The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
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International Atomic
Energy Agency

SMR.1738 - 29

WINTER COLLEGE
on
QUANTUM AND CLASSICAL ASPECTS
of
INFORMATION OPTICS

30 January - 10 February 2006

Cavity qed - couple stationary to flying qubits.

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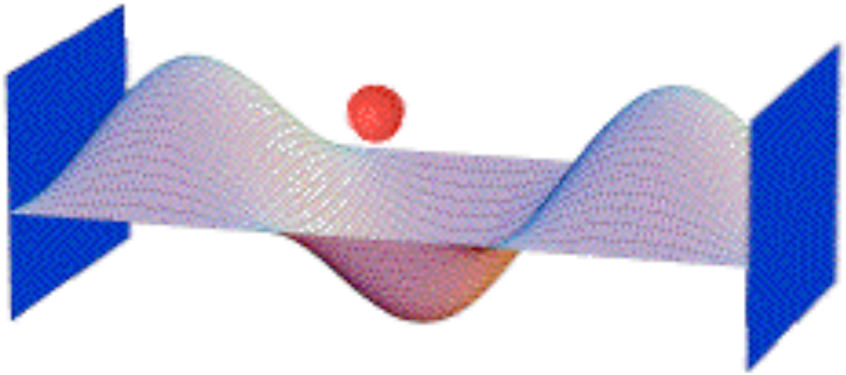
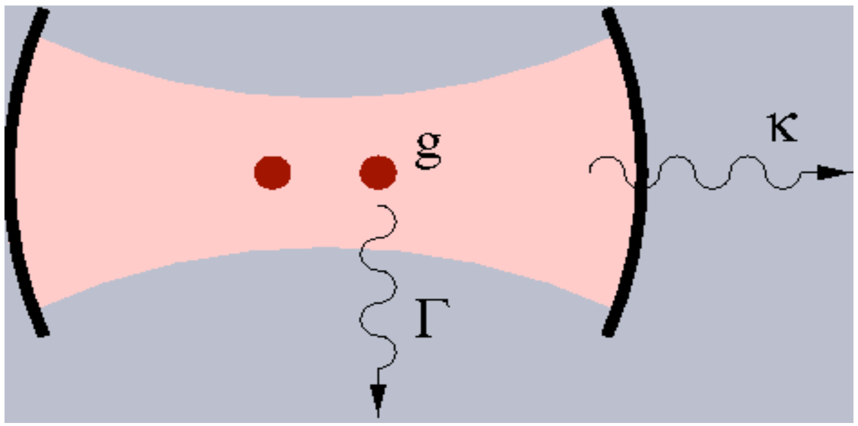
Lecture 3 Cavity qed - couple stationary to flying qubits.

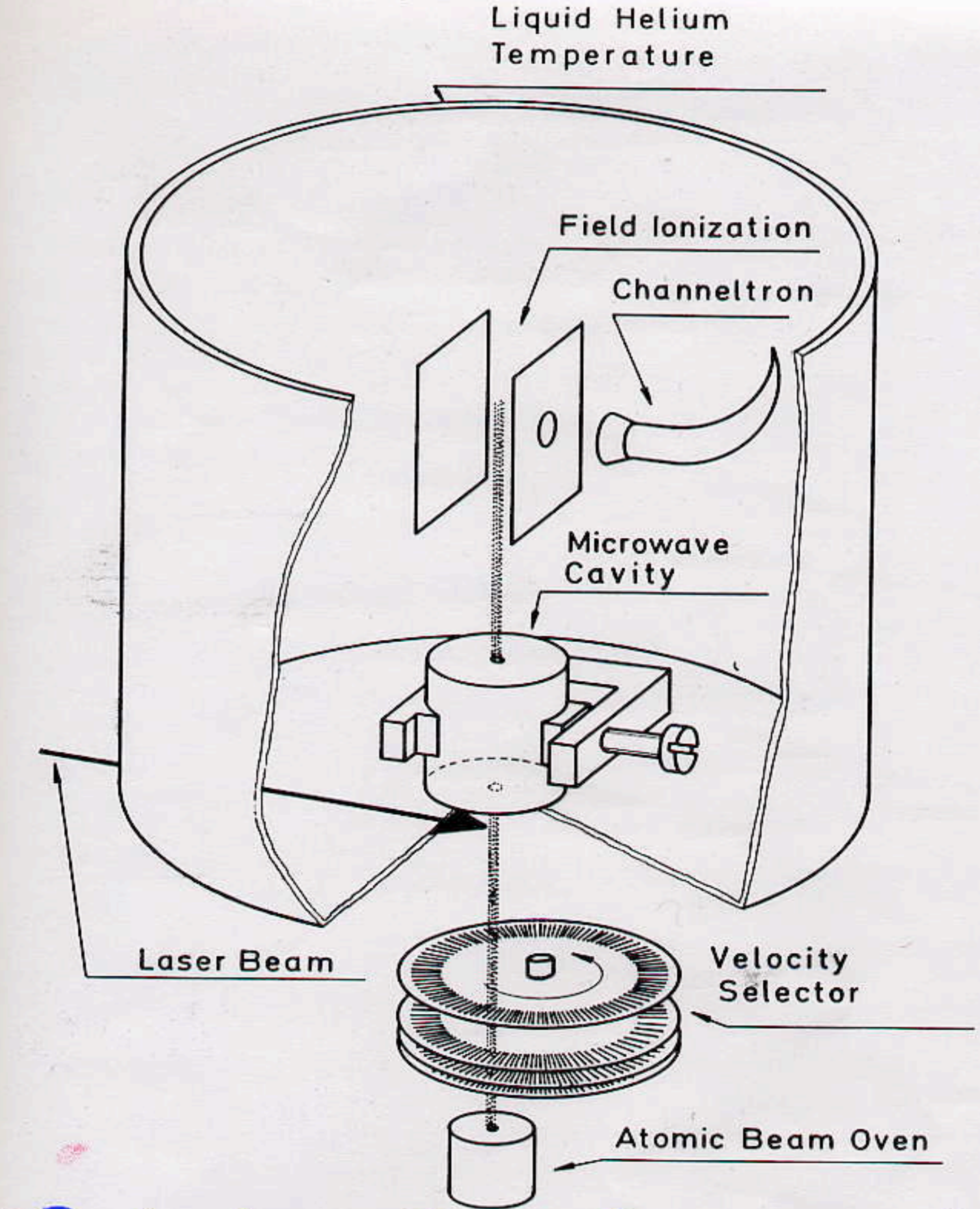




What will this lecture cover?

- Cavity qed, Purcell factor and Rabi oscillations
- Collapses and revivals
- Atom-field entanglement: coupling flying to stationary qubits
- Effects of dissipation
- Entanglement through Bell state measurements

enhance
just one mode
in high Q
cavity

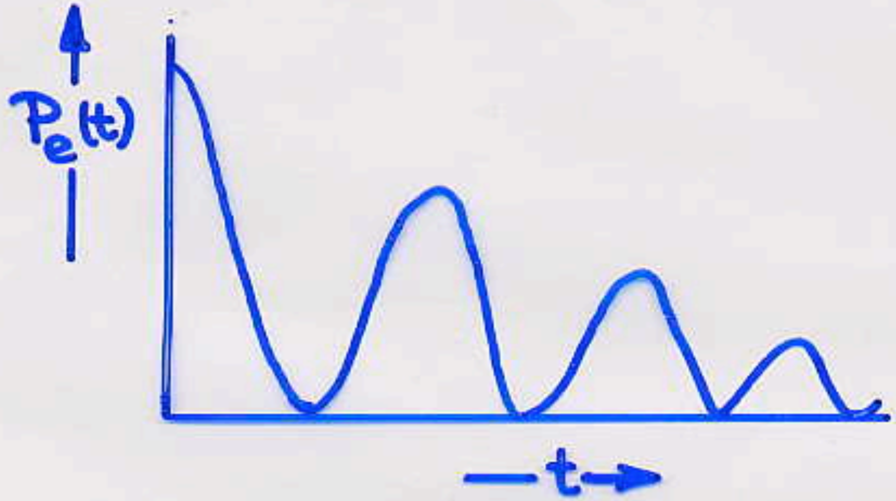


- Realization of one atom – one field mode interaction: Walther et al, München,

1 : Rempe, Walther, Klein   Haroche et al, Paris.

