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Unitarity Corrections to Pion Pion Partial Waves from Dispersion Relation Formalism

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Outline

Hard Meson Method

Unitarization Program

Chiral Perturbation Theory

Comparing O(p^4) and O(p^6) ChPT with UPCA

Analytical Expressions

Tools for Improvement

Conclusion and Perspectives

Hard Meson Method

Current Algebra

$$\left[A_0^a(x), A_\mu^b(0)\right]_{x_0=0} = i\epsilon^{abc} V_\mu^c(x)\delta(x),$$

$$\left[A_0^a(x), \partial^\lambda A_\lambda^b(0)\right]_{x_0=0} = \sigma^{ab}\delta(x)$$

sigma model

Ward Identities

$$\partial^\mu \langle 0|T\left(j_\mu(x) j_1(x_1)\ldots j_n(x_n)\right)|0\rangle = \langle 0|T\left(\partial^\mu j_\mu(x) j_1(x_1)\ldots j_n(x_n)\right)|0\rangle$$

$$+\sum_{i=1}^{n}\langle 0|T\left([j_0(x), j_i(x_i)]_{x_0=x_{i0}} j_1(x_1)\ldots j_{i-1} j_{i+1}\ldots j_n(x_n)\right)|0\rangle$$

Diagonalized N-point functions

$$A^a_\mu(x) \rightarrow A^a_\mu(x) - i\frac{q_\mu}{m}\partial^\lambda A^a_\lambda(x)$$

0^-

1^+

on shell

Starts from

$$M^{abcd}_{\mu\nu\lambda\sigma} = \int e^{i(p_1x+p_2y+p_3z)}\langle 0|T\left(A^a_\mu(x)\,A^b_\nu(y)\,A^c_\lambda(z)\,A^d_\sigma(0)\right)|0\rangle$$

Example

$$iC^A p_4^\sigma M^{abcd}_{\mu\nu\lambda\sigma} = f_\pi^2 m_\pi^2 M^{abcd}_{\mu\nu\lambda} + \Delta^{-1A}_{\mu\alpha}\frac{p_1^\alpha}{m}\Gamma^{abcd}_{\Sigma\nu\lambda}(p_1+p_4,p_2) +$$

$$\varepsilon^{adf}\varepsilon^{bcf}\Delta^{-1A}_{\mu\alpha}\Delta^V_{\alpha\beta}(p_1+p_4)\Gamma^\beta_{\nu\lambda\alpha}(p_2,p_3) + crossed$$

no underlying theory

Pion Pion Scattering Amplitude

$$T^{abcd}(s,t,u) = f_\pi^4 m_\pi^8 M^{abcd}(p_1, p_2, -p_3) \quad \left(p_i^2 = m_\pi^2\right)$$

Tree Structure

Propagators $\Delta_0\ \Delta_2\ \Delta_V$

Form Factors $F_0\ \ F_2\ \ F_1$

$$T^{abcd} = \bar{h}_0(s)\, P_0^{ab,cd} + \bar{h}_2(s)\, P_2^{ab,cd} + (t-u)\bar{h}_1(s)\, P_1^{ab,cd} + crossed$$

$$\bar{h}_I(s) = (F_I(s) - 1)^2 \Delta_I^{-1}(s) \qquad \bar{h}_1(s) = (F_1(s) - \mathrm{K})^2 \Delta_V^{-1}(s) - \frac{C_V}{4 f^4}$$

$$T^{abcd}(s,t,u) = A(s,t,u)\delta^{ab}\delta^{cd} + A(t,s,u)\delta^{ac}\delta^{bd} + A(u,t,s)\delta^{ad}\delta^{bc}$$

Low Energy Estimates

$$F_I(x) \approx 1 + f_I^{(1)}(x) + f_I^{(2)}(x) \qquad \Delta_I(x) \approx \delta_I^{(1)}(x) + \delta_I^{(2)}(x) \qquad \Delta_V(x) \approx C_V + \delta_v^{(1)}(x)$$

$$A(s,t,u) \simeq A^{CA}(s,t,u) + A^{(1)}(s,t,u) + \ldots \qquad s,t,u \simeq m_\pi^2$$

$$A^{CA}(s,t,u) = \frac{1}{f^2}\left(s - m^2\right)$$

Weinberg **soft pion**

How to go beyond this approximation ?

UPCA

Elastic Unitarity Constraints for Form factors and Propagators

Partial-wave Elastic Unitarity Constraint

$$\mathrm{Im}\, t_l^I(s) = \frac{1}{32\pi}\rho(s)\left|t_l^I(s)\right|^2, \quad \rho(s) = \sqrt{\frac{s-4m^2}{s}}$$

phase space factor

$$\mathrm{Im}\, F_I(s) = \frac{1}{32\pi}\rho(s)\, t_I^*(s) F(s), \qquad \mathrm{Im}\,\Delta_I(s) = \frac{1}{32\pi}\rho(s)\left|F_I(s)\right|^2$$

First Order Correction

$$t_0^{CA}(x) = \frac{1}{f_\pi^2}(2x - m_\pi^2)$$

$$\mathrm{Im}\, h_I^{(1)}(x) = \frac{1}{32\pi}\rho(x)\; t_I^{CA}(x)$$

$$t_1^{CA}(x) = \frac{1}{3f_\pi^2}(x - 4m_\pi^2)$$

$$t_2^{CA}(x) = \frac{1}{f_\pi^2}(2m_\pi^2 - x)$$

First Order Corrected Amplitude

I=1

$$h_1^{(1)}(s) = s\, h_1^{'}(0) + \frac{s^2}{32\pi^2} \int_{4m^2}^{\infty} \frac{\rho(x)\, t^{CA}(x)}{x(x-s)}\, dx$$

$$G_1(s) = \frac{s-4m^2}{32\pi^2} \int_{4m^2}^{\infty} \frac{\rho(x)}{(x-4m^2)(x-s)}\, dx = \frac{\rho(s)}{32\pi^2}\left(i\pi + L(s)\right)$$

