



SMR.1751 - 33

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PERSPECTIVES IN HADRONIC PHYSICS
Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies

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**Heavy Ion Collisions at Relativistic Energies:
Testing a Nuclear Matter at High Baryon and Isospin Density**

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These are preliminary lecture notes, intended only for distribution to participants

Heavy Ion Collisions at Relativistic Energies: Testing a Nuclear Matter at High Baryon and Isospin Density

Differential Flows

Pion/Kaon Production

Deconfinement Precursors?

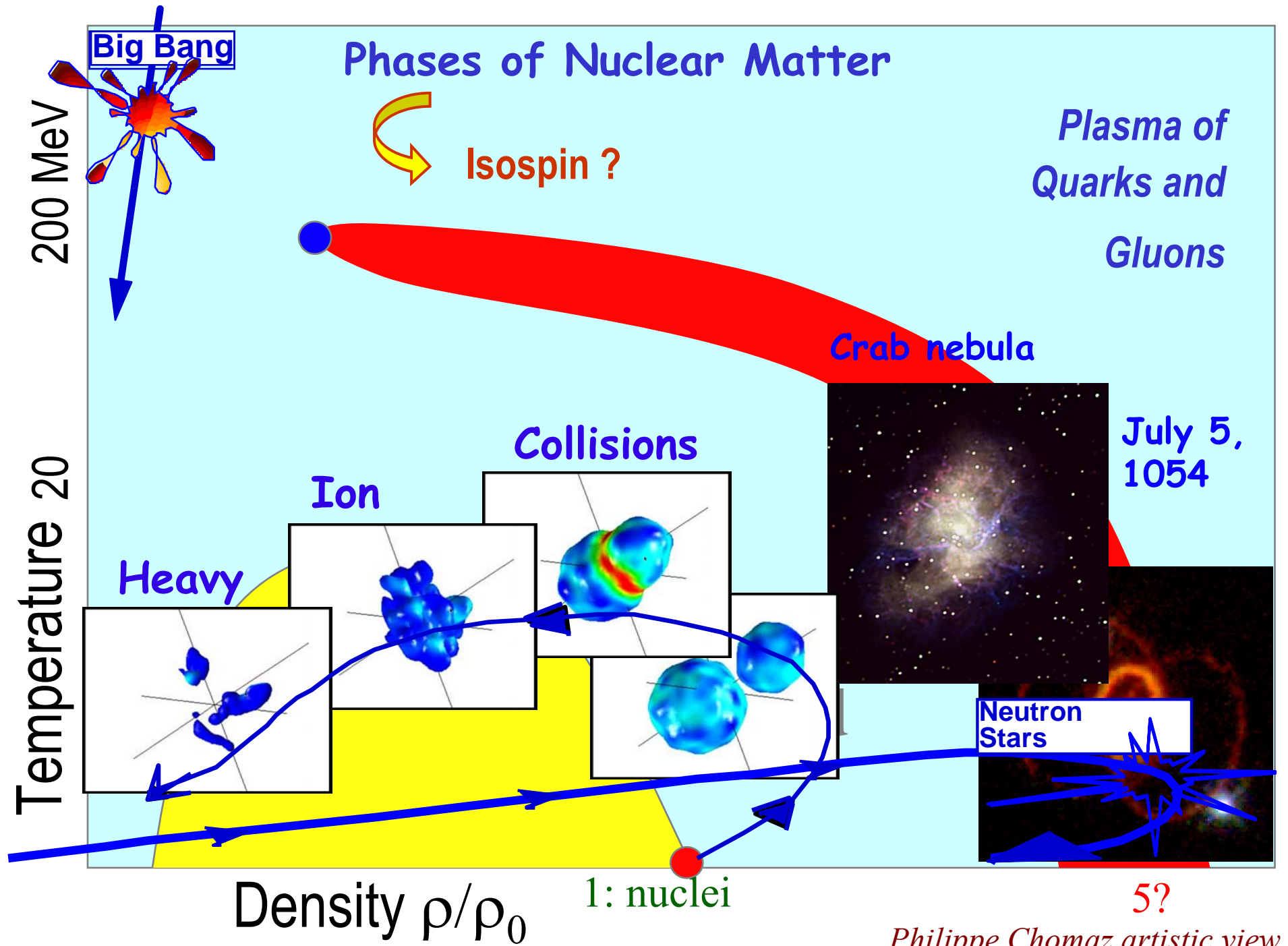
- HIC → probing:
- different densities
 - high momenta
 - covariant structure:
genuine rel. effects
 - limits of the hadronic
picture

Mean Fields

Effective Masses

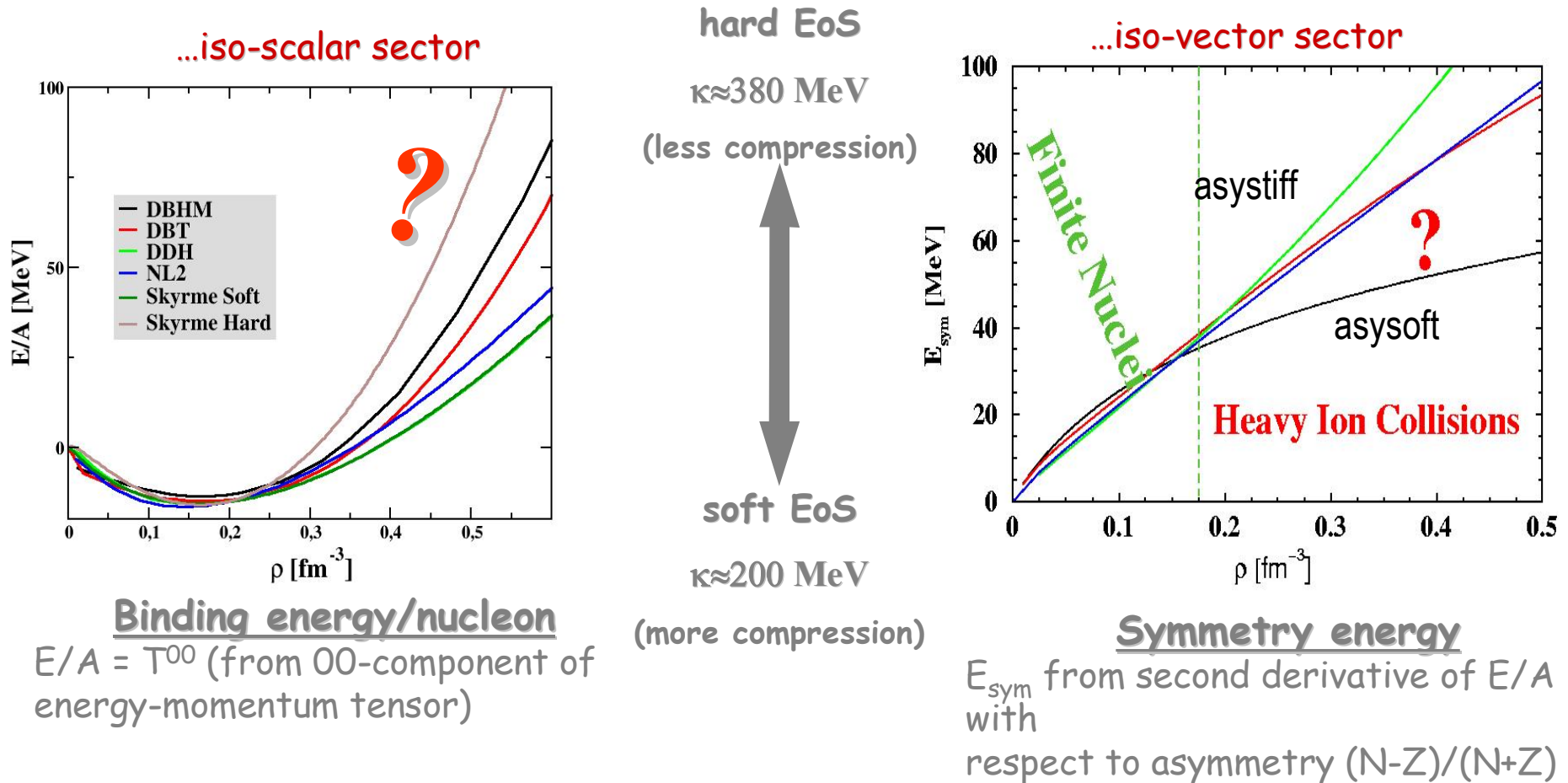
In-medium cross sections

Self-energies → Relativistic Transport Dynamics



Philippe Chomaz artistic view

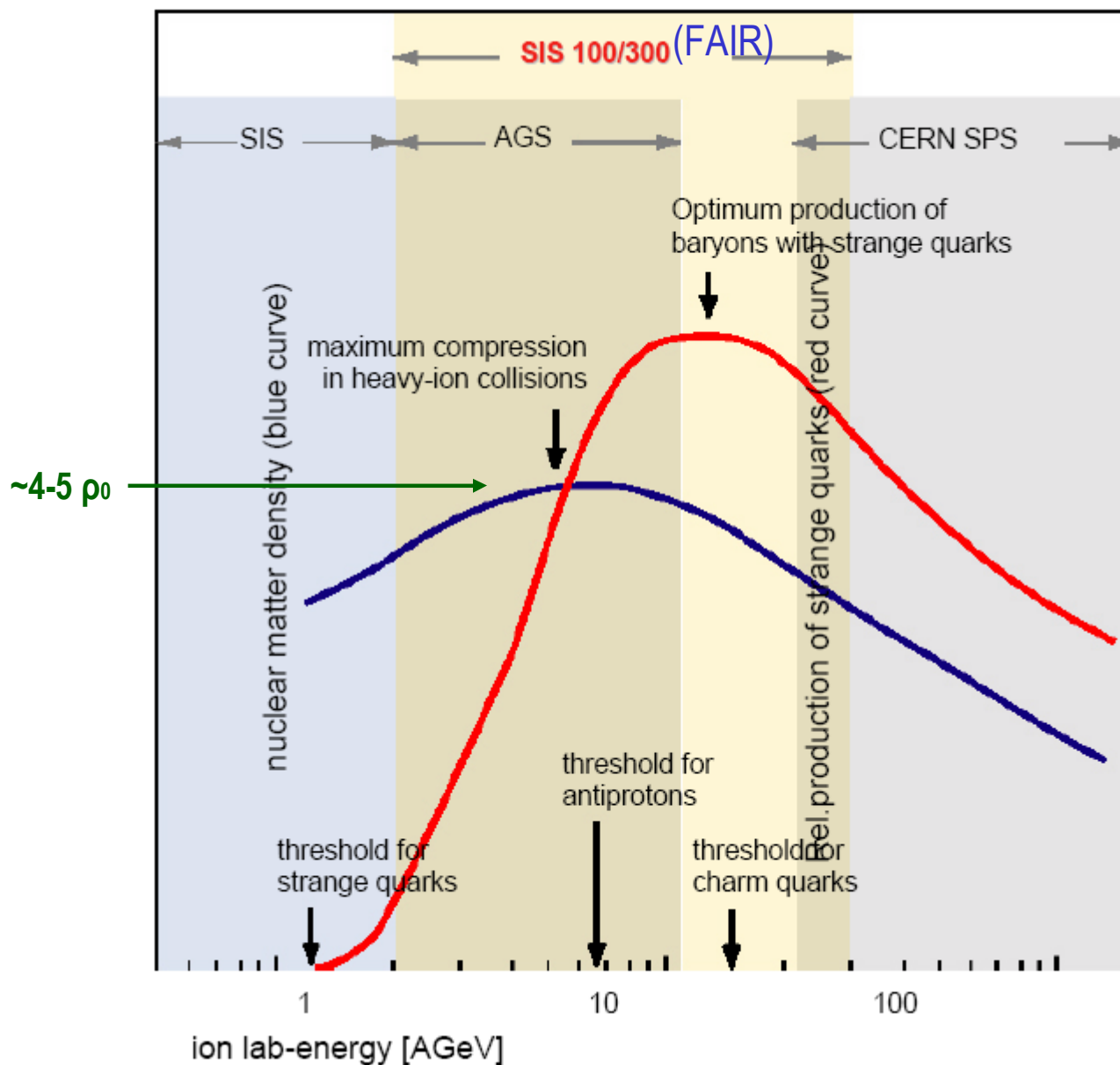
The Nuclear EOS Uncertainties



Nuclear matter at supra-normal densities not fixed (crucial differences between models)

High density symmetry energy → neutron star: - structure (mass/radius, hybrid)
 - cooling (proton fraction → direct URCA)

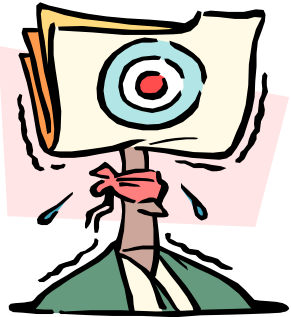
Physics at High Baryon (and Isospin?) Density



Quantum Hadrodynamics (QHD)

NN scattering $\xrightarrow{\text{OBE}}$ nuclear interaction from meson exchange:
main channels (plus correlations)

$\sigma(0^+,0)$	$\omega(1^-,0)$	$\delta(0^+,1)$	$\rho(1^-,1)$
$\underbrace{\hspace{10em}}_{\text{Scalar}} \quad \underbrace{\hspace{10em}}_{\text{Vector}}$		$\underbrace{\hspace{10em}}_{\text{Scalar}} \quad \underbrace{\hspace{10em}}_{\text{Vector}}$	
Isoscalar		Isovector	



Nuclear interaction by Effective Field Theory
as a covariant Density Functional Approach

⊕ Attraction & Repulsion \longrightarrow Saturation

$$L = \bar{\Psi} \left[\gamma_\mu (i\partial^\mu - g_\omega \hat{V}^\mu) - (M - g_\sigma \hat{\Phi}) \right] + \frac{1}{2} (\partial^\mu \hat{\Phi} \partial_\mu \hat{\Phi} - m_\sigma^2 \hat{\Phi}^2) - \frac{1}{4} \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} + \frac{1}{2} m_\omega^2 \hat{V}_\mu \hat{V}^\mu$$

$$\sigma: (\partial_\mu \partial^\mu + m_\sigma^2) \hat{\Phi} = g_\sigma \bar{\Psi} \Psi = g_\sigma \hat{\rho}_S$$

$$\omega: \partial_\mu \hat{W}^{\mu\nu} + m_\omega^2 \hat{V}^\nu = g_\omega \bar{\Psi} \gamma^\nu \Psi = g_\omega J^\nu$$

