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Generalized Parton Distributions in a Meson-Cloud Model

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Generalized Parton Distributions in a Meson-Cloud Model

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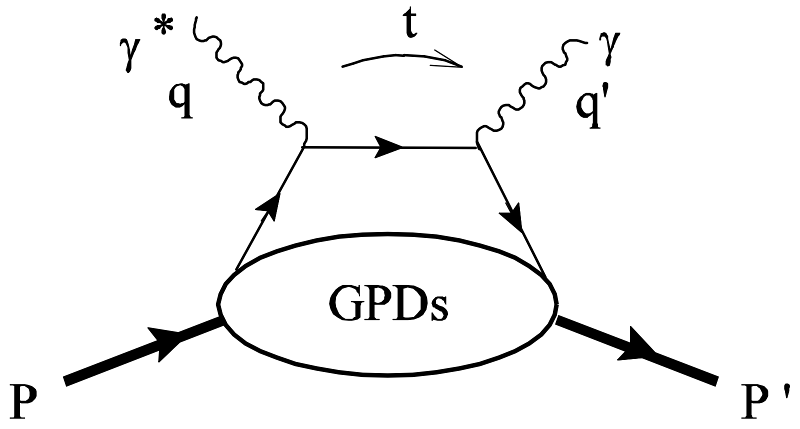
Outline

- GPDs in the light cone wave function overlap representation
- Meson-cloud model for nucleon wave function
- Convolution formalism for the GPDs in the meson-cloud model
- Results for the unpolarized GPDs

Deeply Virtual Compton Scattering



$Q^2 >>$

$s >>$

$t = (q - q')^2 <<$

Unpolarized GPDs $\sim \langle P', S' \, | O_q | \, P, S \rangle$

$$O_q = \int \frac{\mathrm{d}z^-}{4\pi} \bar{q}(-\frac{z}{2}) \, \gamma^+ \, q(\frac{z}{2}) \, e^{ixP^+z^-}$$

Density operator of quark fields

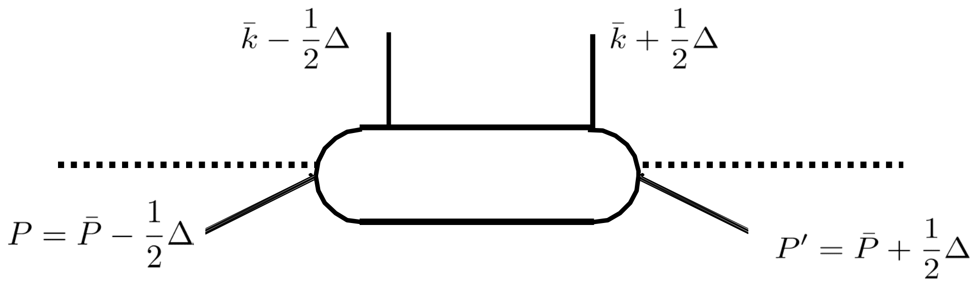
Polarized GPDs $\sim \langle P', S' \, | \tilde{O}_q | \, P, S \rangle$

$$\tilde{O}_q = \int \frac{\mathrm{d}z^-}{4\pi} \bar{q}(-\frac{z}{2}) \, \gamma^+ \gamma_5 \, q(\frac{z}{2}) \, e^{ixP^+z^-}$$

Difference of density operator for left and right-handed quark fields

GPDs are related to off-diagonal matrix elements of the momentum-density matrix and measure the quark-momentum correlations in the nucleon

Generalized Parton Distributions



- $P \neq P' \Rightarrow$ GPDs depend on two momentum fractions $\bar{x}$, ξ, and t

$$\bar{x} = \frac{(k+k')^+}{(P+P')^+} = \frac{\bar{k}^+}{\bar{P}^+} \qquad \xi = \frac{(P-P')^+}{(P+P')^+} = -\frac{\Delta^+}{2\bar{P}^+} \qquad t = (P-P')^2$$

average fraction of the longitudinal momentum carried by partons

skewness parameter ~ $x_1 - x_2$

t-channel momentum transfer squared

- **Unpolarized GPDs**

$$\mathcal{H}^q_{\lambda'\lambda} \equiv \sum_c \int \frac{dz^-}{4\pi}\, e^{i\,\bar{x}\,\bar{P}^+ z^-} \langle P', \lambda' | \overline{\psi}^c_q(-z/2)\,\gamma^+\,\psi^c_q(z/2)\,|P, \lambda\rangle$$

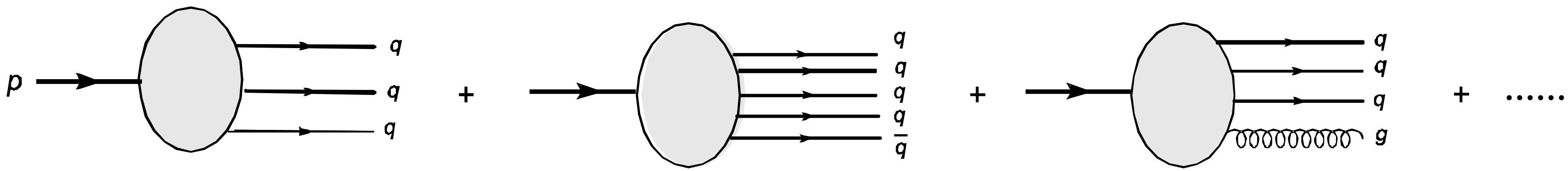
$$= \frac{\overline{u}(P',\lambda')\gamma^+ u(P,\lambda)}{4\bar{P}^+} H^q(\bar{x},\xi,t) + \frac{\overline{u}(P',\lambda') i\sigma^{+\alpha}\Delta_\alpha u(P,\lambda)}{4M\,\bar{P}^+} E^q(\bar{x},\xi,t)$$

H^q: conserves proton helicity

E^q: responsible for proton helicity flip

Light-Cone Fock Expansion

$$| \, \Psi \rangle \quad = \quad \Psi_{3q} | \, qqq \rangle + \Psi_{3q\,q\bar{q}} | \, 3q \, q\bar{q} \rangle + \Psi_{3qg} | \, qqqg \rangle + \cdots$$



