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Nucleon form factors and N- Δ transitions in a hypercentral constituent quark model

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Nucleon form factors and N- Δ transitions in a hypercentral constituent quark model

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Contents

Recent problems

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- **1), $\mu_p G_E^p(q^2)/G_M^p(q^2)$ is monotonically decreasing**
- **Electron-to proton polarization transfer** $\vec{e}^- + p \to e^- + \vec{p}$
- $$\frac{p_t}{p_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}}\frac{G_E}{G_M}$$
- **Traditional Rosenbluth separation:**

$$\langle N(p')|J^\mu(0)|N(p)\rangle = e\bar{u}(p')\left[G_M(Q^2)\gamma^\mu - F_2(Q^2)\frac{(p+p')^\mu}{2M}\right]u(p),$$

- **$G_M=F_1+F_2$; $G_E=F_1-\tau F_2$ (Space-like $Q^2>0$)**

$$d\sigma_B = C_B(Q^2,\varepsilon)\left[G_M^2(Q^2) + \frac{\varepsilon}{\tau}G_E^2(Q^2)\right],$$

- **Sensitive to uncertain radiative corrections(RS) (two-photon)**

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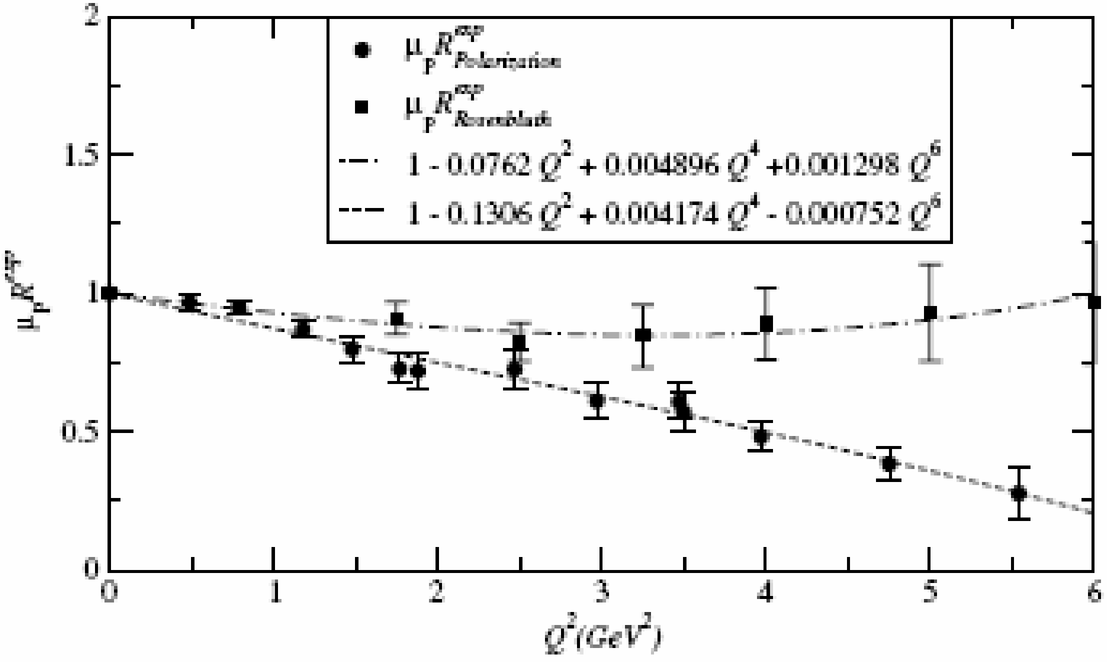

FIG. 1. Experimental values of $R^{exp}_{Rosenbluth}$ [4] and $R^{exp}_{polarization}$ [5,6] and their polynomial fits.

❖$G_E^p(q^2)$ falls faster than $G_M^p(q^2)$

- **2), Quark-hadron Duality**
- **Strong interaction: Two end points**
- **Two languages**

1), nQCD, Confinements : Resonance

2), pQCD, Asympototic freedom

Connection of pQCD and nQCD

Duality for the structure functions

Observable can be explained by two different kinds of Languages (R, S)

- Bloom-Gilman Duality(F2 ,1970):

Resonance region data oscillate around the scaling curve.

smooth scaling curve seen at high $\mathbf{Q^2}$ was an accurate average over the resonance bumps at a low:$\mathbf{Q^2(4GeV^2)}$

By I. Niculescu et al. Phys. Rev. Lett. 85, 1182, 1186 (2000),

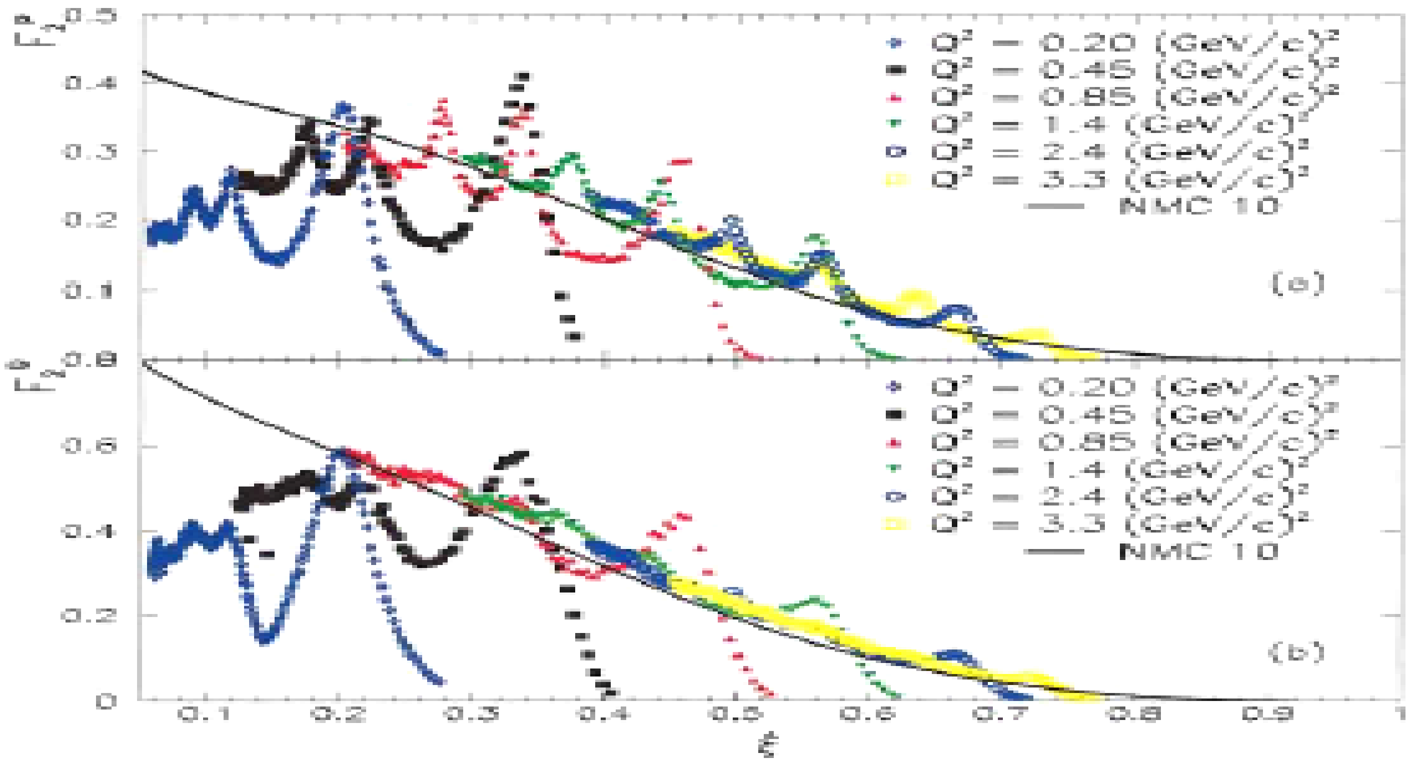

FIG. 1 (color). Extracted F_2 data in the nucleon resonance region for hydrogen (a) and deuterium (b) targets, as functions of the Nachtmann scaling variable ξ. For clarity, only a selection of the data is shown here. The solid curves indicate the result of the NMC fit to deep inelastic data for a fixed $Q^2 = 10\ (GeV/c)^2$ [16].