No linear

No perturbative

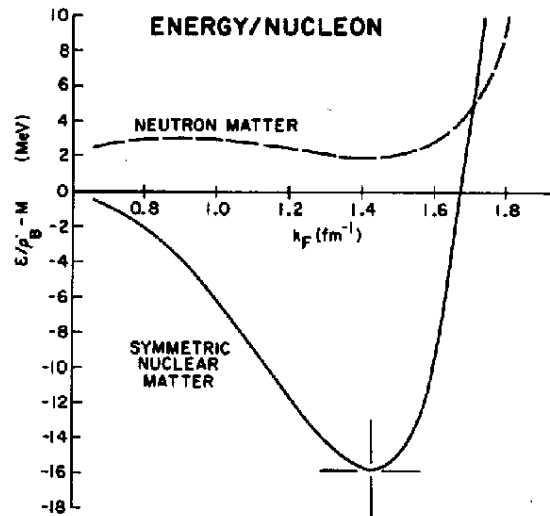
QHD Equation of State

$$\varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) n_i(k) + \frac{1}{2} \left(\frac{g_\sigma^2}{m_\sigma^2} \right) \rho_s^2 + \frac{1}{2} \left(\frac{g_\omega^2}{m_\omega^2} \right) \rho_B^2 \quad \text{QHD-I}$$

\mathbf{f}_σ \mathbf{f}_ω

$$\varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} E_i^*(k) n_i(k) + \frac{1}{2} m_\sigma^2 \Phi^2 + U(\Phi) + \frac{1}{2} \left(\frac{g_\omega^2}{m_\omega^2} \right) \rho_B^2 + \frac{1}{2} \left(\frac{g_\rho^2}{m_\rho^2} \right) \rho_{B3}^2 + \frac{1}{2} \left(\frac{g_\delta^2}{m_\delta^2} \right) \rho_{S3}^2$$

\mathbf{f}_σ \mathbf{f}_ω \mathbf{f}_ρ \mathbf{f}_δ



QHD-I

- ☐ Self-interacting terms (NL models)
- ☐ Charged effective mesons (QHD-II)

- ✓ soft EoS ($K=220 \text{ MeV}$)
- ✓ $E_{\text{bind}} = -16 \text{ MeV}$
- ✓ $\rho_0 (\text{fm}^{-3}) \approx 0.15$
- ✓ $m^*/M = 0.75$
- ✓ $a_4 = 30.7 \text{ MeV}$

Isospin degrees of freedom in QHD

➤ $\sigma - \omega$ model ➔ Only kinetic contribution to E_{sym}

➤ Charged “mesons” :

$\vec{\delta}[a_0(980)]$	(scalar isovector)
$\vec{\rho}(770)$	(vector isovector)

$$N: [\gamma_\mu i\partial^\mu - g_V \gamma_0 V^0 - g_\rho \gamma_0 \tau_3 b^0 - (M - g_S \Phi - g_\delta \tau_3 \delta_3)] \Psi = 0$$

↙ Splitting n & p M^*

$$\vec{\rho}: b_0 = \frac{g_\rho}{m_\rho^2} (\rho_p - \rho_n)$$

$$\vec{\delta}: \delta_0 = \frac{g_\delta}{m_\delta^2} (\rho_{sp} - \rho_{sn})$$

**Relativistic structure also
in isospin space !**

$$E_{sym} = \text{cin.} + (\rho\text{-vector}) - (\delta\text{-scalar})$$

Nuclear Matter

$$f_{\rho,\delta} \equiv \left(\frac{g_{\rho,\delta}}{m_{\rho,\delta}} \right)^2$$

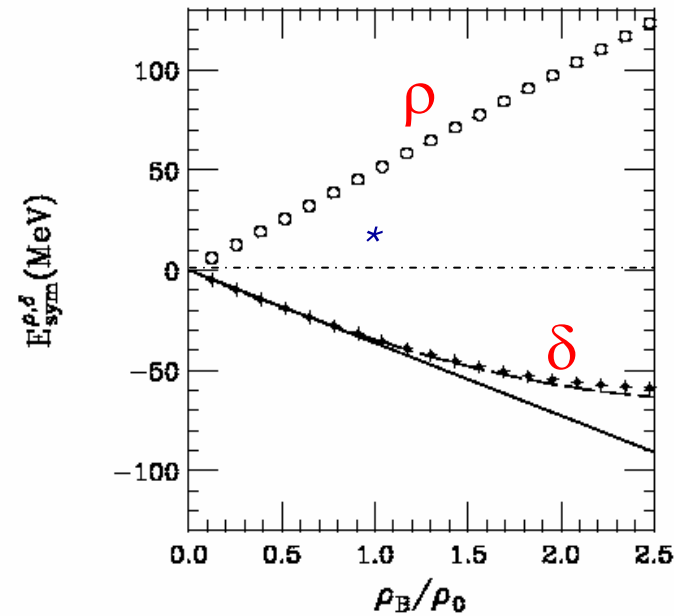
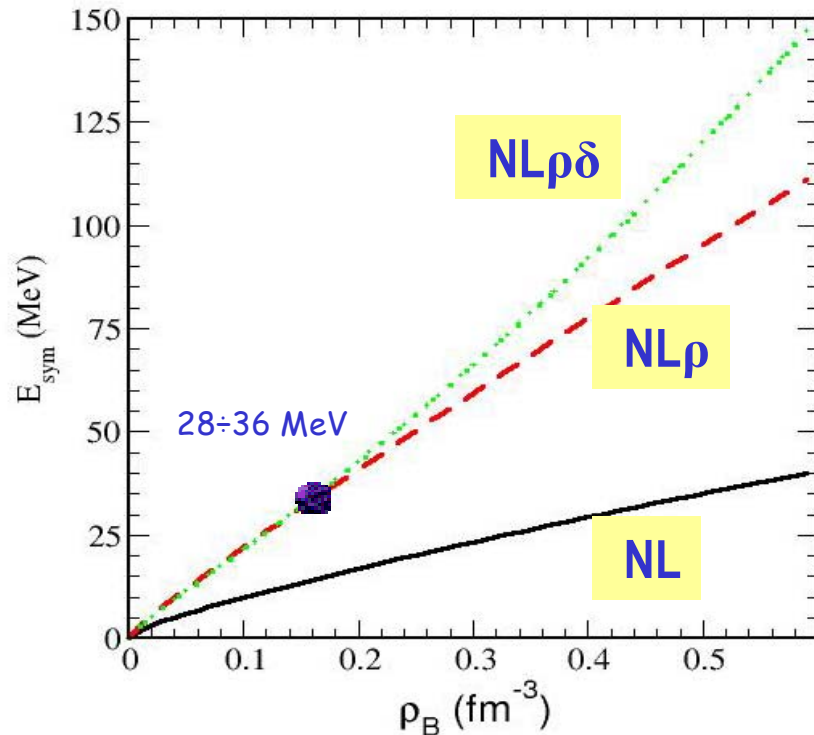
RMF Symmetry Energy: δ – contrib.

$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^{*2}} + \frac{1}{2} \left[f_\rho - f_\delta \left(\frac{M^*}{E^*} \right)^2 \right] \rho_B$$

\rightarrow No δ $\rightarrow f_\rho \cong 1.5 f_\rho^{FREE}$
 $\rightarrow f_\delta = 2.5 \text{ fm}^2$ $\rightarrow f_\rho \cong 5 f_\rho^{FREE}$

$a_4 = E_{sym}(\rho_0)$ fixes (f_ρ, f_δ)

DBHF } $f_\delta \approx 2.0 \div 2.5 \text{ fm}^2$
 DHF }



Self-Energies: kinetic momenta and (Dirac) effective masses

$$k_i^{*\mu} \equiv k_i^\mu - \Sigma_i^\mu$$

$$m_i^* \equiv M - \Sigma_{s,i}$$

$$\Sigma_s(n, p) = f_\sigma \sigma(\rho_s) \mp f_\delta \rho_{s3}$$

$$\Sigma^\mu(n, p) = f_\omega j^\mu \mp f_\rho j_3^\mu$$

Upper sign: n

Dirac dispersion relation: single particle energies

$$(\rho, j)_3 \equiv (\rho, j)_p - (\rho, j)_n$$

$$\rho_{B3} \equiv \rho_{Bp} - \rho_{Bn} < 0, n\text{-rich}$$

$$\varepsilon_i + M = +\Sigma_i^0 + \sqrt{k^2 + m_i^{*2}}$$

Chemical Potentials (zero temp.)

$$\mu_i = \sqrt{k_F^2 + m_i^{*2}} + f_\omega \rho_B \mp f_\rho \rho_{B3}$$

Symmetry Energy

$$\mu_n - \mu_p = 4E_{sym} I$$

$$I \equiv \frac{\rho_n - \rho_p}{\rho}$$

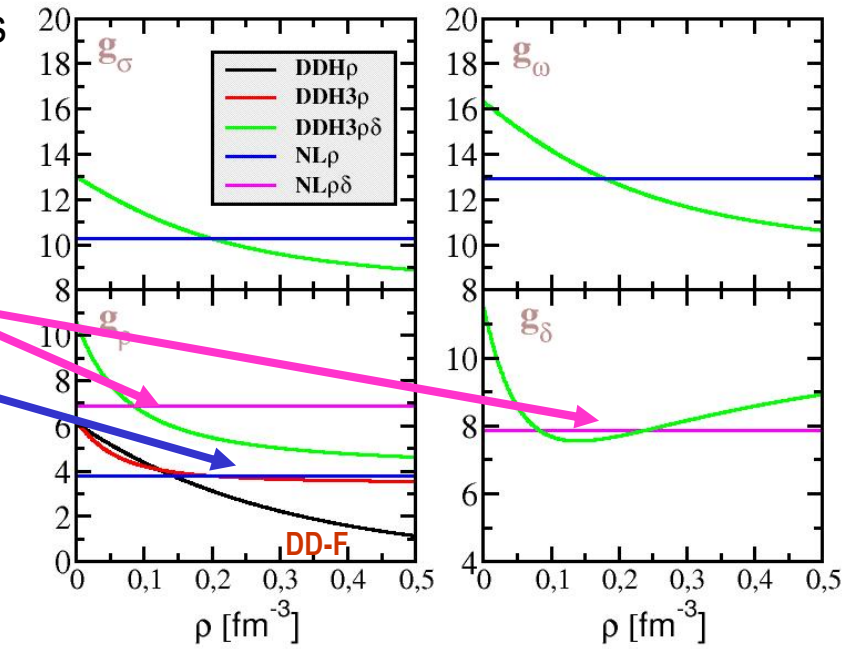
Asymmetry parameter

A $\rho\delta$ parametrization of the isovector dependence

$\sigma, \omega, \rho, \delta$ couplings

NL $\rho\delta$

NL ρ

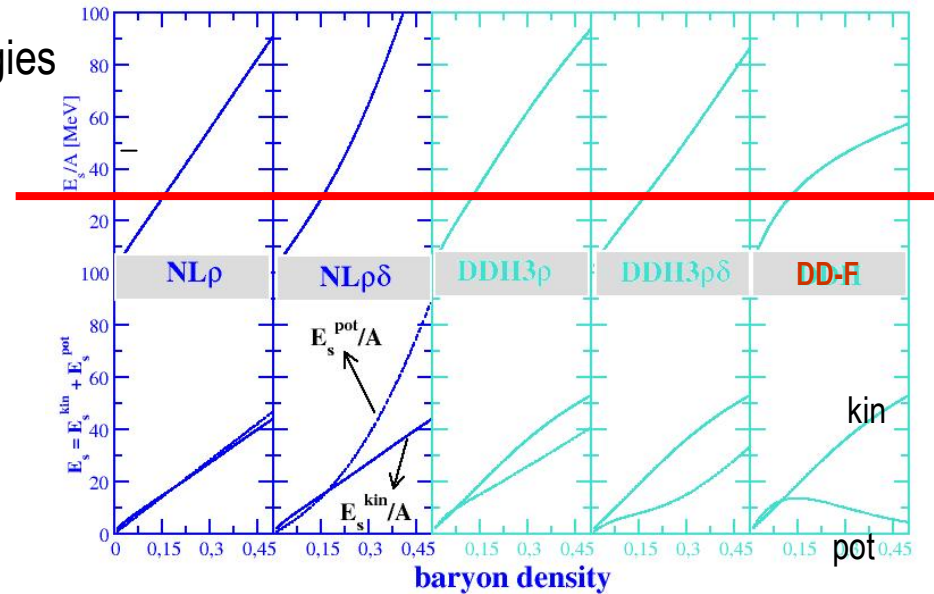


Walecka-Type (NL)

DDH Models

Symmetry energies

$$a_4 = E_{\text{sym}}(\rho_0) \approx 33 \text{ MeV}$$



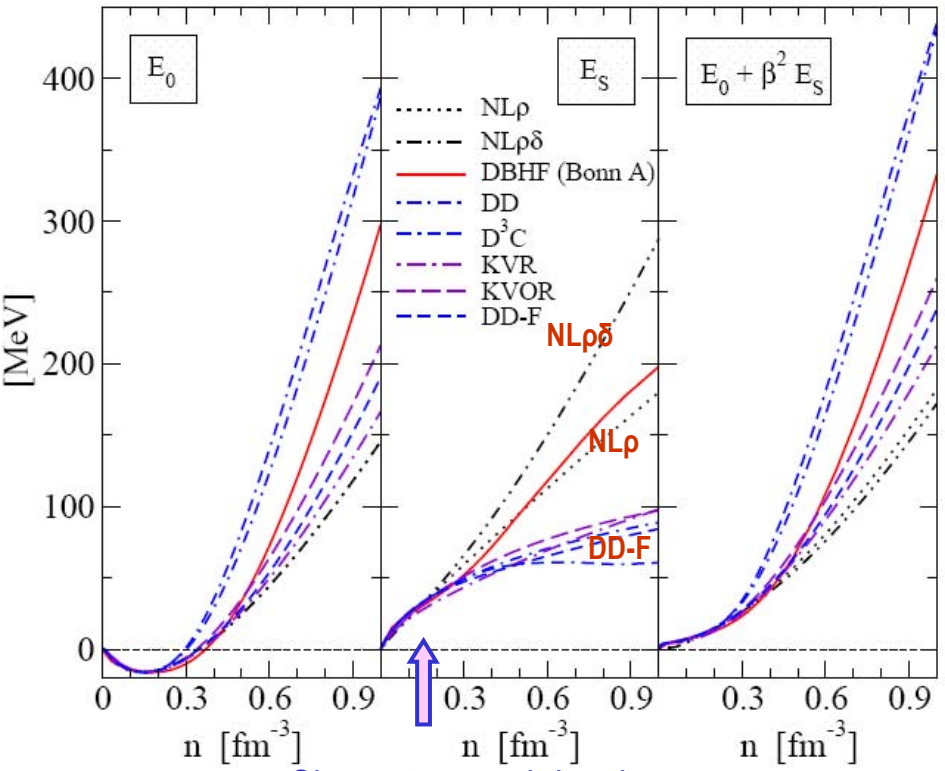
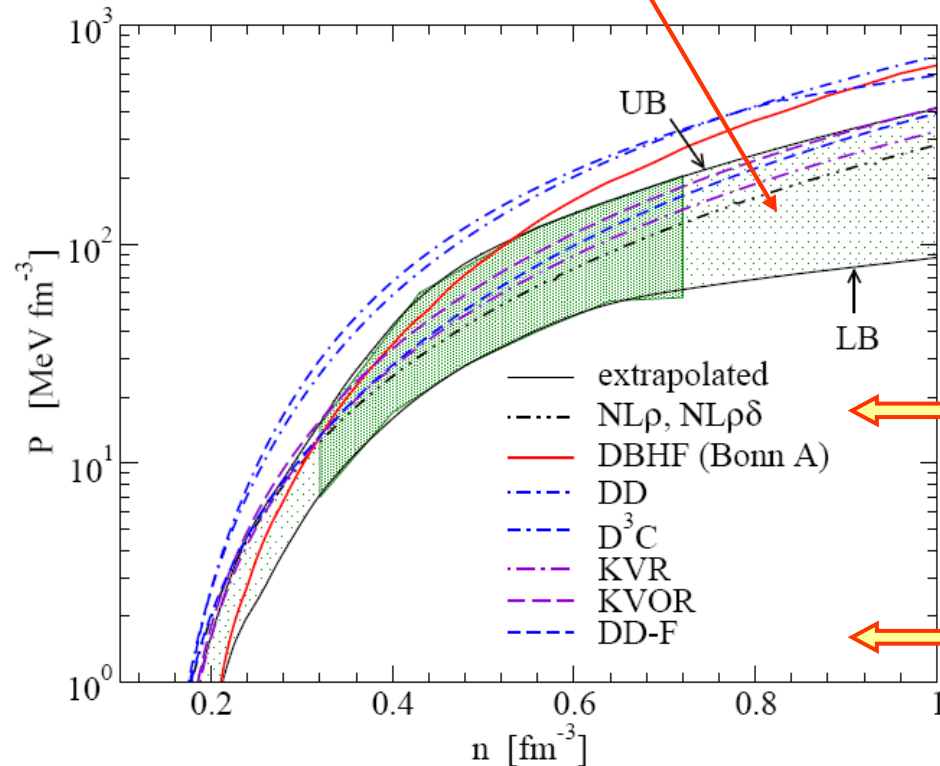
Collection of EOS "Realistic" Covariant Models