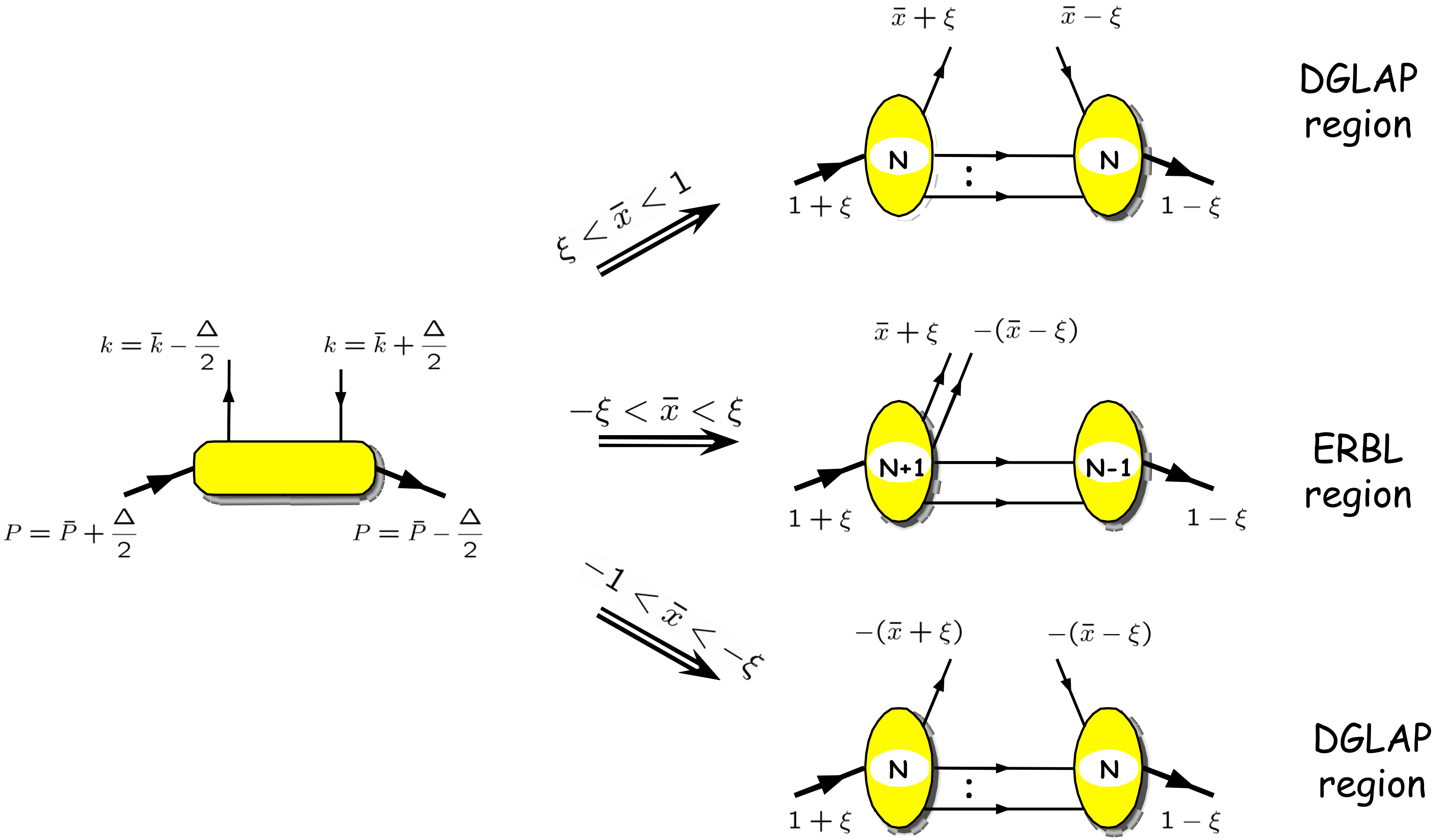
$$| \, (P^+, \vec{P}_\perp), \lambda \rangle \quad = \quad \sum_{N,\beta} \int [dx]_N \, [d\vec{k}_\perp]_N \, \Psi^f_{\lambda,N,\beta}(x_i, \vec{k}_{\perp,i}) \, | \, N, \beta; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp,i}, \lambda_i \rangle$$

- internal variables: $x_i = \dfrac{p_i^+}{P^+}$ $\qquad \sum_{i=1}^{N} x_i = 1 \qquad \sum_{i=1}^{N} \vec{k}_{\perp\, i} = \vec{0}_\perp$

- $\Psi^f_{\lambda,N,\beta}(x_i, \vec{k}_{\perp,i})$: frame INdependent

➡ probability amplitude to find the N parton configuratiion with the complex of quantum number β in the nucleon with helicity λ

Light-Cone Wave Function Overlap Representation



➢ Parton interpretation: amplitude that parton is taken out of the nucleon with long. momentum ($\bar{k}$-$\Delta/2$) and inserted back into the nucleon with long. momentum transfer $\Delta^+= -2\xi P^+$ and momentum transfer $\Delta_\perp$

Diehl, Feldmann, Jakob, Kroll, NPB596, 2001

Meson-Cloud Model

the physical nucleon $\tilde{N}$ is made of a bare nucleon N dressed by a surrounding meson cloud

$$| \tilde{N}\rangle = \Psi^{N}_{(3q)}| N(qqq)\rangle + \sum_{B,M} \Psi^{(BM)}_{(3q)(q\bar{q})}| B(qqq)M(q\bar{q})\rangle + \cdots$$

Light cone Hamiltonian:

$$H_{LC} = \sum_{B,M} \left[H_0^B(q) + H_0^M(q) + H_I(N,BM)\right]$$

$$H_{LC}|\tilde{p}_N,\lambda;\tilde{N}\rangle = \frac{\mathbf{p}^2_{N\perp} + M_N^2}{p_N^+}|\tilde{p}_N,\lambda;\tilde{N}\rangle$$