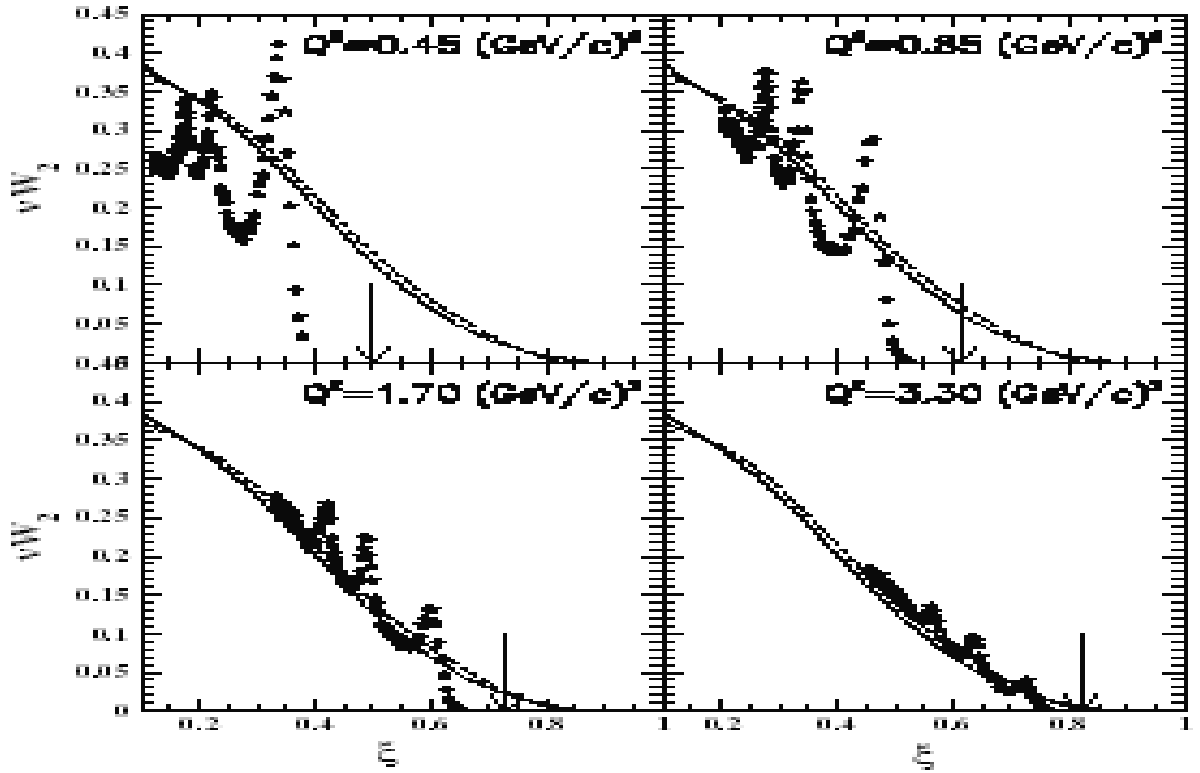

FIG. 1. Sample hydrogen νW_2 structure function spectra obtained at $Q^2 = 0.45, 0.85, 1.70$, and 3.30 $(\mathrm{GeV}/c)^2$ and plotted as a function of the Nachtmann scaling variable ξ. Arrows indicate elastic kinematics. The solid [dashed] line represents the NMC fit [23] of deep inelastic structure function data at $Q^2 = 10\ (\mathrm{GeV}/c)^2$ $[Q^2 = 5\ (\mathrm{GeV}/c)^2]$.

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Hyper central potential model

- **Conventional two-body interaction (Cornell Potential)**
- **(Isgur-Karl, Chiral model),**
-
- **Three-body force can play an important role in**
- **hadrons (Y-type interaction)**
- **(non-abelian nature of QCD leads to g-g coupling,**
- **which can produce three-body forces)**
- **Hyper-central potential model, which amounts**
- **to average any two-body potential for the baryon**
- **over the hyperangle ζ**

Previous works on Hyper-central model

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- **J. –M. Richard, Phys. Rept. C212 (1992)**
- **M. Fabre de la Ripelle and J. Navarro, Ann. Phys. 123 (1979), 185.**
- **Application to the nucleon resonance properties**
- **By Genova Group (M. M. Giannini, E. Santopinot, M. Aillo, M. Ferraris, A. Vassallo et al.)**
- **EPJ A1, 187; EPJ A1, 307; EPJ A2, 403; EPJ A12, 447**
- **PRC62, 025208; PLB387, 215**

- **Spectroscopy of non-strange baryons**
- **Electromagnetic form factors of nucleon**
- **Electromagnetic transition amplitudes**

Frame-work of Hypercentral potential model

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- **The potential is assumed to be the function of hyper-radius x**
- Jacobi coordinates

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2) \qquad \vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3)$$

- Hyper-spherical coordinates:x and

$$\Omega_\rho=(\theta_\rho,\phi_\rho) \qquad \Omega_\lambda=(\theta_\lambda,\phi_\lambda)$$

$$x=\sqrt{\vec{\rho}^{\,2}+\vec{\lambda}^{2}} \qquad \xi=arctg\left(\frac{\rho}{\lambda}\right)$$

- For a baryon, the Hamiltonian is

$$H=\frac{\vec{P}^{\,2}{}_\rho}{2m}+\frac{\vec{P}^{\,2}{}_\lambda}{2m}+V(x)$$

Frame work of HCPM

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- The kinetic energy is

$$-\frac{1}{2m}(\Delta_\rho + \Delta_\lambda) = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \xi)}{x^2}\right),$$

- The quadratic Casimir operator of the six dim. Rotation group O(6)

$$L^2(\Omega_\rho, \Omega_\lambda, \xi) Y_{[\gamma] l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) = \gamma(\gamma+4) Y_{[\gamma] l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi),$$

- With the grant-angular quantum number $\gamma = 2\upsilon + l_\rho + l_\lambda \qquad \upsilon = 0,1,\ldots$
- The hyper-radial wave function

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_\gamma(x) = -2m[E - V(x)]\psi_\gamma(x).$$

Potentials and wave functions

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- **Tow typical examples which can be solved analytically**

$$V(x)_{h.o.} = \frac{3}{2}kx^2 \qquad V_{hyc}(x) = -\frac{\tau}{x}$$

- **(six-dimensional harmonic oscillator, Coulomb potentials)**

$$E = -\frac{\tau^2 m}{2[\omega(\omega+5)+25/4]}, \ \omega = 0, 1, \ldots\infty.$$

- **The principal quantum number n=ω+5/2 (ω =γ+n')**

$$\psi_{\omega\gamma}(x) = \left[\frac{(\omega-\gamma)!(2g)^6}{(2\omega+5)(\omega+\gamma+4)!^3}\right]^{\frac{1}{2}} \times (2gx)^{\gamma} e^{-gx} L_{\omega-\gamma}^{2\gamma+4}(2gx), \qquad g = \frac{\tau m}{\omega+\frac{5}{2}}$$

- **An interesting property is the degeneracy of the first exciting L=0 and the L=1**

Hyperfine interactions

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- **Confinement:** $V(x) = -\frac{\tau}{x} + \alpha x,$

- **Other interactions:**

$$V^S(x) = Ae^{-\alpha x} \sum_{i<j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = Ae^{-\alpha x}[2S^2 - \frac{9}{4}],$$

$$V^T(x) = B\frac{1}{x^3} \sum_{i<j} \left[\frac{(\boldsymbol{\sigma}_i \cdot (\boldsymbol{r}_i - \boldsymbol{r}_j))\,(\boldsymbol{\sigma}_j \cdot (\boldsymbol{r}_i - \boldsymbol{r}_j))}{|\boldsymbol{r}_i - \boldsymbol{r}_j|^2} - \frac{1}{3}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right]$$

