ICTP school 2006
X- Ray Detectors

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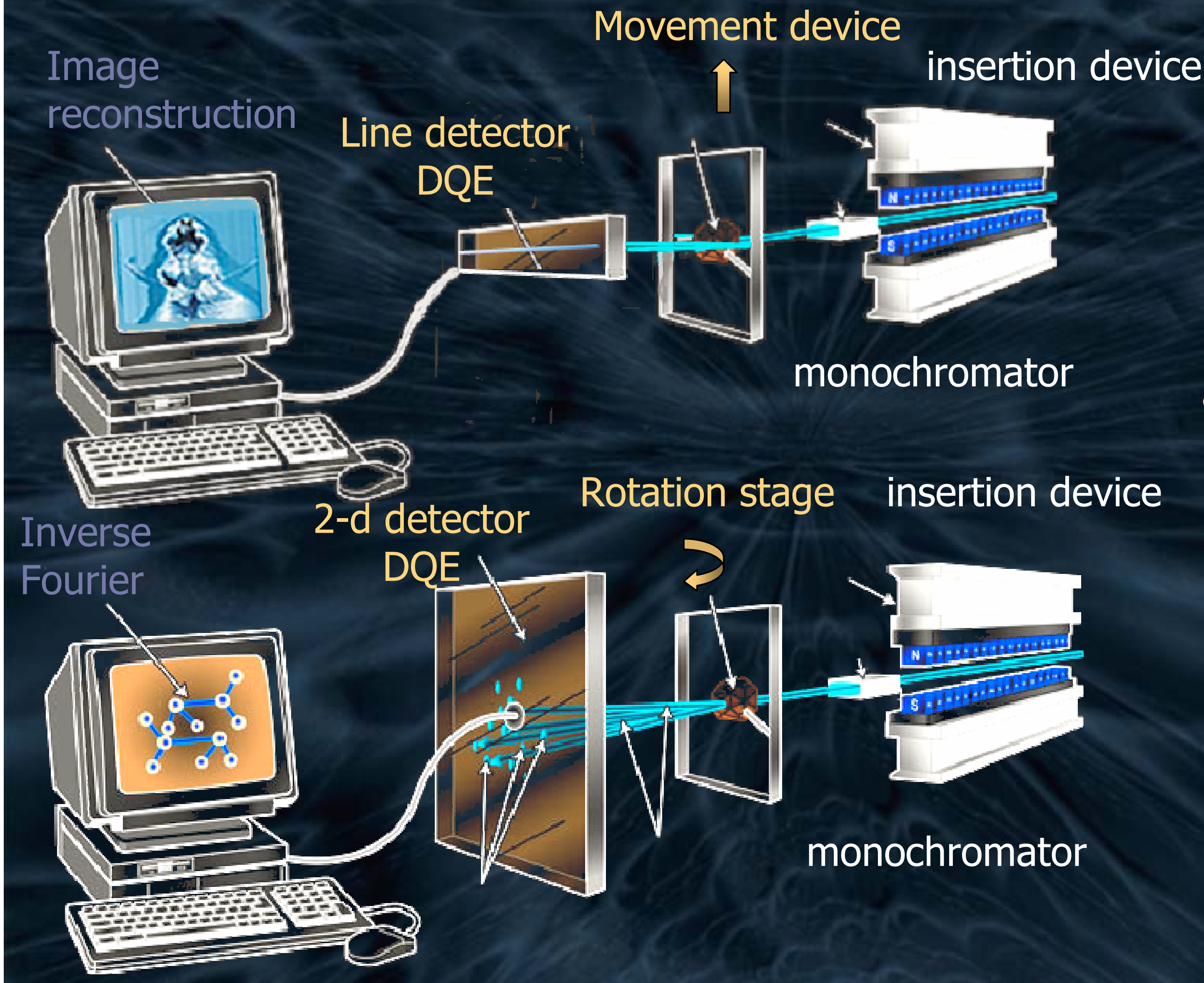
Imaging set up with X-rays



Direct imaging

Length scale

0.5 μm – 30 cm

most of pixels see approx same photon

~ 10^4 for 0.1%

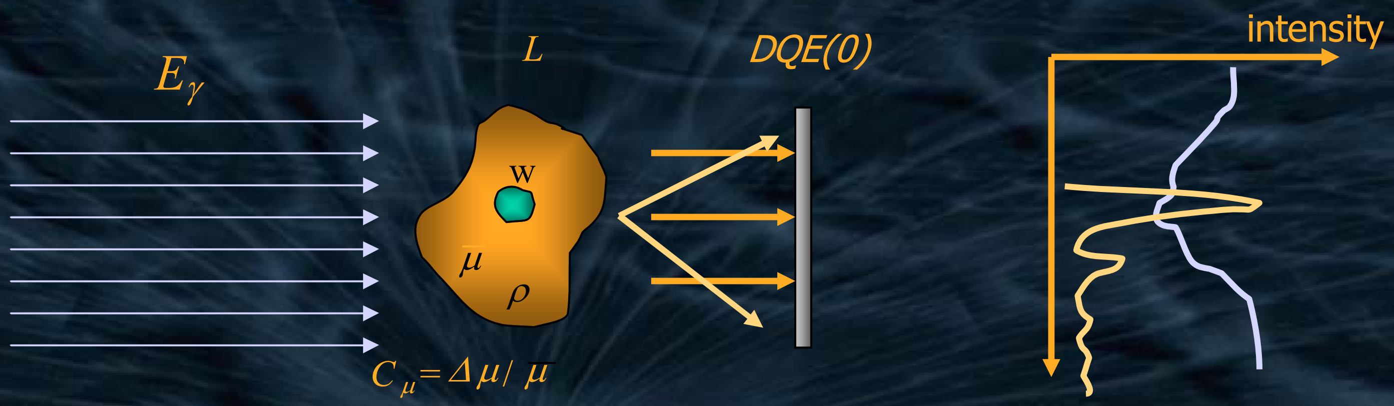
Fourier indirect imaging

Length scale

15 Å –5000Å

most of pixels see ~ 0 photons

Dose considerations



Direct Imaging

$$D_{skin} = \frac{2 \cdot L \cdot e^{\mu \cdot L} \cdot SNR_{out}^{2}}{DQE(f) \cdot \mu^{2} \cdot w^{4} \cdot C_{\mu}^{2}} \cdot E_{\gamma} \cdot \left(\frac{\mu}{\rho}\right)$$

$$D_{sample} = \frac{\mu \cdot P \cdot h \cdot \nu}{DQE(f) \cdot \rho^{2} \cdot w^{4} \cdot \lambda^{2} \cdot r_{e}^{2}}$$

$$\left.\right\} \approx \frac{1}{w^{4} \cdot DQE(f)}$$

Indirect imaging

Signal to noise &
Detective Quantum Efficiency DQE

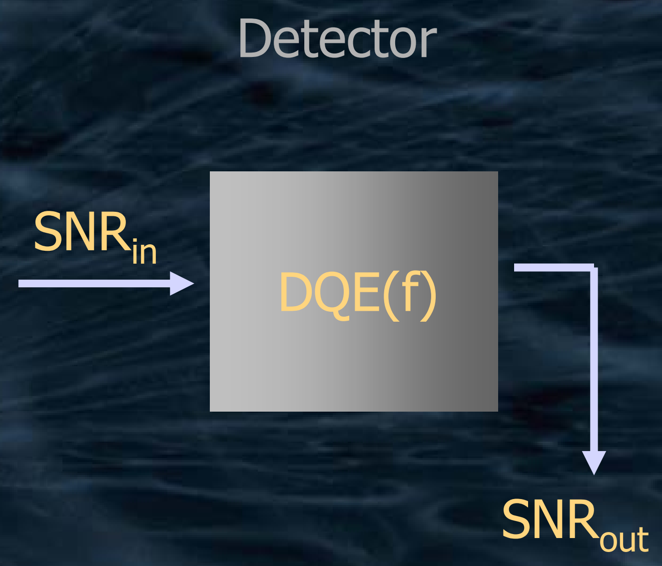
$$SNR \equiv \frac{Signal}{noise} = \frac{S}{\sigma}$$

In case of photons > Poisson statistics

$$S = N,\ \ \sigma = \sqrt{N}\ \ \Rightarrow SNR^2 = N$$

$$DQE(f) \equiv \frac{SNR_{out}^2}{SNR_{in}^2} = \frac{SNR_{out}^2}{N}$$

$$DQE \subset [0,1]$$



Your measurement!

