



The Abdus Salam  
International Centre for Theoretical Physics



SMR.1738 - 20

WINTER COLLEGE  
on  
QUANTUM AND CLASSICAL ASPECTS  
of  
INFORMATION OPTICS

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**Photonic Quantum Logic**

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# Photonic Quantum Logic

Trieste Winter School 8  
FEB 2006

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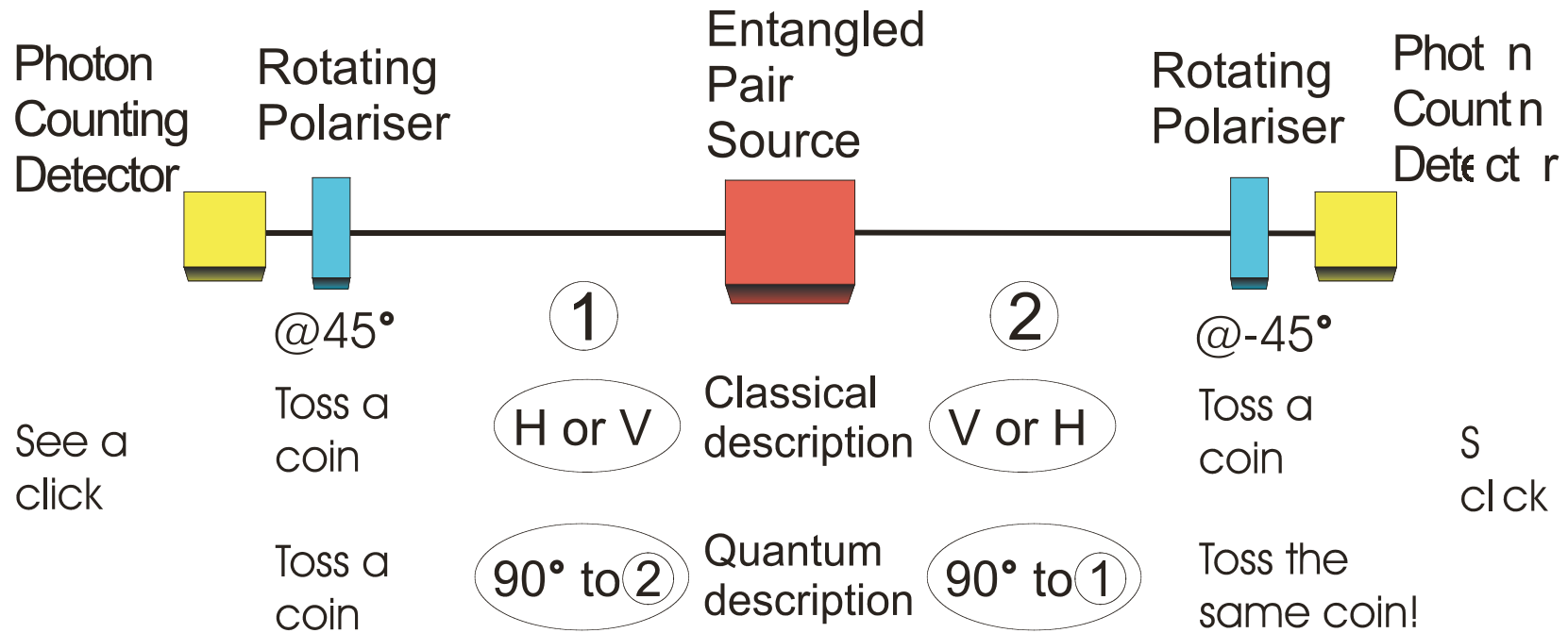
**EPSRC**  
*1-phot*

**FP6:IP**  
**SECOQC**  
**& QAP**

Bristol: Daniel Ho, J. Fulconis, J. Duligall, C. Hu, R. Gibson, O Alibart

# Philosophical questions

## Non-locality of entangled photon pairs



# Photonic Quantum Information.

- single photon coding for quantum key distribution = a (secure, non-local) random number generator.
- Can we develop more general quantum information processing using quantum effects with photons?

Have:

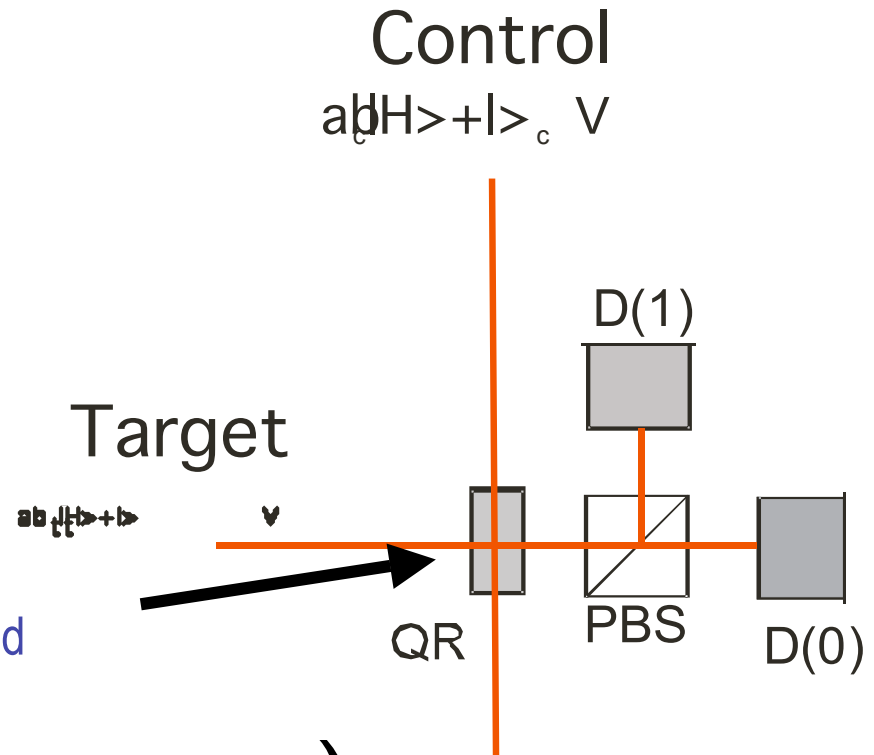
- low error single qubit manipulation  $QBER < 10^{-4}$ ,
- Little decoherence (but loss)

Need:

- low error rate and high efficiency controlled NOT gate (**hard**).
- Can we do this with 'off-the-shelf items':
- linear optical elements (efficient, scalable?)
- single photon sources (efficiency, pure state?)
- entangled state sources (on demand, pure state?)
- single photon detectors (efficiency, photon number resolving?)

# 2QUBIT logic: Photonic CNOT Gate.

QR is a quantum polarisation rotator  
 Rotates polarisation if control is vertically polarised  
 Does nothing if control is Horizontally polarised



$$|\Psi\rangle_{in} = (\alpha|0\rangle_t + \beta|1\rangle_t) (\alpha_c|0\rangle_c + \beta_c|1\rangle_c)$$

$$|\Psi\rangle_{out} = \alpha\alpha_c|0\rangle_t|0\rangle_c + \alpha\beta_c|1\rangle_t|1\rangle_c + \beta\alpha_c|1\rangle_t|0\rangle_c + \beta\beta_c|0\rangle_t|1\rangle_c$$

