The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1738 - 27

WINTER COLLEGE

on

QUANTUM AND CLASSICAL ASPECTS

of

INFORMATION OPTICS

30 January - 10 February 2006

AMO realizations

Drawing on work from the EU QBITS and QGATES Networks

(First Part)

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Lecture 1: AMO realizations
Trieste Winter School, February 2006

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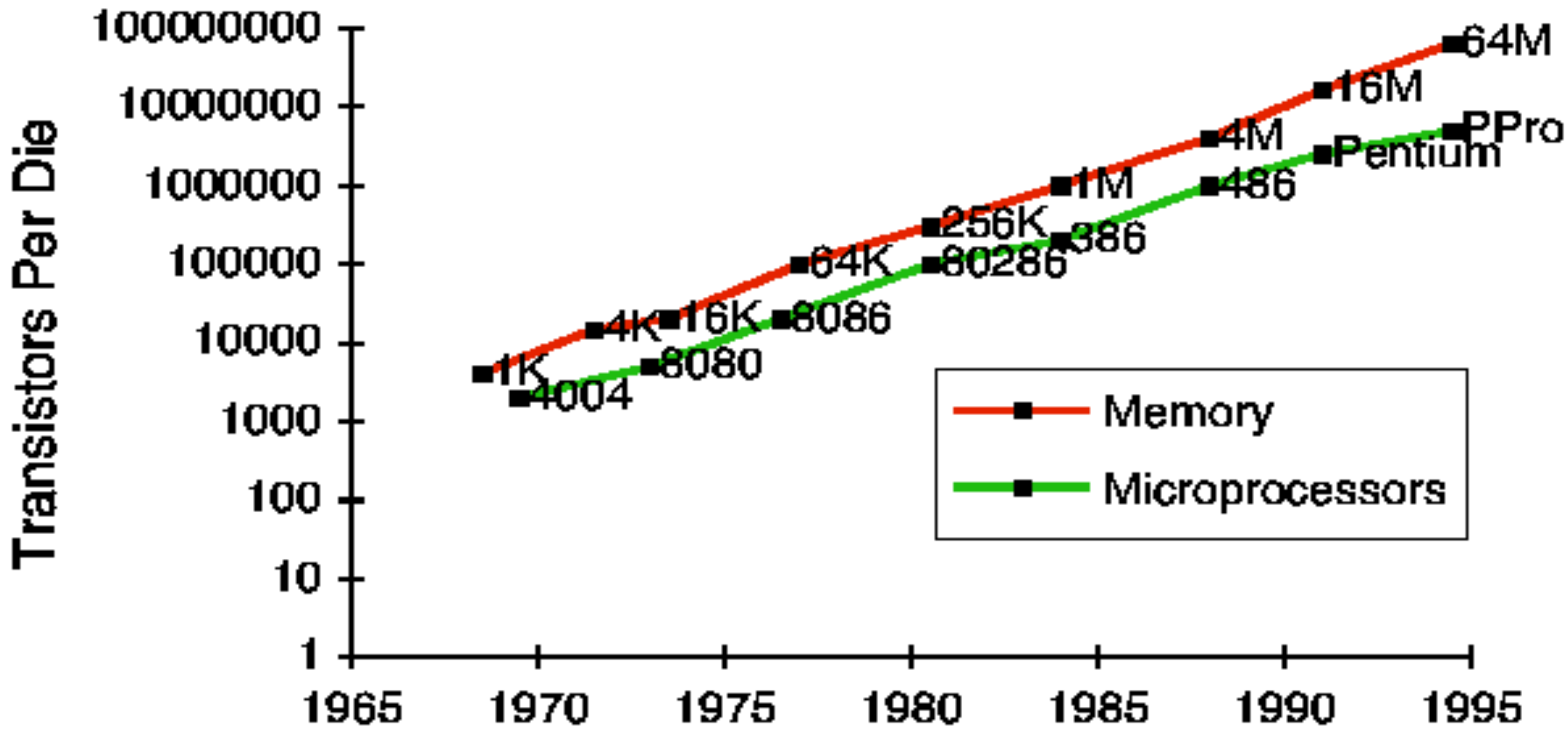
Drawing on work from the EU QBITS and QGATES Networks

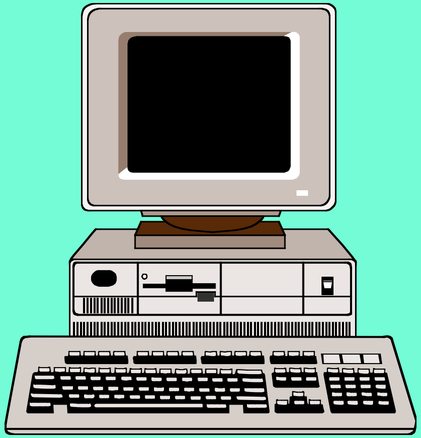
Coverage in this set of lectures

- Section 1
  - Basics of quantum gates etc
  - AMO realizations
  - DiVincenzo criteria
- Section 2: Ions
- Section 3: atoms, lattices and chips
  - Cold atoms
  - Optical lattices
  - Mott transition
  - Atom chips and decoherence



Moore’s Law: Growth in chips and shrinking space. What when/if get to one electron/gate?

Computation = physical process



Hardware obeys the laws of physics-
but nature is quantum mechanical

So what would a quantum computer
look like?

“Computers of the future may weigh no more than 1.5 tons”

Popular mechanics, 1949!

Quantum Computing History

Initial Ideas - *quantum more powerful than classical*

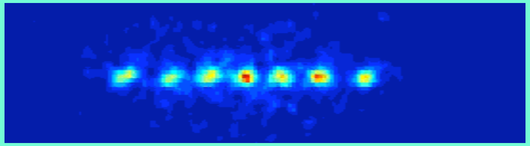
Benioff - 1982, Feynman - 1984

Quantum Parallelism - *oracles, Hadamards...*

Deutsch-Jozsa (92)/ Bernstein-Vazirani (93) / Simon (93)

Quantum Factoring- *explosion of interest*

Shor (94)



Implementations- *hardware, gates, decoherence*

Cirac-Zoller (94)

Wineland, Kimble, Haroche, Hughes, Blatt,….