To be or not to be in nature

is here the question of the detector….

Detective Quantum Efficiency DQE

What looks DQE like?

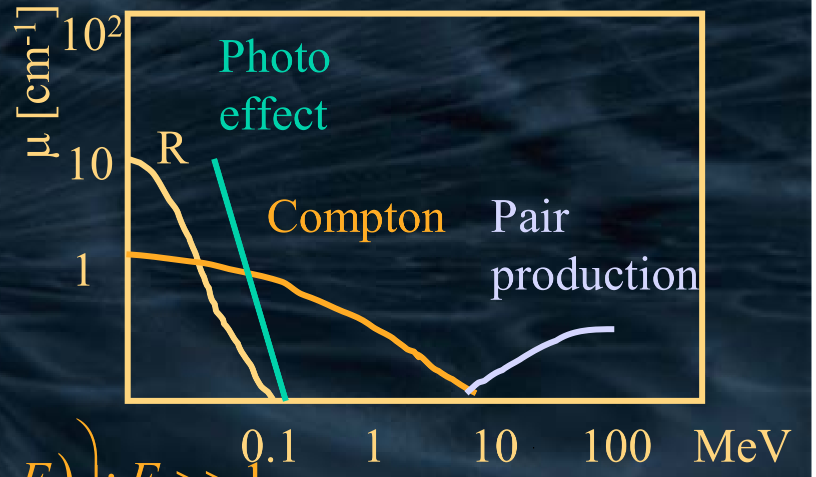
To answer this questions one as to understand the underlying detection principle

Bottom line: Convert photons to free charges and measure those

$$\mu \approx \frac{Z^5}{E^{\frac{7}{2}}}$$

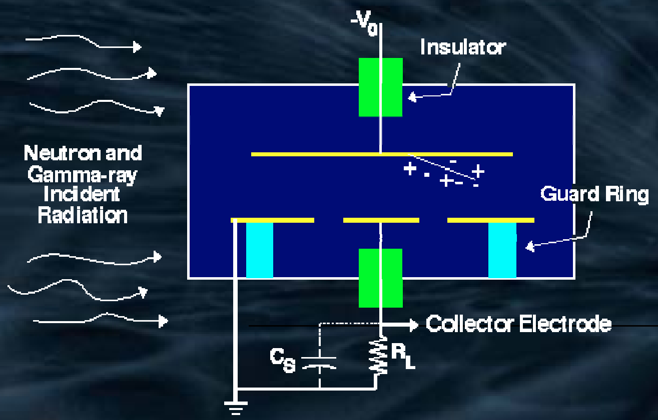
$$\mu \approx Z \cdot \frac{1}{E} \cdot \left(\frac{1}{2} + \ln\left(2 \cdot E\right) \right); E >> 1$$

$$\mu \approx Z^2 \cdot \left(\ln\left(2 \cdot E\right) \right); 1 < E < \frac{137}{Z^{1/3}}$$



Charge Collection

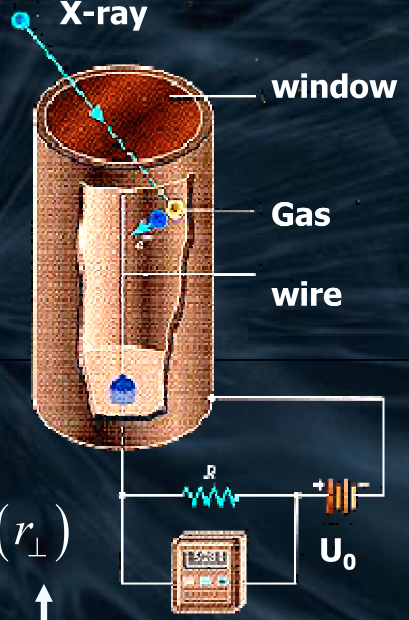
Integrating detectors



Gas	W_{ion}[eV]
Ar	26
Kr	24
Xe	22

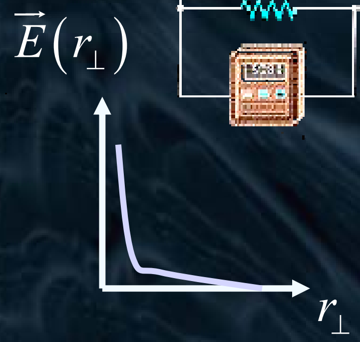
$$Q(E_\gamma) = \frac{E_\gamma}{W_{ion}} \cdot \varepsilon(E_\gamma) \cdot N \cdot e^-$$

Counting detectors



$$\Delta E_{kin} = e^- \cdot \int_{r_1}^{r_2} \vec{E}(\vec{r}_\perp) \cdot d\vec{r}_\perp$$

$$= e^- \cdot U_0 \cdot \frac{\ln\left(\frac{r_2}{r_1}\right)}{\ln\left(\frac{r_a}{r_i}\right)}$$



$\Delta E_{kin} > W_{ion} \Rightarrow$ *gas amplification*

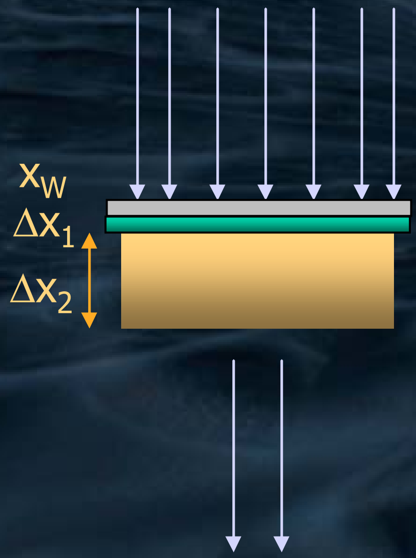
$$A = e^{\int \alpha(r_\perp) \cdot dr_\perp}$$

α Townsend Coefficient

$$Q(E_\gamma) = \varepsilon(E_\gamma) \cdot N \cdot \frac{E_\gamma}{W_{ion}} \cdot e^- \cdot A \cdot \delta(t)$$