Symmetric Matter | Symmetry Energy | Neutron Matter

compact stars & heavy ion data
T.Klaen et al. nucl-th/0602038

Pressure

Exp boundaries (flow, kaon multiplicities)



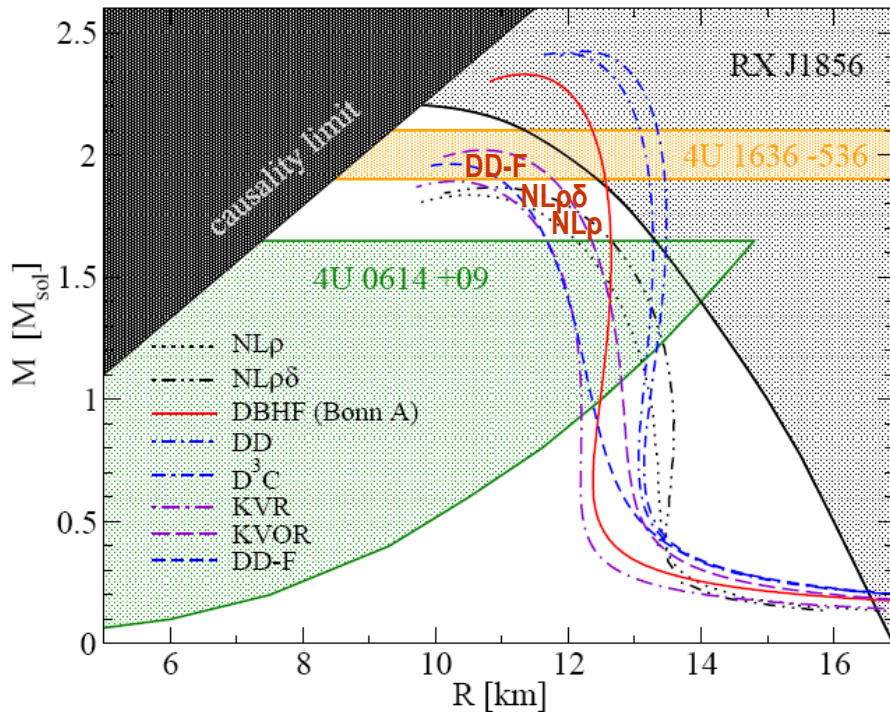
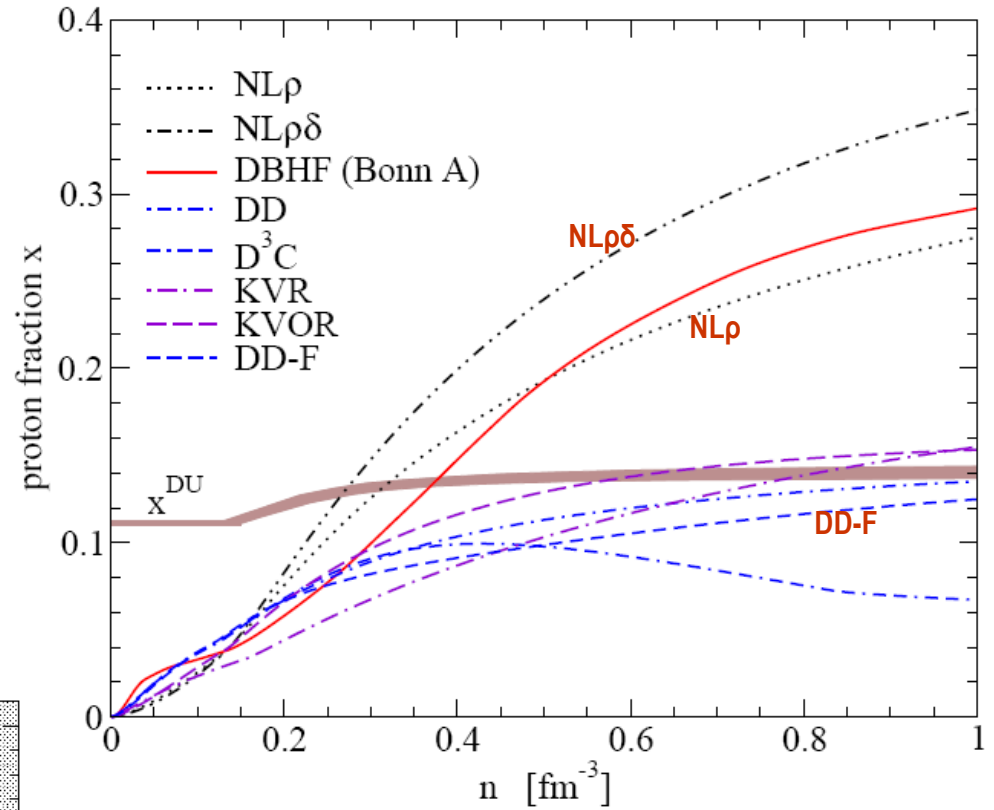
Slope at normal density:
Isospin transport at Fermi energies

← Soft Symm Matter, Stiff (super-stiff) Symmetry Energy

← Soft Symm Matter, Soft Symmetry Energy

Neutron Star ($npe\mu$) properties

Direct URCA threshold



Mass/Radius relation

compact stars & heavy ion data
T.Klaen et al. nucl-th/0602038

RBUU transport equation

Wigner transform \cap Dirac + Fields Equation \Rightarrow Relativistic Vlasov Equation + Collision Term...

$$\left[\frac{p_i^{*\mu}}{M_i^*} \partial_\mu + \left(\frac{p_{\nu i}^*}{M_i^*} \mathcal{F}_i^{\mu\nu} + \partial^\mu M_i^* \right) \partial_\mu^{(p^*)} \right] f_i(x, p^*) = \mathcal{I}_c$$

drift

mean field

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_r f + \vec{\nabla}_r U \cdot \vec{\nabla}_p f = I_{coll}$$

Non-relativistic Boltzmann-Nordheim-Vlasov, Landau-Vlasov...

“Lorentz Force” \rightarrow Vector Fields
pure relativistic term

$$k_i^{*\mu} \equiv k_i^\mu - \Sigma_i^\mu$$

$$m_i^* \equiv M - \Sigma_{s,i}$$

$$F^{\mu\nu} = \partial^\mu \Sigma^\nu - \partial^\nu \Sigma^\mu$$

Collision term:

$$\mathcal{I}_c = \frac{g}{(2\pi)^3} \int \frac{dp_2^*}{p_2^{*0}} \frac{dp_3^*}{p_3^{*0}} \frac{dp_4^*}{p_4^{*0}} \int d\Omega (p^* + p_2^*)^2 \frac{d\sigma}{d\Omega} \delta^4(p^* + p_2^* - p_3^* - p_4^*) \\ \times \{f_3 f_4 [1 - f][1 - f_2] - f f_2 [1 - f_3][1 - f_4]\}$$

Relativistic Landau Vlasov Propagation

Discretization of $f(x, p^*) \rightarrow$ Test particles represented by covariant Gaussians in xp -space

$$f(x, p^*) = \sum_{i=1}^{AN_{test}} \int_{-\infty}^{+\infty} d\tau \, g(x - x_i(\tau)) g(p^* - p_i^*(\tau))$$

\rightarrow **Relativistic** Equations of motion for x^μ and $p^{*\mu}$ for centroids of Gaussians

$$\frac{d}{d\tau} x_i^\mu = \frac{p_i^*(\tau)}{M_i^*(x_i)},$$

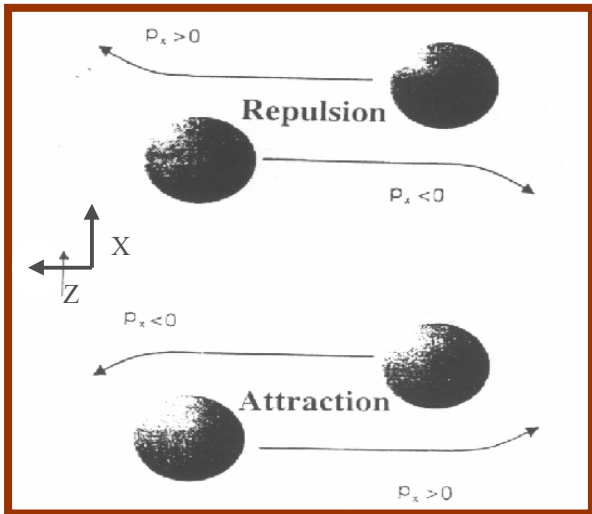
$$\frac{d}{d\tau} p_i^{*\mu} = \frac{p_{i\nu}^*(\tau)}{M_i^*(x_i)} \mathcal{F}_i^{\mu\nu}(x_i(\tau)) + \partial^\mu M_i^*(x_i)$$

u_ν Test-particle 4-velocity \rightarrow Relativity: - momentum dependence always included due to the Lorentz term $(u_\nu F^{\mu\nu})$
 - E^*/M^* boosting of the vector contributions

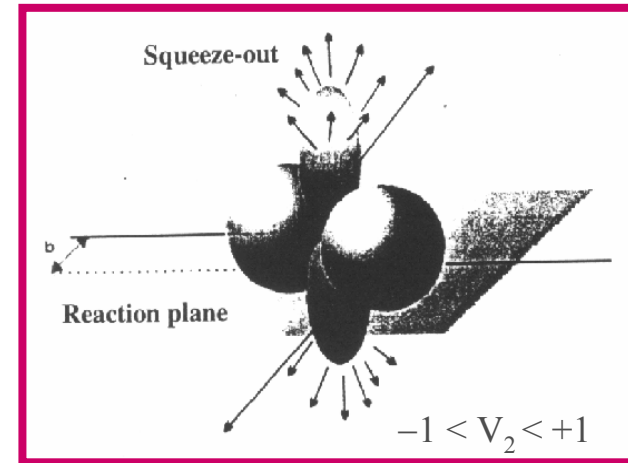
Collision Term: local Montecarlo Algorithm imposing an average Mean Free Path plus Pauli Blocking
 \rightarrow **in medium reduced Cross Sections**

Collective flows

In-plane



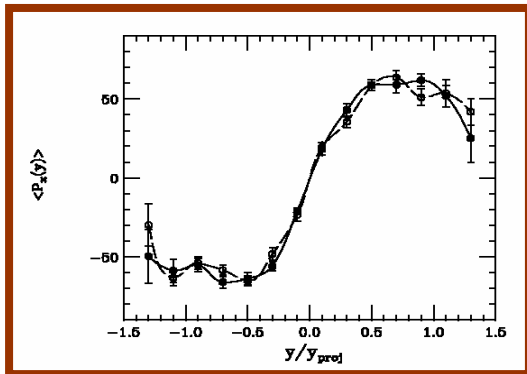
Out-of-plane



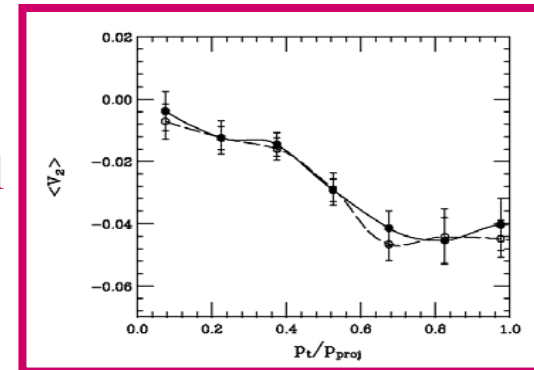
$$V_1(y, p_t) = \langle p_x \rangle / \langle p_t \rangle$$

$y = \text{rapidity}$
 $p_t = \text{transverse momentum}$

$$V_2(p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$



$V_2 \begin{cases} = -1 & \text{full out} \\ = 0 & \text{spherical} \\ = +1 & \text{full in} \end{cases}$



$$V_1^{p-n}(y, p_t) = V_1^p(p_t) - V_1^n(p_t)$$

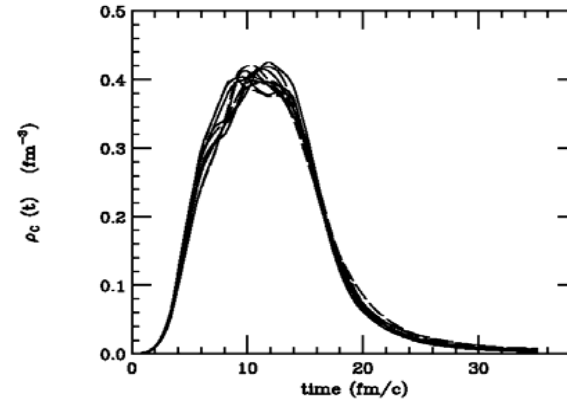
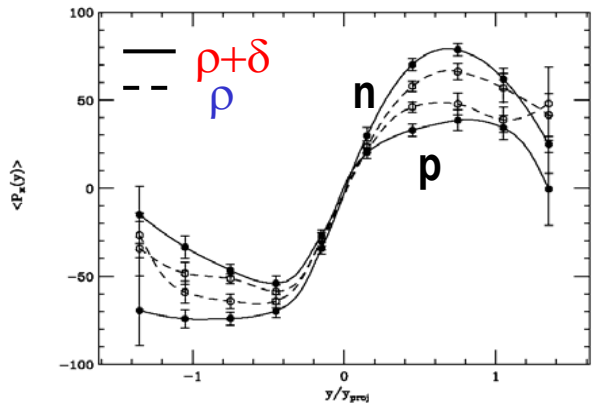
Isospin