$$L(s) = \ln\left(\frac{1-\rho}{1+\rho}\right)$$

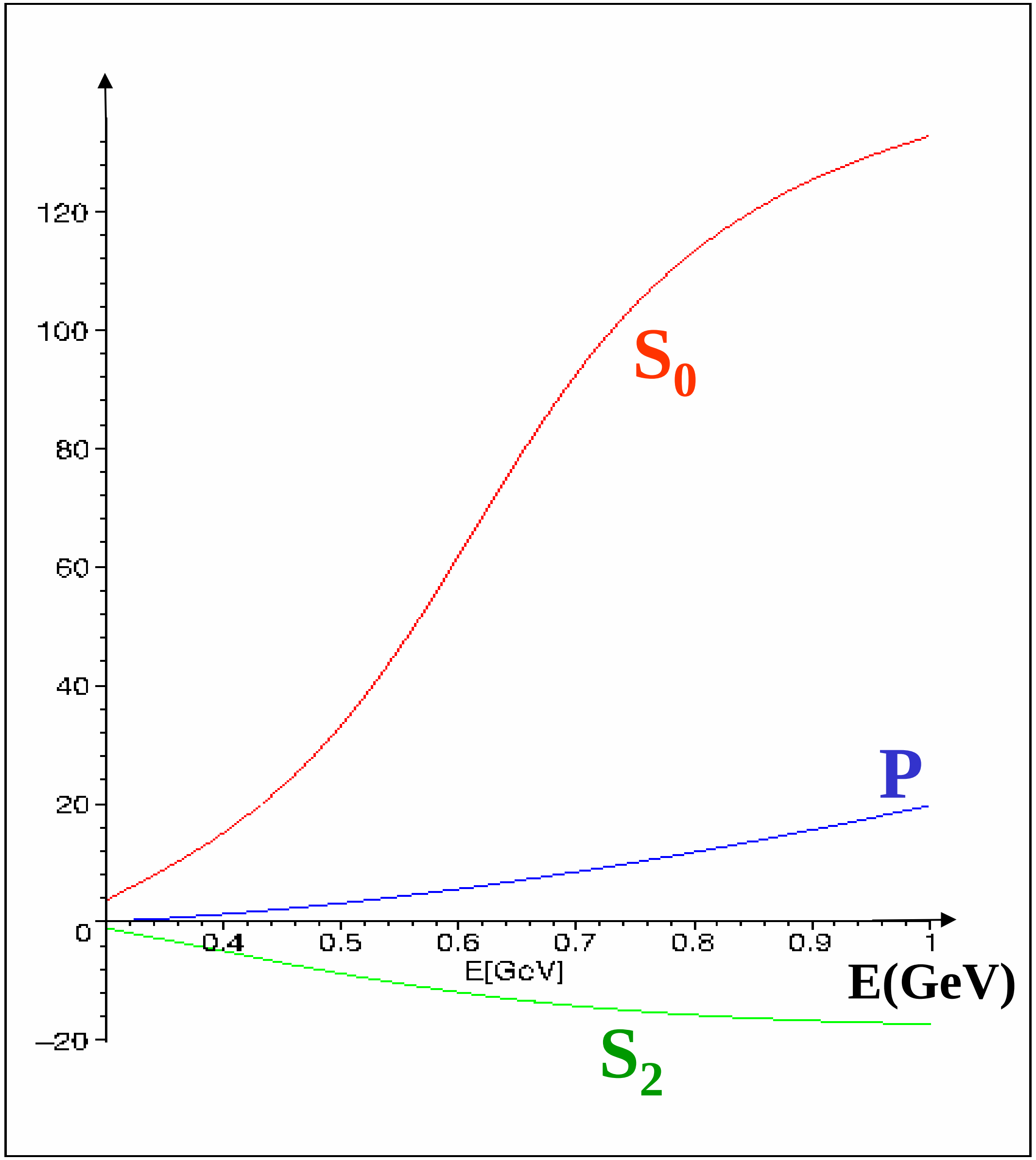
$$A^{(1)} = \frac{1}{f^4}(s^2 - m^4)\, G_1(s) + \frac{2}{3f^4}\left(t - 10m^2 t - 4m^2 s + 14m^4\right) G_1(t) + (t \leftrightarrow u)$$

$$+ P(\lambda_1, \lambda_2, \lambda_3) \qquad \lambda_3 = h_1^{'}(0)$$

Amplitude phases

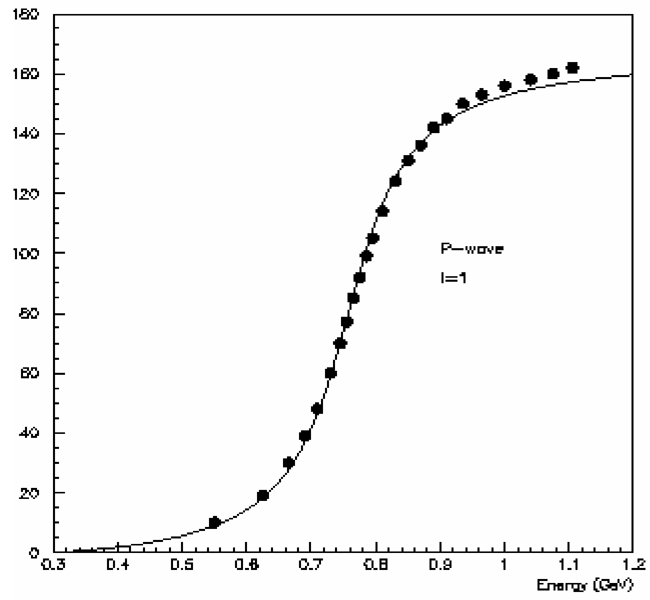
UPCA (first order)

