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Optical Communications and It's Quantum Limits

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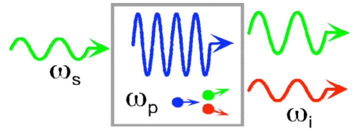

Optical Communications and It's Quantum Limits

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Winter College on
Quantum and Classical Aspects of Information Optics
ICTP, Trieste, Italy
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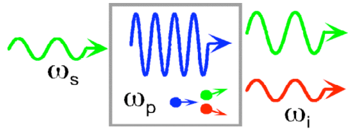
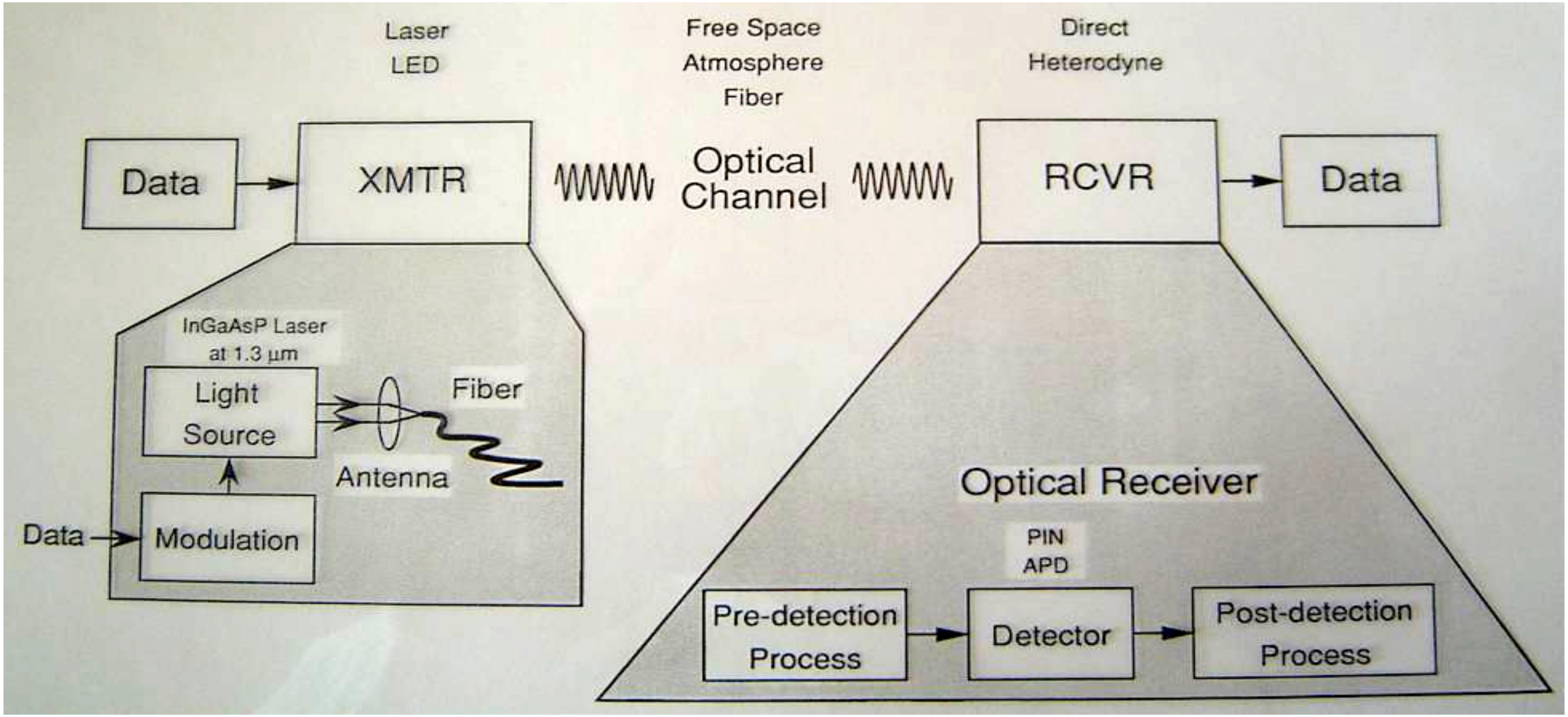
Topics for Lecture 1