Perturbation theory: we expand the nucleon wave function in terms of eigenstates of the bare Hamiltonian $H_0 = \sum_{B,M} H_0^B + H_0^M$

$$|\tilde{p}_N,\lambda;\tilde{N}\rangle = \sqrt{Z}\left(|\tilde{p}_N,\lambda;N\rangle + {\sum_{n_1}}' \frac{|n_1\rangle\langle n_1|H_I|\tilde{p}_N,\lambda;N\rangle}{E_N - E_{n_1} + i\epsilon} + {\sum_{n_1,n_2}}' \frac{|n_2\rangle\langle n_2|H_I|n_1\rangle\langle n_1|H_I|\tilde{p}_N,\lambda;N\rangle}{(E_N - E_{n_2} + i\epsilon)(E_N - E_{n_1} + i\epsilon)} + \cdots\right)$$

Sullivan, PRD5, 1972;
Speth and Thomas, Adv. Nucl. Phys. 24, 1998

One-meson approximation: we truncate the series expansion to first order in H_I

$$|\tilde p_N, \lambda; \tilde N\rangle = \boxed{\sqrt{Z}|\tilde p_N, \lambda; N\rangle} + \sum_{B,M} \int \frac{\mathrm{d}y\mathrm{d}^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{\sqrt{y(1-y)}} \sum_{\lambda',\lambda''} \boxed{\phi^{\lambda\,(N,BM)}_{\lambda'\lambda''}(y,\mathbf{k}_\perp)}$$
$$\times \boxed{|yp^+_N, \mathbf{k}_\perp + y\mathbf{p}_{N\perp}, \lambda'; B\rangle\, |(1-y)p^+_N, -\mathbf{k}_\perp + (1-y)\mathbf{p}_{N\perp}, \lambda''; M\rangle}$$

$|\tilde p_N, \lambda; N\rangle$ state of the bare nucleon

Z: probability to find the bare nucleon in the physical nucleon

$|yp^+_N, \mathbf{k}_\perp + y\mathbf{p}_{N\perp}, \lambda'; B\rangle\, |(1-y)p^+_N, -\mathbf{k}_\perp + (1-y)\mathbf{p}_{N\perp}, \lambda''; M\rangle$ Baryon-Meson fluctuation

Splitting function: $$\phi^{\lambda\,(N,BM)}_{\lambda'\lambda''}(y,\mathbf{k}_\perp) = \frac{1}{\sqrt{y(1-y)}} \frac{V^{\lambda}_{\lambda',\lambda''}(N,BM)}{M^2_N - M^2_{BM}(y,\mathbf{k}_\perp)}$$

probability amplitude for a nucleon with helicity λ to fluctuate into a (BM) system with the baryon having helicity λ' and momentum (yp^+_N, k_B) and the meson having helicity λ'' and momentum ($p^+_N(1-y)$, $-k_B$)

Partonic content of the nucleon wave function

❖ Bare Nucleon: N (qqq) – 3 valence quarks

$$| \tilde{p}_N, \lambda; N\rangle = \sum_{\tau_i,\lambda_i} \int \left[\frac{\mathrm{d}x}{\sqrt{x}}\right]_3 [\mathrm{d}^2\mathbf{k}_\perp]_3 \Psi_\lambda^{3q,[f]}(\{x_i, \mathbf{k}_{\perp i}; \lambda_i, \tau_i\}_{i=1,2,3}) \prod_{i=1}^{3} | \, x_i p_N^+, \, \mathbf{p}_{i\perp}, \lambda_i, \tau_i; q\rangle$$

$\tilde{\Psi}_\lambda^{3q,\,[f]}$: probability amplitude to find the 3 valence quarks in the nucleon with helicity λ

❖ Baryon – Meson fluctuation: B(qqq) M(q$\bar{\mathrm{q}}$)

$$\begin{aligned} |\tilde{p}_N, \lambda; N(BM)\rangle \quad = \quad & \int \mathrm{d}y \, \mathrm{d}^2\mathbf{k}_\perp \int_0^y \prod_{i=1}^{3} \frac{\mathrm{d}\xi_\mathrm{i}}{\sqrt{\xi_i}} \int_0^{1-y} \prod_{i=4}^{5} \frac{\mathrm{d}\xi_\mathrm{i}}{\sqrt{\xi_i}} \int \prod_{i=1}^{5} \frac{\mathrm{d}\mathbf{k}'_{\mathrm{i}\perp}}{[2(2\pi)^3]^4} \\ & \times \delta\left(y - \sum_{i=1}^{3} \xi_i\right) \delta^{(2)}\left(\mathbf{k}_\perp - \sum_{i=1}^{3} \mathbf{k}'_{i\perp}\right) \delta\left(1 - \sum_{i=1}^{5} \xi_i\right) \delta^{(2)}\left(\sum_{i=1}^{5} \mathbf{k}'_{i\perp}\right) \\ & \times \sum_{\lambda_i,\tau_i} \tilde{\Psi}_\lambda^{5q,\,[f]}(y, \mathbf{k}_\perp; \, \{\xi_i, \mathbf{k}'_{i\perp}; \lambda_i, \tau_i\}_{i=1,\ldots,5}) \\ & \times \prod_{i=1}^{5} |\xi_i p_N^+, \, \mathbf{k}'_\perp + \xi_i \mathbf{p}_{N\perp}, \, \lambda_i, \, \tau_i; q\rangle \end{aligned}$$

$\tilde{\Psi}_\lambda^{5q,\,[f]}$: probability amplitude for finding in the nucleon a configuration of 5 quarks composed by two clusters of (qqq) and (q$\bar{\mathrm{q}}$) with momentum p_B and p_M

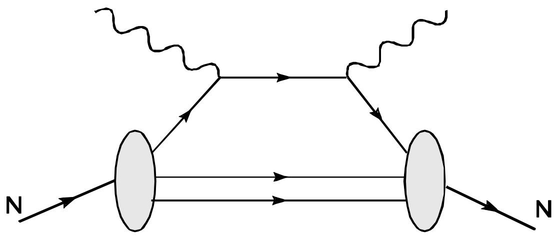
GPDs in the region $\xi < \bar{x} < 1$: describe the emission of a quark with momentum fraction $\bar{x}+\xi$ and its reabsorption with momentum fraction $\bar{x} - \xi$

