Insertion Device Radiation

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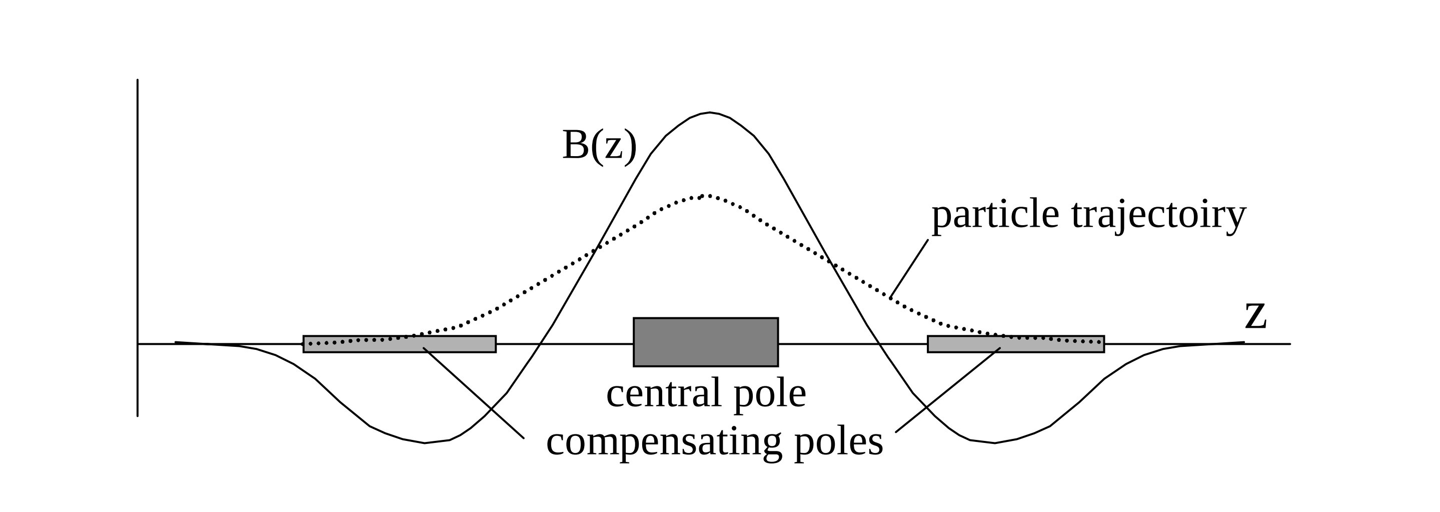
Insertion devices do not change the shape of the storage ring!

$$\int_{-\infty}^{+\infty} B_y(y=0,z)\,\mathrm{d}z = 0$$

- **Wavelength shifter**
- **Wiggler magnet**
- **Undulators**
- **Super bends**

Purpose: **harden radiation**
increase intensity
high brightness monochromatic radiation
elliptically polarized radiation

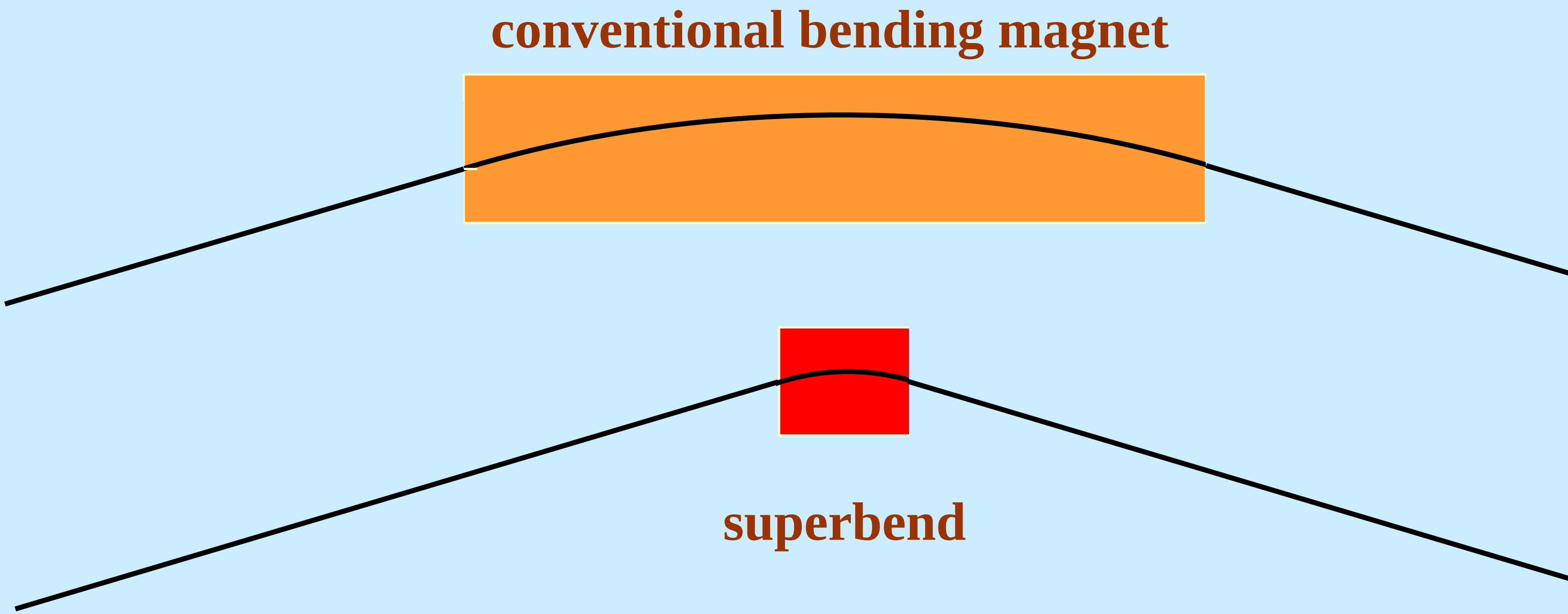
wave length shifter



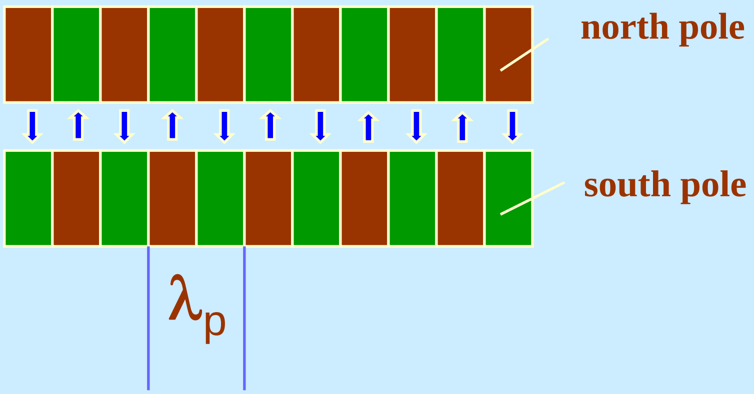
$$\int_{-\infty}^{+\infty} B_y(y = 0, z)\mathrm{d}z = 0$$

Super bends:

replace conventional bending magnets with super conducting bending magnets causing the same deflection angle.



Periodically deflecting magnets:



Wiggler magnets, strong field
Undulators, weak field

Wiggler magnets produce ordinary, broad band synchrotron radiation;
Intensity increased by factor N_p (# of poles)

