



The Abdus Salam
International Centre for Theoretical Physics



SMR.1738 - 19

WINTER COLLEGE
on
QUANTUM AND CLASSICAL ASPECTS
of
INFORMATION OPTICS

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Free Space Quantum Cryptography

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ESA: QIPS

Trieste lecture 2

Free Space Quantum Cryptography

7th Feb 2006

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FP6: IP
SECOQC
www.secoqc.net

IP: QAP

Overview

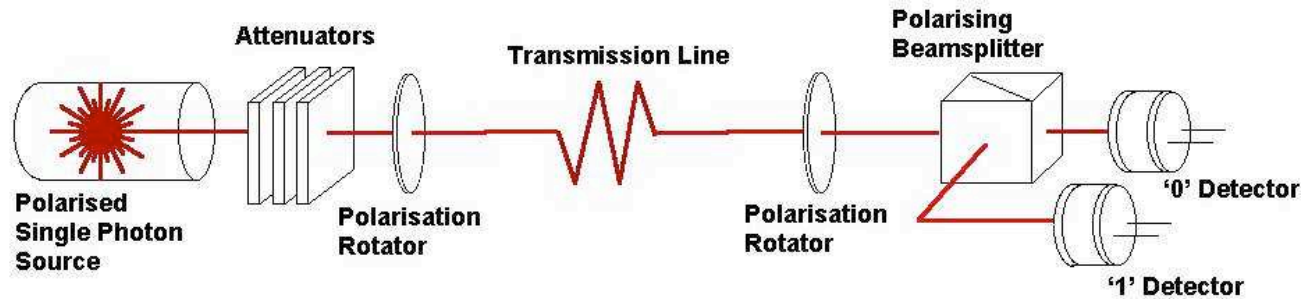
- Introduction to quantum cryptography
 - Faint pulse
 - Entangled state
- Experiments
 - Faint pulse
 - Heralded entangled photon
- Performance limits
- Future experiments

Bennett and Brassard 1984

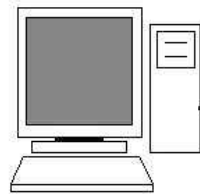
Secure key exchange using quantum cryptography

Sends

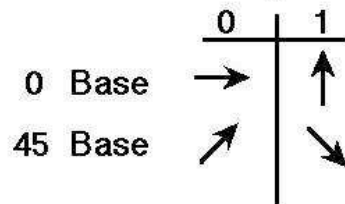
no.	bit	pol.
1	1	45
2	0	45
3	0	0
4	1	45
5	1	0
6	0	45
7	1	45
...		
1004	0	45
1005	1	0
....		
3245	1	45
...		