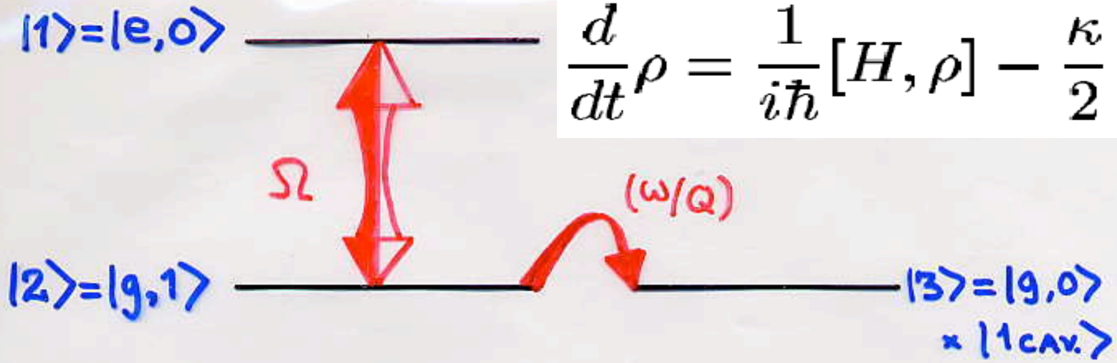
(i) In an Ultra-high Q cavity

Cavity decay rate $\frac{\omega}{Q} < 2\Omega$, then see oscillations at $\sim \Omega$, damped at rate $\frac{\omega}{2Q}$



Spontaneous emission is reversible : Rabi flopping in the atom's own field at frequency given by $\frac{\langle e,0 | \underline{d}.\hat{\underline{E}} | g,1 \rangle}{\hbar}$.

$$|1\rangle = |\uparrow\rangle|0\rangle,$$
$$|2\rangle = |\downarrow\rangle|1\rangle,$$
$$|3\rangle = |\downarrow\rangle|0\rangle.$$



$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] - \frac{\kappa}{2}\left\{a^\dagger a, \rho\right\} + \kappa a \rho a^\dagger$$

$$\frac{d}{dt}\rho_{11} = \frac{i}{2}\Omega\,(\rho_{12} - \rho_{21}) \equiv \frac{i}{2}\Omega V$$
$$\frac{d}{dt}\rho_{22} = -\left(\frac{\omega}{Q}\right)\rho_{22} - \frac{i}{2}\Omega V$$
$$\frac{d}{dt}V = i\Omega W - \frac{1}{2}\left(\frac{\omega}{Q}\right)V$$

Bloch equations with a "one-photon" Rabi frequency Ω.

$W = (\rho_{11} - \rho_{22})$.

OR

$$\frac{d}{dt}\begin{bmatrix}\rho_{11}\\ \rho_{22}\\ V\end{bmatrix} = \begin{bmatrix}0 & 0 & i\Omega/2\\ 0 & -\omega/Q & -i\Omega/2\\ i\Omega & -i\Omega & -\omega/2Q\end{bmatrix}\begin{bmatrix}\rho_{11}\\ \rho_{22}\\ V\end{bmatrix}$$

This has 3 eigenvalues, solutions of

$$\left(\frac{\omega}{2Q} + \lambda\right)\left(\lambda^2 + \frac{\omega}{Q}\lambda + \Omega^2\right) = 0$$

ie $\lambda_0 = -\omega/2Q,$

$$\lambda_\pm = -\frac{\omega}{2Q} \pm \frac{\omega}{2Q}\left(1 - \frac{4\Omega^2 Q^2}{\omega^2}\right)^{1/2}.$$

(ii) Strong Cavity Field Damping?

When $\frac{\omega}{Q} > 2\Omega$, the roots $\lambda_{\pm}$ are real.

λ_+ is smallest and governs evolution:

$$\lambda_+ \simeq -\Omega^2 (Q/\omega)$$

$$\Gamma_{CAV.} = \frac{2 d_{eg}^2 Q}{\hbar \epsilon_0 V} = \frac{\pi d_{eg}^2}{\epsilon_0 \hbar^2} \times N^{CAV}(\omega) \times \hbar\omega$$

where $N^{CAV}(\omega) = \frac{2}{\pi}\left(\frac{Q}{\omega}\right)\left(\frac{1}{V}\right)$ = density of modes/unit frequ/unit volume.

In free space, $V \to \infty \ldots \Rightarrow \Gamma_0$

$$\frac{\gamma_c}{\gamma_f} = \frac{\rho_c(\omega_0)}{\rho_f(\omega_0)} = \frac{2\pi Q}{V_c \omega_0^3} = \frac{Q\lambda_0^3}{4\pi^2 V_c},$$

which is an old nmr result (Purcell).

- IF CAVITY MODE IS RESONANT ($\Delta = 0$)

$$E(\pm) = -i\frac{\omega}{4Q} \pm i\frac{\omega}{4Q}\left[1 - 4\Omega^2\frac{Q^2}{\omega^2}\right]^{1/2}$$

$$\Omega = 2\,|\langle e,0|\underline{d}.\hat{\underline{E}}\,|g,1\rangle|/\hbar.$$

$$|C_0|^2 = \cos^2\Omega t/2\ \exp(-\omega t/2Q) \qquad \frac{\omega}{Q} \ll \Omega$$

AND SPONTANEOUS DECAY IS REVERSIBLE.

- IF $(\omega/Q) > 2\Omega$, OVERDAMPED:

$$E(+) \simeq -i(\omega/4Q) + i(\omega/4Q)\left[1 - 2\Omega^2Q^2/\omega^2\right]$$

$$= -i\,(\Omega^2Q^2/2\omega) = -\frac{i}{2}\gamma_0 \quad \text{AS BEFORE.}$$

$$\frac{\gamma_0(Q)}{\gamma_0(\text{free space})} = \frac{3}{4\pi^2}Q\left(\frac{\lambda_0^3}{V}\right)$$

- OBSERVED:

GABRIELSE + DEHMELT (1985)

GOY et al (1983)

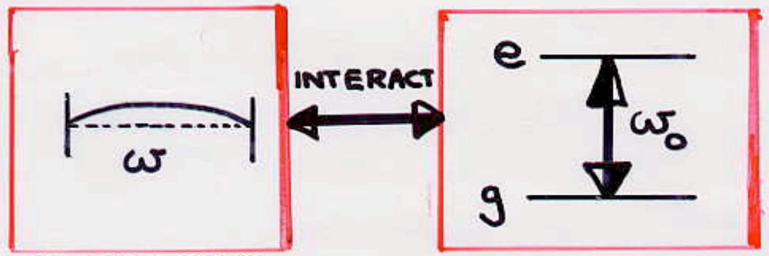
Jaynes-Cummings model and single-mode-atom interactions.

SINGLE ATOM – SINGLE FIELD MODE INTERACTION

SIMPLEST MODEL OF QUANTUM OPTICAL RESONANCE, YET

- EXHIBITS NON-PERTURBATIVE EVOLUTION
- IS SENSITIVE TO PHOTON STATISTICS
- CAN BE REALIZED IN THE LAB.

MODEL CONTAINS



SINGEE FIELD MODE IN CAVITY.

SINGLE TWO-LEVEL ATOM.

In qm $\mathcal{H}_Q = \frac{1}{2}\hbar\omega_0\sigma_3 + \hbar\omega a^{+}a + \hbar\lambda(a^{+}\sigma_- + \sigma_+ a)$

(OR $= \frac{1}{2}\hbar\omega_0\sigma_3 + \hbar\lambda(a^{+}e^{i\omega t}\sigma_- + \sigma_+ a e^{-i\omega t})$)

Note conservation of excitation.

In semiclassical theory

$a \rightarrow \alpha$

$a^{+} \rightarrow \alpha^{*}$.

COHERENT STATE RABI MODEL

$$W_Q(\alpha) = -\sum_{n=0}^{\infty} e^{-\bar{n}} \frac{\bar{n}^n}{n!} \cos 2\lambda\sqrt{n}\,t$$

- NO KNOWN FINITE EXPRESSION
- IF $\bar{n} \gg 1$, USUALLY TAKE $a \rightarrow \alpha$, $a^{\dagger} \rightarrow \alpha^*$

DOES $W_Q(\alpha) \rightarrow -\cos 2\lambda\sqrt{\bar{n}}\,t = W_c(\alpha)$?

