”.
- We have not considered the photomaterial dispersion. For conservation of the internal energy, we may add the condition:

$$\frac{d\varepsilon(\omega)}{d\omega} > 0$$

Diffraction efficiency

- The diffraction efficiency of a holographic grating is defined:

$$\eta = \frac{P_{-1}}{P_{in}}$$

P_{-1} : Power flux of the 1st-order wave. P_{in} : Power of the incident wave

- The corrected diffraction efficiency:

$$\eta = \frac{P_{-1}}{\sum_i P_i} = \frac{P_{-1}}{P_{in} - (P_a + P_s + P_f)}$$

P_i : power flux of the i -th order wave. P_a : absorption losses. P_s : scattering losses.
 P_f : Fresnel reflection losses.

- From the coupled wave theory:

α' : angle of the reconstruction wave

$$\eta = \exp(-\alpha d_{ef}) \sin^2 \left[\left(\frac{\pi \Delta n d}{\lambda \cos \alpha'} \right) \right]$$

Diffraction efficiency and absorption influence

When there is a non zero modulated conductivity:

$$\sigma = \sigma_0 + \sigma_1 \cos(\bar{K} \cdot \bar{r} + \phi)$$

$$\omega \ll \frac{\sigma}{\epsilon}; \text{ non negligible absorption: } \alpha \neq 0$$

$$\alpha = \frac{\sigma_0}{2} \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}$$

The coupled wave theory leads to a new definition of the diffraction efficiency with a dependence on absorption.

(a): Influence of the dephasing term for fixed α absorption values in reflection grating.

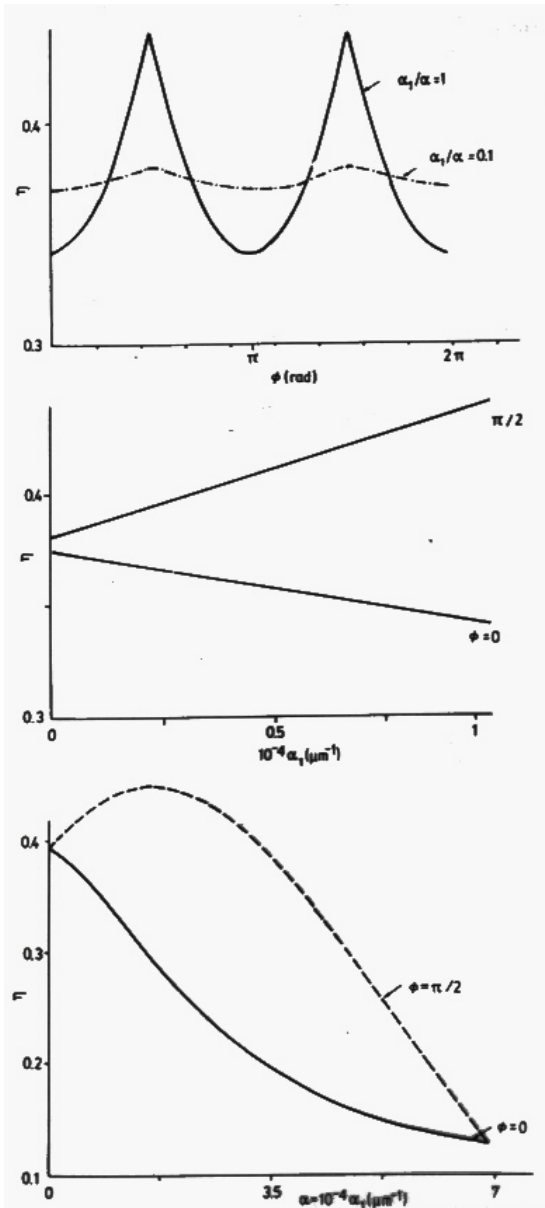
(b): Influence of the α absorption coefficient for fixed phase values in the same grating.

