the **abdus salam**
international centre for theoretical physics

Optical path function

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School on Synchrotron Radiation and Applications
in memory of J.C. Fuggle and L. Fonda
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Outline

- Main properties of SR, brilliance

- Main properties of VUV, EUV and soft x-rays mirrors and gratings

- Conserving brilliance up to the experiment

- Determining optical paths from Fermat's principle: general theory of aberrations

Main properties of Synchrotron Radiation

- Very broad and continuous spectral range, from infrared up to soft and hard x-rays

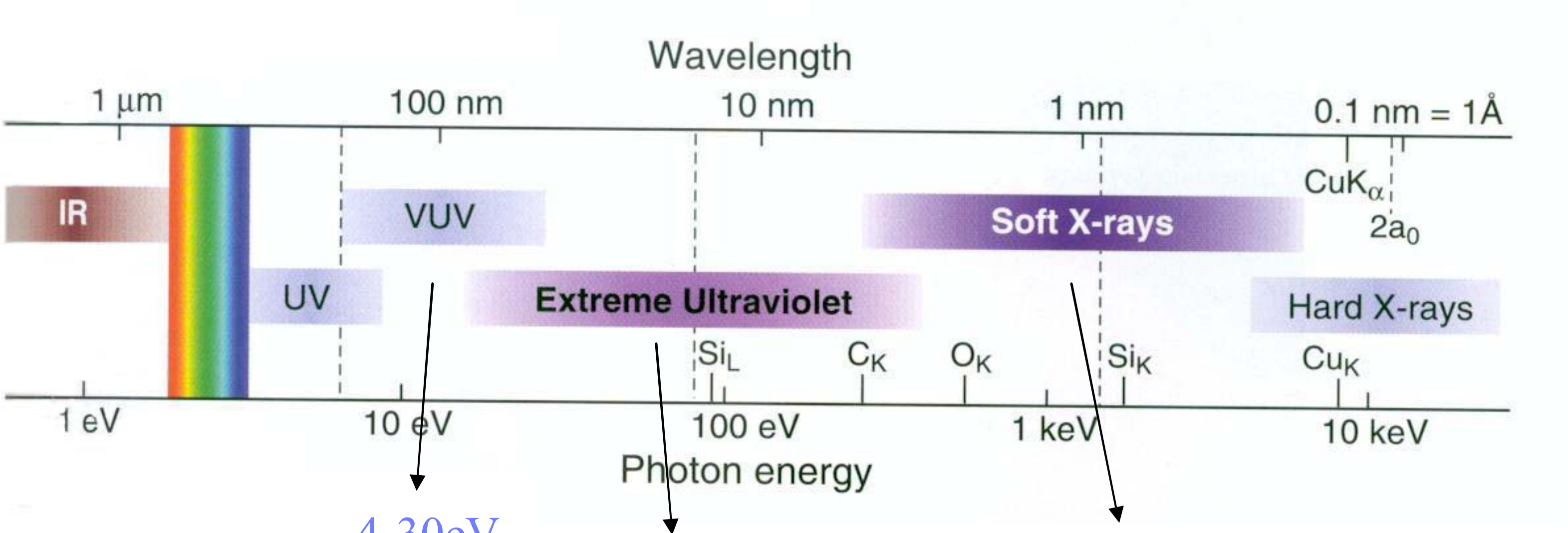
- High intensity

- The emitted radiation is highly collimated and emanates from a very small source: the electron beam

- Pulse time structure

- High degree of polarization

Spectral range



4-30eV
300-40nm

30-250eV
40-5nm

250eV - several keV

D.Attwood, "Soft x-rays and extreme ultraviolet radiation", Cambridge University Press, 1999

Brilliance

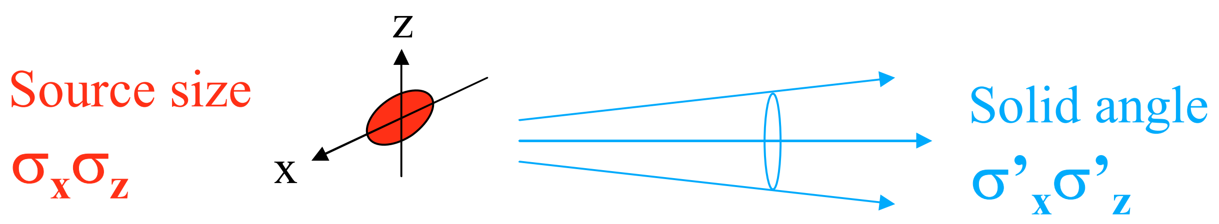
$$Brilliance = \frac{photon\ flux}{I} \frac{1}{\sigma_x \sigma_z \sigma'_x \sigma'_z BW}$$

I = electron current in the storage ring

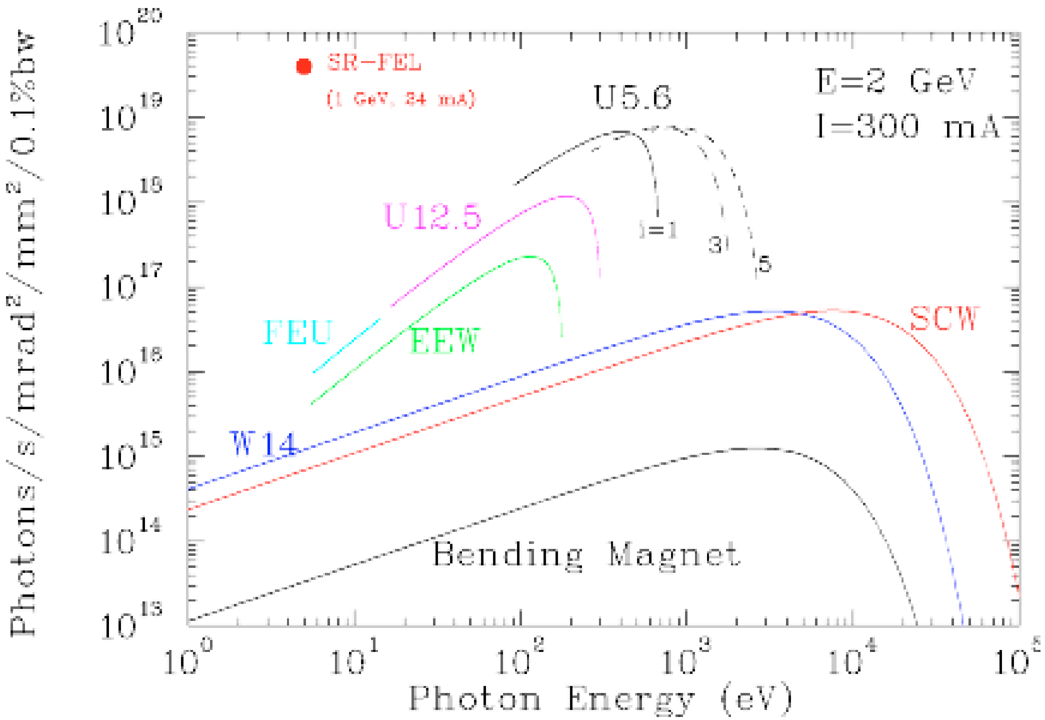
$\sigma_x \sigma_z$ = transverse area from which SR is emitted

$\sigma'_x \sigma'_z$ = solid angle into which SR is emitted

BW = spectral bandwidth, usually: $\frac{\Delta E}{E} = 0.1\%$



SR brilliance at ELETTRA



Why is brilliance important? (1)

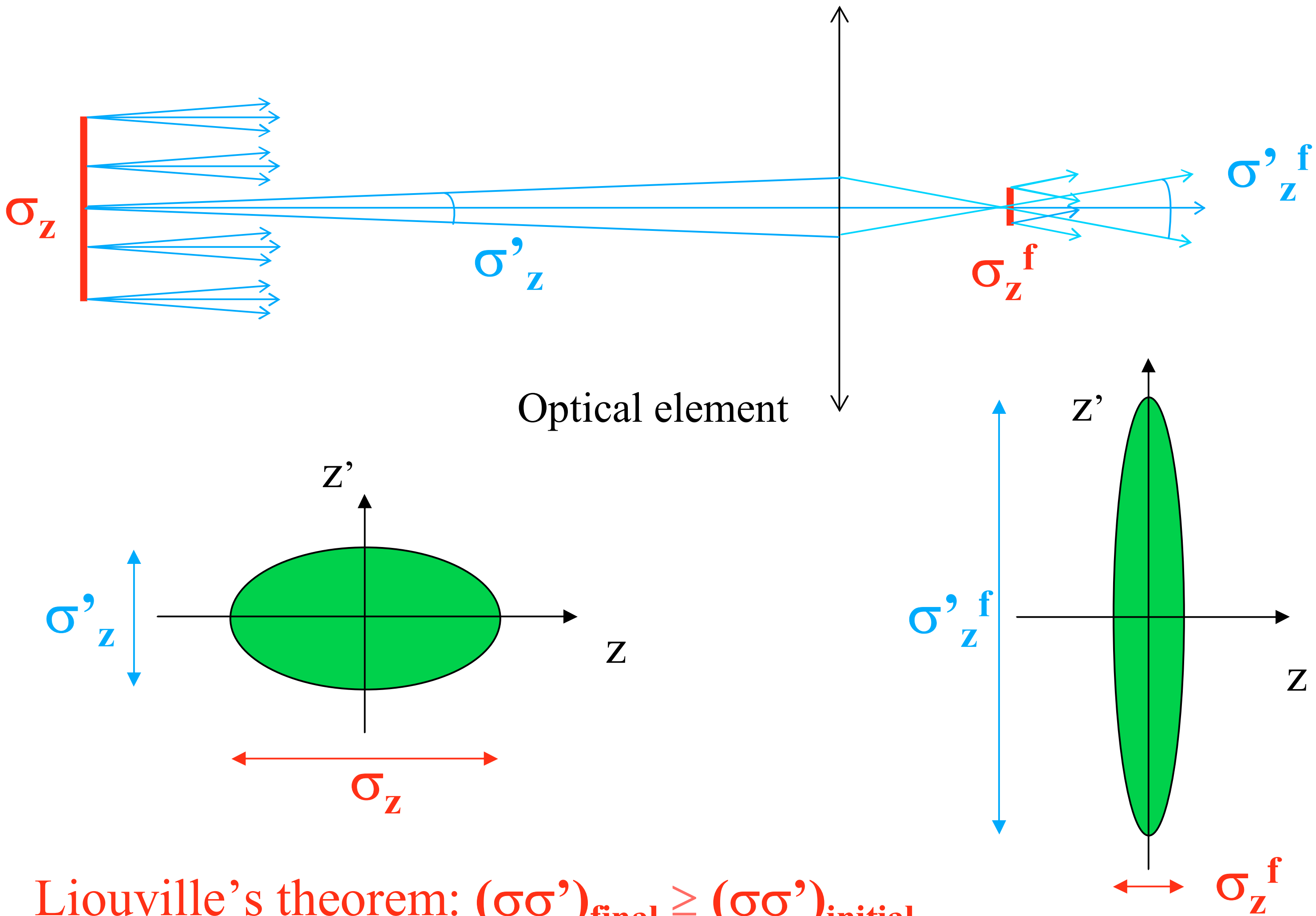
$$Brilliance = \frac{photon\ flux}{I} \frac{1}{\sigma_x \sigma_z \sigma'_x \sigma'_z BW}$$