Quantum efficiency ε

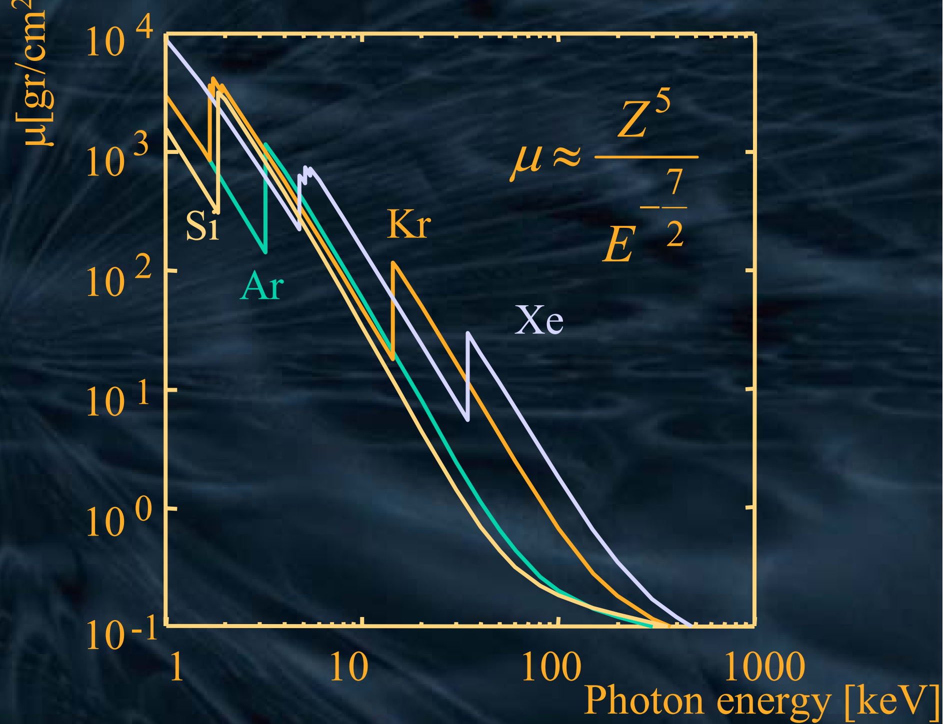
$$\varepsilon := \frac{\text{no of photons that interact in the detector volume}}{\text{no of photons in front of the detector}}$$



I_0

Detector window and/or 'dead volume'

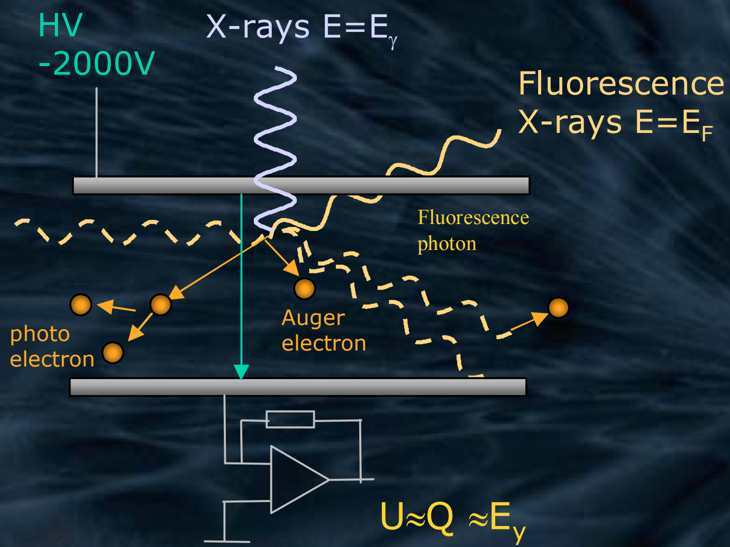
Conversion volume defined by ρ and μ



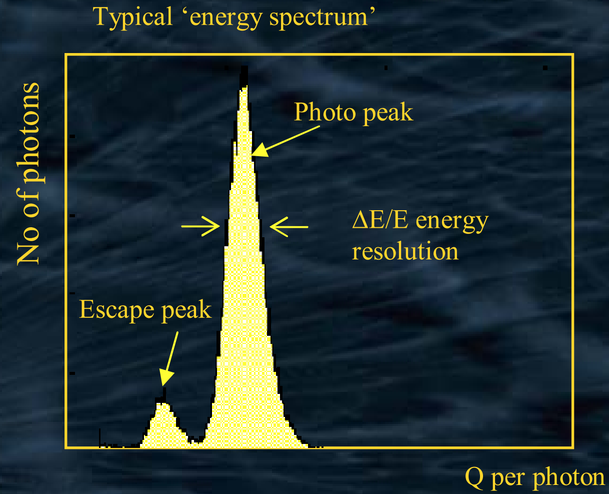
$$\varepsilon(E_\gamma) = \underbrace{e^{-\mu_w(E_\gamma)\cdot\rho_w\cdot x_w}}_{\text{Transmission window}} \cdot \underbrace{e^{-\mu_d(E_\gamma)\cdot\rho_d\cdot\Delta x_1}}_{\text{Transmission dead volume}} \cdot \left(\underbrace{1-e^{-\mu(E_\gamma)\cdot\rho\cdot\Delta x_2}}_{\text{Absorption in detector volume}}\right)$$

Energy resolution

Example: gaseous detector-. Proportional counter



Energy resolution for gaseous detectors $\Delta E/E \sim 10\%$

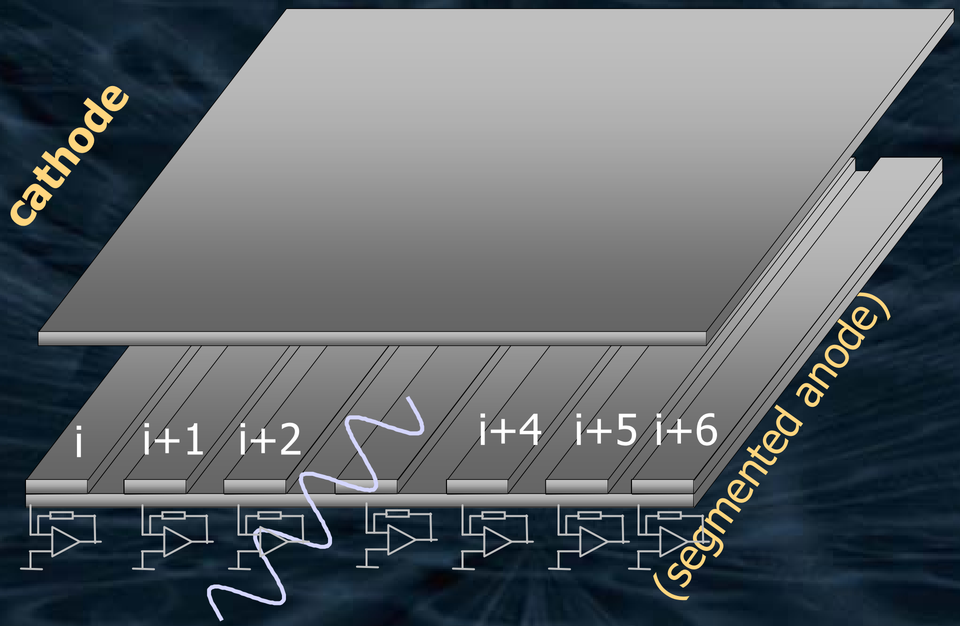


Process	Energy
photo electron	$E_p = E_\gamma - E_b$
Fluorescence photon	$E_f = E_i - E_j$
Auger electron	$E_a = E_k - 2E_l$ for photo effect on k-shell

$$\frac{\Delta E}{E} = \frac{\int_0^\infty n(E)\cdot E^2 \cdot dE - \left(\int_0^\infty n(E)\cdot E\cdot dE\right)^2}{\left(\int_0^\infty n(E)\cdot E\cdot dE\right)^2}$$