Error Correction- *the conquest of decoherence*

Shor, Steane

David Deutsch

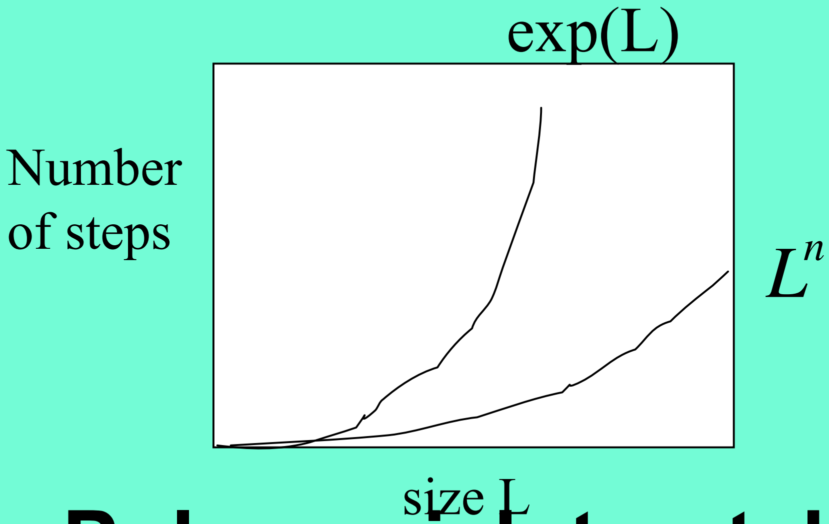
Peter Shor

Andy Steane

Pioneers:

1. ***Deutsch: basic ideas, parallelism, first quantum algorithms***
2. ***Shor: factorisation algorithm, error correction***
3. ***Steane: error correction***

Complexity and tractable problems



I/P size ~ amount of info in bits needed to specify I/P

Then evaluate number of steps needed as f(size)

- **Polynomial: tractable, complexity class "P"**
- **Non-polynomial: difficult to prove, easy to verify, complexity class NP**
- **Exponential: intractable, complexity class E**

 → QM changes this

Qubits & Quantum Registers

Classical Bit

0 or 1

Quantum Bit

0 or 1 or 0 1

Classical register

101

Quantum register

000 001 010 011
100 101 110 111

Quantum Logic I

Define a quantum XOR => Quantum **CNOT** gate

State 1	State 2	Out 1	Out 2
|0>	|0>	|0>	|0>
|0>	|1>	|0>	|1>
|1>	|0>	|1>	|1>
|1>	|1>	|1>	|0>

Looks the same as before!

Differences?

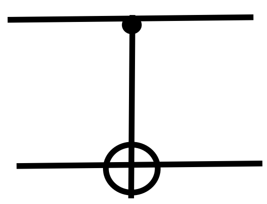
↓

Basic input |0> and |1> unit called **qubit**

Quantum Mechanics allows for **superpositions** of states!

Map superpositions of states into entangled states!

(|0> + |1>) |0> → |00> + |11>



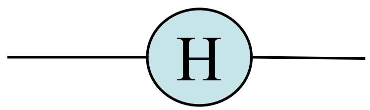
Quantum Logic II

We need gates that make quantum superpositions.

The Hadamard gate

$H|0\rangle \rightarrow (|0\rangle + |1\rangle) / \sqrt{2}$

$H|1\rangle \rightarrow (|0\rangle - |1\rangle) / \sqrt{2}$



General single qubit rotations

$|0\rangle \longrightarrow \cos x\, |0\rangle + \exp(iy) \sin x\, |1\rangle$

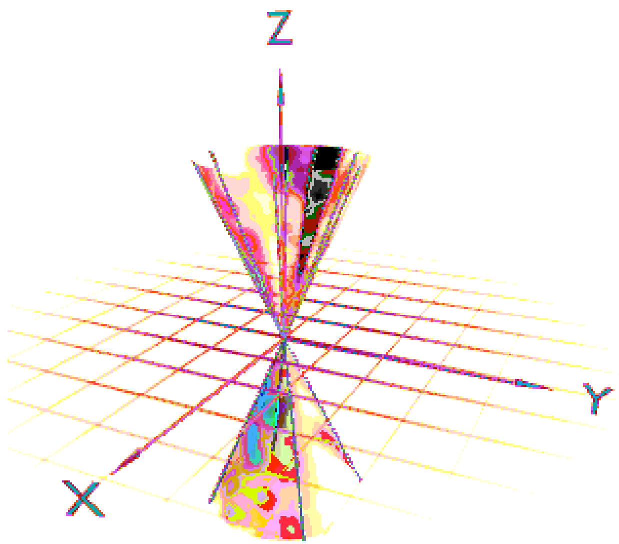
$|1\rangle \longrightarrow -\sin x\, |0\rangle + \exp(-iy) \cos x\, |1\rangle$

Quantum Logic I: one-bit Hadamard Gates

Consider a k-bit string: $|0\rangle|0\rangle\ldots\ldots|0\rangle$

Apply one bit (Hadamard) rotation S to each bit

$$S = (1/\sqrt{2})\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \Rightarrow$$



$$(2^{-k/2})(|0\rangle+|1\rangle)(|0\rangle+|1\rangle)\ldots.(|0\rangle+|1\rangle)$$

$$= (2^{-k/2})\sum_i |i\rangle \quad \text{sum over all} \quad 2^k \quad \text{k-bit strings}$$