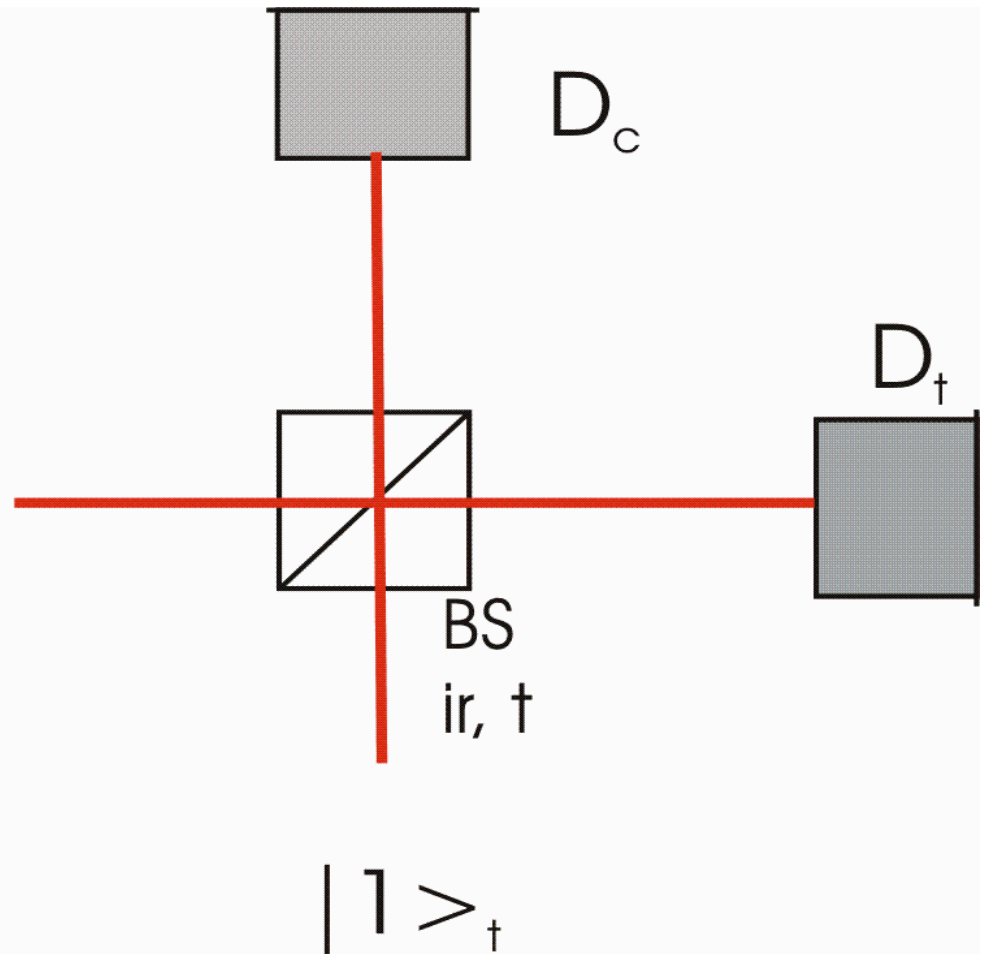
Requires non-linearity at single photon level:

Atoms: Turchette and Kimble PRL 1995,

Solid state: J. P. Reithmaier/ A. Forchel, NATURE 432, Nov 2004.

# Hong Ou Mandel interference effect

$$|1\rangle_t$$



$$|1\rangle_t$$

## Hong - Ou - Mandel Dip

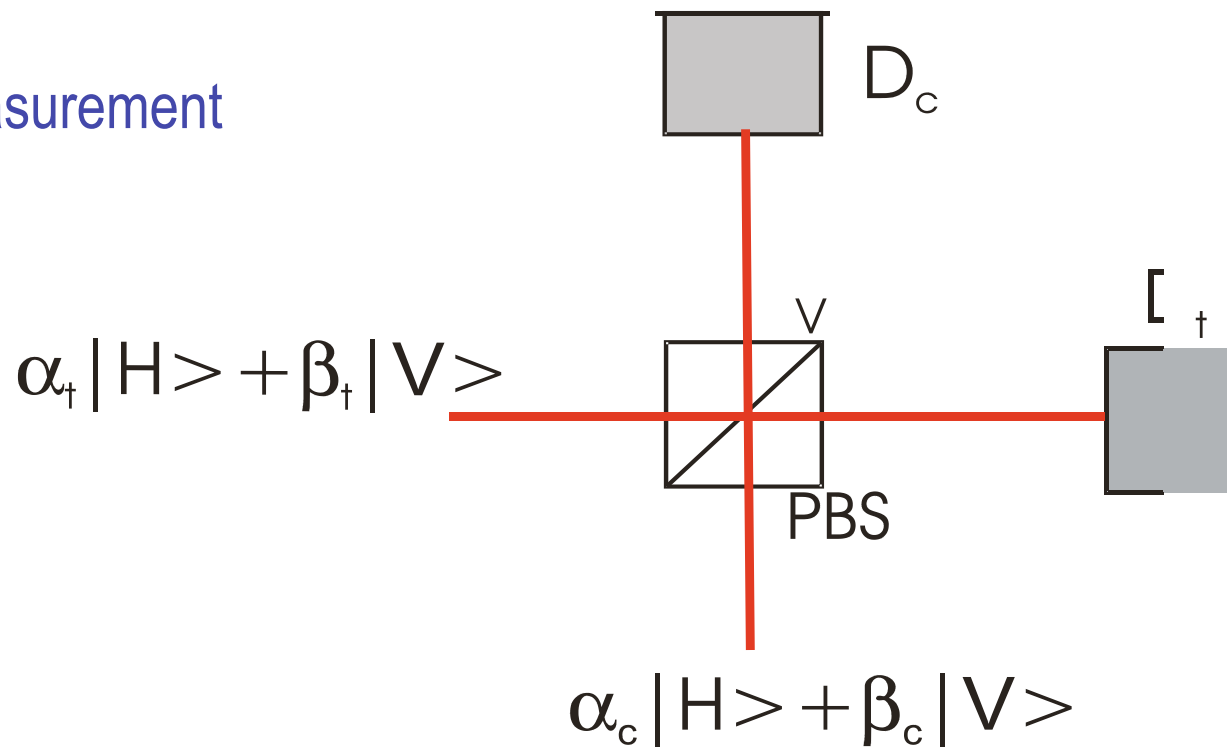
$$|\Psi_{in}\rangle = |1\rangle_t |1\rangle_c$$

$$|1\rangle_t \rightarrow t|1\rangle_t + ir|1\rangle_c : |1\rangle_c \rightarrow t|1\rangle_c + ir|1\rangle_t$$

$$|\Psi_{out}\rangle = \left( (t^2 - r^2) |1\rangle_t |1\rangle_c + irt |1,1\rangle_t + irt |1,1\rangle_c \right)$$

Hong, Ou, Mandel  
PRL 1987

# Parity Measurement

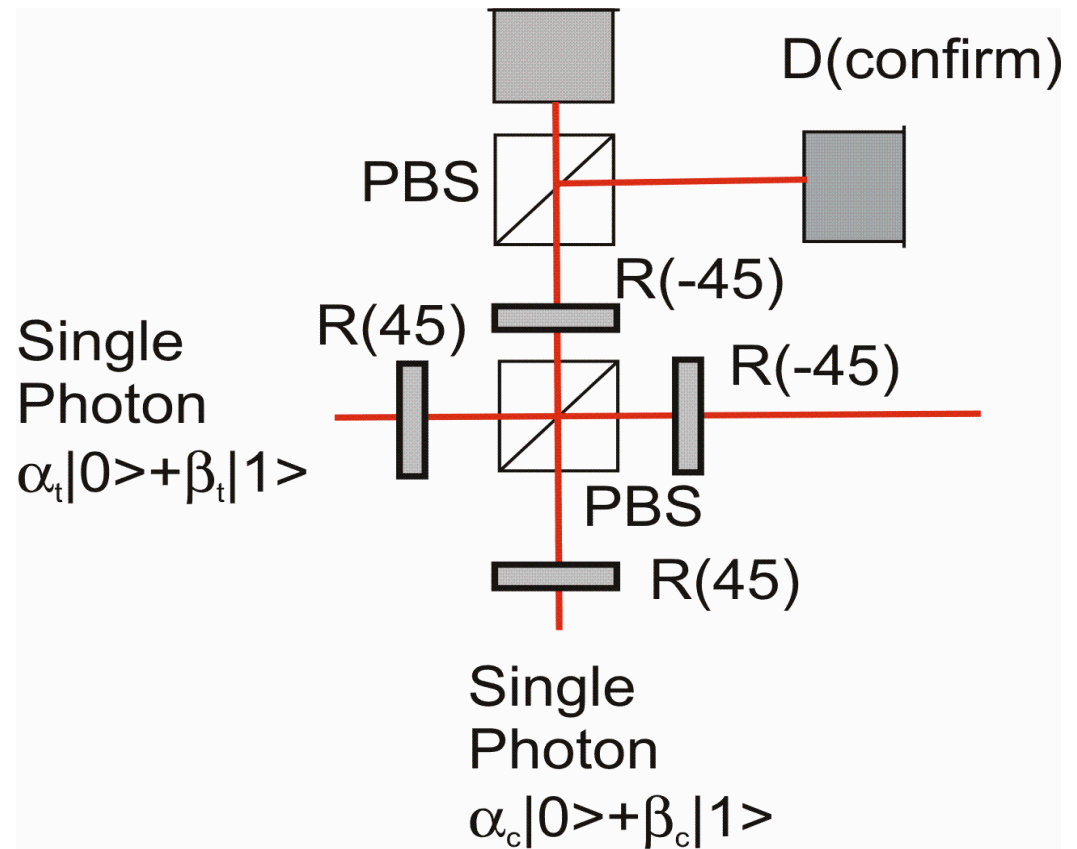


## Linear conditional CNOT gate

Knill et al Nature 409, 46–52 (2001)