deflection angle per half-pole

$$B_y(z) = B_0 \cos k_\mathrm{p} z$$

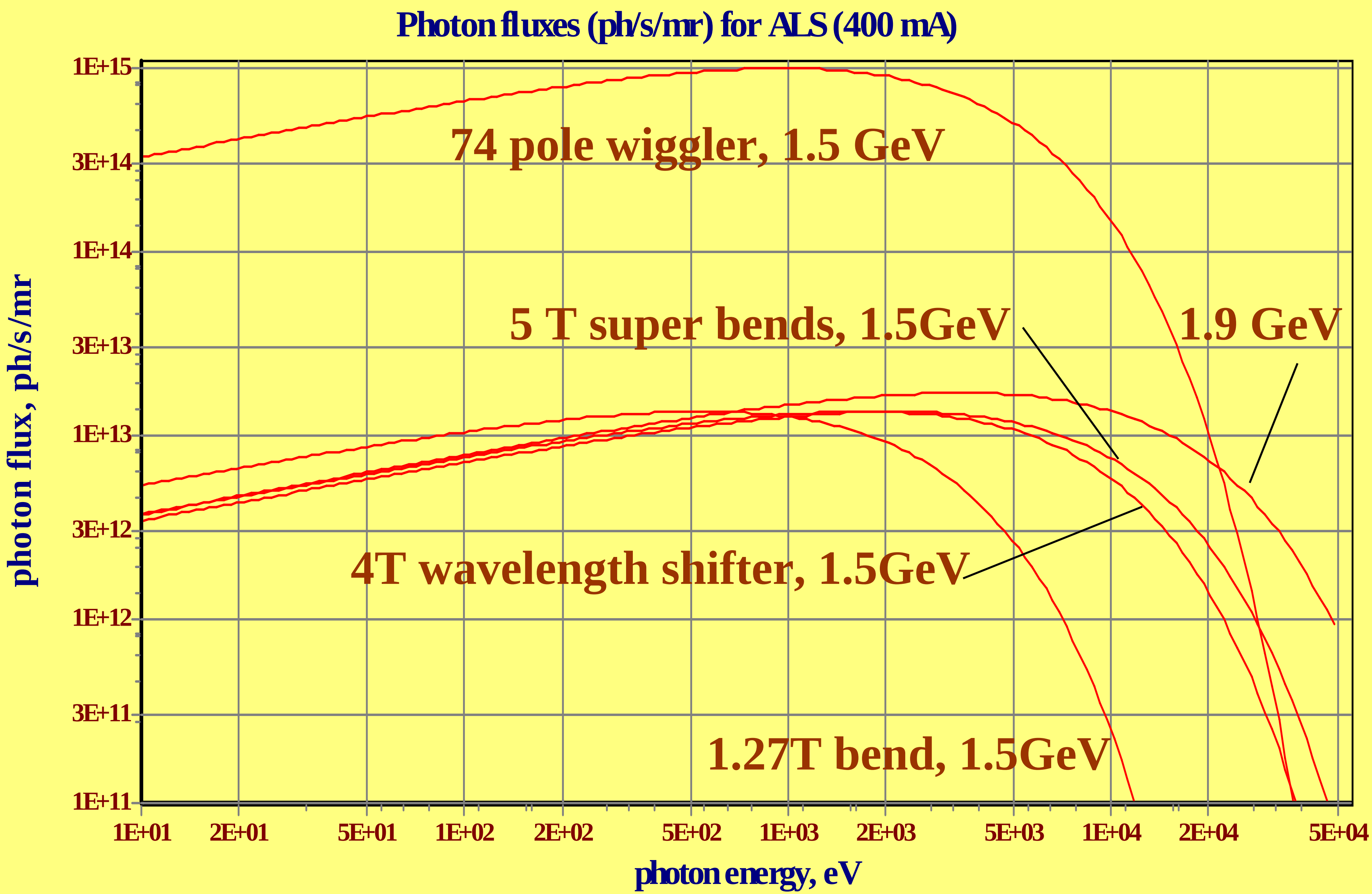
$$d\theta = \frac{dz}{\rho} = \frac{eB}{cp}dz \quad \Rightarrow \quad \theta = \int \frac{dz}{\rho} = \frac{eB}{cp}\int_0^{\lambda_\mathrm{p}/4} \cos k_\mathrm{p} z\, dz = \frac{eB_0\lambda_\mathrm{p}}{2\pi cp}$$

$$\theta = \frac{eB_0\lambda_\mathrm{p}}{2\pi cp} = \frac{K}{\gamma}$$

K: undulator/wiggler strength parameter

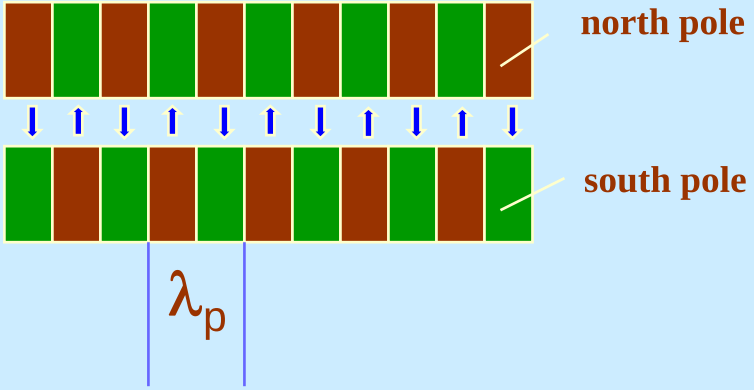
$$K = \frac{eB_0\lambda_\mathrm{p}}{2\pi mc^2\beta} = 0.934\, B_0(\mathrm{T})\, \lambda_\mathrm{p}(\mathrm{cm})$$

Insertion Device Radiation



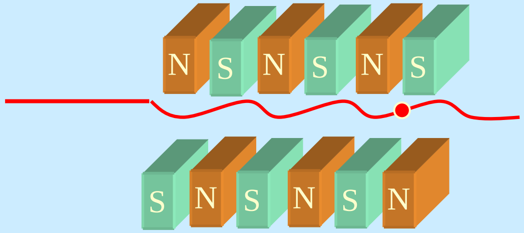
Insertion Device Radiation

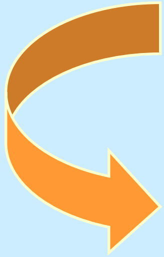
assume a magnetic field $B_y(z) = B_0 \cos k_p z$



$$k_p = \frac{2\pi}{\lambda}$$

electron performs sinusoidal oscillations





sinusoidal perturbation of field lines

sinusoidal perturbation of field lines

N_p undulator periods ⟷ N_p field oscillations

$$E(t) = \begin{cases} E_0 \sin\omega_0 t & \text{for } -\tfrac{1}{2}N_p T_0 < \omega_0 t < \tfrac{1}{2}N_p T_0 \\ 0 & \text{elsewhere} \end{cases}$$

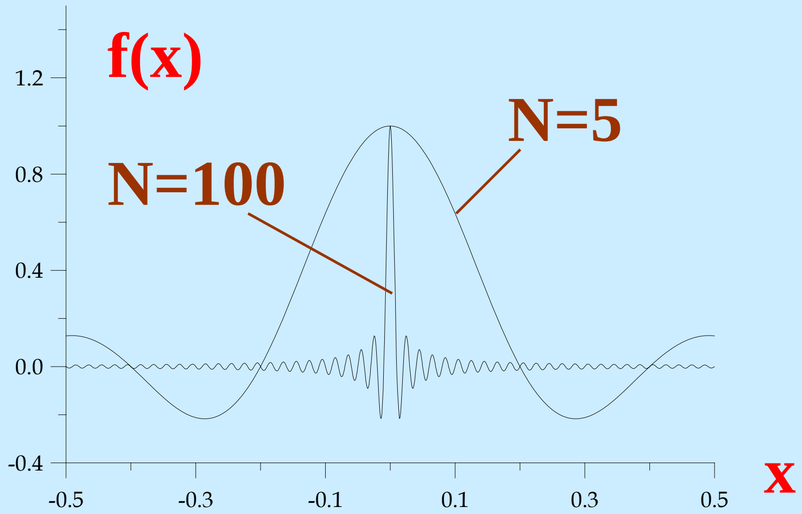
spectrum

$$E(\omega) = \int E(t)\, e^{-i\omega t} dt = \int_{-\infty}^{\infty} E_0 \sin\omega_0 t\, e^{-i\omega t} dt = E_0 \int_{-\infty}^{\infty} e^{-i(\omega_0-\omega)t} dt$$

$$E_0 \int_{-\frac{1}{2}N_p T_0}^{\frac{1}{2}N_p T_0} e^{-i(\omega_0-\omega)t} dt = E_0 \frac{e^{i(\omega_0-\omega)\frac{1}{2}N_p T_0} - e^{-i(\omega_0-\omega)\frac{1}{2}N_p T_0}}{i(\omega_0-\omega)} = E_0 N_p T_0 \frac{\sin(\omega_0-\omega)\frac{1}{2}N_p T_0}{(\omega_0-\omega)\frac{1}{2}N_p T_0}$$

$$E(\omega) = E_0 N_p T_0 \frac{\sin(\omega_0-\omega)\frac{1}{2}N_p T_0}{(\omega_0-\omega)\frac{1}{2}N_p T_0}$$

Sinc-function: $f(x) = \dfrac{\sin \pi N x}{\pi N x}$



f(0)=1 and

f(y)=0 for y=1/N
or for

$$(\omega_0 - \omega)\tfrac{1}{2} N_p T_0 = \pi$$

line width: $\dfrac{\delta\omega}{\omega_0} = \pm\dfrac{\omega_0 - \omega}{\omega_0} = \dfrac{2\pi}{T_0}\dfrac{1}{N_p}\dfrac{1}{\omega_0} = \dfrac{1}{N_p}$

$$\boxed{\frac{\delta\omega}{\omega_0} = \pm\frac{1}{N_p}}$$

What is ω_0 ?