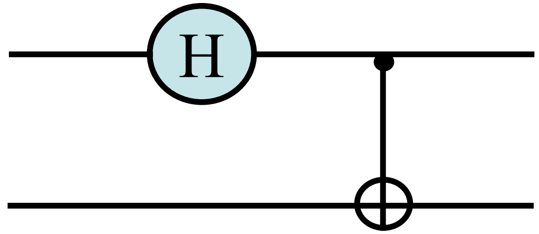
But entangle? 2 qubit tensor product not same as two classical strings each in superposition

Quantum Logic III

Make entanglement

$|0\rangle$

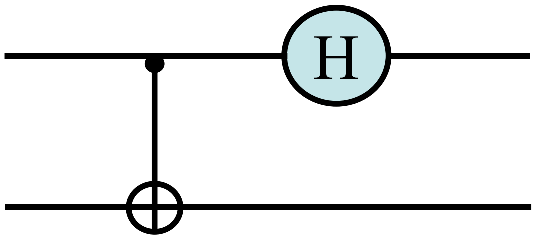
$|0\rangle$



$|00\rangle + |11\rangle$

Measure entanglement

$|00\rangle + |11\rangle$



$|0\rangle$

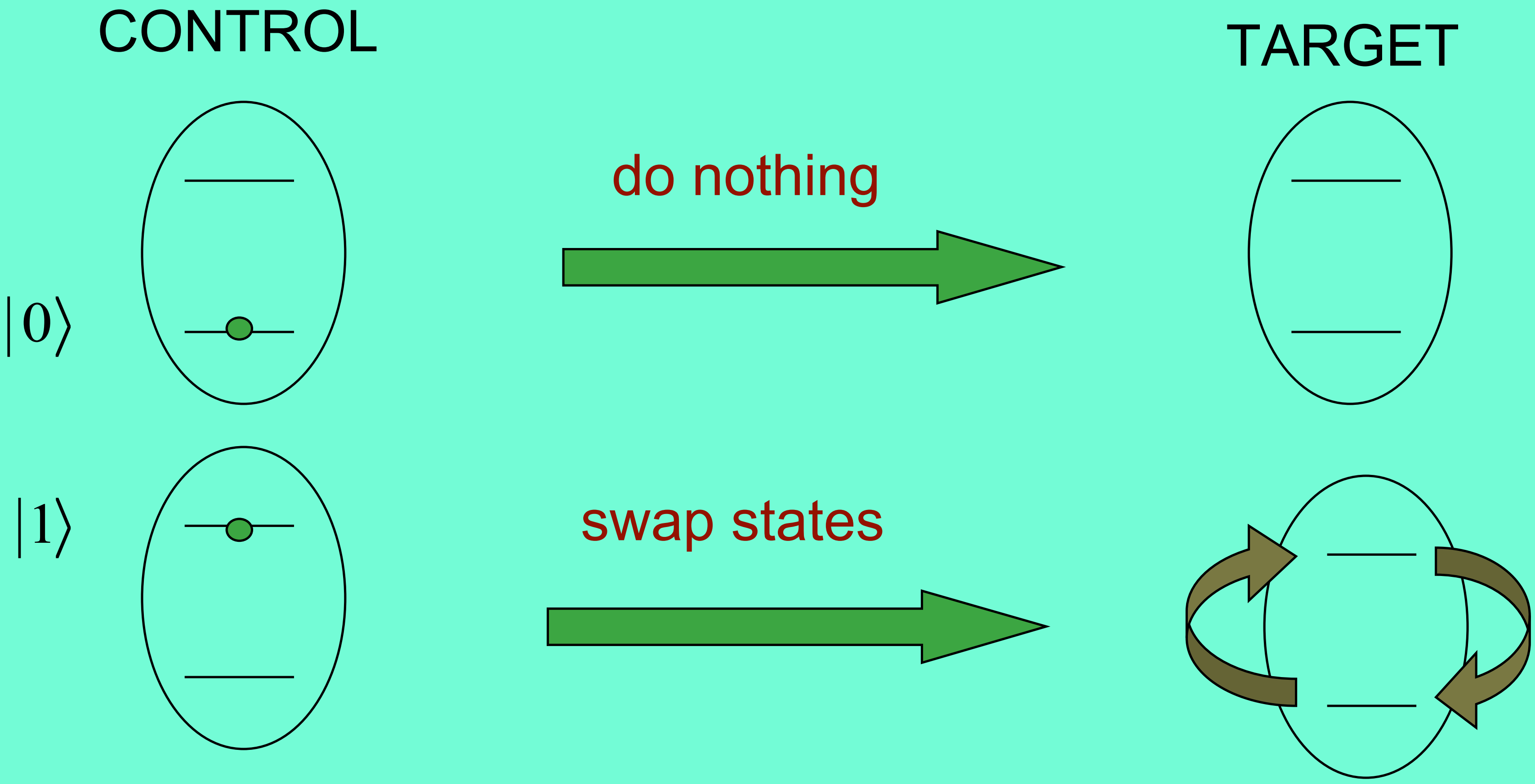
$|0\rangle$

$$|00\rangle - |11\rangle \mapsto |1\rangle|0\rangle$$

$$|01\rangle + |10\rangle \mapsto |0\rangle|1\rangle$$

$$|01\rangle - |10\rangle \mapsto |1\rangle|1\rangle$$

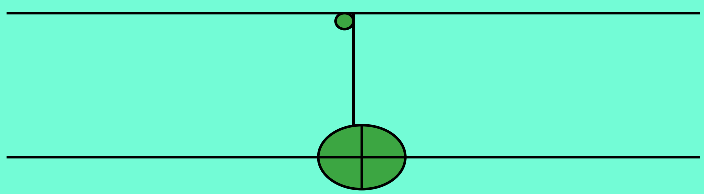
Controlled NOT



$$|0><0|\otimes 1+|1><1|\otimes\sigma_x$$

Controlled NOT as entangler

A



A control

B

$A \otimes B$ target

Superpositions → entangled states

$$(a|0\rangle + b|1\rangle)|0\rangle \Rightarrow a|00\rangle + b|11\rangle$$

Truth table for CNOT

$\|00\rangle$		$\|00\rangle$
$\|01\rangle$	→ CNOT	$\|01\rangle$
$\|10\rangle$		$\|11\rangle$
$\|11\rangle$		$\|10\rangle$

3 CNOTs swap

Quantum Logic IV

Want a quantum processor to compute function F(x)!