NO

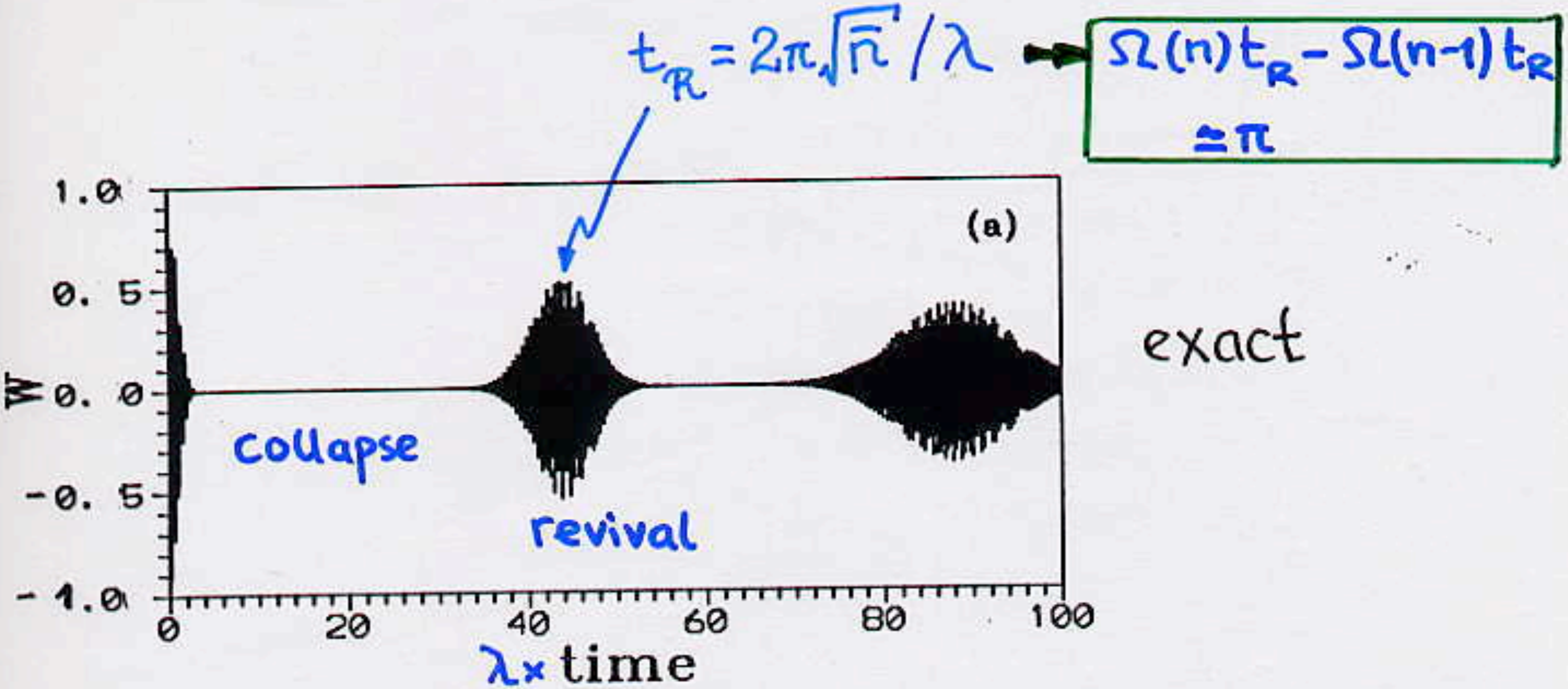
- WHERE IS SEMICLASSICAL LIMIT?
- NOTE POISSON SPREAD OF RABI FREQUENCIES

→ DEPHASING OF RESONANT OSCILLATIONS

→ DECAY OF COHERENCE

Reviewed by B.W. Shore and P.L. Knight
J. Mod. Opt. 40, 1195 (1993)