$= single\ event\ energy\ resolution$

Spatial resolution



Bottom line: connect each strip to pream. and collect charges release

Integrating detectors

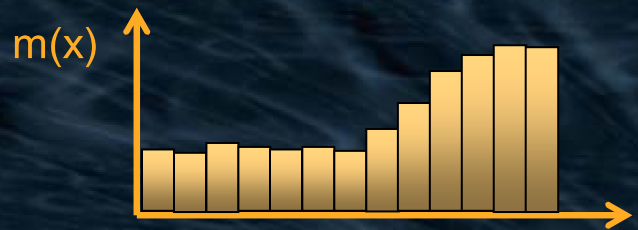
$$Q_i\left(E_\gamma\right) = \frac{E_\gamma}{W_{ion}} \cdot \varepsilon\left(E_\gamma\right) \cdot N \cdot e^-$$

Counting detectors

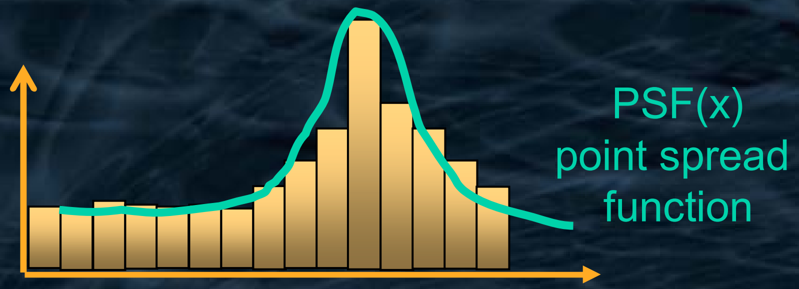
$$Q_i\left(E_\gamma\right) = \varepsilon\left(E_\gamma\right) \cdot N \cdot \frac{E_\gamma}{W_{ion}} \cdot e^- \cdot A \cdot \delta(t)$$

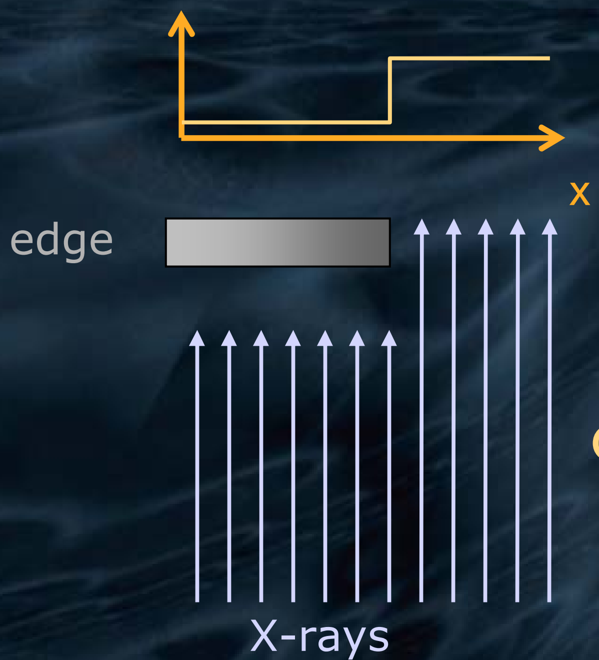
$$\langle x \rangle = \sum_i i \cdot Q_i\left(E_\gamma\right) \Big/ \sum_i Q_i\left(E_\gamma\right)$$

Spatial resolution: PSF



$$\frac{\partial m}{\partial x} \Rightarrow$$





$$m(x) = \int_{-\infty}^{\infty} PSF(x') \cdot \Theta(x - x') \cdot dx'$$

$$\frac{\partial m(x)}{\partial x} = \int_{-\infty}^{\infty} \frac{\partial}{\partial x}\left(PSF(x')\right) \cdot \Theta(x - x') \cdot dx' + \int_{-\infty}^{\infty} PSF(x') \cdot \frac{\partial}{\partial x} \Theta(x - x') \cdot dx'$$

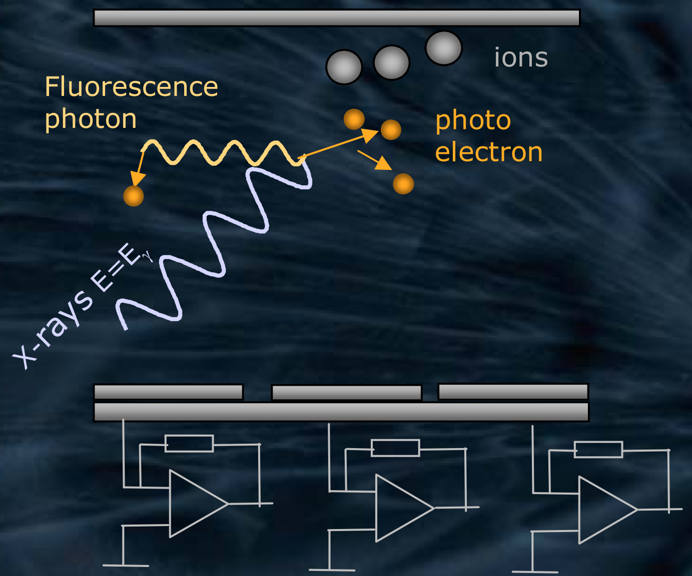
$$= \int_{-\infty}^{\infty} PSF(x') \cdot \delta(x - x') \cdot dx' = PSF(x)$$

$$\Theta(x - x') = \begin{cases} 0 \; for \; x < x' \\ 1 \quad else \end{cases};$$

$$\frac{\partial \Theta(x - x')}{\partial x} = \delta(x - x')$$

Signal smearing is due to
- the process of charge generation
- and the discrete pixel size

Spatial resolution: PSF



Contributions to the spatial resolution

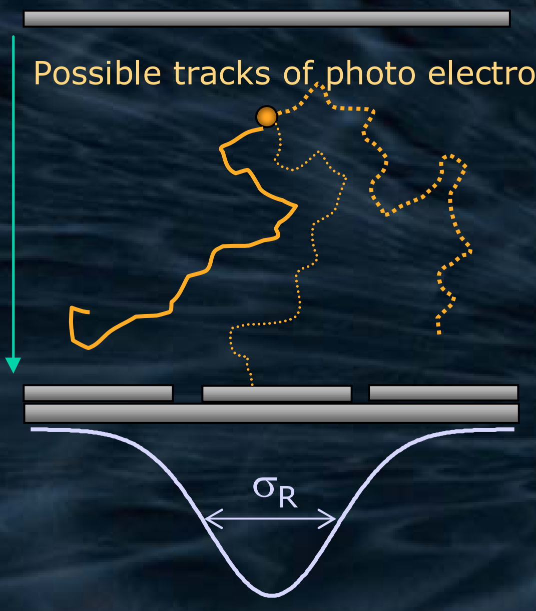
- Range of photo electrons
- diffusion of the electron components
- range of the fluorescence
- pixel size of the segmentation
- electronics cross talk
- induction of ion component
- etc

$$PSF(x) = \int_{-\infty}^{\infty} ..\left[..\left[......\int_{-\infty}^{\infty} \delta\left(x^{'......'}\right)\cdot g_1\left(x^{'......'}\right)dx^{'......'}.\right]..\right]\cdot g_n\left(x-x'\right)\cdot dx'$$

