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Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

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WINTER COLLEGE
on
QUANTUM AND CLASSICAL ASPECTS
of
INFORMATION OPTICS

30 January - 10 February 2006

AMO realizations

Drawing on work from the EU QBITS and QGATES Networks

(Second Part)

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Lecture 2: AMO realizations
Trieste Winter School, February 2006

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Drawing on work from the EU QBITS and QGATES Networks

Coverage in this set of lectures

- Section 1
  - AMO realizations
  - DiVincenzo criteria
- Section 2: Ions
- Atom-phonon interactions
  - Gates
  - Algorithms
  - Error correction
- Section 3: atoms, lattices and chips
  - Cold atoms
  - Optical lattices
  - Mott transition
  - Atom chips and decoherence

Munro et al: Contemp Phys (2005)

Approach	Qubit	Preparation	Decoherence	Gates	Measurement	What has been done?
Linear optics	Photon polarization or dual rail	Photons from down-conversion	Photon loss in fibres	Photon bunching, measurement	Photo-detectors	CNOT gate between two qubits
Non-linear optics	Photon polarization or dual rail	Photons from down-conversion	Photon loss in fibres, dephasing in atomic systems	Photons interact through atomic systems	Photo-detectors	EIT seen in certain atomic systems for classical fields
Continuous variables	Qunat encoded in quadratures of coherent light pulse	Weak coherent light source or vacuum	Photon loss in fibres	Non-linear medium giving Hamiltonians polynomial in quadrature operators	Homodyne or heterodyne detection	Teleportation of a continuous variable
Ions in traps	Energy levels of ion	Optical pumping and laser cooling	Fluctuating fields, level lifetimes	Collective vibrations and external lasers	Resonance fluorescence	Deutsch-Jozsa algorithm and teleportation
Neutral atoms in optical lattices	Energy levels or motional states of atom	Optical pumping and laser cooling	Fluctuating fields, level lifetimes	Dipole-dipole coupling or collisions	Resonance fluorescence	Mott transition loading of a lattice

DiVincenzo Criteria

- 1. A collection of well-characterised qubits is needed. One at a time will do for cryptography although entangled pairs useful; controlled interactions between a few qubits for small scale processing; scalability in number is necessary for full blown quantum computing.
- 2. Preparation of known initial states for the qubits must be possible. The purer the better.
- 3. The quantum coherence of the system(s) must be maintained to a high degree during the evolution stage, giving a decent fidelity for the final state. For few-qubit processing it may su_ce to have a straight shot at the process with good qubits and gates; for large scale quantum computation error correction will almost certainly be needed. For fault-tolerant operation the fidelity of individual gates probably needs to be 0.999 or better.
- 4. unitary quantum evolution must be realisable. A universal set of elementary gates must be possibl
- 5. High fidelity quantum measurements on specific qubits must be possible, in order to readout the result.
- 6. Interconvert stationary (processing or memory) qubits and flying (communication) qubits.
- 7. Need to transmit flying qubits coherently between specified locations.

Ions

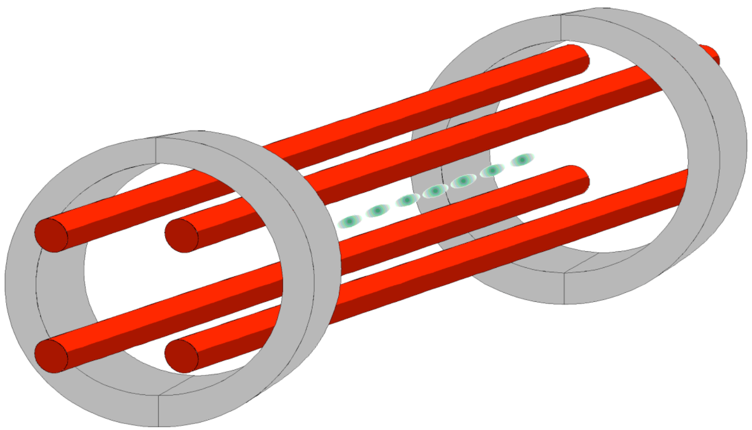
Currently the best realisation:

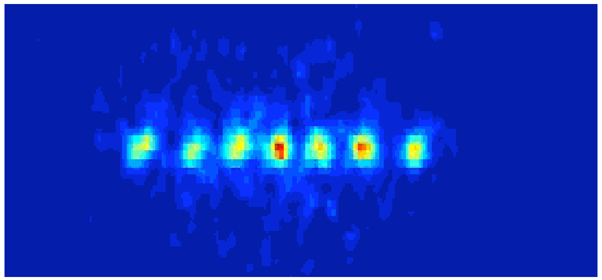
- Addressable
- Can be initialised in near-pure states
- Can realise 1 & 2 bit gates
- Have been used to realise simple quantum algorithms

Implementation of quantum computing with trapped ions

GATES AND NETWORKS REQUIRED	SPECIFICATIONS NEEDED	COLD IONS IN TRAPS?
candidate qbits	long coherence times compared with gate times	Isolated in high vacua
qbit registers	array of addressable qbits	linear ion traps support ion strings
1 & 2 qbit gates	communication between qbits, operations on individual qbits	Coulomb repulsion between ions spatial separation allows us to address individual ions
Initialization	state preparation	laser cooling, or sympathetic cooling and then state preparation with laser pulses
Computation	Unitary state manipulation (although cluster computation possible driven by measurements)	Internal (2-level or Raman) & external (eg motional) excitations,
Readout/measurement	state measurement (need near-100% efficiency)	Quantum jumps in fluorescence

Ion Trap Quantum Information Processing
Cirac and Zoller, PRL 74, 4091 (1995)

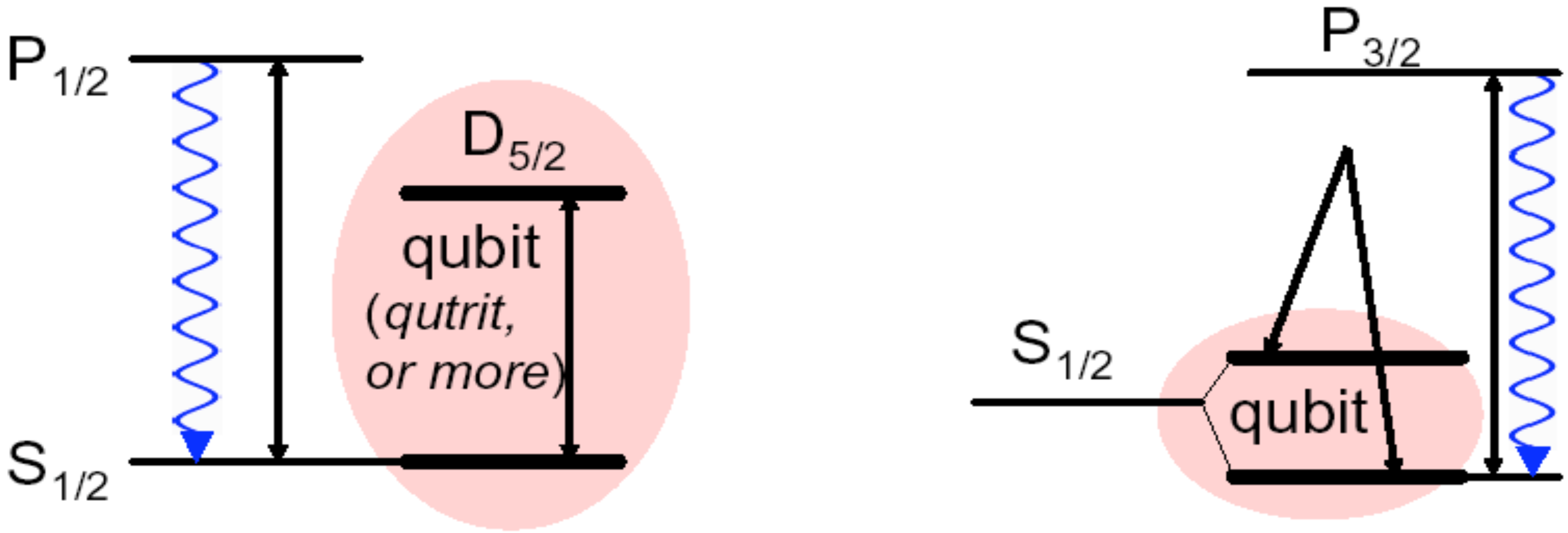




Figures courtesy of Rainer Blatt (Innsbruck)

