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Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies

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Universal Nuclear Dependence in High Energy Nuclear Collisions

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These are preliminary lecture notes, intended only for distribution to participants

Universal Nuclear Dependence in High Energy Nuclear Collisions

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ICTP, Trieste, Italy, May 22-26, 2006

Outline of the talk

- ❑ Hard probe and its probing size
- ❑ Source of nuclear dependence
- ❑ Coherent multiple scattering and power corrections
  - —Resummation of power corrections to DIS SFs
- ❑ Universal nuclear dependence in nPDFs
  - —Resummation of power corrections to nPDFs
- ❑ Summary and outlook

Hard probe and its probing size

- Hard probe – process with a large momentum transfer:

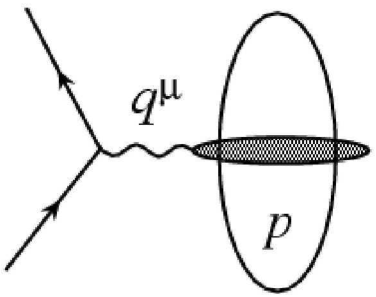
$$q^\mu \quad \text{with} \quad Q \equiv \sqrt{|q^2|} \gg \Lambda_{\mathrm{QCD}}$$

- Size of a hard probe is very localized and much smaller than a typical hadron at rest:

$$\frac{1}{Q} \ll 2R \sim \mathrm{fm}$$

- But, it might be larger than a Lorentz contracted hadron:

$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2R\left(\frac{m}{p}\right) \quad \text{or equivalently} \quad x \ll x_c \equiv \frac{1}{2mR} \sim 0.1$$



If an active parton *x* is small enough
the hard probe could cover several nucleons
In a Lorentz contracted large nucleus!

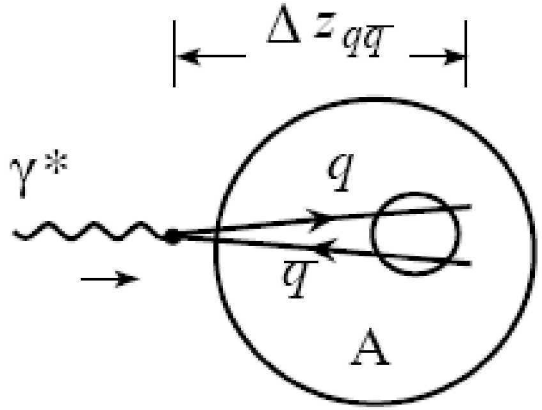
Coherence length in different frames

- **Use DIS as an example – in target rest frame:**
  virtual photon fluctuates into a q-qbar pair
  - Lifetime of the $q\bar{q}$ state:

$$\Delta E_{q\bar{q}} \sim \nu - E_{q\bar{q}} \sim \frac{Q^2}{2\nu}\left[1+\mathcal{O}\left(\frac{m_{q\bar{q}}^2}{Q^2}\right)\right]$$

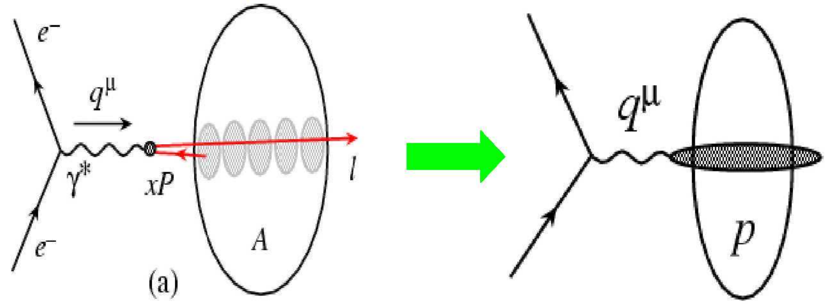
$$\Delta z_{q\bar{q}} \sim \frac{1}{\Delta E_{q\bar{q}}} \sim \frac{2\nu}{Q^2} = \frac{1}{m x_B}$$