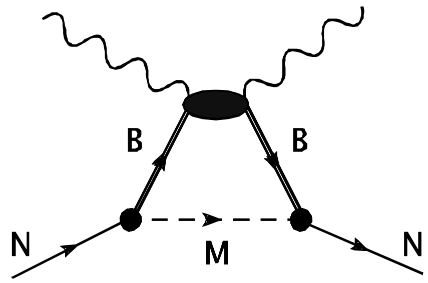
- during the interaction with the hard photon, there is no interaction between the partons in a multiparticle Fock state
- the photon can scatter either on the bare nucleon (N) or one of the constituent in the higher Fock state component (BM)

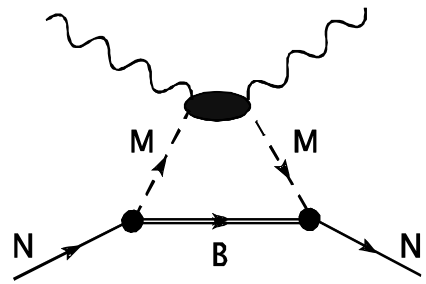
$$F^{q}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) = Z\,F^{q,bare}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) + \delta F^{q}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp)$$

valence quark

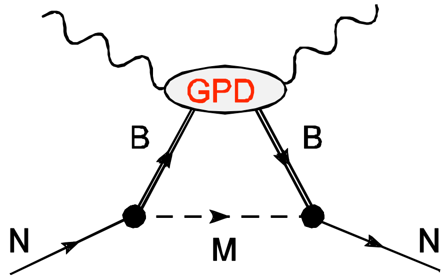
baryon-meson substate



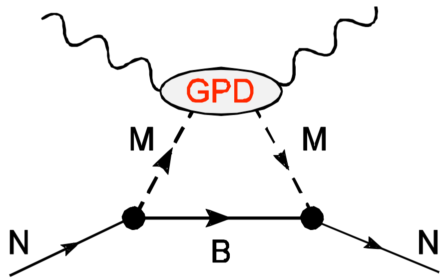




Convolution formalism for the contribution from the (BM) substate



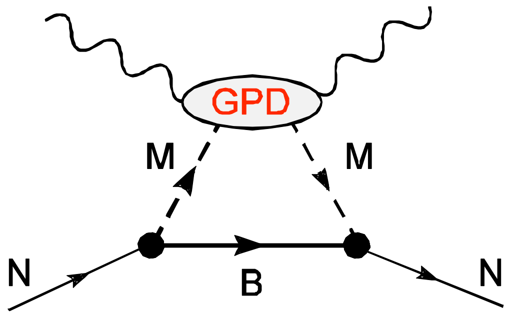
$$\delta F^{q/BM}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}}\sum_M \sum_{\lambda,\lambda',\lambda''}\int_{\overline{x}}^1 \frac{\mathrm{d}\overline{y}_B}{\overline{y}_B}\int \frac{\mathrm{d}^2\overline{\mathbf{p}}_{B\perp}}{2(2\pi)^3} F^{q/B}_{\lambda'\lambda}\left(\frac{\overline{x}}{\overline{y}_B},\frac{\xi}{\overline{y}_B},\mathbf{\Delta}_\perp\right)$$
$$\times\, \phi^{\lambda_N\,(N,BM)}_{\lambda\lambda''}(\tilde{y}_B,\tilde{\mathbf{k}}_{B\perp})\,[\phi^{\lambda'_N\,(N,BM)}_{\lambda'\lambda''}(\hat{y}'_B,\hat{\mathbf{k}}_{B\perp})]^*$$



$$\delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}}\sum_B \sum_{\lambda,\lambda',\lambda''}\int_{\overline{x}}^1 \frac{\mathrm{d}\overline{y}_M}{\overline{y}_M}\int \frac{\mathrm{d}^2\overline{\mathbf{p}}_{M\perp}}{2(2\pi)^3} F^{q/M}_{\lambda'\lambda}\left(\frac{\overline{x}}{\overline{y}_M},\frac{\xi}{\overline{y}_M},\mathbf{\Delta}_\perp\right)$$
$$\times\, \phi^{\lambda_N\,(N,BM)}_{\lambda''\lambda}(1-\tilde{y}_M,-\tilde{\mathbf{k}}_{M\perp})\,[\phi^{\lambda'_N\,(N,BM)}_{\lambda''\lambda'}(1-\hat{y}'_M,-\hat{\mathbf{k}}_{M\perp})]^*$$

GPDs in the region -1 < $\bar{x}$ < -ξ: describe the emission of an antiquark from the nucleon with momentum fraction −($\bar{x}$+ξ) and its reabsorption with momentum fraction −($\bar{x}$ -ξ)

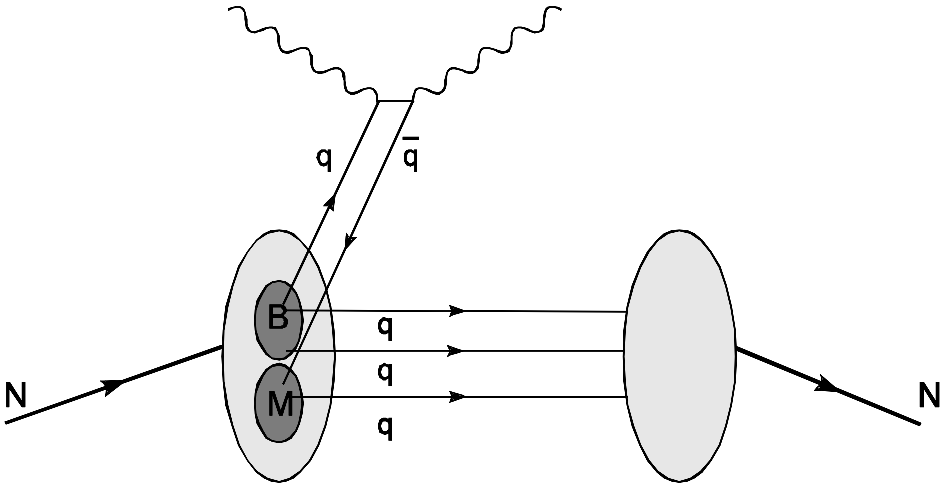
$$F^{q}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) = \delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp)$$



$$\delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) \quad = \quad \frac{1}{\sqrt{1-\xi^2}}\sum_B \sum_{\lambda,\lambda',\lambda''}\int_{-\overline{x}}^{1}\frac{\mathrm{d}\overline{y}_M}{\overline{y}_M}\int\frac{\mathrm{d}^2\overline{\mathbf{p}}_{M\perp}}{2(2\pi)^3}F^{q/M}_{\lambda'\lambda}\left(\frac{\overline{x}}{\overline{y}_M},\frac{\xi}{\overline{y}_M},\mathbf{\Delta}_\perp\right)$$
$$\times\,\phi^{\lambda_N\,(N,BM)}_{\lambda''\lambda}(1-\tilde{y}_M,-\tilde{\mathbf{k}}_{M\perp})\,[\phi^{\lambda'_N\,(N,BM)}_{\lambda''\lambda'}(1-\hat{y}'_M,-\hat{\mathbf{k}}_{M\perp})]^*$$