- Qubits are stored in long-lived electronic states of laser-cooled trapped ions
- >99% efficient readout using electron shelving techniques
- Quantum gates are realised by illuminating the ions with properly tuned laser beams
  - Single-qubit rotations obtained by driving the ionic transition
  - Two-qubit gates require coupling internal states to a vibrational motional mode (‘data bus’)
- Relatively low ratio between gate switching rate and decoherence rate

Internal qubits of ions



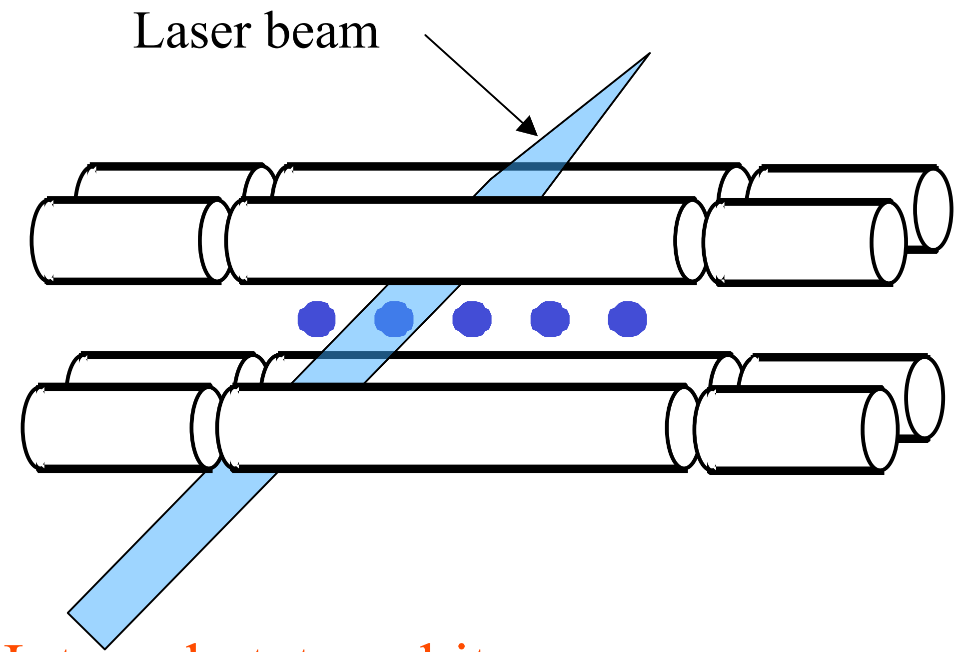
2-level system

Raman system

Ion traps and DiVincenzo Criteria

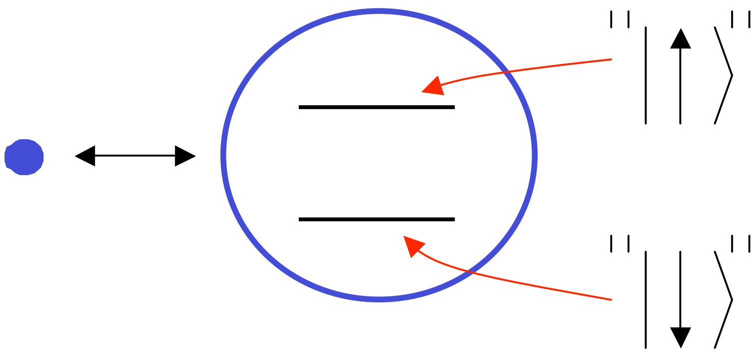
- Two energy levels of a trapped ion used as an effective qubit. The number of qubits in a linear ion trap cannot be scaled up to large numbers, but scaling proposed through cavity QED or connections through an array of microtraps coupled through a common moving head, and through coherent moving of ions in and out of a processor region.
- State preparation achieved by optical pumping and laser cooling techniques.
- Simple gate sequences have been performed to realise the Deutsch-Jozsa algorithm before decoherence intervenes. If external noise sources (fluctuating fields injecting heat) can be reduced, the ultimate limits of qubit decay are far enough away that fault-tolerant QIP should be possible.
- Single qubit gates can be achieved through external laser excitation, as the ions sit microns apart and can be individually addressed. Qubit-qubit coupling (capable of generating entanglement) is achieved through vibrations of the ion crystal acting as a data bus. Universal QIP is possible.
- Projective measurements of an ion in the energy eigenstate basis can be performed with 99.99 percent efficiency by use of resonance fluorescence. One state scatters light and the other is "dark".
- Cavity QED techniques provide a route for coupling stationary ion processing qubits to travelling photon qubits. The first experiments in this area are just beginning to produce results.
- Distant communication of quantum information between ions will likely involve interconversion with photon modes. Small scale (maybe less than a metre) movement of quantum information may be possible by simply moving ions around coherently within a single trap complex.

Ion Trap QC: Proposal: J. I. Cirac and P. Zoller, PRL **74**, 4091 (1995)



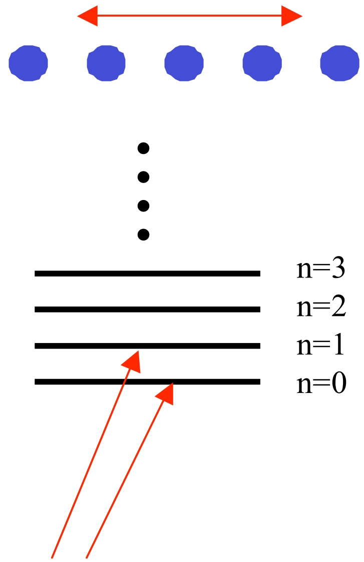
Internal-state qubit

($t_{decoherence}$ > 30 min)



Motion "data bus"

(e.g., center-of-mass mode)



Stay in two lowest motional states (motion qubit)
($t_{decoherence}$ ~ 0.01 – 100 ms)

Frequencies of trapped ion motion

How do you trap an ion in 3D-need to circumvent Earnshaw's theorem?

$\vec{r}$ binding force $\vec{F} \sim -\vec{r}$, that is $\vec{F} = e\vec{E} = -e\nabla\Phi \Rightarrow \Phi \sim \vec{r}^{\,2}$

Make a quadrupole potential
and rotate!

$$\Phi = \frac{\Phi_0}{r_0^2}(x^2 + y^2 - 2z^2)$$

RF Paul trap: $\Phi_0 = U_0 + V_0 \cos \Omega t$

Penning trap: $\Phi_0 = U_0 +$ axial magnetic field

equation of motion in a Paul trap:

$$\ddot{x} + (a - 2q\cos\Omega t)\frac{\Omega^2}{4}x = 0$$

$$a \sim U_0 , q \sim V_0$$

This is a MATHIEU EQUATION

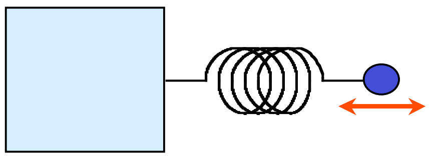
frequencies of **secular motion**: $\omega_x, \omega_y, \omega_z$

superimposed is **micromotion** with: Ω

$$\omega \approx (a + \tfrac{1}{2}q^2)\Omega$$

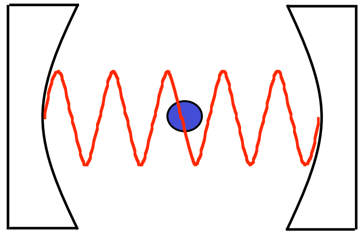
Trapped ions and cavity qed without the cavity

Cold trapped ions



quantized oscillator =
mode of motion
(can also do with neutral atoms too:
See next lecture on lattices

Cavity-QED



quantized oscillator =
mode of electromagnetic field
(Jaynes Cummings model used by
Haroche, Kimble, Walther,
Rempe, Orozco, Hinds….