undulator period: λ_p

in electron rest system: $\lambda_p^* = \frac{\lambda_p}{\gamma}$

in lab system (Doppler effect): $\omega = \omega_p^* \gamma (1 + \mathbf{n}_z^* \beta)$

or $\lambda = \frac{\lambda_p}{\gamma^2 (1+\mathbf{n}_z^* \beta)}$

with $\mathbf{n}_z = \frac{\beta + \mathbf{n}_z^*}{1+\mathbf{n}_z^* \beta}$ ⟹ $\lambda = \frac{\lambda_p}{\gamma^2} \frac{\mathbf{n}_z}{\beta + \mathbf{n}_z^*} = \frac{\lambda_p}{\gamma^2} \frac{\cos\theta}{1+\cos\theta^*}$

$\sin\theta = \frac{\sin\theta^*}{\gamma(1+\beta\cos\theta^*)}$ ⟹ $\theta \approx \frac{\sin\theta^*}{\gamma(1+\beta\cos\theta^*)}$

or

$\gamma^2\theta^2 = \frac{\sin^2\theta^*}{(1+\beta\cos\theta^*)^2} = \frac{1-\cos\theta^*}{1+\cos\theta^*}$ ⟹ $\cos\theta^* = \frac{1-\gamma^2\theta^2}{1+\gamma^2\theta^2}$

$$\lambda = \frac{\lambda_p}{\gamma^2}\frac{\cos\theta}{1+\cos\theta^*} = \frac{\lambda_p}{\gamma^2}\frac{1}{1+\frac{1-\gamma^2\theta^2}{1+\gamma^2\theta^2}} = \frac{\lambda_p}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$

$$\theta^2 = (\theta_{und} + \theta_{obs})^2 = \theta_{und}^2 + 2\theta_{und}\theta_{obs} + \theta_{obs}^2$$

$$\theta_{und} = \frac{K}{\gamma}\cos k_p z$$ $\langle\theta_{und}\rangle = 0$ and $\left\langle\theta_{und}^2\right\rangle = \frac{1}{2}\frac{K^2}{\gamma^2}$

fundamental undulator wavelength:

$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1+\frac{1}{2}K^2+\gamma^2\theta_{obs}^2\right)$$

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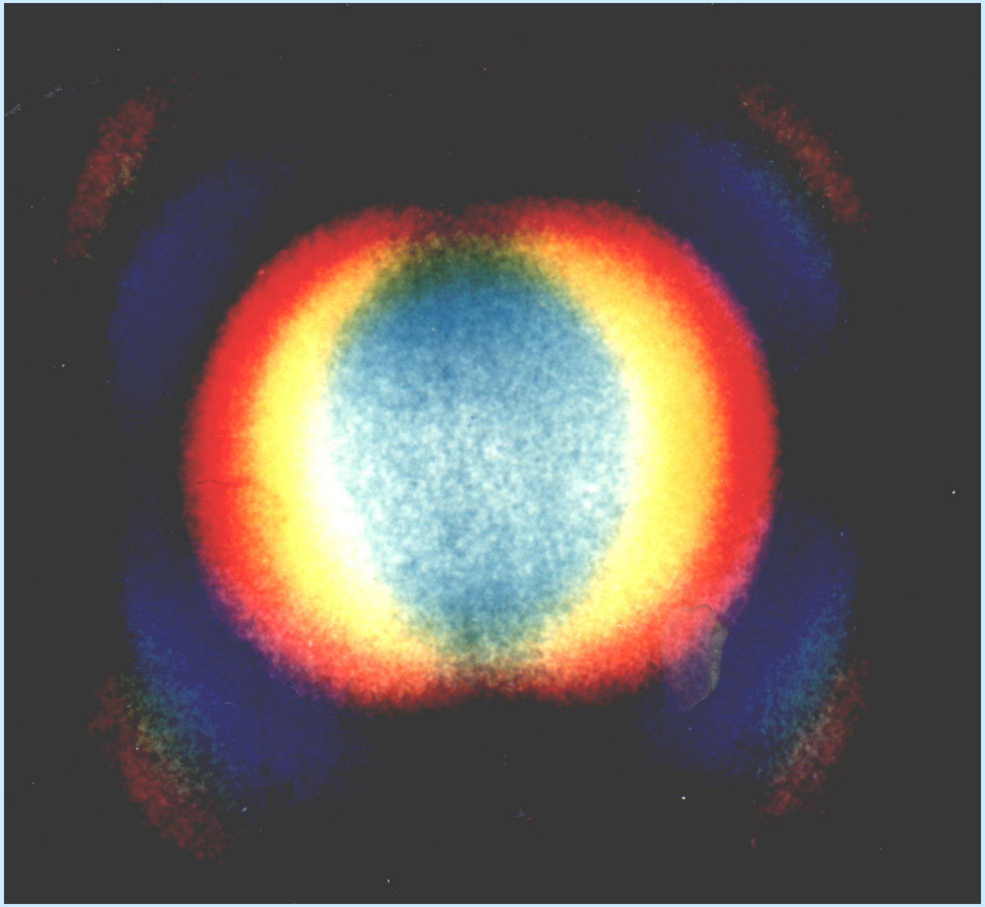


$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta^2_{obs}\right)$$

Insertion Device Radiation

$$\lambda_i = \frac{\lambda_p}{2i\gamma^2}\left[1 + \frac{1}{2}K^2 + \gamma^2(\theta^2 + \psi^2)\right]$$

$$\lambda_i(\text{Å}) = 1305.6\frac{\lambda_p}{iE^2}\left(1 + \frac{1}{2}K^2\right)$$

$$\varepsilon_i(\text{eV}) = 9.4963 \cdot \frac{iE^2}{\lambda_p\left(1+\frac{1}{2}K^2\right)}$$

$$\sigma_\theta = \frac{1}{\gamma}\sqrt{\frac{1+\frac{1}{2}K^2}{2iN_p}}$$

$$B_y(z) = B_0 \cos k_p z$$ **this is what we want**

Maxwell tells us what we can get!

$$B_y(y,z) = B_0\, b(y) \cos k_p z$$

$\nabla \times \mathbf{B} = \mathbf{0}$ ⟹ $$\frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z} = -B_0\, b(y)\, k_p \sin k_p z$$

and $B_y = -B_0\, b(y)\,(1 - \cos k_p z)$

$\nabla \cdot \mathbf{B} = 0$, $\mathbf{B} \neq \mathbf{B}(\mathbf{x})$ ⟹ $$\frac{\partial B_z}{\partial z} = -B_0 \frac{\partial b(y)}{\partial y} \cos k_p z$$

form $\dfrac{\partial^2 B_z}{\partial y \partial z}$ ⟹ $$\frac{\partial^2 b(y)}{\partial^2 y} = k_p^2\, b(y)$$ ⟹ $$b(y) = a_1 \cosh k_p y + a_2 \sinh k_p y$$

$$B_x = 0$$
$$B_y = B_0 \cosh k_p y \cos k_p z$$
$$B_z = -B_0 \sinh k_p y \sin k_p z$$

beam dynamics

$$\frac{d^2\mathbf{r}}{ds^2} = \frac{\mathbf{n}}{\rho} = \frac{e}{mc^2\gamma}\left[\frac{\mathbf{v}}{v}\times\mathbf{B}\right]$$

$$\frac{d^2x}{dt^2} = -\frac{eB_0}{mc\gamma}\frac{dz}{dt}\cos k_p z$$

$$\frac{d^2z}{dt^2} = +\frac{eB_0}{mc\gamma}\frac{dx}{dt}\cos k_p z$$

$$\frac{dx}{dt} = -c\beta\frac{K}{\gamma}\sin k_p z$$

$$\frac{dz}{dt} = +c\beta\left(1-\frac{K^2}{2\gamma^2}\sin^2 k_p z\right)$$

drift velocity

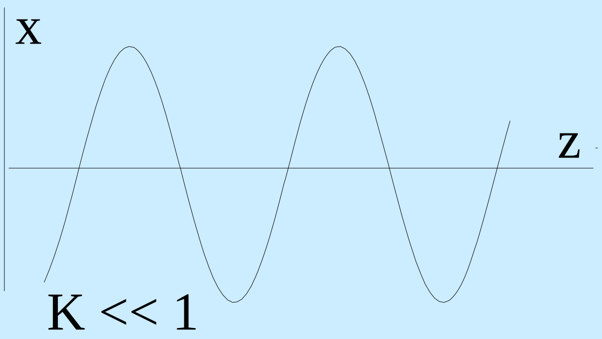
$$\bar{\beta} = \beta\left(1-\frac{K^2}{4\gamma^2}\right)$$