Pittman et al (2002) PRL 88, 257902

Not 100% efficient but  
Up to 50%



### Notes

Target  $V \rightarrow H+V$  Control  $V \rightarrow H+V$

Parity  $\rightarrow HH+VV$  -45  $\rightarrow H(H+V)-V(H-V)$

Confirm click is  $H \rightarrow (H-V)$  out -45  $\rightarrow |H\rangle$

Confirm click is  $V \rightarrow (H+V)$  out -45  $\rightarrow |V\rangle$

Target  $V \rightarrow H+V$  Control  $H \rightarrow H-V$

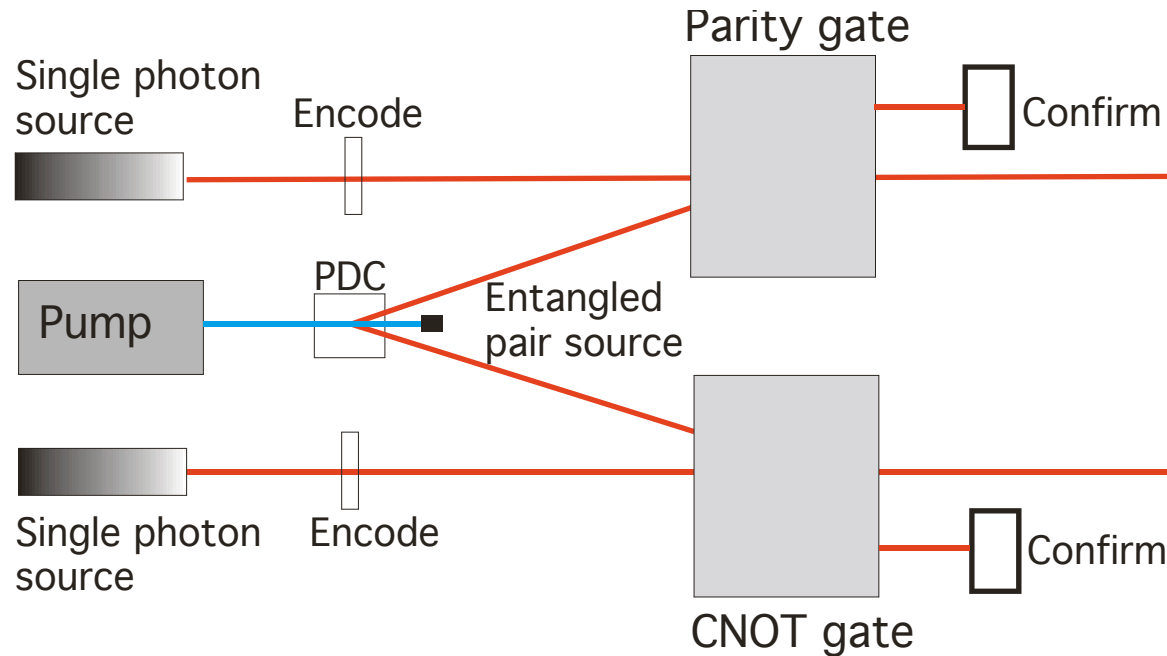
Parity  $\rightarrow HH-VV$  -45  $\rightarrow H(H+V)+V(H-V)$

Confirm click is  $H \rightarrow (H+V)$  out -45  $\rightarrow |V\rangle$



# Possible Scalable gate?

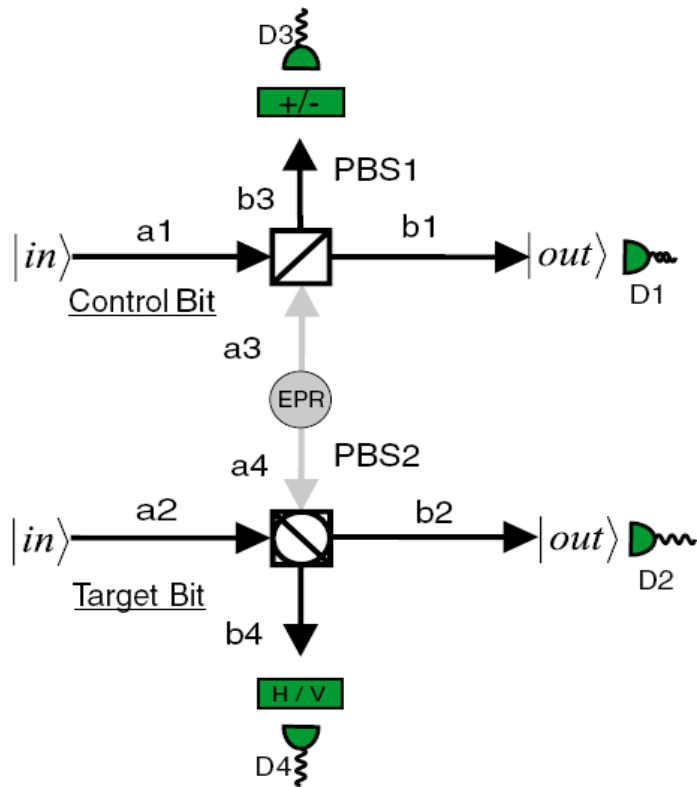
Franson et al 2003  
Zeilinger et al 2004



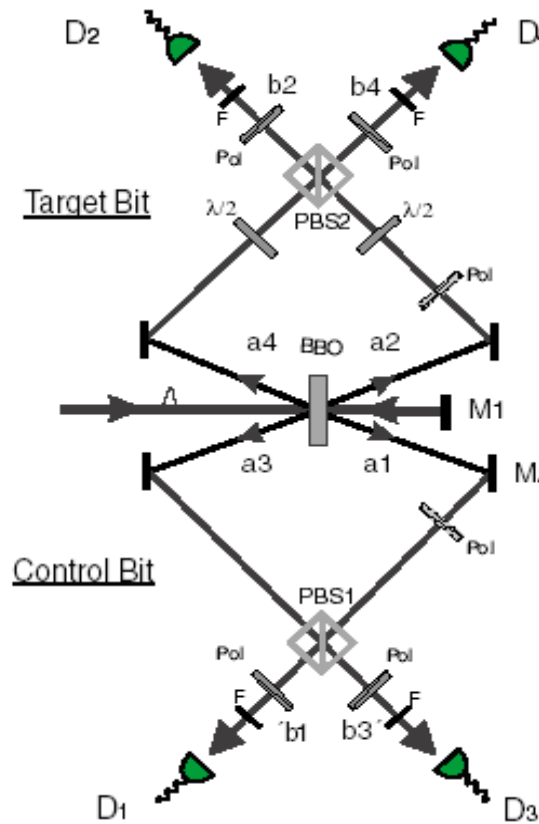
Using teleportation to make  
non-destructive gates