No parameters



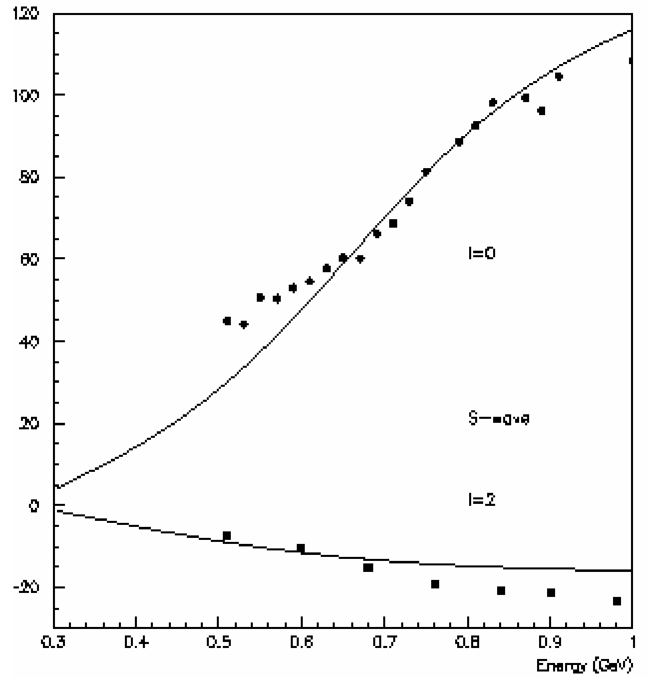
P-wave phase

fix λ_1 λ_2



S-wave phase

fix λ_3



Effective Lagrangians

The current algebra method was used in early calculations of muti-Goldstone-boson amplitudes. Unfortunately, such calculations are tedious, especially when three or more Goldstone bosons are involved ...

For this reason a different and more physical calculational technique was introduced, based on the use of effective Lagrangians:

We simply calculate the Goldstone boson amplitude by the methods of perturbation theory ...

Weinberg - Modern Applications

Ch P T

Momentum expansion

Counting scheme: infinities absorbed

Lowest Order

$$L_2 = \frac{F^2}{4} D_\mu U \; D^\mu U^+ \; + \frac{F^2 B}{2} (\chi U^+ + \chi^+ U)$$

NLσ model

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \qquad U \rightarrow g_L U g_R^+$$

External fields

$$\chi = s + ip$$

$$F = F_\pi \qquad F^2 B = - \langle \bar{u} u \rangle_0 \qquad m_\pi^2 = B(m_u + m_d)$$

G.O.R.

Pion-Pion Amplitude

$$\boldsymbol{A^{CA}(s,t,u) = \frac{1}{f^2}\left(s - m^2\right)}$$

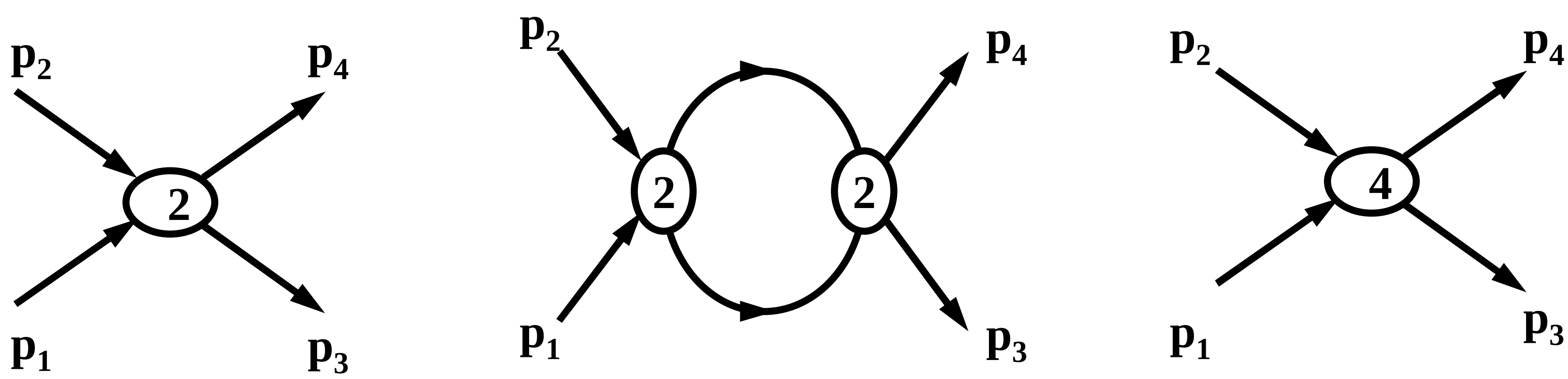
$O(p^4)$ One Loop Calculation

Building Blocks

$$L_4 = \ell_1 \left\langle D_\mu U^+ D^\mu U \right\rangle^2 + \ell_2 \left\langle D_\mu U^+ D_\nu U \right\rangle \left\langle D^\mu U^+ D^\nu U \right\rangle +$$

$$\ell_3 \left\langle D_\mu U^+ D^\mu U \, D_\nu U^+ D^\nu U \right\rangle + \ell_4 \left\langle D_\mu U^+ D^\mu U \right\rangle \left\langle \chi^+ U + \chi U^+ \right\rangle$$

How to fix **LECs** $\bar{\ell}_1 \quad \bar{\ell}_2 \quad \bar{\ell}_3 \quad \bar{\ell}_4$ **?**



same structure as UPCA !!

UPCA second order approximation

'84

from $f^{(2)}$ and $\delta^{(2)}$

$$\text{Im}\, h_I^{(2)}(x) = \frac{1}{16\pi}\rho(x)\, t_{rI}^{(1)}(x) \qquad x \approx m^2$$

$$t_1^{(1)}(s) = \frac{1}{9f^4}(s-4m)G_1(s) + \frac{m^4}{48f^4\pi^2}\frac{3s^2-13m^2s-6m^4}{(s-4m^2)^2}L(s)^2$$

$$-\frac{1}{288f^4\pi^2}\frac{s^3-16s^2m+72sm^4-36m^6}{s-4m^2}\frac{L(s)}{\rho} - \frac{1}{864f^4\pi^2}\frac{s^3+37s^2m^2-149sm^4+120m^6}{s-4m^2}$$

$$+\frac{1}{288f}(s-4m)(\lambda_1 s + m^2\lambda_2)$$

$$G_2(s) = \frac{s-4m^2}{32\pi^2}\int\frac{\rho(x)\, G_{r1}(x)}{(x-4m^2)(x-s)}\,dx$$

$$Y(s) = \frac{s-4m^2}{32\pi^2}\int\frac{L(x)}{(x-4m^2)(x-s)}\,dx \qquad Z(s) = \frac{s-4m^2}{32\pi^2}\int\frac{\rho(x)\, L^2(x)}{(x-4m^2)(x-s)}\,dx$$