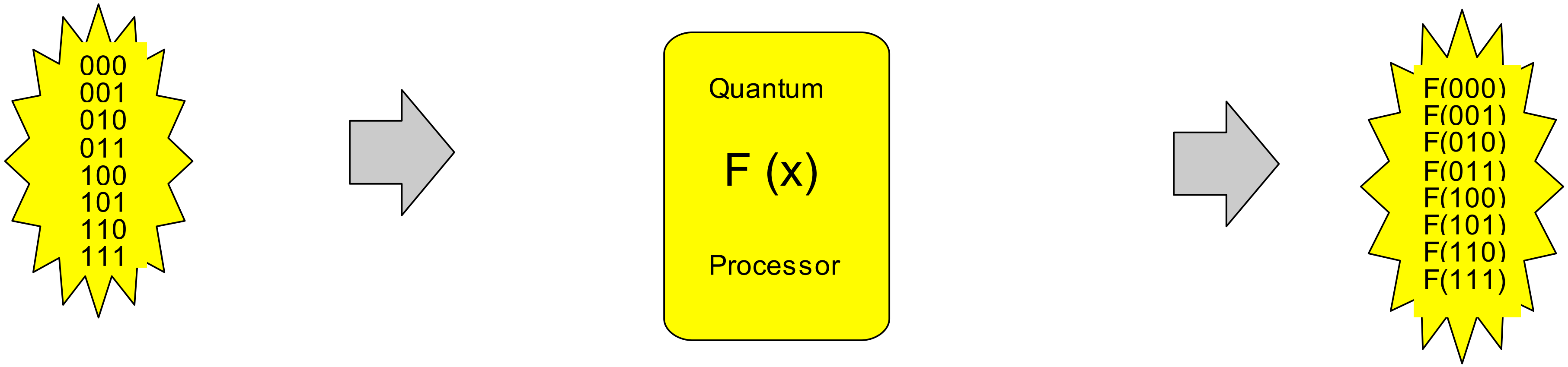
Have state |x> of many qubits which represents number x in binary notation (Example: |1>|1>|1> = |111> represents 7!)

Wish U|x> = |F(x)> ! BUT that's not always unitary.

Make Computation reversible:

$$|x\rangle|0\rangle \Longleftrightarrow |x\rangle|F(x)\rangle$$

There is a unitary operation that implements this for any x!



Reversible Quantum Computation

*classical

$$0 \rightarrow f(0); 1 \rightarrow f(1)$$

$$x = 00001001$$

$$and$$

$$f(x) = x + 1$$

$$\Rightarrow x = 00001010$$

one input

one computation

one result

*quantum

Computation is unitary, reversible

$$|x> \neq |f(x)>$$

:if f is not a 1:1 mapping, ie if f(x)=f(y) for some x not equal to y, then 2 orthogonal kets |x>, |y> can be evolved into same ket |f(x)>=|f(y)> and so violate unitarity! Need 2 registers!

Dual registers and possible outputs

Each input x ⇒ |x>, quantum state of first register

Each possible output y=f(x) ⇒ |y>, quantum state of 2nd register

Function evaluation determined by unitary U acting on both registers

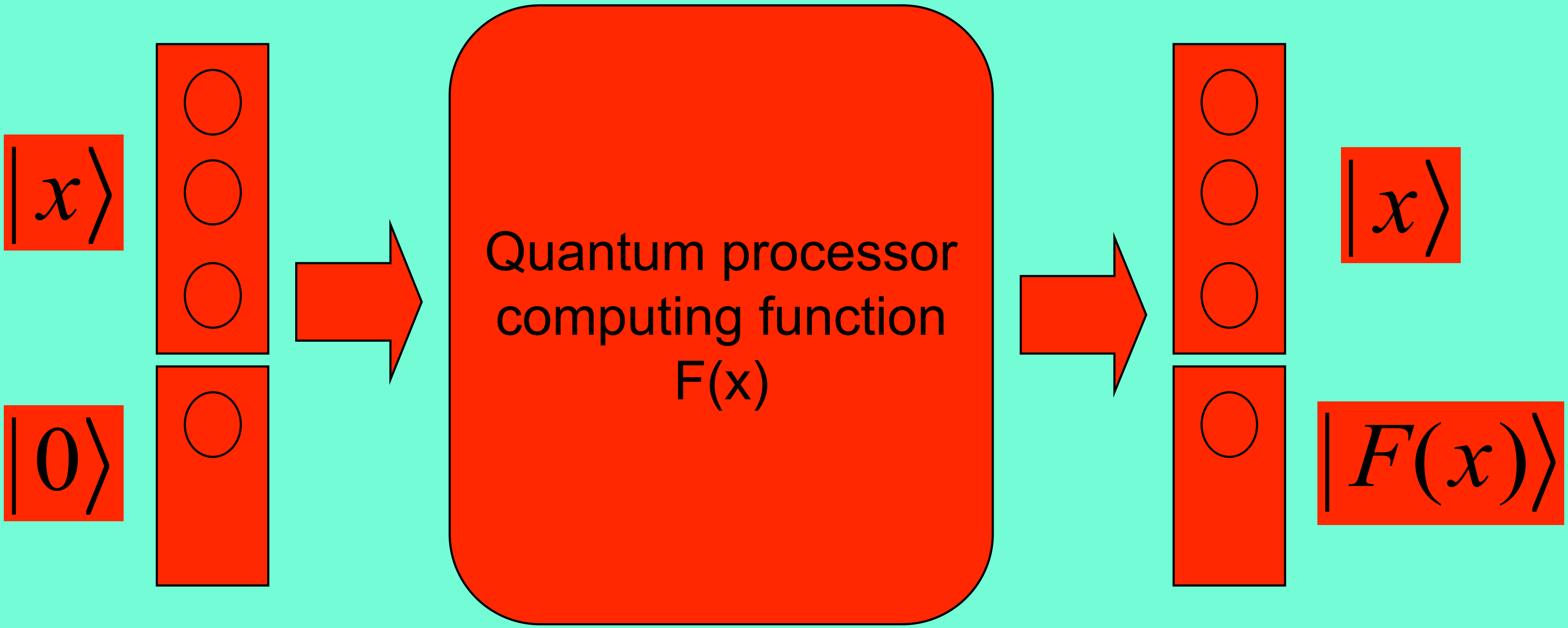
$$U_f |x>|0> = |x>|f(x)>$$

Eg |00>+|01>+… f(x)=x+1 |01>+|10>+...