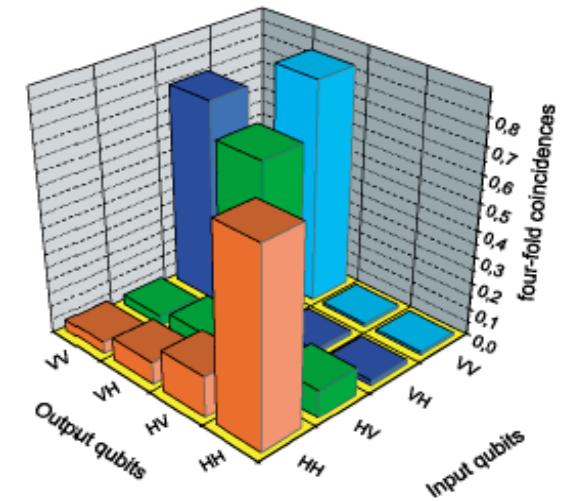
# A 'scalable' 2-qubit CNOT gate



In the proposal

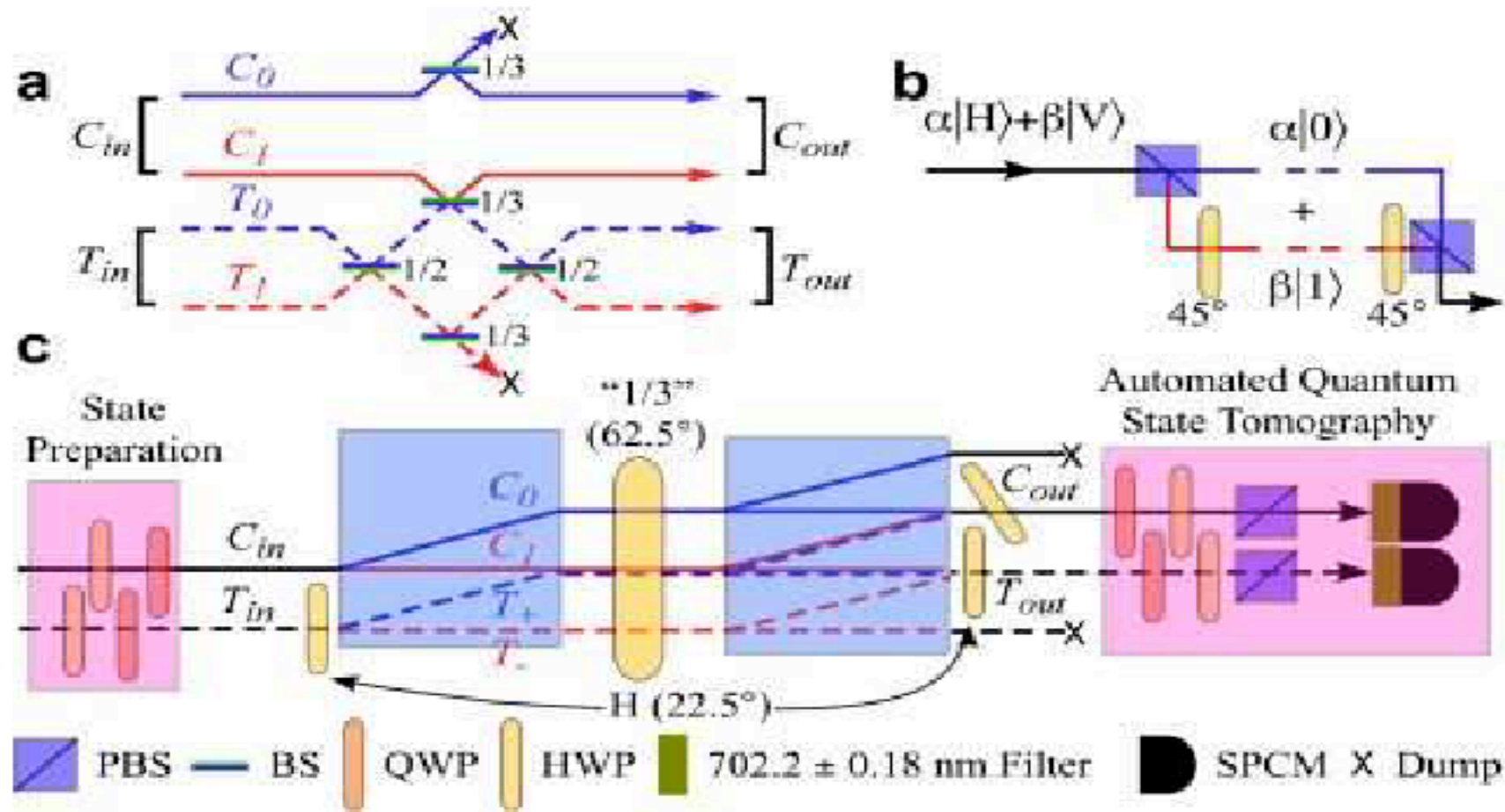


Actual realisation



Truth table  
Fidelity ~0.8

# Demonstration of an all-optical quantum controlled-NOT gate

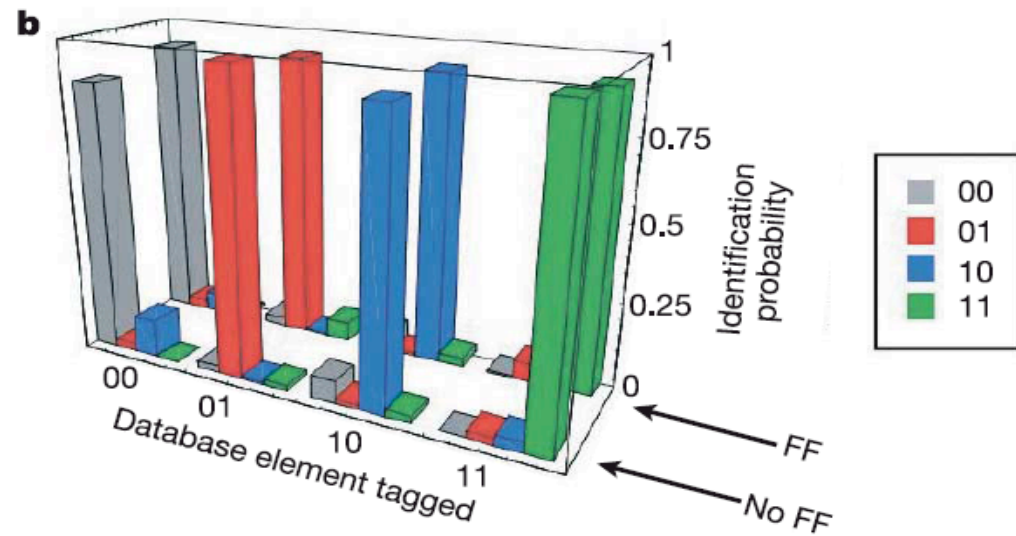
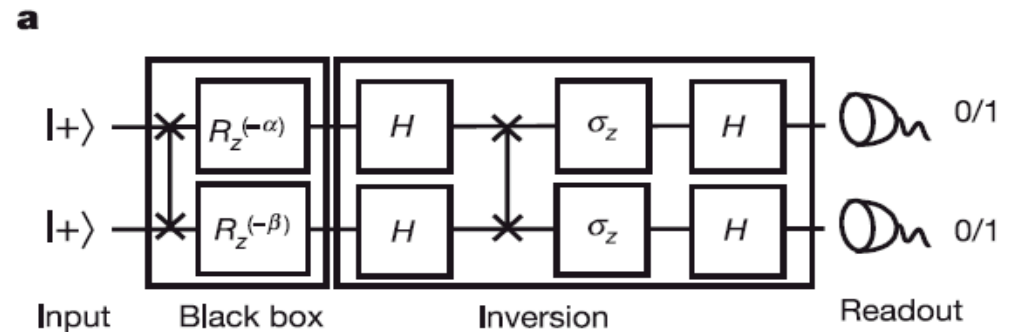
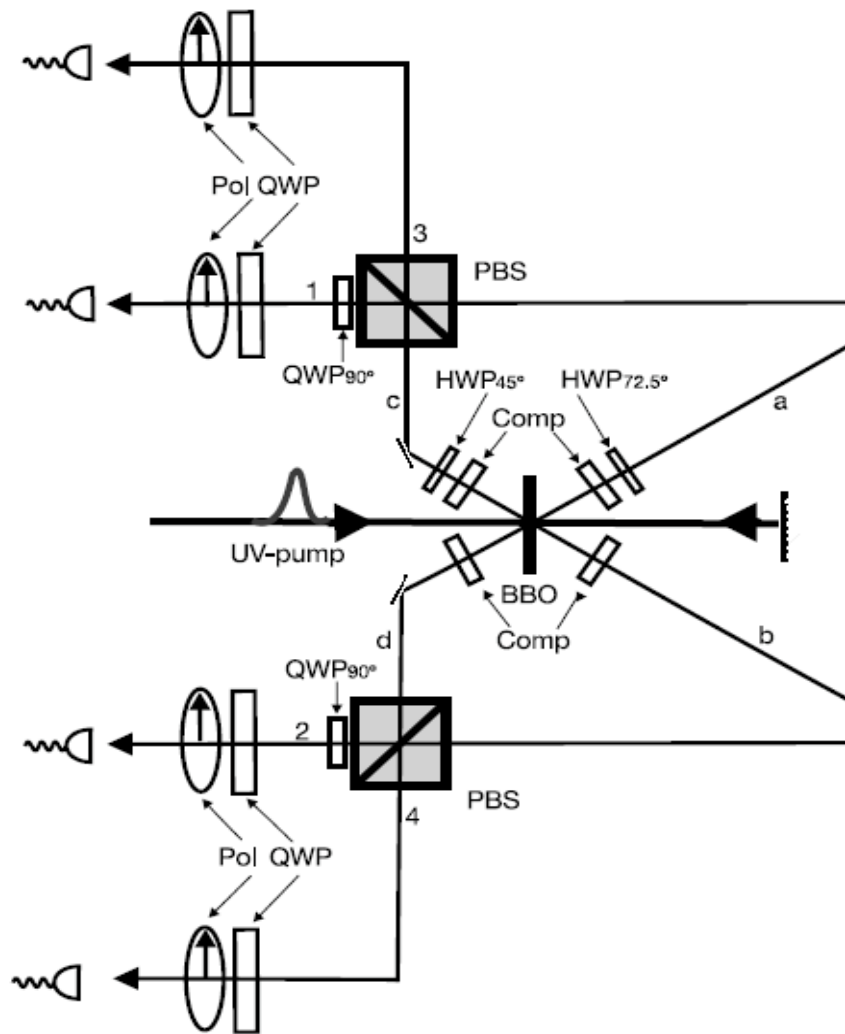