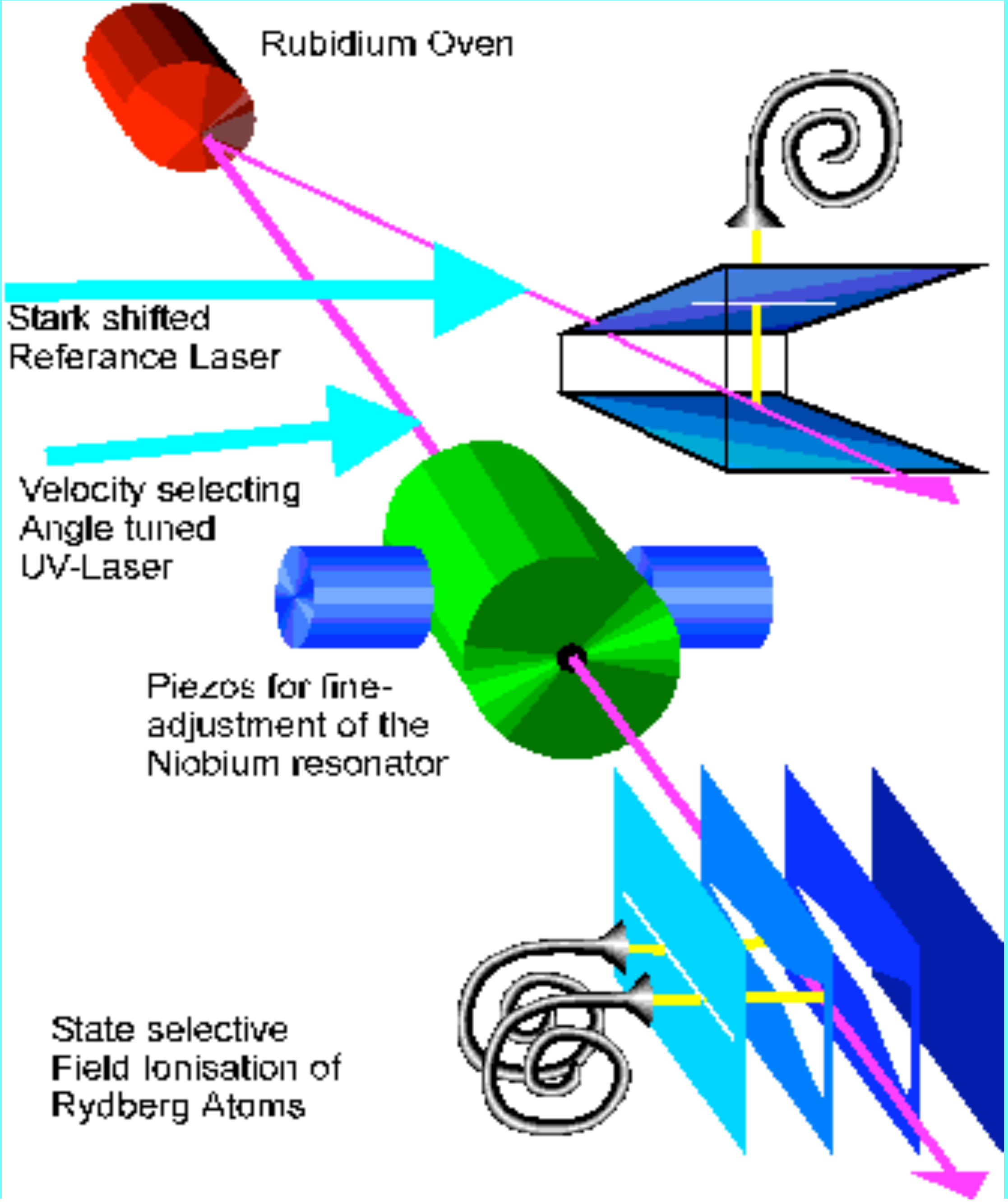
Atomic Collapse and Revival



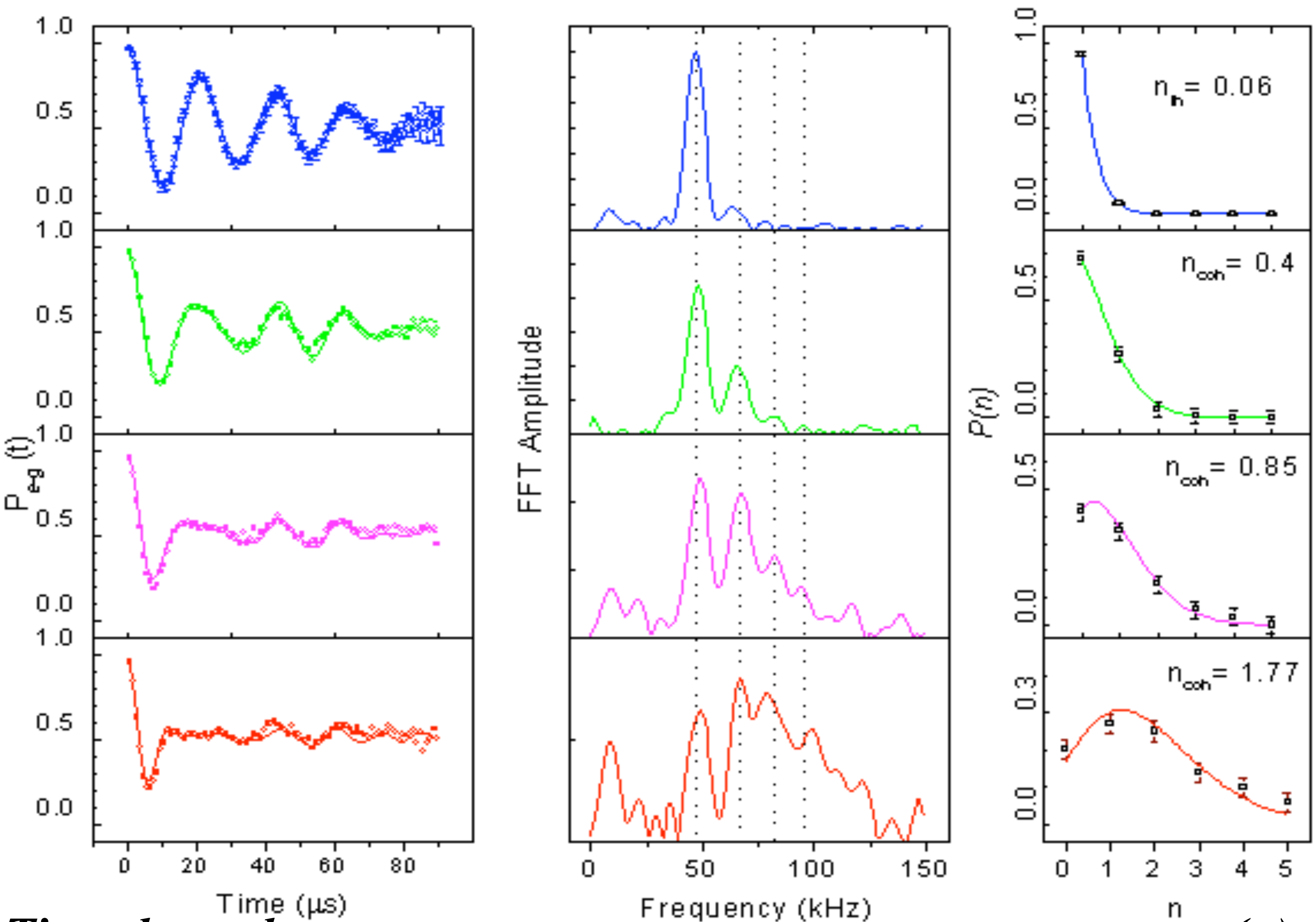
- $\bar{n} = 49$: collapse of inversion

$$W = -\sum_{n=0}^{\infty} P(n) \cos \lambda \sqrt{n}\, t$$

MPQ micromaser set up

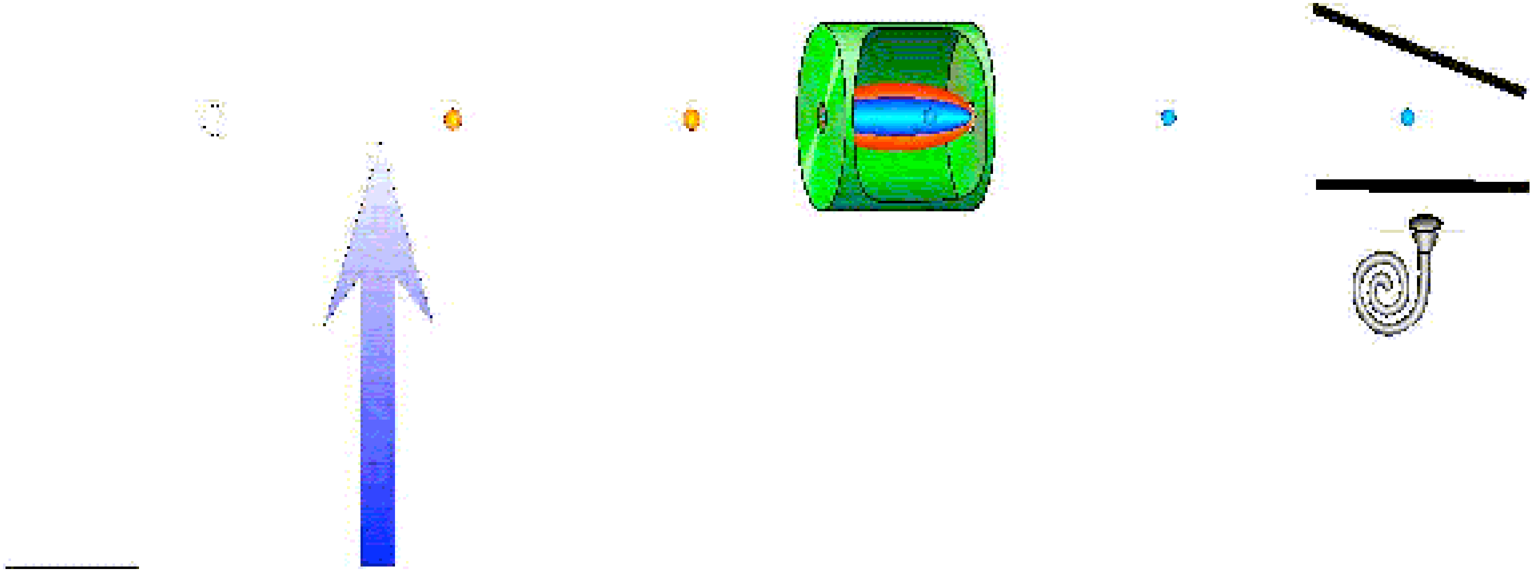


ENS: quantum Rabi oscillations



Time dependence ***spectra*** ***p(n)***

CQED Qubits:Atoms and photons in cavities



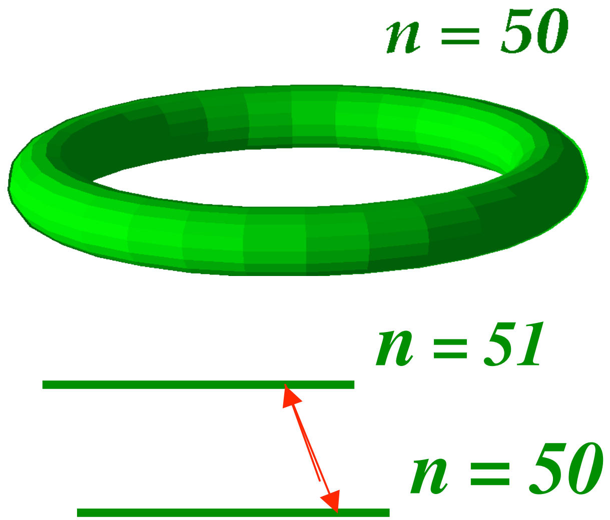
Experiments at ENS, Paris. Also Garching, Caltech..

Qubit realized either by atom (in e or g state) or by photon field (0 or 1 photon in cavity)

Increasing complexity one atom or one photon at a time: from microscopic to mesoscopic world

Raimond, Brune and Haroche, RMP, July 2001

Two essential ingredients



Circular Rydberg atoms

Large circular orbit

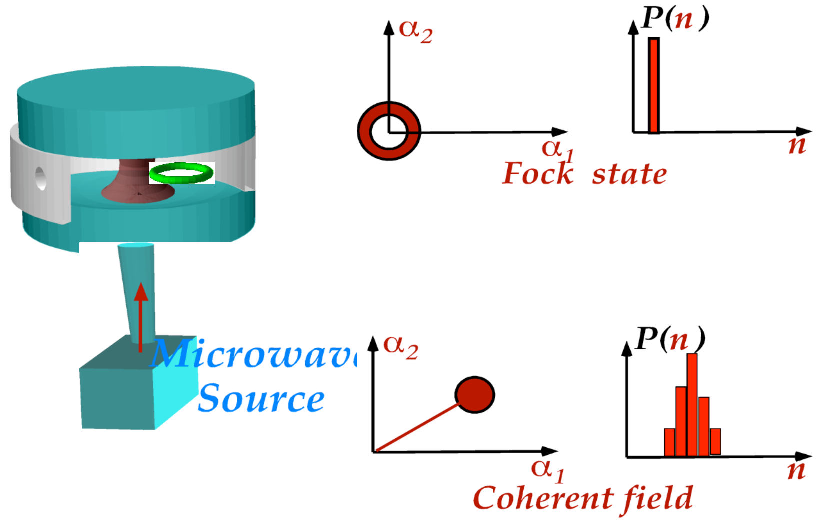
Strong coupling to microwaves

Long radiative lifetimes (30ms)

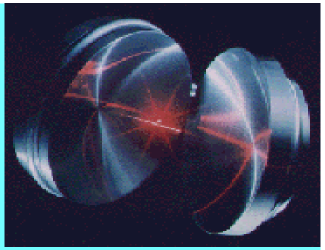
Level tunability by Stark effect

Easy state selective detection

Quasi two-level systems



Superconducting mirror cavity



Gaussian field mode with 6mm waist

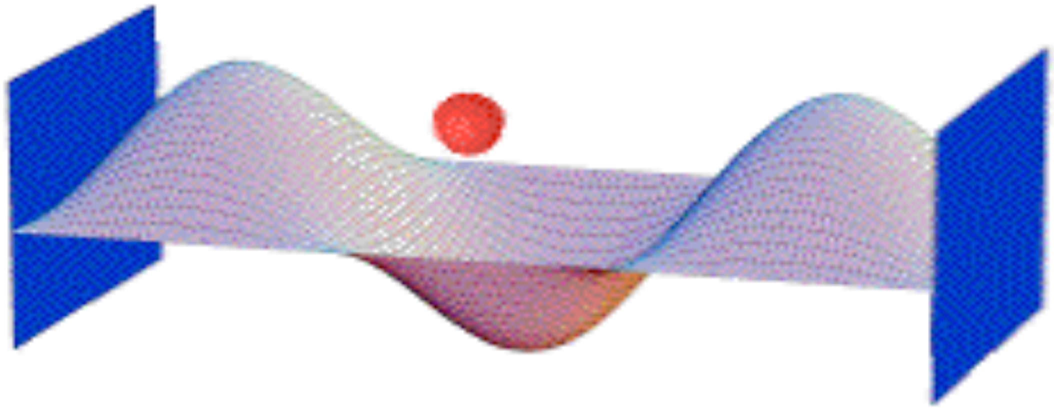
Large field per photon

Long photon life time improved by ring around mirrors (1ms)