$O(p^6)$ Two Loop Calculations

Building Blocks

Terms with six D_μ

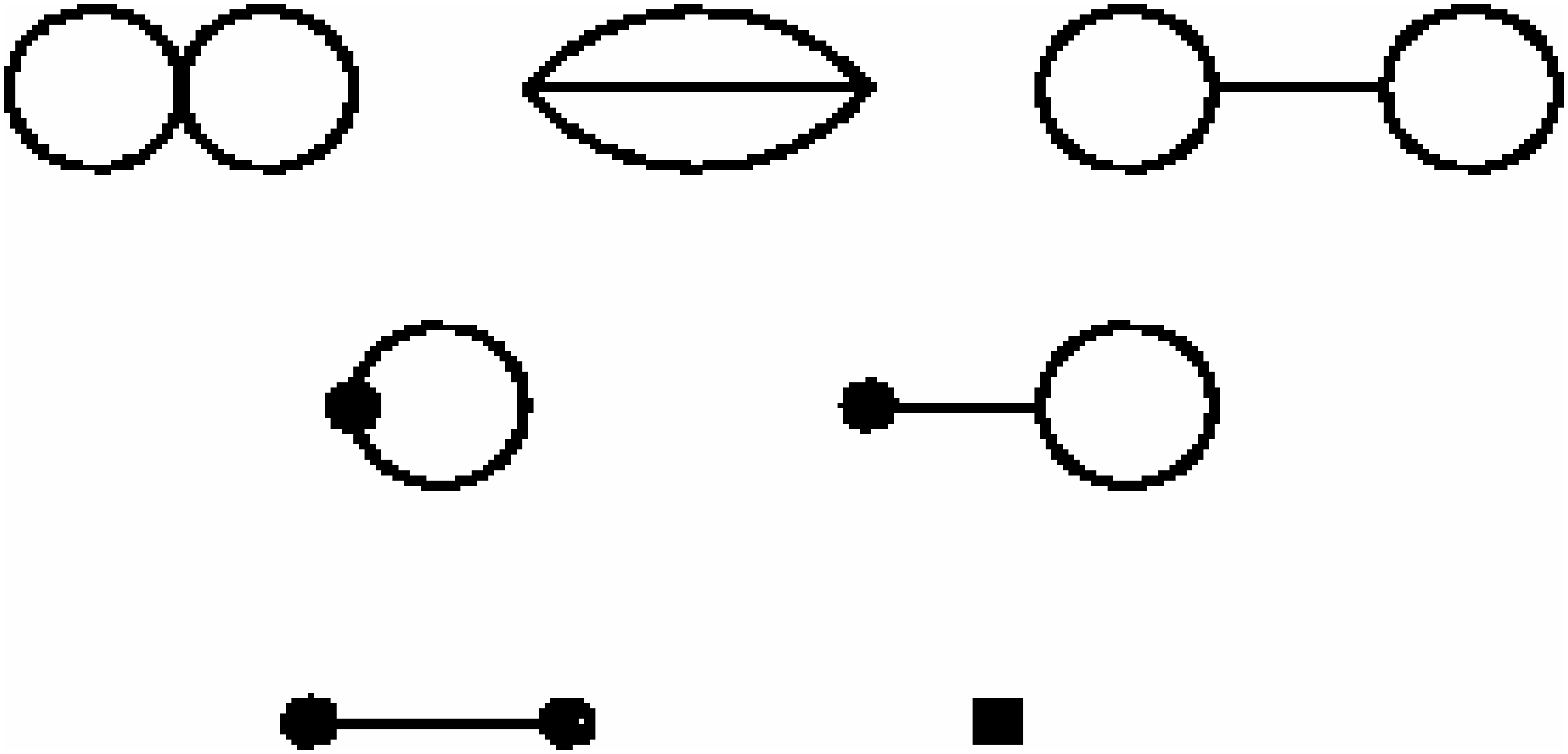
Terms with four D_μ and one $\chi_{\mu\nu}$ ($\approx f_{\mu\nu}$)

Terms with two D_μ and two $\chi_{\mu\nu}$

Terms with three $\chi_{\mu\nu}$

independent couplings $L_2(2)$ $L_4(10)$ $L_6(90)$

Skeleton Diagrams of $O(p^6)$



full lowest-order tree structures to be attached to propagators and vertices.