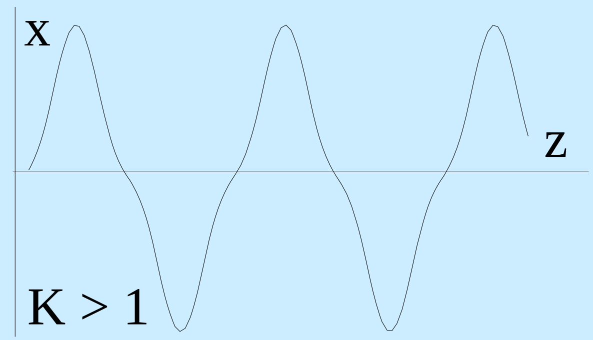
$$x(t) = a\cos(k_p c\bar{\beta}t)$$

$$z(t) = c\bar{\beta}t + \tfrac{1}{8}k_p a^2\sin(2k_p c\bar{\beta}t)$$

$$a = \frac{K}{\gamma k_p}$$

Stronger undulator field



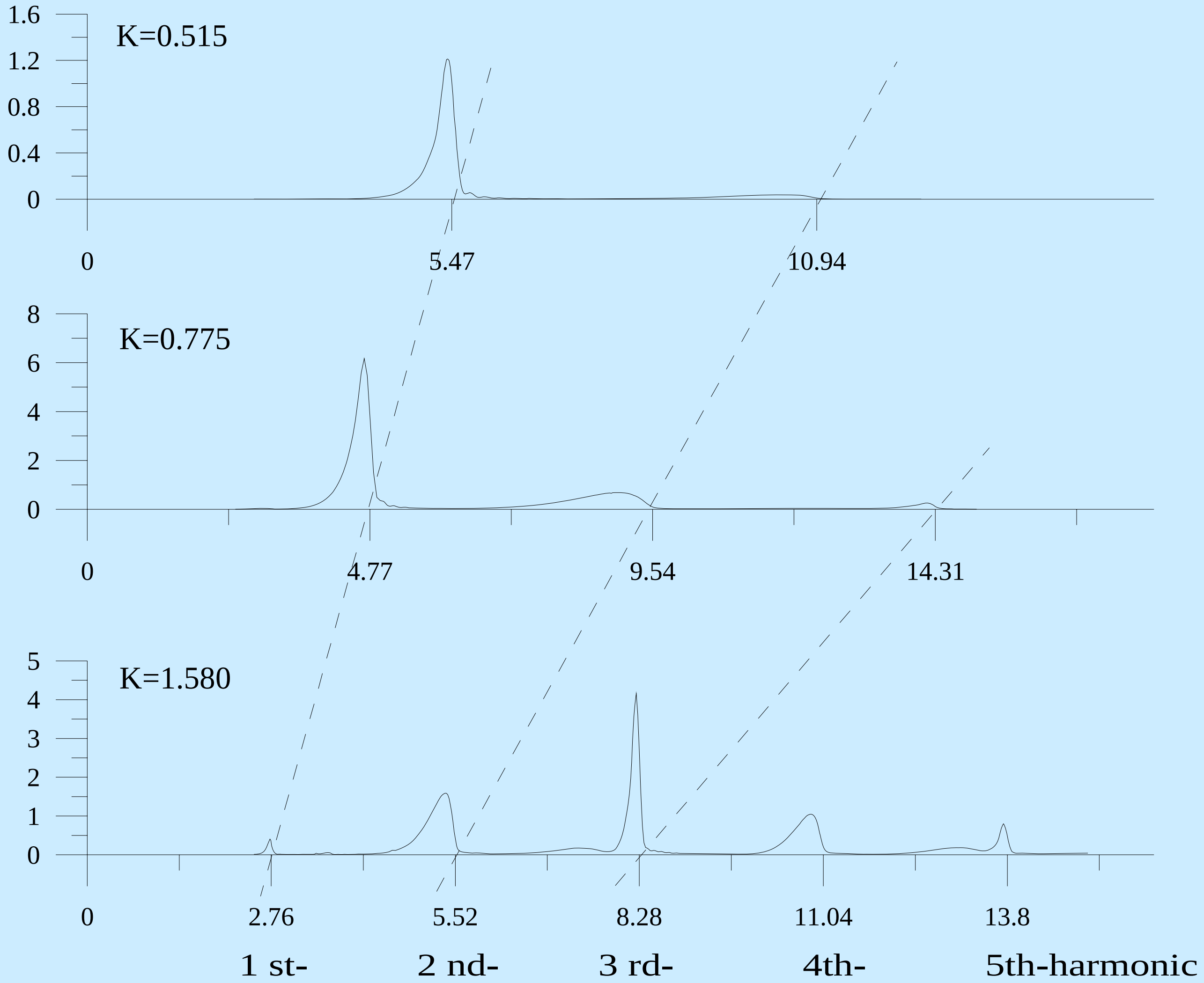


transverse motion completely non-relativistic

relativistic effect on transverse motion

source of higher harmonics (only odd harmonics !)

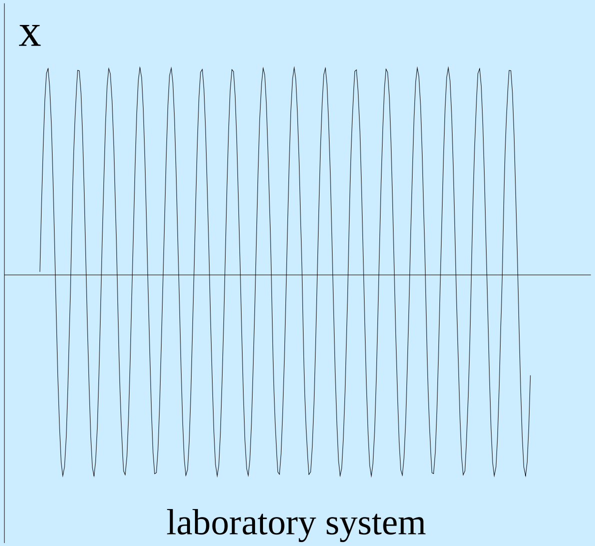
PEP-Undulator: 77mm, 27 periods, 7.1 GeV

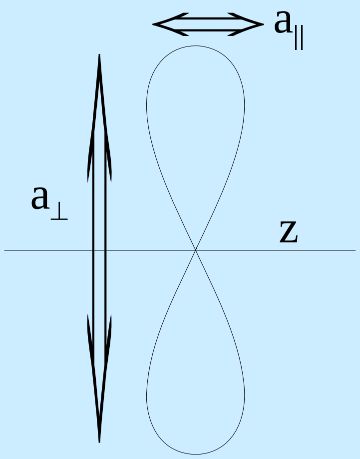


now we increase strength parameter K

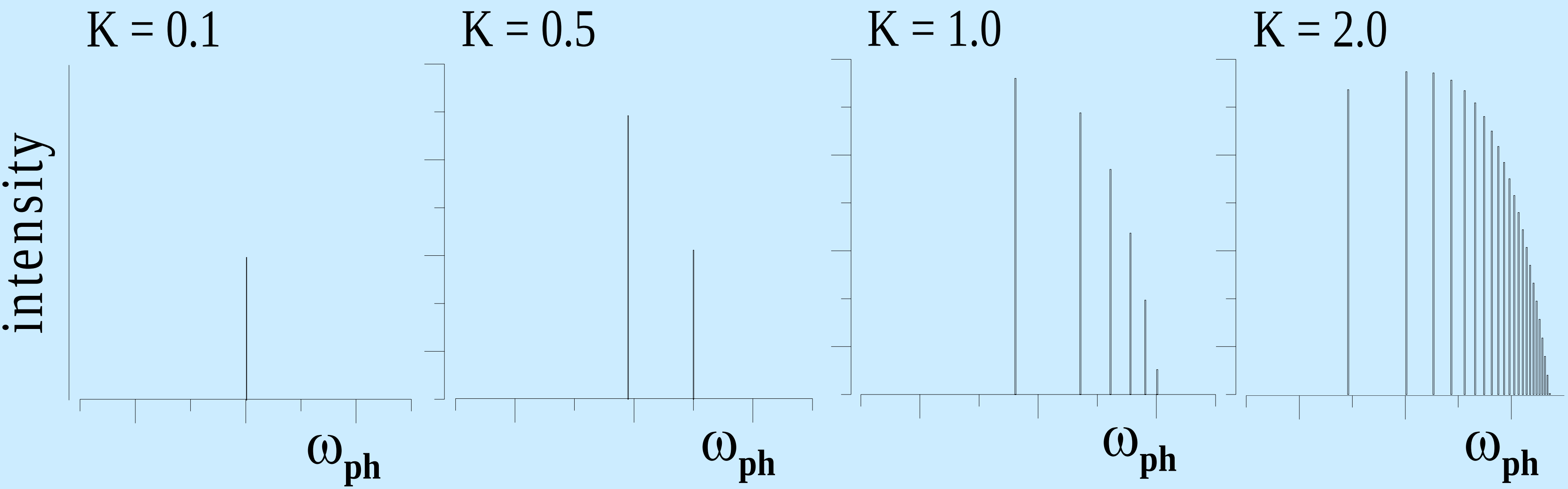
$$x(t) = a\cos(k_p c\bar{\beta} t)$$

$$z(t) = c\bar{\beta} t + \frac{1}{8} k_p a^2 \sin(2k_p c\bar{\beta} t)$$





transition from undulator to wiggler radiation



energy loss per undulator pass

$$\Delta E_{\mathrm{rad}} = \frac{1}{3} r_{\mathrm{c}} mc^2 \gamma^2 K^2 k_{\mathrm{p}}^2 L_{\mathrm{u}}$$