more flux → more signal for the experiment

But why combining the flux with geometrical factors?

Liouville's theorem: for an optical system the occupied phase space volume cannot be decreased along the optical path (without loosing photons) → $(\sigma\sigma')_{final} \geq (\sigma\sigma')_{initial}$

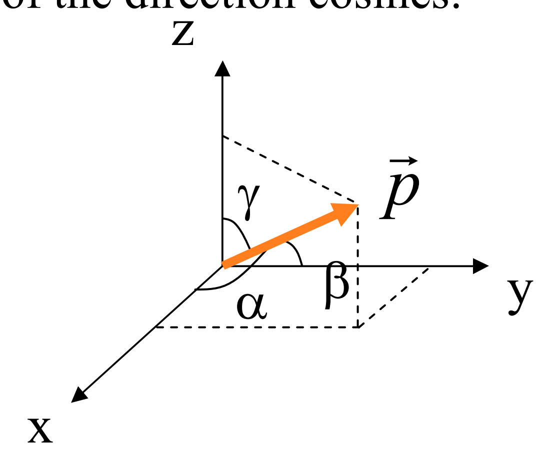
Example : a focusing beam



Liouville's theorem: $(\sigma\sigma')_{final} \geq (\sigma\sigma')_{initial}$

Meaning of x’,z’

- transverse momenta p_x, p_z can be expressed in terms of the direction cosines:



$$p_x = p \cos\alpha$$

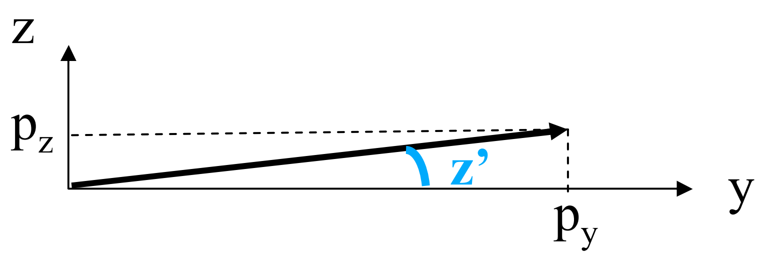
$$p_z = p \cos\gamma$$

- defining: $x' = \cos\alpha \qquad z' = \cos\gamma$

and assuming: $p_y >> p_x, p_z$ → $p \cong p_y$,

the transverse momenta are proportional to x', z' :

$$p_x = p_y x' \qquad p_z = p_y z'$$



Why is brilliance important? (2)

Liouville's theorem: $(\sigma\sigma')_{final} \geq (\sigma\sigma')_{initial}$

→ to focus a beam in a small spot (which is needed for achieving energy and/or spatial resolution) one must accept an increase in the beam divergence

High beam divergence along the beamline:
→ large optical devices
→ high costs and low optical qualities
With a not brilliant source the spot size can be made small only reducing the photon flux.

The high brilliance of the radiation source allows the development of monochromators with high energy resolution and high throughput and gives also the possibility to image a beam down to a very small spot on the sample with high intensity.

The beamline

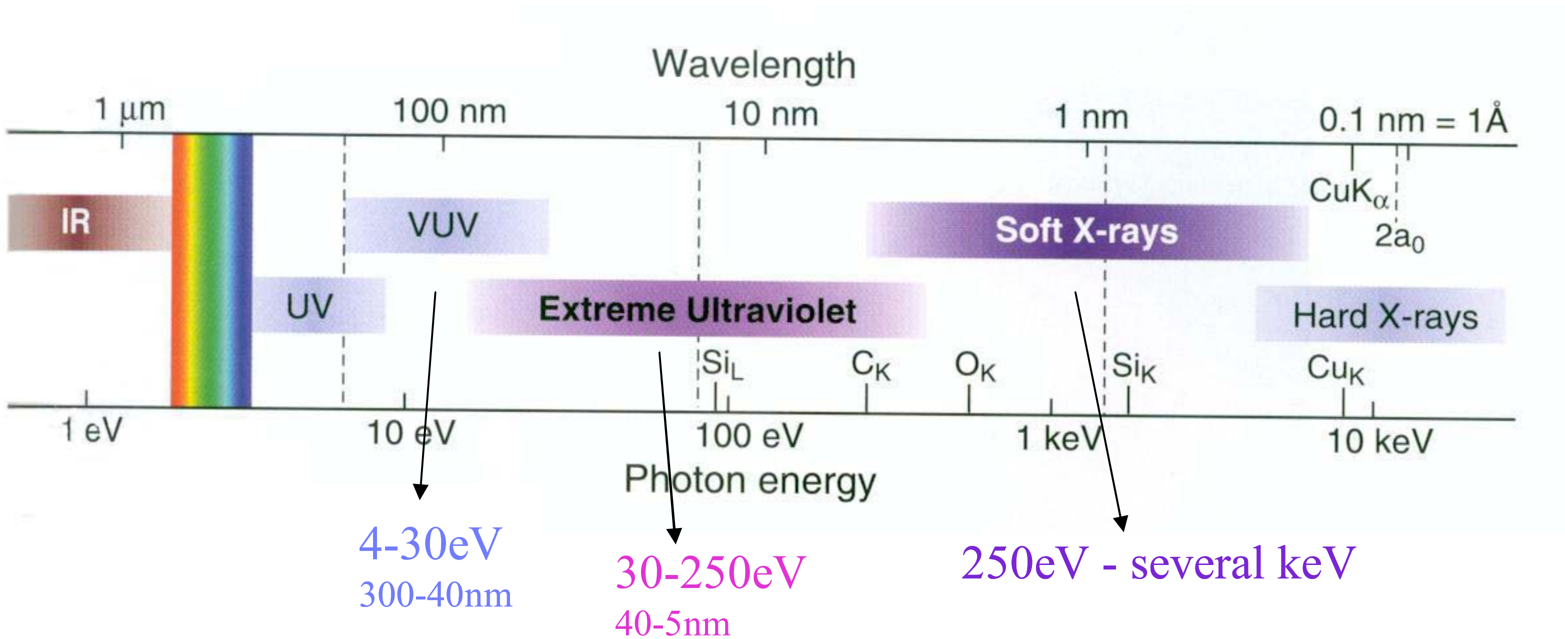
The researcher needs at his experiment a certain number of photons/second into a phase volume of some particular characteristics. Moreover, these photons have to be monochromatized.

The beamline:

- is the means of bringing radiation from the source to the experiment transforming the phase volume in a controlled way: it demagnifies, monochromatizes and refocuses the source onto a sample
- must preserve the excellent qualities of the radiation source

VUV, EUV and soft x-rays

We restrict ourselves to photon energies from 10 to 2000 eV.



These regions are very interesting because are characterized by the presence of the absorption edges of most low and intermediate Z elements

→ photons with these energies are **a very sensitive tool** for elemental and chemical identification

But… these regions are difficult to access.