Easy tunability

Possibility to prepare Fock or coherent states with controlled mean photon number

Atom field entanglement via vacuum Rabi oscillation



Realizes controlled atom-field entanglement which survives after atom leaves cavity (EPR correlated)

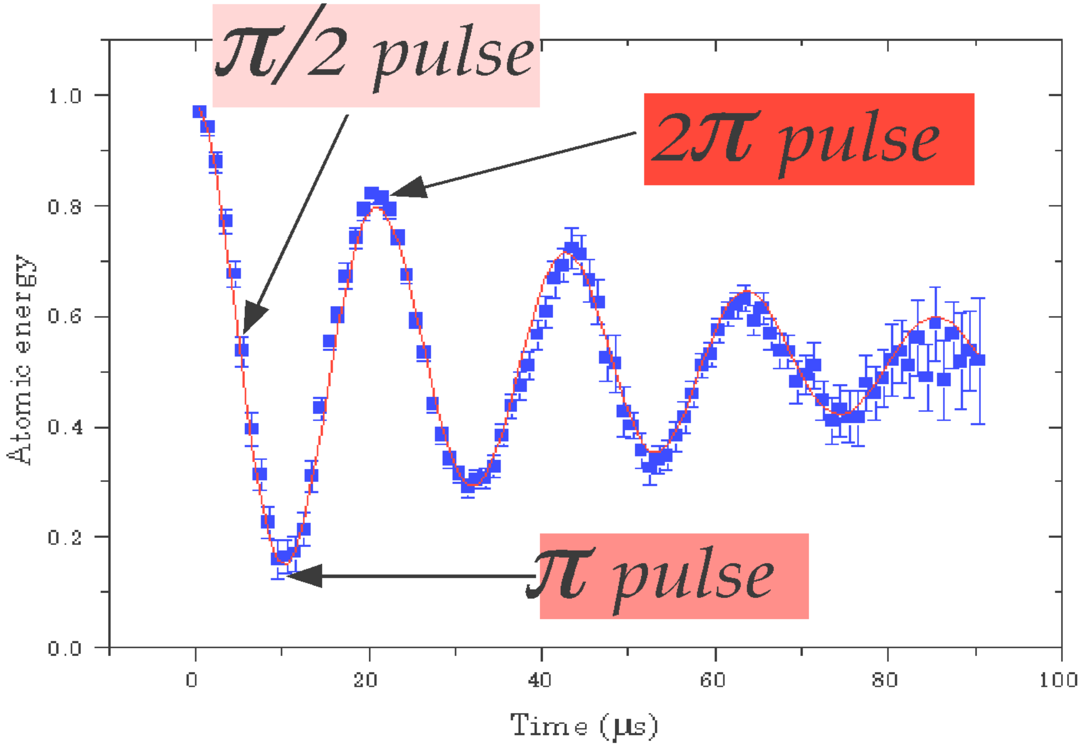
$$|\psi(t)\rangle = \cos(\Omega\, t/2)\,|e,0\rangle - i\sin(\Omega\, t/2)\,|g,1\rangle$$

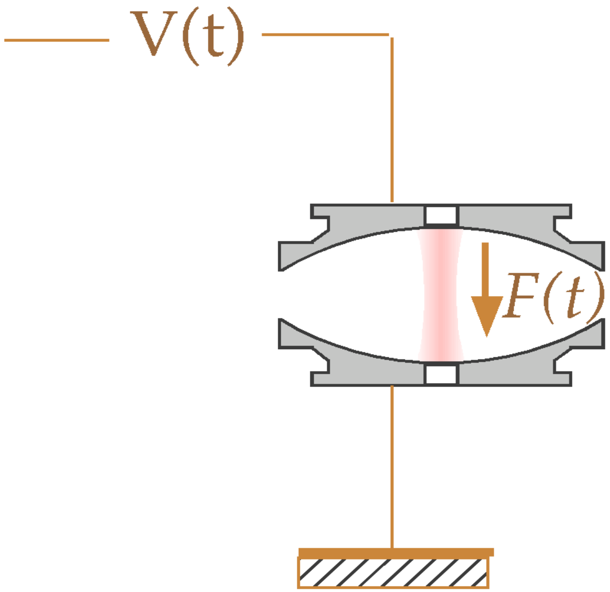
Strong coupling: vacuum Rabi frequency larger than field and atomic decoherence times in Paris experiments

$$\Omega \gg 1/T_{cav},\ 1/T_{at}:$$

$$3.10^5\ s^{-1} \gg 10^3\ s^{-1},\ 30\ s^{-1}$$

Haroche group: controlled generation of superpositions & entanglement



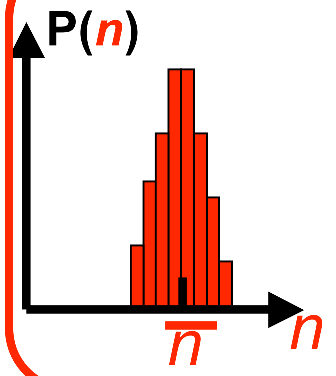


Electric field F(t) used to tune atoms in resonance with C for a determined time, realizing proper Rabi pulse conditions...

Coupling atomic eigenstates to mesoscopic coherent field (1st order in $n-\bar{n}$)

$$\left(\frac{|e\rangle+|g\rangle}{\sqrt{2}}\right)\otimes|\alpha\rangle \rightarrow e^{-2i\bar{n}\varphi(t)}|\psi_+^{at}(t)>\otimes|\alpha_+(t)>$$

$$\frac{1}{\sqrt{2}}\left(e^{-i\varphi(t)}|e>+|g>\right) \qquad |\alpha e^{-i\varphi(t)}>$$



$$\left(\frac{|e\rangle-|g\rangle}{\sqrt{2}}\right)\otimes|\alpha\rangle \rightarrow e^{2i\bar{n}\varphi(t)}|\psi_-^{at}(t)>\otimes|\alpha_-(t)>$$

$$\frac{1}{\sqrt{2}}\left(e^{i\varphi(t)}|e>-|g>\right) \qquad |\alpha e^{i\varphi(t)}>$$

The atomic eigenstates induce opposite phase rotations on coherent fields. Field reacts back on atom and induces a correlated atomic dipole phase shift. Effect vanishes at classical limit ($n\rightarrow\infty$)

$$\varphi(t)=\frac{\Omega\ t}{4\sqrt{\bar{n}}}$$

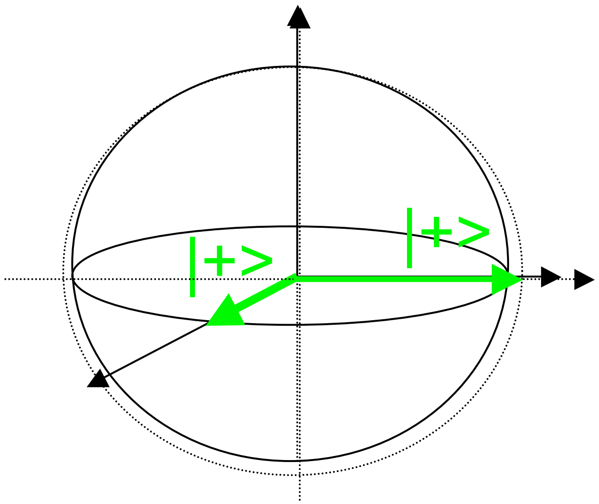
Exact calculation possible (ensemble of 2 level systems)