$$\Delta E_{\mathrm{rad}}(\mathrm{eV}) = 0.07257 \frac{\mathrm{E}^2 \mathrm{K}^2}{\lambda_{\mathrm{p}}^2} L_{\mathrm{u}}$$

tot. radiation power

$$P(\mathrm{W}) = 0.07257 \frac{\mathrm{E}^2 \mathrm{K}^2 \mathrm{N I}}{\lambda_{\mathrm{p}}}$$

$$\frac{d\dot{N}_{ph}(\omega)}{d\Omega} = \alpha\gamma^2 N_p^2 \frac{\Delta\omega}{\omega}\frac{I}{e} \times \sum_{i=1}^{\infty} i^2\, \mathrm{Sinc}\,(F_\sigma^2 + F_\pi^2)$$

$$\mathrm{Sinc} = \left(\frac{\sin \pi N_p \Delta\omega_i/\omega_1}{\pi N_p \Delta\omega_i/\omega_1}\right)^2$$

$$F_\sigma = \frac{2\gamma\theta\Sigma_1 \cos\varphi - K\Sigma_2}{1+\frac{1}{2}K^2+\gamma^2\theta^2} \qquad F_\pi = \frac{2\gamma\theta\Sigma_1 \sin\varphi}{1+\frac{1}{2}K^2+\gamma^2\theta^2}$$

$$\Sigma_{1,i} = \sum_{m=-\infty}^{\infty} J_{-m}(u)\, J_{i-2m}(v)$$

$$\Sigma_{2,i} = \sum_{m=-\infty}^{\infty} J_{-m}(u)\left[J_{i-2m-1}(v) + J_{i-2m+1}(v)\right]$$

$$u = \frac{\omega}{\omega_1}\frac{\bar{\beta}K^2}{4(1+\frac{1}{2}K^2+\gamma^2\theta^2)} \qquad v = \frac{\omega}{\omega_1}\frac{2\bar{\beta}K^2\gamma\theta\cos\varphi}{1+\frac{1}{2}K^2+\gamma^2\theta^2}$$

pin hole radiation

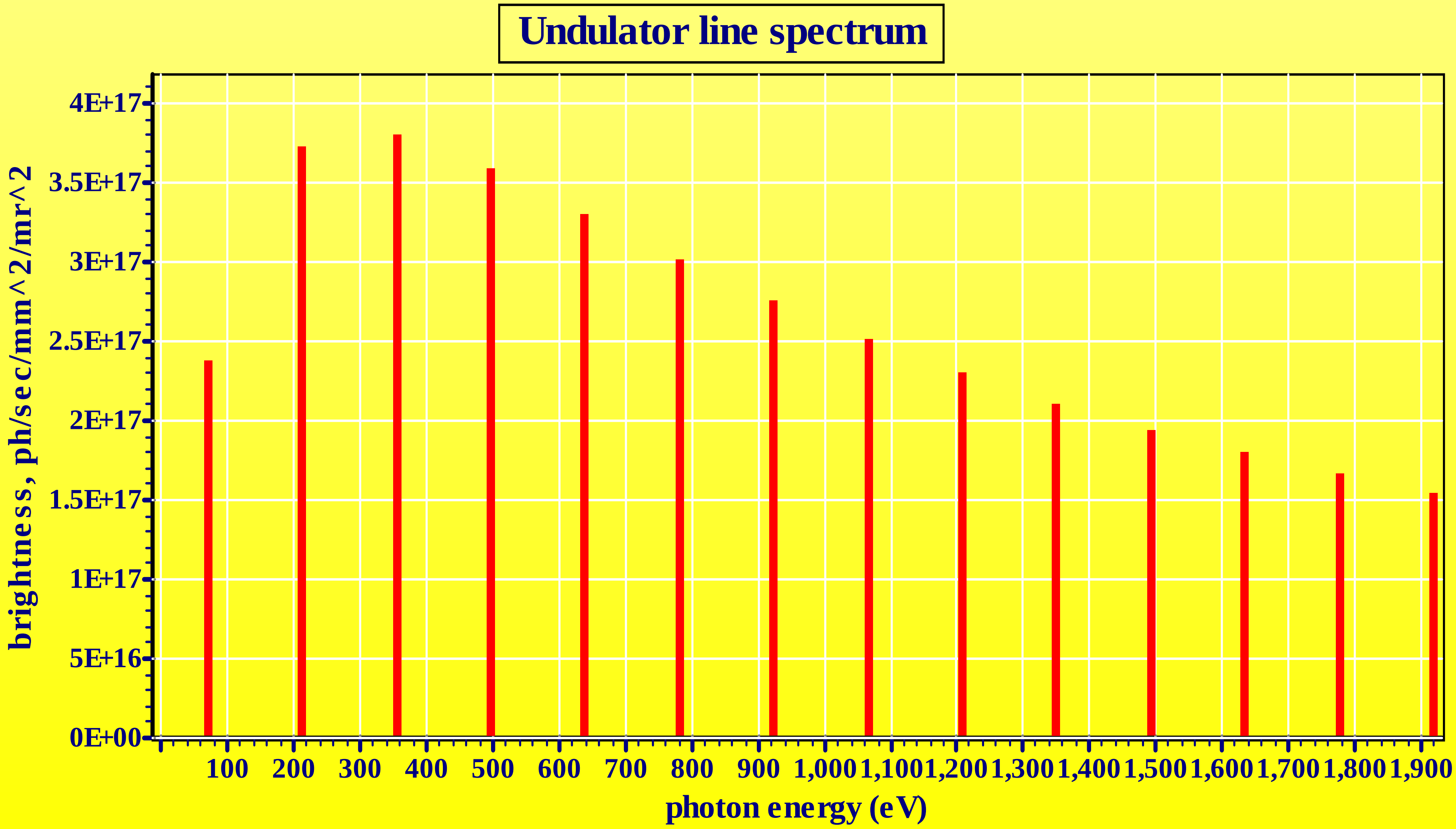
$$\left.\frac{d\dot{N}_{ph}(\omega)}{d\Omega}\right|_i = \alpha\gamma^2 N_p^2 \frac{\Delta\omega}{\omega}\frac{I}{e}\frac{i^2 K^2 [JJ]^2}{\left(1+\frac{1}{2}K^2\right)^2}$$

$$= 1.7466 \cdot 10^{23} E^2(\mathrm{GeV}^2) I(\mathrm{A}) N_p^2 \frac{\Delta\omega}{\omega} f_i(K),$$

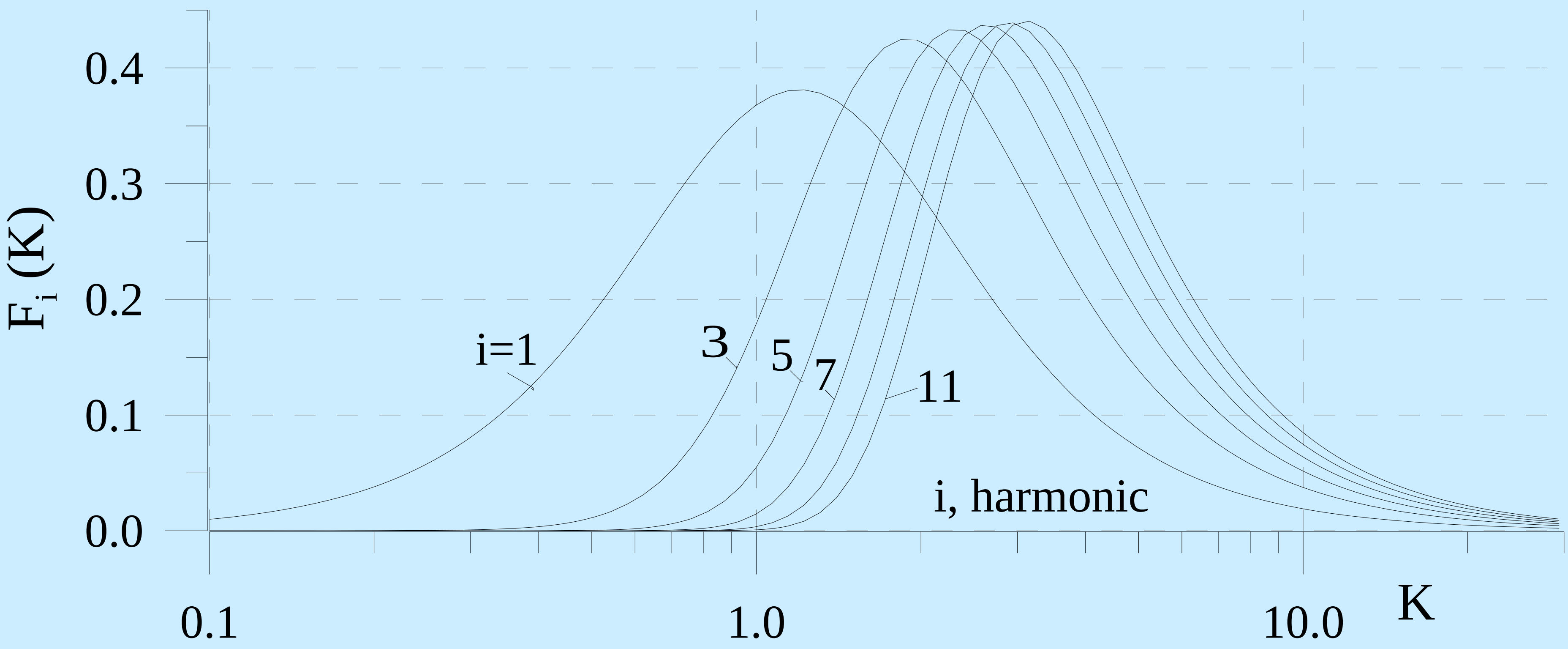
$$f_i(K) = \frac{i^2 K^2 [JJ]^2}{\left(1+\frac{1}{2}K^2\right)^2}$$