Differential flows

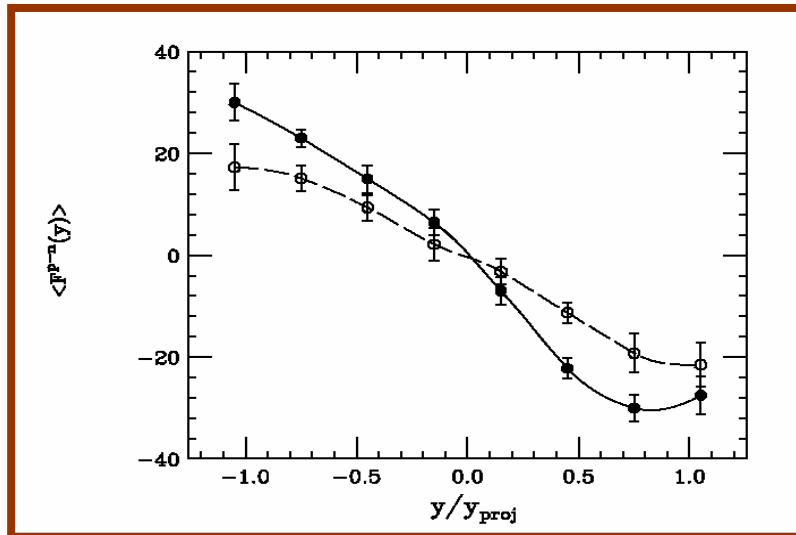
$$V_2^{p-n}(p_t) = V_2^p(p_t) - V_2^n(p_t)$$

Differential Transverse Flow

$^{132}\text{Sn} + ^{132}\text{Sn} @ 1.5 \text{ AGeV } b=6\text{fm}$



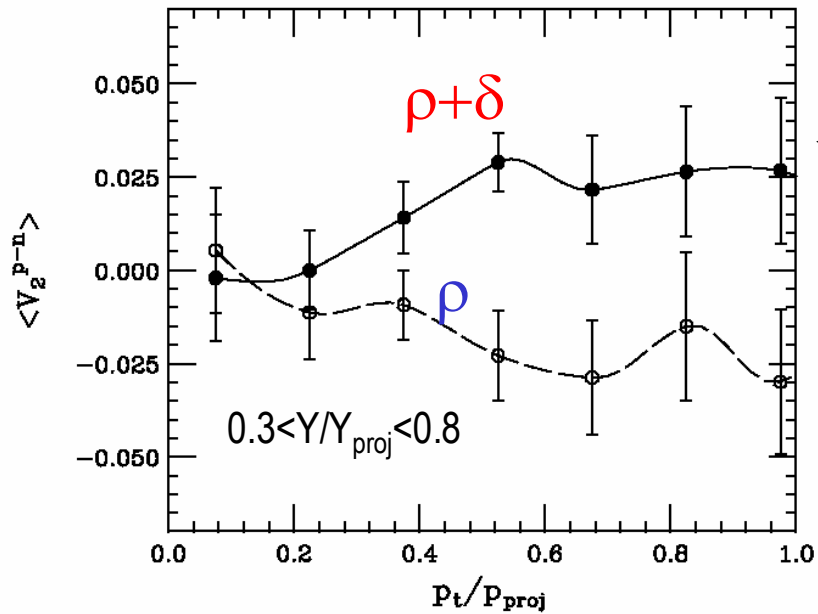
- Sensitivity to the isovector part of the mean field
 → E_{sym} around $\rho \approx 3-4\rho_0$



○ NLH- ρ
 ● NLH-($\rho+\delta$)
 Greater E_{sym}
 ↓
 stiffer $\langle F^{\text{p-n}} \rangle$

Elliptic flow

132Sn+132Sn, 1.5A GeV, b=6fm: Test with NL- ρ & NL- $(\rho+\delta)$



✳ Difference at high p_t \leftrightarrow first stage

← High p_t neutrons are emitted “earlier”

Equilibrium (ρ, δ) dynamically broken
Importance relativistic structure

approximations

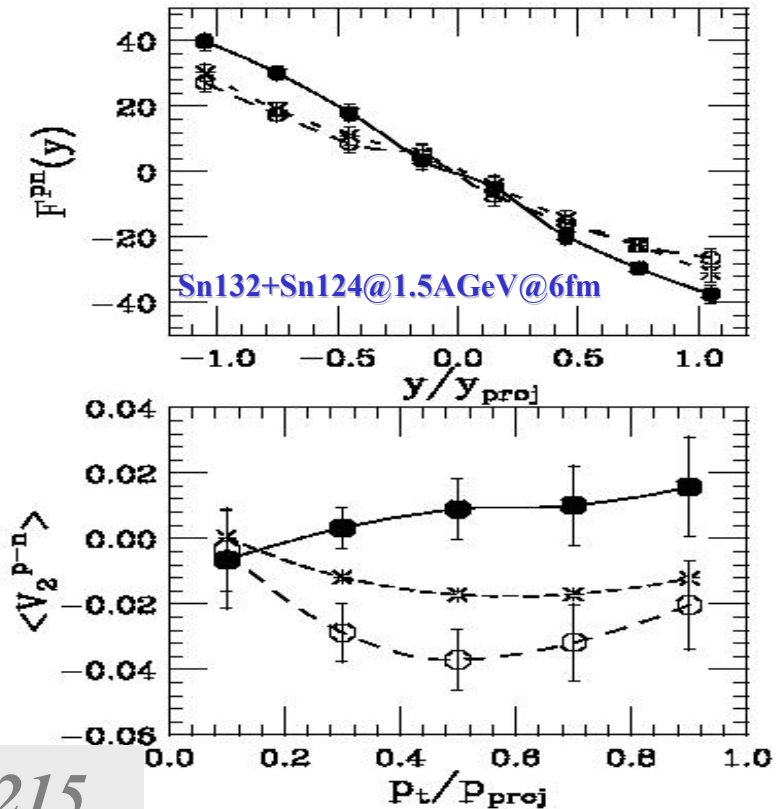
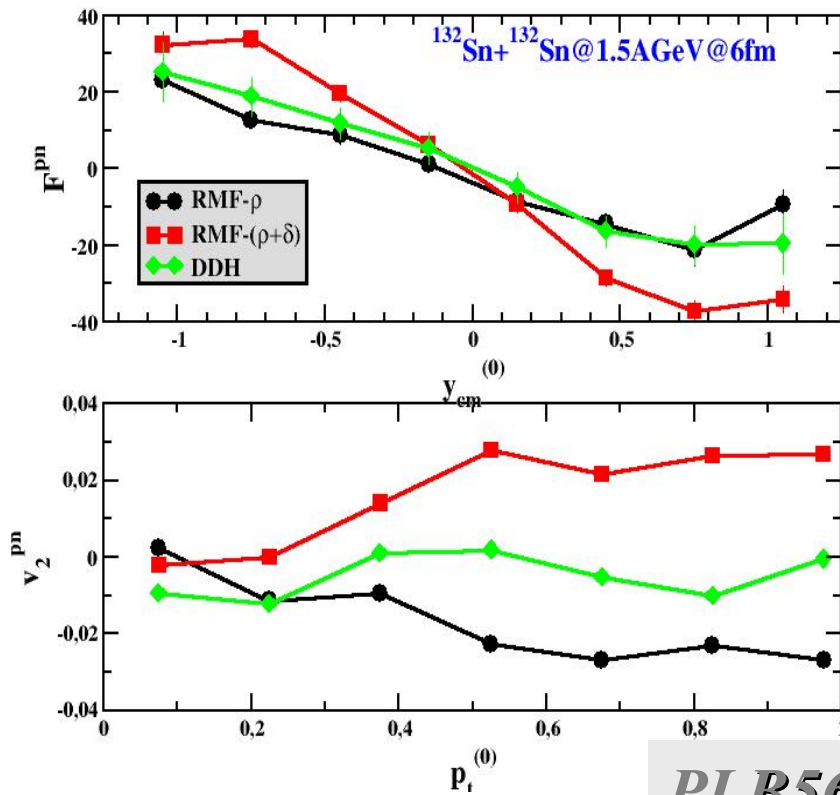
$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \vec{\nabla} \rho_3 = \frac{4}{\rho_B} E_{sym}^* \vec{\nabla} \rho_3$$

$$2 \left[f_\rho - f_\delta \frac{M^*}{E_F^*} \right] = \frac{4}{\rho_B} E_{sym}^{pot}$$

Dynamical boosting of the vector contribution

PLB562(2003)

Not just a symmetry energy effect: DDH \rightarrow density dep. f_{ρ} to reproduce the same symmetry term of $NL\rho\delta$



PLB562(2003)215

Strong isospin dependence of isospin flow

\rightarrow p_t -dependence: Chronometer of collision (high p_t 's reflect earlier high compression)

\rightarrow $NL\rho\delta$: more I-Flow due to Lorentz decomposition of iso-vector channel: $\frac{d\bar{p}_p^*}{d\tau} - \frac{d\bar{p}_n^*}{d\tau} \approx 2 \left[\gamma f_{\rho} - \frac{f_{\delta}}{\gamma} \right] \bar{v}_{\rho i}$

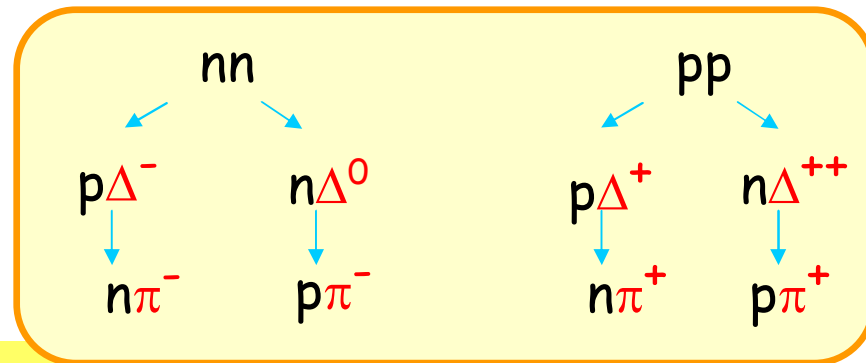
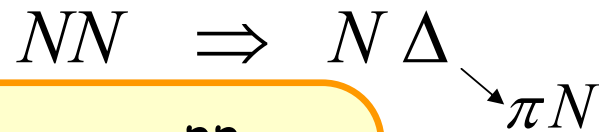
ρ -meson enhanced by γ δ -meson suppressed by scalar density

One needs neutron (light isobars) detection from experiments!

PION PRODUCTION

NPA762(2005) 147

Main mechanism



$n \rightarrow p$ "transformation"

$$\Rightarrow \frac{\pi^-}{\pi^+}$$

1. C.M. energy available: "threshold effect"

$$\varepsilon_{n,p} = E_{n,p}^* + f_\omega \rho_B \mp f_\rho \rho_{B3} \rightarrow \begin{matrix} s_{nn}(NL) < s_{nn}(NL\rho) < s_{nn}(NL\rho\delta) \\ s_{pp}(NL) > s_{pp}(NL\rho) > s_{pp}(NL\rho\delta) \end{matrix}$$

$\pi(-)$ enhanced
 $\pi(+)$ reduced



2. Fast neutron emission: "mean field effect"

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\Delta^{0,-}) \downarrow}{Y(\Delta^{+,++}) \uparrow} \Rightarrow \frac{\pi^-}{\pi^+} \downarrow \Rightarrow \text{decrease} : NL \rightarrow NL\rho \rightarrow NL\rho\delta$$

Compensation
in "open" systems:
HIC



Vector self energy more repulsive for neutrons and more attractive for protons

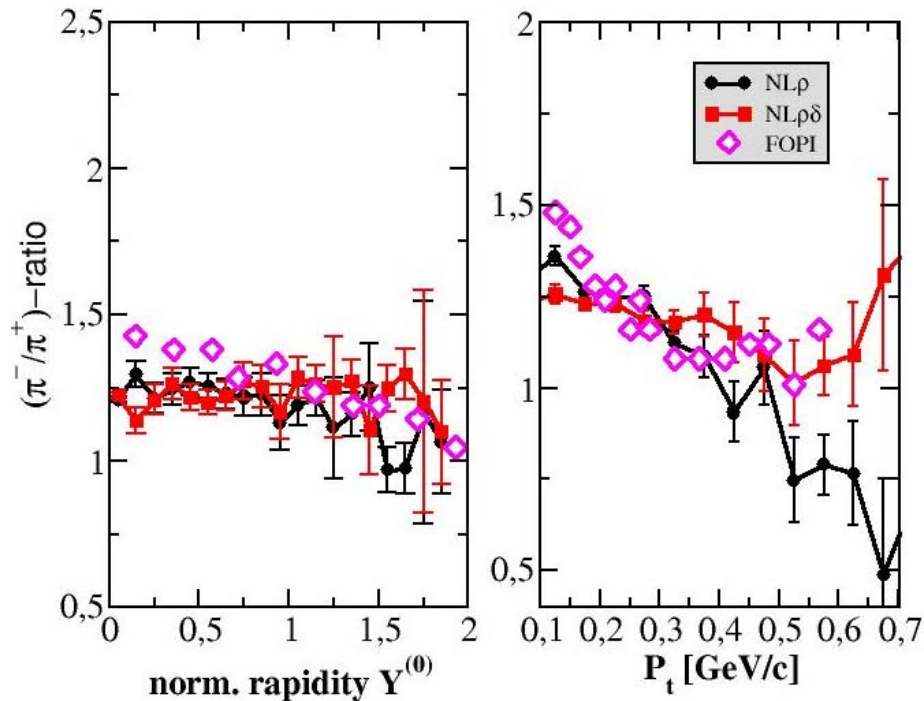
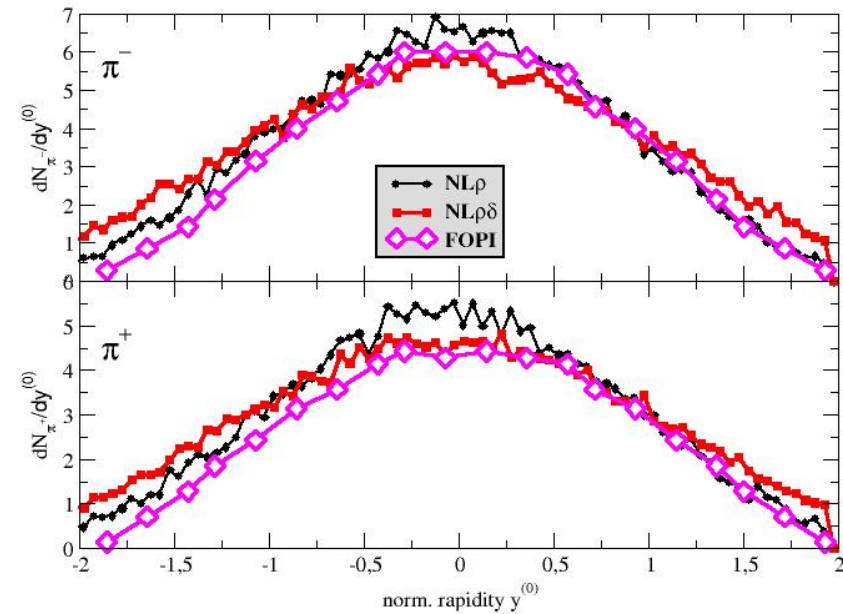
3. Pion absorption