many inputs — one computn — many results

Prepare superposition of inputs, run computation U just once and get all 2^m output values f(0), f(1), ….f(2^m-1): but can you read them all? One measurement: look for global property...

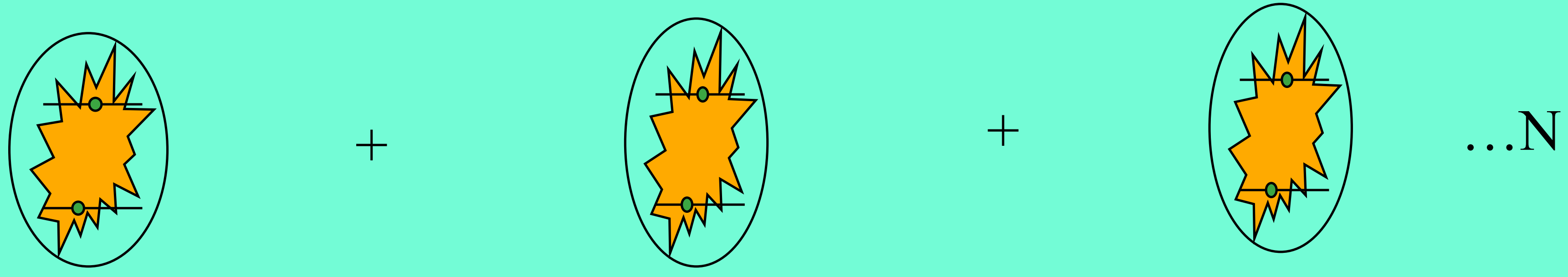
Quantum Computation



$$|\psi_{IN} >= \sum_x | x >|0 > \Rightarrow | \psi_{OUT} >= \sum_x | x >| F(x) >$$

Qubits and resources

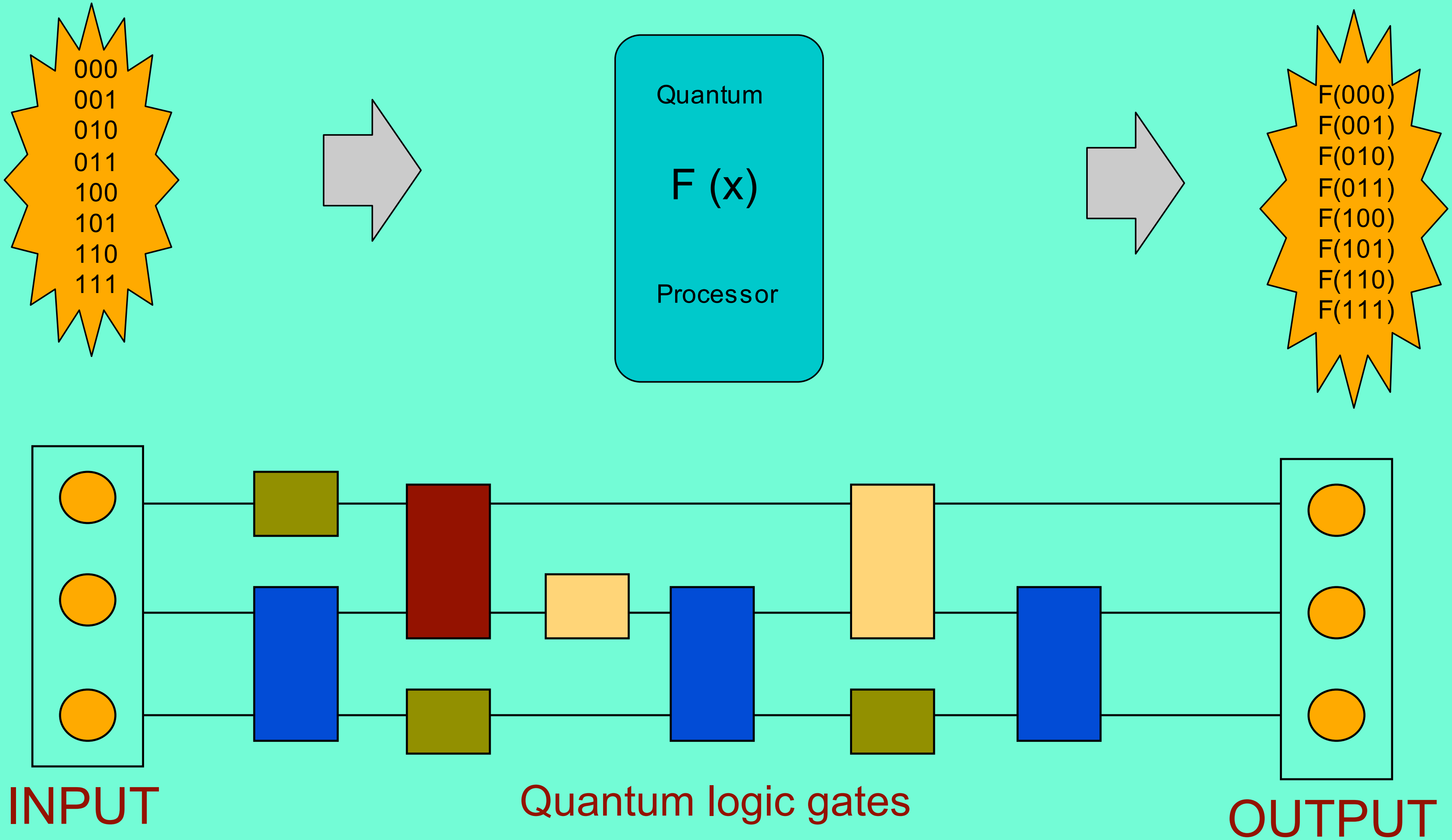
N 2-level systems:uses nE energy in 2^n states



N-level system (unary representation) can also have 2^n states but needs $2^n E$ energy and cannot always get fast access (eg sho……)

See Seth Lloyd quant-ph/9903057 for discussion of role of entanglement. Also recent wavepacket experiments of Bucksbaum..