Ultra-high vacuum

VUV, EUV and soft x-rays have a high degree of absorption in all materials

→ No windows → The entire optical system must be kept under vacuum

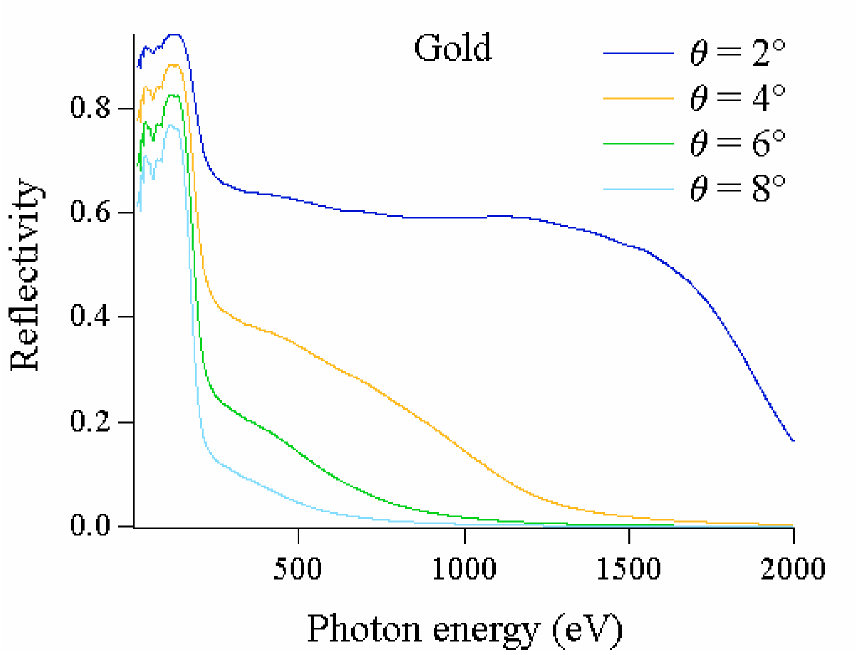
Ultrahigh vacuum conditions ($P=1-2\times10^{-9}$ mbar) are required:

- Not to disturb the storage ring and the experiment
- To avoid photon absorption in air
- To protect the optical surfaces from contamination (especially from carbon)

Grazing incidence optics

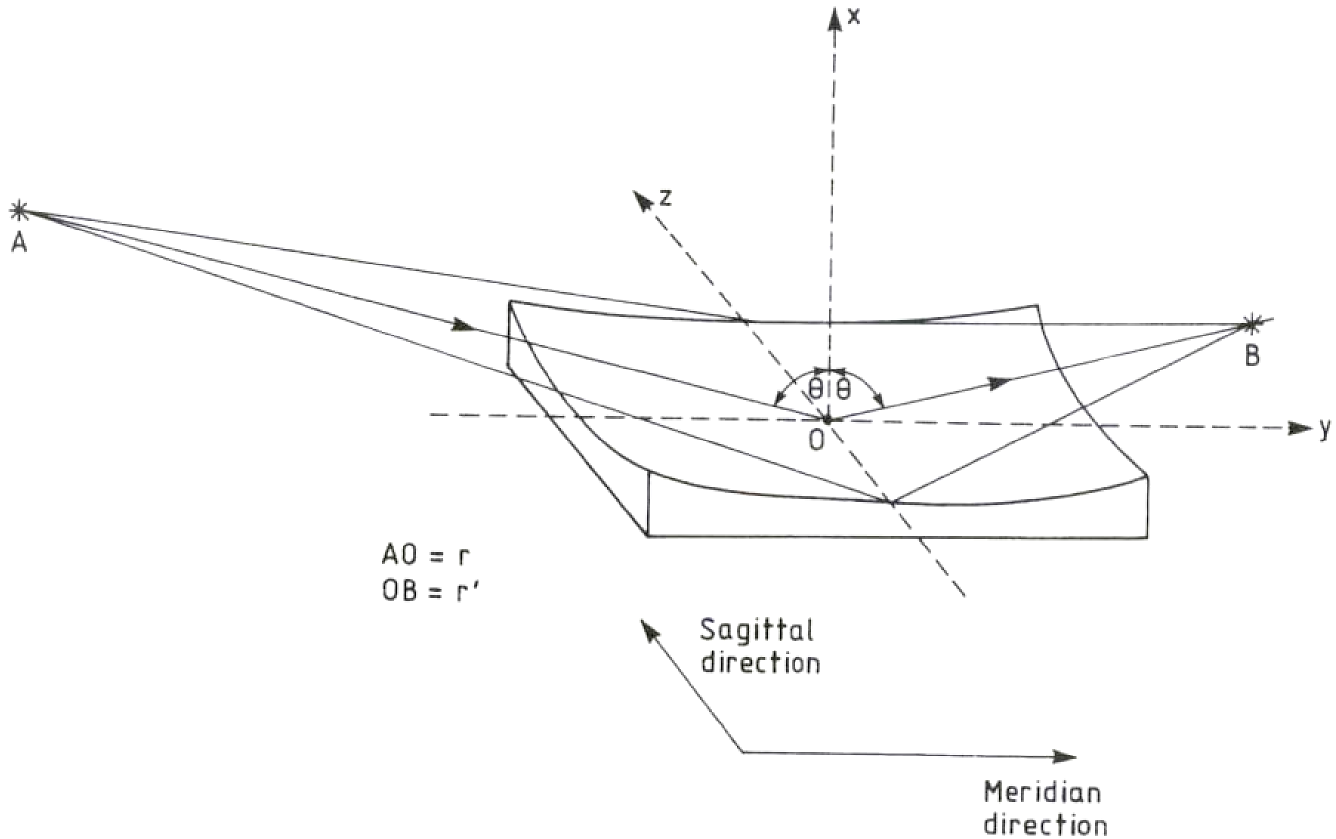
Strong absorption of radiation by all materials:
→ no lenses: the only optical elements that can work are mirrors and diffraction gratings, used in reflection

Reflectivities drop down fast with the increasing of the grazing incidence angle
→ only reflective optics at grazing incidence angles (1-2 degrees)



Focusing properties

The meridional or tangential plane contains the central incidence ray and the normal to the surface. The sagittal plane is the plane perpendicular to the tangential plane and containing the normal to the surface.



Paraboloid

Rays traveling parallel to the symmetry axis OX are all focused to a point A.

Conversely, the parabola collimates rays emanating from the focus A.

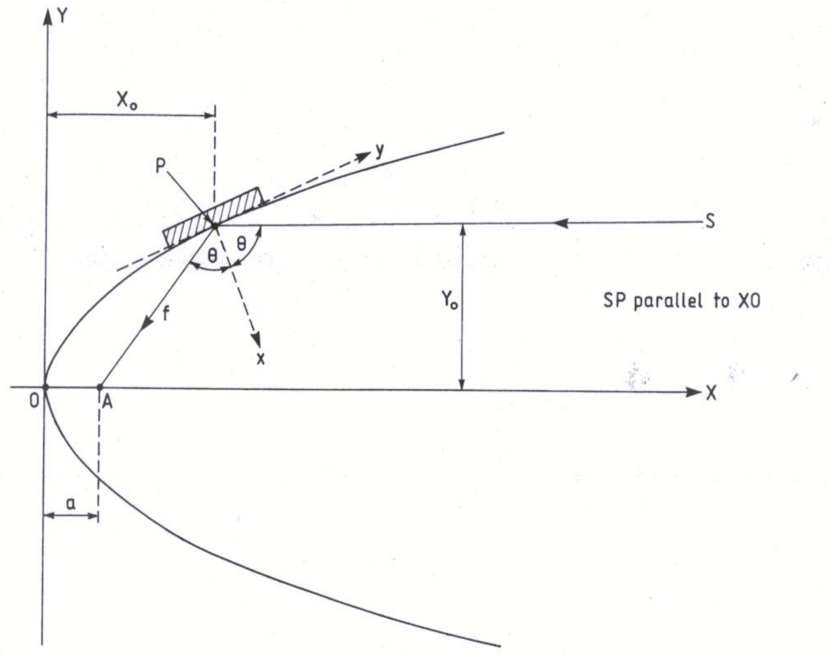
Line equation: $Y^2 = 4aX$

Paraboloid equation: $Y^2 + Z^2 = 4aX$

where: $a = f\cos^2\vartheta$

Position of the pole P:

$$X_o = a\tan^2\vartheta$$
$$Y_o = 2a\tan\vartheta$$



Paraboloid equation:

$$x^2\sin^2\vartheta + y^2\cos^2\vartheta + z^2 - 2xy\sin\vartheta\,\cos\vartheta - 4\,ax\sec\vartheta = 0$$

J.B. West and H.A. Padmore, Optical Engineering, 1987

Ellipsoid

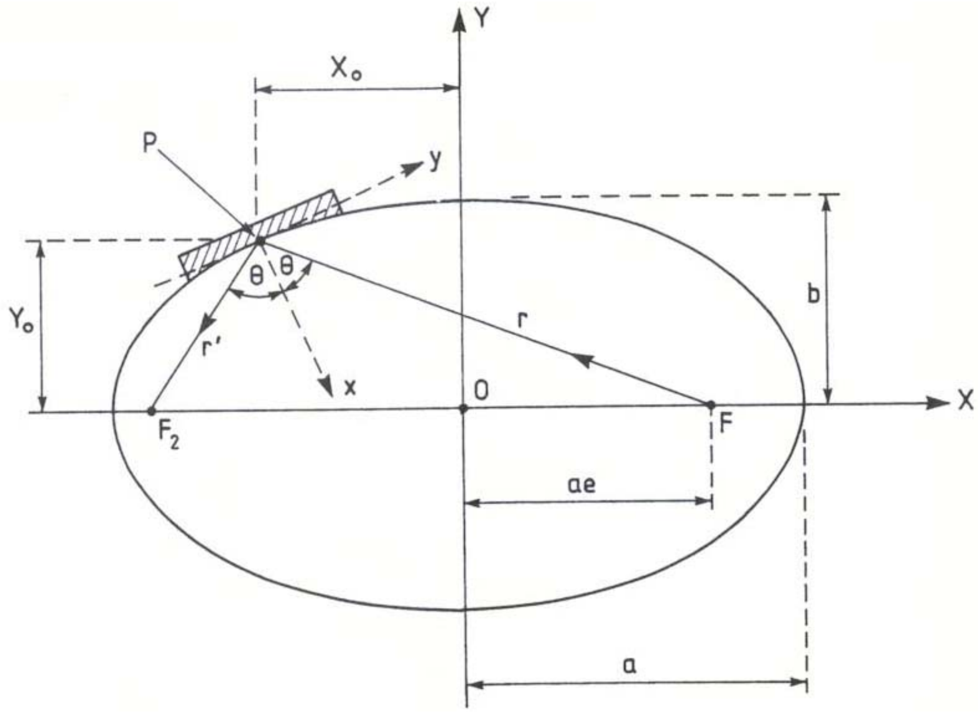
Line equation: $$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1$$

Ellipsoid equation:

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}+\frac{Z^2}{b^2}=1$$

where: $a=\frac{r+r'}{2}$; $b=a\sqrt{1-e^2}$

$$e=\frac{1}{2a}\sqrt{r^2+r'^2-2rr'\cos(2\vartheta)}$$



Rays from one focus F_1 will always be perfectly focused to the second focus F_2.