At low energies pi(-) more absorbed since more energy is available in their production

Pion production at SIS energies: 96Ru+96Ru at 1.53A GeV

Central selection

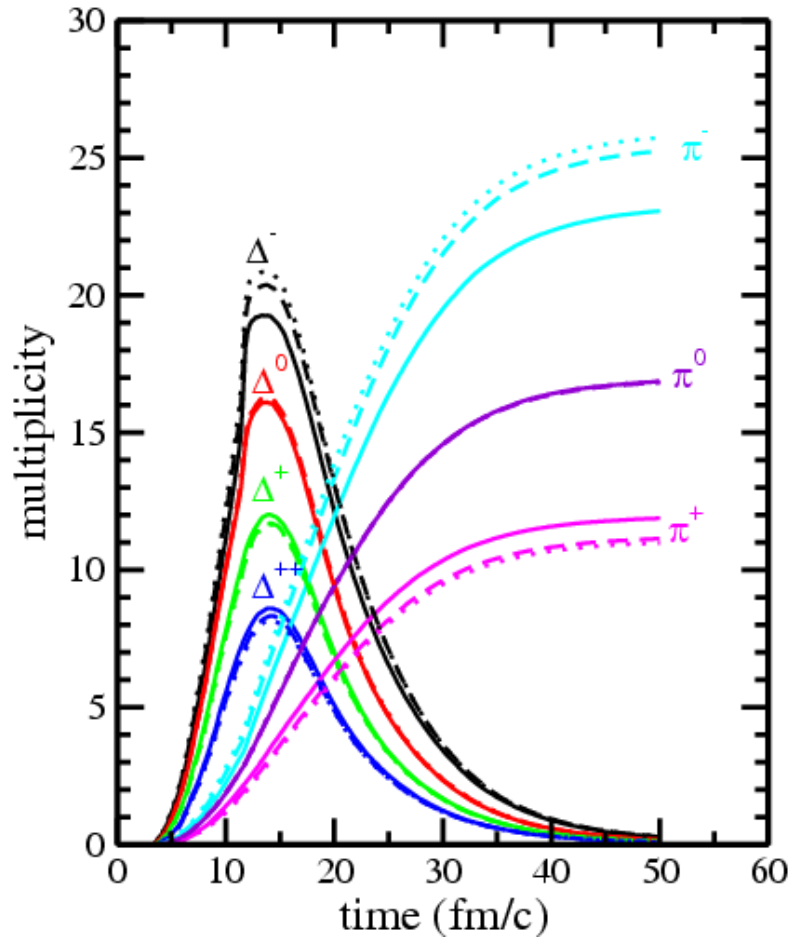
N/Z=1.18, still some Iso-EOS sensitivity



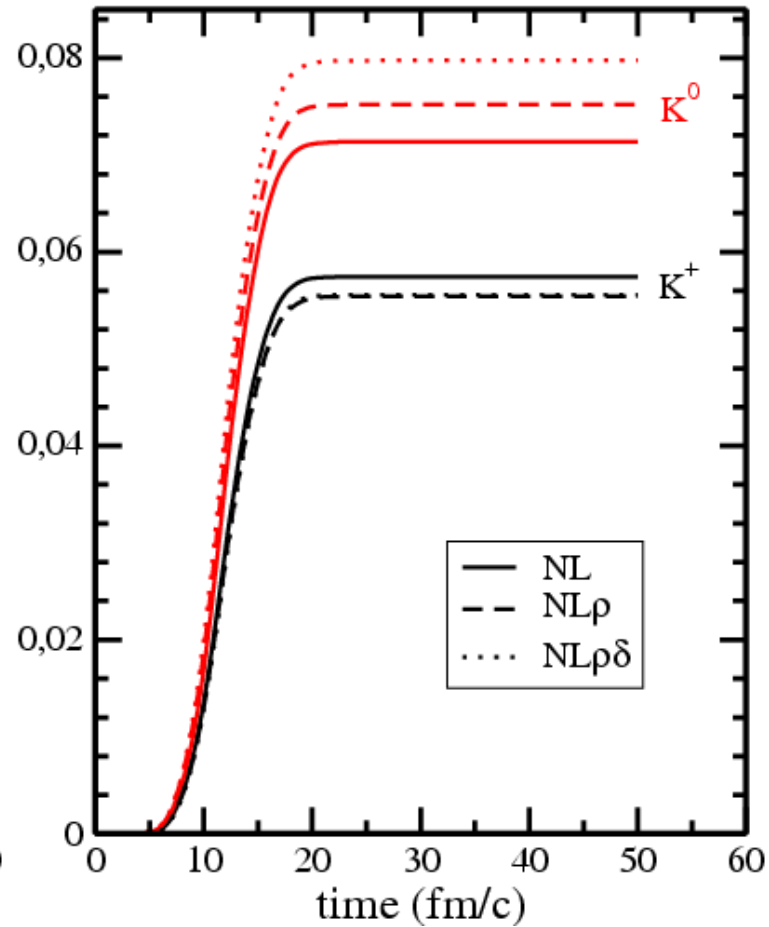
NL\rho\delta more efficient in “transforming” neutrons into protons at high density, producing π^-

Coulomb effect: less π^+ present in the high density region

Pion/Kaon production in “open” system: Au+Au 1A GeV, central



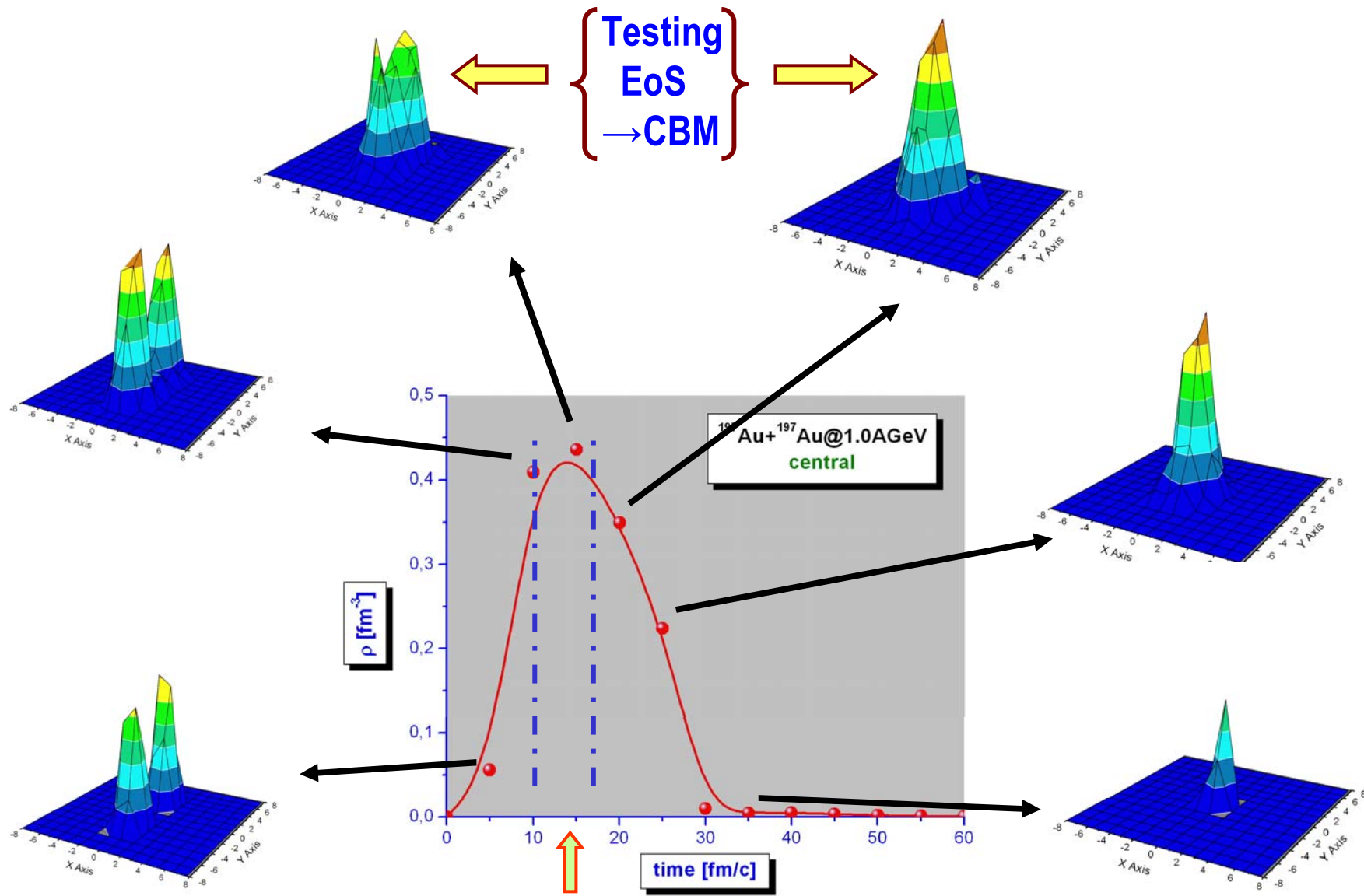
Pions: compensation 



Kaons: 

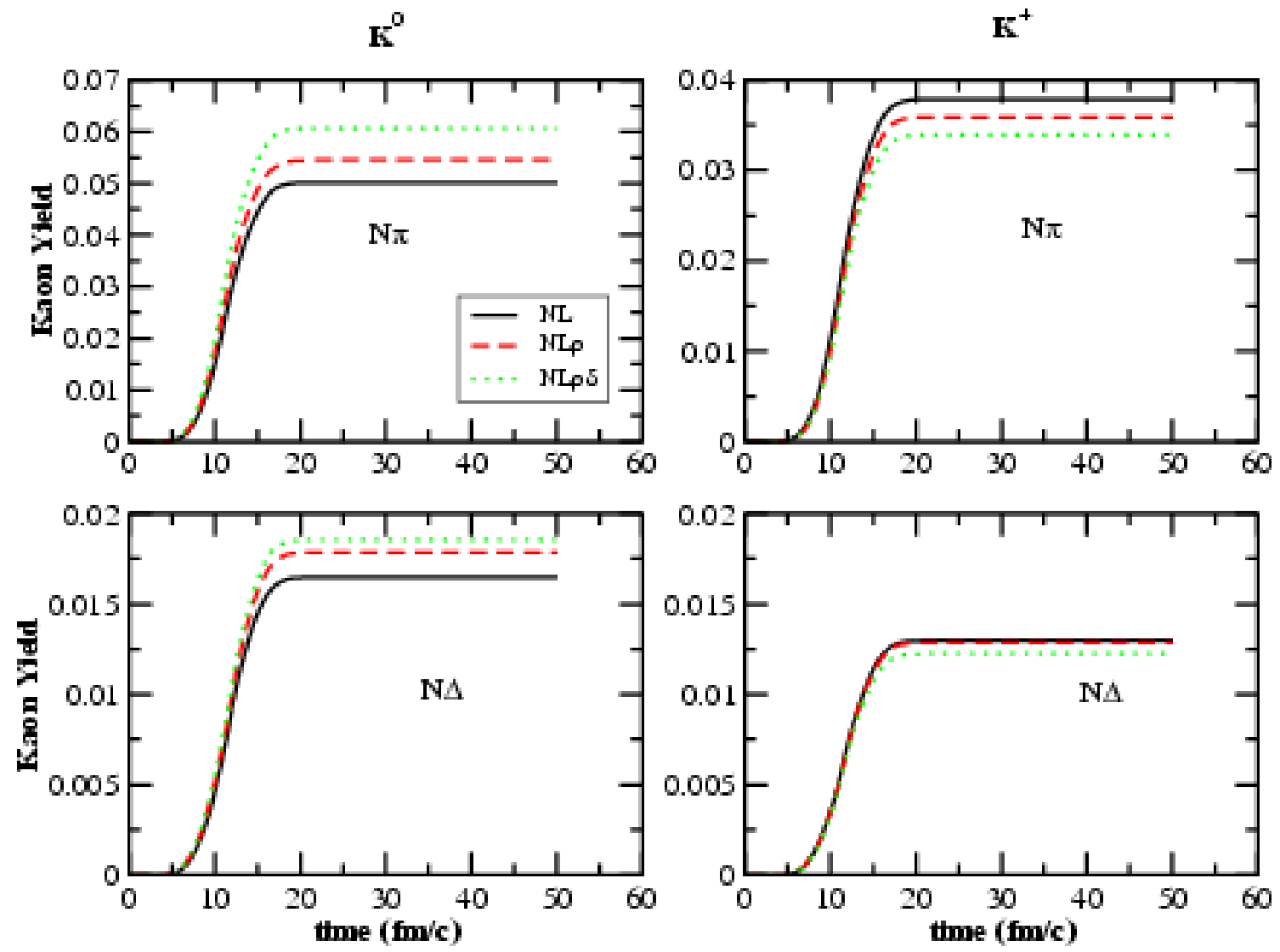
- early production: high density phase
- isovector channel effects \rightarrow
- but mostly coming from second step collisions...*
- \rightarrow *reduced asymmetry of the source*