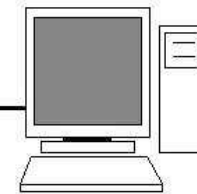
Alice



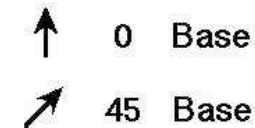
Encodes using



Bob



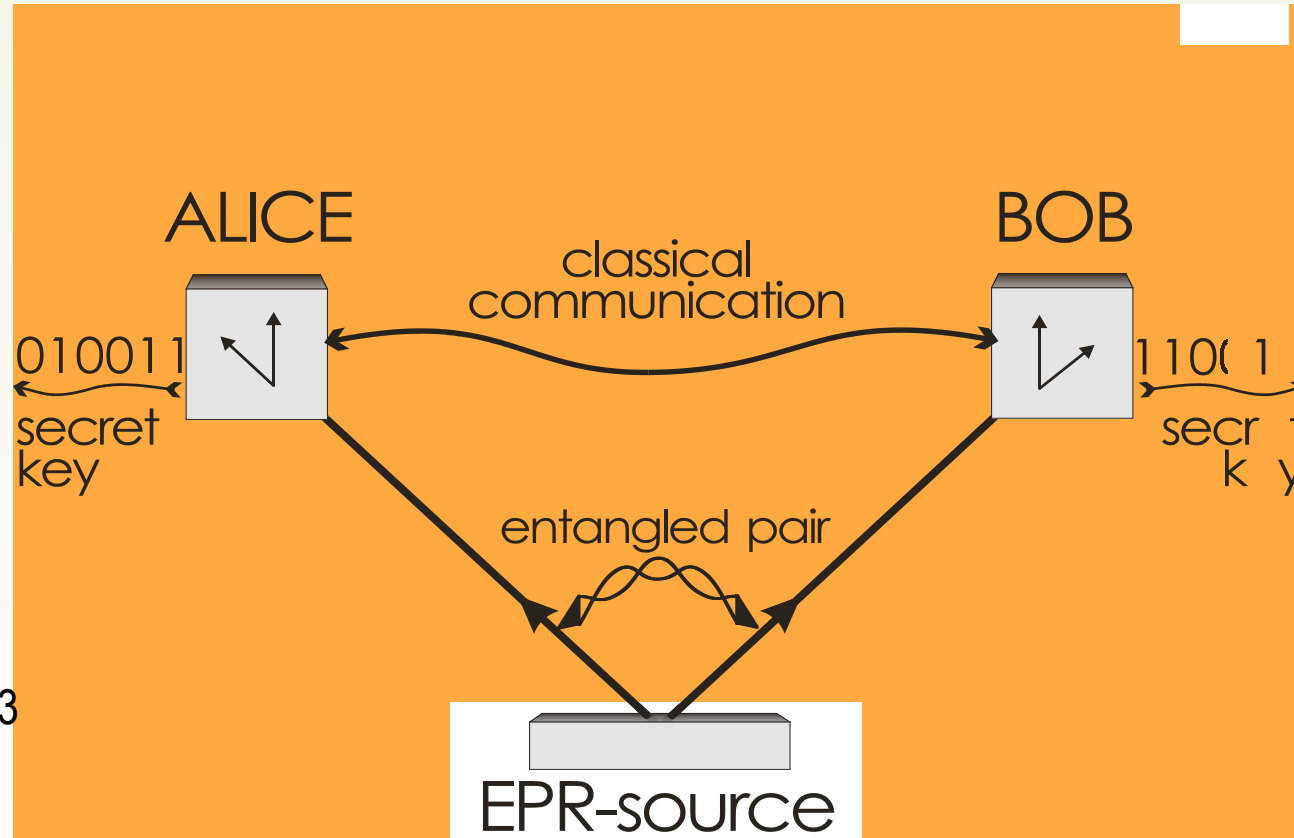
Analyses using



Receives

no.	Bit	Pol.
246	1	45
1004	0	45
2134	0	0
3245	0	0
4765	1	0
5698	0	45

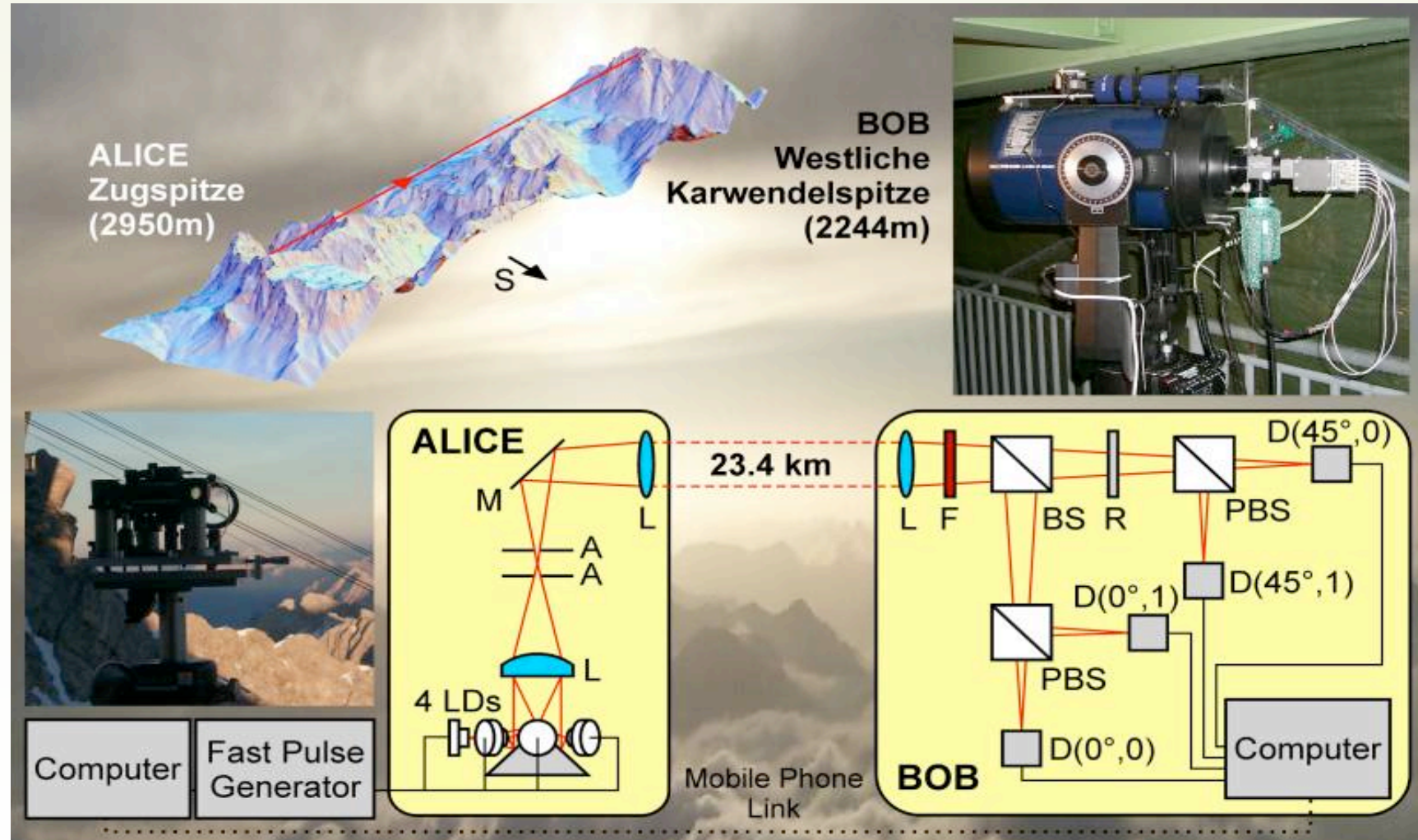
Entanglement and key exchange



Aspelmeyer et al
Science August 2003

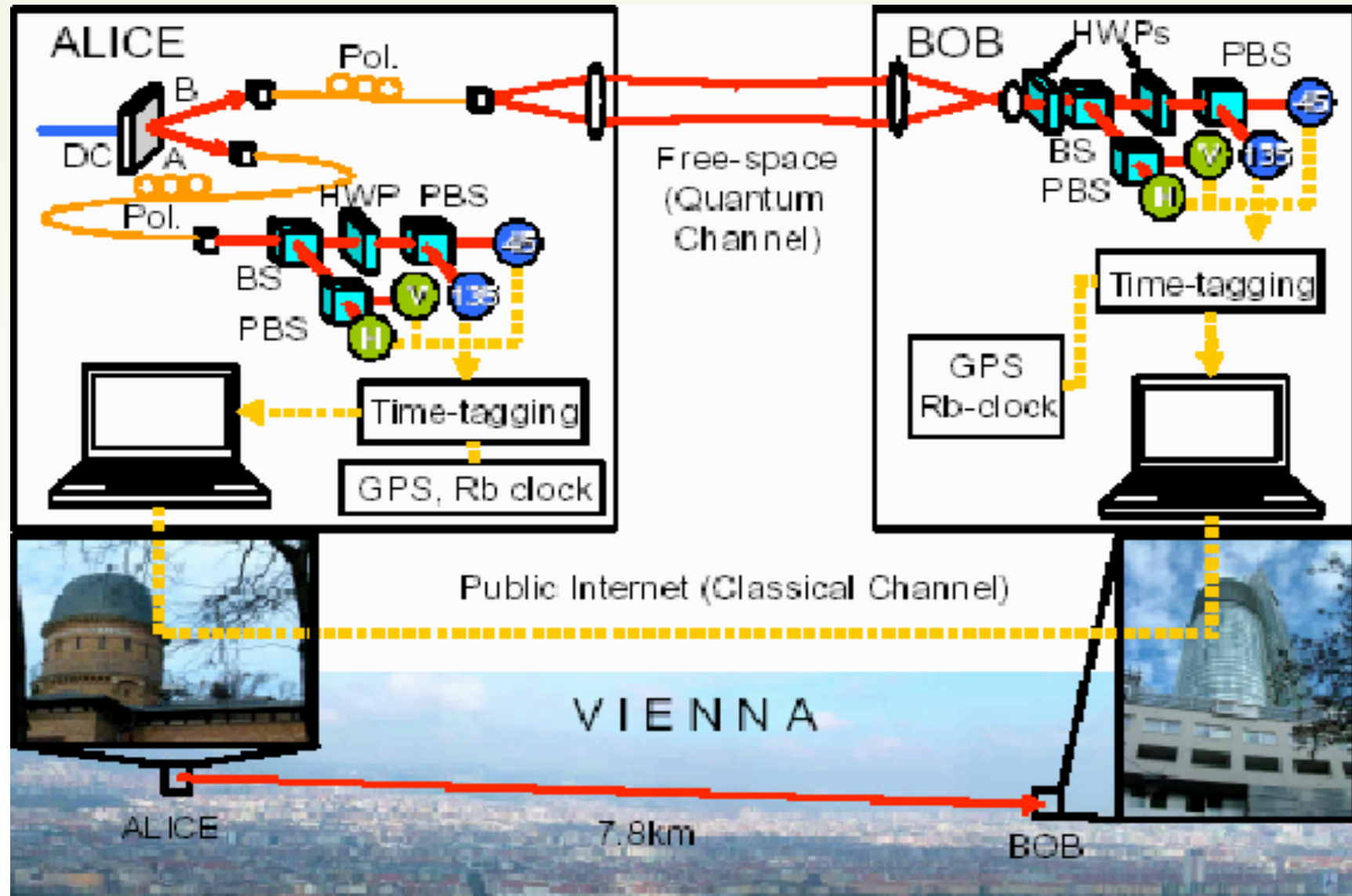
$$\begin{aligned}
 |\Psi\rangle &= \frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B) \\
 &\equiv \frac{1}{\sqrt{2}} (|'45'\rangle_A |'45'\rangle_B + |'-45'\rangle_A |'-45'\rangle_B)
 \end{aligned}$$

Previous faint pulse free space experiment over 23.4km



Kurtsiefer et al, (2002), *Nature*, **419**, 450.