-

Spectrum of the model

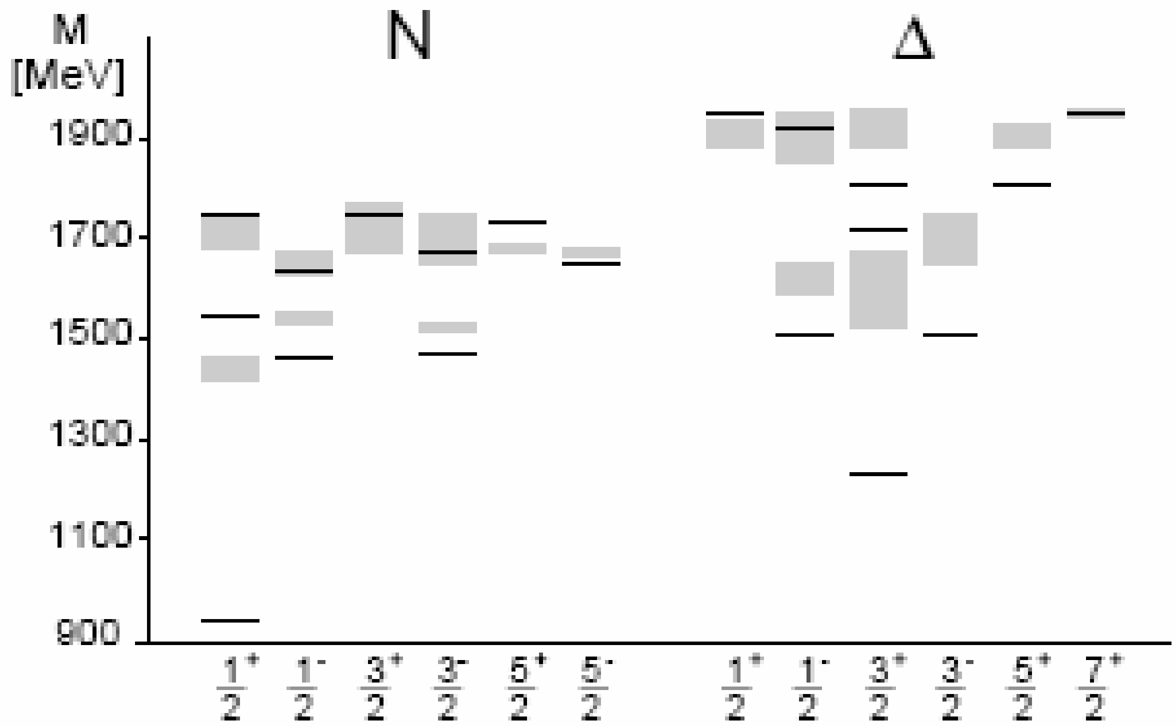

Fig. 1. The spectrum obtained with the hypercentral potential, eq. (8), and the spin-dependent term, eq. (9). The fitted parameters are $\alpha = 1.58\,\mathrm{fm}^{-2}$, $\tau = 4.98$, $A_S = 38.4\,\mathrm{fm}^2$, $\sigma_S = 0.8\,\mathrm{fm}$.

Transition amplitudes S

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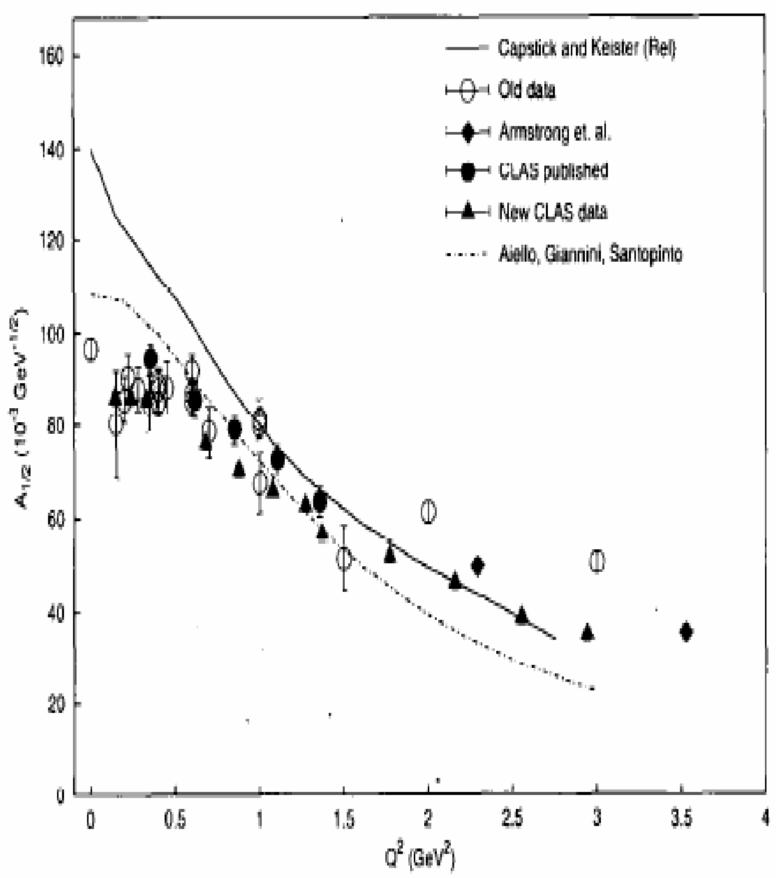
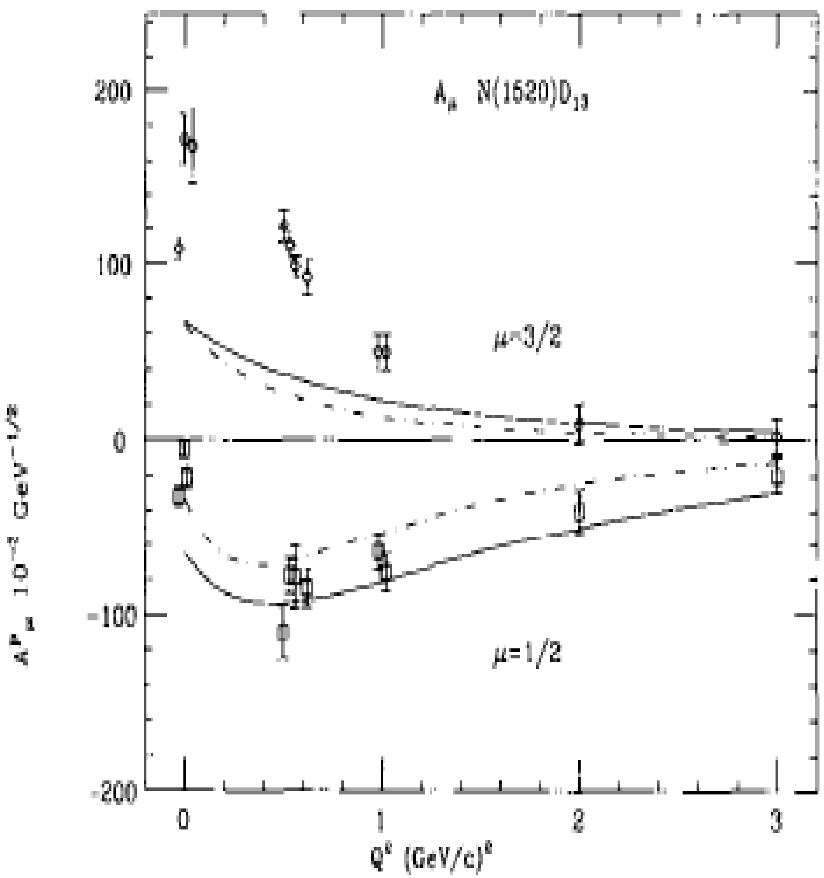
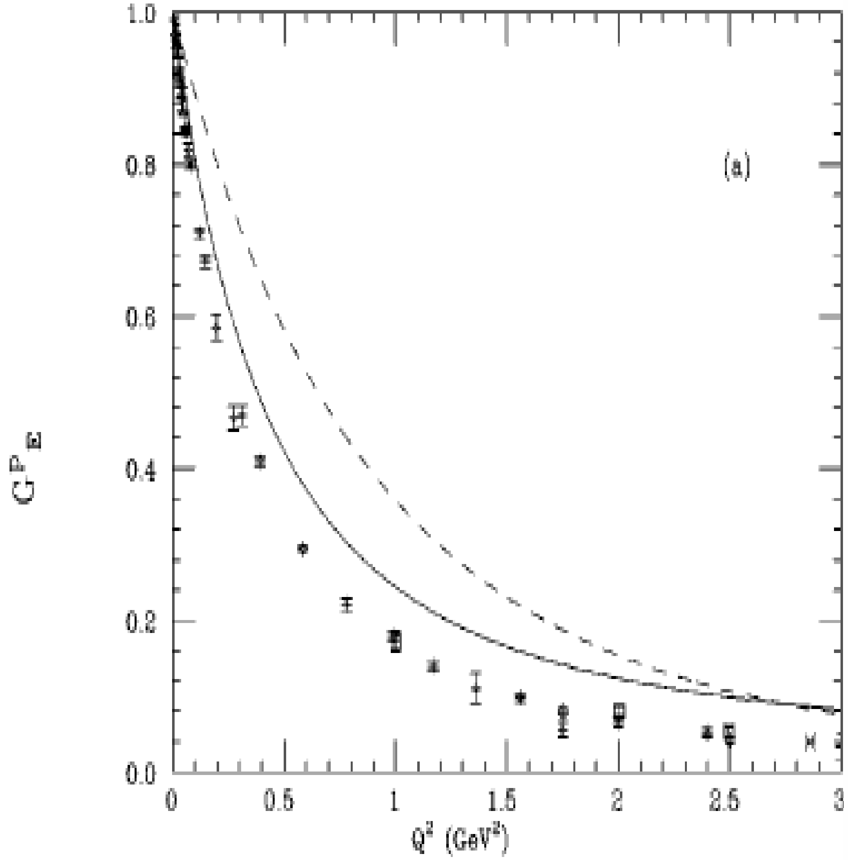
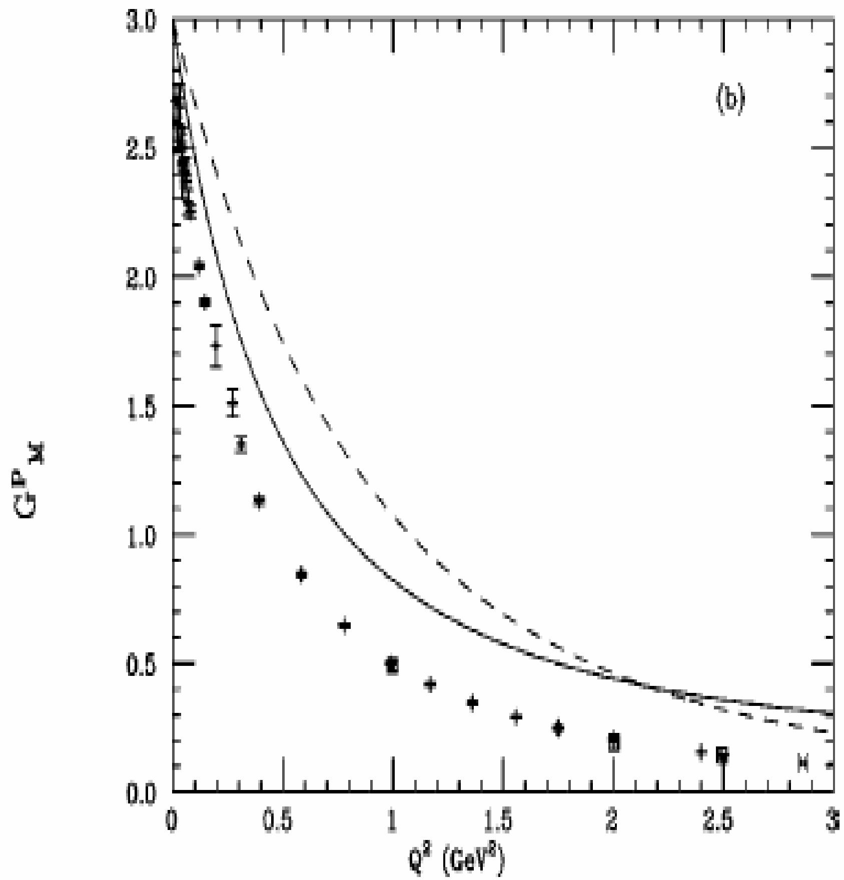