$$x^2\left(\frac{\sin^2\vartheta}{b^2}+\frac{1}{a^2}\right)+y^2\left(\frac{\cos^2\vartheta}{b^2}\right)+\frac{z^2}{b^2}-x\left(\frac{4f\cos\vartheta}{b^2}\right)-xy\left[\frac{2\sin\vartheta\sqrt{e^2-\sin^2\vartheta}}{b^2}\right]=0$$

where: $$f=\left(\frac{1}{r}+\frac{1}{r'}\right)^{-1}$$

J.B. West and H.A. Padmore, Optical Engineering, 1987

Toroid (1)

The bicycle tyre toroid is generated by rotating a circle of radius ρ in an arc of radius R. In general, two non-coincident focii are produced: one in the meridional plane and one in the sagittal plane

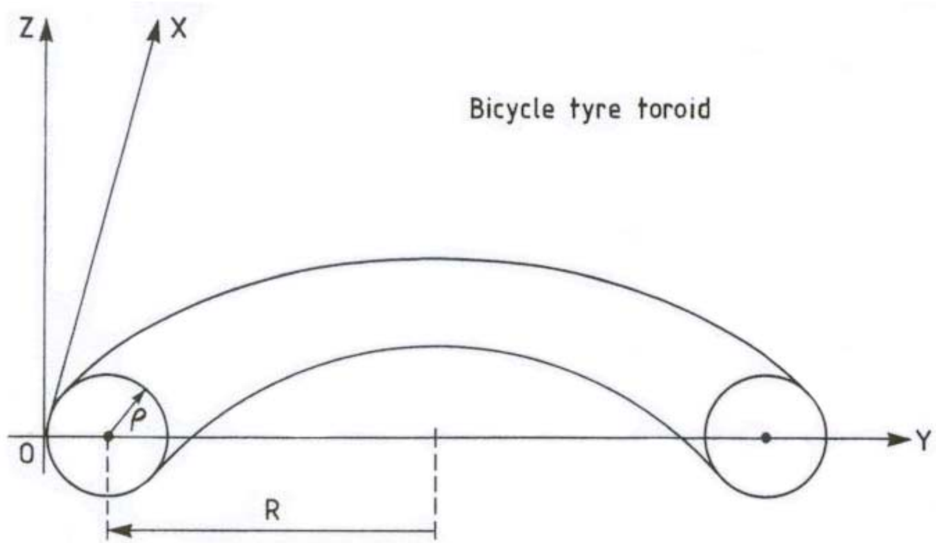
Tangential focus:

$$\left(\frac{1}{r}+\frac{1}{r'_t}\right)\frac{\cos\vartheta}{2}=\frac{1}{R}$$

Sagittal focus:

$$\left(\frac{1}{r}+\frac{1}{r'_s}\right)\frac{1}{2\cos\vartheta}=\frac{1}{\rho}$$

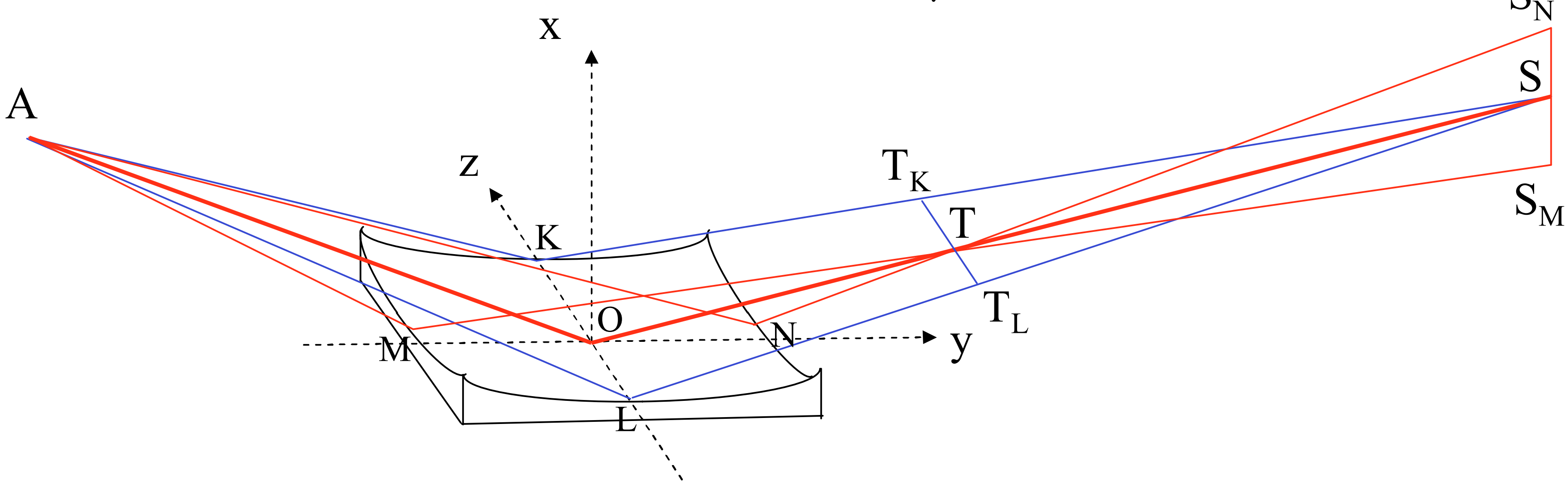
Stigmatic image: $$\frac{\rho}{R}=\cos^2\vartheta$$



J.B. West and H.A. Padmore, Optical Engineering, 1987

Toroid (2)

$$x^2 + y^2 + z^2 = 2Rx - 2R(R-\rho) + 2(R-\rho)\sqrt{(R-x)^2 + y^2}$$



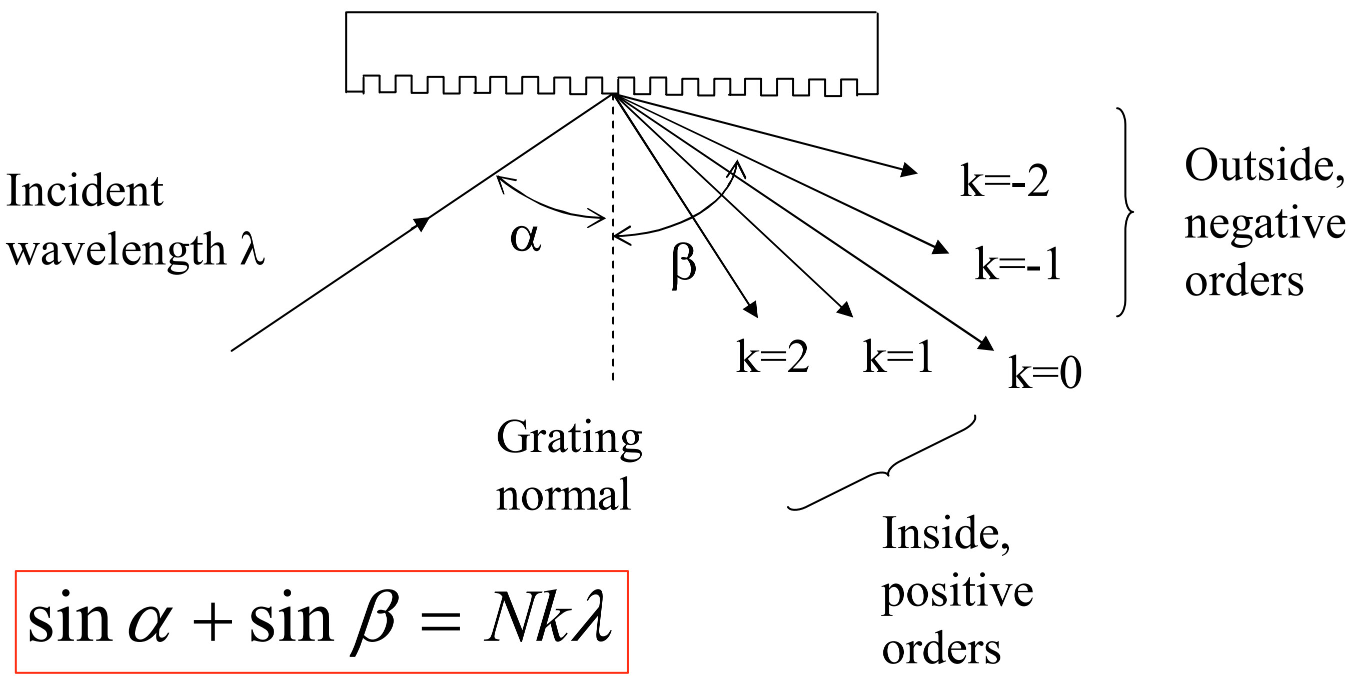
For ρ=R → spherical mirror

A stigmatic image can only be obtained at normal incidence.

For a vertical deflecting spherical mirror at grazing incidence the horizontal sagittal focus is always further away from the mirror than the vertical tangential focus. The mirror only weakly focusses in the sagittal direction.

Gratings

The diffraction grating separates the different components of the spectrum by redirecting the radiation by an amount which depends upon the wavelength.



$$\sin\alpha + \sin\beta = Nk\lambda$$

N=1/d is the groove density, k is the order of diffraction (±1,±2,...)