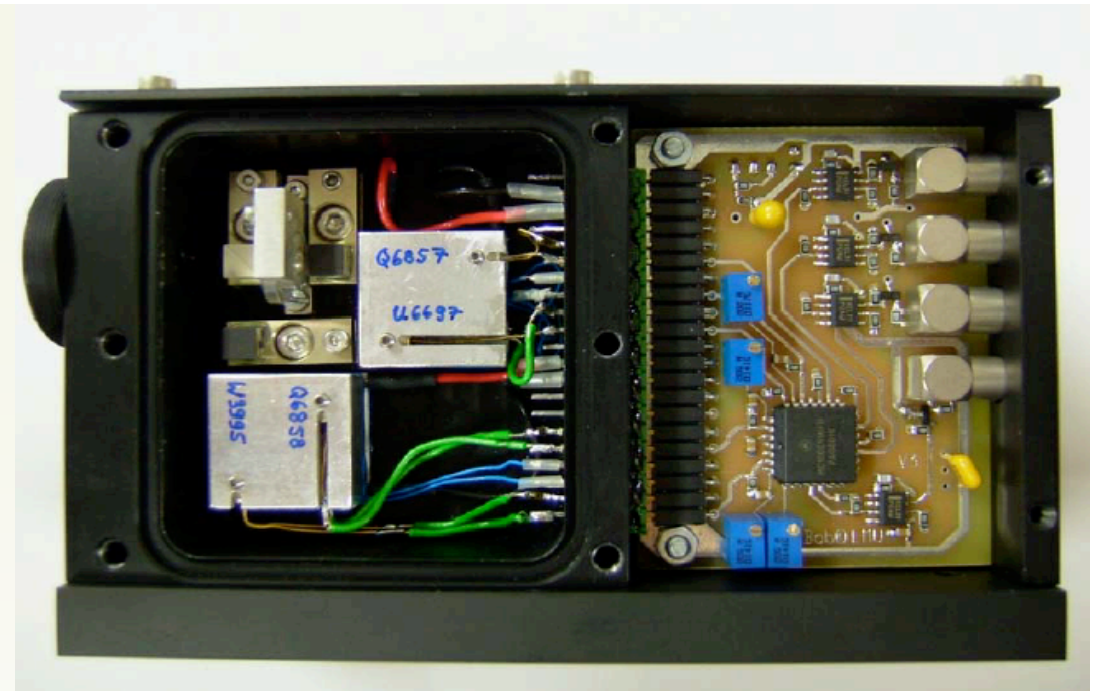
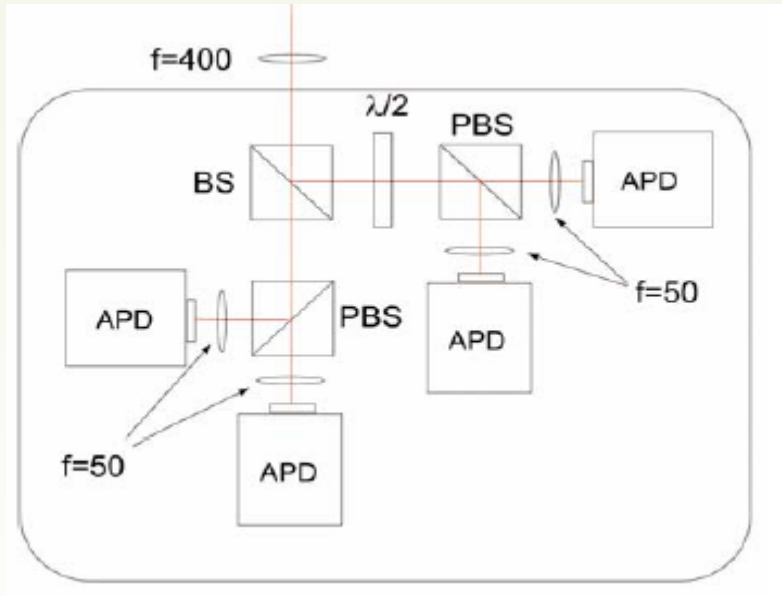
Entangled state/heralded single photon system



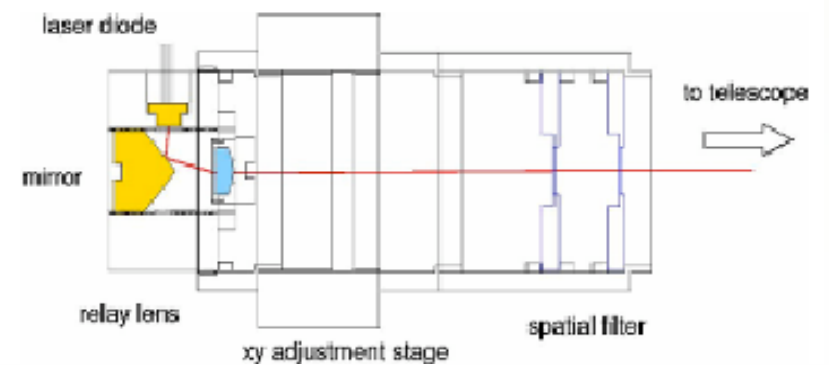
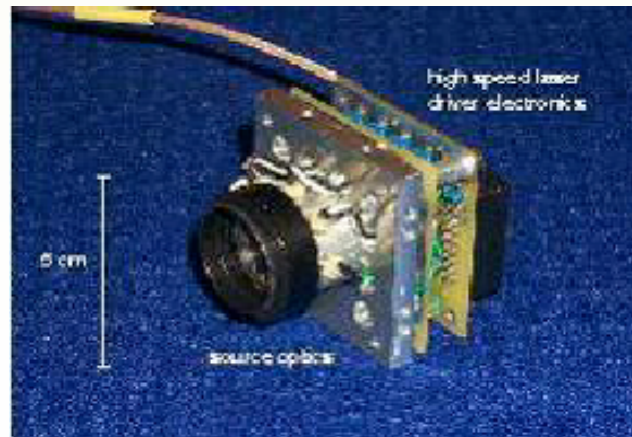
K. J. Resch, Optics Express 13, 202-9 (2005)

Miniature Bob detector unit:

Medium Bob (F5 optics)