Form factors





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A short summary

- 1), The simple hyper-central potential model
- simple 3-body quark model

- 2), The spectrum of the non-strange nucleon
- resonances

- 3), Transition amplitudes S_{11}(1535), D_{13}(1520)

Meson cloud effect

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- **To include the meson cloud, the total Lagrangian densitiy with π qq coupling,**

$$\mathcal{L} = i\overline{\psi}_q(x)\gamma^\mu\partial_\mu\psi_q(x) - m_Q\overline{\psi}_q(x)\psi_q(x) + \frac{1}{2}(\partial_\mu\pi(x))^2 - \frac{1}{2}m_\pi^2\pi^2(x)$$
$$-ig\overline{\psi}_q(x)\gamma_5\psi_q(x)\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x).$$

- **The total electromagnetic current is**

$$J^\mu(x) = j_q^\mu(x) + j_\pi^\mu(x),$$

- **where**

$$j_q^\mu(x) = \sum_a Q_a e\overline{\psi}_a(x)\gamma^\mu\psi_a(x),$$
$$j_\pi^\mu(x) = -ie[\pi^\dagger(x)\partial^\mu\pi(x) - \pi\partial^\mu\pi(x)^\dagger(x)].$$

Electromagnetic interaction

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$$
\begin{aligned}
\mathcal{H}_{em}^{+} &= \sum_{a=1}^{3} -e_a J_a^{-} A_a^{+}, \\
\mathcal{H}_{em}^{0} &= \sum_{a=1}^{3} e_a \left[J_a^0 A_a^0 - J_a^3 A_a^3 \right].
\end{aligned}
$$

- **Pion meson coupling, a baryon state is written as**

$$|A\rangle = \sqrt{Z_2^A}\left[1 + (E_A - H_0 - \Lambda \mathcal{H}_{int} \Lambda)^{-1} \mathcal{H}_{int}\right] |A_0\rangle ,$$

- **The interaction for the process of emission and absorption of pions is**

$$\mathcal{H}_{int} = \sum_j \int d^3k \{ V_{0j}(\boldsymbol{k}) a_j(\boldsymbol{k}) + V_{0j}^{\dagger}(\boldsymbol{k}) a_j^{\dagger}(\boldsymbol{k}) \}$$

$$V_{0j}(\boldsymbol{k}) = \sum_{A_0, B_0} A_0^{\dagger}\, v_{0j}^{AB}(\boldsymbol{k})\, B_0,$$

$$v_{0j}^{AB}(\boldsymbol{k}) = \frac{i f_0^{AB}}{m_\pi} \frac{u(k)}{[2\omega_k (2\pi)^3]^{1/2}}\, C_{s_B\ m\ S_A\ s_A}^{S_B\ 1} (\hat{S}_m^{*} \cdot \boldsymbol{k})\, C_{t_B\ n\ T_A\ t_A}^{T_B\ 1} (\hat{t}_n^{*} \cdot \hat{e}_j), \qquad f_0 = \frac{3}{2}\, g\, \frac{m_\pi}{m_N}.$$

Parameters and calculations

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- **a),** $g = 0.585,$ **then** $g^2_{\pi NN}/(4\pi) = 13.6,$
- **b), wave functions (HC, and HO potentials)**

$$u(k)_{Hyc} = \frac{1}{(1+k^2a^2)^{7/2}},$$
$$u(k)_{HO} = exp\left[-\frac{1}{6}\frac{k^2}{m\omega}\right]$$

$a = 5\sqrt{2/3}/(4\tau m), m\omega = \alpha^2$, where τ and α are parameters of the two potentials.