Quantum Networks



Applications &Algorithms

1 Deutsch-Jozsa Algorithm

2 Shor Factorization

3 Grover Search

4 Lloyd/Feynman Quantum Simulation

5 Wineland/Huelga Frequency Standards

Factoring and Data Security

- Factoring number N of L digits takes time ~ $10^{\sqrt{L}}$

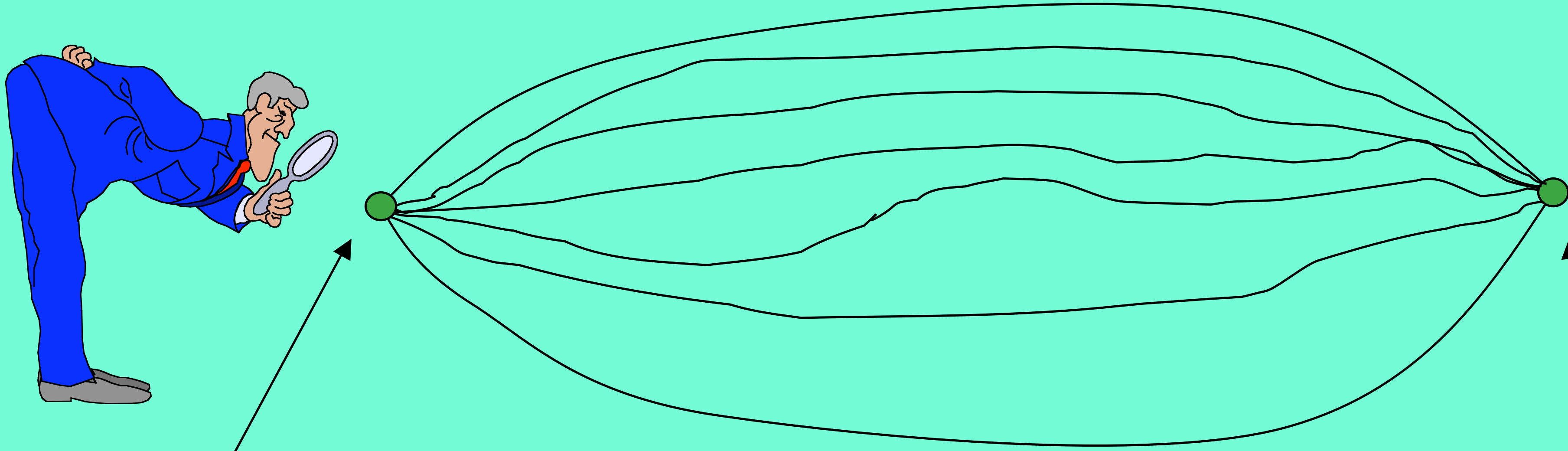
For L=400, execution time ~ 10^{20} *seconds*

Age of Universe ~ 10^{17} seconds

Quantum Computer could take seconds

☛ data security problems

Quantum Parallelism: preserve phases, no decoherence



$$|\psi^{IN}\rangle = \sum_x |x\rangle|0\rangle \Rightarrow |\psi^{OUT}\rangle = \sum_x |x\rangle|F(x)\rangle$$

Initial state, superpositions of classical inputs

Final state, superposition of corresponding outputs

Requirements: DiVincenzo Checklist

1. State Space Control

Identify qubits, addressable, scalable

2. Cold States

Accurate preparation of initial conditions

3. Isolation

Fidelity $$1 - f = 1 - \langle \psi | \rho | \psi \rangle \sim 10^{-4}$$

4. Controlled Time Evolution

5. Projective State Measurements Possible

Error correction?

Encode logical bits using set of bits: eg 3 bit code

$$|0\rangle \Rightarrow (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) = |\tilde{0}\tilde{0}\tilde{0}\rangle$$

$$|1\rangle \Rightarrow (|0\rangle - |1\rangle)(|0\rangle - |1\rangle)(|0\rangle - |1\rangle) = |\tilde{1}\tilde{1}\tilde{1}\rangle$$

Phase error in normal basis = amplitude error in tilda basis

$$\sigma_z\left[|\tilde{0}\rangle\right] = |\tilde{1}\rangle \qquad \sigma_z\left[|\tilde{1}\rangle\right] = |\tilde{0}\rangle$$

detect 100 in tilda basis - know there was phase error in first qubit. Proper error correction will need ~ 5 bit codes

Successes?