Motional states

- Two level ion, cold and confined in trap

$$\hat{H} = \hat{H}_0 + [\mathcal{D}E^{(-)}(\hat{x}, t)\,\hat{\sigma}_- + H.c.]$$

$$\hat{H}_0 = \frac{\hbar\omega_0}{2}\hat{\sigma}_3 + \hbar\nu\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

$$E^{(-)}(\hat{x}, t) = E_0 \exp[i(\omega_L t - k_L\hat{x} + \phi)]$$

$$\hat{x} = \sqrt{\frac{\hbar}{2\nu M}}(\hat{a} + \hat{a}^\dagger) \quad E^{(-)}(\hat{x}, t) = E_0 e^{i(\phi+\omega_L t)} e^{-i\eta(\hat{a}+\hat{a}^\dagger)}$$

$\eta \equiv k_L(\hbar/2\nu M)^{1/2}$ is the so-called Lamb–Dicke parameter.

Motional JCM

$$\hat{H}_{\rm I} = \hat{U}^\dagger \hat{H} \hat{U} + i\hbar \frac{d\hat{U}^\dagger}{dt}\hat{U}, \qquad \hat{U} = \exp(-i\,\hat{H}_0 t/\hbar)$$

$$= \mathcal{D}E_0 e^{i\phi} e^{i\omega_{\rm L} t} \exp[-i\,\eta(\hat{a}\,e^{i\nu t} + \hat{a}^\dagger\, e^{-i\nu t})]\hat{\sigma}_- e^{-i\omega_0 t} + H.c.$$

As η is small, we expand to first order

$$\exp[-i\,\eta(\hat{a}\,e^{i\nu t} + \hat{a}^\dagger\, e^{-i\nu t})] \approx 1 - i\,\eta(\hat{a}\,e^{i\nu t} + \hat{a}^\dagger\, e^{-i\nu t}),$$

$$\hat{H}_{\rm I} \approx \mathcal{D}E_0 e^{i\phi}[e^{i(\omega_{\rm L}-\omega_0)\,t} - i\,\eta(\hat{a}\,e^{i(\omega_L-\omega_0+\nu)\,t} + \hat{a}^\dagger\, e^{i(\omega_L-\omega_0-\nu)\,t})]\hat{\sigma}_- + H.c.$$

Suppose now that the laser is tuned such that $\omega_{\rm L} = \omega_0 + \nu$.

$$\hat{H}_{\rm I} \approx \mathcal{D}E_0 e^{i\phi}[e^{i\nu t} - i\,\eta(\hat{a}\,e^{i2\nu t} + \hat{a}^\dagger)]\hat{\sigma}_- + H.c.$$

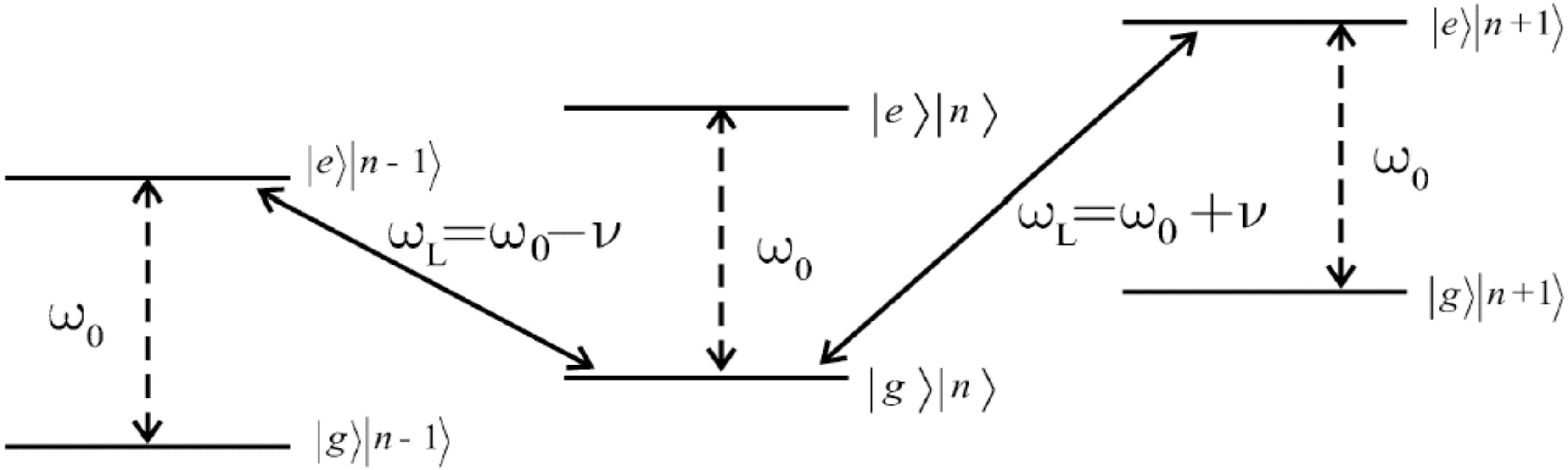
$$\hat{H}_{\rm I} \approx -i\,\hbar\eta\,\Omega\, e^{i\phi}\hat{a}^\dagger\hat{\sigma}_- + H.c.,$$

Slowly varying, RWA

$\omega_{\rm L} = \omega_0 - \nu$ we obtain the interaction

$$\hat{H}_{\rm I} \approx -i\,\hbar\eta\,\Omega\, e^{i\phi}\hat{a}\,\hat{\sigma}_- + H.c.$$

Motional JCM



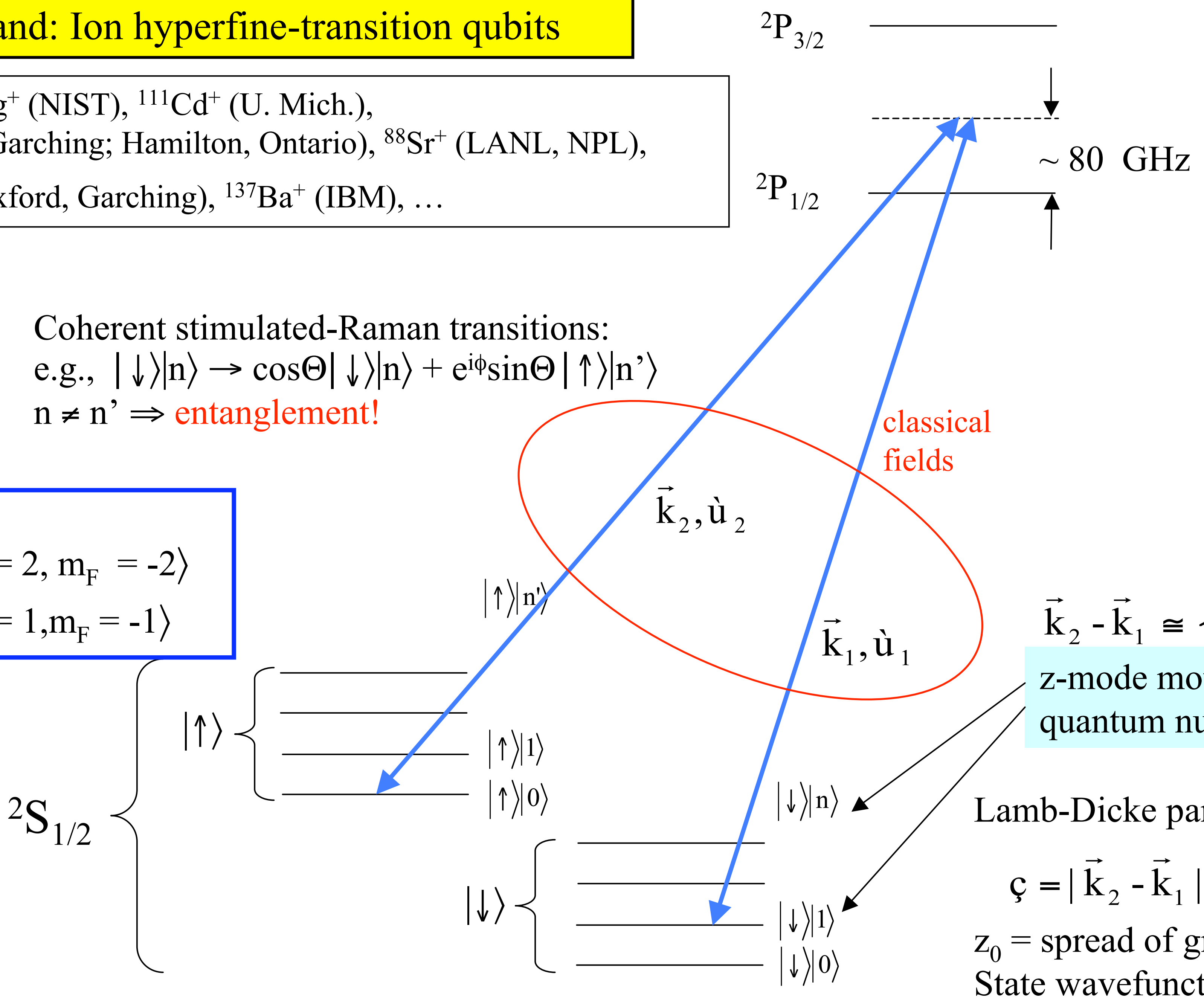
- Chose red detune: JCM
- Chose blue detune: anti-JCM