Knill et al Nature 409, 46–52 (2001)

J L O'Brien et al, **Nature** 426, 264 (2003) / quant-ph/0403062

# Optical Cluster State Computing

P. Walther et al Nature **434**, 169-176 (2005)



# Futures of optical quantum logic

- Higher numbers of entangled photons 6-fold entanglement in view
- Still require efficiency improvements in deterministic sources and detectors
- Weakly non-linear gates with enhanced efficiency and scalability
- Fully non-linear gates

17-Nov-2004

# Strong coupling of a single QD in 3D microcavity

## Cavity Quantum Electrodynamics (CQED)

Coupling of QD with Cavity

$$g = \sqrt{\frac{1}{V_{eff}} \frac{\hbar^2}{4\pi\epsilon_r\epsilon_0} \frac{\pi e^2 f}{m}}$$

Oscillator Strength

$$f = \frac{|\langle \Psi_{ex} | \hat{e} \cdot \hat{P} | 0 \rangle|^2}{m\hbar\omega/2} \approx 10 - 50 \quad \text{for Q-dots}$$

Broadening of QD emission  $\gamma_e$ : dephasing time and lifetime

Broadening of cavity mode

$$\gamma_c = \frac{\hbar\nu}{Q}$$

# Cavity Quantum Electrodynamics (CQED)

Mixed states of QD and photon

$$E_{\pm} = E_0 - i \frac{\gamma_e + \gamma_c}{4} \pm \sqrt{g^2 - \left( \frac{\gamma_e - \gamma_c}{4} \right)^2}$$

✿ Strong coupling (  $g > |\gamma_c - \gamma_e|/4$  )

Rabi oscillations — reversible spontaneous emission

Rabi splitting 
$$h\Omega = 2\sqrt{g^2 - \left( \frac{\gamma_e - \gamma_c}{4} \right)^2}$$

Applications in QIP

Single-photon switch

Exciton-photon entanglement

Micro-pillar: Nature 432, 197(2004)

PhC cavity: Nature 432, 200(2004)

Microdisk: PRL 95, 067401(2005)




# Cavity Quantum Electrodynamics (CQED)


❁ Weak coupling (  $g < |\gamma_c - \gamma_e|/4$  )

Enhanced spontaneous emission

$$\frac{\tau_{free}}{\tau_{cavity}} = \frac{3Q(\lambda/n)^3}{4\pi^2 V_{eff}} \frac{\Delta\omega_c^2}{4(\omega_e - \omega_c)^2 + \Delta\omega_c^2} \frac{|\vec{E}(\vec{r})|^2}{|\vec{E}_{max}|^2} \left( \frac{\vec{d} \cdot \vec{E}(\vec{r})}{|\vec{d}| |\vec{E}(\vec{r})|} \right)^2$$



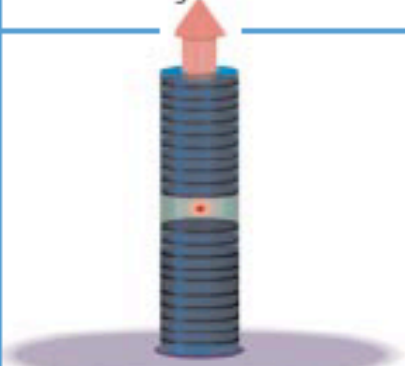
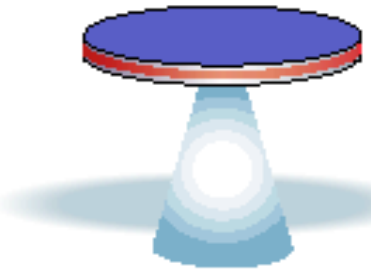
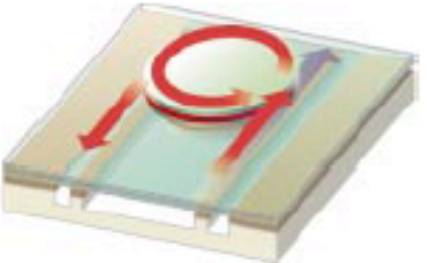
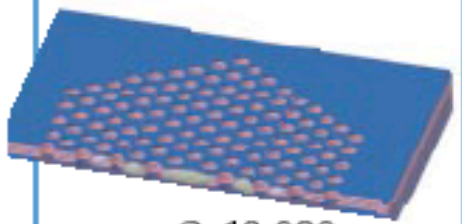
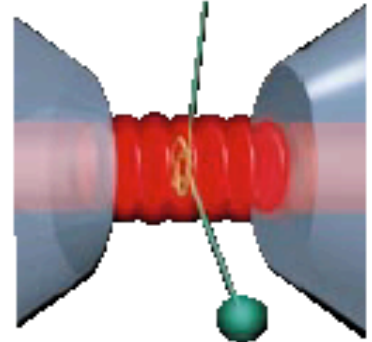
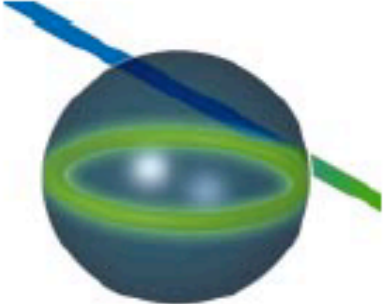

Purcell factor  $F_p$       detuning      position      orientation

Large  $\frac{Q}{V_{eff}}$   large  $F_p$

Improve efficiency of light emitter

# Typical 3D Optical Microcavities

K.J. Vahala, Nature 424, 839 (2003)

	Fabry-Perot	Whispering gallery	Photonic crystal
High $Q$	 <p><math>Q: 2,000</math> <math>V: 5 (\lambda/n)^3</math></p>	 <p><math>Q: 12,000</math> <math>V: 6 (\lambda/n)^3</math></p>  <p><math>Q_{III-V}: 7,000</math> <math>Q_{Poly}: 1.3 \times 10^5</math></p>	 <p><math>Q: 13,000</math> <math>V: 1.2 (\lambda/n)^3</math></p>
Ultrahigh $Q$	 <p><math>F: 4.8 \times 10^5</math> <math>V: 1,690 \mu\text{m}^3</math></p>	 <p><math>Q: 8 \times 10^9</math> <math>V: 3,000 \mu\text{m}^3</math></p>  <p><math>Q: 10^8</math></p>	

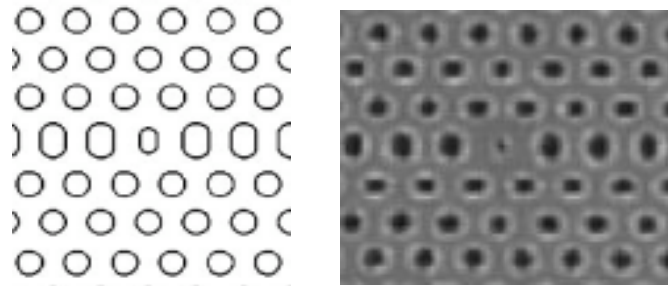
Theoretical limit of modal volume  $\sim 0.125 \times (\lambda/n)^3$