1. Ions

Qubit gates: Wineland

2. Atoms-Photons

Large Phase Shifts: Kimble

3. Photon-Photon

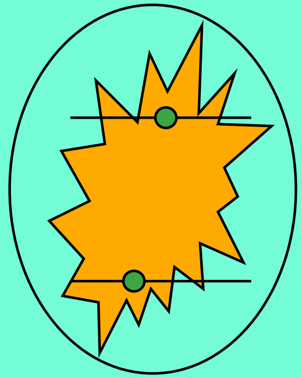
Nonlinear Phase Shifts? Franson

4. NMR

Deutsch-Jozsa algorithm realized

Grover search algorithm realized

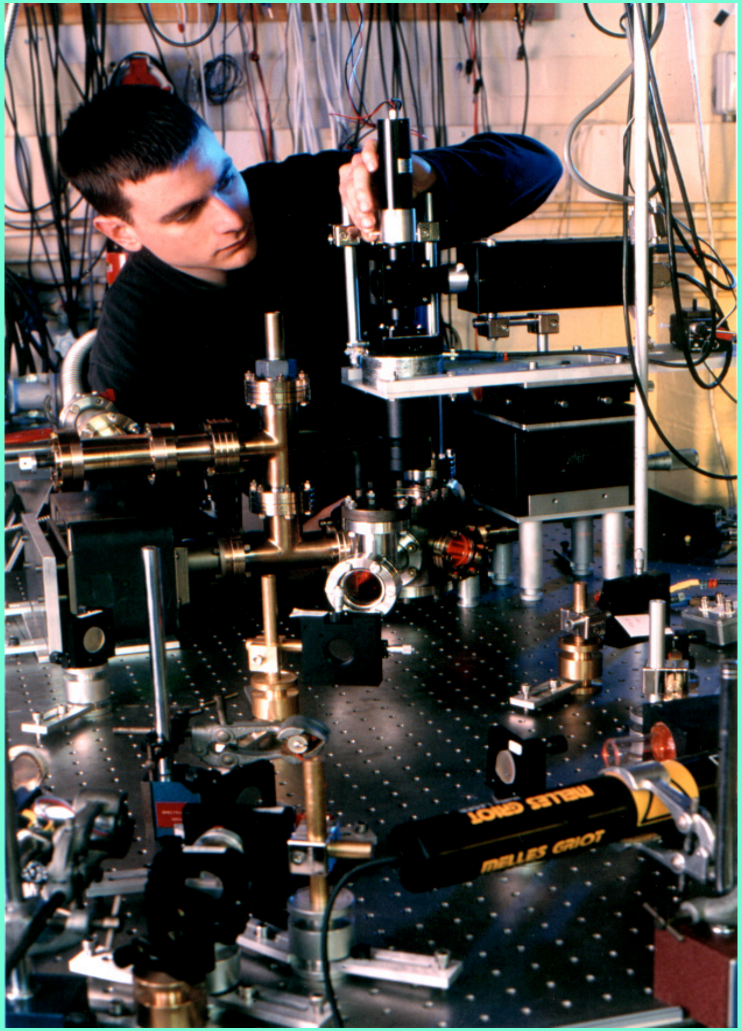
Three Qubit error correction realized

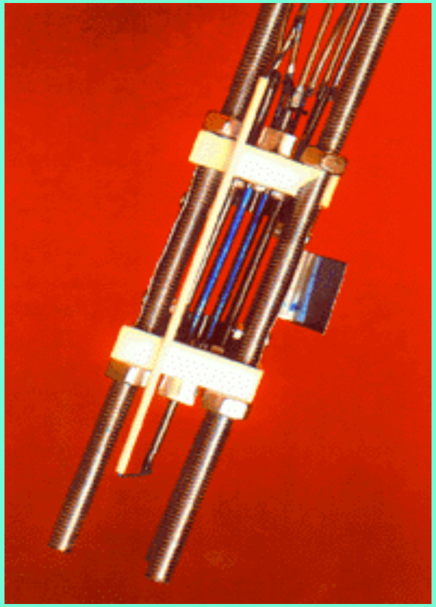


Munro et al

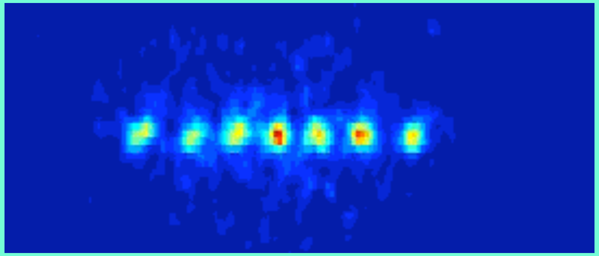
Approach	Qubit	Preparation	Decoherence	Gates	Measurement	What has been done?
Linear optics	Photon polarization or dual rail	Photons from down-conversion	Photon loss in fibres	Photon bunching, measurement	Photo-detectors	CNOT gate between two qubits
Non-linear optics	Photon polarization or dual rail	Photons from down-conversion	Photon loss in fibres, dephasing in atomic systems	Photons interact through atomic systems	Photo-detectors	EIT seen in certain atomic systems for classical fields
Continuous variables	Qunat encoded in quadratures of coherent light pulse	Weak coherent light source or vacuum	Photon loss in fibres	Non-linear medium giving Hamiltonians polynomial in quadrature operators	Homodyne or heterodyne detection	Teleportation of a continuous variable
Ions in traps	Energy levels of ion	Optical pumping and laser cooling	Fluctuating fields, level lifetimes	Collective vibrations and external lasers	Resonance fluorescence	Deutsch-Jozsa algorithm and teleportation
Neutral atoms in optical lattices	Energy levels or motional states of atom	Optical pumping and laser cooling	Fluctuating fields, level lifetimes	Dipole-dipole coupling or collisions	Resonance fluorescence	Mott transition loading of a lattice