- $\tau = \tau_1 = 6.39$, $\tau = \tau_2 = 4.59$; $\alpha = \alpha_1 = 0.410$, $\alpha = \alpha_2 = 0.229\ GeV$
- **c), Nucleon form factors**

$$G_N^M(Q^2) = \frac{1}{\sqrt{2Q^2}}\left\langle N, S_z = \frac{1}{2}\middle|\mathcal{H}^+_{em}\middle| N, S_z = -\frac{1}{2}\right\rangle,$$
$$G_N^E(Q^2) = \left\langle N, S_z = \frac{1}{2}\middle|\mathcal{H}^0_{em}\middle| N, S_z = \frac{1}{2}\right\rangle.$$

Transition amplitudes(1)

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- **For nucleon resonance, the electro-production amplitudes are**

$$A_{3/2} = \frac{1}{\sqrt{2\omega_\gamma}} \left\langle \Delta; s_\Delta = 3/2 \left| \mathcal{H}_{em}^{+} \right| N; s_N = 1/2 \right\rangle ,$$

$$A_{1/2} = \frac{1}{\sqrt{2\omega_\gamma}} \left\langle \Delta; s_\Delta = 1/2 \left| \mathcal{H}_{em}^{+} \right| N; s_N = -1/2 \right\rangle$$

- **To calculate in the Breit frame,**

$$|\boldsymbol{q}|^2 = Q^2 + \frac{M^2 - m^2}{2(M^2 + m^2) + Q^2},$$

$$q_0 = \sqrt{M^2 + |\boldsymbol{q}|^2/4} - \sqrt{m^2 + |\boldsymbol{q}|^2/4},$$

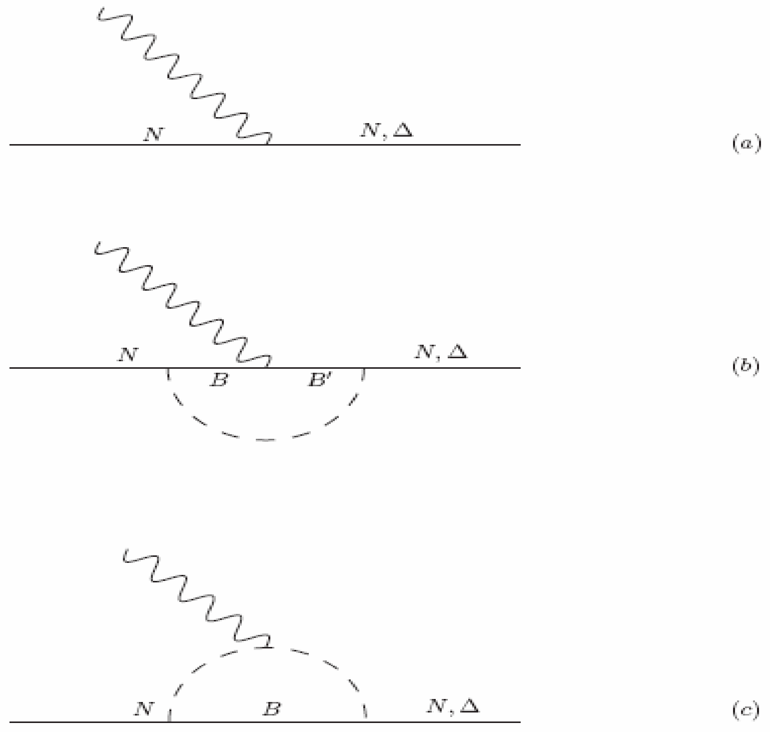
- **The form factors are defined as**

-

$$G_\Delta^M(Q^2) = \sqrt{\frac{1}{4\pi\alpha} \frac{m}{M} \omega_\gamma} \left[1 + \frac{Q^2}{(M+m)^2} \right]^{1/2} \frac{m}{|\boldsymbol{q}|} \left[A_{1/2}^N + \sqrt{3} A_{3/2}^N \right],$$

$$G_\Delta^E(Q^2) = \sqrt{\frac{1}{4\pi\alpha} \frac{m}{M} \omega_\gamma} \left[1 + \frac{Q^2}{(M+m)^2} \right]^{1/2} \frac{m}{|\boldsymbol{q}|} \left[A_{1/2}^N - \frac{1}{\sqrt{3}} A_{3/2}^N \right].$$

Transition amplitudes (2)



Diagrams illustrating the various contributions included in the calculation. The intermediate baryons B and B' are restricted to the N and Δ here.

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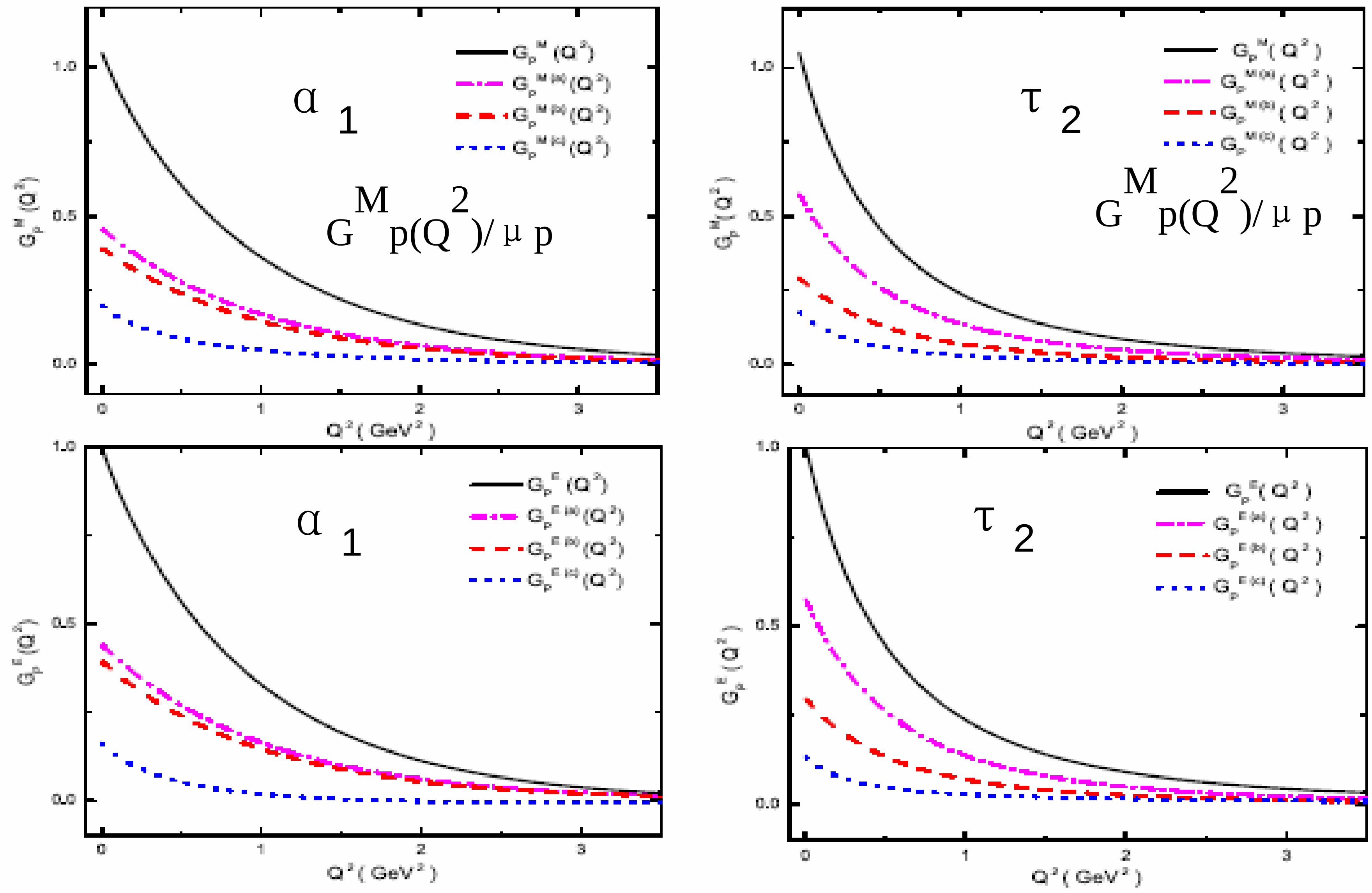
Individual contribution