  - $\Delta z_{q\bar{q}} \gg 2$ fm, inter-nuclear distance, if $x_B \ll 0.1$



- **If $x_B \ll 0.1$, the probe – q-qbar state of the virtual photon can interact with who hadron/nucleus coherently.**

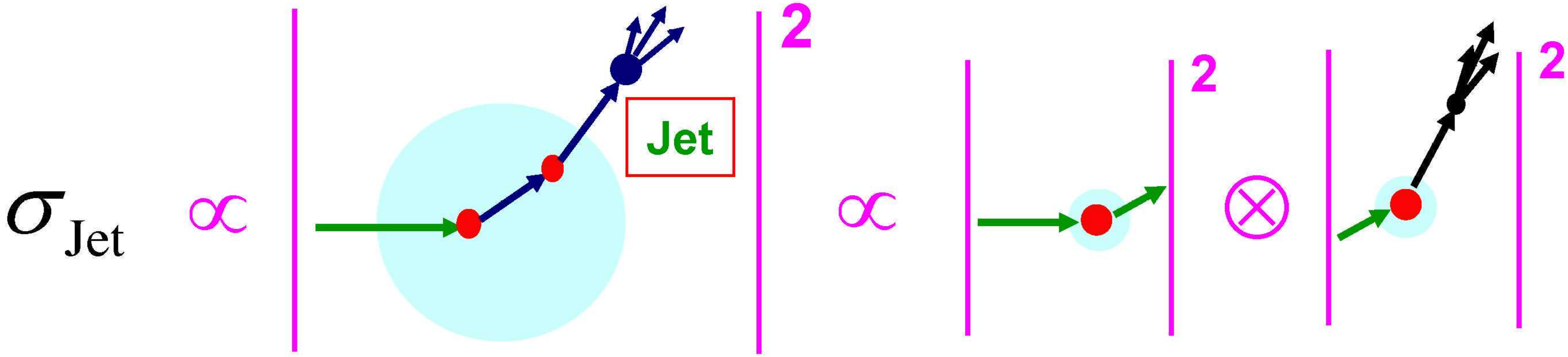
- **In Breit frame:**
  coherent final-state rescattering



The conclusion is frame independent

Incoherent/independent multiple scattering

- **Weak quantum interference between scattering centers**

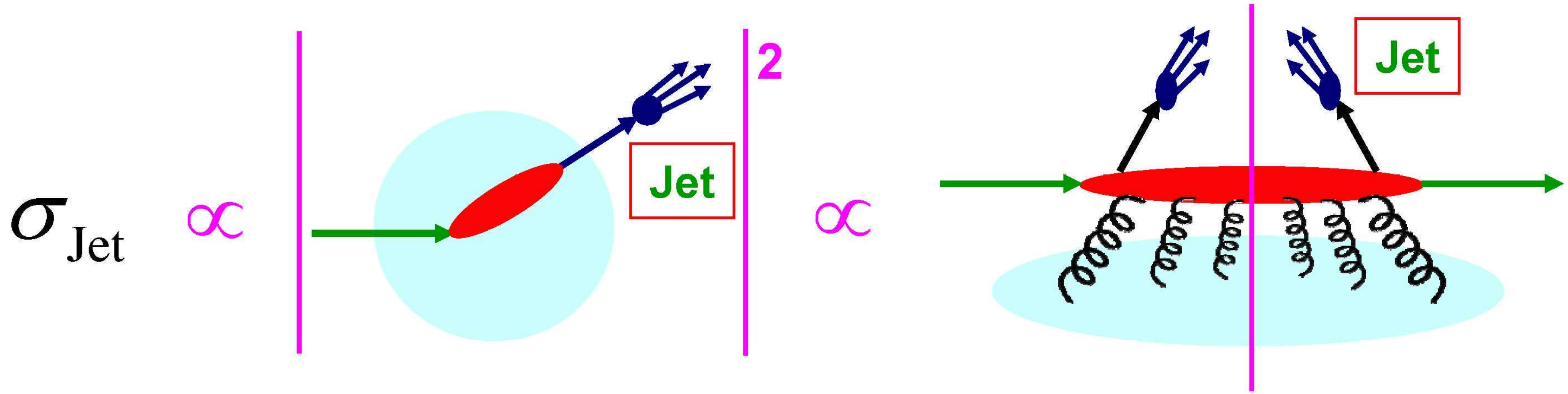


- **Modify jet spectrum without changing the total rate**
- **Nuclear dependence from the scattering centers'**
  - density
  - number
  - momentum distribution and cut-off (new scale)
  - etc