GPDs in the region $-\xi < x < \xi$: describe the emission of a quark-antiquark pair from the initial nucleon

❖ non-diagonal contribution in the Fock-space expansion: the initial nucleon has the same parton content as the final state plus an additional quark-antiquark pair



❖ in the meson cloud model, we have the contribution from the overlap of the 5q component of the nucleon wave function (the (BM) substate) and the 3q valence component (the bare nucleon)

Model Calculation

✓ we consider only the pion cloud contribution of the proton

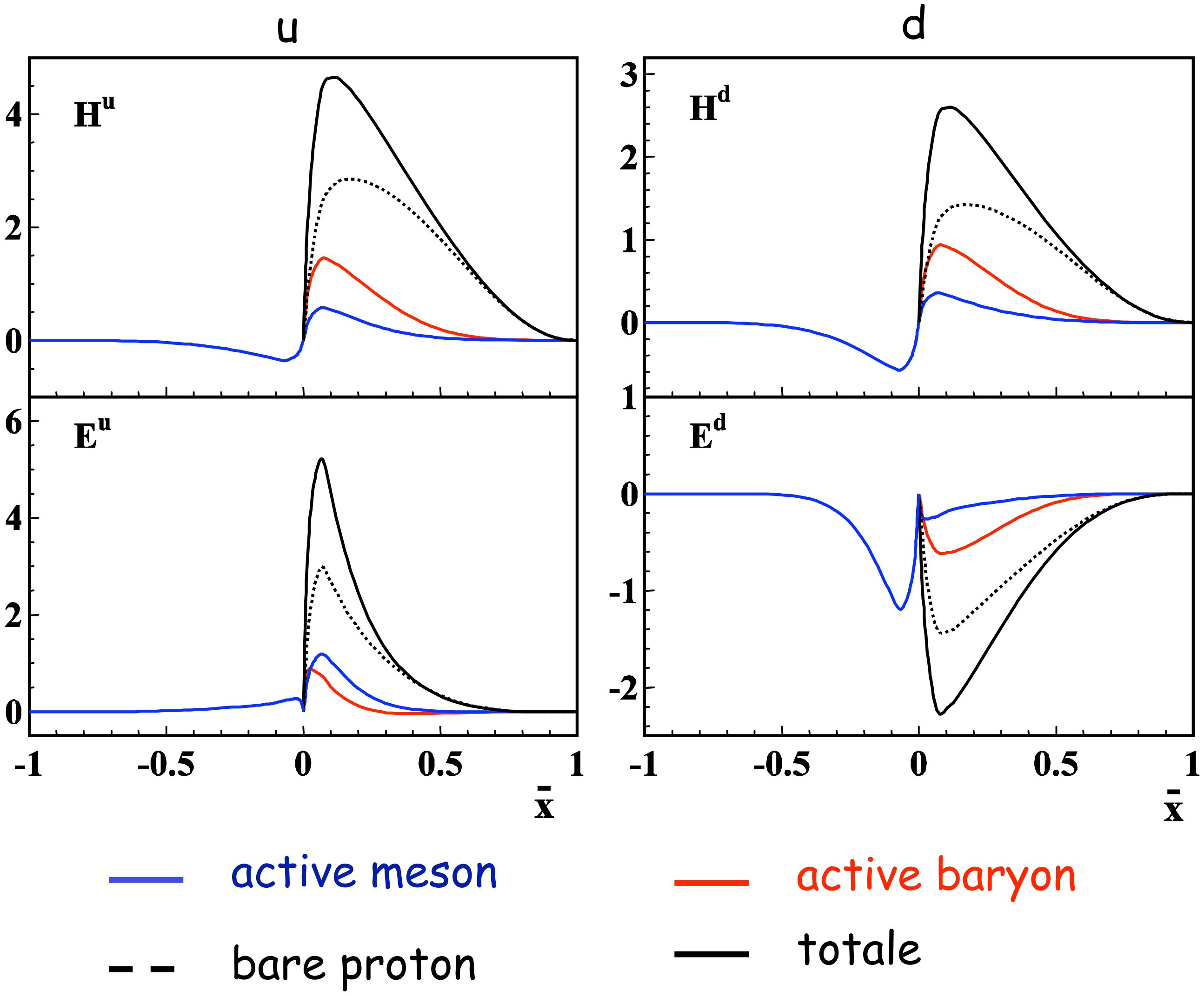
✓ the baryon in the (BM) fluctuation is a N or a Δ

$$|\tilde{N}\rangle = \Psi^{N}_{(3q)}|\,N(qqq)\rangle + \Psi^{N}_{(3q)} \otimes \Psi^{\pi}_{(q\bar{q})}|\,N(qqq)\pi(q\bar{q})\rangle + \Psi^{\Delta}_{(3q)} \otimes \Psi^{\pi}_{(q\bar{q})}|\,\Delta(qqq)\pi(q\bar{q})\rangle + \cdots$$

✓ the wave function of the bare baryon and pion are modeled in a light-front relativistic constituent quark model $\longrightarrow$ SU(6) symmetric wave functions