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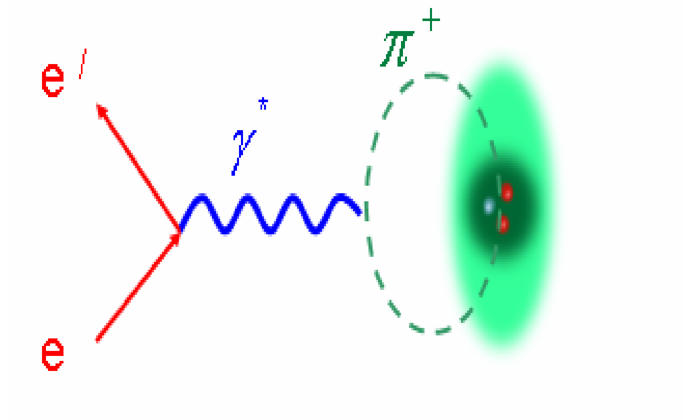
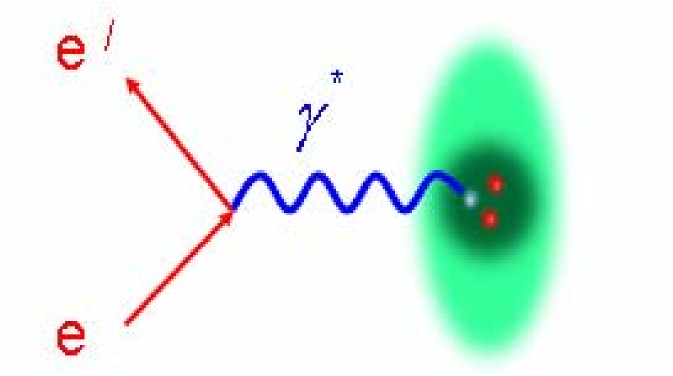


Low energy region



High energy region

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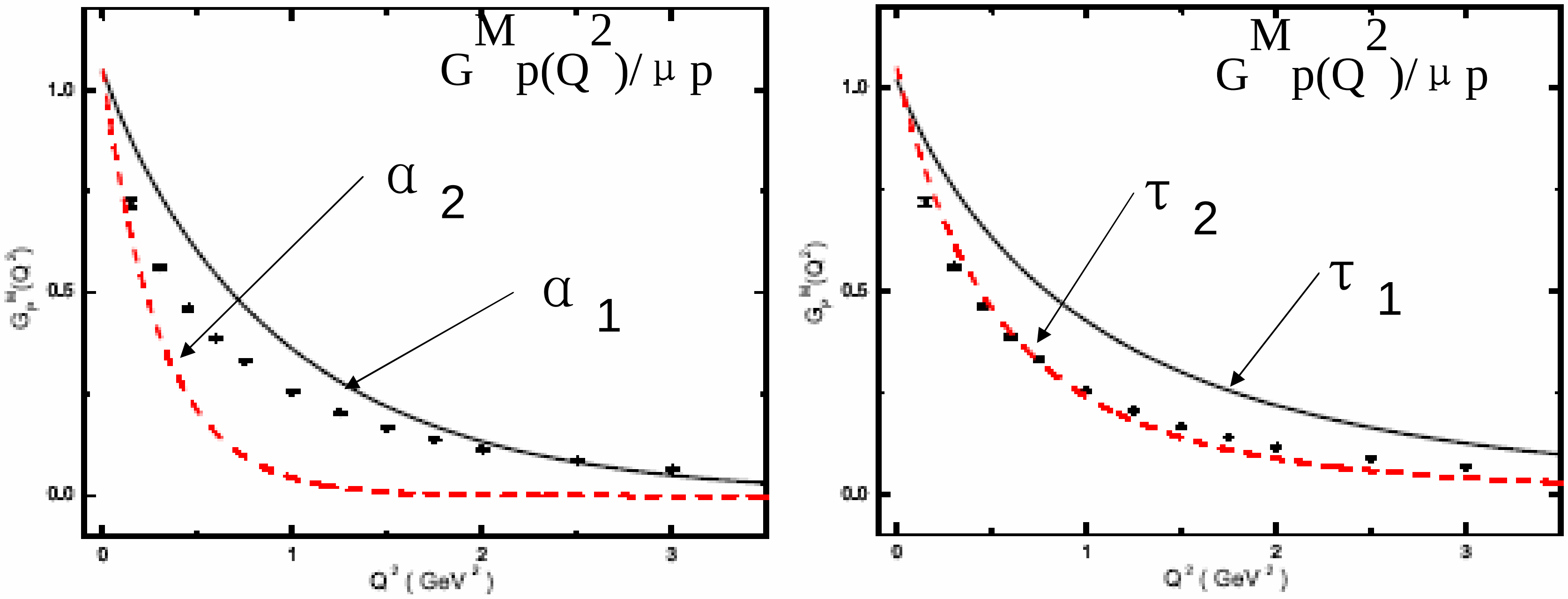
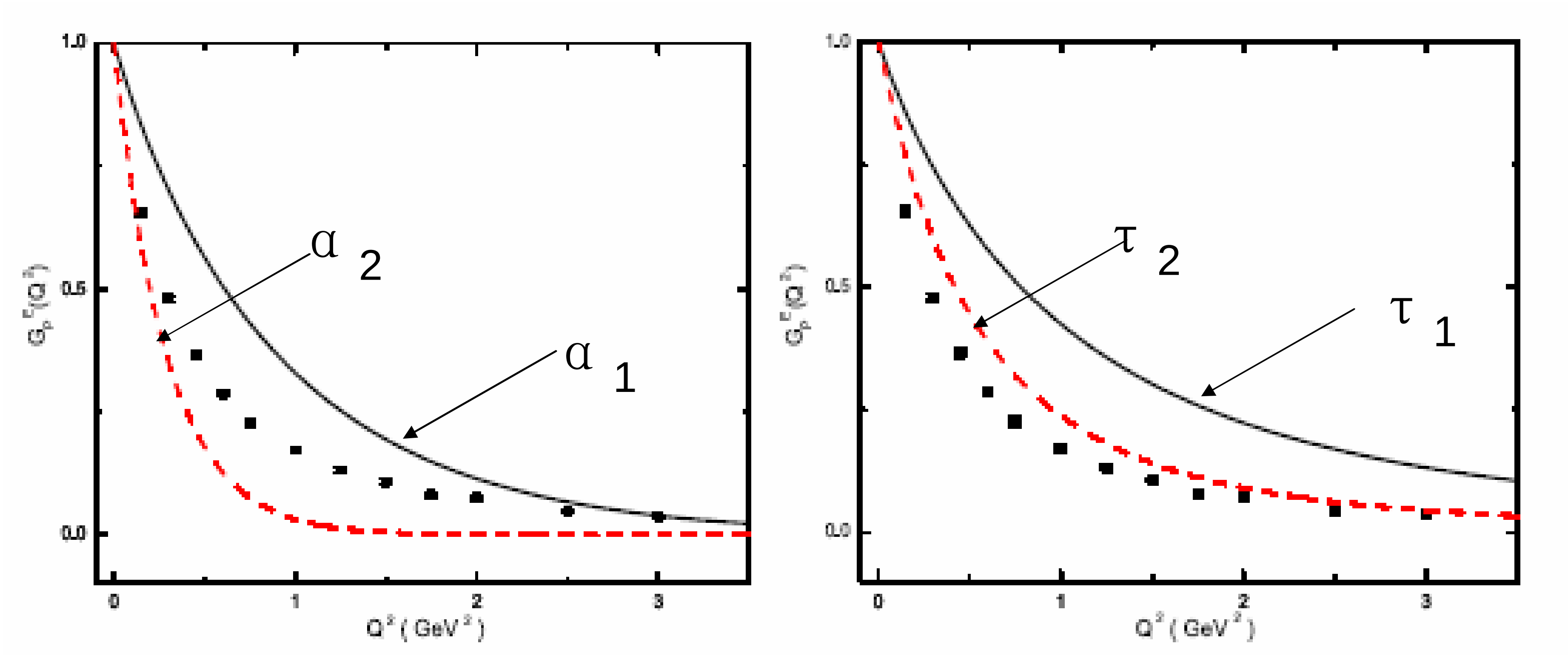
Magnetic form factor of proton