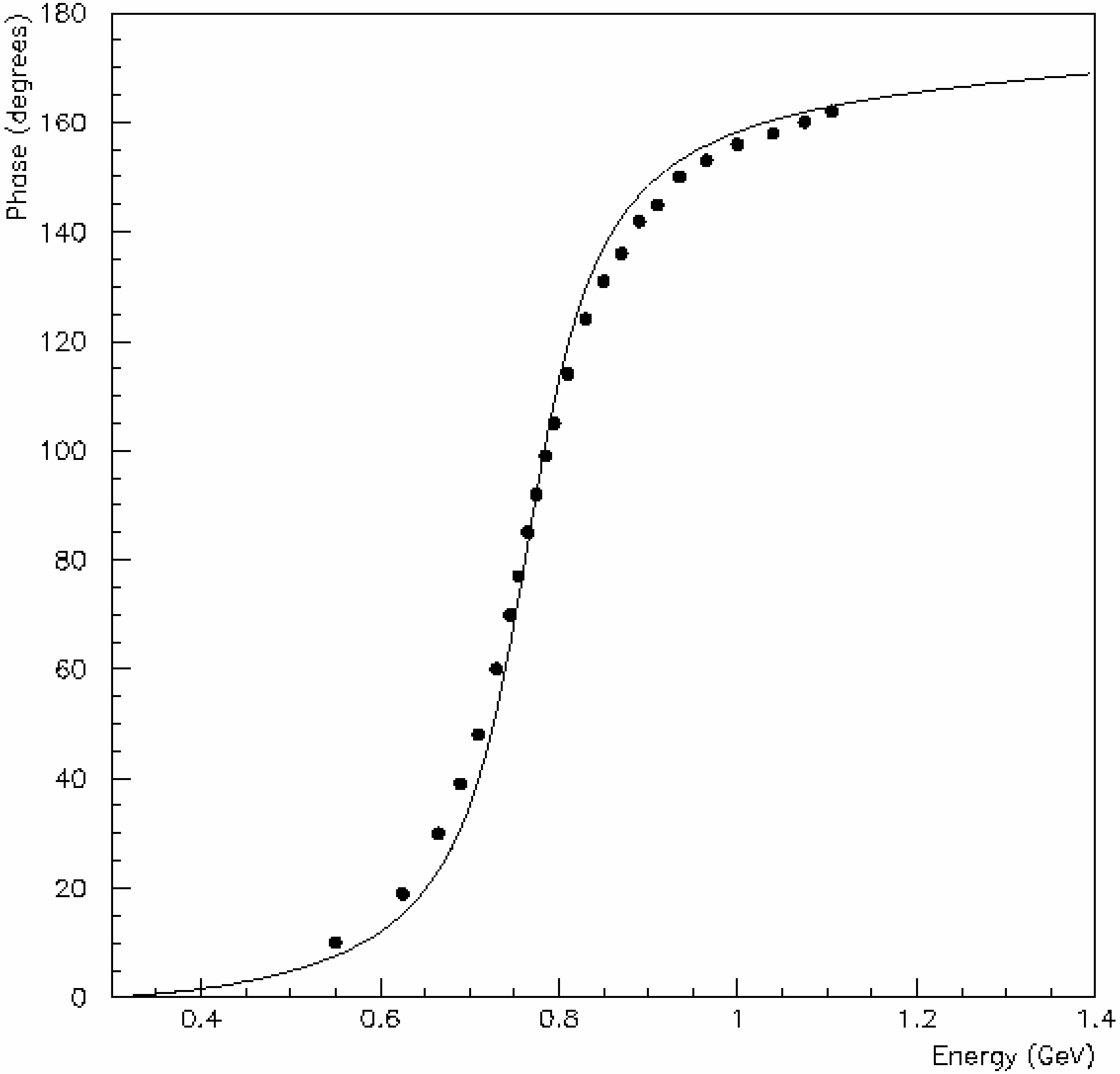
Comparison

	ChPT	UPCA	
Functions	$\bar{J}$	G_1	$O(p^4)$
Parameters	$\bar{\ell}_1 \; \bar{\ell}_2 \; \bar{\ell}_3 \; \bar{\ell}_4$	$\lambda_1 \lambda_2 \lambda_3$	
Functions	$\bar{J} \; K_1 K_2 K_3$	$G_2 \; Y \; Z$	$O(p^6)$
Parameters	$b_1 \ldots b_6 \quad \mu$	$\lambda_1 \lambda_2 \lambda_3$	

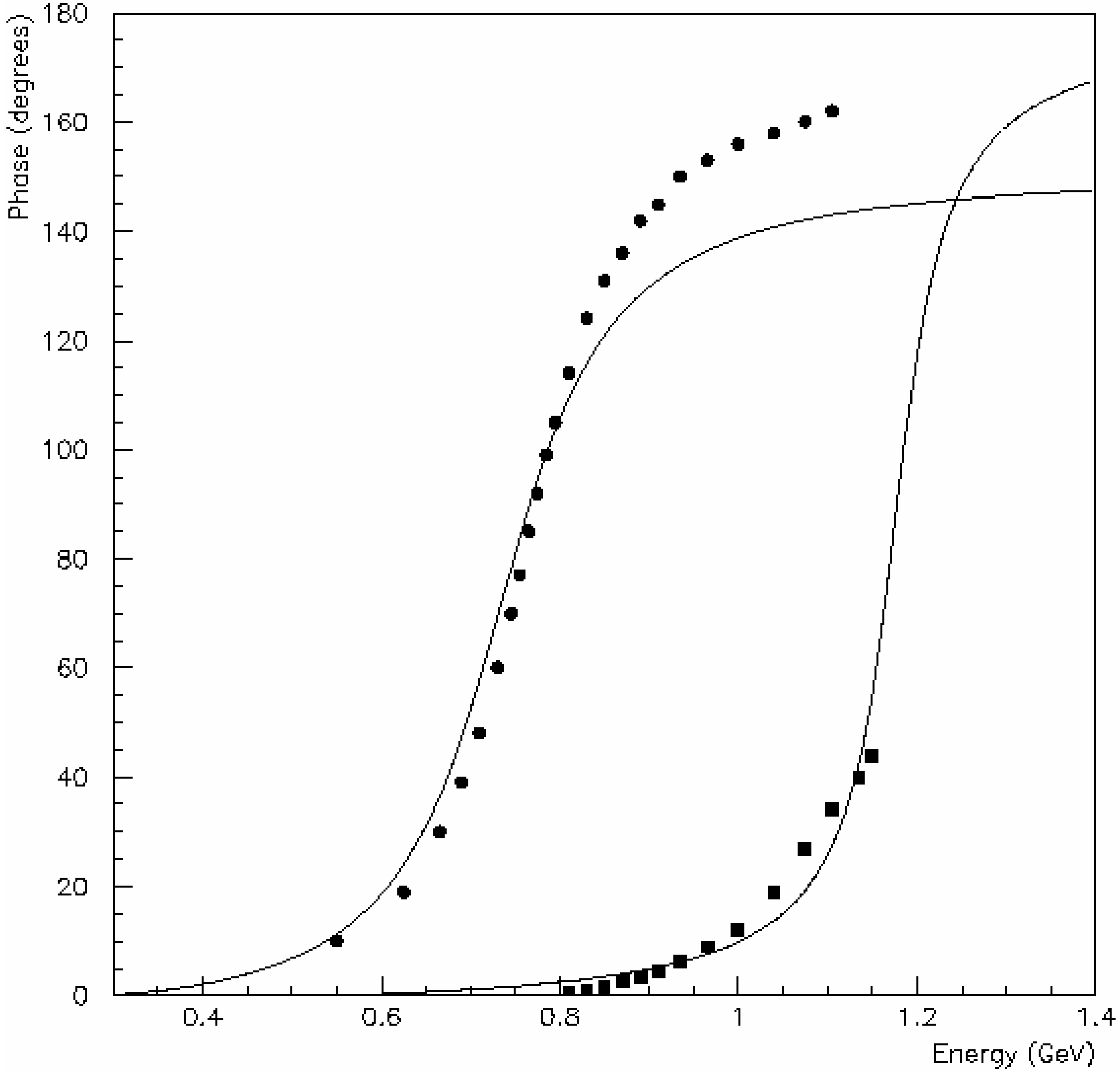
same structure !!