Wineland: Ion hyperfine-transition qubits

$^9Be^+$, Mg^+ (NIST), $^{111}Cd^+$ (U. Mich.),
$^{25}Mg^+$ (Garching; Hamilton, Ontario), $^{88}Sr^+$ (LANL, NPL),
$^{40}Ca^+$ (Oxford, Garching), $^{137}Ba^+$ (IBM), …

Coherent stimulated-Raman transitions:
e.g., $|\downarrow\rangle|n\rangle \rightarrow \cos\Theta|\downarrow\rangle|n\rangle + e^{i\phi}\sin\Theta|\uparrow\rangle|n'\rangle$
$n \neq n' \Rightarrow$ entanglement!

$^9Be^+$
$|\downarrow\rangle \equiv |F = 2, m_F = -2\rangle$
$|\uparrow\rangle \equiv |F = 1, m_F = -1\rangle$



$\vec{k}_2 - \vec{k}_1 \cong \sqrt{2}|\vec{k}|\hat{z}$

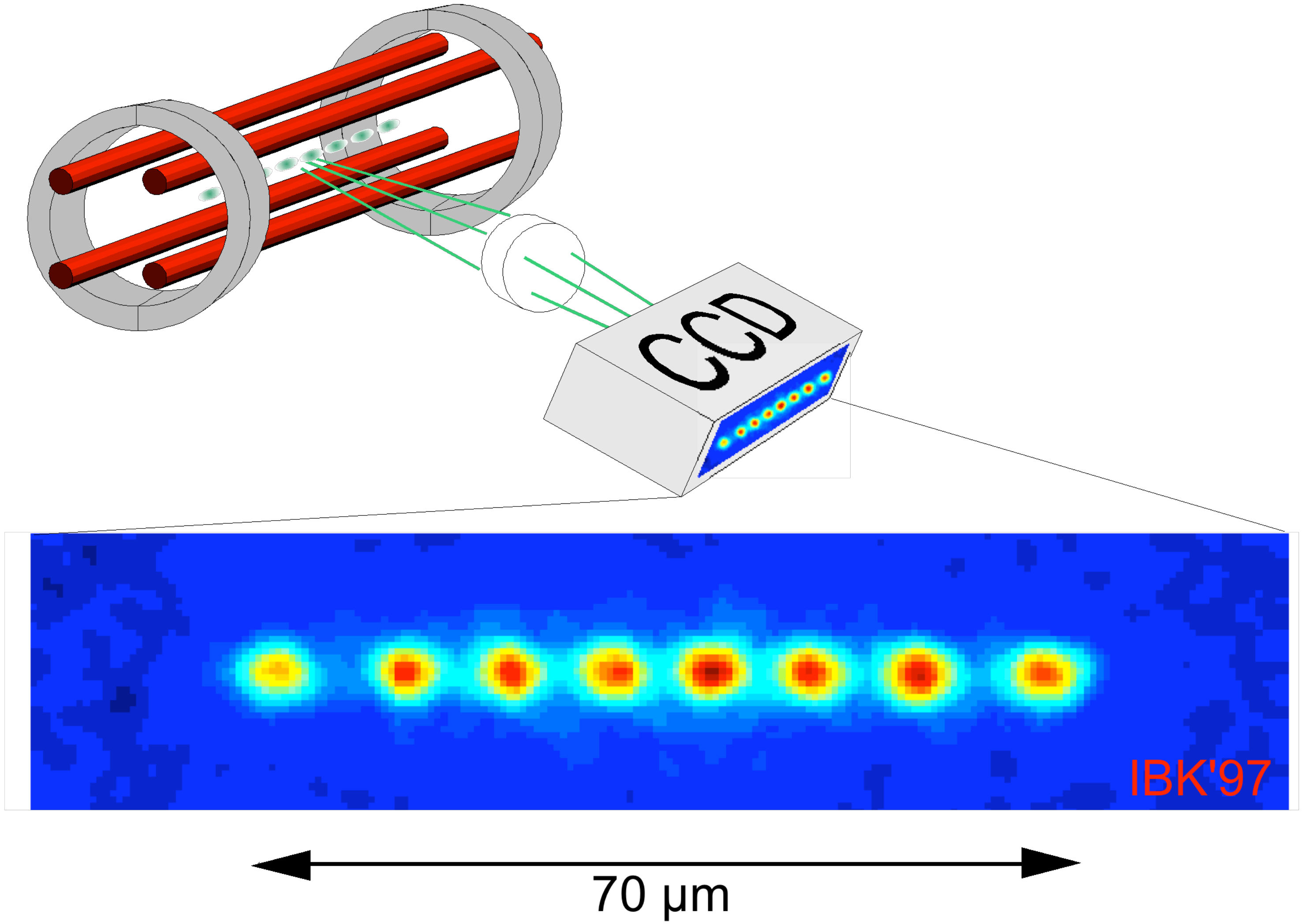
z-mode motional quantum numbers

Lamb-Dicke parameter:

ç $= |\vec{k}_2 - \vec{k}_1| z_0$

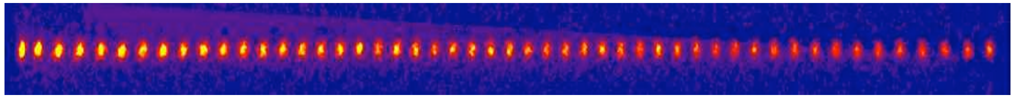
z_0 = spread of ground-State wavefunction

Innsbruck string of Ca^+ ions in a linear Paul trap



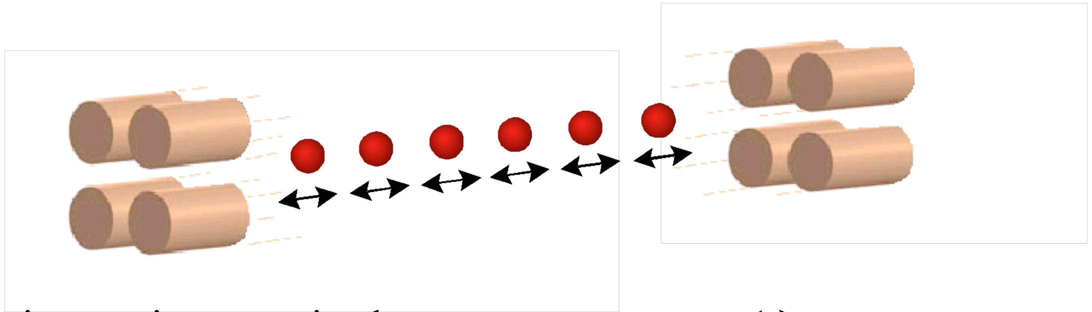
Linear Ion Trap

- Linear ion trap (Raizen, Walther....)