- **Two sets of parameters; H O:** α_1=0.410GeV, α_2=0.229 GeV
- **HYC:** τ_1=6.39 , τ_2=4.59



Charge form factors of proton:



❖Electromagnetic form factors of neutron:

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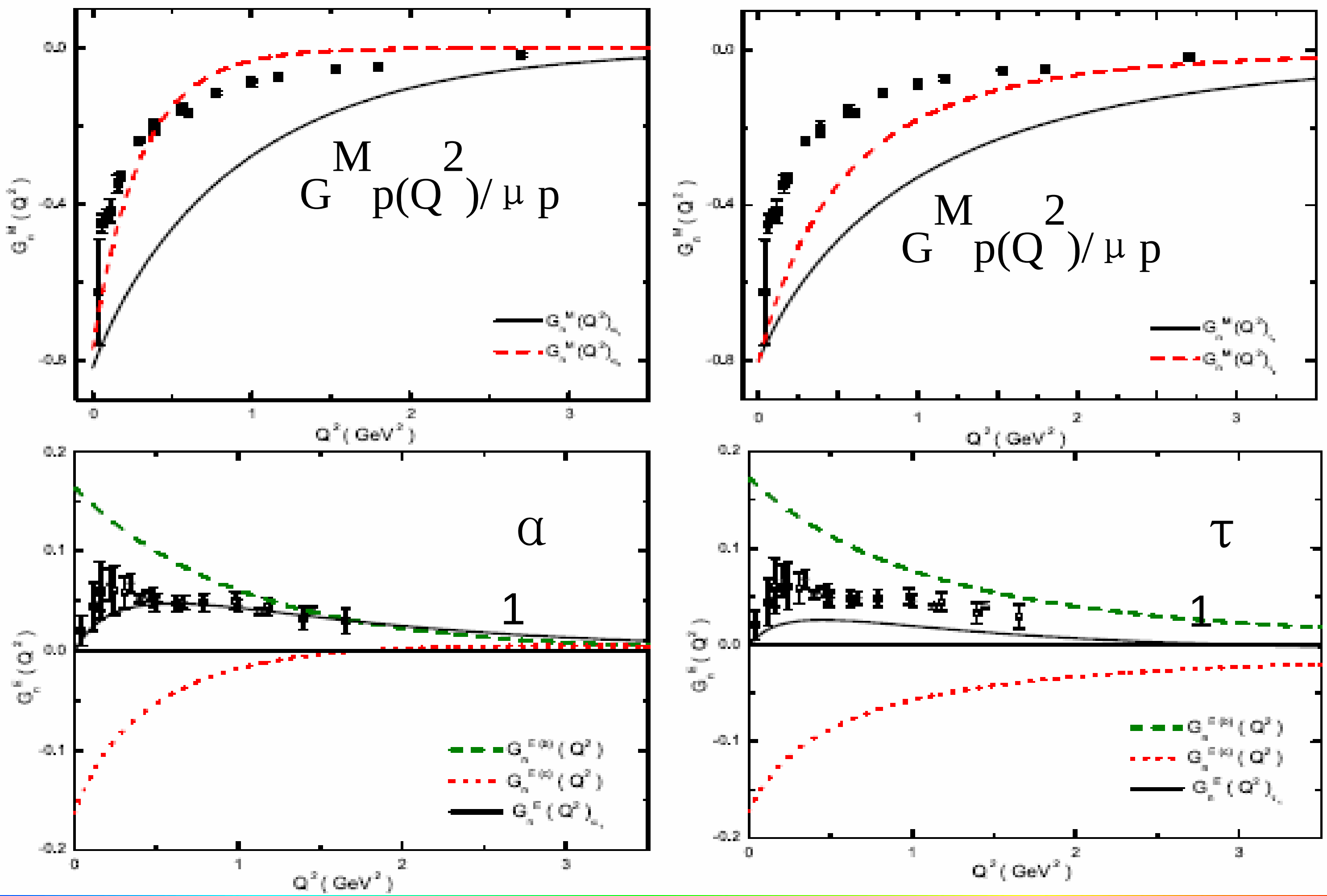
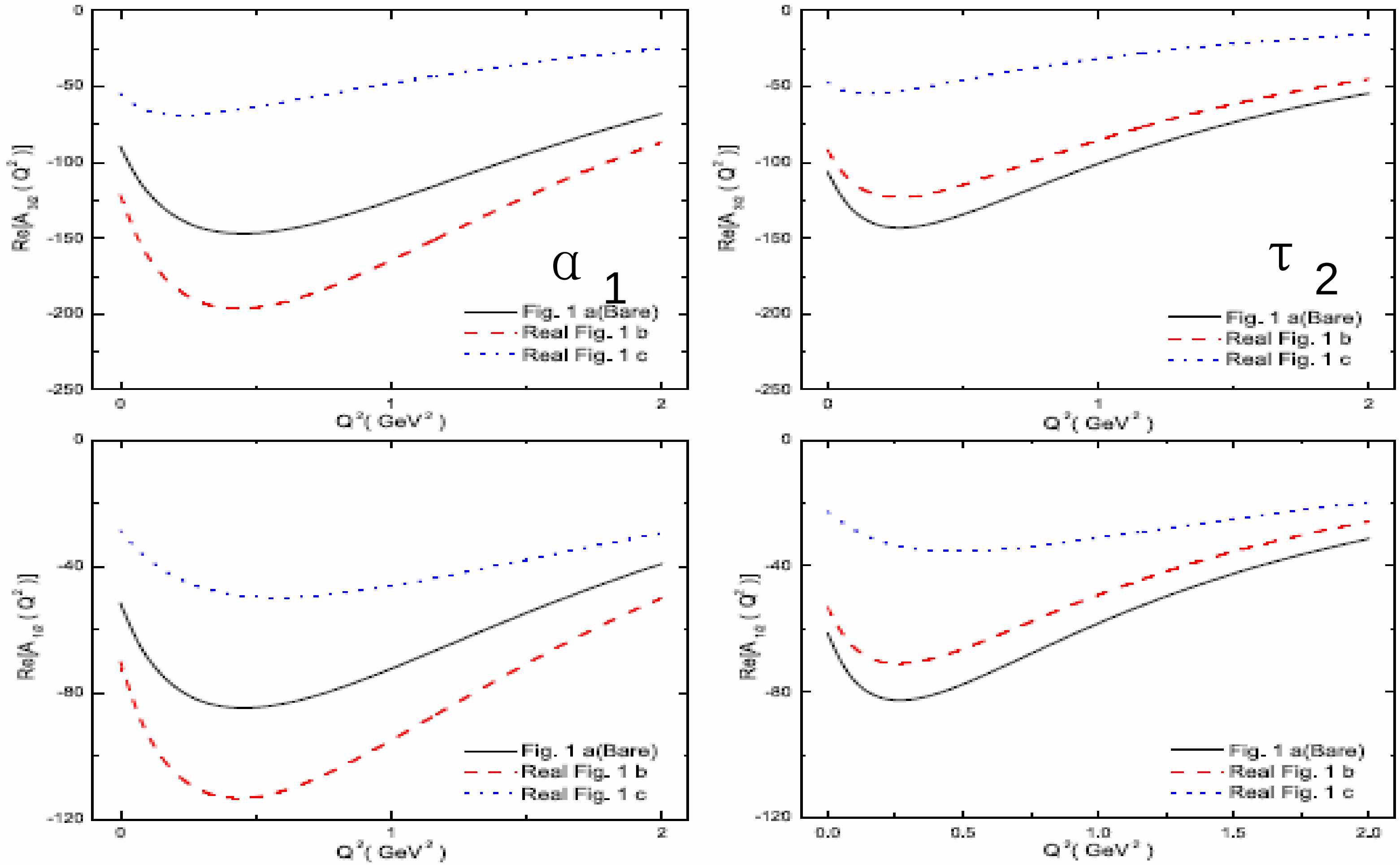
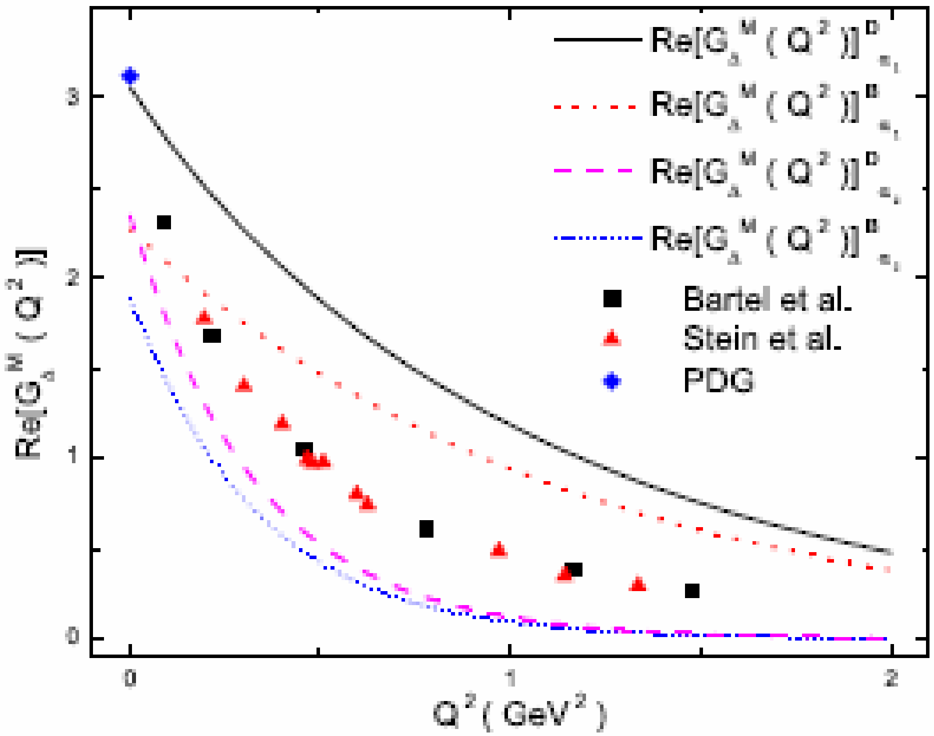
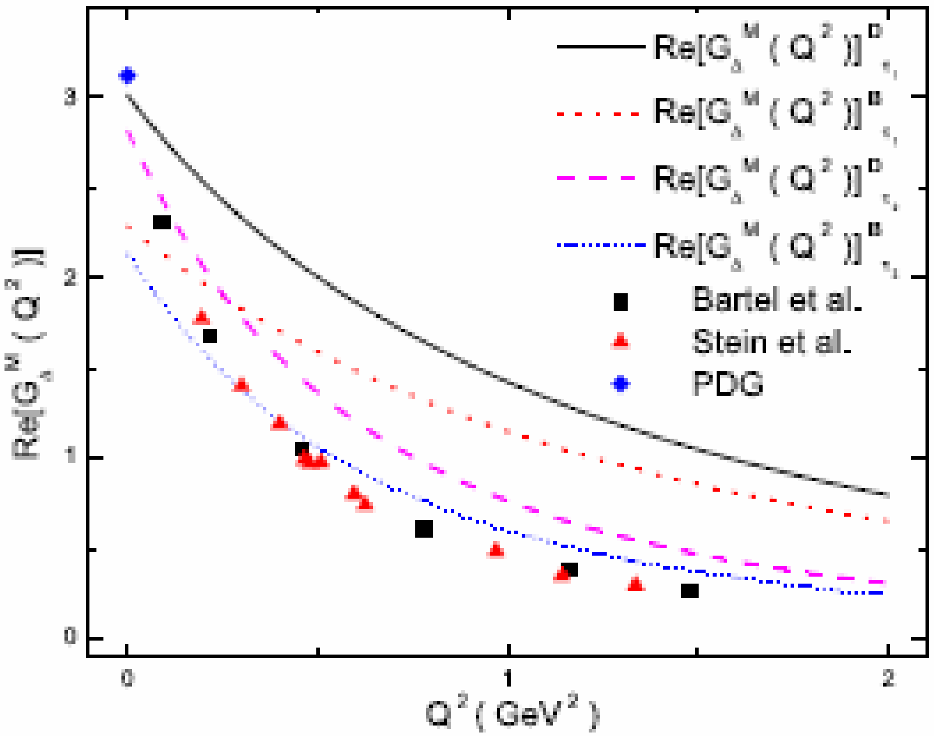