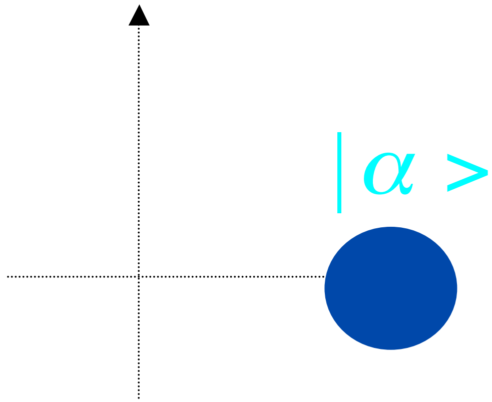
Representation in phase space

Atomic state in equatorial plane of Bloch sphere

Representation in the same plane

Coherent field in phase space





Equatorial plane of Bloch sphere

Phase correlation

↕

Atomic dipole and field « aligned »

$$|e,\alpha\rangle = \left(\frac{1}{\sqrt{2}}\right)\left(|+,\alpha\rangle + |-,\alpha\rangle\right) \rightarrow$$

$$\frac{1}{\sqrt{2}}\left(e^{-2i\bar{n}\varphi(t)}\,|\psi_+^{at}(t)>|\alpha_+(t)> + e^{2i\bar{n}\varphi(t)}\,|\psi_-^{at}(t)>|\alpha_-(t)>\right)$$

$$|+,\alpha> \rightarrow |\psi_+^{at}(t),\alpha_+(t)>$$

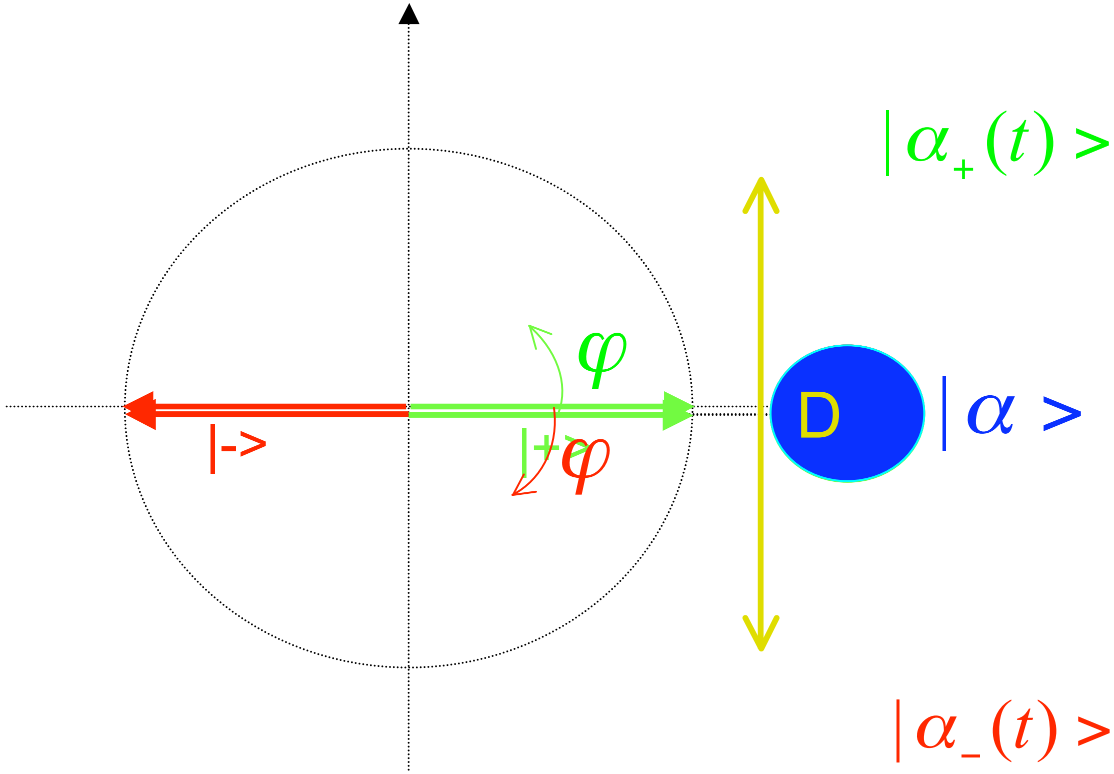
$$|-,\alpha> \rightarrow |\psi_-^{at}(t),\alpha_-(t)>$$

Single atom-field mesoscopic entanglement

Fast quantum phase responsible for Rabi oscillation

A microscopic object leaves its imprint on a mesoscopic one

Schrödinger-cat situation



"Size" of the cat=D

$$D = 2\sqrt{\bar{n}}\sin\left(\frac{\Omega_0 t}{4\sqrt{\bar{n}}}\right)$$

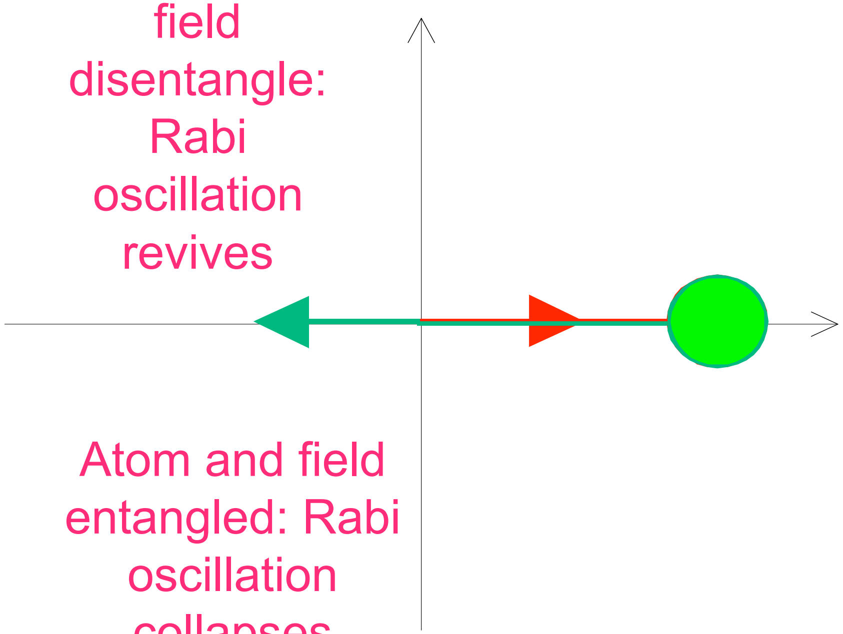
The field acts as a Which-Path detector

Contrast of Rabi oscillations

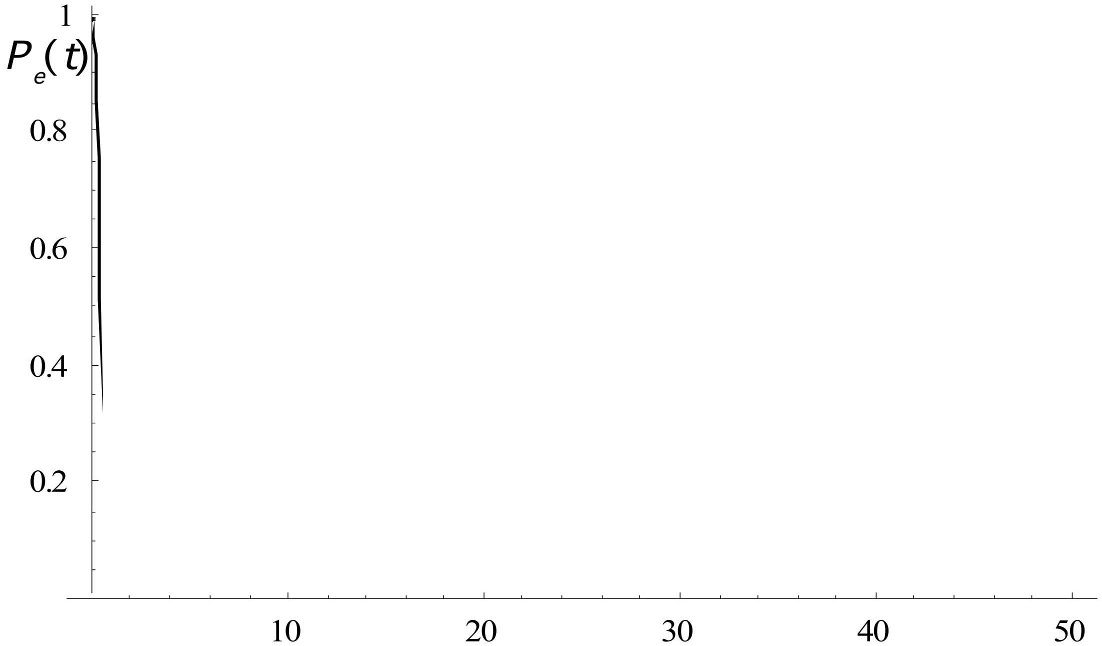
$$C(t) = |<\alpha_+(t)\,|\,\alpha_-(t)>| = e^{-D^2(t)}$$

Rabi oscillation in mesoscopic field collapses and revives as field components separate and recombine

Atom and field disentangle: Rabi oscillation revives



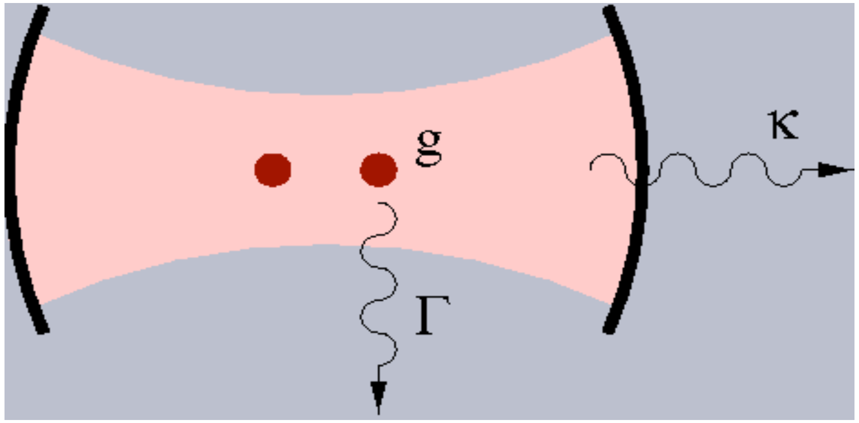
Atom and field entangled: Rabi oscillation collapses



Complementarity in action!