- Modern optical transmitters
- Optical receivers,
- Performance criteria,
- Quantum limits on optical communications
- Linear communication systems
- Nonlinear systems

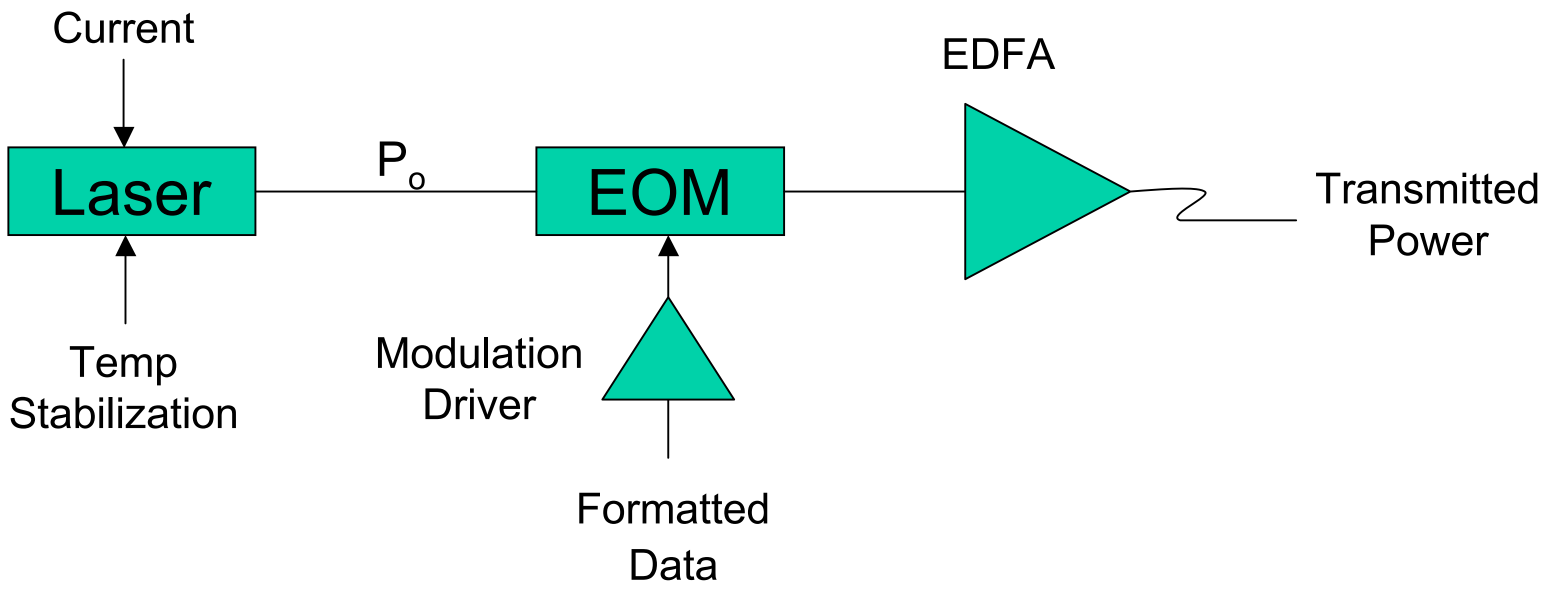
A Generic Optical Communication System



This is an old schematic, but now a days in most high-end fiber-optic systems the lasers are at 1.5 mm wavelength.