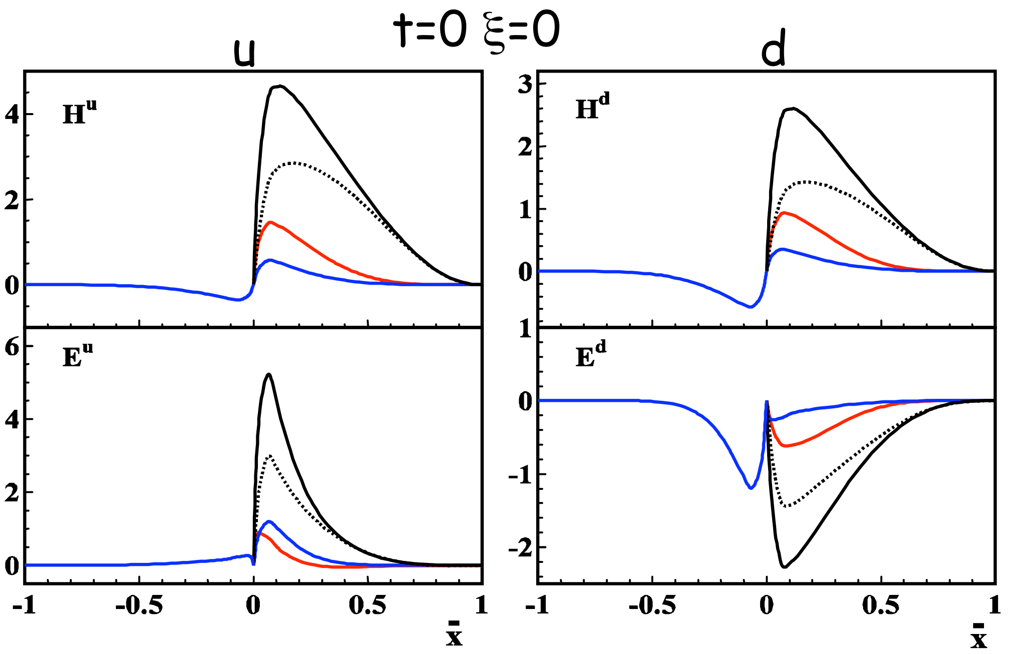
Nucleon and Δ wf: Faccioli, Traini, Vento, NPA656 (1999)
Pion wf: H.M. Choi, C.R, Ji, PRD59 (1999)

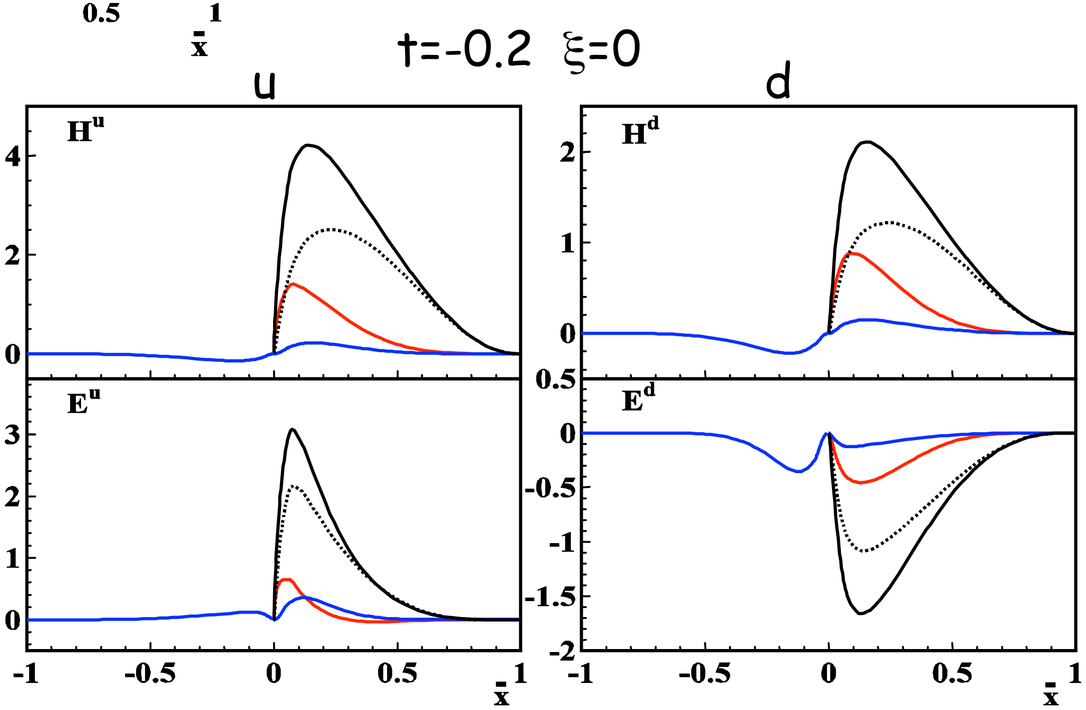
Forward limit (ξ = 0, t=0)