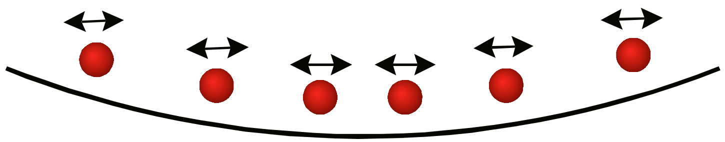


  - up to about 30 ions in a string but not yet under complete control: packing instabilities

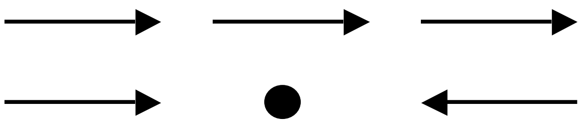
- - ions separated by about 10 – 20 µm



- motional collective motion quantized

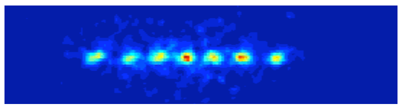


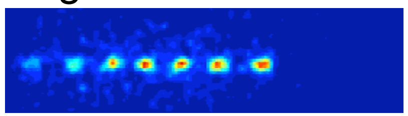
  - anisotropic motion: $n_z << n_x, n_y$
  - ion motion coupled by Coulomb repulsion
  - eigenmodes only weakly dependent on the number of ions



center of mass mode $\nu_1 = \nu_z$

breathing mode $\nu_2 = \sqrt{3}\,\nu_z$





Regimes of ion-trap gates

- "Cold" ions
  - Requires cooling the ions to their motional ground-state
  - Gates are relatively fast but sensitive to motional heating
  - Cirac + Zoller (1995); Monroe *et al,* PRA 55, 2489 (1997); Leibfried, PRA 60 3335 (1999);

- "Hot" ions
  - Work even in the presence of a few 'phonons'
  - Gates are relatively slow
  - Can be sensitive to heating during gate operations
  - Poyatos *et al,* PRL 81, 1322 (1998); Sørensen + Mølmer, PRL 82, 1835 (1999); PRL 82, 1971 (1999); quant-ph/0002024 (2000); Schneider *et al*, JMO 47, 499 (2000)

Cirac - Zoller gate scheme

- 1 - qubit gates: laser resonant with the atomic transition freq. ω_a

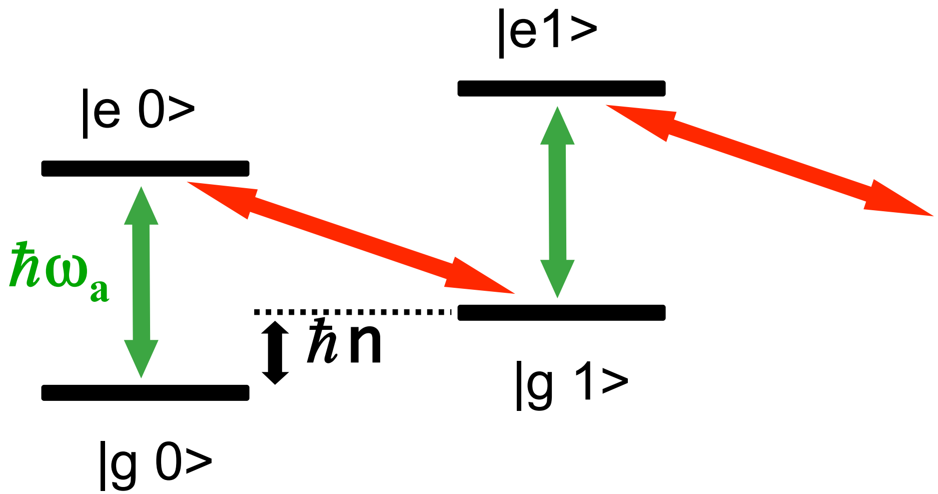
$$H_{res} \approx \Omega\left(\sigma_+ + \sigma_-\right)$$

- 2 - qubit ion-mode gates: laser resonant with the first red sideband transition freq. $\omega_a - \nu$

$$H_{1rs} \approx i\eta\Omega\left(a\sigma_+ - a^+\sigma_-\right)$$

- $\eta = \sqrt{\frac{E_R}{\hbar\nu}}$ is the $<< 1$

Lamb-Dicke parameter



A 2-qubit gate between different ions is realised by preparing the mode in the ground state |0|, then applying a 3-step sequence

- SWAP state of ion 1 into mode
- Entangle mode and ion 2
- SWAP mode state back to ion 1

Experimental obstacles to realising CZ's scheme

- 'Technical'
  - Requires cooling to the collective motional ground state (by e.g. sideband cooling)

    ⇨ Feasible with a 'strong' trap
    (King *et al*, PRL 81, 1525 (1998); Roos *et al*, PRL 83, 4713 (1999))

    but also

  - Requires individual laser access to each ion

    ⇨ Feasible with a 'weak' trap
    (Naegerl *et al*, PRA 60, 145 (1999))

- 'Intrinsic'
  - Decoherence is mainly from heating / dephasing of the 'data bus' (Wineland *et al* 1998).

    Gate steps via the motion must switch much faster than decoherence rates

    but also

  - Gate-generating Hamiltonians valid only if off-resonant couplings can be neglected (RWA)

    Requires very small switching rates

Speed limits for Cirac-Zoller gates

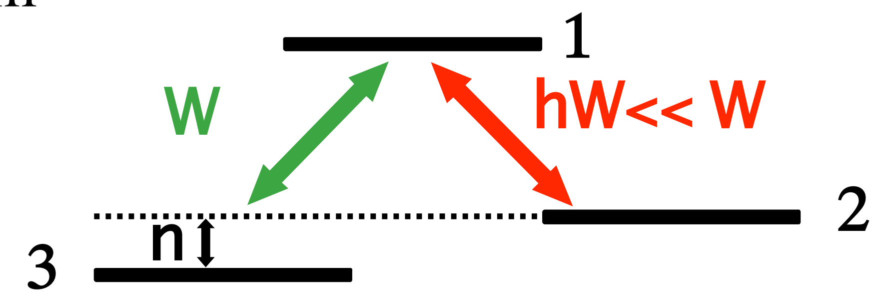
D. Jonathan *et al,* quant-ph/0002092; also A. Steane *et al,* quant-ph/0003087

- Overall processor speed is limited by the switching rate of 2-qubit gates, R = hW. But how large can R be?

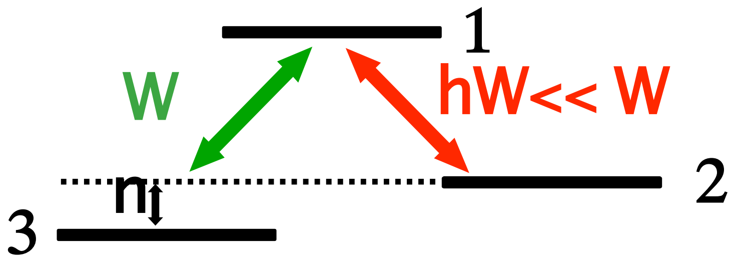
- When driving the red sideband (with travelling-wave radiation), the most important off-resonant correction comes from the strong carrier transition

$$H_{1rs} \approx i\eta\Omega\left(a\sigma_{+} - a^{+}\sigma_{-}\right) + \Omega\left(e^{ivt}\sigma_{+} + e^{-ivt}\sigma_{-}\right)$$

- Situation analogous to a competition between a strong off-resonant coupling and a weak resonant coupling in a 3 -level system