Individual contributions to the transition amplitude of Δ(1232)



$\Delta \rightarrow \gamma N$ Amplitudes





Other results:

	HO				Hyc				Expt. Value
	$\alpha_1(B)$	$\alpha_1(D)$	$\alpha_2(B)$	$\alpha_2(D)$	$\tau_1(B)$	$\tau_1(D)$	$\tau_2(B)$	$\tau_2(D)$	
μ_P/μ_0	1	1.046	1	1.048	1	1.02	1	1.04	0.93
μ_n/μ_0	-2/3	-0.81	-2/3	-0.76	-2/3	-0.80	-2/3	-0.80	-0.64
$\langle r^2\rangle_P$	0.23	0.30	0.74	0.85	0.21	0.26	0.41	0.46	0.76
$\langle r^2\rangle_n$	0	-0.07	0	-0.06	0	-0.05	0	-0.04	-0.116

-250 ± 8

	HO					Hyc			
	a	b	c	$A_{3/2}$		a	b	c	$A_{3/2}$
α_1	-89	-121-10i	-54 -18i	-264-28i	τ_1	-76	-130-8i	-54 -15i	-263-23i
α_2	-126	-47-20i	-31 -20i	-204-40i	τ_2	-106	-91-11i	-46-21i	-243-33i

−0.015±0.004

	HO					Hyc			
	c	$A_{1/2}$	$Re[E_2/M_1]$	μ_{Δ^+}		c	$A_{1/2}$	$Re[E_2/M_1]$	μ_{Δ^+}
α_1	-29-26i	-150-26i	-0.007	3.05	τ_1	-29-22i	-147-27i	-0.006	3.01
α_2	-12-21i	-112-21i	-0.020	2.34	τ_2	-22-28i	135-34i	-0.012	2.81

Conclusions

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- 1), Meson cloud effect is considered.

- 2), Its effect on the EM transition of nucleon and its
- resonances is stressed.

- 3), The size is enlarged (for the helicity amplitude
- and E2/M1)

- Relativistic version +configuration mixing effect

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Thank you!

Rujula, Georgi and Politzer

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The resonance strengths average to a global scaling curve resembling the curve of DIS, as the higher-twist effect is not large, if averaged over a large kinematics region.

Bloom-Gilman duality was offered by De Rujula, Georgi, and Politzer [6] in terms of the moments $M_n(Q^2)$ of the nucleon structure function $F_2(\xi,Q^2)$:

$$M_n(Q^2)=\int_0^1 d\xi\,\xi^{n-2}F_2(\xi,Q^2), \qquad (1)$$

where ξ is the Nachtmann variable (cf. [7]),

$$\xi=\frac{2x}{1+\sqrt{1+4m^2x^2/Q^2}}. \qquad (2)$$

Using the operator product expansion (OPE) the authors of Ref. [6] argued that

$$M_n(Q^2)=A_n(Q^2)+\sum_{k=1}^{\infty}\left(n\frac{\gamma^2}{Q^2}\right)^k B_{nk}(Q^2), \qquad (3)$$

where γ^2 is a scale constant. The first term $A_n(Q^2)$ in Eq. (3) is the result of perturbative QCD, while the remaining terms $B_{nk}(Q^2)$ are higher twists related to parton-parton correla-

- $G_E^p(q^2)$ falls faster than $G_M^p(q^2)$
  (spacelike Q^2=-q^2)
- ➔F2/F1 falls more slowly than $1/Q^2$ (1/Q)
- PQCD and dimension counting
  rules➔F1($1/Q^4$,Dirac),
- F2(Pauli)/F1(Dirac)➔$1/Q^2$