$$= \delta * g_1 * .... * g_n$$

Point Spread Function /Line Spread Function

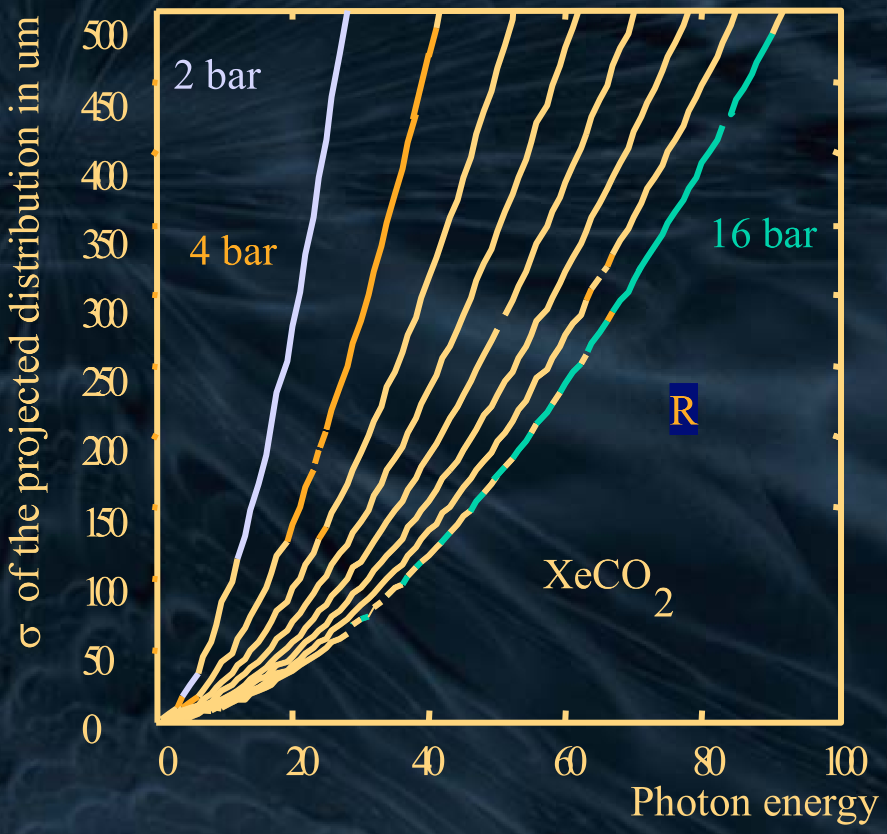
range of photo electrons



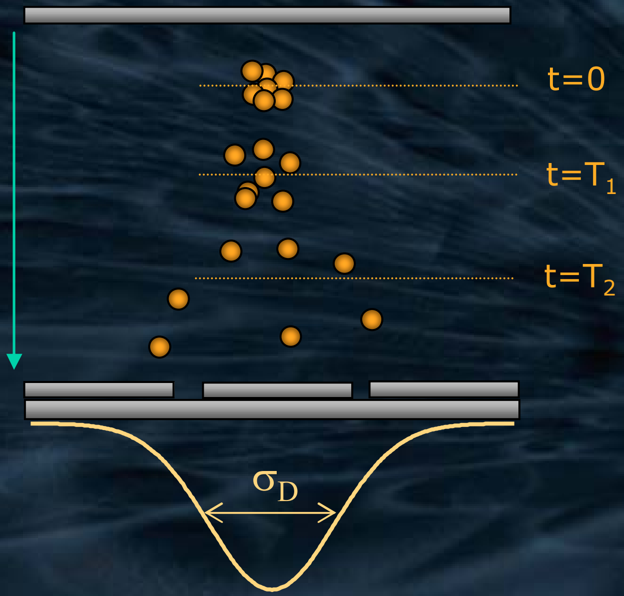
Projected distribution of photo electrons on the segmented electrode

$$r_P(x) = e^{-\frac{x^2 \cdot \rho_{gas}^2}{2 \cdot \sigma_R^2}}$$

$\sigma_R = 1.5\ 10^{-3}\ E^{1.75}$ dim (σ_R) = [mgr/cm^2] dim(E) = [keV]



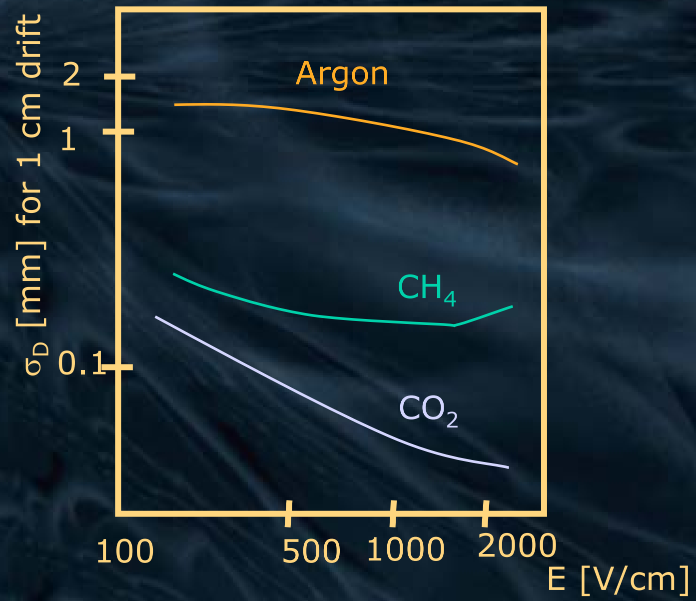
diffusion of electrons



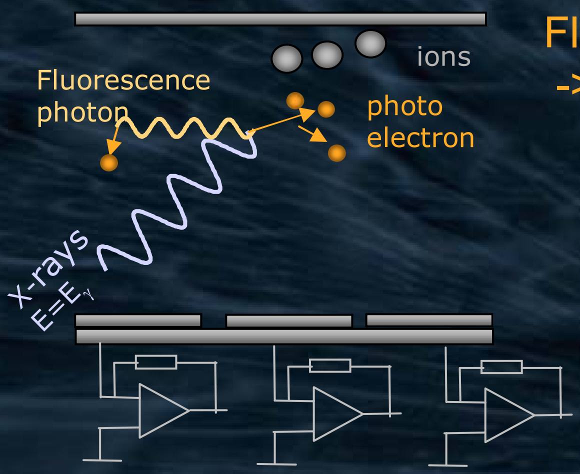
$$d(x) = e^{-\frac{x^2}{4\cdot\sigma_D^2}}$$

$$\sigma_D = \sqrt{2\cdot D_t \cdot t} = \sqrt{\frac{2\cdot D_t \cdot z_{drift}\cdot P}{\mu^- \cdot E}}$$

- D_t Diffusion constant
- E electrical field
- P pressure
- u- mobility
- z_{drift} drift distance
- t drift time