Result: a trade-off between gate speed and fidelity



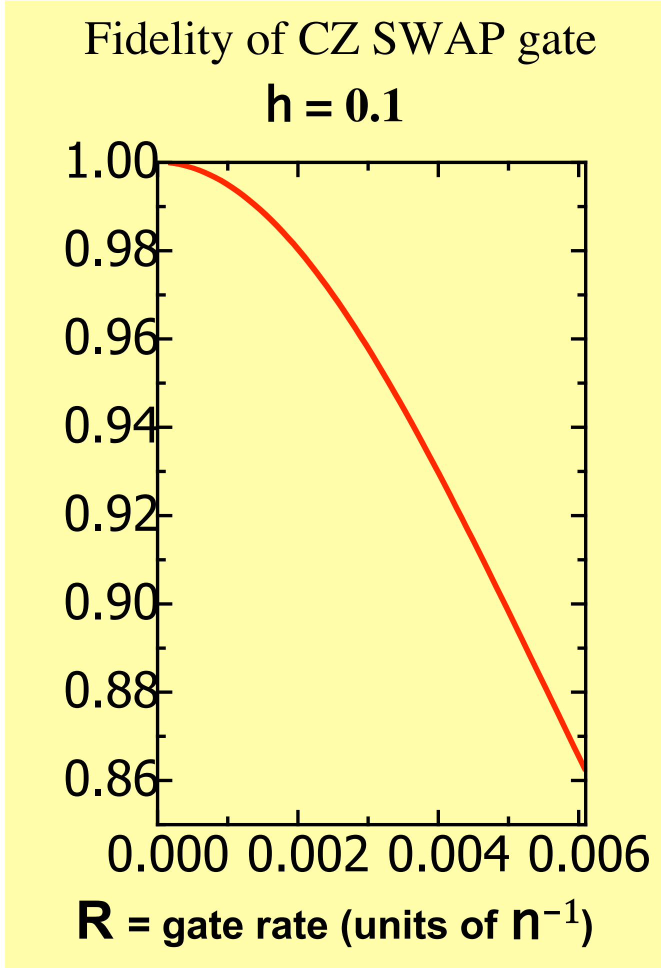
- Level 3 can be disregarded only if

$$\frac{\Omega^2}{\nu} << \eta\Omega \Rightarrow \Omega << \eta\nu$$

$$\Rightarrow R << \eta^2\nu = \frac{1}{\hbar} E_R$$

- More precisely (A. Steane *et al*): the population leakage is $L \approx$ (W/ hn)^2

- Fidelity: $F \approx 1 - L \approx 1 - (R\,/\,h^2\,n)^2$



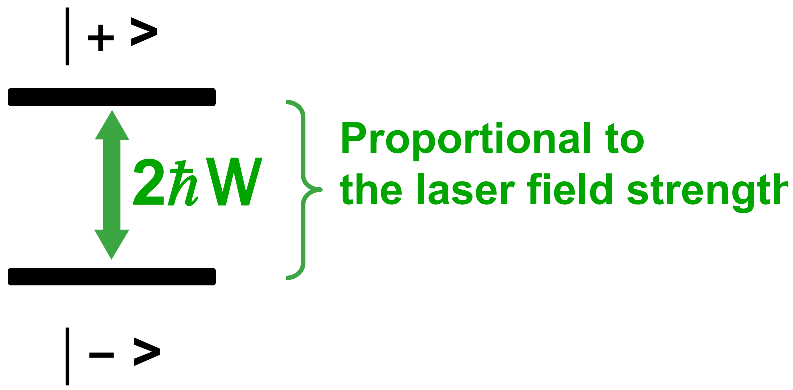
Beating the speed limit: ways to obtain faster cold-ion gates

- (Cirac + Zoller 1995): Use standing-wave radiation, with the target ion held in one field node. Interference between travelling-wave components cancels the off-resonant carrier
  - Pro: high gate speeds (up to ~10^{-1} n at *F* = 99%)
  - Con: hard to implement experimentally
- (Steane *et al* 2000): Off-resonant transition can also be seen as causing a shift in the desired sideband resonance. Can be partially compensated by a slight retuning of the laser beam.
  - Gate speeds up to R ~ 10^{-1} n
  - (Jonathan *et al* 2000): Use light resonant with the carrier. At *specific* beam intensities, the resulting AC Stark-shift (lightshift) can be used to drive a 2-qubit gate
  - Gate speeds up to ~ $5\mathrm{x}10^{-2}$ n at *F* = 99%)

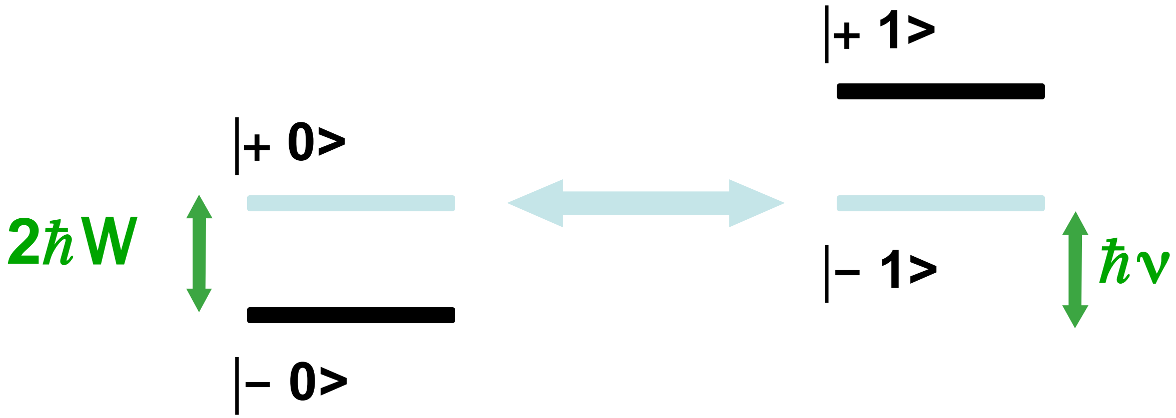
Lightshift-induced 2-qubit gates
D. Jonathan *et al* quant-ph/0002092

- Resonant driving is equivalent (in the interaction picture) to splitting the (semiclassical) atomic dressed states

$$|\pm\rangle \equiv (1/\sqrt{2})(|g\rangle \pm |e\rangle)$$



- When the splitting equals the mode level spacing (2W= ν), states with different phonon numbers become degenerate, and oscillations are induced between them



→ **2-qubit ion-mode gates**

Lightshift-induced 2-qubit gates - the maths

- Oscillations are driven via the off-resonant sideband transition

$$H_{res} \approx \Omega\left[\sigma_+ + \sigma_- + i\eta(\sigma_+ - \sigma_-)\left(ae^{-i\nu t} + a^\dagger e^{i\nu t}\right)\right]$$

- New term causes $|\pm\rangle$ to become nonstationary. Effect can be seen by moving into the 'dressed-state' picture defined by

$$V(t) = \exp\left[i\Omega t\sigma'_z\right] \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$H_{res} \to V(t)H_{res}V^+(t) =$$

$$= \eta\Omega\left(i\sigma'_+\left[ae^{it[2\Omega-\nu]} + a^+e^{it[2\Omega+\nu]}\right] + h.c.\right)$$

(where, e.g., $\sigma'_+ = |+\rangle\langle-|, \sigma'_z = |+\rangle\langle+| - |-\rangle\langle-|$

Lightshift-induced 2-qubit gates (cont.)