P-wave phase

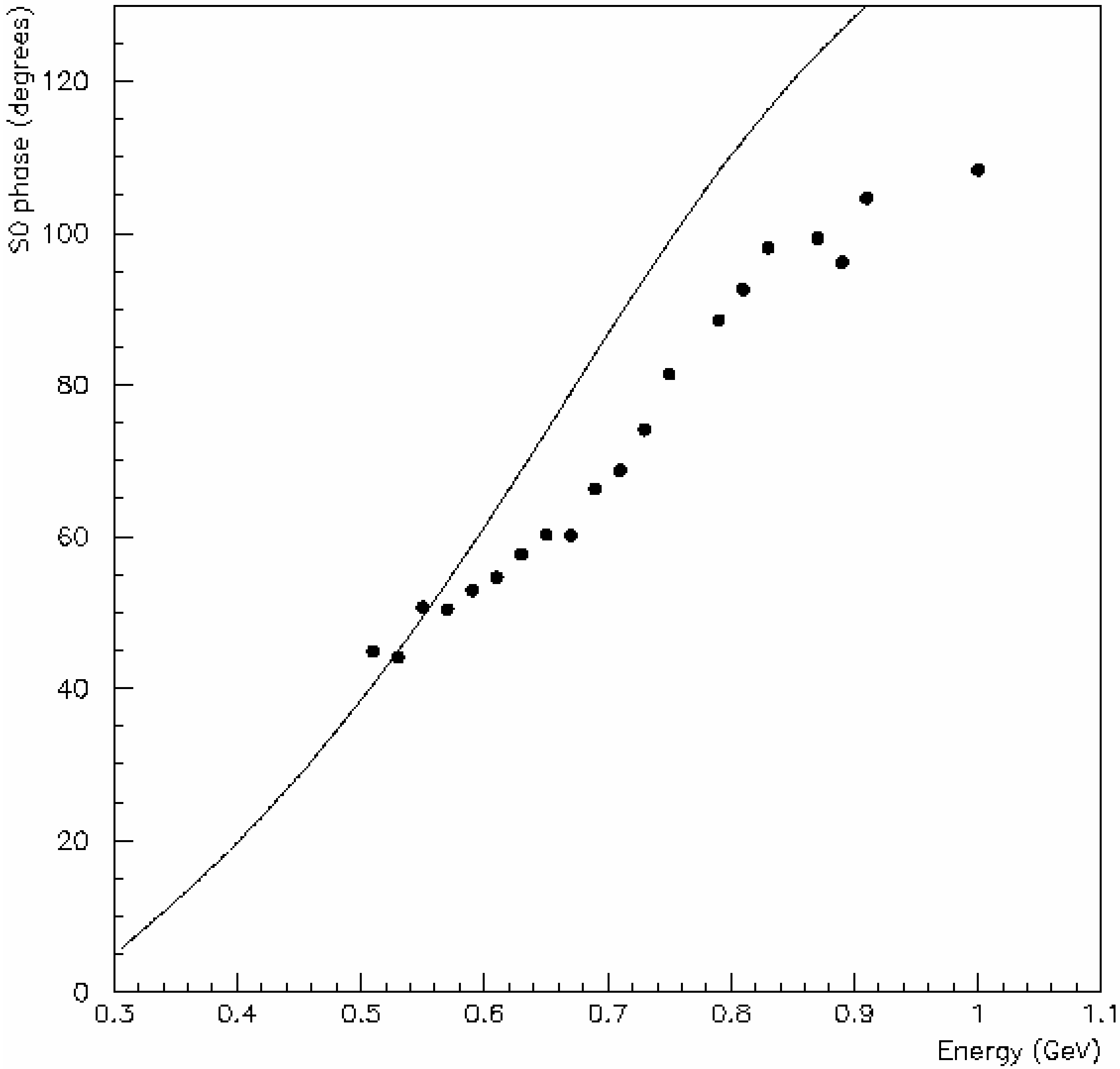
(fix λ_3)



P- and D-wave phases (λ_1 λ_2 λ_3)



S-wave phase



S0 phase (degrees)
120
100
80
60
40
20
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
Energy (GeV)

Analytical Expressions

O(p^4)

$$A_\ell^n(s) = \frac{1}{2}\int_{-1}^{+1} d(\cos\theta)\ t^n \mathbf{G}_1(t) P_\ell(\cos\theta)$$

O(p^6)

$$B_\ell^n(s) = \frac{1}{2}\int_{-1}^{+1} d(\cos\theta)\ t^n \mathbf{G}_2(t) P_\ell(\cos\theta)$$

$$C_\ell^n(s) = \frac{1}{2}\int_{-1}^{+1} d(\cos\theta)\ t^n \mathbf{Y}_1(t) P_\ell(\cos\theta)$$

$$D_\ell^n(s) = \frac{1}{2}\int_{-1}^{+1} d(\cos\theta)\ t^n \mathbf{Z}_1(t) P_\ell(\cos\theta)$$

trivial!!

Solving Dispersion Relation Integrals

$$G_2(s) = -\frac{\rho^2}{32\pi^3}\left(\mathrm{di}\log\left(\frac{2\rho}{\rho+1}\right) + \mathrm{di}\log\left(\frac{2\rho}{\rho-1}\right) + \frac{\pi^2}{6}\right)$$

$$G_3(s) = -\frac{1}{(32\pi^2)^2}\frac{\rho^3}{\pi}\left(\mathrm{poly}\log\left(3,\frac{1-\rho}{1+\rho}\right) + \mathrm{poly}\log\left(3,\frac{1+\rho}{1-\rho}\right)\right) + \ldots$$

$$\mathrm{poly}\log(a, z) = \sum_{n=1}^{\infty}\frac{z^n}{n^a}$$

combine polylog

Higher Orders

$$\mathrm{Im}\, h^{(3)}(x) = \frac{\rho}{32\pi}\left(t^{CA} h_r^{(2)} + t_r^{(1)} h_r^{(1)} + t_r^{(2)} + \rho^2 t^{3\ CA}\right)$$

$$G_3(s) = \frac{s-4m^2}{32\pi^2}\int \frac{\rho(x)\ G_{r2}(x)}{\left(x-4m^2\right)(x-s)}\, dx$$

right hand side discontinuities

$$G_2(s) = \frac{\rho^2}{\left(32\pi^2\right)^2}\left(\frac{L^2}{2} - \frac{\pi^2}{2} + i\pi L\right)$$

$$G_3(s) = \frac{\rho^3}{\left(32\pi^2\right)^3}\left(\frac{L^3}{6} - \frac{5}{6} L + i\pi \frac{L^2}{2} - i\frac{\pi^2}{2} - \frac{4\rho^2}{3}\right)$$

Conclusions and Perspectives

Program

- .$O(p^6)$ dispersion relation integrals
- . Scheme for higher orders
- . Extending to different masses

Motivation

Pionic Atoms Decay - Dirac

Scalar resonances

CP violation kaonic system

A C U rigorous relations!!