At classical limit, collapse and revival times rejected to t = ∞

CQED Entanglement generation and dissipation.



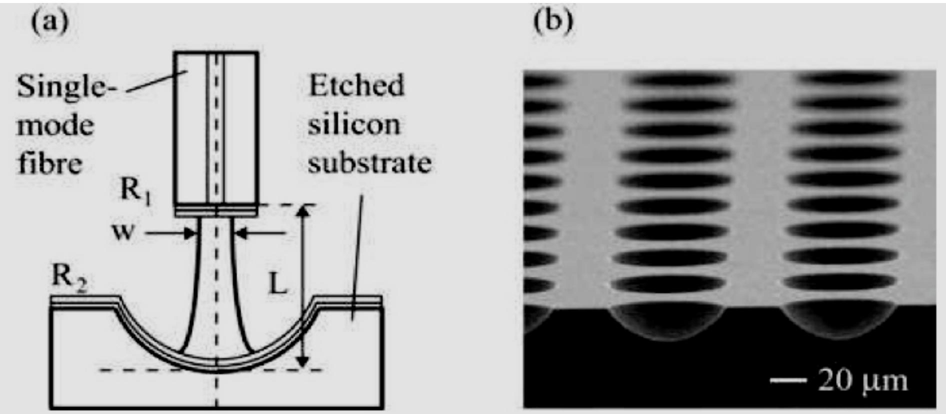


FIG. 3: The optical microcavity. (a) Plano-concave optical microcavity with length L and cavity waist w. The plane mirror with reflectivity R_1 is formed on the fibre tip by applying a pull-off coating. The concave mirror with reflectivity R_2 is formed by sputtering Au onto the etched silicon wafer (see main text for details). (b) SEM micrograph of an array of isotropically etched spherical mirrors on a silicon wafer.

Hinds group atom chip cavities

Origin of Noise

Interaction with an unobserved environment!

System becomes entangled with the environment. Tracing out unobserved environment leads to loss of coherence and entanglement.

Example: Two atoms suffer random phase flips, initially EPR state then becomes separable mixture.

$$(|eg\rangle + |ge\rangle)_S \otimes |0\rangle_E \rightarrow (|eg\rangle + |ge\rangle)_S \otimes |0\rangle_E + (|eg\rangle - |ge\rangle)_S \otimes |1\rangle_E$$

State of system: $\rho_S = \frac{1}{2}(|ge\rangle\langle ge| + |eg\rangle\langle eg|)$ is disentangled.

Ways to Deal with Noise

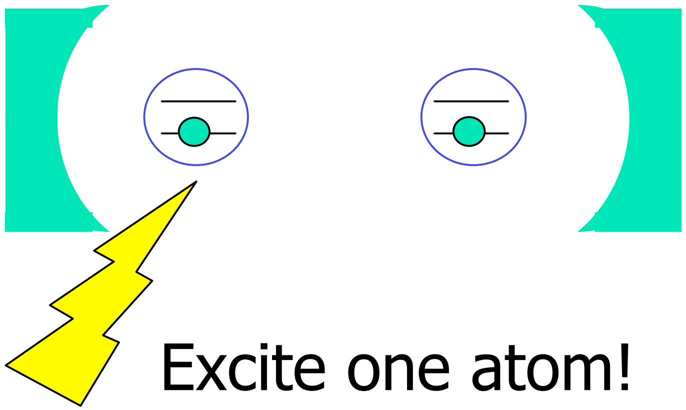
- **Active Stabilization**
- Quantum error correction (in computation)
- Entanglement purification (in communication)

- **Passive stabilization**
- Intrinsically fault tolerant system (eg geometric phases)

- **Employ noise constructively!**
- Using noise to generate entanglement e.g. for use in communcation and computation

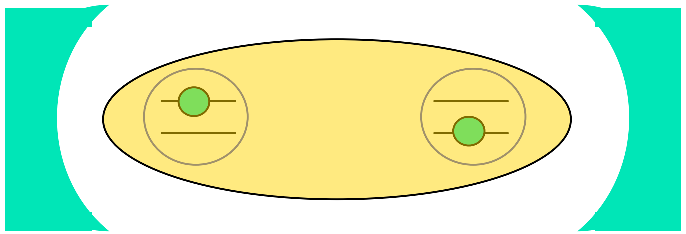
Two atoms in a cavity: entanglement via decay

M.B. Plenio et al, Phys. Rev. A 59, 2468 (1999)



Cavity in vacuum state, with two atoms in their ground state.

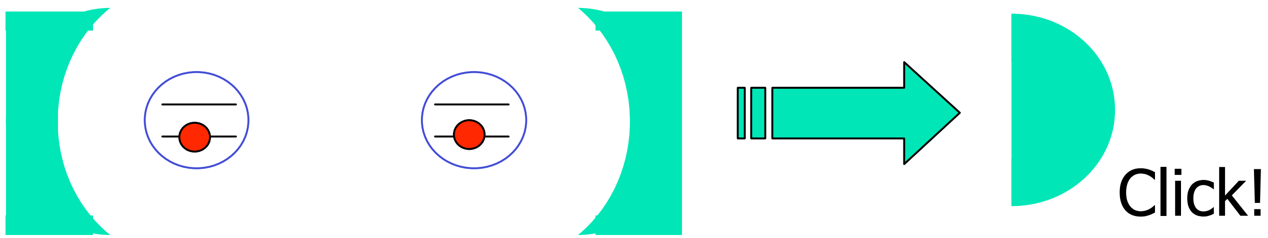
Excite one atom!



Exchange of excitation between the atoms and the cavity mode.

Two atoms in a cavity cont'd

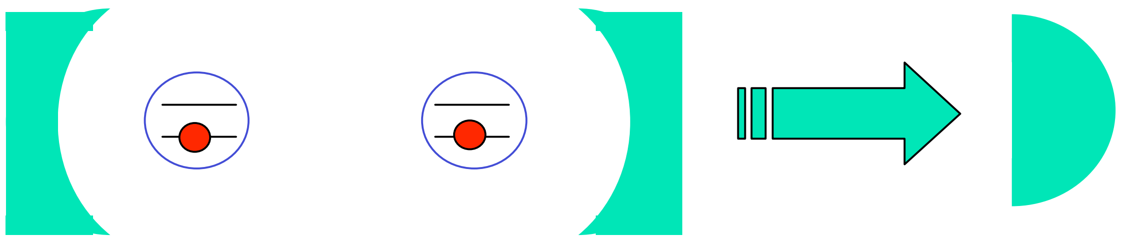
First possibility:



A photon may escape the cavity and be detected outside. In that case both system are in the ground state → no entanglement.

Two atoms in a cavity cont'd

Second possibility:



No photon is ever detected outside. In that case the cavity mode is in the ground state, and the two atoms are in the anti-symmetric singlet state

$|\Psi^{-}\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}$ → entanglement.

The maths

Hamilton operator for symmetrically placed atoms:

$$H = \mathrm{h} g(a\sigma_+^{(a)} + a^+\sigma_-^{(a)}) + \mathrm{h} g(a\sigma_+^{(b)} + a^+\sigma_-^{(b)})$$

Stable state:

$$|\Psi\rangle = \frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes |0_{photons}\rangle$$

$$H|\Psi\rangle = \frac{a^+\sigma_-^a|eg\rangle - a^+\sigma_-^b|ge\rangle}{\sqrt{2}} \otimes |0_{Photons}\rangle = \frac{|gg\rangle - |gg\rangle}{\sqrt{2}} \otimes |1_{Photons}\rangle = 0$$

The maths cont'd

Initial state:

$$|\Psi_{ini}\rangle = |eg\rangle \otimes |0_{photons}\rangle$$

$$= \frac{1}{\sqrt{2}}\left(\frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes |0_{photons}\rangle + \frac{|eg\rangle + |ge\rangle}{\sqrt{2}} \otimes |0_{photons}\rangle\right)$$

State remains invariant: If the detector never clicks, then this is equivalent to a projection onto this state.