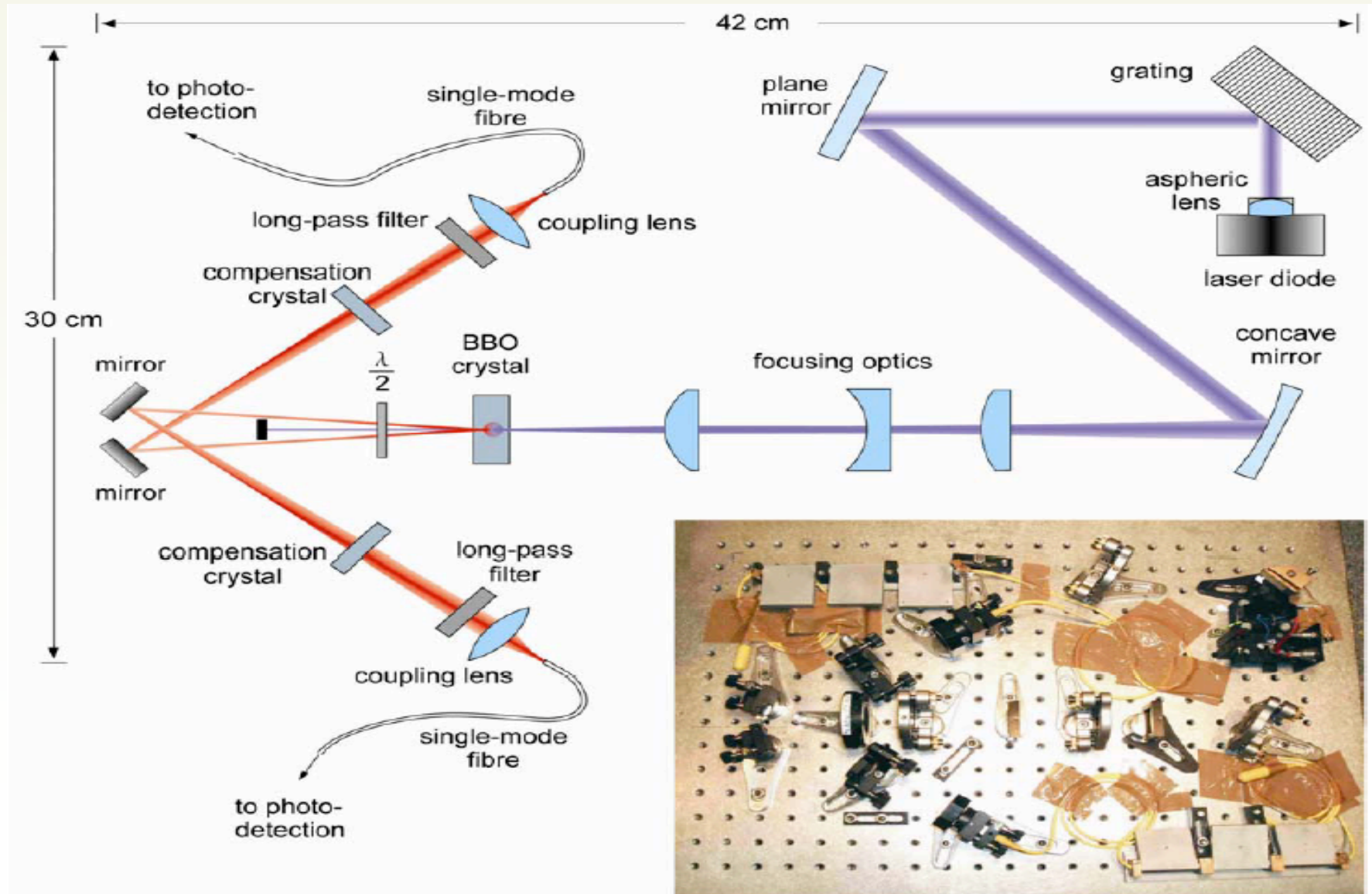
Miniature Alice



Portable Crystal based entangled pair source

150mW diode

70000 detected coincidences/sec



Transmission limitations of any system

Background count error probability per pulse

$$P_b = Bt/4 \quad (\text{half lead to errors})$$

The photo-count probability

$$P_p = \mu T \eta / 2 \quad (\text{BB84})$$

μ = photons per bit (0.1 faint pulse, ~0.75 heralded)

T = transmission (variable)

η = efficiency of receiver (20%)

B = background rate (~2000)

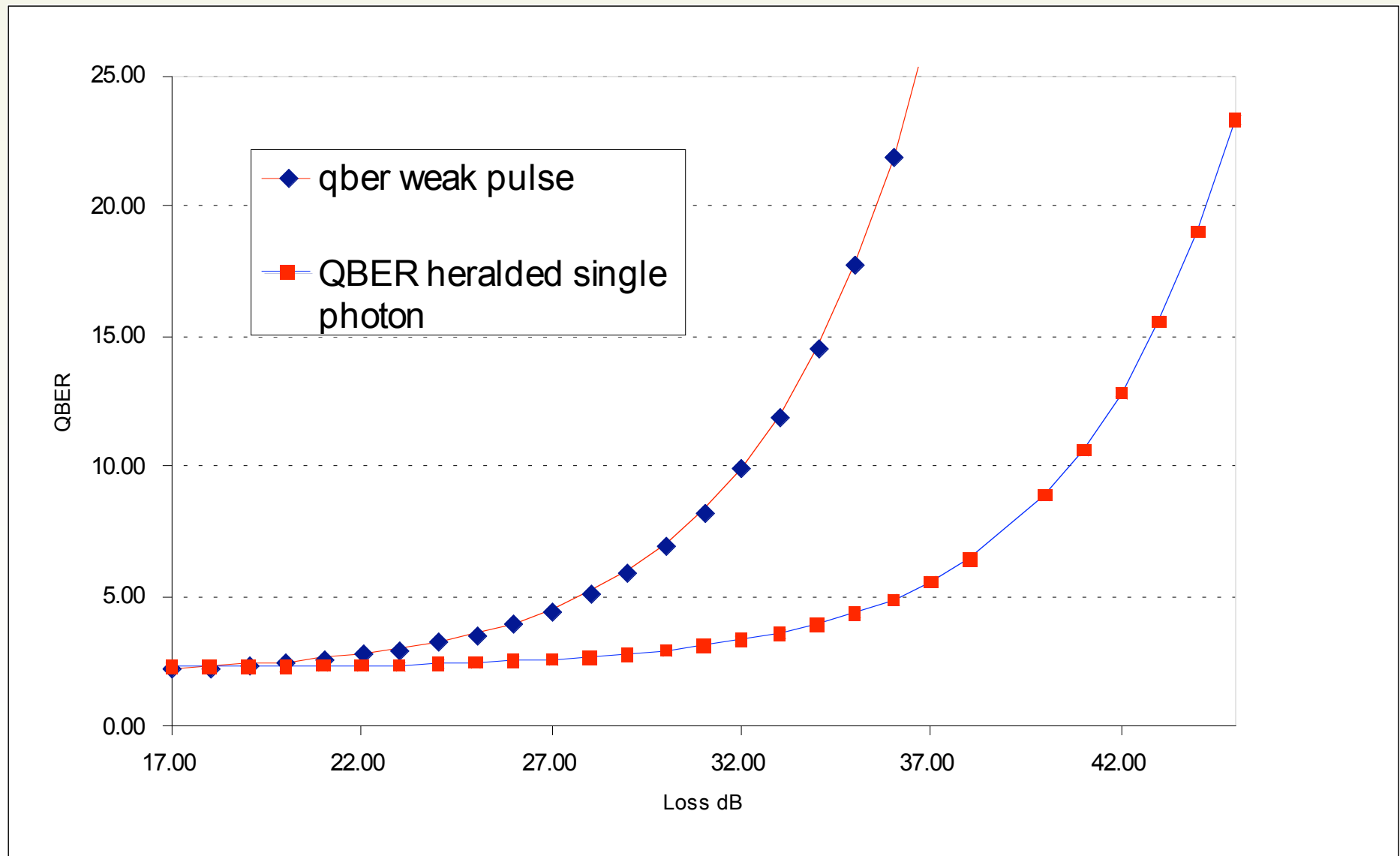
t = gate width (~1ns)

Error rate (QBER)

$$E = E_0 + Bt / 2\mu T \eta$$

E_0 is the technical error

QBER as a function of loss



Secret bit yields, after error correction and privacy amplification

Raw bit rate

$$K = 10^{-L/10} \mu \eta_B R$$

Number of sifted bits in time T

$$n_{rec} = KT / 2$$

Number of bits revealed in error correction [1]

$$n_E = n_{rec} (1 - E)$$

$$E = \varepsilon \left(\sqrt{2} - \log_2(1 - \varepsilon) \right)$$

Every error could be a result of eavesdropping
Information leakage [2]:

$$n_{leak} = n_{rec} \left(\log_2 \left(1 + 4\varepsilon - 4\varepsilon^2 \right) \right)$$