(c): Influence of the full absorption modulation:

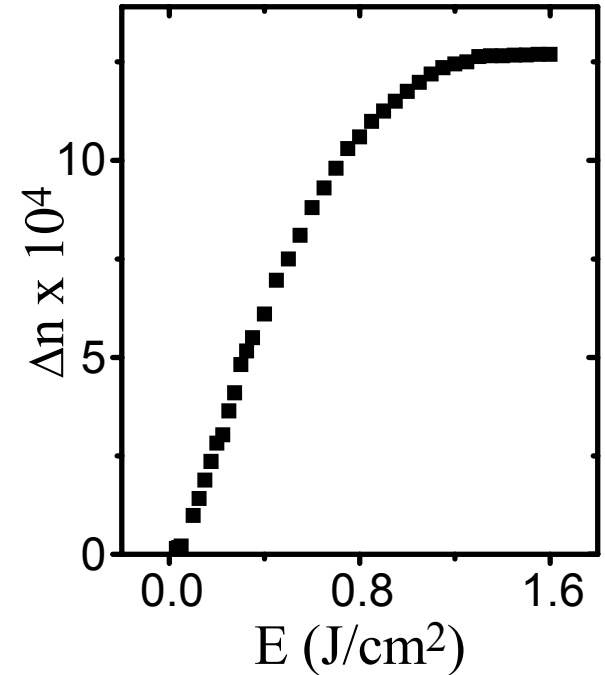
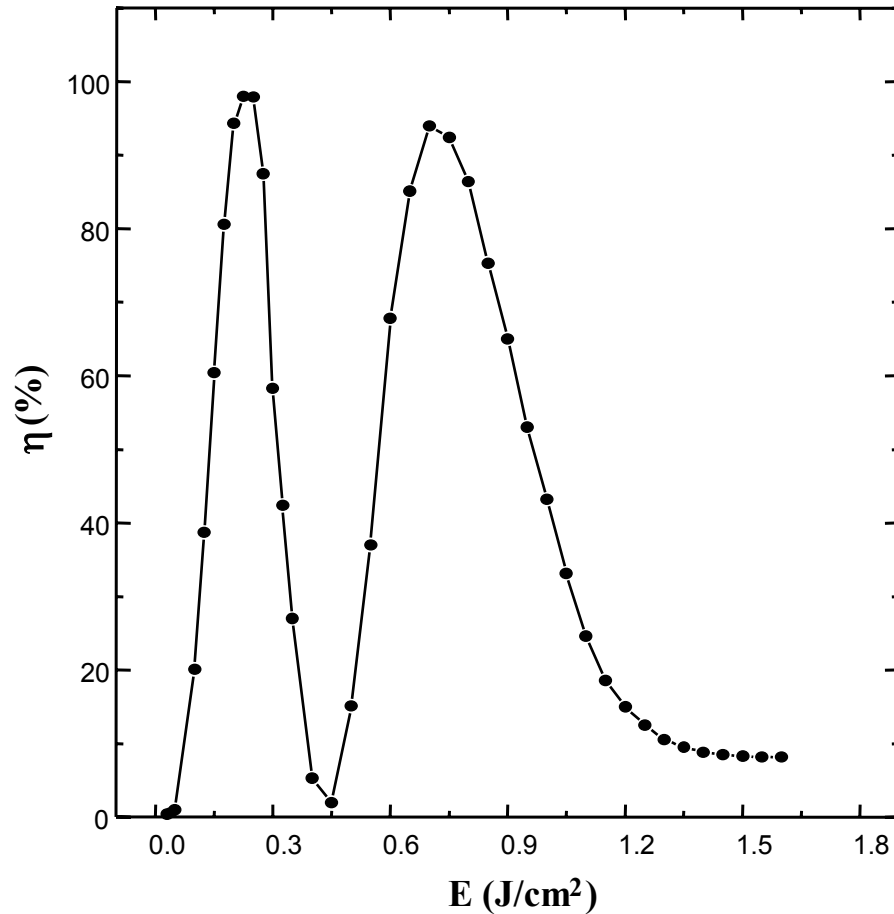
$$\sigma_0 \approx \sigma_1$$

and fixed dephasing values.

After: Guibelalde E., Calvo M.L., Opt. and Quant. Electr., **18**, 213(1986).



Diffraction efficiency and modulation



After: P. Cheben and M. L. Calvo, Appl. Phys. Lett. **78**, 1490-1492 (2001)

I.7.- Conclusions

- The phenomenon of the interaction of light with a periodic dielectric medium can be rigorously treated within the electromagnetic theory framework.
- The so-called dynamical diffraction theory is applicable to the study of the modulation of the electromagnetic field generated by the interaction of light with a volume holographic grating.
- The scalar and eikonal approximations provide simple and useful solutions, the so called Kogelnik's solutions that are rather easy to analyze.
- This analysis leads to the introduction of certain relevant physical parameters of interest for the applications of volume holographic gratings to data storage.

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