# PhC Slab

Taken from Johnson's  
lecture LEOS-05

## Cavities: $Q$ vs. $V$

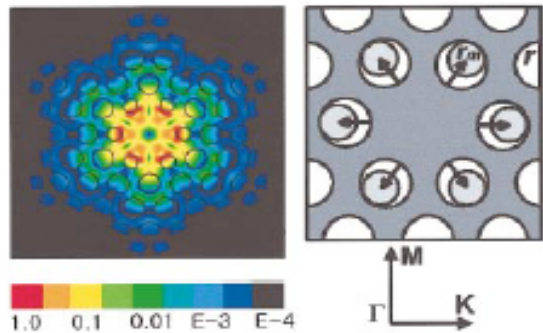
[ Loncar, *APL* **81**, 2680 (2002) ]



H1

$Q \sim 10,000$  ( $V \sim 4 \times \text{optimum}$ )  
 $= (l/2n)^3$

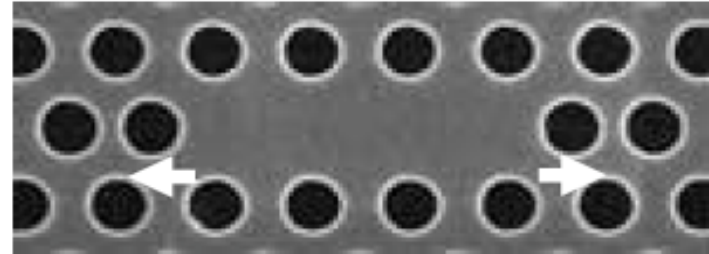
[ Ryu, *Opt. Lett.* **28**, 2390 (2003) ]



H1

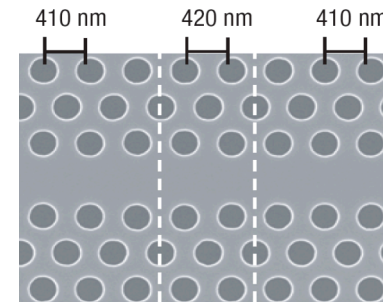
$Q \sim 10^6$  ( $V \sim 11 \times \text{optimum}$ )

[ Akahane, *Nature* **425**, 944 (2003) ]

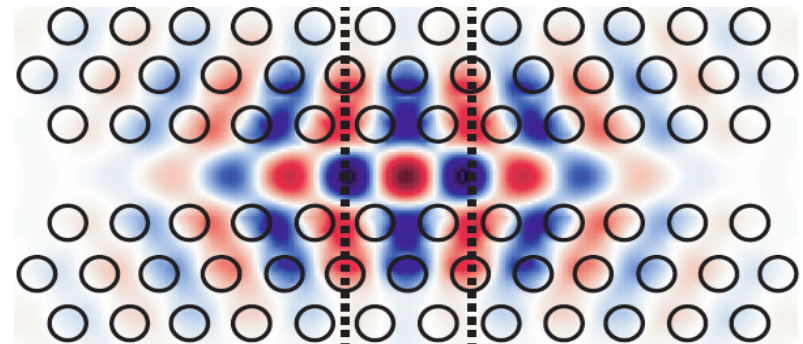


L3

$Q \sim 45,000$  ( $V \sim 6 \times \text{optimum}$ )

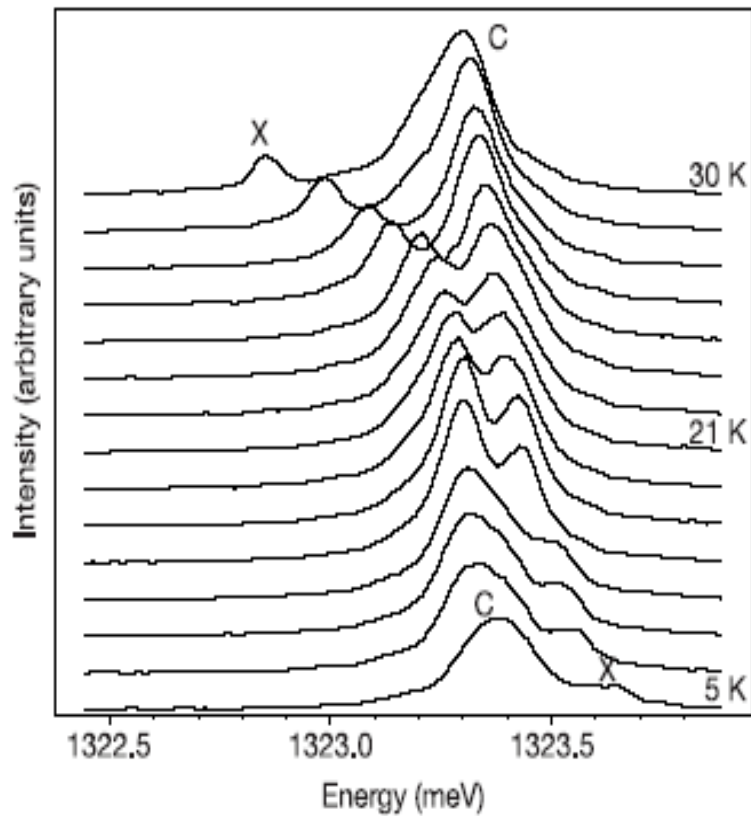


[ Song, *Nature Mat.*  
**4**, 207 (2005) ]



$Q \sim 600,000$  ( $V \sim 10 \times \text{optimum}$ )

## Forchel's group



QD: In<sub>0.3</sub>Ga<sub>0.7</sub>As QD of size 100nm×30nm

Oscillator strength:  $f=50$

Fabrication: circular micropillars by ICP

Pillar size:  $d_c = 1.5 \mu\text{m}$  with maximal  $Q/d_c$  selected

Q-factor: 8800

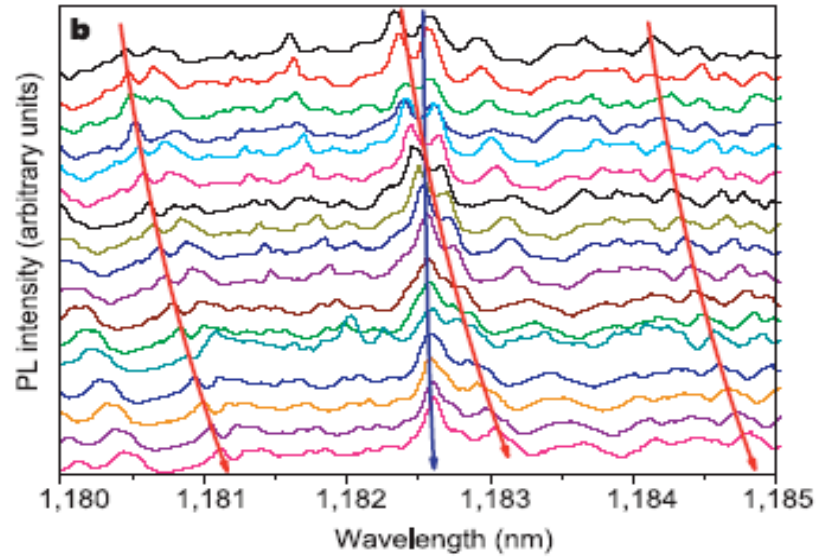
Mode volume:  $0.3 \mu\text{m}^3$

Coupling constant:  $g=0.08 \text{ meV}$

Vacuum Rabi splitting:  $\hbar\Omega=0.14 \text{ meV}$

Nature 432, 197(2004)

## Scherer's group



QD: conventional InAs QD, oscillator strength  $f=10$

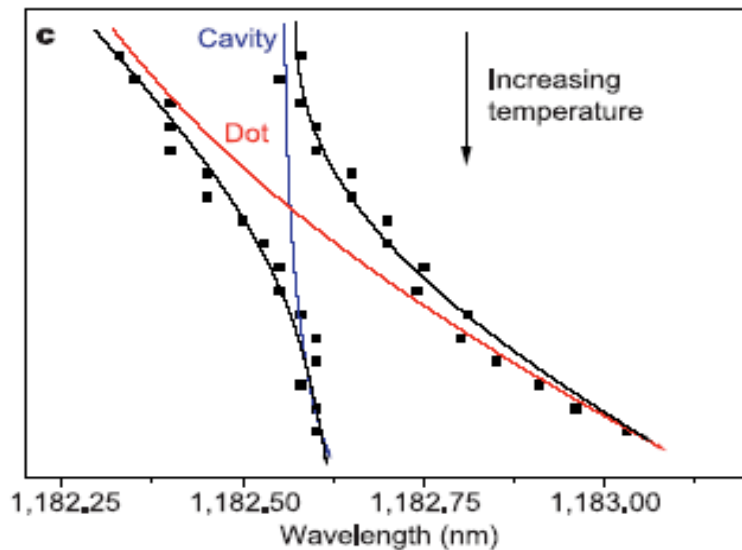
Fabrication: photonic crystal defect cavity by ICP