VUV, EUV and soft x-rays beamline

Basic elements:

- mirrors to focus, deflect and filter
- gratings to diffract
- slits to spatially select the radiation

Optical elements have to preserve the quality (brilliance) of the radiation

Conserving brilliance

Brilliance decreases because of:

- roughness and slope errors on optical surfaces

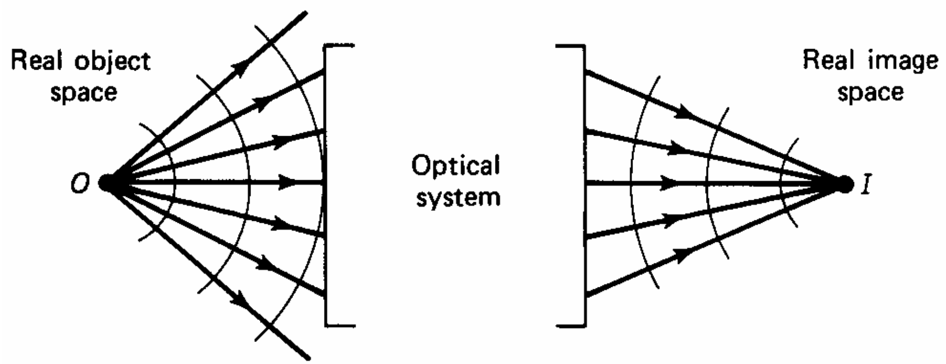
- thermal deformations of optical elements due to heat load produced by the high power radiation

- aberrations of optical elements

In the following we will consider OEs with theoretical surface shapes.

Perfect imaging and aberrations

An ideal optical element is able to perform perfect imaging if all the rays originating from a single object point cross at a single image point.



Deviations from perfect imaging are called aberrations.

Aberrations theory

Image quality is essential for achieving high energy and spatial resolution → knowledge of aberrations theory is necessary
It shows what the different aberration terms are and how they play a role in the image formation → it teaches how aberrations can be reduced

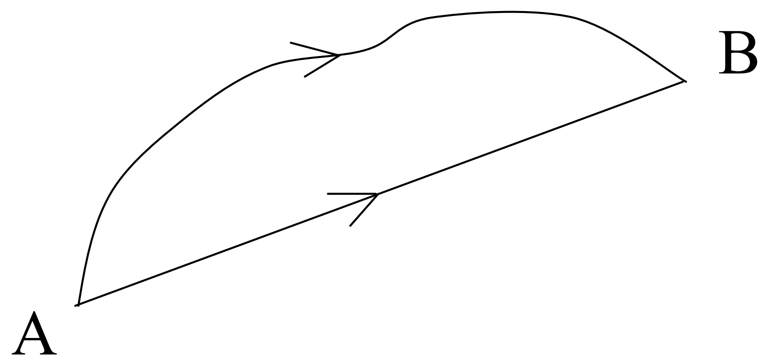
Goal: understand in general terms how to treat mathematically the focusing properties of a concave optical element.
We will study the case of a grating.

The general theory of aberrations of diffraction gratings applies Fermat's principle to derive expressions for the aberration coefficients.

Fermat's principle

Light-rays choose their paths to minimize the optical length

$$\int_A^B n(\vec{r})dl$$



$n(\vec{r})$: index of refraction of the medium

dl : line segment along the path

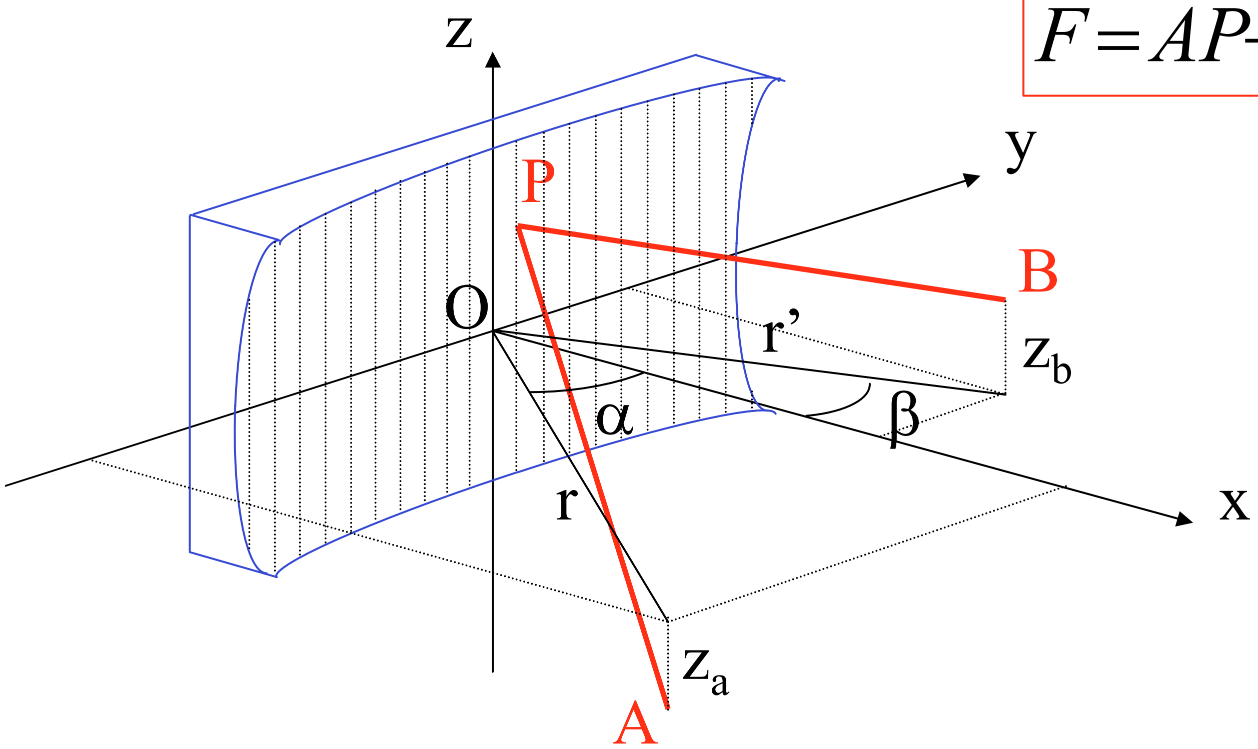
A more accurate statement:

a light-ray going from A to B must traverse an optical path length which is stationary with respect to small variations of that path

Theory of conventional diffraction gratings

For a classical grating with rectilinear grooves parallel to z with constant spacing d, the optical path length is:

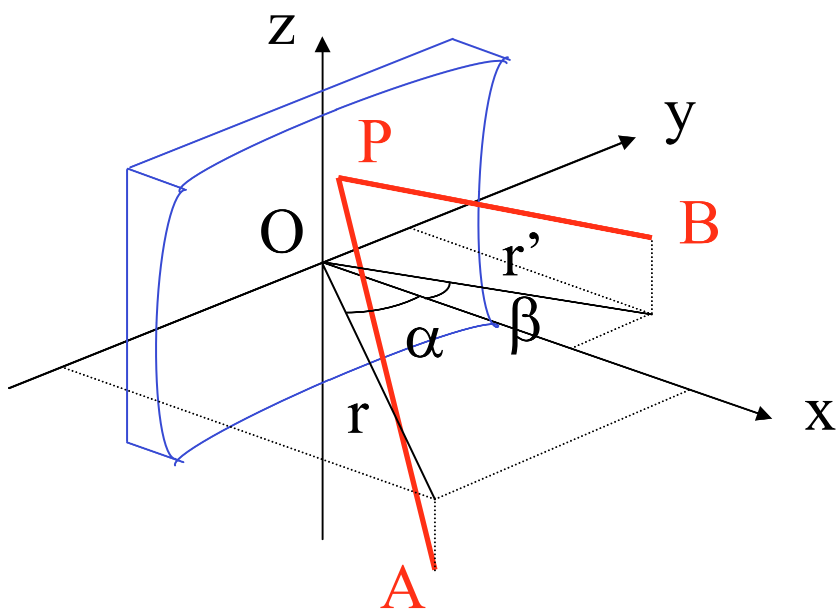
$$F = \overline{AP} + \overline{PB} + kN\lambda y$$



where λ is the wavelength of the diffracted light, k is the order of diffraction (±1,±2,...), N=1/d is the groove density

Perfect focus condition (1)

Let us consider some number of light rays starting from A and impinging on the grating at different points P. Fermat's principle states that if the point A is to be imaged at the point B, then all the optical path lengths from A via the grating surface to B will be the same.



B is the point of a perfect focus if:

$$\frac{\partial F}{\partial y} = 0 \qquad \frac{\partial F}{\partial z} = 0$$

for any pair of (y,z)

Perfect focus condition (2)

Equations:

$$F = \overline{AP} + \overline{PB} + kN\lambda y \quad + \quad \frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0 \quad \textit{for any pair of } (y,z)$$

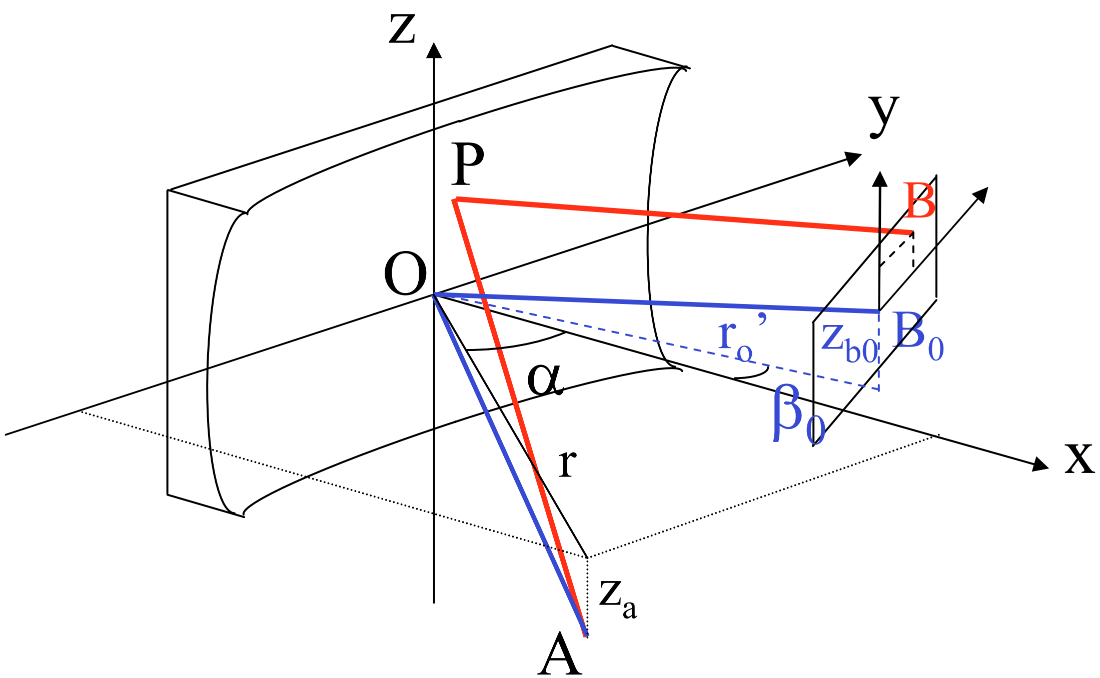
can be used to decide on the required characteristics of the diffraction grating:

- the shape of the surface
- the grooves density
- the object and image distances

Aberrated image

In general, $\frac{\partial F}{\partial y}$ and $\frac{\partial F}{\partial z}$ are functions of y and z and can not be made zero for any y,z

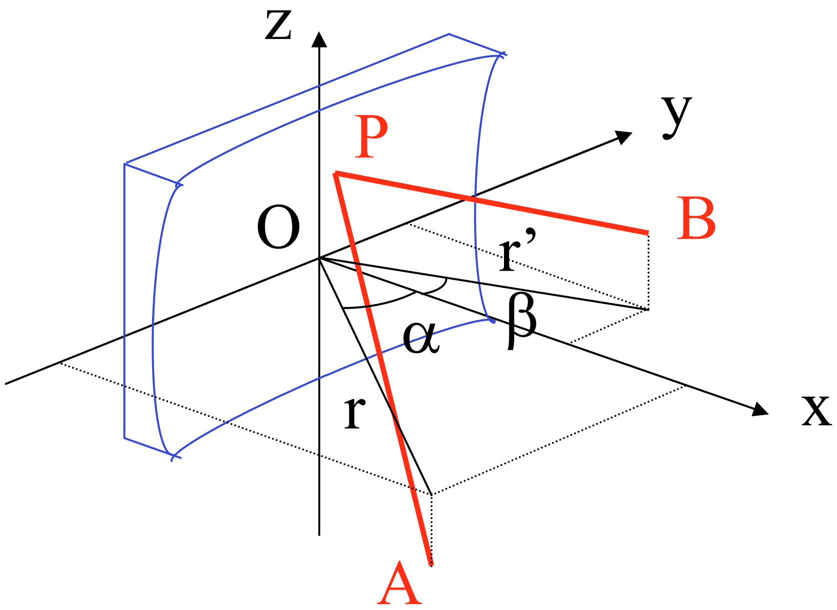
→ when the point P wanders over the grating surface, diffracted rays fall on slightly different points on the focal plane and an aberrated image is formed



- B_0: gaussian image, produced by the central ray
- B: ray diffracted by the generic point P on the grating surface
- Aberrations: displacements of B with respect to B_0

Grating surface

The grating surface may in general be described by a series expansion:



$$x = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} y^i z^j$$

$a_{00} = a_{10} = a_{01} = 0$ because of the choice of origin

j = even if the xy plane is a symmetry plane

Giving suitable values to the coefficients a_{ij}'s we obtain the expressions for the various geometrical surfaces.

a_{ij} coefficients (1)

Toroid

$$a_{02} = \frac{1}{2\rho}; \quad a_{20} = \frac{1}{2R}; \quad a_{22} = \frac{1}{4R^2\rho}; \quad a_{40} = \frac{1}{8R^3};$$

$$a_{04} = \frac{1}{8\rho^3}; \quad a_{12} = 0; \quad a_{30} = 0$$

Sphere, cylinder and plane are special cases of toroid:

R=ρ → **sphere**

R= ∞ → **cylinder**

R=ρ= ∞ → **plane**

Paraboloid

$$a_{02} = \frac{1}{4f\cos\vartheta}; \quad a_{20} = \frac{\cos\vartheta}{4f}; \quad a_{22} = \frac{3\sin^2\vartheta}{32f^3\cos\vartheta};$$

$$a_{12} = -\frac{\tan\vartheta}{8f^2}; \quad a_{30} = -\frac{\sin\vartheta\cos\vartheta}{8f^2}$$

$$a_{40} = \frac{5\sin^2\vartheta\cos\vartheta}{64f^3}; \quad a_{04} = \frac{\sin^2\vartheta}{64f^3\cos^3\vartheta}$$

a_{ij} coefficients (2)

Ellipsoid

$$a_{02} = \frac{1}{4f\cos\vartheta}; \quad a_{20} = \frac{\cos\vartheta}{4f}; \quad a_{04} = \frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{\sin^2\vartheta}{b^2} + \frac{1}{a^2}\right];$$

$$a_{12} = \frac{\tan\vartheta}{8f^2\cos\vartheta}\sqrt{e^2 - \sin^2\vartheta}; \quad a_{30} = \frac{\sin\vartheta}{8f^2}\sqrt{e^2 - \sin^2\vartheta} \; ;$$

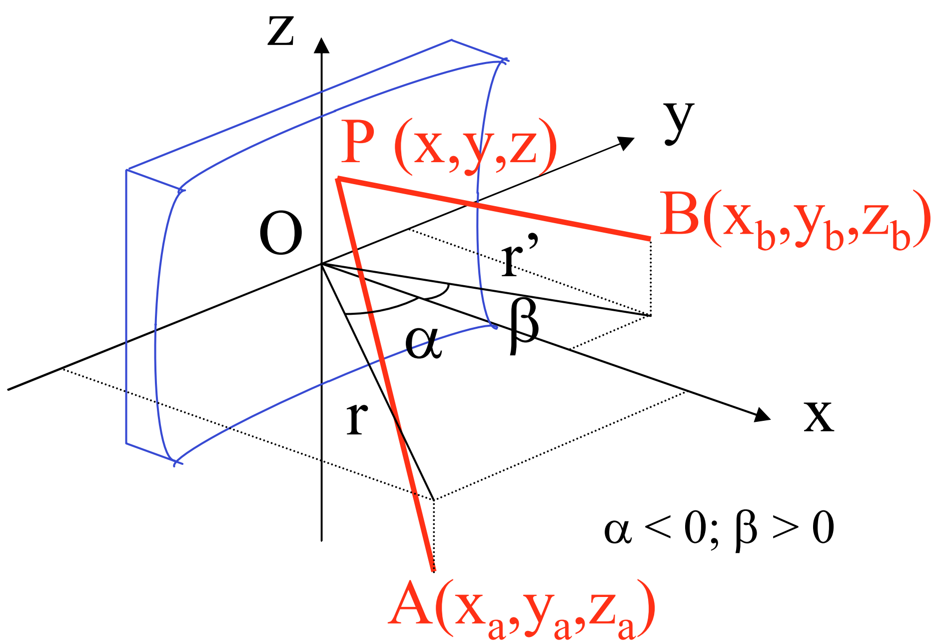
$$a_{40} = \frac{b^2}{64f^3\cos^3\vartheta}\left[\frac{5\sin^2\vartheta \ \cos^2\vartheta}{b^2} - \frac{5\sin^2\vartheta}{a^2} + \frac{1}{a^2}\right];$$

$$a_{22} = \frac{\sin^2\vartheta}{16f^3\cos^3\vartheta}\left[\frac{3}{2}\cos^2\vartheta - \frac{b^2}{a^2}\left(1 - \frac{\cos^2\vartheta}{2}\right)\right]$$

$$where \quad f = \left[\frac{1}{r} + \frac{1}{r'}\right]^{-1}$$

http://xdb.lbl.gov/Section4/Sec_4-3Extended.pdf

Optical path function (1)



$$F = \overline{AP} + \overline{PB} + kN\lambda y$$

$$\overline{AP} = \sqrt{(x_a - x)^2 + (y_a - y)^2 + (z_a - z)^2}$$

$$\overline{PB} = \sqrt{(x_b - x)^2 + (y_b - y)^2 + (z_b - z)^2}$$

$$x_a = r\cos\alpha \qquad y_a = r\sin\alpha$$

$$x_b = r'\cos\beta \qquad y_b = r'\sin\beta$$

Optical path function (2)

$$F = \sum_{ijk} F_{ijk} y^i z^j$$

$$= F_{000} + yF_{100} + zF_{011} + \frac{1}{2} y^2 F_{200} + \frac{1}{2} z^2 F_{020} + \frac{1}{2} y^3 F_{300}$$

$$+ \frac{1}{2} yz^2 F_{120} + \frac{1}{8} y^4 F_{400} + \frac{1}{4} y^2 z^2 F_{220} + \frac{1}{8} z^4 F_{040}$$

$$+ yzF_{111} + \frac{1}{2} yF_{102} + \frac{1}{4} y^2 F_{202} + \frac{1}{2} y^2 zF_{211} + \ldots$$

$$F_{ijk} = z_a^k C_{ijk}(\alpha, r) + z_b^k C_{ijk}(\beta, r') + Nk\lambda f_{ijk}$$

$$f_{ijk} = \begin{cases} 1 & \text{when ijk} = 100 \\ 0 & \text{otherwise} \end{cases}$$

Perfect focus condition (3)

$$\frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0 \quad \text{for any pair of } (y,z)$$

⬇

$$F_{ijk} = 0 \quad \text{for all } ijk \neq (000)$$

Each term $F_{ijk} y^i z^j$ in the series (except F_{000} and F_{100}) represents a particular type of aberration