Chiral Ward Identities

The effective Lagrangians method was used in calculations of pion-pion scattering amplitude.

Unfortunately, such calculations are tedious, especially when two or more loops are involved ...

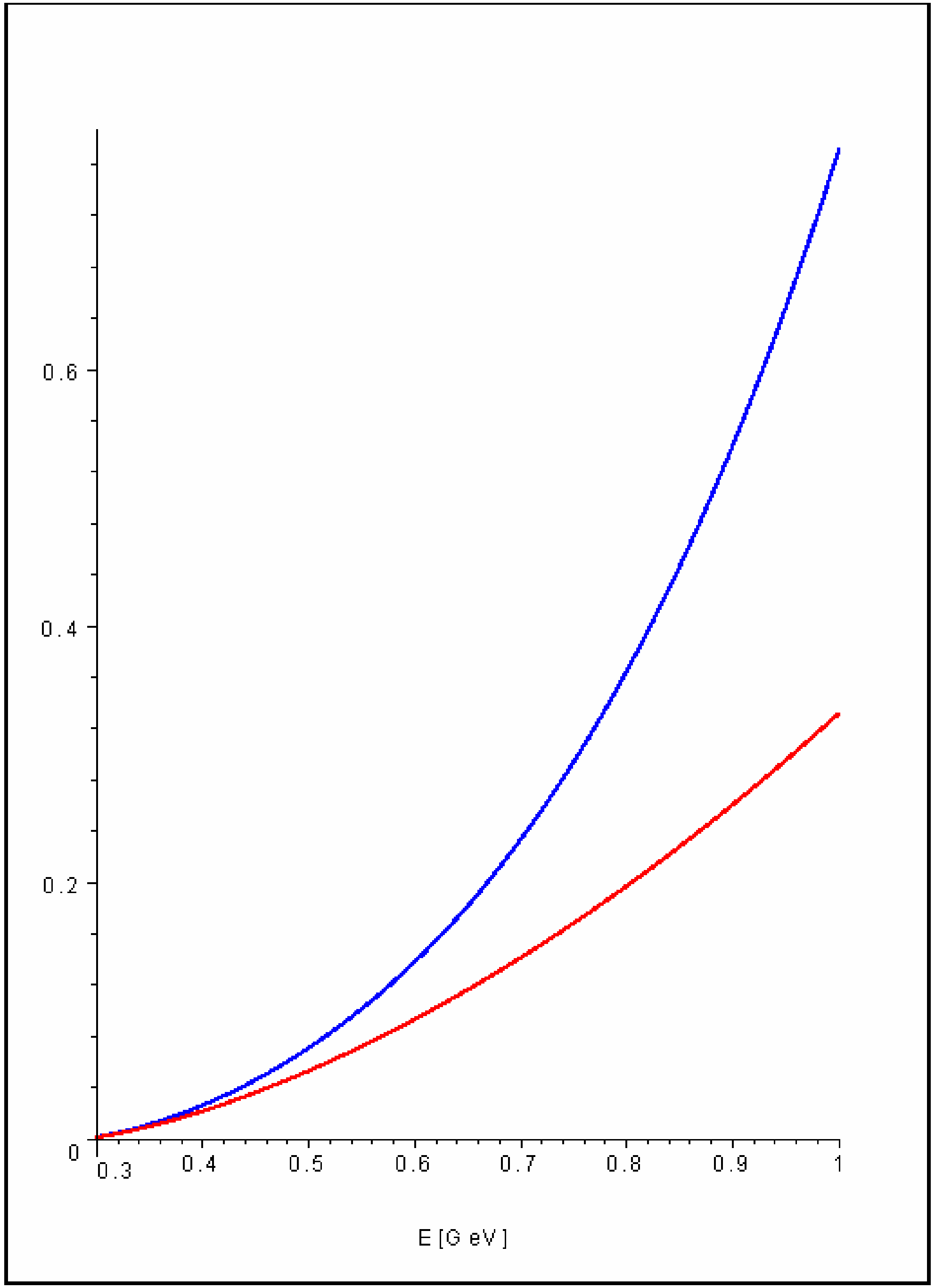
For this reason a method was re-introduced, based on the use of the chiral Ward Identities:

We simply calculate the amplitude by the technique of dispersion relations ...