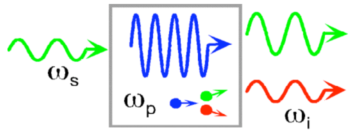
Modern Optical Transmitters

- Transmitter (XMTR) Block Diagram



- Data format depends on whether the communication is binary or M'ary

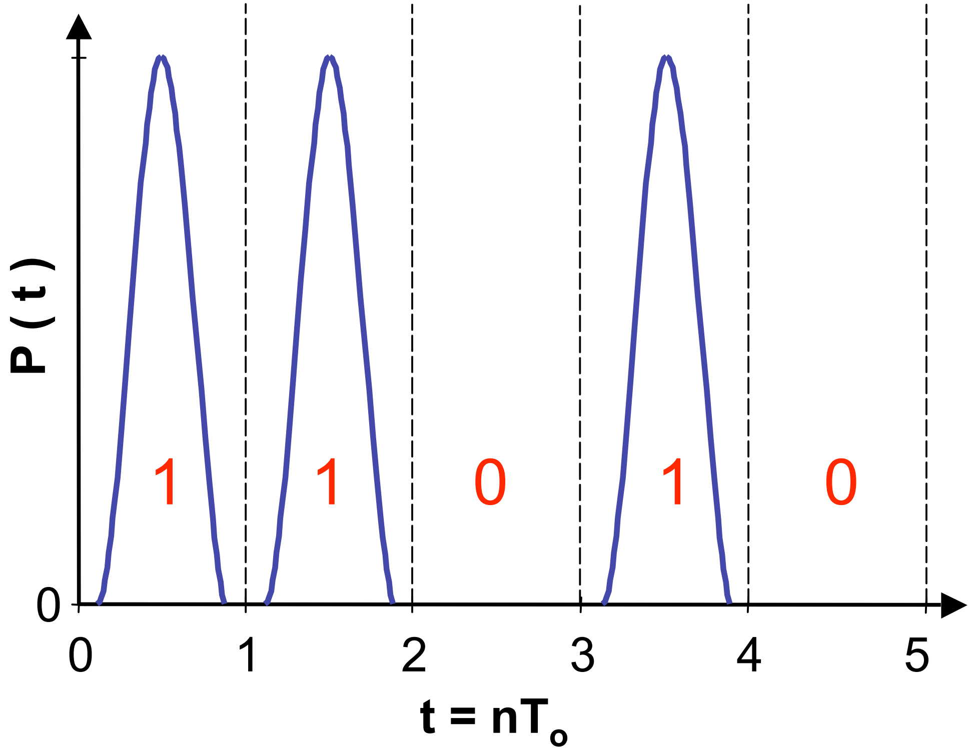
Binary vs. M’ary Data Formats



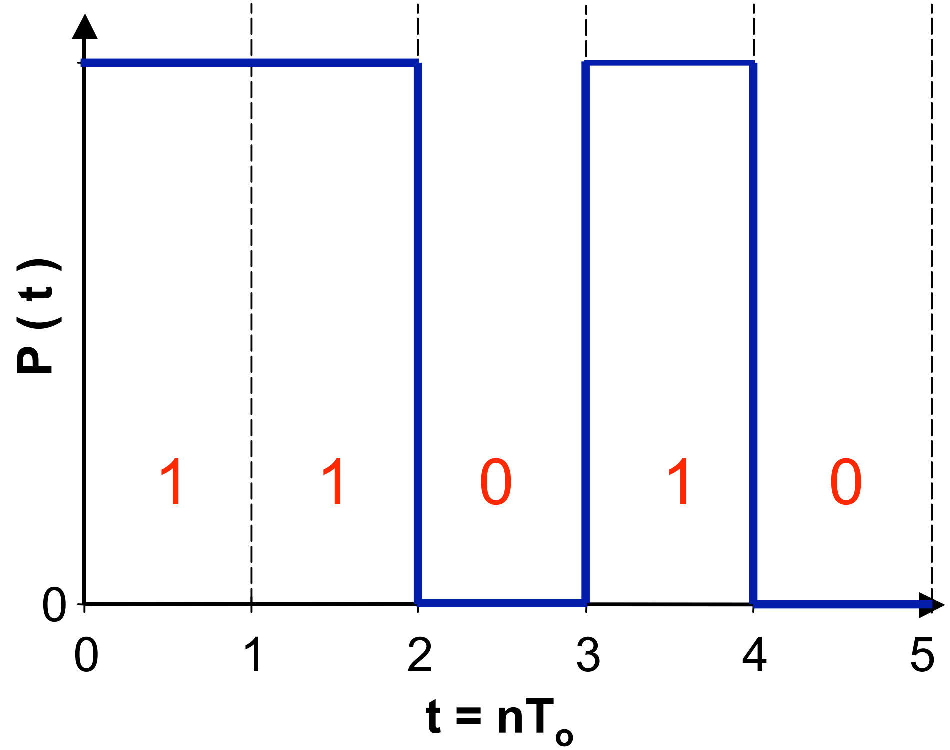
	Binary Communications	M’ary Communications
Decimal data	M = 1	M = 2
0	0000	00
1	0001	01
2	0010	02
3	0011	03
4	0100	10
5	0101	11
6	0110	12
7	0111	13
8	1000	20
9	1001	21
10	1010	22
11	1011	23
12	1100	30
13	1101	31
14	1110	32