Au+Au 1A GeV central: Phase Space Evolution in a CM cell



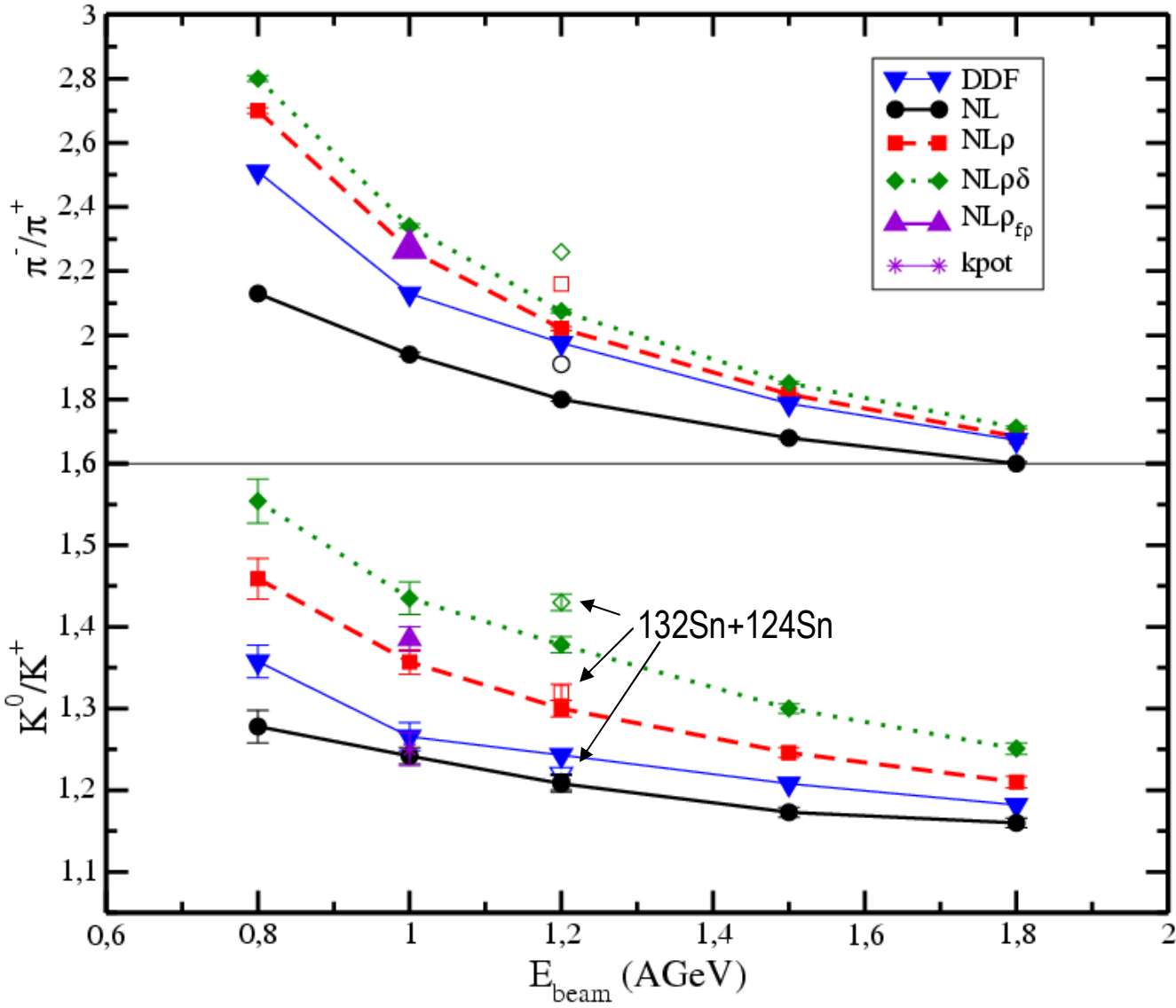
Kaon production in "open" system: Au+Au 1A GeV, central Main Channels

- NN → BYK
-
- NΔ → BY
- ΔΔ → BYK
- πN → YK
- πΔ → YK



opposite contribution of the δ -coupling

Au+Au central: Pi and K yield ratios vs. beam energy



Kaons:
~15% difference between DDF and NL $\rho\delta$

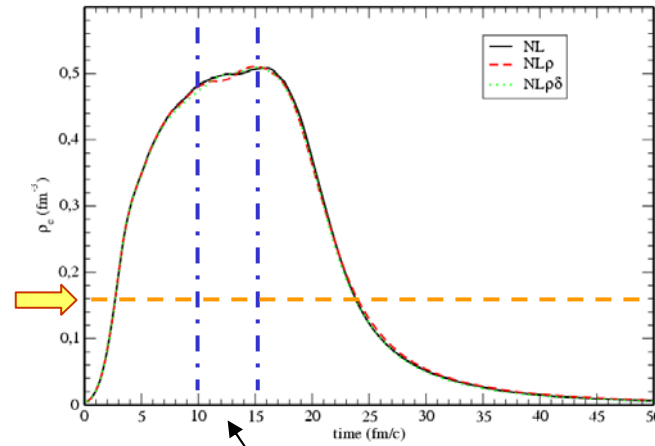
No sensitive to the K-potential

Pions: less sensitivity ~10%, but larger yields

Inclusive multiplicities

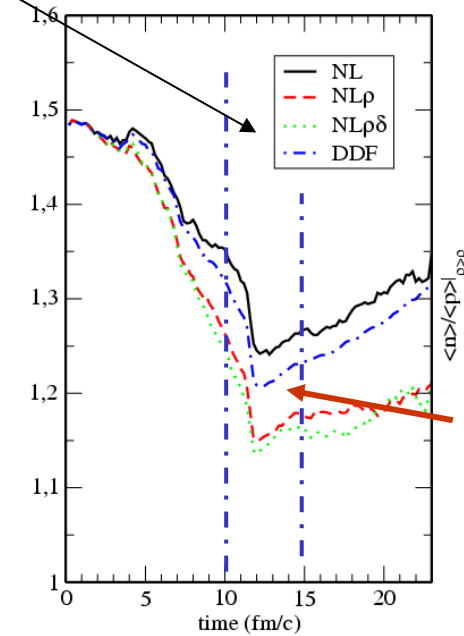
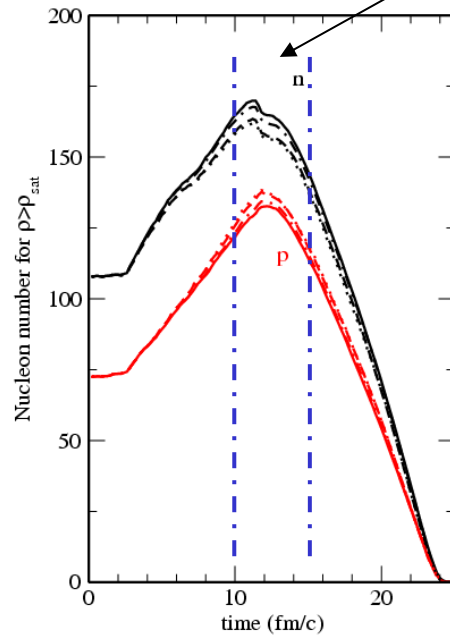
Au+Au 1A GeV: density and isospin of the Kaon source

“central” density



Time interval of Kaon production

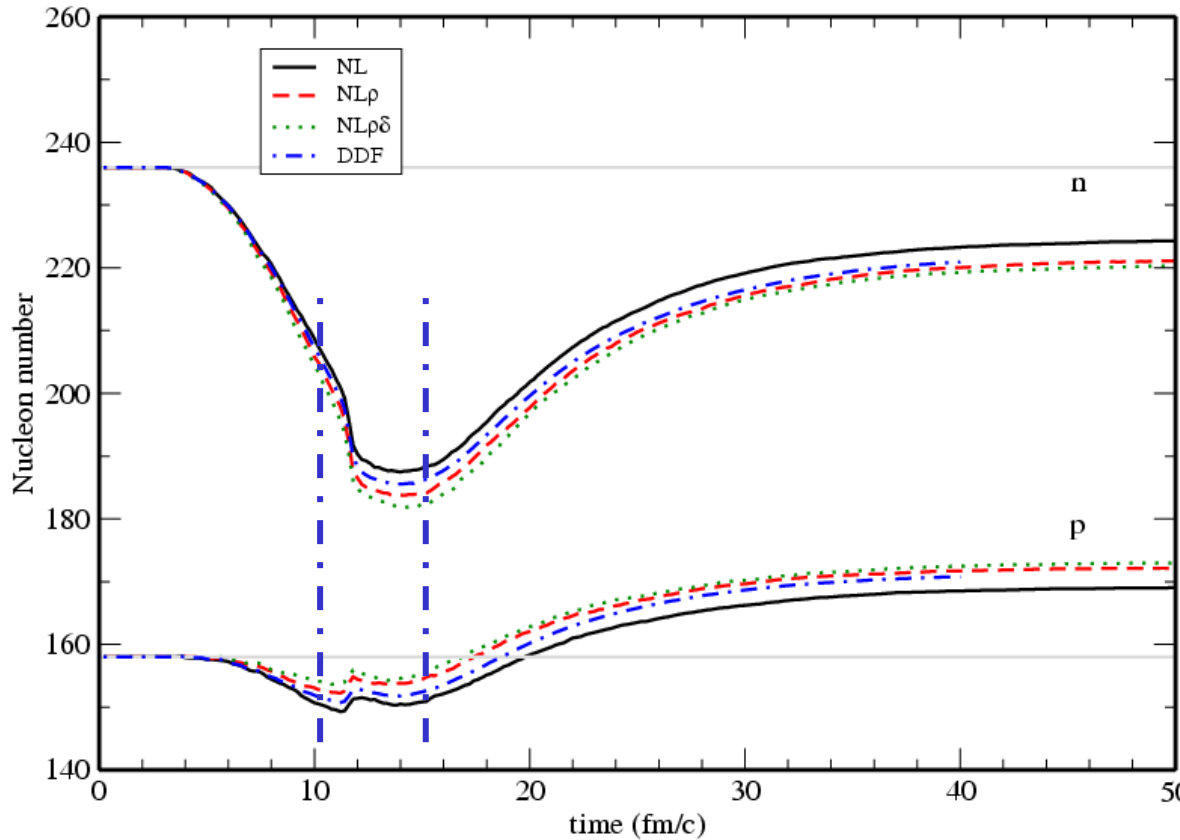
n, p at High density



n/p at High density

Drop:
Competition of fast neutron emission
and
Inelastic channels:
n → *p* transformation

Au+Au 1A GeV: time evolution of the total number of nucleons



Large $n \rightarrow p$ transformation
at early times:
Less asymmetry in the Kaon source

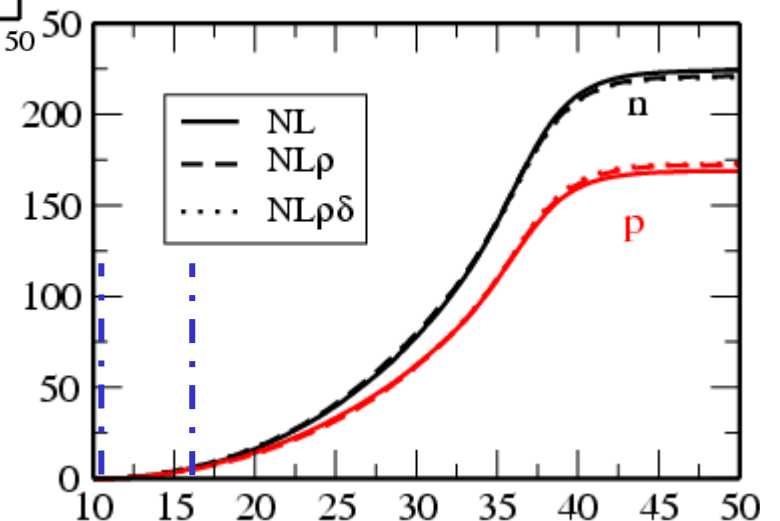
f_ρ increasing sequence
NL < DDF < NL ρ < NL $\rho\delta$

$\rho < 0.02 \text{ fm}^{-3}$

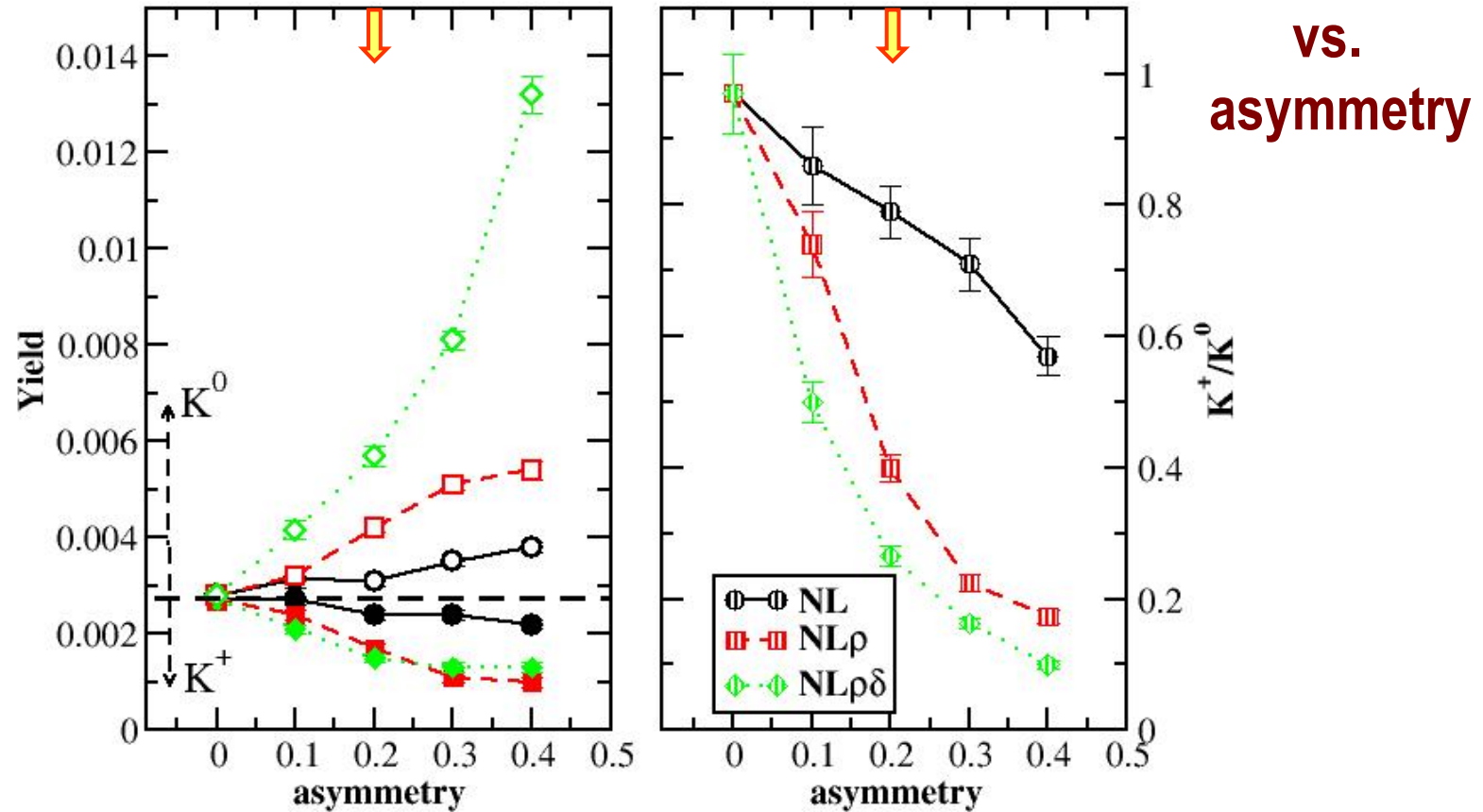
Check:
 π^-/π^+ , free n/p , $K(0)/K(+)$ vs.
emission time (p_t)