➜ Jaynes-Cummings interaction between motional and internal states *with respect to the dressed basis!*

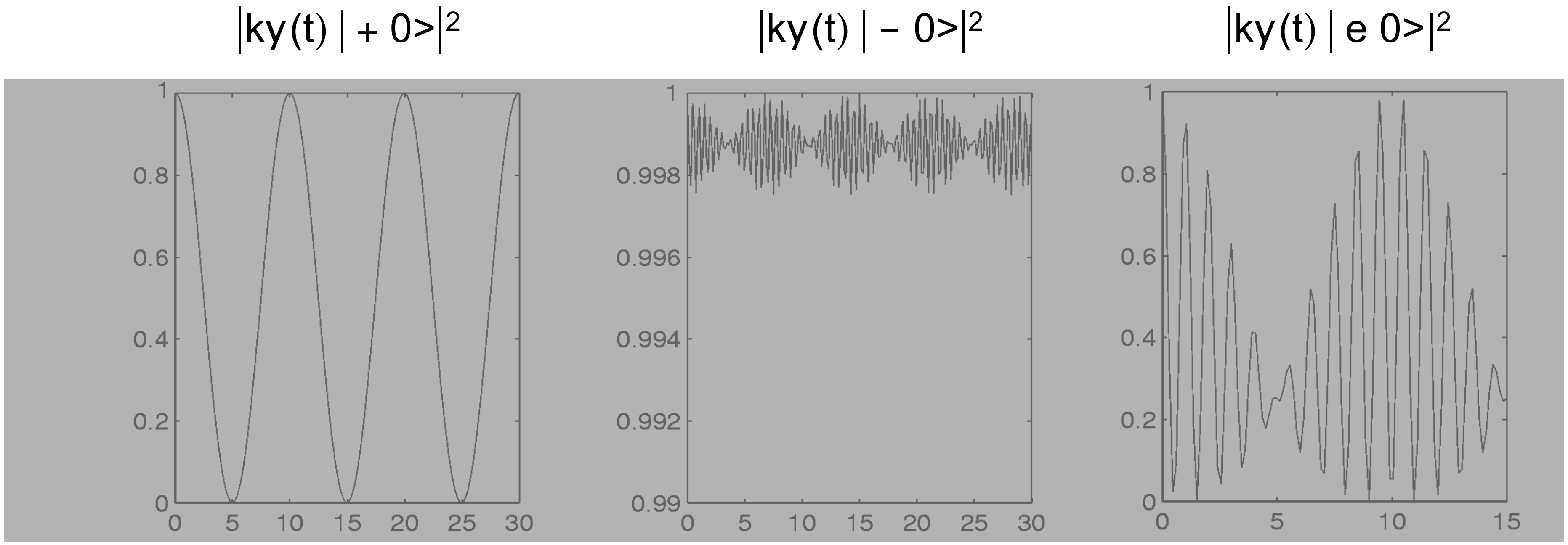
- When $2W= \nu$, the first term is resonant and the second can be neglected (RWA), so

$$H_D = \frac{i\eta\nu}{2}\left(a\sigma'_+ - a^+\sigma'_-\right)$$

- Conclusion: if the laser's frequency and amplitude are *both* tuned to specific values, the off-resonant atomic transitions can lead to entangling dynamics!!

Lightshift-induced 2-qubit gates (cont.)

- Result: Rabi flop - type population exchanges between the mode and the ionic state *in the dressed-state basis,* at frequency **hn**

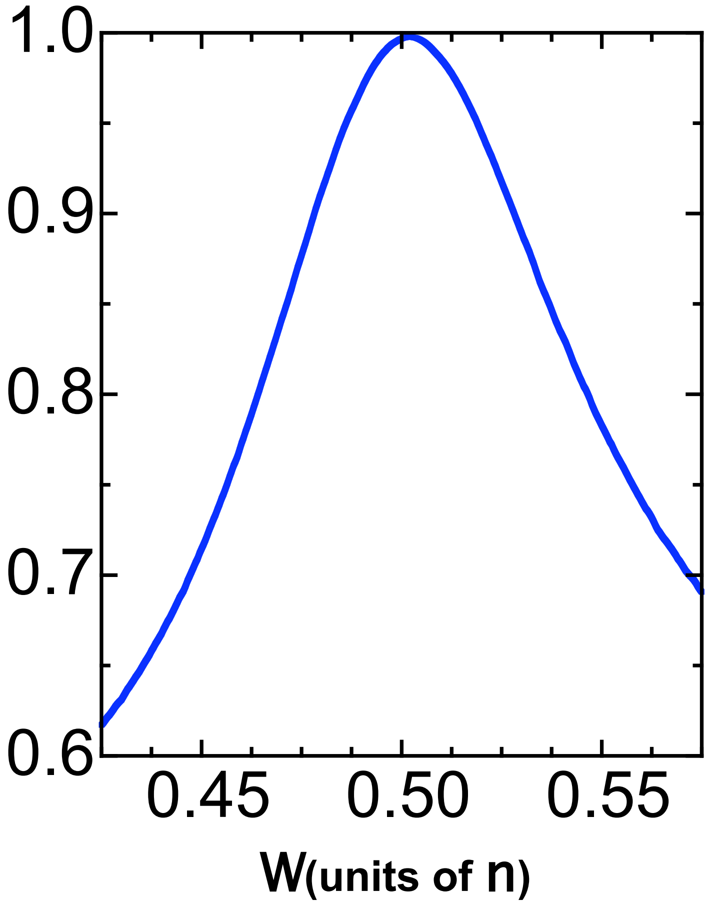


- A full ion-ion gate can be constructed using a CZ-like 3-step procedure
- In the many-ion case, different motional modes are selected by tuning the Rabi frequency to satisfy **2W= ν_j**

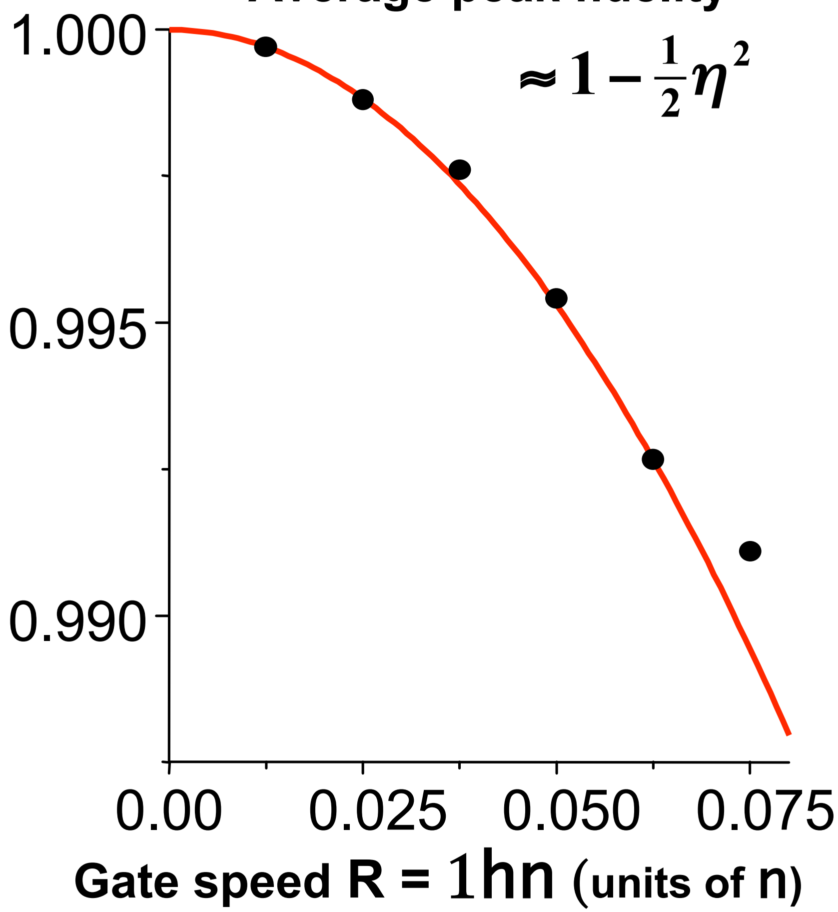
Fidelity of lightshift-induced SWAP gate

Dressed-basis SWAP : |+ 0> ↔ |− 1>

Fidelity vs Rabi freq. (h=0.1)