Not discussed in this talk

Coherence soft multiple scattering

- Strong quantum interference between scattering centers

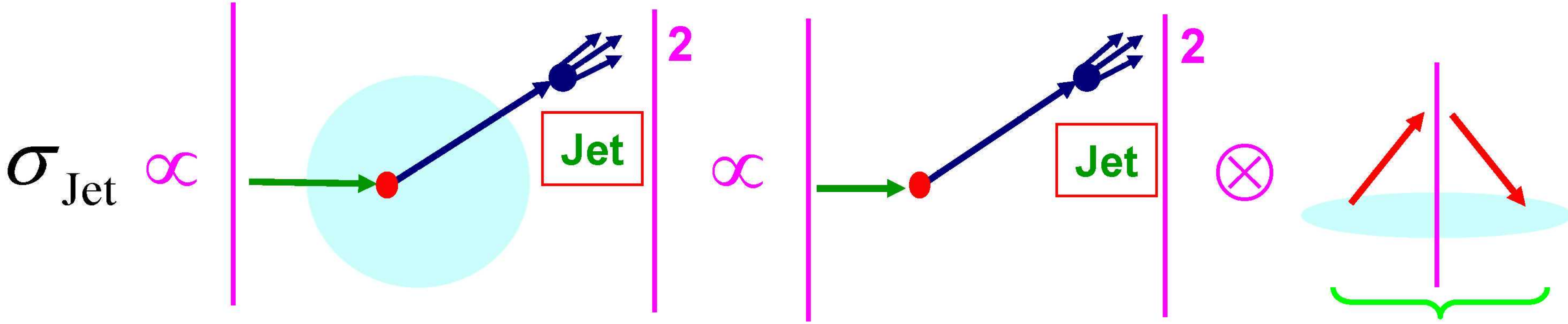


- Modify production rate as well as jet spectrum

- Nuclear dependence from multi-parton correlations
  - ❖ Multi-parton correlation functions are process independent if pQCD factorization can be applied
  - ❖ Fourier transform from momentum to coordinate ⟹ universal matrix elements of multiple fields
  - ❖ no additional scale – power suppressed

Single hard scattering

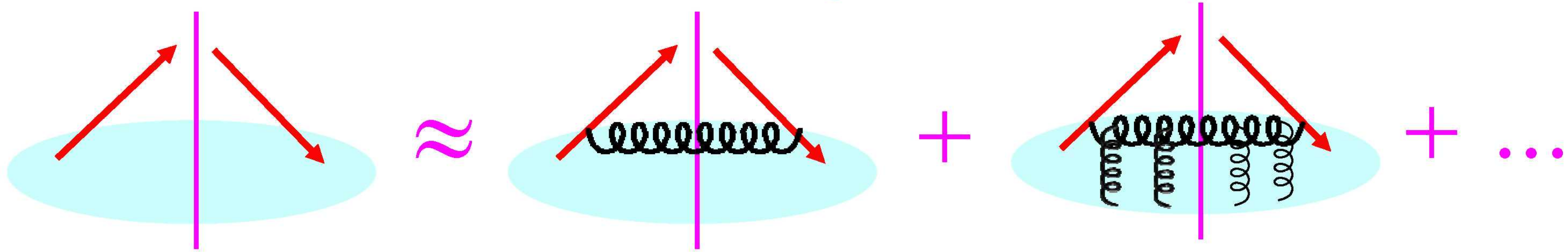
- ❑ Non-perturbative dynamics is effectively frozen



- ❑ Production rate is proportional to the PDFs

PDF

- ❑ Nuclear dependence from nPDFs
  - ❖ modified DGLAP evolution
  - ❖ input nPDFs for the evolution
  - ❖ nPDFs are universal and process independent