More on Data Formats: On-Off Keying (OOK)

RZ — Return to Zero



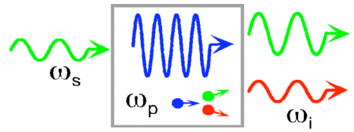
NRZ — Non-Return to Zero



T_o = Bit Period

- NRZ requires less bandwidth compared to RZ, and hence is preferred.

More on Data Formats: Phase-Shift Keying (PSK)



- Let the electric field of the lightwave be:

$$e(t) = \mathrm{Re}[\varepsilon(t) e^{-i\omega_0 t}]$$

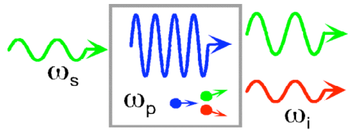
- Then the power carried by the lightwave is:

$$P(t) \propto |\varepsilon(t)|^2$$

- The amplitude and phase are defined through:

$$\varepsilon(t) = \sqrt{P(t)}\, e^{i\phi(t)}$$

amplitude (pointing to $\varepsilon(t)$) — phase



PSK vs. Differential PSK

- a) PSK:

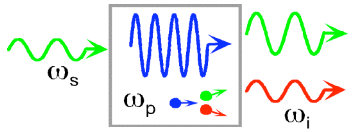
Data:	1	1	0	1	0	0
f =	π	π	0	π	0	0

- b) Differential PSK (DPSK)

Data:	0	1	1	0	1	0	0
f =	↑ Start string of 0s	π	0	π	π	π	0

* Some pre & post processing is needed for DPSK.

Summary of Modern Transmitters



- Basically, a modern transmitter emits a certain amount of power over a sequence of bit slots, which are encoded in the amplitude or the phase of laser light.

- The output of a well stabilized laser can be modeled as emitting a coherent state of light over the bit slot.

- # of photons/slot $= \dfrac{P_o T_o}{h\nu_o} \equiv n_o$

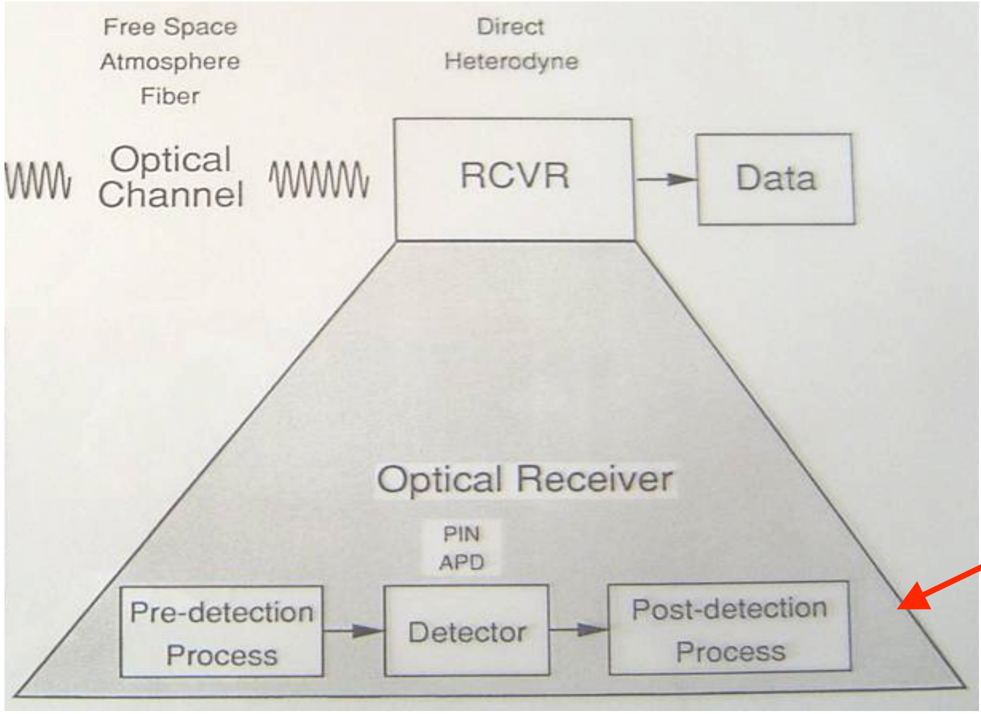
- In a coherent state, photons are randomly distributed with Poisson law. Therefore, $\Delta n = \sqrt{n_o}, \quad \text{with } \Delta n = \left[\langle (n - n_o)^2 \rangle\right]^{1/2}$