Q-factor: 13,300

Mode volume:  $0.04 \mu\text{m}^3$

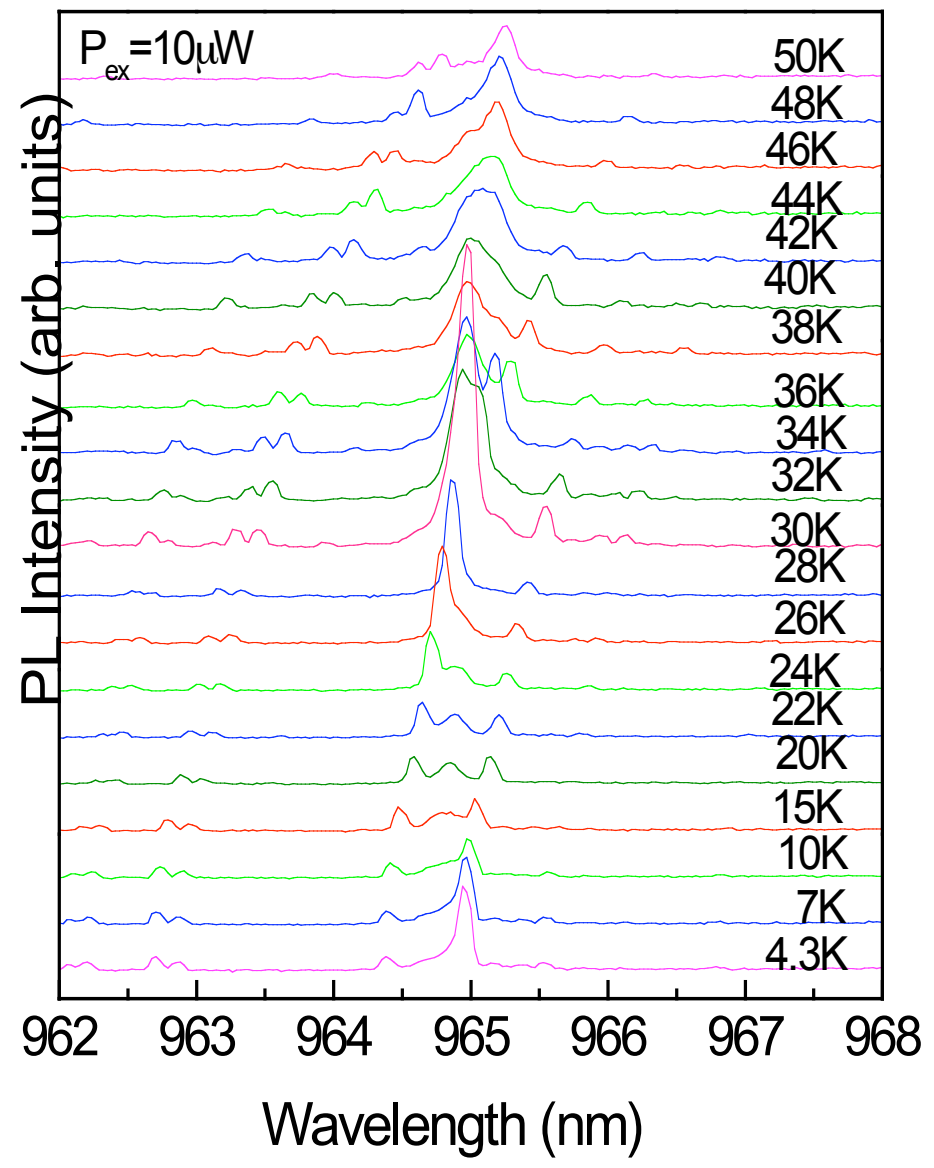
Coupling constant:  $g=0.085 \text{ meV}$

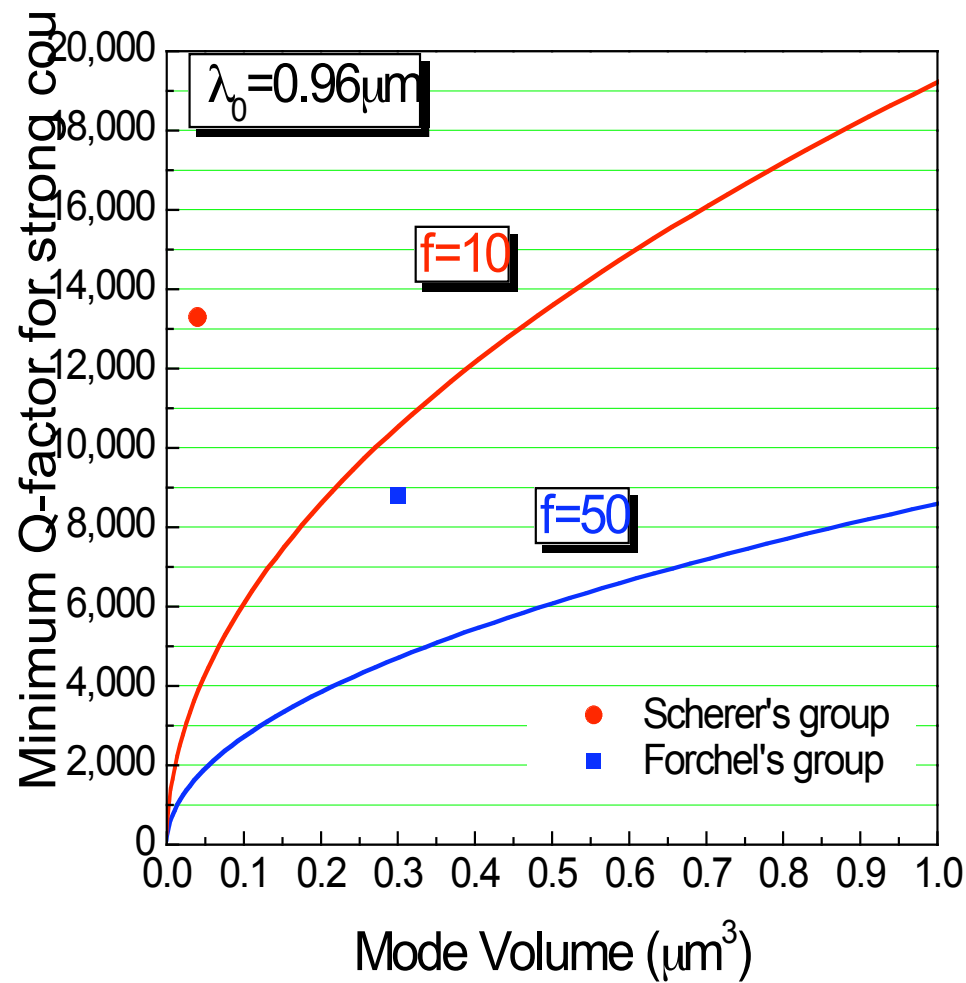
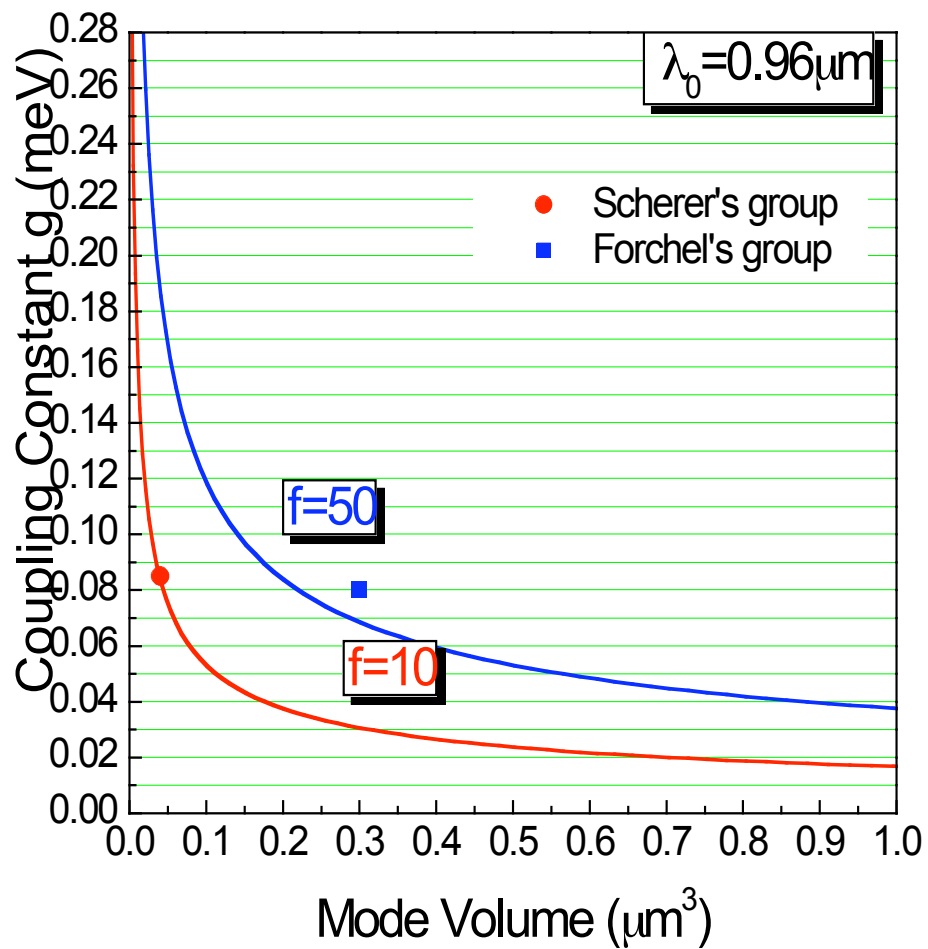
Vacuum Rabi splitting:  $\hbar\Omega=0.17 \text{ meV}$