F_{ijk} coefficients (1)

for r,r’ >> z_a,z_b

$$F_{000} = r + r'$$

$$F_{100} = Nk\lambda - (\sin\alpha + \sin\beta)$$

$$F_{200} = \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta}{r'}\right) - 2a_{20}(\cos\alpha + \cos\beta)$$

$$F_{020} = \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta)$$

$$F_{300} = \left[\frac{T(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{T(r',\beta)}{r'}\right]\sin\beta - 2a_{30}(\cos\alpha + \cos\beta)$$

$$F_{120} = \left[\frac{S(r,\alpha)}{r}\right]\sin\alpha + \left[\frac{S(r',\beta)}{r'}\right]\sin\beta - 2a_{12}(\cos\alpha + \cos\beta)$$

where $T(r,\alpha) = \dfrac{\cos^2\alpha}{r} - 2a_{20}\cos\alpha$ and $S(r,\alpha) = \dfrac{1}{r} - 2a_{02}\cos\alpha$

and analogous expressions for $T(r',\beta)$ and $S(r',\beta)$

F_{ijk} coefficients (2)

$$F_{400} = \left[\frac{4T(r,\alpha)}{r^2}\right]\sin^2\alpha - \left[\frac{T^2(r,\alpha)}{r}\right] + \left[\frac{4T(r',\beta)}{r'^2}\right]\sin^2\beta - \left[\frac{T^2(r',\beta)}{r'}\right]$$
$$-8a_{30}\left[\frac{\sin\alpha\cos\alpha}{r} + \frac{\sin\beta\cos\beta}{r'}\right] - 8a_{40}(\cos\alpha + \cos\beta) + 4a_{20}^2\left[\frac{1}{r} + \frac{1}{r'}\right]$$

$$F_{220} = \left[\frac{2S(r,\alpha)}{r^2}\right]\sin^2\alpha + \left[\frac{2S(r',\beta)}{r'^2}\right]\sin^2\beta - \left[\frac{T(r,\alpha)S(r,\alpha)}{r}\right] - \left[\frac{T(r',\beta)S(r',\beta)}{r'}\right]$$
$$+4a_{20}a_{02}\left[\frac{1}{r} + \frac{1}{r'}\right] - 4a_{22}(\cos\alpha + \cos\beta) - 4a_{12}\left[\frac{\sin\alpha\cos\alpha}{r} + \frac{\sin\beta\cos\beta}{r'}\right]$$

$$F_{040} = 4a_{02}^2\left[\frac{1}{r} + \frac{1}{r'}\right] - 8a_{04}(\cos\alpha + \cos\beta) - \left[\frac{S^2(r,\alpha)}{r}\right] - \left[\frac{S^2(r',\beta)}{r'}\right]$$

F_{ijk} coefficients (3)

$$F_{011} = -\frac{z_a}{r} - \frac{z_b}{r'}$$

$$F_{111} = -\frac{z_a \sin\alpha}{r^2} - \frac{z_b \sin\beta}{r'^2}$$

$$F_{102} = \frac{z_a{}^2 \sin\alpha}{r^2} + \frac{z_b{}^2 \sin\beta}{r'^2}$$

$$F_{202} = \left(\frac{z_a}{r}\right)^2 \left[\frac{2\sin^2\alpha}{r} - T(r,\alpha)\right] + \left(\frac{z_b}{r'}\right)^2 \left[\frac{2\sin^2\beta}{r'} - T(r',\beta)\right]$$

$$F_{211} = \frac{z_a}{r^2}\left[T(r,\alpha) - \frac{2\sin^2\alpha}{r}\right] + \frac{z_b}{r'^2}\left[T(r',\beta) - \frac{2\sin^2\beta}{r'}\right]$$

Gaussian image point (1)

If we apply Fermat's principle to the central ray: $\left(\frac{\partial F}{\partial y}\right)_{y=0,z=0} = 0 \quad \left(\frac{\partial F}{\partial z}\right)_{y=0,z=0} = 0$

$F_{100} = 0 \Rightarrow \sin\alpha + \sin\beta_0 = Nk\lambda$ **grating equation**

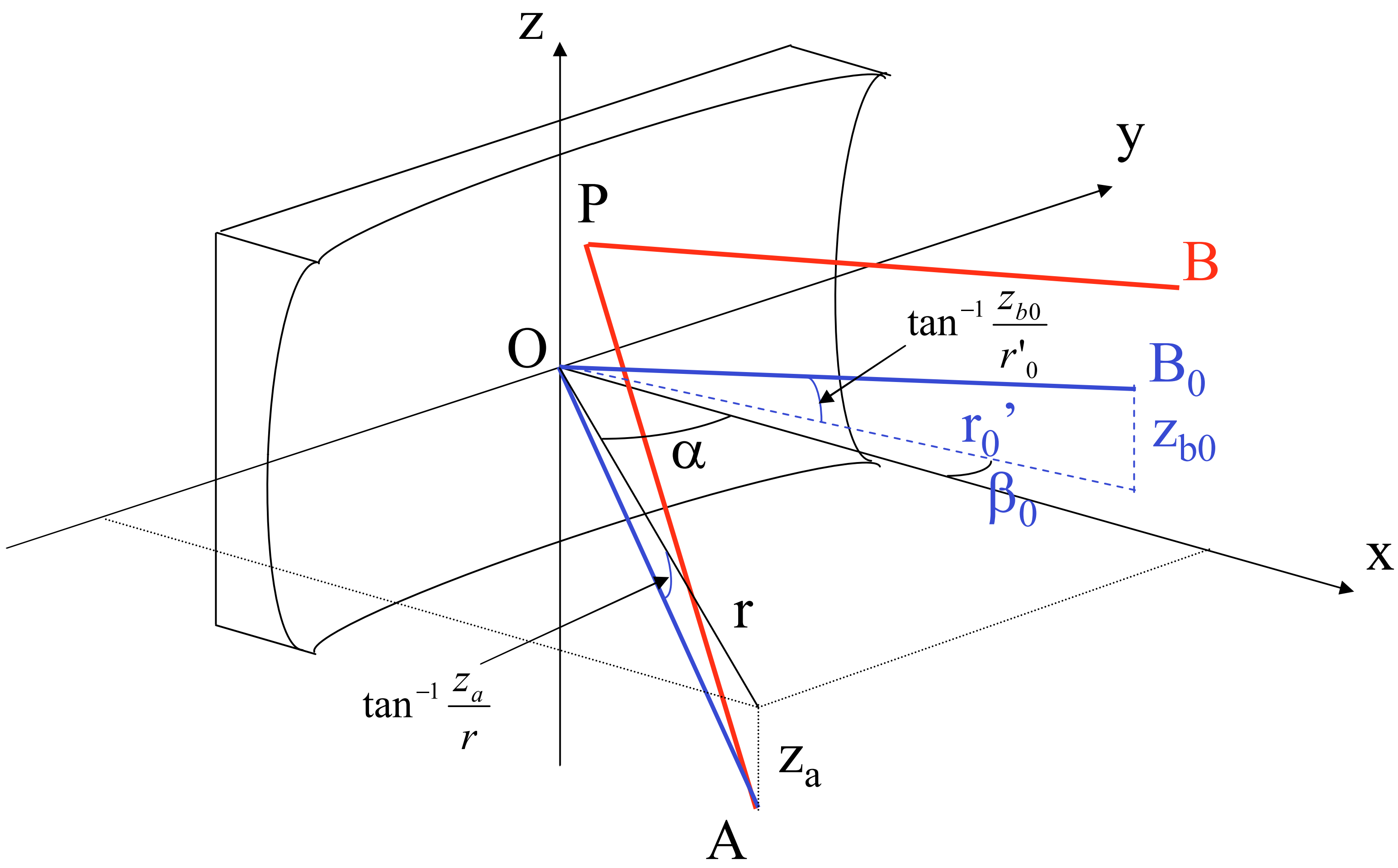
$F_{011} = 0 \Rightarrow \frac{z_a}{r} = -\frac{z_{b0}}{r'_0}$ law of magnification in the sagittal direction

The tangential focal distance r'_0 is obtained by setting:

$F_{200} = 0 \Rightarrow \left(\frac{\cos^2\alpha}{r} + \frac{\cos^2\beta_0}{r'_0}\right) - 2a_{20}(\cos\alpha + \cos\beta_0) = 0$ **tangential focusing**

The three above equations determine the Gaussian image point $B_0(r'_0, \beta_0, z_{b0})$

Gaussian image point (2)



Sagittal focusing

While the second order aberration term F_{200} governs the tangential focusing, the second order term F_{020} governs the sagittal focusing:

$$F_{020} = 0 \quad \Longrightarrow \quad \frac{1}{r} + \frac{1}{r'} - 2a_{02}(\cos\alpha + \cos\beta) = 0$$

sagittal focusing

Example: toroidal mirror

Substituting $a_{02} = \frac{1}{2\rho}; \quad a_{20} = \frac{1}{2R}$ in $F_{200} = 0; \quad F_{020} = 0$

and imposing $\alpha = -\beta = \theta$

$$\Longrightarrow \quad \left(\frac{1}{r} + \frac{1}{r_t'}\right)\frac{\cos\vartheta}{2} = \frac{1}{R} \qquad \left(\frac{1}{r} + \frac{1}{r_s'}\right)\frac{1}{2\cos\vartheta} = \frac{1}{\rho}$$