$$\Delta n \Delta \phi \;\square\; 1$$

- From the Heisenberg uncertainty relation:

- Thus photon number and phase intrinsically fluctuate

Modern Optical Receivers



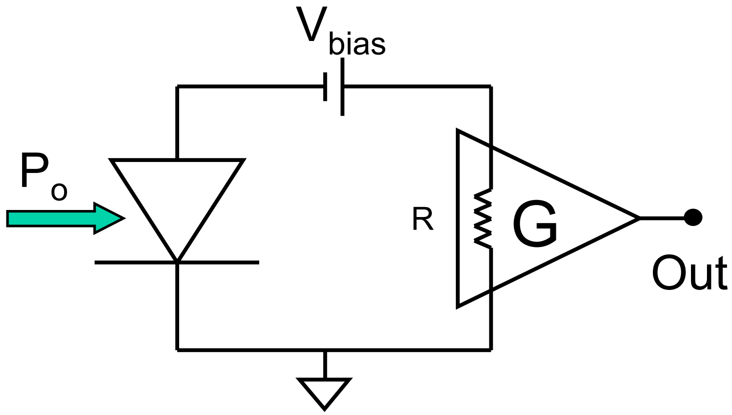
Mixing with a Local Oscillator (LO) as in Homodyne or Heterodyne detection, or delayed self-mixing as in a DPSK receiver

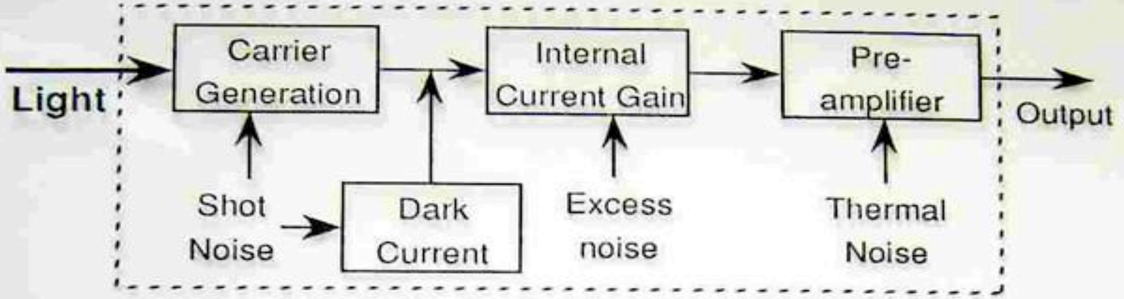
This entails processing of detected photo current or voltage in some fashion prior to making a bit decision.

For example, decisions are made based on measured values of two successive time slots in DPSK encoded systems.