Ion experiments



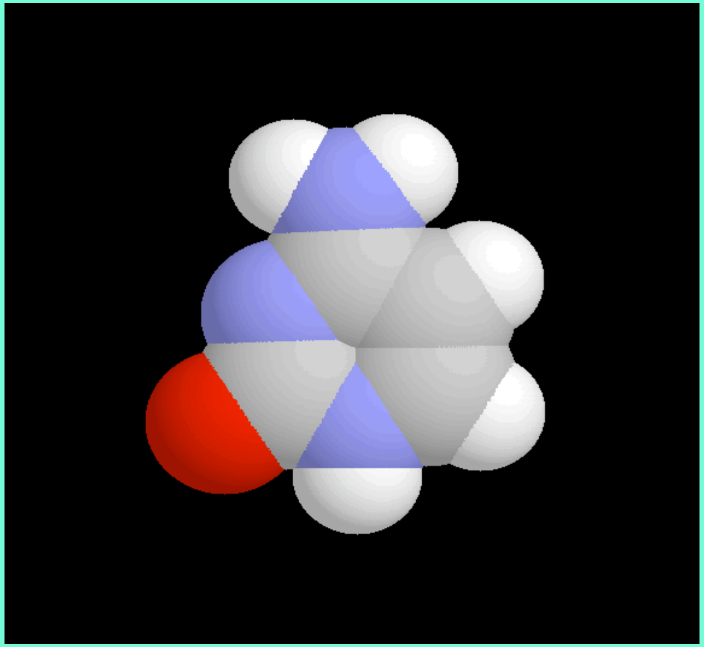


Ion trap
(Oxford)

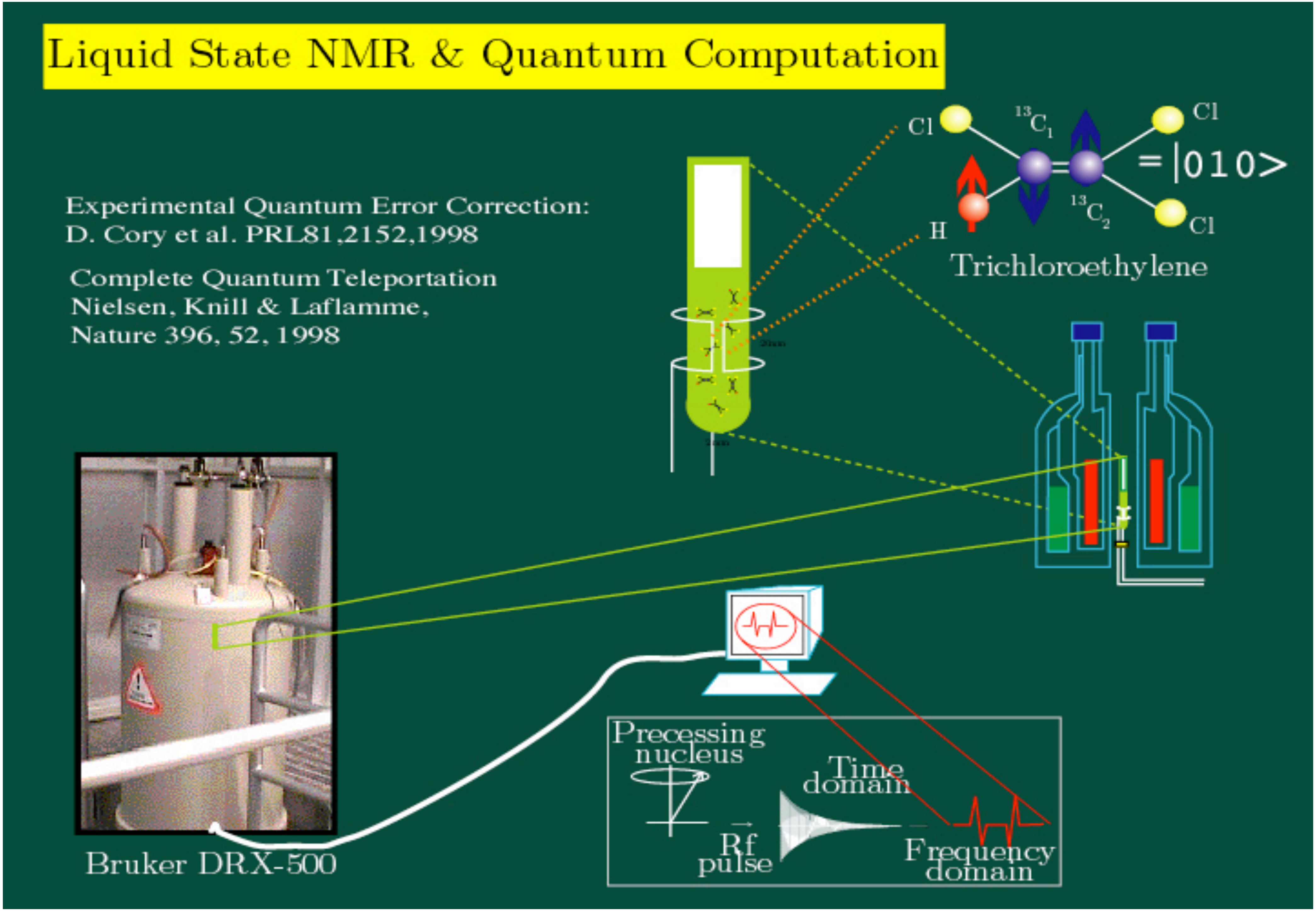


7 trapped ions (Innsbruck)

NMR experiments: computing within a molecule



Cory et al;
Gershenfeld & Chuang;
Jones et al - Cytosine:


Liquid State NMR & Quantum Computation
Experimental Quantum Error Correction:
D. Cory et al. PRL81,2152,1998
Complete Quantum Teleportation
Nielsen, Knill & Laflamme,
Nature 396, 52, 1998
Cl
13C1
Cl
= |010>
H
13C2
Cl
Trichloroethylene
Bruker DRX-500
Precessing nucleus
Time domain
Rf pulse
Frequency domain

Further reading

1. Barenco, Cont Phys 37, 375 ‘96
2. Bennett, Phys Today 48, 24, Oct 95
3. Steane Rep Prog Phys 61, 117 (98)
4. Zeilinger, Physics World March 98, 35
5. Phoenix&Townsend Cont Phys36,165 ‘95
6. Hughes et al Cont Phys 36, 149 ‘95
7. Vedral& Plenio Cont Phys 39, 431 ‘96
8. Vedral & Plenio Prog Qu El 22, 1 ‘98
9. Haroche, Phys Today, July ‘98 p36
10. *More each day in quant-ph!!*