Final key length thus given by:

$$\begin{aligned} n_{fin} &= n_{rec} - n_E - n_{leak} - n_s \\ &= n_{rec} \left(E - \log_2 \left(1 + 4\varepsilon - 4\varepsilon^2 \right) \right) - n_s \end{aligned}$$

n_s = excess reduction to enhance security

For Poisson sources all multiphoton pulses are insecure

$$n_{fin} = n_{rec} a \left(E - \log_2 \left(1 + 4 \frac{\varepsilon}{a} - 4 \left(\frac{\varepsilon}{a} \right)^2 \right) \right) - n_s$$

$$a = \frac{e^{-\mu} \cdot \mu}{1 - e^{-\mu}}$$

Is the fraction of pulses containing photons that contain only one

Infinite technology Eve

Multi-photon pulses are split and sent on through a loss free channel to Bob.

$$n_{fin} = n_{rec} b \left(E - \log_2 \left(1 + 4 \frac{\varepsilon}{b} - 4 \left(\frac{\varepsilon}{b} \right)^2 \right) \right) - n_s$$

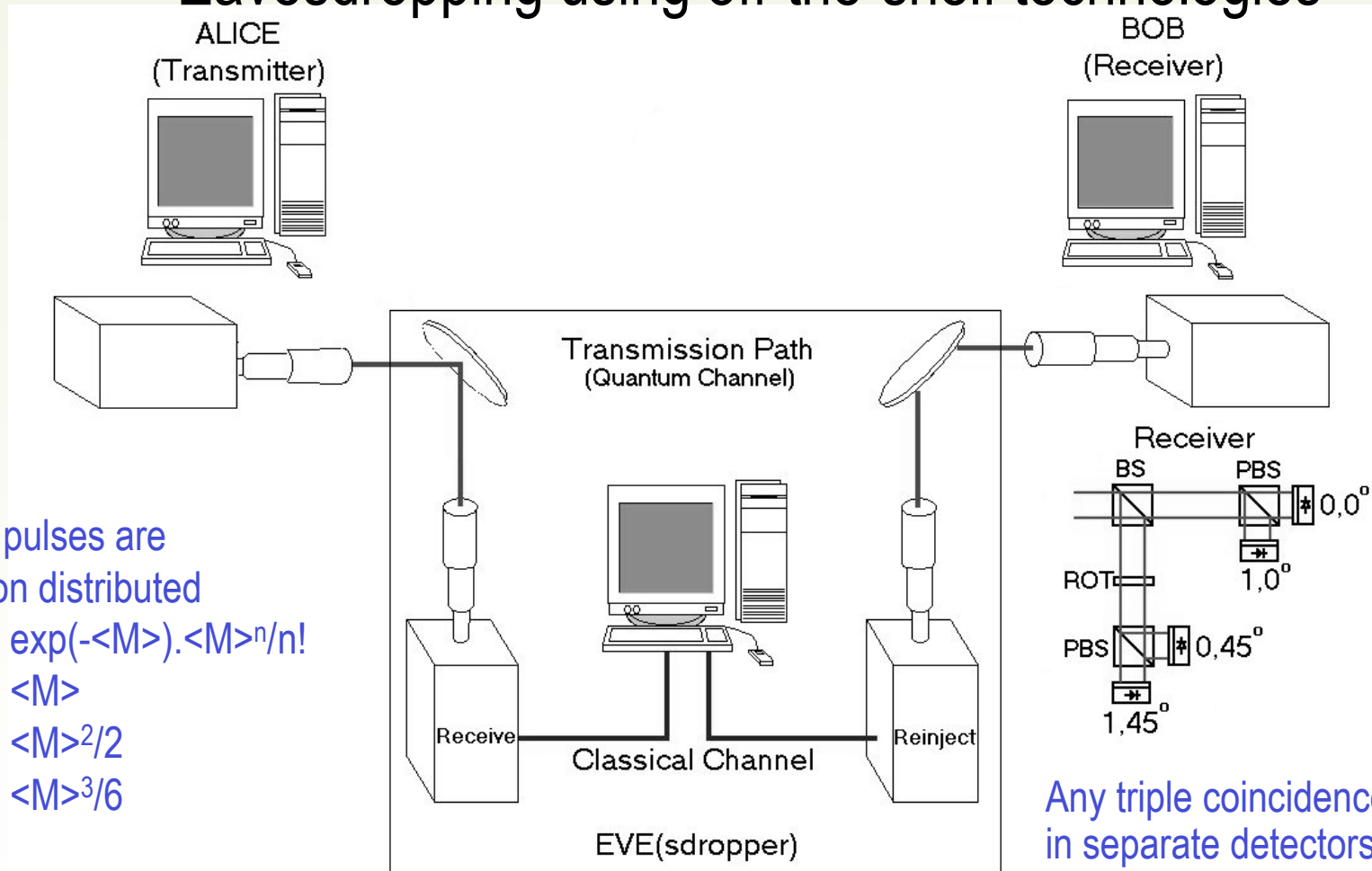
$$b = 1 - \frac{(1-a)}{10^{-L/10}} \quad \mu > 10^{-L/10}, b < 0$$

Eve can only get at >3 photon pulses

$$a' = a(1 + M/2) \quad b = 1 - \frac{(1-a')}{4T}$$

1. L. Tancevski, et al. Proc. SPIE, **3228**, 322-331 (1997).
2. N. Lutkenhaus, Phys. Rev. A 59, 3301–3319 (1999).
3. G. Brassard, et al Phys.Rev.Letts, **85**, pp1330-1333 (2000).

Eavesdropping using off-the-shelf technologies



Weak pulses are
Poisson distributed
 $P(n) = \exp(-\langle M \rangle) \cdot \langle M \rangle^n / n!$
 $P(1) \sim \langle M \rangle$
 $P(2) \sim \langle M \rangle^2 / 2$
 $P(3) \sim \langle M \rangle^3 / 6$