Signal-to-Noise Model of a Detector

Receiver input end:





- Photocurrent due to received power:

$$i_r = \frac{\eta P_r e G}{h\nu_0}. \qquad (3)$$

- Shot Noise:

$$\langle i_n^2 \rangle = 2eG^2 i_r B. \qquad (4)$$

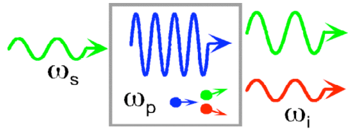
- Thermal Noise:

$$\langle i_{th}^2 \rangle = \frac{4k_B T B}{R}. \qquad (5)$$

The various symbols are defined on the next page.

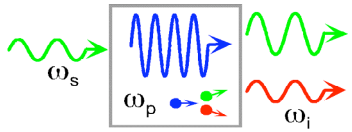
- Main Photodetector Types:
  Si for $0.4 - 0.85\,\mu$m and InGaAs for $1.3 - 1.6\,\mu$m.
  - PIN photodiode ($G = 1, \eta \simeq 0.9$),
  - avalanche photodiode or APD ($G \simeq 100, x \simeq 0.5, \eta \simeq 0.9$).

Various Definitions



P_r = received optical power,
P_b = background light on the detector,
B = electrical bandwidth of the receiver,
G = photodetector gain,
I_d = photodetector dark current,
η = quantum efficiency of the photodetector,
x = photodetector excess-noise factor,
$\nu_0 = c/\lambda$ = center frequency of light,
R = input impedance of the preamplifier,
T = photodetector temperature,
h = Planck's constant,
k_B = Boltzman's constant,
e = electron charge.

Direct Detection Carrier-to-Noise



The carrier-to-noise ratio (CNR) is given by

$$\mathrm{CNR} = \frac{(\eta P_r / h\nu_0 B)}{G^x[1 + (P_b + P_d)/P_r] + P_{\mathrm{th}}/P_r}, \qquad (6)$$

where the equivalent dark and thermal-noise powers are defined as

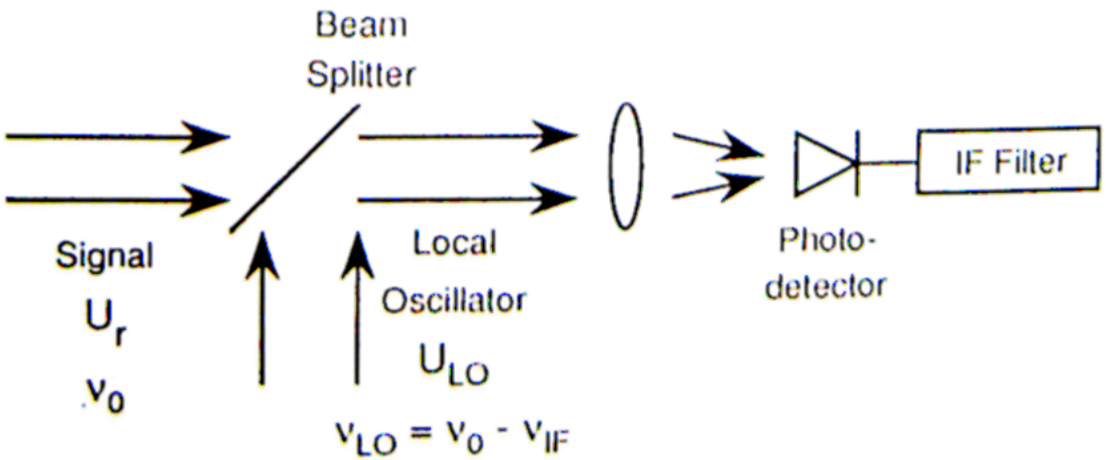
$$P_d \equiv \frac{h\nu_0}{\eta e G} I_d, \qquad (7)$$

$$P_{\mathrm{th}} \equiv \frac{2 k_B T h\nu_0}{R e^2 G^2 \eta P_r}, \qquad (8)$$

- Quantum Limited CNR $= \dfrac{P_r}{h\nu_o B} = \dfrac{P_r T_o}{h\nu_o} \equiv n_o\,, \quad B = 1/T_o$

Heterodyne / Homodyne Detection

It is possible to obtain quantum-limited CNR without having large P_r or G. A local oscillator (LO) at the receiver provides the mixing gain.