Dynamics leads to photons in cavity which eventually decay and are detected.

$$|\Psi_{ini}\rangle = \frac{1}{\sqrt{2}}\left(\frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes |0_{photons}\rangle + \frac{|eg\rangle + |ge\rangle}{\sqrt{2}} \otimes |0_{photons}\rangle\right)$$

Time evolution

$$|\Psi_{ini}\rangle = \frac{1}{\sqrt{2}}\left(\frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes |0_{phot}\rangle + \sqrt{1-\varepsilon}\,|\varphi\rangle|0_{phot}\rangle + \sqrt{\varepsilon}\,|gg\rangle|1_{phot}\rangle\right)$$

No photon detected

$$|\Psi_{ini}\rangle = \sqrt{1+\frac{\varepsilon}{2}}\,\frac{|eg\rangle - |ge\rangle}{\sqrt{2}} \otimes |0_{phot}\rangle + \sqrt{1-\frac{\varepsilon}{2}}\,|\varphi\rangle|0_{phot}\rangle$$

Weight of singlet state increased!

One photon detected

$$|\Psi_{ini}\rangle = |gg\rangle|1_{phot}\rangle$$

End of evolution!

Entanglement between distant cavities via spontaneous decay

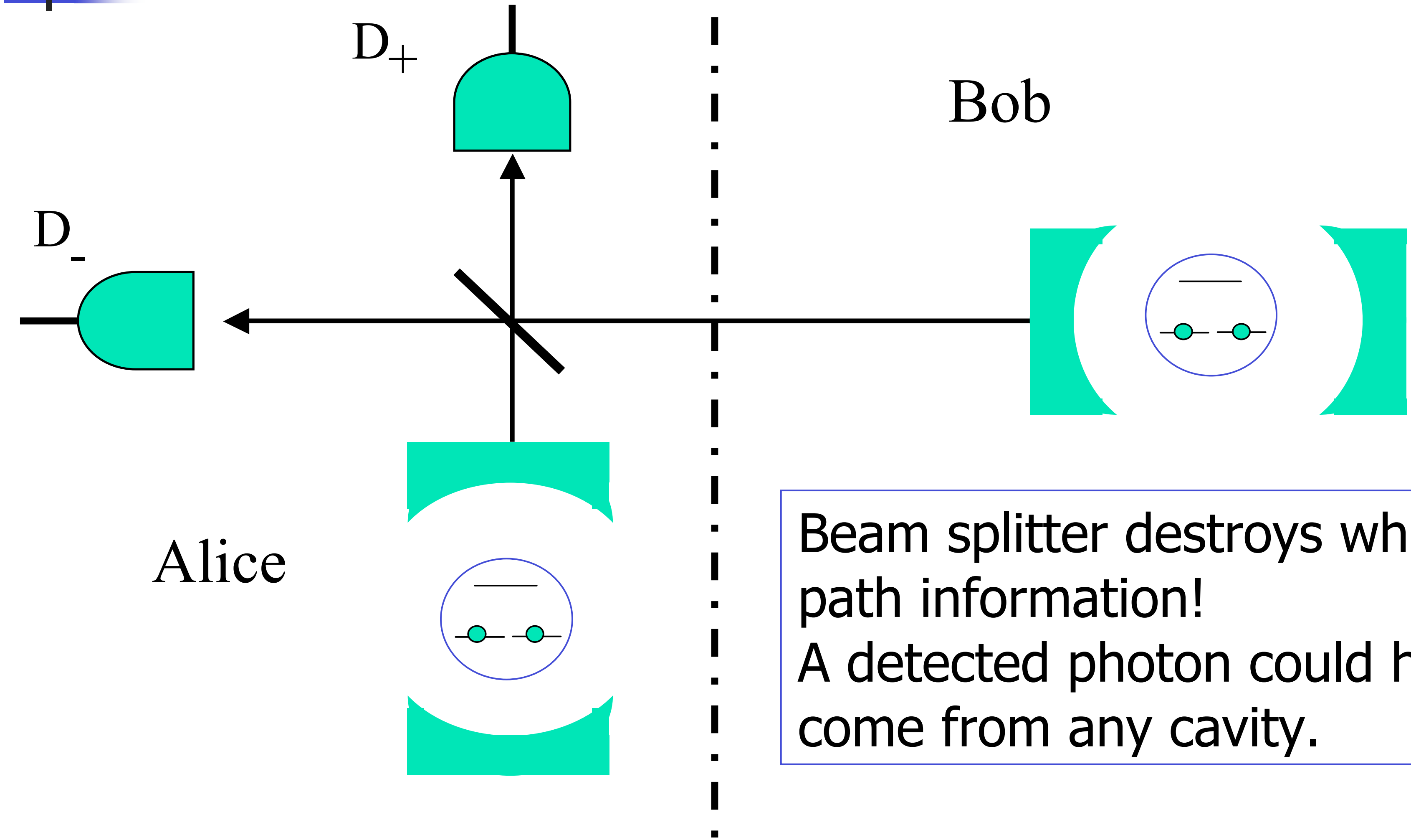
Entangled atoms in a cavity may look a bit boring.

Entanglement between different cavities, however, would allow to build a quantum communication network.

Again, use the spontaneous decay from the cavity mirrors as a means to create entanglement between distant atoms.

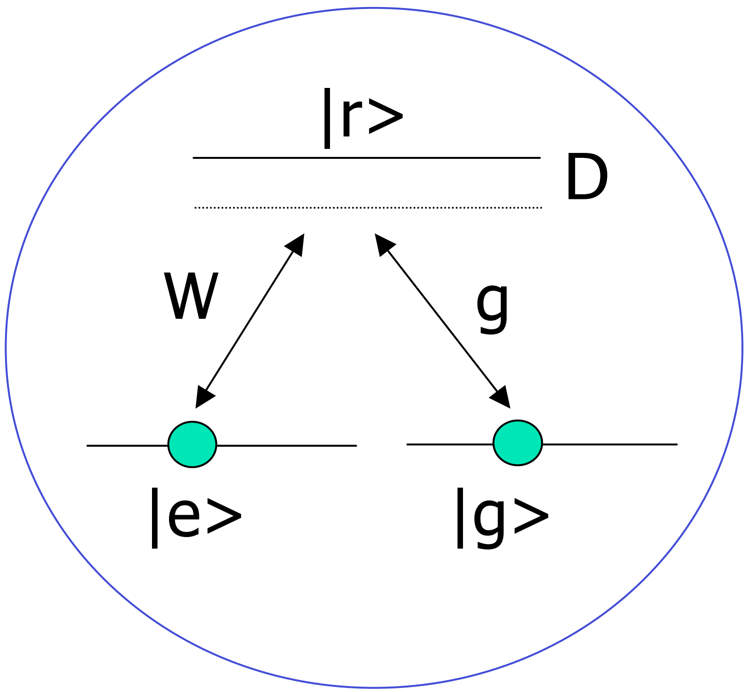
Entanglement between distant cavities.

Generalization of S. Bose, P.L. Knight, M.B. Plenio and V. Vedral, PRL 58, 5158 (1999)



Beam splitter destroys which-path information!
A detected photon could have come from any cavity.

The protocol



Effective Hamilton operator:

$$H = \frac{g\Omega}{\Delta}\left(a|e\rangle\langle g| + a^{+}|g\rangle\langle e|\right)$$

Switching on g,Wmaps atomic state

$$|\Psi_{atom}\rangle = |e\rangle$$

into field mode!

$$|\Psi_{atom-field}\rangle = \frac{1}{\sqrt{2}}\left(|e0\rangle + i|g1\rangle\right)$$

Bob starts with atom in state $|\Psi_{atom}\rangle = |e\rangle$

Entangle atom with cavity mode

$$|\Psi_{atom-cavity}\rangle = \frac{1}{\sqrt{2}}(|e0\rangle + i|g1\rangle)$$

Both Alice and Bob end their action at the same time!
Total state:

$$|\Psi_{tot}\rangle = \frac{1}{2}(|ee00\rangle + |gg11\rangle + i|ge10\rangle + i|eg01\rangle)$$

$$|\Psi_{tot}\rangle = \frac{1}{2}\left(|ee00\rangle + |gg11\rangle + i|ge10\rangle + i|eg01\rangle\right)$$

Alice looks for detection events:
No-photons or two photons are failures and require rerun!

Click '+' : Project modes onto state |01>+|10>

$$|\Psi_{atoms}\rangle = \frac{1}{\sqrt{2}}\left(|eg\rangle + |ge\rangle\right)$$

Click '-' : Project modes onto state |01> -|10>

$$|\Psi_{atoms}\rangle = \frac{1}{\sqrt{2}}\left(|eg\rangle - |ge\rangle\right)$$

With success rate of 50 percent on holds maximally entangled atoms!

Summary and conclusion

- Entanglement is sensitive to noise.
- Can fight noise actively.
- Can also use dissipation to generate entanglement.
- Even thermal noise as only driving force can generate entanglement (Plenio & Huelga; Bose et al..)

Further reading

- refs here