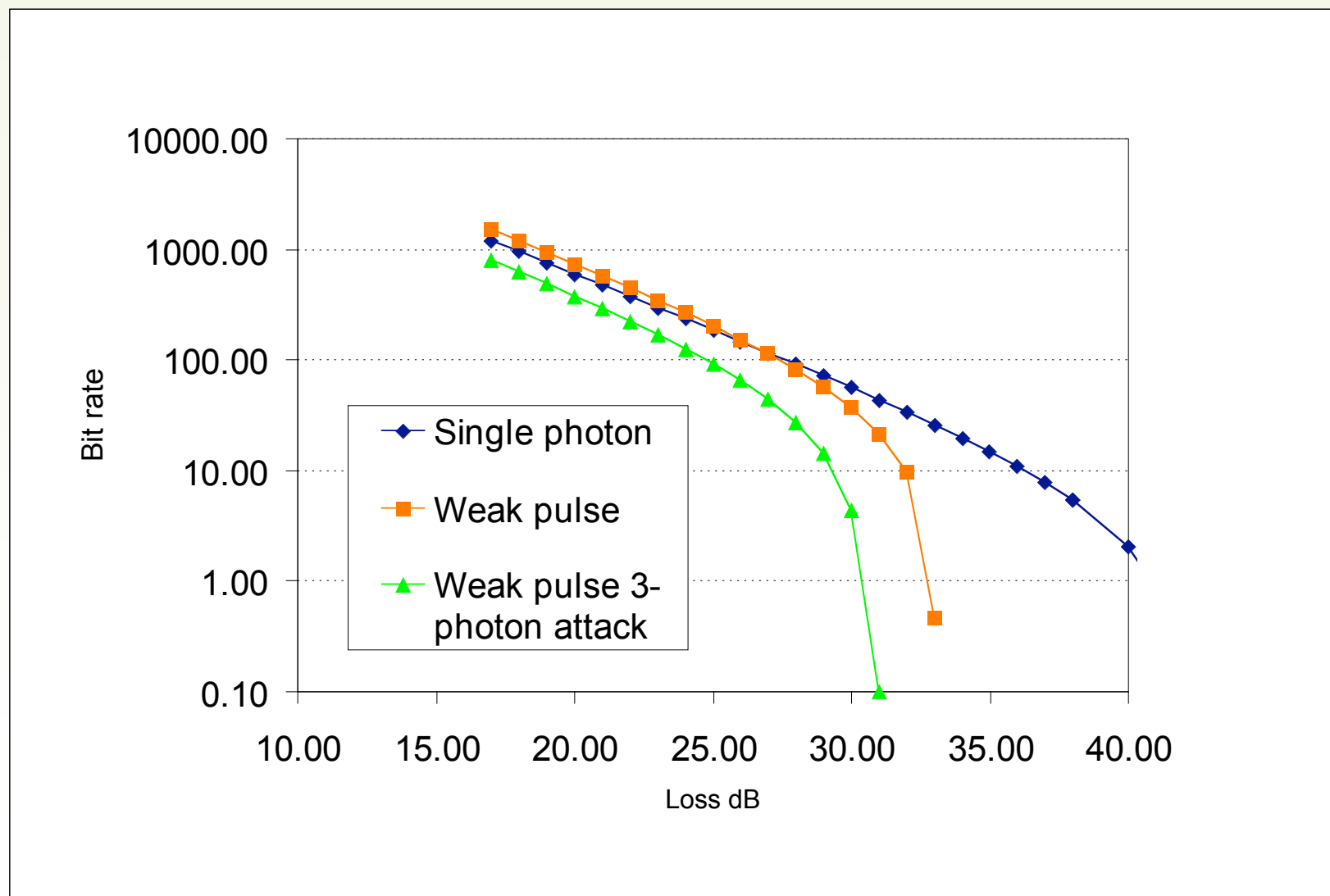
$$P(3)/P(1) = \langle M \rangle^2 / 6$$

Any triple coincidence
in separate detectors
uniquely determines
the state!!

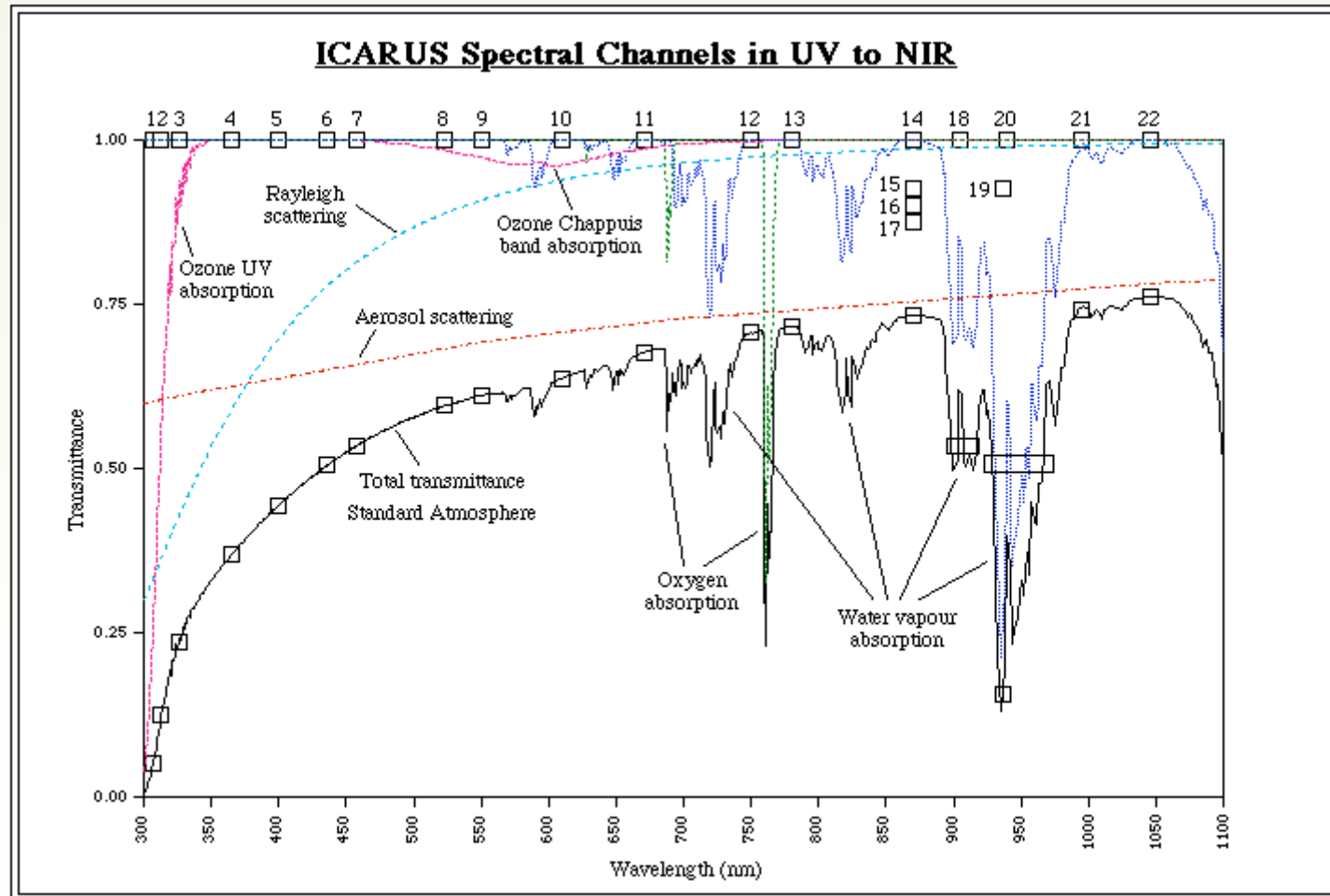
Safe if $M^2 < 24T$

$M \sim 0.1$ $T > -33\text{dB}$

Expected Secret bit yields as a function of loss after error correction and privacy amplification



Atmospheric Transmission Windows for long range.



Chosen trial site:

1 - The Observatorio del Roque de los Muchachos (Island of La Palma)

<http://www.iac.es/gabinete/orm/indice.html>

2 - The Observatorio del Teide (Island of Tenerife)

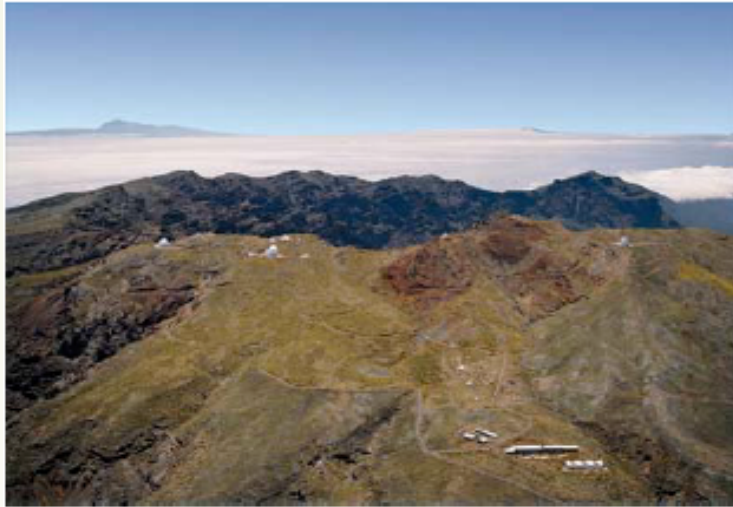
<http://www.iac.es/ot/indice.html>



The design and loss budget

- 10 MHz pulse rate, 0.1 photons/pulse
- Entangled source with $>5 \cdot 10^6$ pairs/sec ($>100000/s$ locally)
- 700/810/850nm wavelength
- $<20 \mu\text{R}$ beam divergence
- Diffraction spot <3 m diameter after 144 km
- Expect $\sim 40 \mu\text{R}$ from turbulence + ~ 10 dB absorption/scatter
- Collection and atmospheric transmission, 25-35dB loss
- Aim for background counts <2000 cps/detector through 10nm filter at night