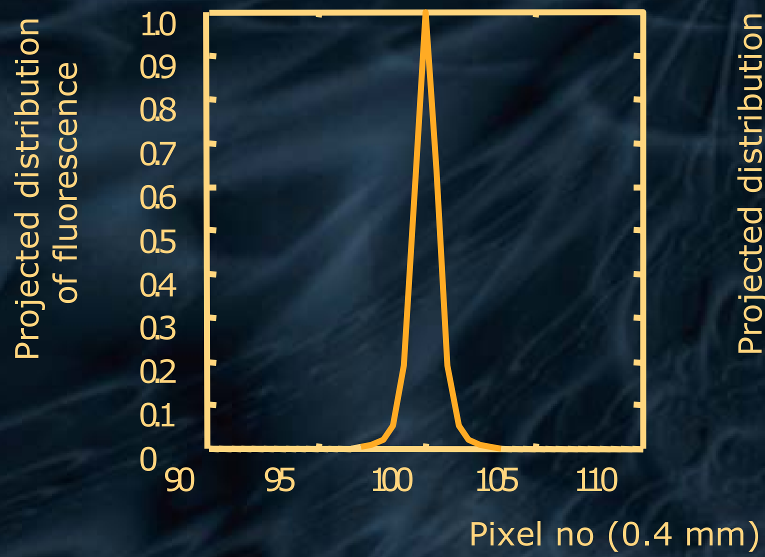
Fluorescence



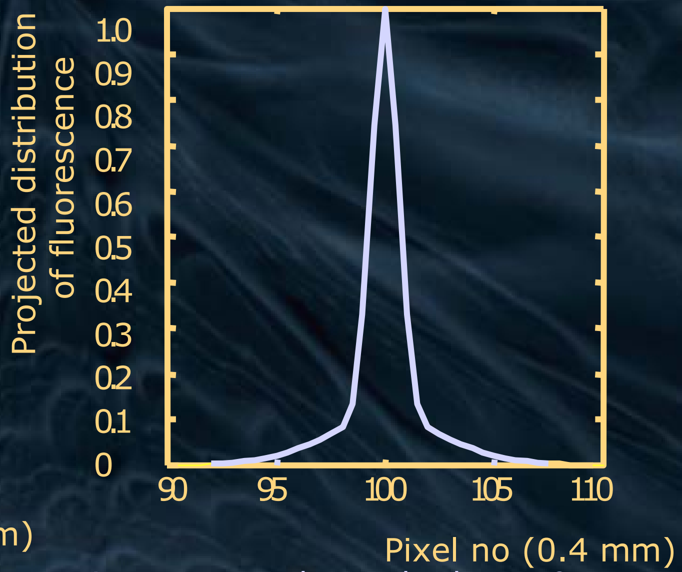
Fluorescence strongly depends on the geometry gas, energy etc.
-> no analytical expression -> Monte Carlo

- Dice position of incident photon
- Dice primary ionization $e^{-\mu\Delta z}$
- Dice effect (Auger or fluorescence
- If fluorescence
  - dice φ and dcos(θ) [solid angle]
  - dice conversion position according to $e^{-\mu\Delta r}$
  - projection on x-axis
  - apply segmentation
- endif & go to beginning

For Kr-CO_2 filled Ionization chamber with a pixel size of 0.4mm and E_γ= 33.174 keV

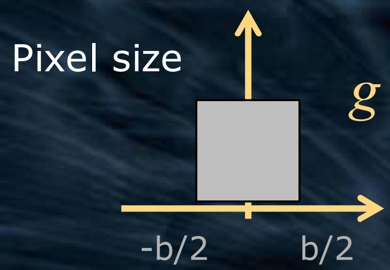


Distance anode -cathode = 3 mm



Distance anode -cathode = 10 mm

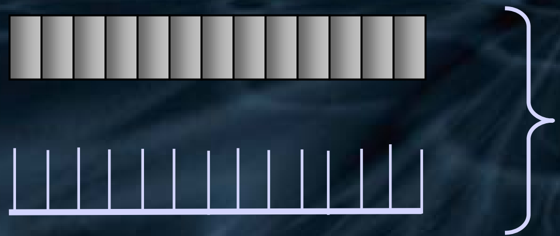
Spatial resolution: pixel size



$$g(x) = \begin{cases} \frac{1}{b} & for -\frac{b}{2} < x < \frac{b}{2} \\ 0 & else \end{cases}$$

$$\sigma_p = \frac{b}{\sqrt{12}}$$

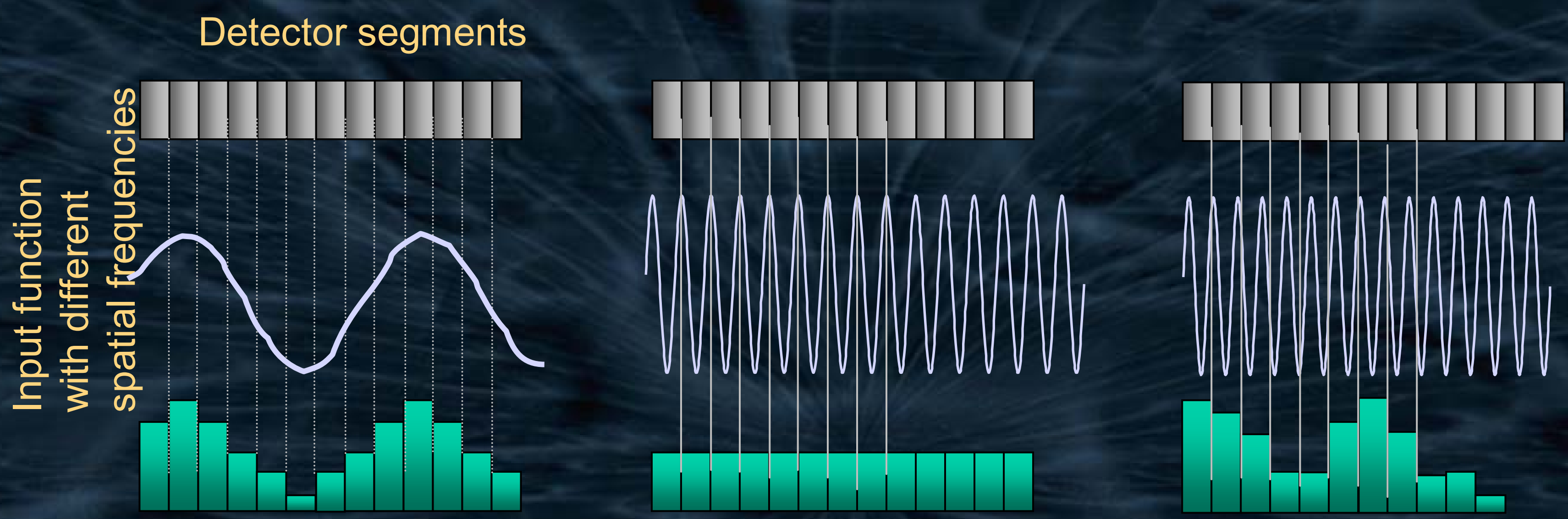
Periodic repetition of pixels in real detector