$$[JJ] = \left[J_{\frac{i-1}{2}}(x) - J_{\frac{i+1}{2}}(x)\right]$$

$$x = \frac{iK^2}{4+2K^2}$$


Undulator line spectrum
brightness, ph/sec/mm^2/mr^2
4E+17
3.5E+17
3E+17
2.5E+17
2E+17
1.5E+17
1E+17
5E+16
0E+00
100
200
300
400
500
600
700
800
900
1,000
1,100
1,200
1,300
1,400
1,500
1,600
1,700
1,800
1,900
photon energy (eV)

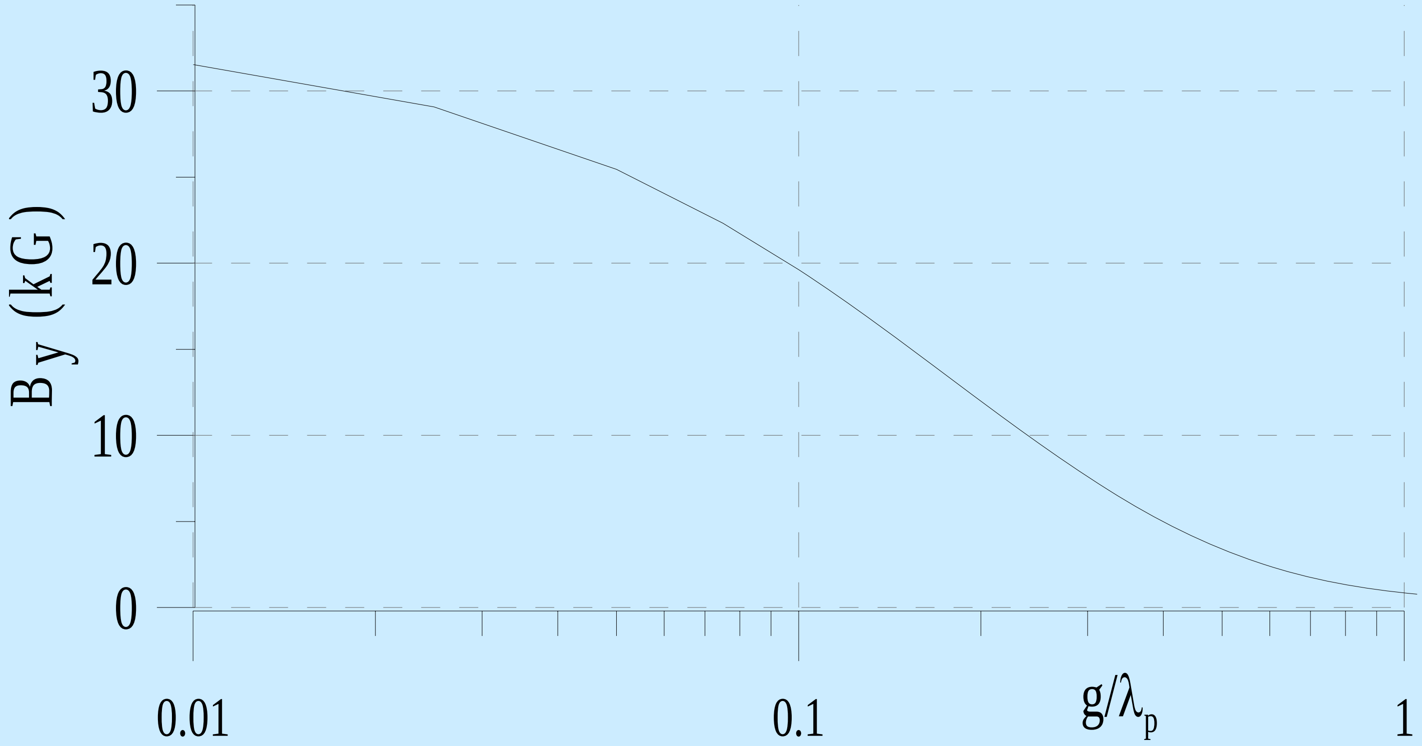
Insertion Device Radiation



vary field strength by varying gap. For hybrid undulator:

$$B(\mathrm{T}) = 3.3\exp\left[-\frac{g}{\lambda_p}\left(5.74 - 1.8\frac{g}{\lambda_p}\right)\right]$$

K.Halbach



$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta^2_{obs}\right)$$

make λ_p very short → to get x-rays !?

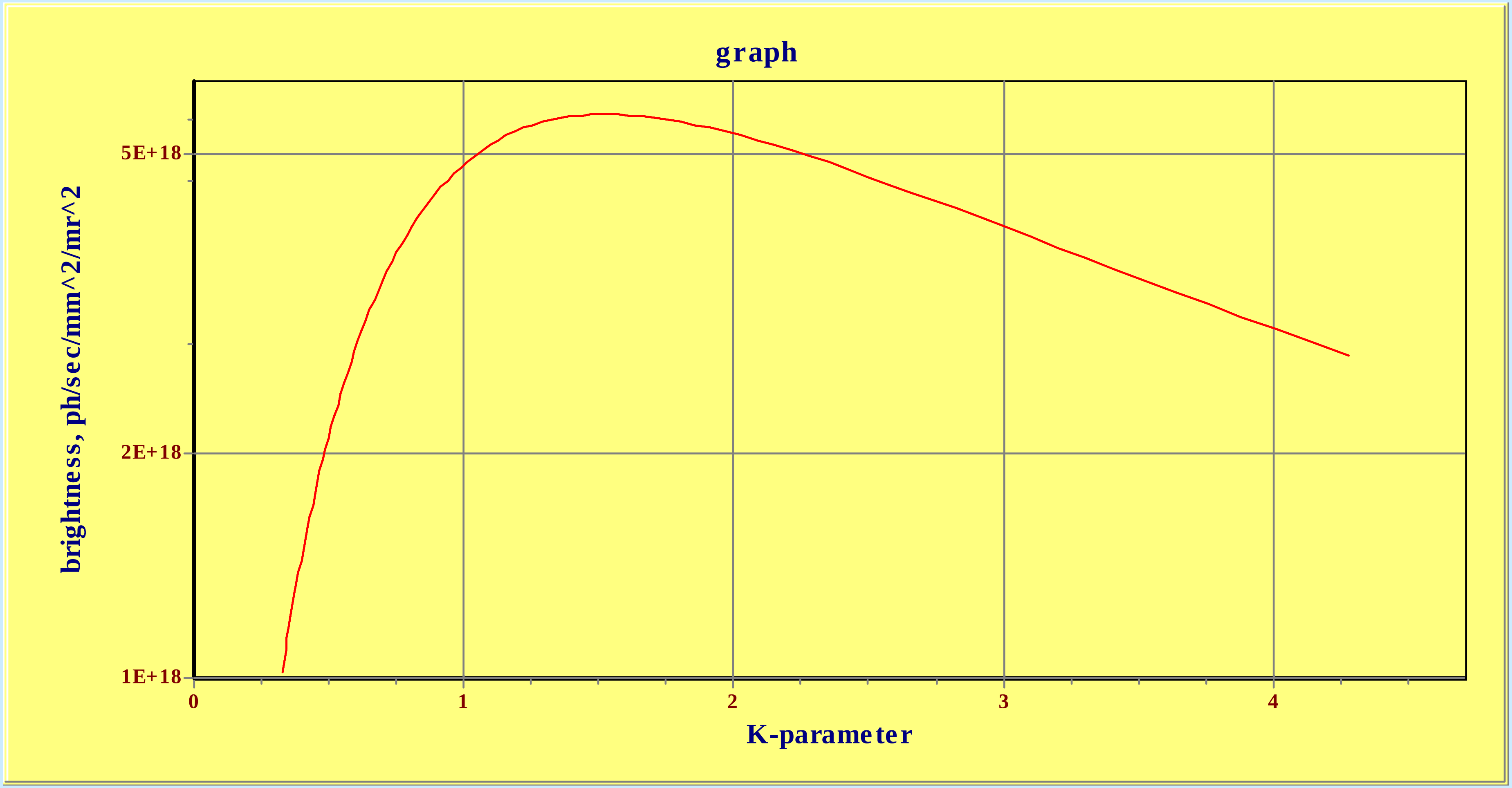
does not work well !