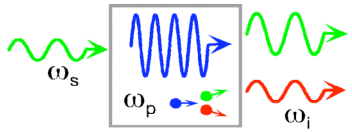


$$P_{\text{det}} = \left|U_r + U_{\text{LO}} e^{-i\omega_{\text{IF}} t}\right|^2 = \left|U_r\right|^2 + \left|U_{\text{LO}}\right|^2 + 2U_r U_{\text{LO}} \cos\left(\omega_{\text{IF}} t + \phi\right)$$

$$= P_r + P_{\text{LO}} + 2\sqrt{P_r P_{\text{LO}}} \cos\left(\omega_{\text{IF}} t + \phi\right)$$

A radio-frequency filter in the receiver separates the IF photocurrent

Quantum Limited Detection w/ Heterodyne



- **Mode-Matching Efficiency (MME):**

  Overlap of the signal and LO wave fronts at the photodetector is determined by the MME defined as

$$\begin{aligned} M &= \iint_{A_d} dx\, dy\, U_r(x,y)\, U_{LO}^*(x,y) \\ &\leq 1. \end{aligned} \tag{11}$$

- **Heterodyne CNR:**

  For $P_{LO} \gg P_r$, $P_b + P_d$, and P_{th}, the heterodyne CNR is given by

$$\text{SNR}_{\text{het}} = \frac{\eta P_r}{h\nu_0} \frac{1}{B}. \tag{12}$$

  Thus, heterodyne detection achieves the quantum limited CNR. The price one pays is in the complexity of the system. A mode-matched LO is needed at the receiver.

Performance Criteria: Bit-Error Rate

$$\text{BER} = \text{EP} \times \text{BR} \qquad (13)$$

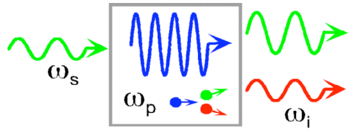
where EP = Error Probability, and BR = Bit Rate. Generally,

$$\text{EP} \simeq e^{-\text{SNR}}. \qquad (14)$$

The actual EP depends upon the detection scheme and the modulation format.

$$\text{EP} = \Pr(0) \times \Pr(1\,|\,0) + \Pr(1) \times \Pr(0\,|\,1)$$

$$= \frac{1}{2}\left[\Pr(1\,|\,0) + \Pr(0\,|\,1)\right], \quad \text{for equally-likely binary}$$



Quantum-Limited Sensitivities

– Using OOK and direct detection,

$$\begin{aligned} \mathrm{BER} &= \mathrm{BR} \times e^{-\eta P_r T / h\nu_0} \\ &= \mathrm{BR} \times e^{-n_r}. \end{aligned} \qquad (15)$$

Here T is the bit duration and n_r is the number of photons detected during the bit duration. *A minimum of* 21 *photons is needed to achieve an error probability of* 10^{-9}.

– Using OOK and heterodyne detection,

$$\begin{aligned} \mathrm{BER} &= \mathrm{BR} \times e^{-\eta P_r T / 4h\nu_0} \\ &= \mathrm{BR} \times e^{-n_r/4}. \end{aligned} \qquad (16)$$

In this case, a minimum of 84 *photons is needed to achieve an error probability of* 10^{-9}.

Modern Optical Transport Systems

- Linear systems employing erbium-doped fiber amplifiers
  - Many wavelength channels are multiplexed
  - ITU frequency/wavelength grid defines channel spacing of 100 GHz, although many systems run channels 50 GHz apart
  - 10 Gbps data rate on each channel, one fiber can carry several terabits per second of data
  - Data signals are periodically amplified to overcome loss due to propagation in the fiber (generally every 80 km or so)
  - ASE (amplified spontaneous emission) from the amplifiers is equivalent to background light on the receiver
  - In multiplexed systems, fiber nonlinearity causes cross-talk penalties, and sets a limit on how far signals can propagate
  - 5 Mm terrestrial systems and 10 Mm submarine systems have been demonstrated