Average peak fidelity



Fastgates summary

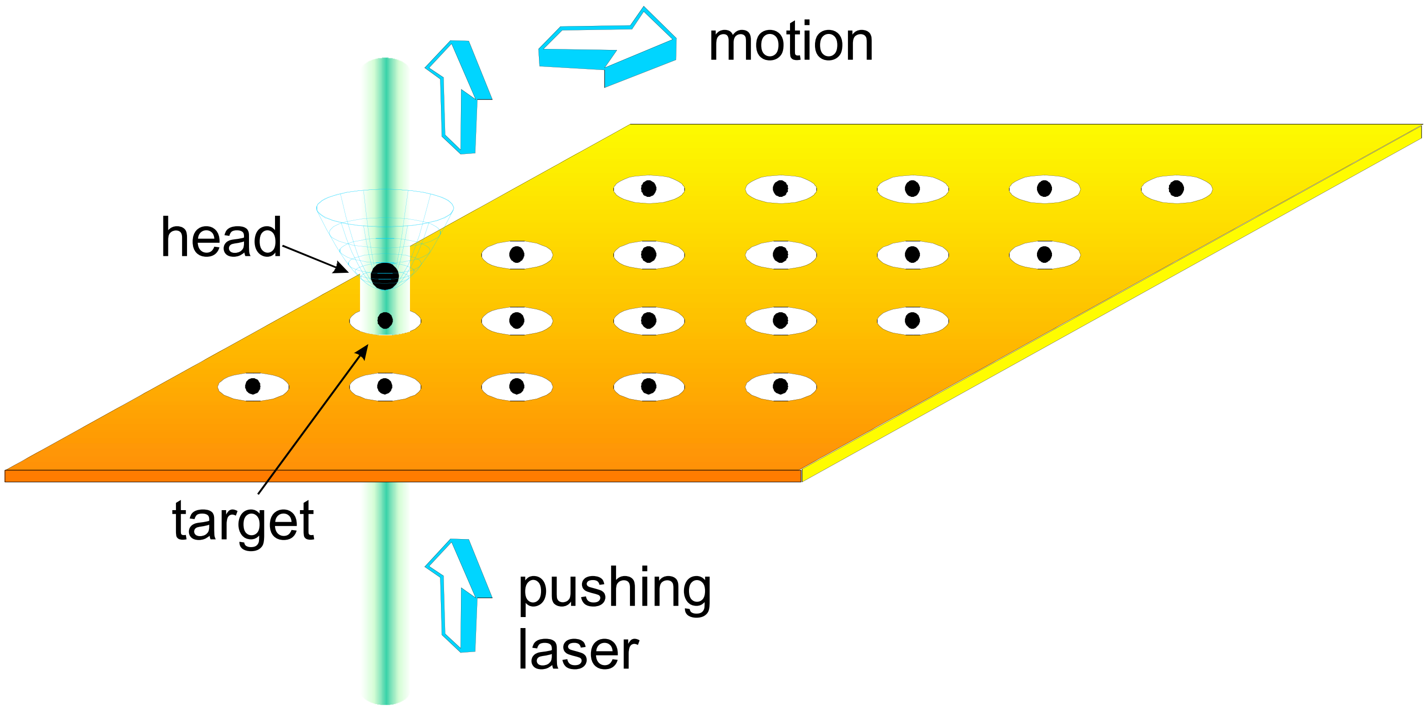
- By driving on the carrier at specific intensities, high- fidelity 2-qubit quantum gates can be generated at up to 30 - 50 times the rate obtained in experiments so far
- The scheme is also simpler since the same laser can be used for generating both 1- and 2 - qubit gates
- An experiment is currently under way at the Innsbruck group

Reference: D. Jonathan *et al,* quant-ph/0002092 (&PRA 62, 42307 (2000))

Vision of quantum computer with ion traps 2000

Cirac and Zoller, Nature 404, 579, (2000)

- quantum optics and nano-technology: scalability



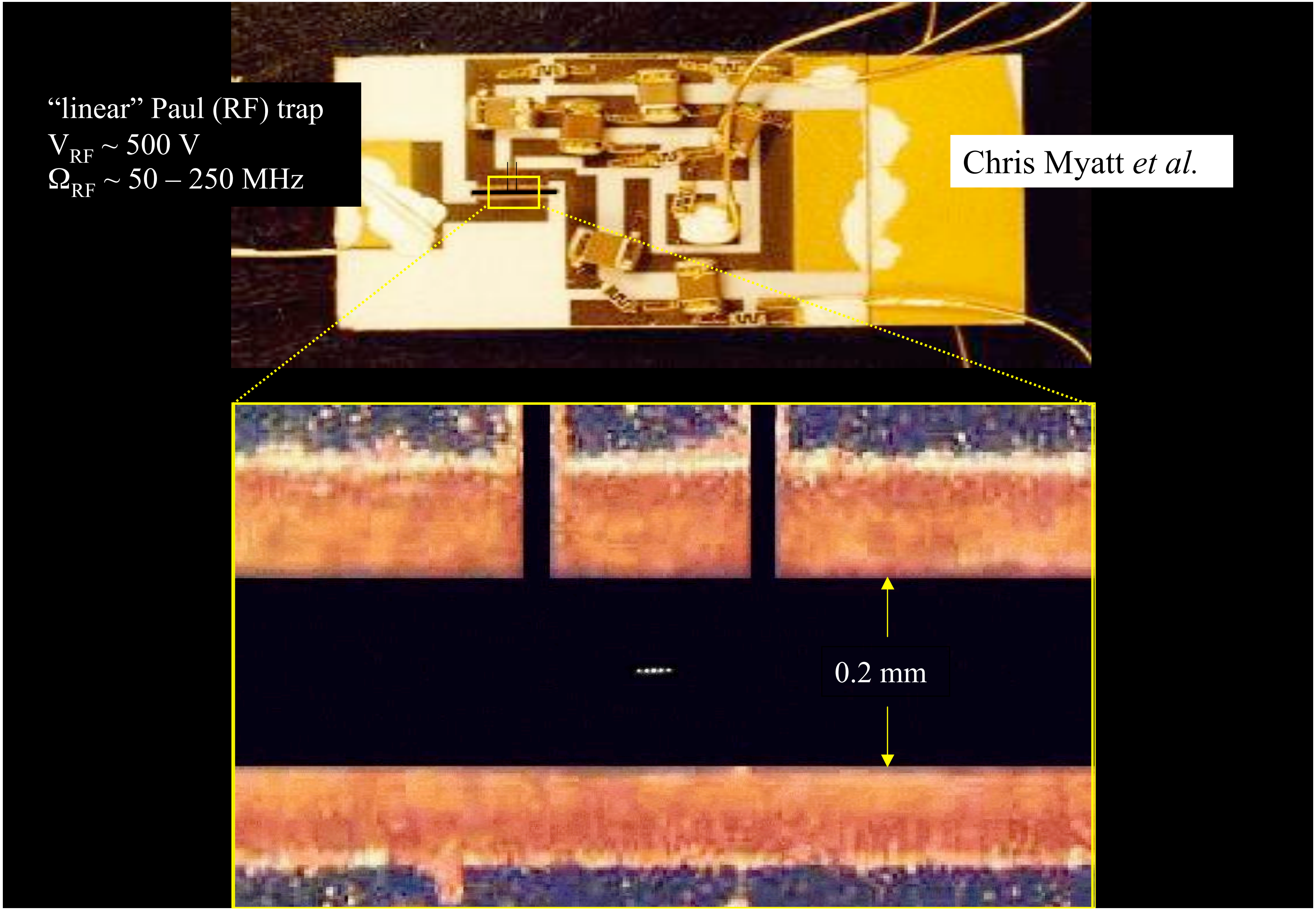
Quantum processing and multiplexing with trapped ions

Goals:

- Find simple, efficient, high-fidelity gates
- Scale up scheme of Cirac and Zoller ('95)
  ⇒ improved trapology, multiplexed traps,…
- QC: fault-tolerant error correction, useful computations …

Summary:

- experimental set-up ($^{9}Be^{+}$ ions)
  + elements of quantum computing
- gates
- scaling up with ion traps
- applications


"linear" Paul (RF) trap
V_{RF} ~ 500 V
Ω_{RF} ~ 50 – 250 MHz
Chris Myatt *et al.*
0.2 mm

Quantum Ion Railway: Wineland group