$$K = 0.934\, B(\mathrm{T})\, \lambda_p(\mathrm{cm})$$

short λ_p leads generally to small value of K !

intensity is low
tuning range is narrow

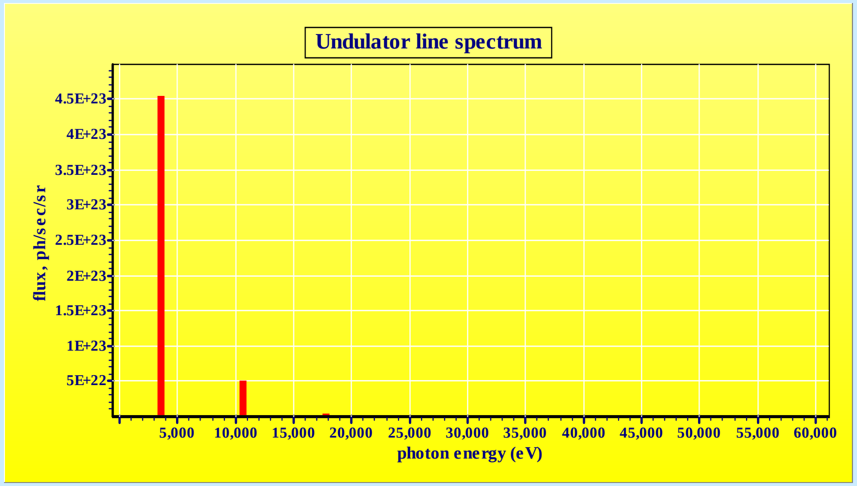
Insertion Device Radiation

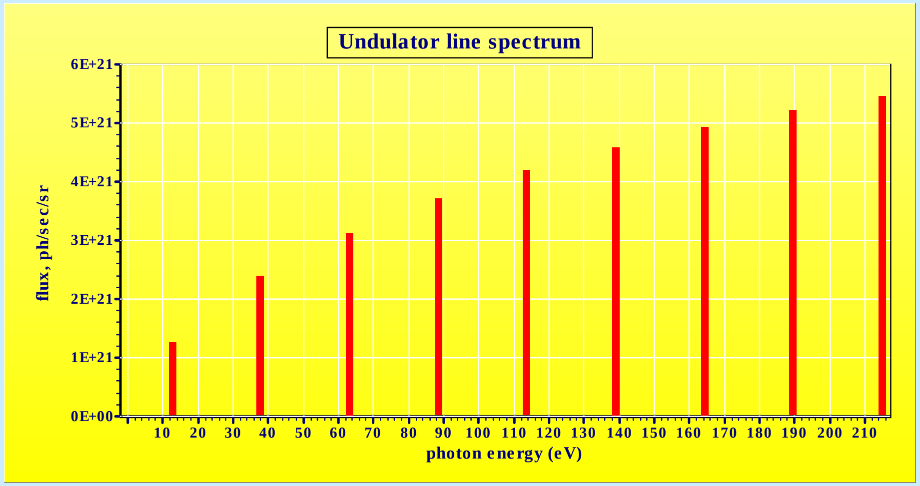


Insertion Device Radiation

tuning range: $$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta_{obs}^2\right)$$

compare two undulators in SPEAR: λ_p= 20 mm and =80 mm

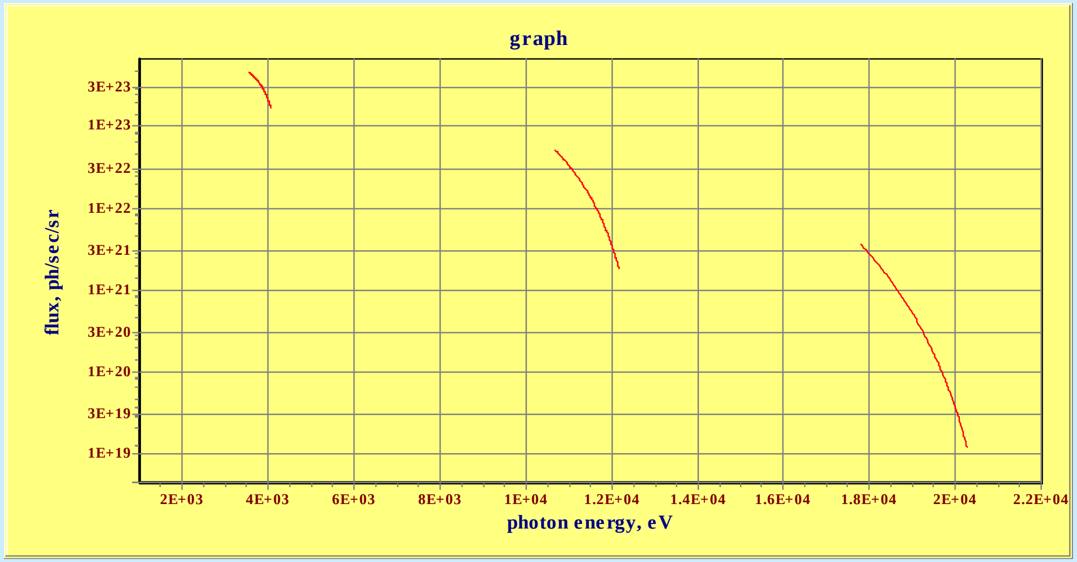




λ_p= 20 mm, 100 periods
0.33 < K < 0.63

λ_p= 80 mm, 25 periods
4.6 < K < 12.9

Insertion Device Radiation



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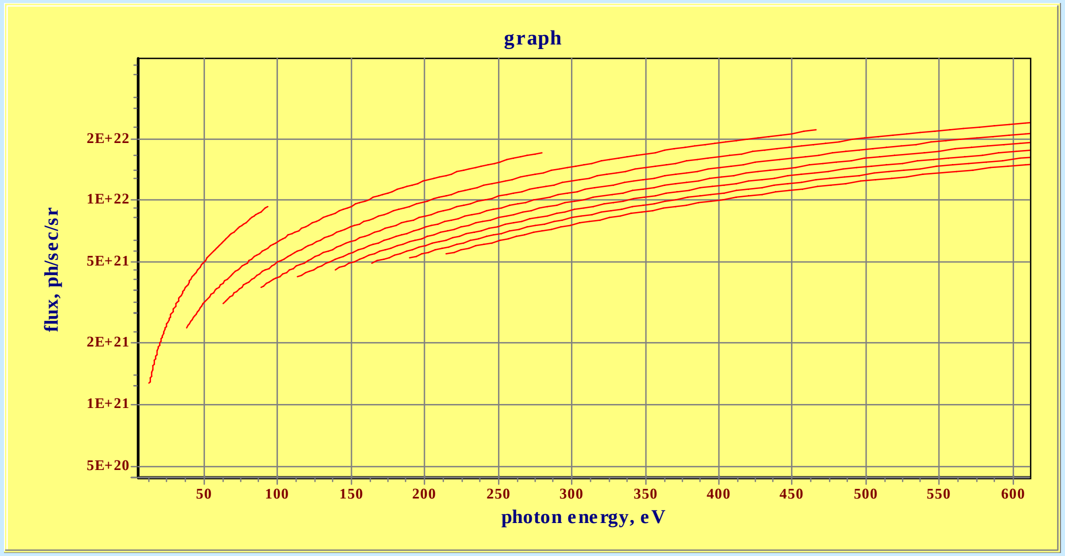