Free nucleons

→ Different behavior at lower energies, reduced inelastic competition



Density and temperature like in Au+Au 1A GeV at max.compression

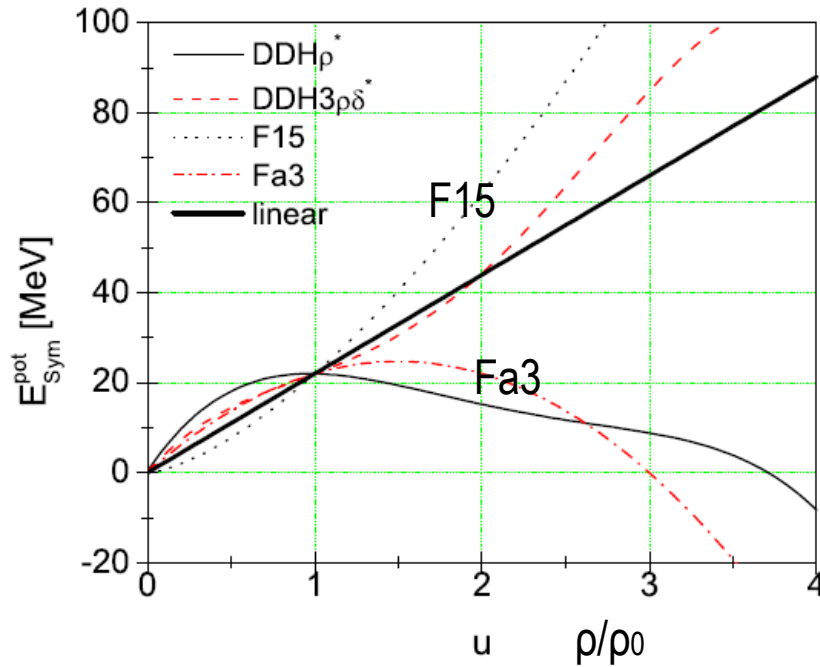


Larger isospin effects: - no neutron escape

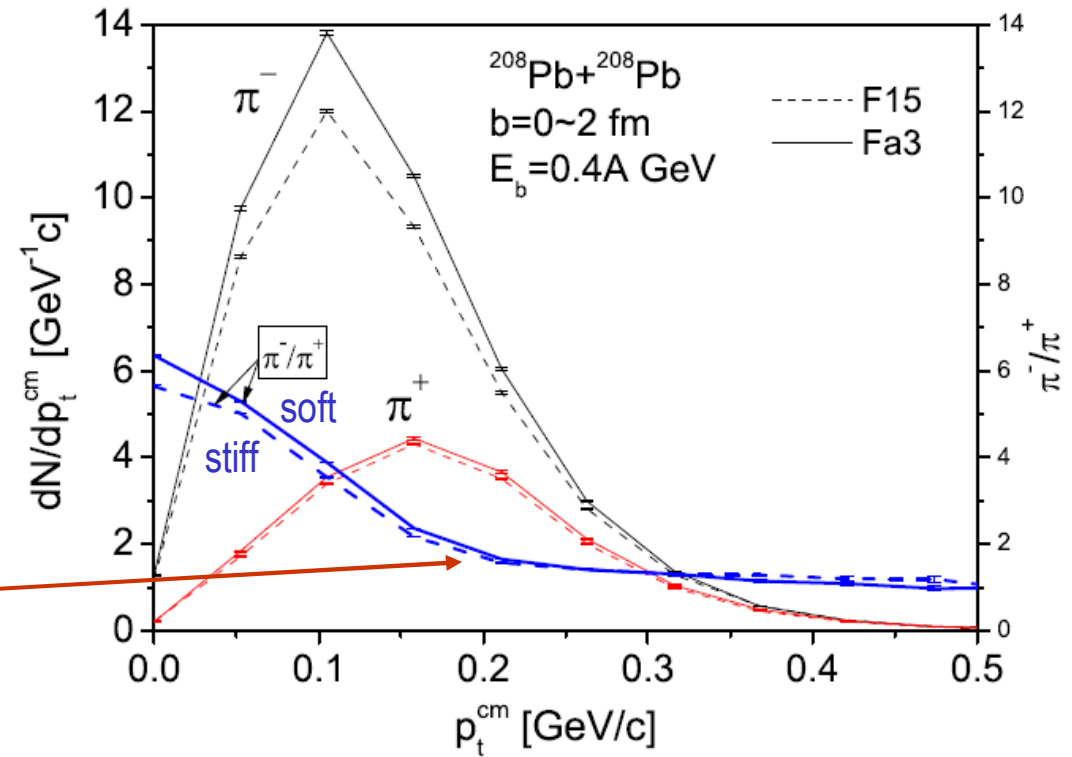
- Δ 's in chemical equilibrium \rightarrow less n-p "transformation"

UrQMD : not fully covariant symmetry term

208Pb+208Pb at 0.4A GeV

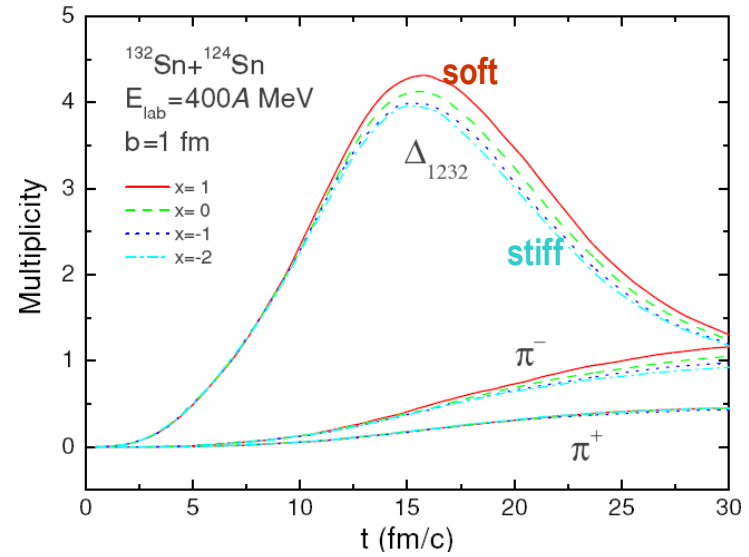
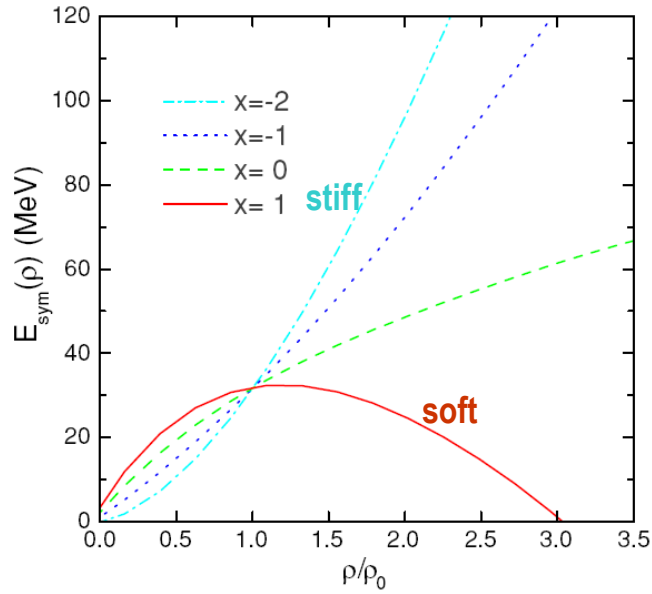


Inelastic channels less important but still crossing at high p_t

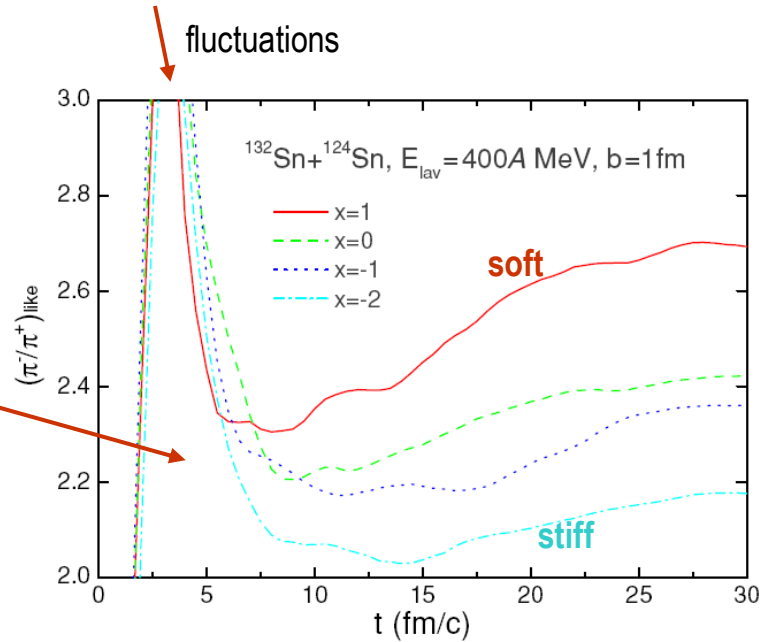


IBUU : not fully covariant symmetry term

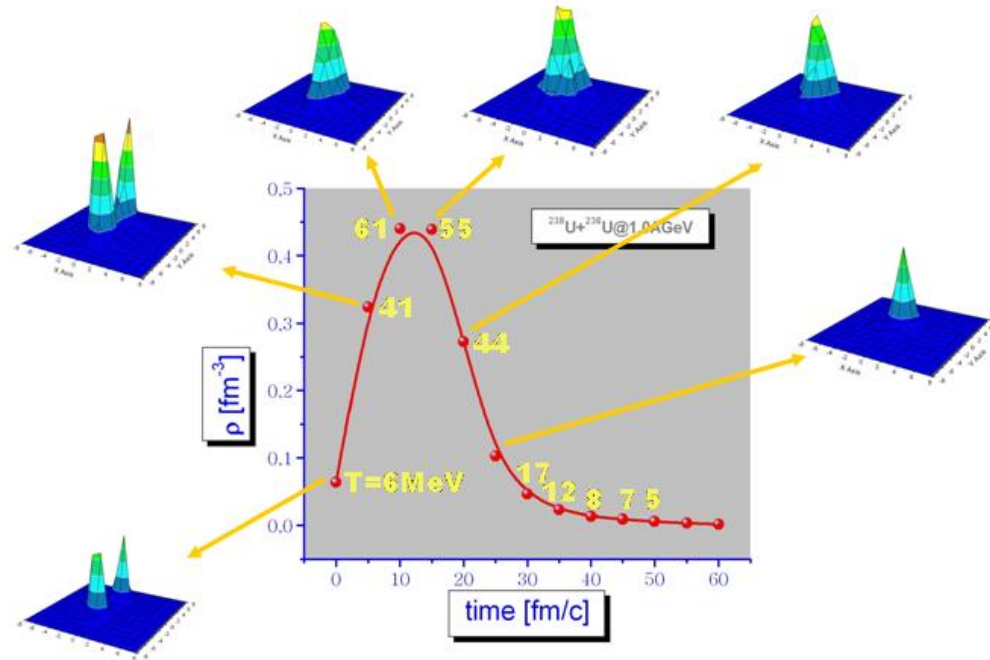
$^{132}\text{Sn}+^{124}\text{Sn}$ at 0.4A GeV



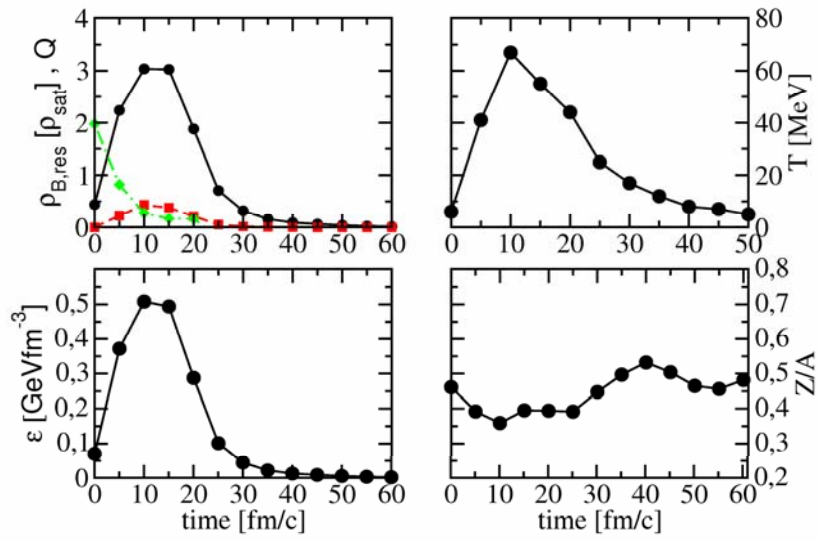
$\pi(-)/\pi(+)$ always decreasing with the iso-stiffness?



System Size Dependence & Equilibration (U+U)



In a C.M. cell



$$^{238}\text{U} + ^{238}\text{U}, 1\text{A GeV}, b = 7\text{ fm}$$

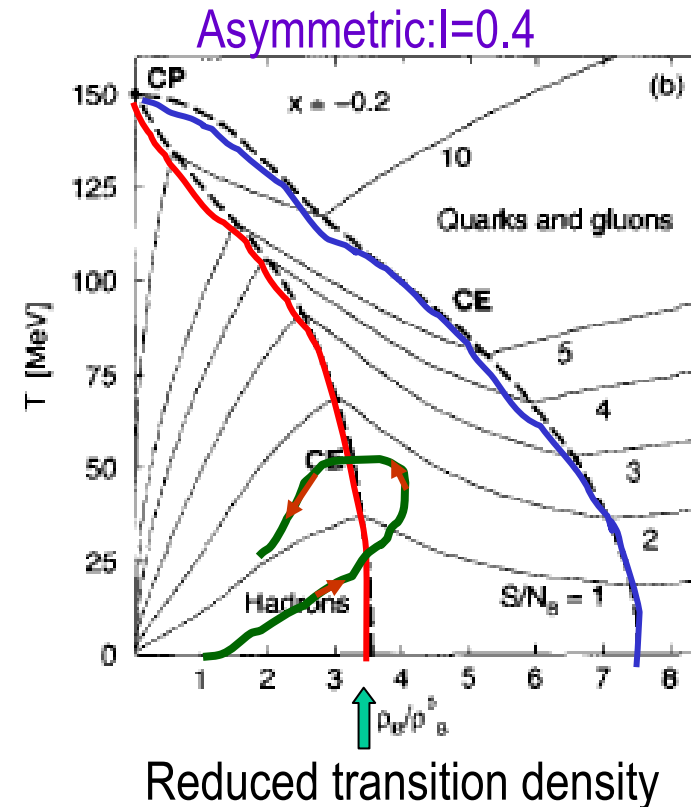
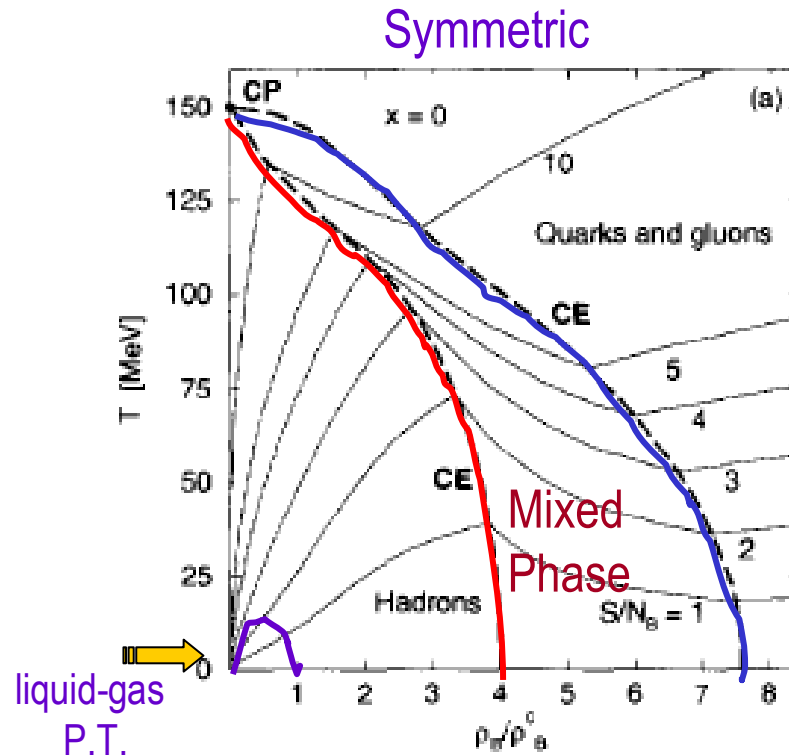
Exotic matter over 10 fm/c ?