B. Pasquini, S. Boffi, PRD73, 2006

t-dependence at ξ = 0





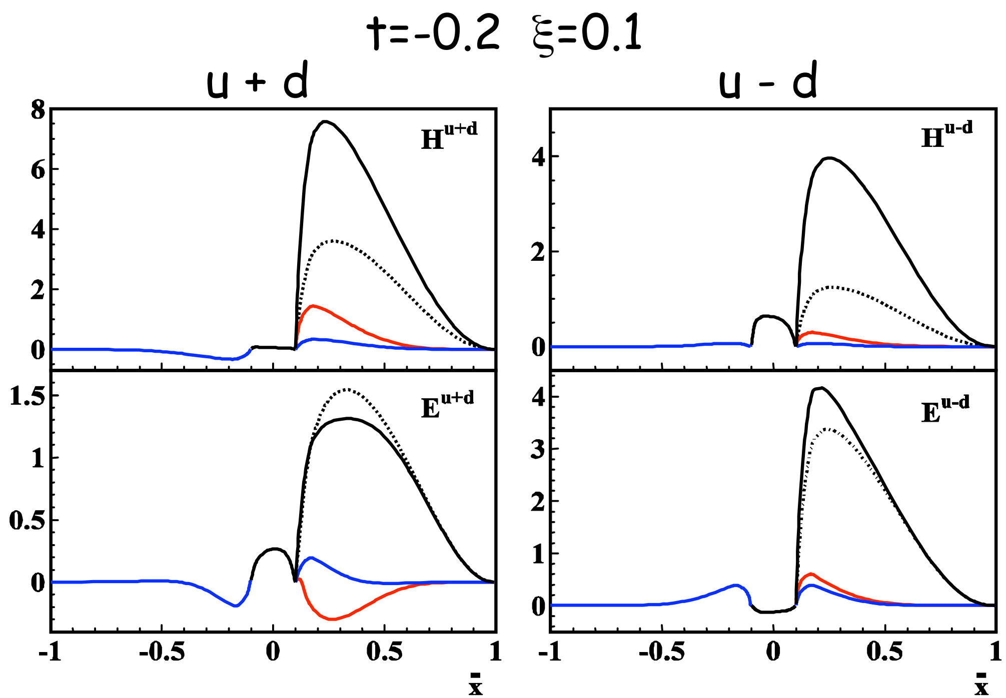
active meson

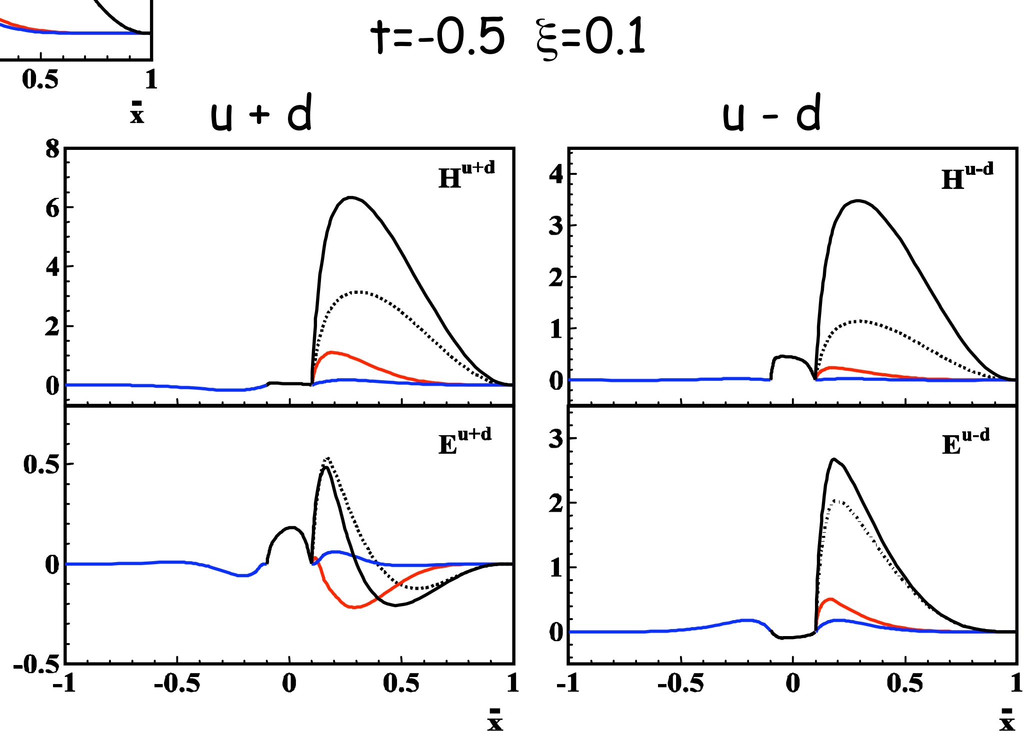
bare proton

active baryon

totale

t-dependence at fixed ξ = 0.1





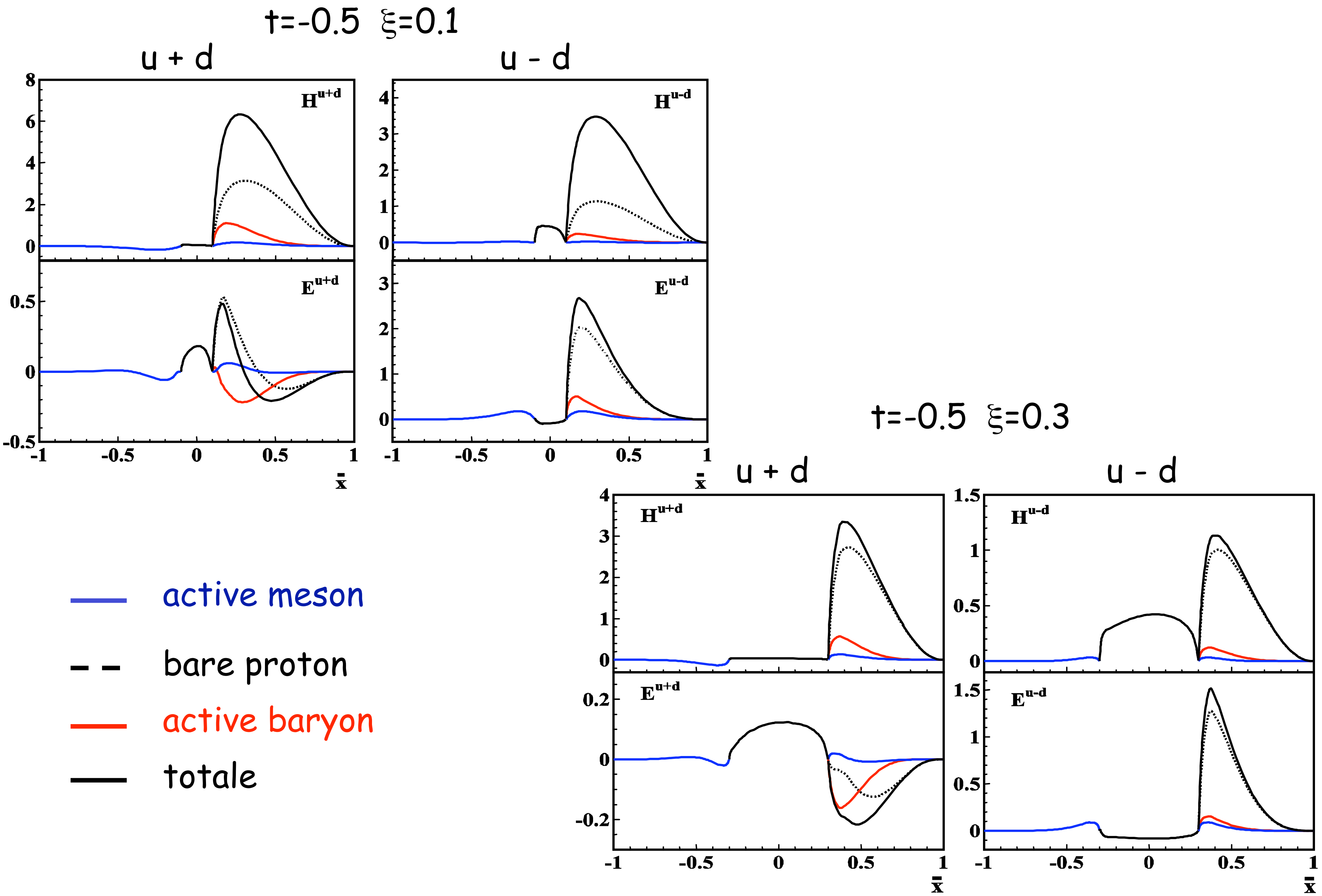
active meson

bare proton

active baryon

totale

ξ–dependence at fixed t= -0.5



Summary

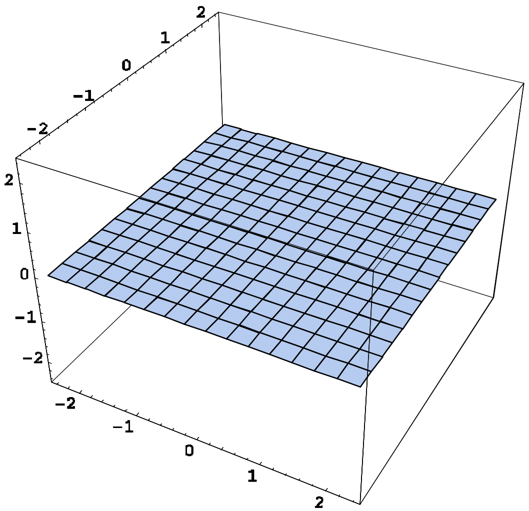
❖ Meson-cloud model from a Fock-space expansion of the nucleon state in terms of 3 valence quarks and a quark-sea contribution coming from baryon-meson fluctuations of the nucleon

❖ Meson-cloud model for parton distributions $\Rightarrow$ Sullivan process capable to describe the sea-quark distributions of the nucleon

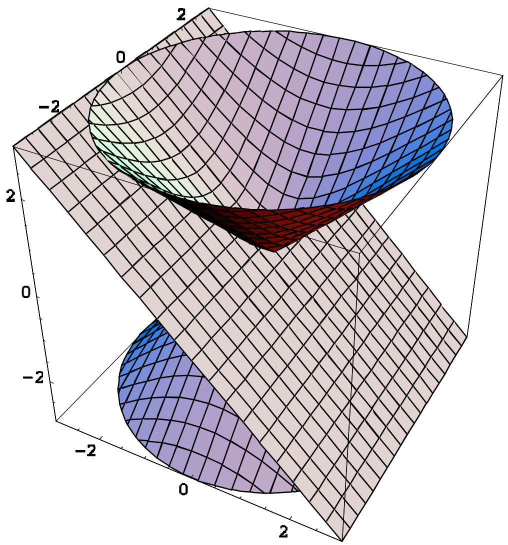
❖ Extension of the model to GPDs $\Rightarrow$ convolution formalism for the GPDs obtained by folding the quark GPDs within bare constituents (nucleons, pion, Δ,….) with the probability amplitudes describing the distributions of these constituents in the dressed nucleon

❖ Predictions for the GPDs at the hadronic scale in the DGLAP and ERBL regions

Instant form



Light-cone form



coordinates

x^0 time

x^1, x^2, x^3 space

$x^+ = x^0 - x^3$ time

$x^- = x^0 + x^3,\ x^1,\ x^2$ space

Hamiltonian

$$H = \sqrt{\vec{P}^2 + M_0^2} + V$$

$$P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

eigenvalue equation

$$(H^2 - \vec{P}^2)|\,\Psi\rangle = M^2|\,\Psi\rangle$$

$\vec{P}$ fixed

many states with $\sum_i \vec{p}_i = \vec{P}$

⇨ vacuum complicated

$$(P^+P^- - P_\perp^2)|\,\Psi\rangle = M^2|\,\Psi\rangle$$

$P^+ > 0, \vec{P}_\perp$ fixed

zero-particle state with $P^+ = 0$

⇨ vacuum trivial

Model Calculation

✓ we consider only the pion cloud contribution of the proton