La Palma: Roque de los Muchachos



Overview of Roque with
Tenerife in background



La Palma from Tenerife

Optical ground station Tenerife

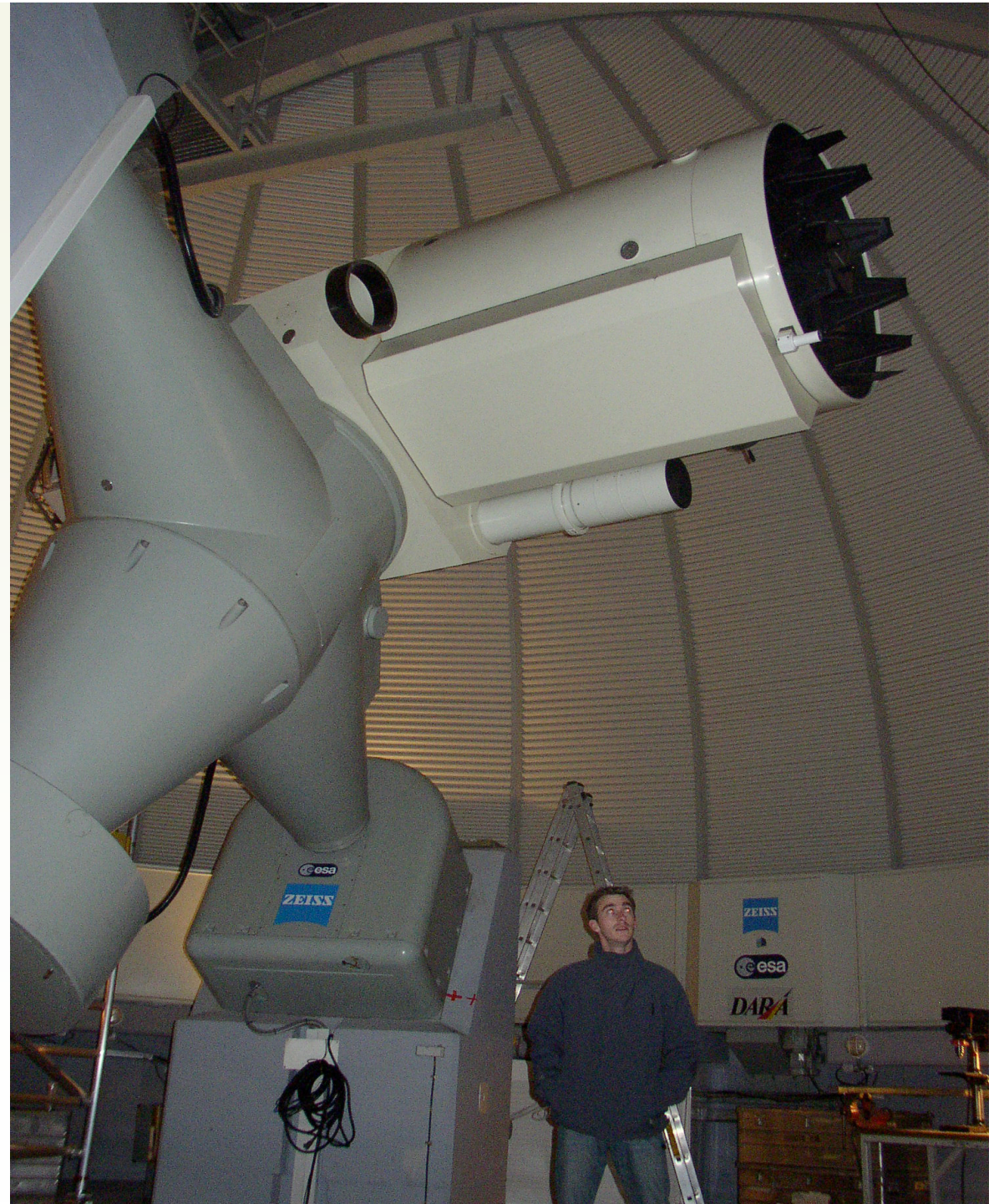


Overview of Izana (Tenerife) site



OGS with mount Tiede in background

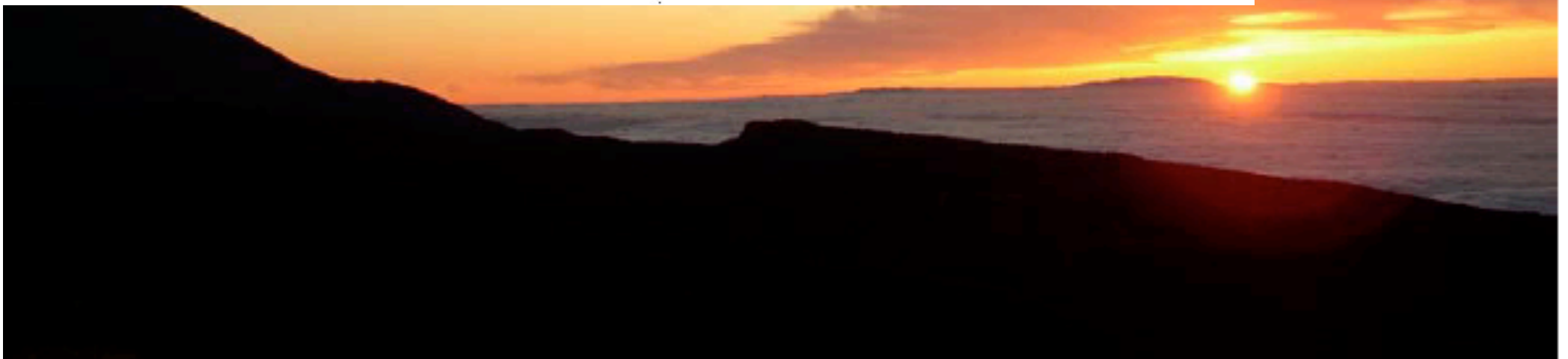
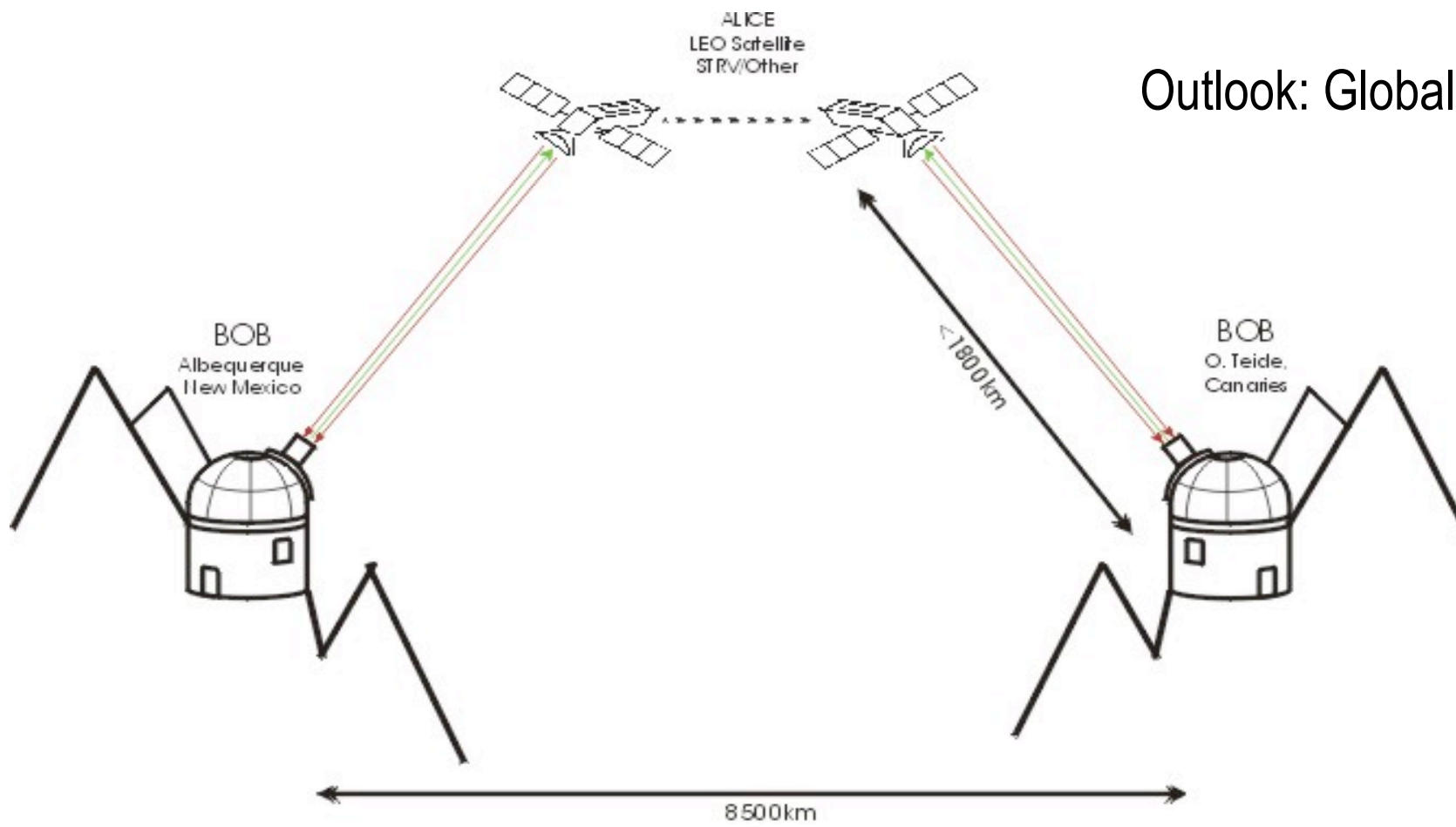
ESA OGS 1 metre telescope (Bob)



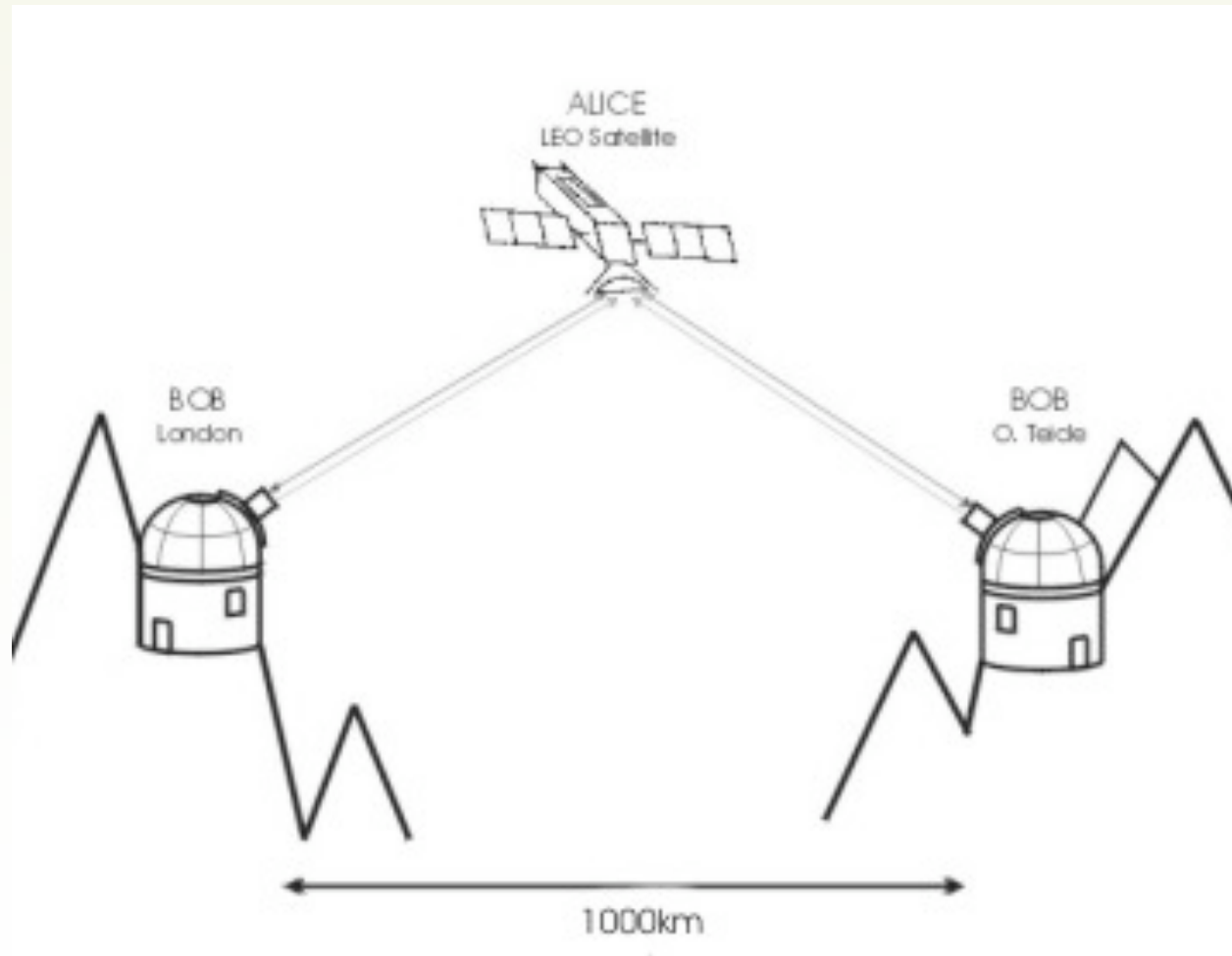
Metrology:

- Cartesian distance OGS - NOT : 143.630 km
- in position angle (North trough West) : 251° .
- Angle TNG-NOT-Teide: $123^\circ.7$ (vertex at NOT).
- Also possible Teide-OGS-La Palma (see TN4)

Outlook: Global key exchange



Coincident key generation at 1000km separations using satellite, also tests the non-locality of QM



Philosophical questions

Non-locality of entangled photon pairs