λ_p= 80 mm, 25 periods

4.6 < K < 12.9

Insertion Device Radiation

SUBARU : 2.3 m undulator, λ_p = 7.6 cm, 30 periods

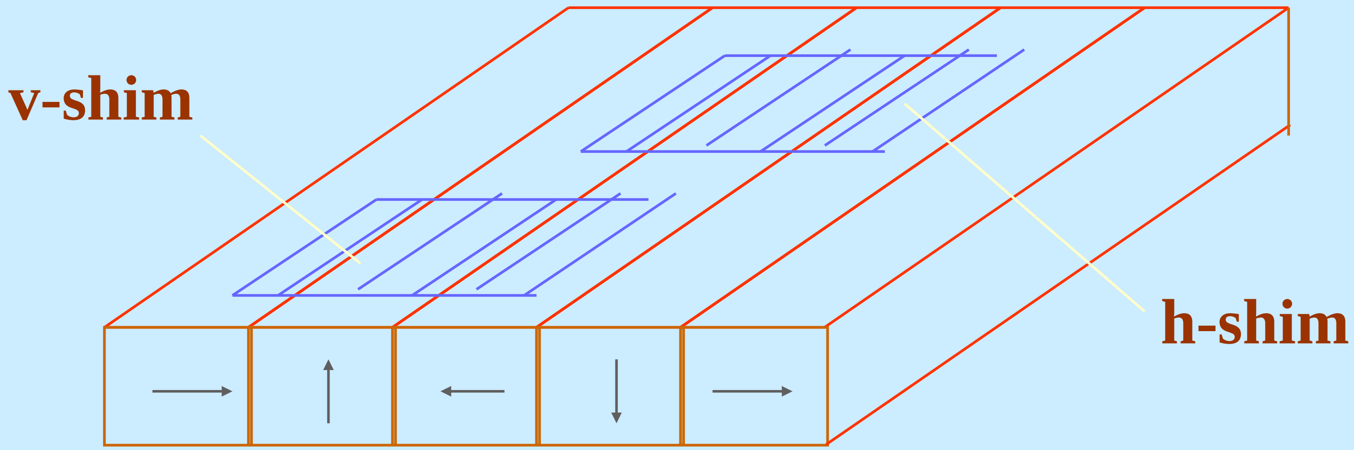
Insertion Device Radiation

SUBARU : 10.8 m undulator, λ_p = 5.4 cm, 200 periods

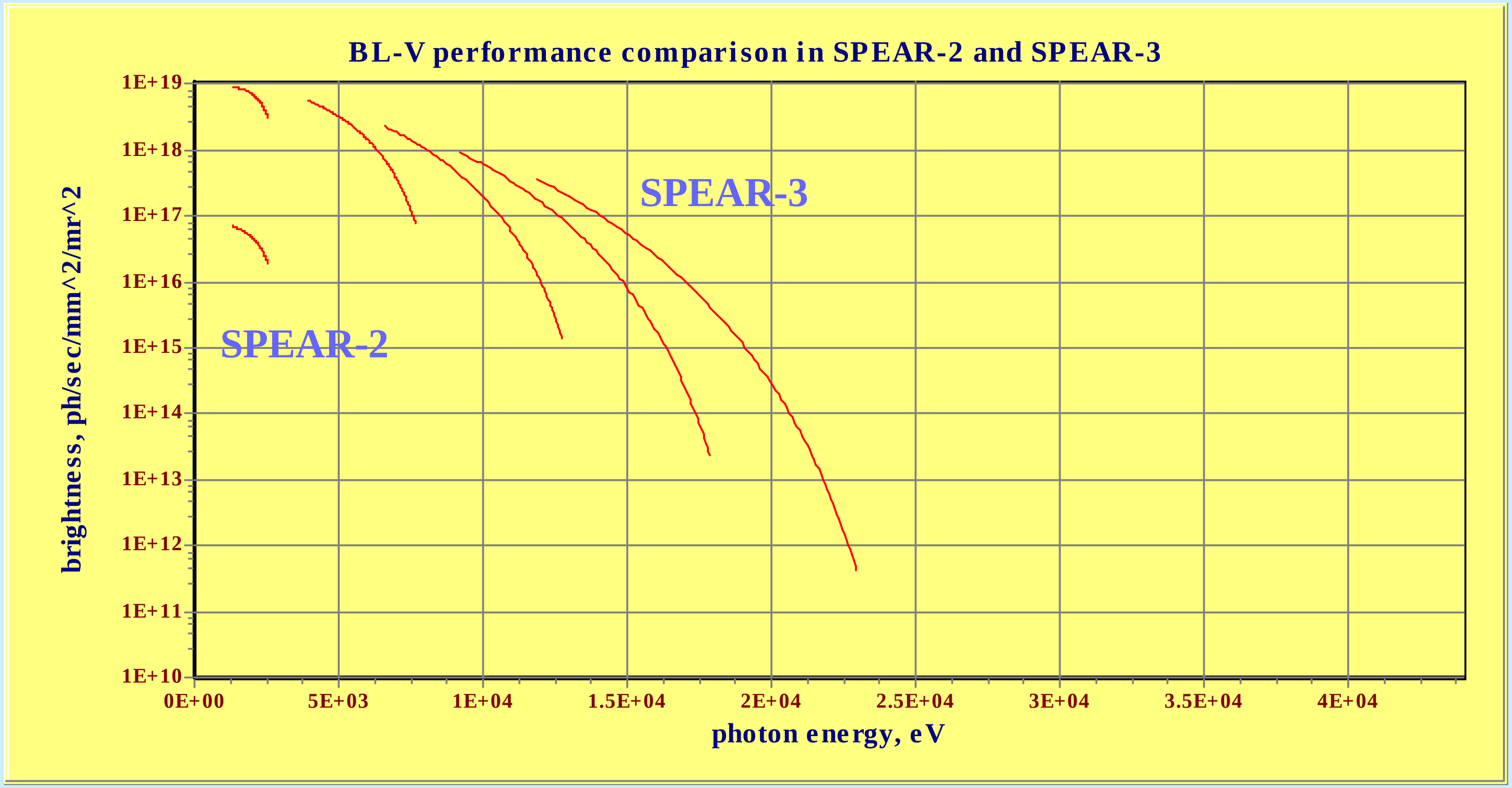
use harmonics to reach high photon energies at low electron energy

requires high undulator precision:

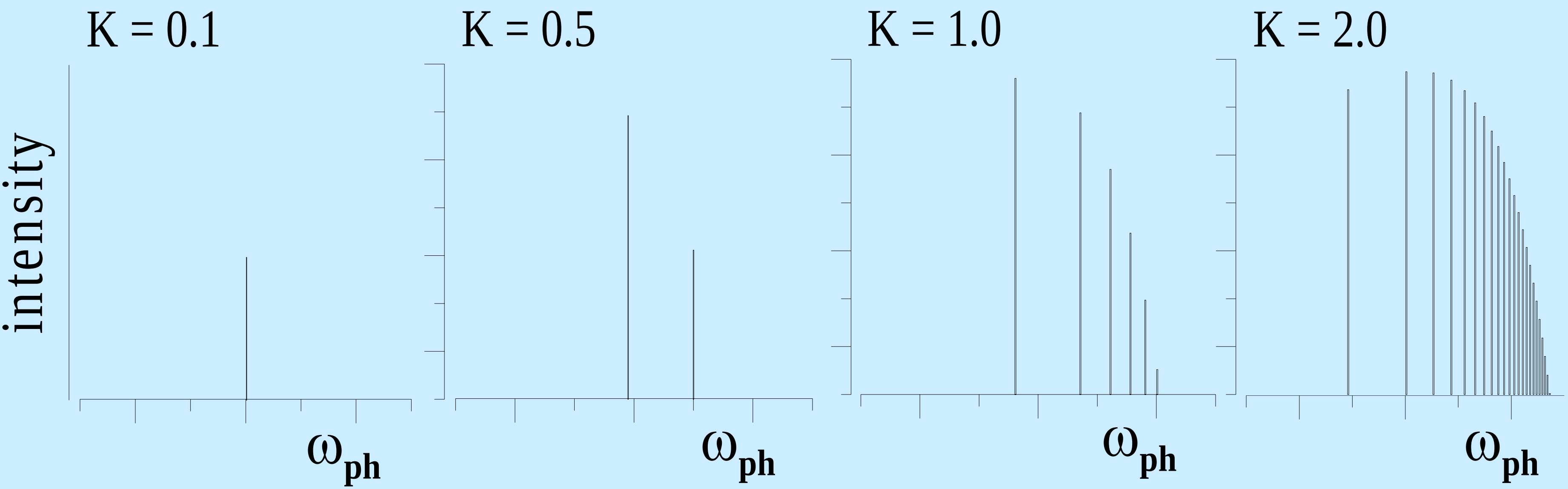
cannot build undulators that precise, but we can fix them by shimming



Insertion Device Radiation



transition from undulator to wiggler radiation



crit. Photon energy from wiggler magnet at angle ψ with axis

$$\varepsilon_c(\psi) = \varepsilon_c(0)\sqrt{1-\left(\frac{\gamma\psi}{K}\right)^2}$$