✓ the baryon in the (BM) fluctuation is a nucleon or a Δ

✓ the wave function of the bare baryon and pion are modeled in a light-front relativistic constituent quark model

✓ we do not consider configuration mixing $\longrightarrow$ SU(6) symmetric wf

Hypercentral Model

Faccioli, Traini, Vento, 1999

Hamiltonian

= relativistic kinetic energy

+ linear confinement potential

+ hyper-Coulomb potential

$$H = \sum_{i=1}^{3} \sqrt{\vec{k}_i^2 + m_i^2} - \frac{\tau}{y} + \kappa y$$

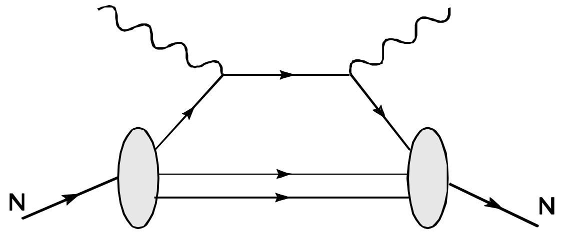
$$\vec{\rho} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}} \qquad \vec{\lambda} = \frac{\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3}{\sqrt{6}} \qquad y = \sqrt{\vec{\rho}^2 + \vec{\lambda}^2}$$

GPDs in the region $\xi < \bar{x} < 1$: describe the emission of a quark with momentum fraction $\bar{x}+\xi$ and its reabsorption with momentum fraction $\bar{x} - \xi$

- during the interaction with the hard photon, there is no interaction between the partons in a multiparticle Fock state
- the photon can scatter either on the bare nucleon (N) or one of the constituent in the higher Fock state component (BM)

$$F^q_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) = Z\,F^{q,bare}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) + \delta F^q_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp)$$

$Z\,F^{q,bare}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp)$: valence quark contribution



$$\delta F^q_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) = \sum_{B,M}\left[\delta F^{q/BM}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) + \delta F^{q/MB}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp)\right]$$