$$\sum_{i=-\infty}^{\infty} \delta(x - i \cdot b) \quad Dirac\ Comb$$

Segmentation is a convolution of g(x) with Dirac comb

Nyquist sampling theorem



Measured response

Measured response no signal transfer

Measured response over sampling

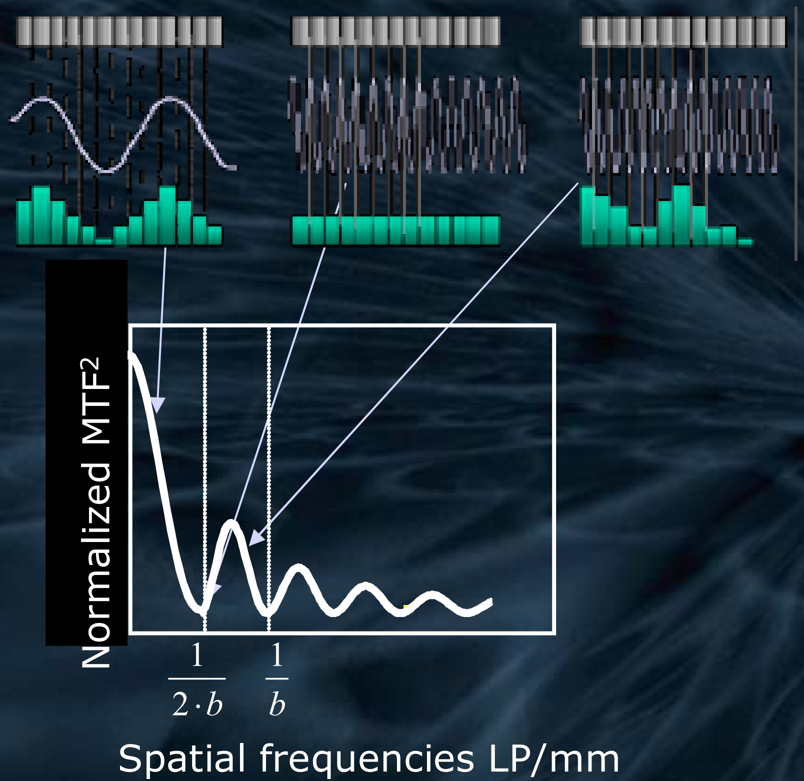
How good are different spatial frequencies transmitted (modulated) by the detector??

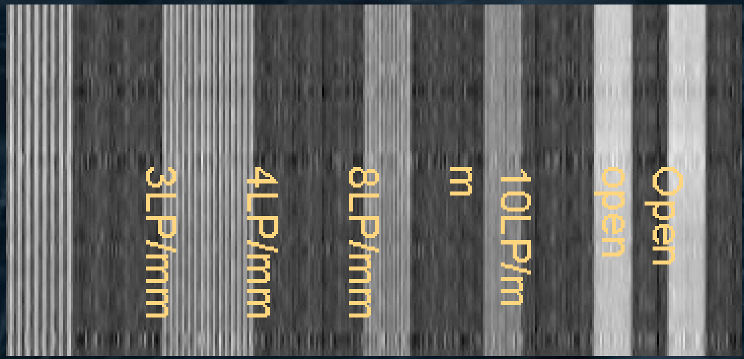
$$PSF(X) = \begin{cases} \frac{1}{b} & for -\frac{b}{2} < x < \frac{b}{2} \\ 0 & else \end{cases}$$

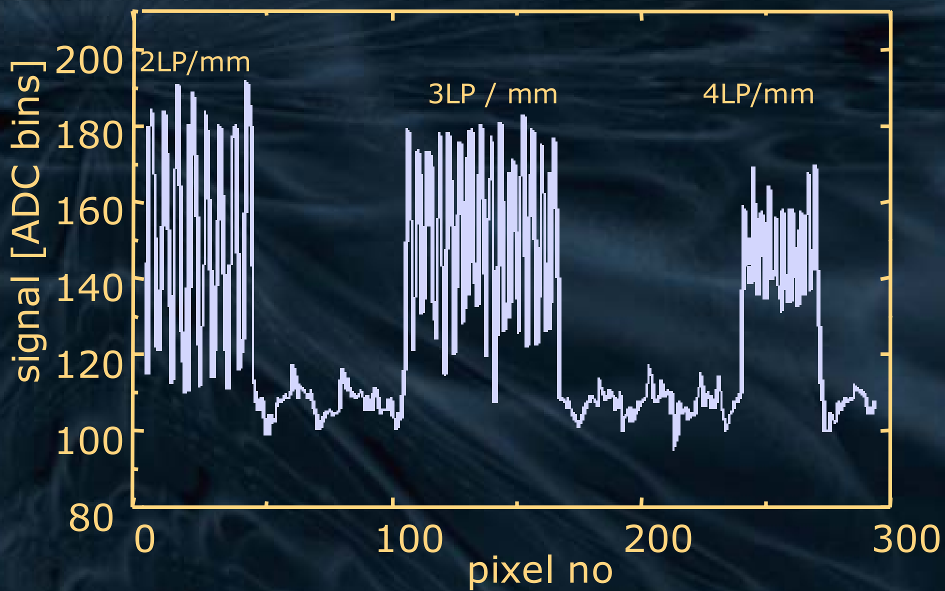
$$MTF(u) = \left| \int_{-\infty}^{\infty} PSF(x) \cdot e^{-2 \cdot i \cdot \pi \cdot u \cdot x} \cdot dx \right| = \left| \frac{\sin(2 \cdot \pi \cdot u \cdot b)}{2 \cdot \pi \cdot u \cdot b} \right|$$

MTF = Modulation Transfer Function

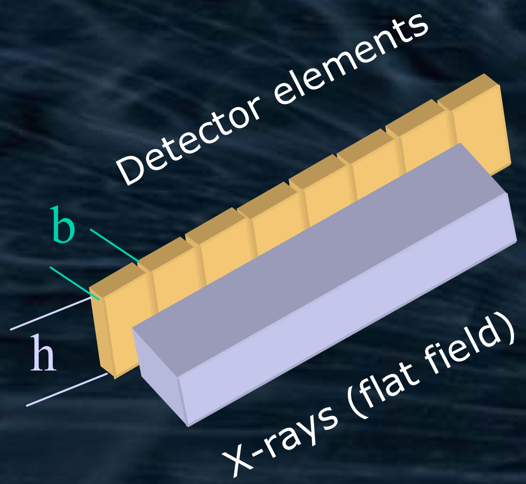
Modulation transfer function







Signal to noise ratio



$$\phi(x,y,t) = \phi_0 = const$$

$$\int_A da = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} dx \cdot dy$$

$$S_{in} = N_{in} = \int_A \int_T \phi(x,y,t) \cdot dt \cdot da = \phi_0 \cdot b \cdot h \cdot T$$

$$\sigma_{in} = \sqrt{\int_A \int_T \phi(x,y,t) \cdot dt \cdot da} = \sqrt{\phi_0 \cdot b \cdot h \cdot T}$$

$$SNR_{in} = \frac{\int_A \int_T \phi(x,y,t) \cdot dt \cdot da}{\sqrt{\int_A \int_T \phi(x,y,t) \cdot dt \cdot da}} = \frac{N}{\sqrt{N}} = \sqrt{N}$$

DQE: Signal to noise ratio for integrating detectors

$$S_{out} = \varepsilon \cdot S_{in}(x) \otimes PSF(x) = \varepsilon \cdot N(x) \otimes PSF(x)$$

$$\sigma_{out} = \sqrt{\underbrace{\varepsilon \cdot N(x) \otimes PSF(x)}_{\text{Poisson noise}} + \underbrace{\sigma^2_{out}}_{\text{Electronics noise of an integrating detector}}}$$

Poisson noise

Electronics noise
of an integrating detector

$$SNR_{out} = \frac{S_{out}}{\sigma_{out}} = \frac{\varepsilon \cdot S_{in}(x) \otimes PSF(x)}{\sqrt{\varepsilon \cdot N(x) \otimes PSF(x) + \sigma^2_{out}}} = \frac{\varepsilon \cdot N(x) \otimes PSF(x)}{\sqrt{\varepsilon \cdot N(x) \otimes PSF(x) + \sigma^2_{out}}}$$

$$SNR_{out} = \sqrt{DQE} \cdot SNR_{in}$$

$$DQE = \left(\frac{SNR_{out}}{SNR_{in}} \right)^2$$

DQE for integrating detectors

$$\Im(DQE) = \Im\left(\left(\frac{SNR_{out}}{SNR_{in}}\right)^2\right) = DQE(f);\ f\ spatial\ frequency$$

$$DQE(f,N) = \Im\left(\left(\frac{\varepsilon \cdot N(x) \otimes PSF(x)}{\sqrt{\varepsilon \cdot N(x) \otimes PSF(x) + \sigma^2_{out}}}\right)^2\right)$$

$$DQE(f,N) = \chi(N) \cdot \frac{\left|MTF(f)\right|^2}{NPS(f)};\ with\ \left|MTF(f=0)\right|^2 = 1\ and$$