Aberrations terms

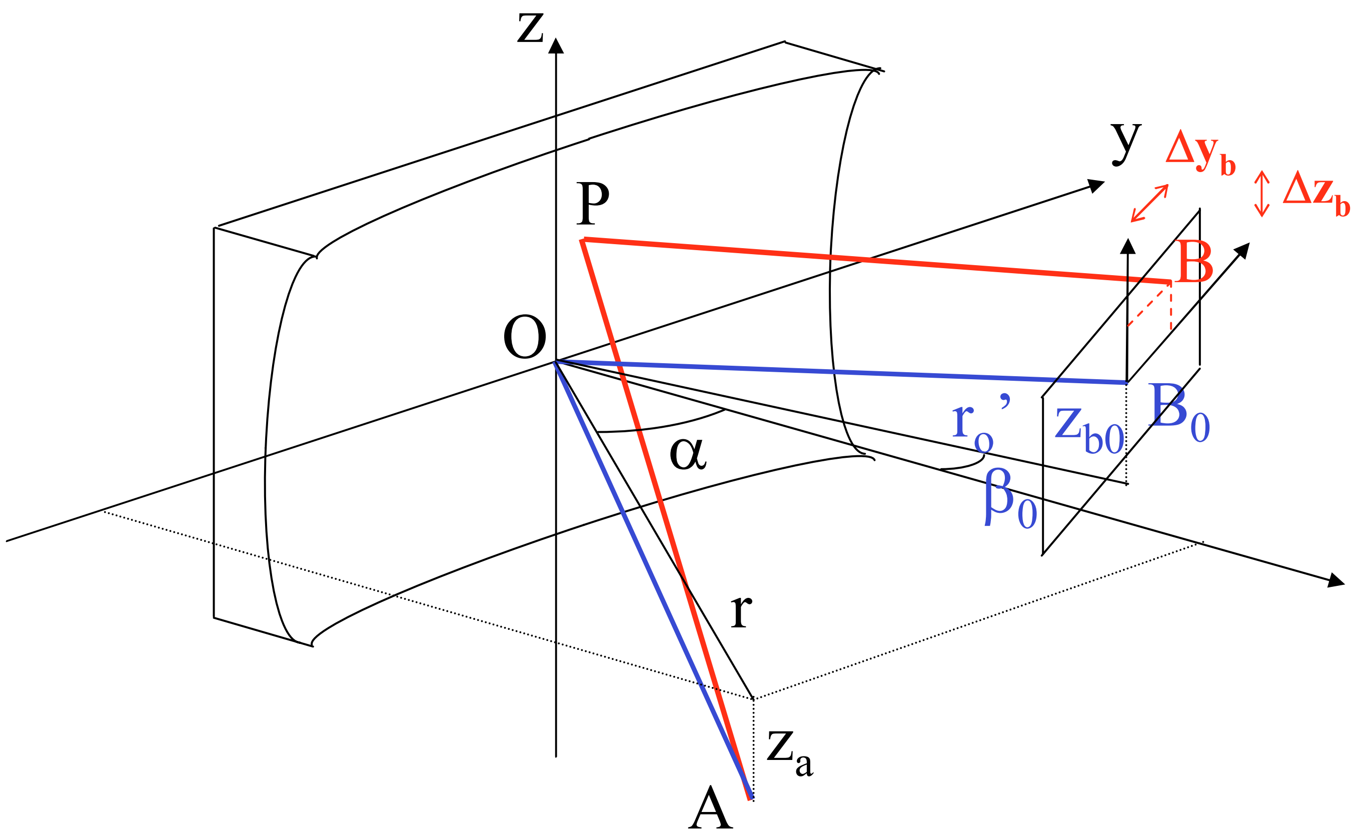
Most important imaging errors:

F_{200}	defocus
F_{020}	astigmatism
F_{300}	primary coma (aperture defect)
F_{120}	astigmatic coma
F_{400} F_{220} F_{040}	spherical aberration

There is an ambiguity in the naming of the aberrations in the grazing incidence case!

Ray aberrations (1)

The generic ray starting from A will arrive at the focal plane at a point B displaced from the Gaussian image point B_0 by the ray aberrations Δy_b and Δz_b:



$$\Delta y_b = \frac{r_0'}{\cos \beta_0} \frac{\partial F}{\partial y}$$

$$\Delta z_b = r_0' \frac{\partial F}{\partial z}$$

Ray aberrations (2)

Substituting the expansion of F , the ray aberrations for each aberration type can be calculated separately:

$$\Delta y_b^{\,ijk} = \frac{r_0'}{\cos\beta_0} F_{ijk}\, i\, y^{i-1}\, z^{j}$$

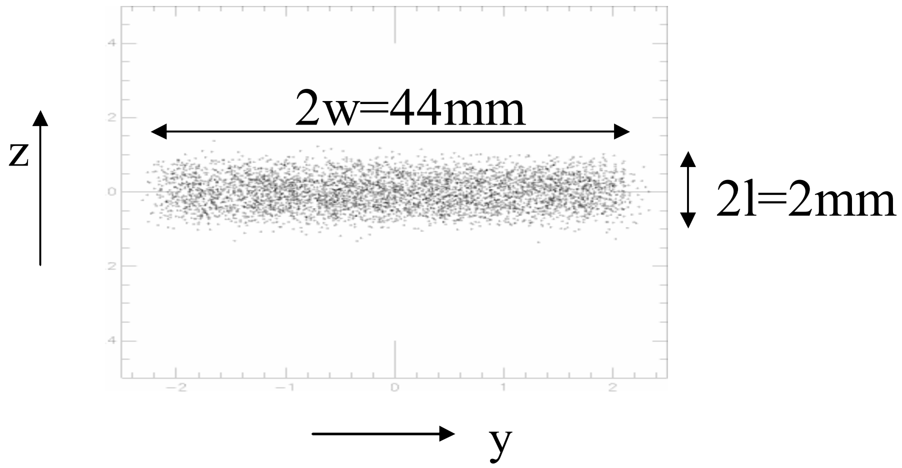
$$\Delta z_b^{\,ijk} = r_0' \, F_{ijk}\, y^{i}\, j\, z^{j-1}$$

Provided the aberrations are not too large, they are additive: they may either reinforce or cancel.

$$\Delta y_b = \sum_{ijk} \Delta y_b^{\,ijk} \qquad \Delta z_b = \sum_{ijk} \Delta z_b^{\,ijk}$$

Aberrated image

Example of footprint on the grating:



Substituting y=±w and z=±l in the ray aberrations Δy_b^{ijk} and Δz_b^{ijk} , we evaluate the contributions of the rays which are more distant from the pole of the grating

→ size ($\Delta y_b * \Delta z_b$) of the resulting aberrated image

Defocus and coma contributions

The defocus contribution is linear in the ruled length (± w) of the grating, the error in the dispersive direction is symmetric about the Gaussian image point:

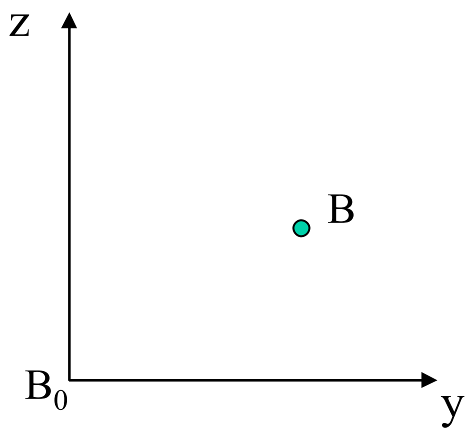
$$\Delta y_b{}^{200}(\pm w) = \pm \frac{r_0'}{\cos\beta_0} F_{200}\, 2\, w$$

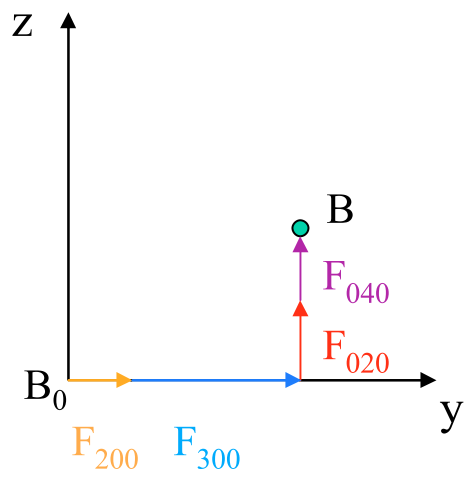
The coma contribution is proportional to w^2 giving a dispersive error which only occurs on one side of the Gaussian image point for rays from both the top and the bottom of the grating (y=±w):

$$\Delta y_b{}^{300}(\pm w) = \frac{r_0'}{\cos\beta_0} F_{300}\, 3\, w^2$$

Comparison ray trace - aberration calculations

Example





Ray trace simple tells us that the ray arrives in a certain point

Aberration-based calculations specify the different contributions

Aberrations contribution to resolution

$$\Delta\lambda = \left(\frac{\partial\lambda}{\partial\beta}\right)_{\alpha=const} \Delta\beta$$

$$= \frac{\cos\beta}{Nk}\Delta\beta$$

Substituting: $\Delta\beta = \frac{\Delta y_b}{r'}$ → $\Delta\lambda = \frac{\cos\beta}{Nk}\frac{\Delta y_b}{r'}$

Substituting: $\Delta y_b = \frac{r_0'}{\cos\beta_0}\frac{\partial F}{\partial y}$ → $\Delta\lambda = \frac{1}{Nk}\frac{\partial F}{\partial y}$

$$\Delta\lambda = \frac{1}{Nk}\sum_{ijk} F_{ijk}\, i\, y^{i-1}\, z^{j}$$

Aberration theory: conclusions

• Perfect focus condition: $\frac{\partial F}{\partial y} = 0 \quad \frac{\partial F}{\partial z} = 0$ for each pair (y,z)
→ all the coefficients F_{ijk} must be zero

• Non-zero values for the coefficients F_{ijk} lead to displacements of the rays arriving in the image plane from the ideal Gaussian image point.

• We have found the expressions for these rays displacements and the corresponding contributions to wavelength resolution. In this way the impact on the imaging and energy resolution properties of a given grating can be evaluated.

• By a proper choice of the grating shape, groove density, object and image distances, the sum of the aberrations may be reduced to a minimum.

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