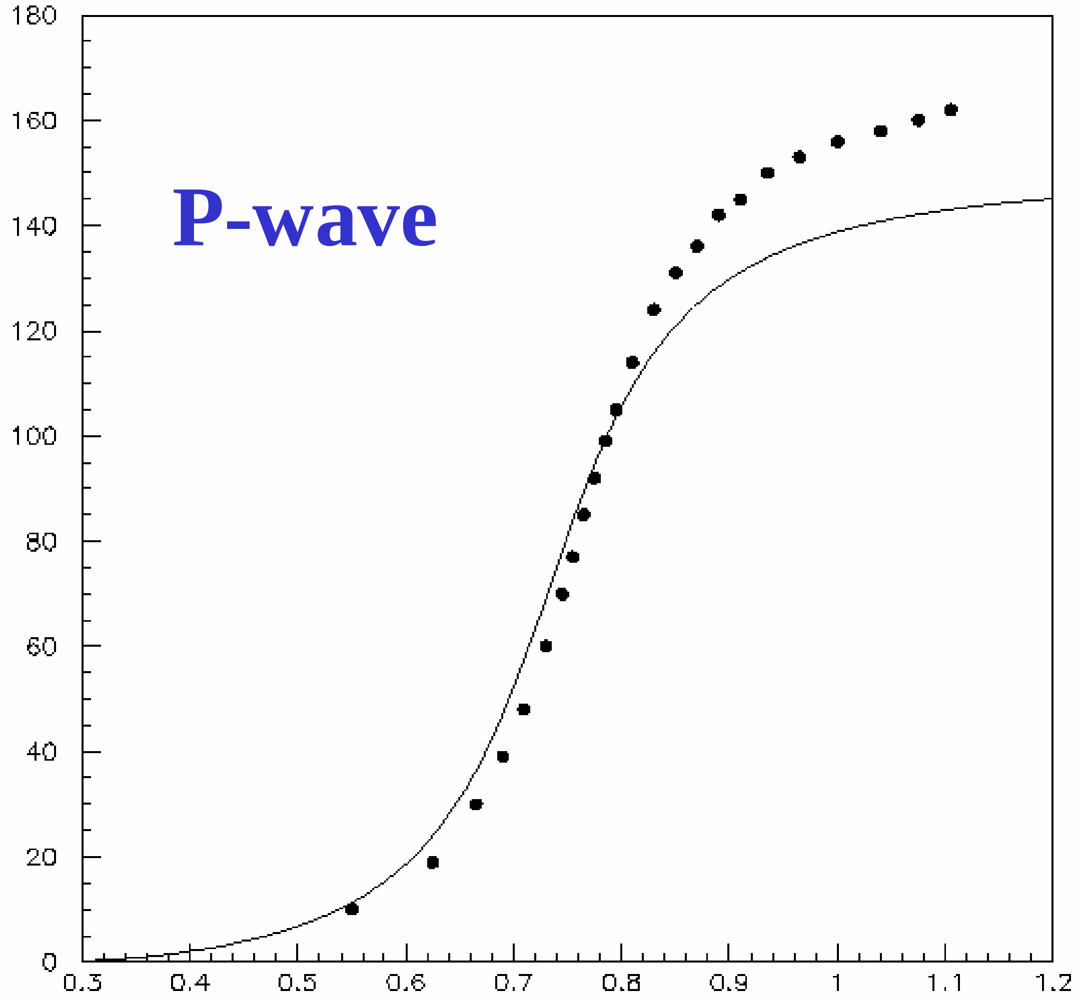
Thank you

Draft

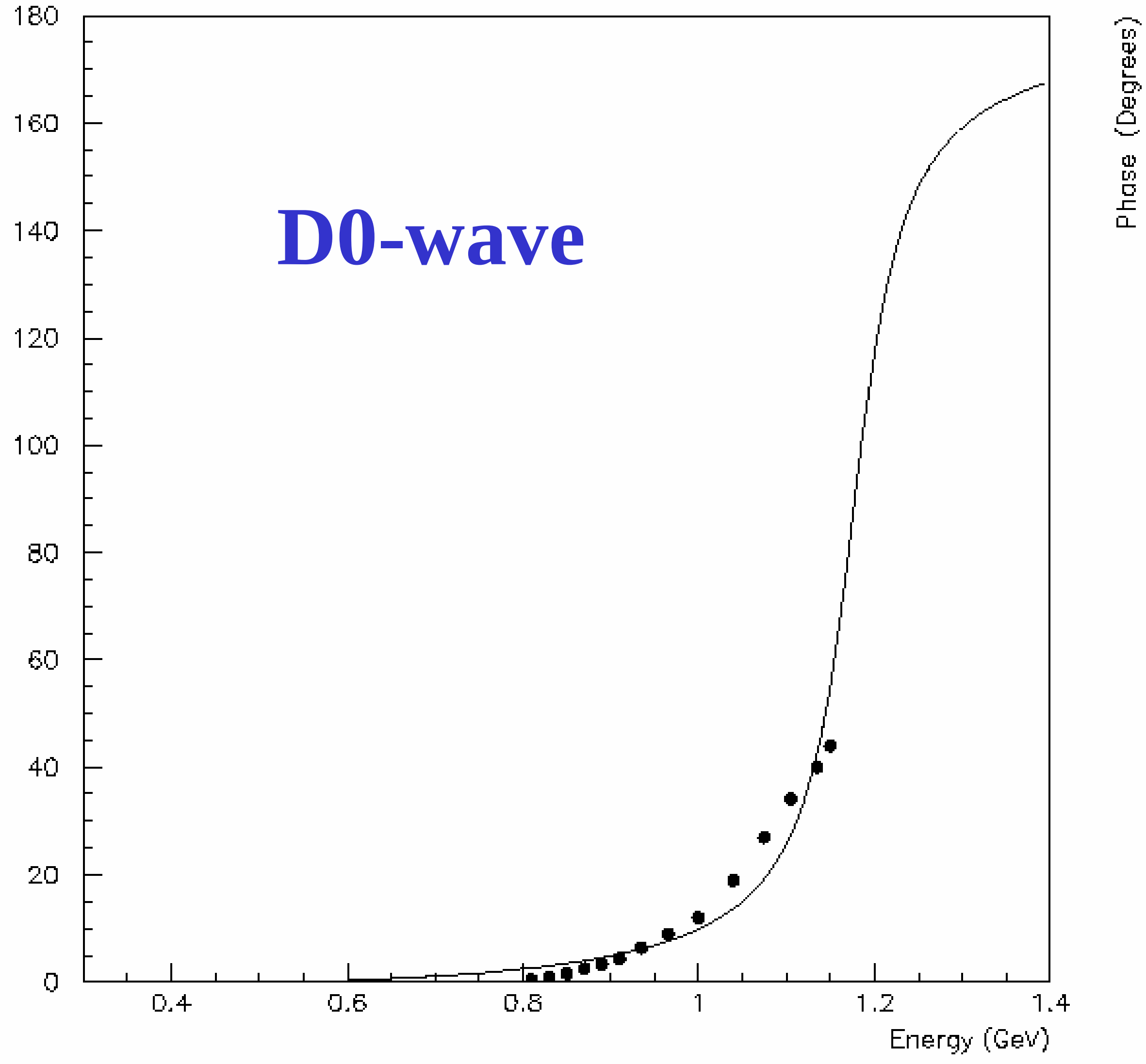
Unitarity corrections to e.m. form factor



P-wave phase



D0-wave phase



S0-wave phase