Transition to deconfined phase at high baryon density

H.Mueller NPA618(1997)

Hadron EOS : QHD

Quark EOS: MIT-Bag Model

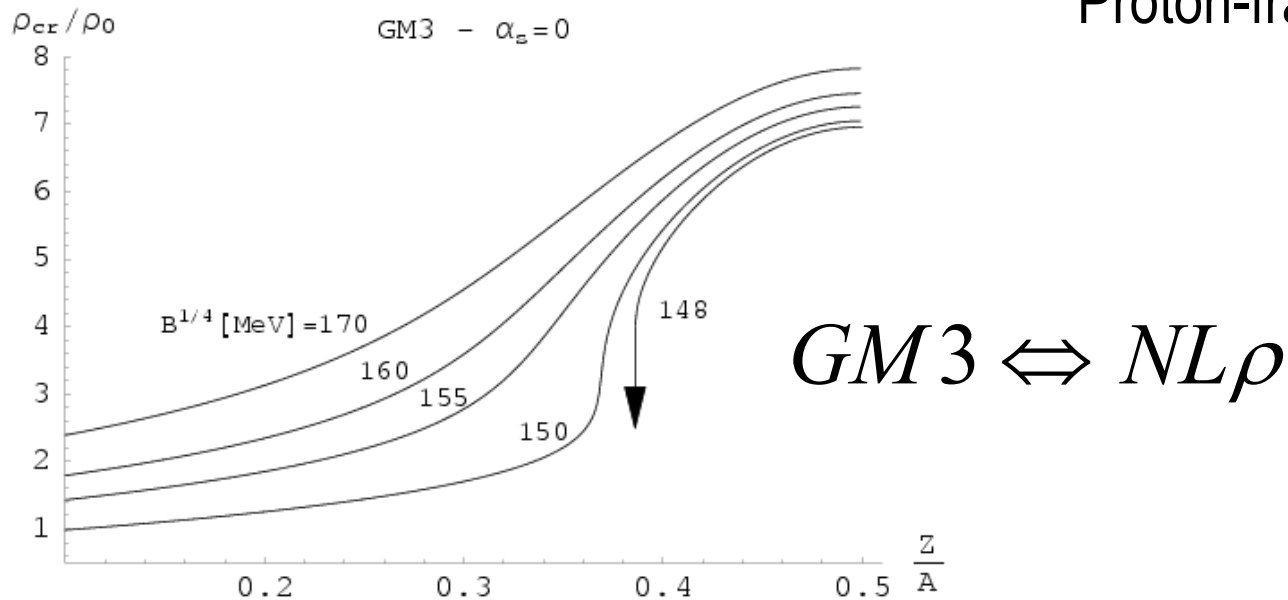
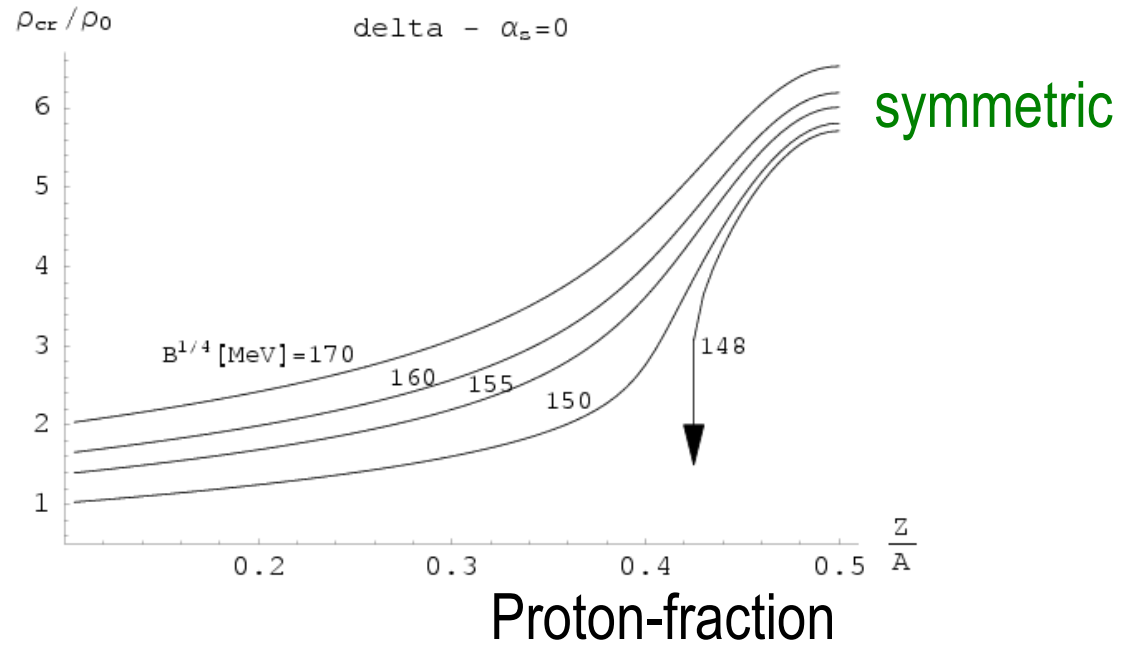


1. Earlier transition at high isospin density
2. Worse model choice? Hadron: rho-meson only, Quark: $B^{1/4}=190\text{MeV}$
large Bag-Pressure

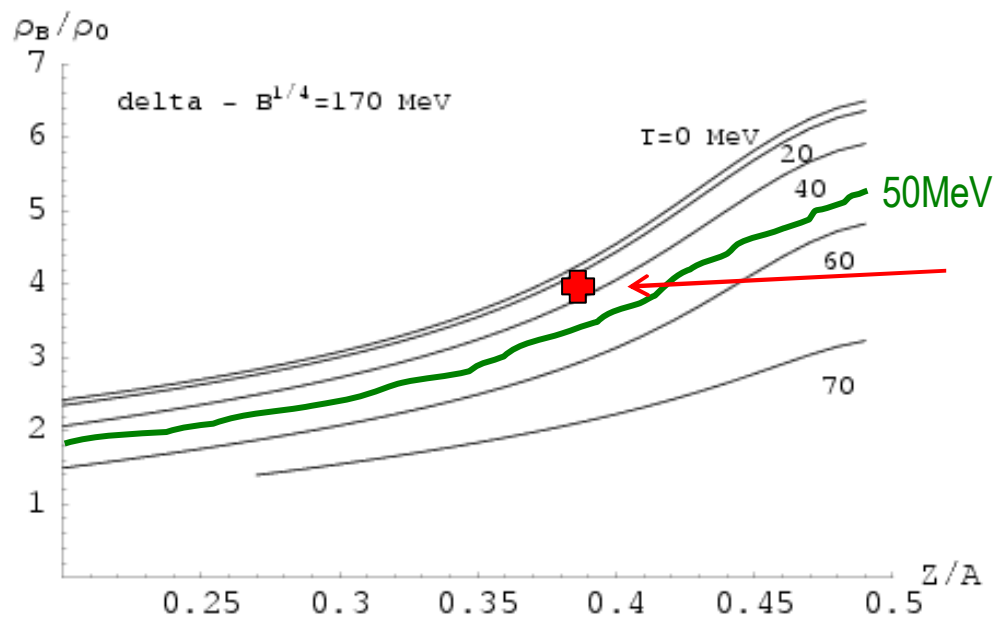
Lower Boundary of the Binodal Surface vs. NM Asymmetry

Hadron : NL $\rho\delta$

vs. Bag-constant choice

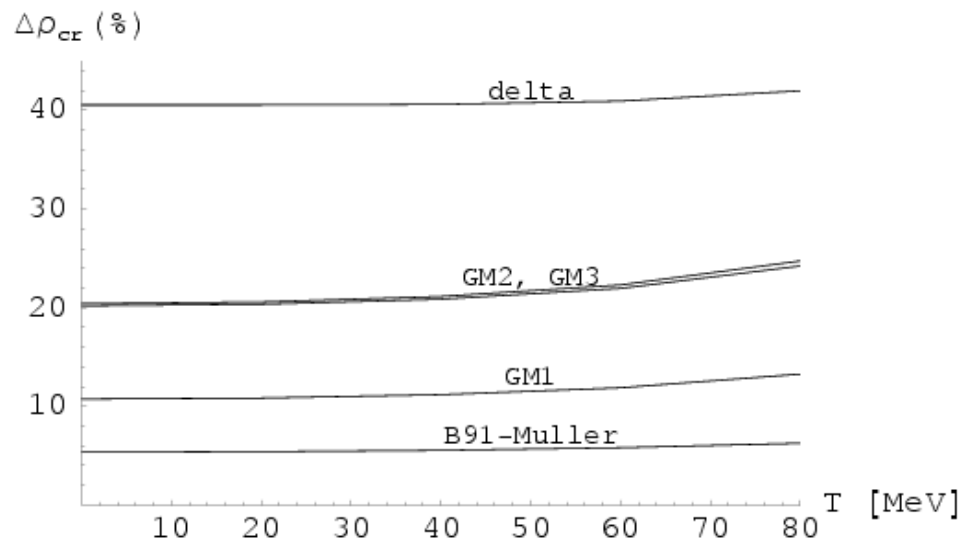


Temperature variation of the crossing density

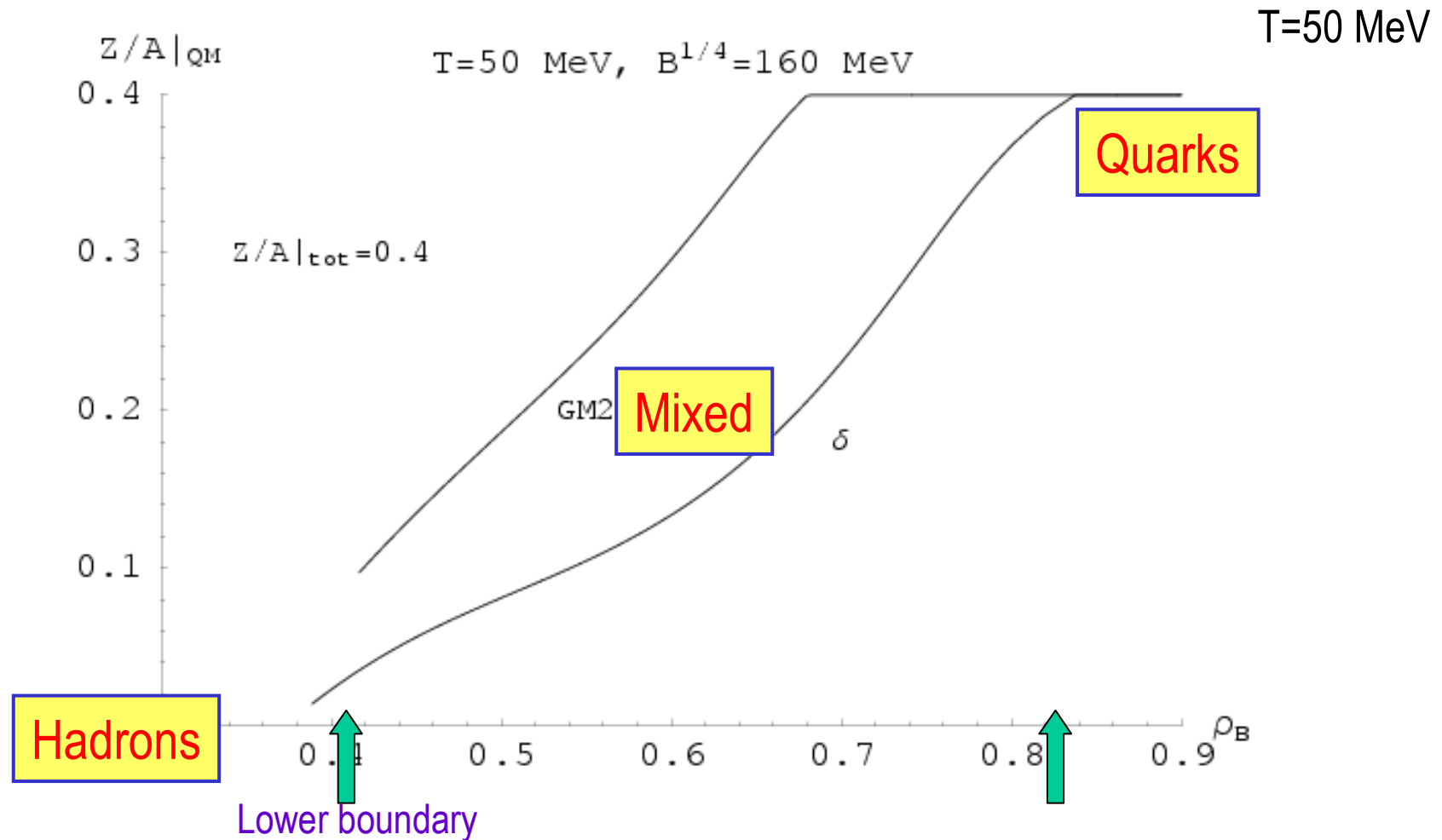


U+U, 1AGeV, semicentral

Reduction of the crossing density vs. T :
 delta-meson very efficient!



Isospin content of the Quark Clusters in the Mixed Phase



Signatures? Neutron migration to the quark clusters (instead of a fast emission)

ISOSPIN IN RELATIVISTIC HIC: EOS - SENSITIVE OBSERVABLES

1. $n - p$ collective flows \longrightarrow light isobar flows
2. Kaon Yields, (π^-/π^+) ?, flows?
3. Deconfinement precursors



Violent Collisions of Relativistic Radioactive Beams?

Genuine relativistic effects: - boosting of vector potentials
- baryon and scalar densities
(vector vs. scalar field competition)
- Dirac masses

People: M.Colonna, M.Di Toro, **G.Ferini**, Ch.Fuchs,
Th.Gaitanos, **V.Greco**, Liu Bo, **V.Prassa**,
E.Santini, **S.Yildirim** and H.H.Wolter
+ **A.Drago**, **A.Lavagno**

Relativistic Transport Dynamics

Effective Lagrangian → Transport Equations → Event simulation

Hadronic: High Baryon and Isospin Densities → New Physics?

- Pion, proton multiplicities (saturation?)
- Meson, baryon spectra vs. transverse momentum
- Elliptic Flows (EOS softening?)
- Isospin structure of particles at high p_t

Partonic: beyond “Cascade”

- Hadronization (coalescence) dynamics
- Hydro limit
- Collective flows
- Spinodal mechanism for hadronization

Why Intermediate Energies?
Proton stopping at mid-rapidity: Au+Au central