Nature 432, 200(2004)

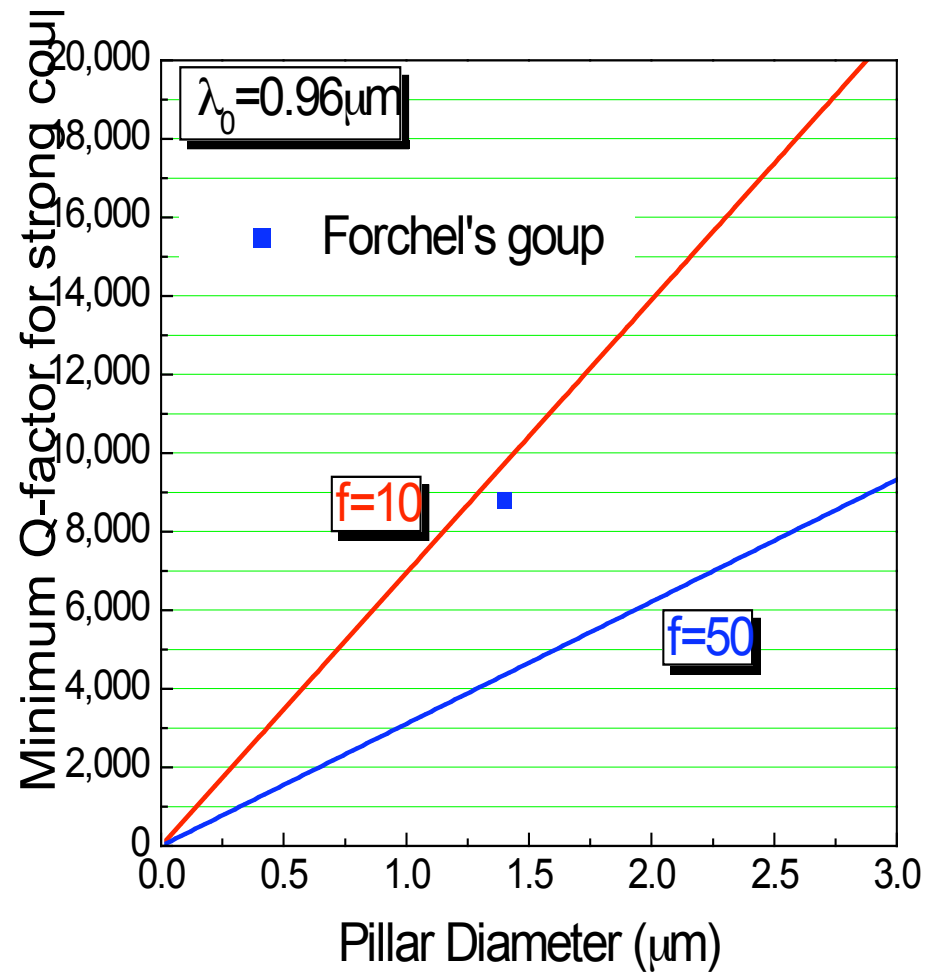
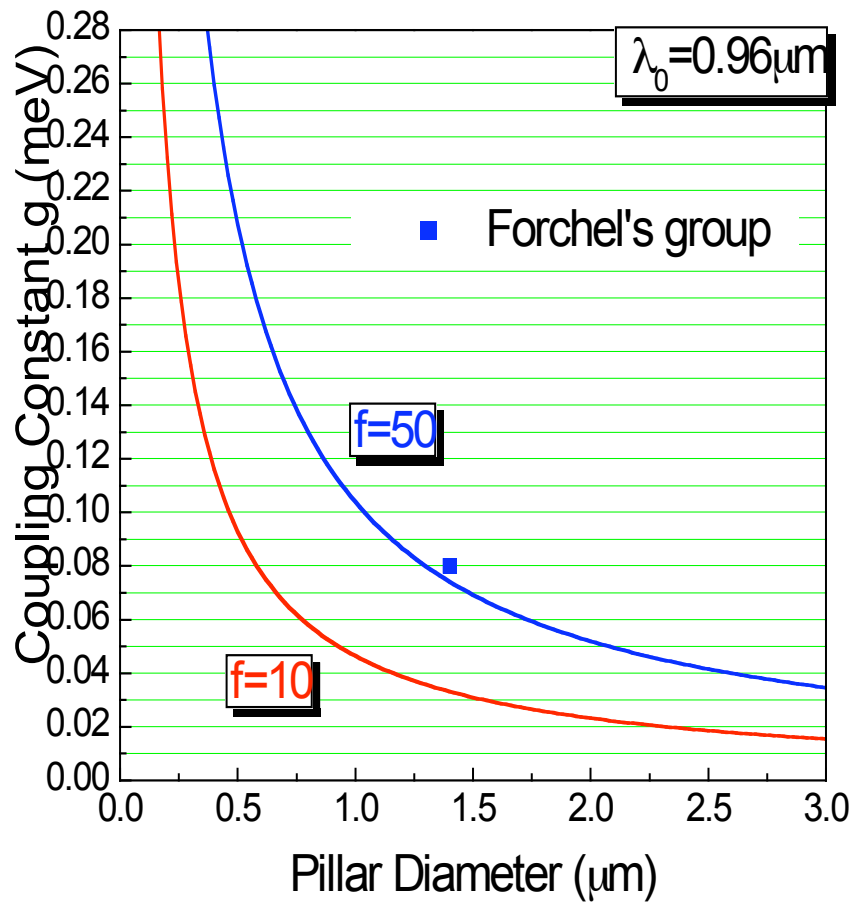
Sample VN87      3 $\mu\text{m}\times 3\mu\text{m}$





Condition for strong coupling:  $g > \Delta\omega_c/4 \Leftrightarrow Q > \omega/4g$

Condition for resolved vacuum Rabi splitting:  $g > 0.05 \text{ meV}$



Condition for strong coupling:  $g > \Delta\omega_c/4 \Leftrightarrow Q > \omega/4g$

Condition for resolved vacuum Rabi splitting:  $g > 0.05 \text{ meV}$



## Challenges for strong coupling of single QD

For smaller InAs QD with  $f=10$

1. Micropillars with diameter  $< 0.9 \mu\text{m}$ , Q factor  $> 8000$
2. Photonic crystal cavity with mode volume  $< 0.1 \mu\text{m}^3$ , Q factor  $> 8000$

For larger InGaAs QD with  $f=50$

3. Micropillars with diameter  $< 2.0 \mu\text{m}$ , Q factor  $> 6000$
4. Photonic crystal cavity with mode volume  $< 0.5 \mu\text{m}^3$ , Q factor  $> 6000$

other conditions:

QD situated at the field maxima of mode

to increase oscillator strength by increasing coherence volume of exciton

QD with large size ( $f \sim 50$ )

interface bound exciton ( $f \sim 100$ )

bound exciton