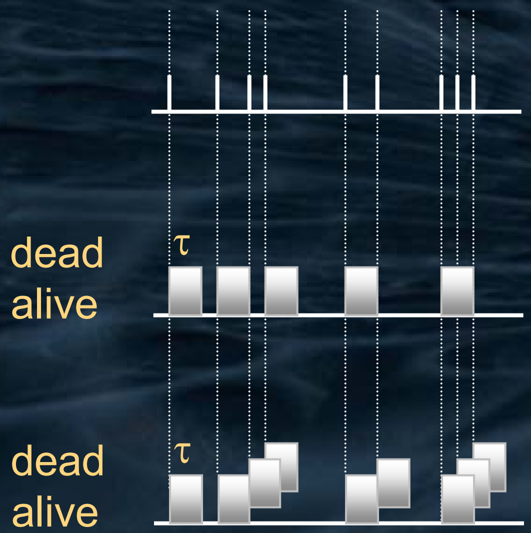
$$NPS(f=0) = 1 + \frac{\sigma^2_{out}}{\varepsilon \cdot N}$$

$$DQE(f=0,N) = \frac{\varepsilon}{1 + \frac{\sigma^2_{out}}{\varepsilon \cdot N}}$$

MTF = modulation transfer function
NPS = noise power spectrum
$\chi(N)$ = zero spatial frequency DQE

DQE: Signal to noise ratio for counting detectors

m = measure rate
n = real rate
τ = dead time
Events in the detector



Non paralyzable

m = n/(1+n τ)

Paralyzable

m = $ne^{-n\tau}$

$$DQE(f=0) = \left(\frac{SNR_{out}}{SNR_{in}}\right)^2$$

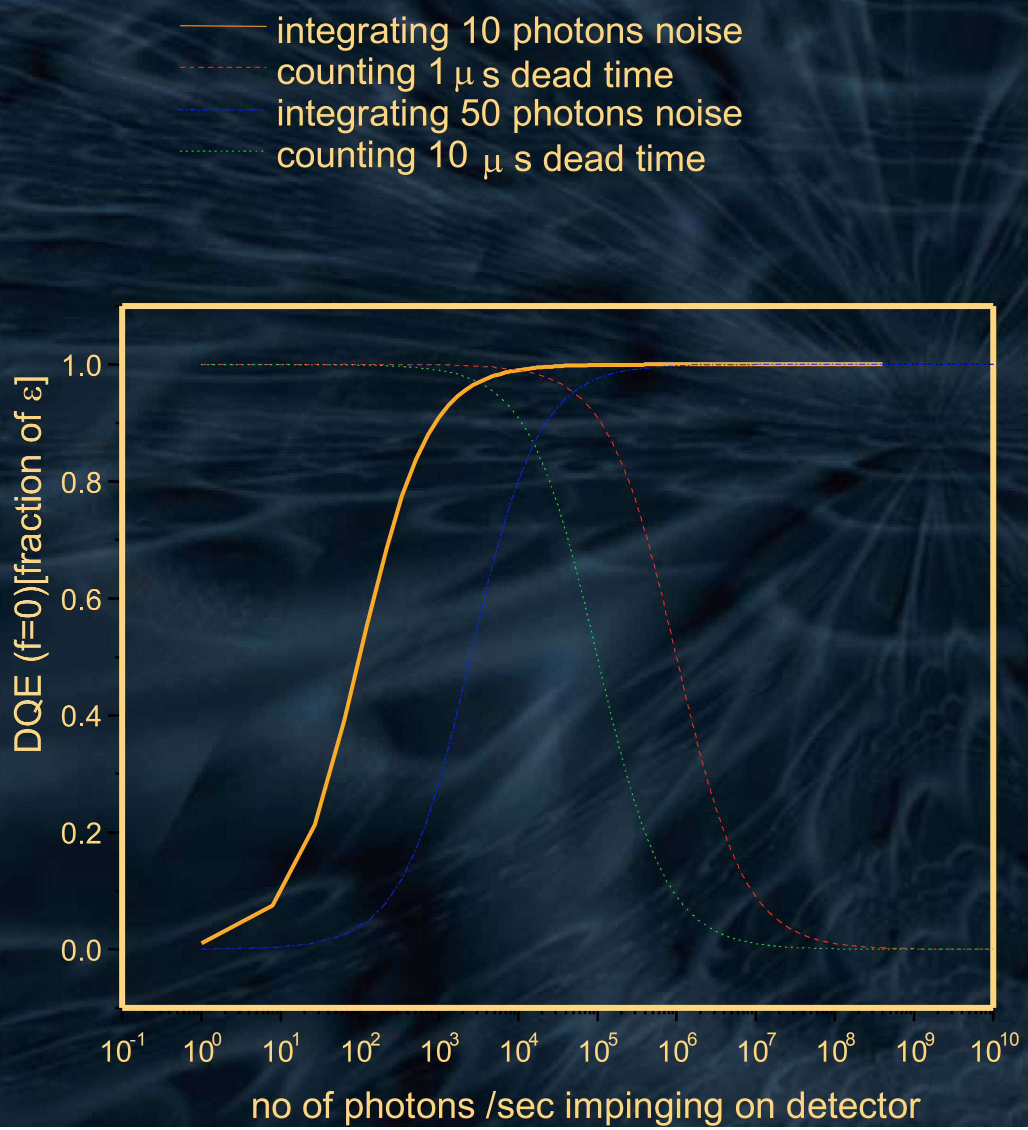
$$S_{out} = \frac{\varepsilon \cdot n}{1 + n \cdot \tau}; \ \sigma_{out} = \sqrt{\frac{\varepsilon \cdot n}{1 + n \cdot \tau}}$$

$$SNR_{out} = \sqrt{\frac{\varepsilon \cdot n}{1 + n \cdot \tau}}$$

$$SNR_{in} = \sqrt{n}$$

$$DQE(f=0) = \frac{\varepsilon}{1 + n\tau}$$

Zero spatial frequency DQE



Counting detectors

$$DQE = \frac{\varepsilon}{1 + N \cdot \tau} \cdot |MTF|^2$$

Integrating detectors

$$DQE = \frac{\varepsilon}{1 + \frac{\sigma_{add}^2}{\varepsilon \cdot N}} \cdot |MTF|^2$$

DQE

Integrating detectors

$$DQE = \frac{\varepsilon}{1 + \frac{\sigma_{add}^2}{\varepsilon \cdot N}} \cdot |MTF|^2$$

Counting detectors

$$\mathrm{DQE} = \frac{\varepsilon}{1 + \mathrm{R} \cdot \tau} \cdot |MTF|^2$$

$$MTF = \Im(PSF)$$

$$PSF = \begin{cases} 1 \text{ for } -b/2 < x < b/2 \\ 0 \text{ else} \end{cases} \Rightarrow MTF \sim \frac{Sin(x)}{x}$$

Integrating: noise 10 photon

Counting: deadtime 10^{-6}s