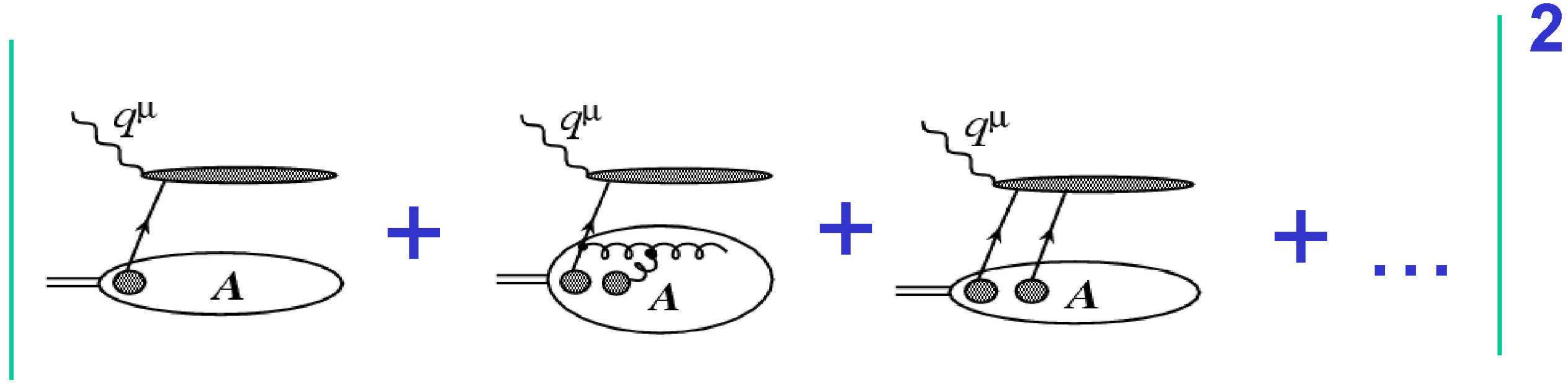


Nuclear dependence of observables

❑ At small x, measured nuclear dependence include both

nuclear dependence from coherent multiple scattering

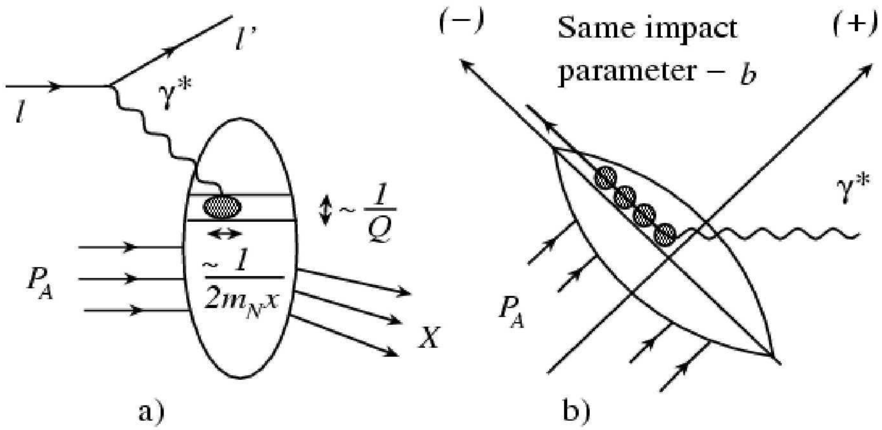
nuclear dependence from nPDFs



❑ Factorization to separate these two contributions

Size of the power correction

- Coherent multiple scattering leads to dynamical power corrections:



2D lightcone dynamics

$$d\sigma \approx d\sigma^{(S)} + d\sigma^{(D)} + \ldots$$

Naïve power counting:

$$\frac{d\sigma^{(D)}}{d\sigma^{(S)}} \sim \alpha_s \frac{1/Q^2}{R^2} \langle F^{+\alpha} F_\alpha^+ \rangle A^{1/3}$$

- Characteristic scale for the power corrections: $\langle F^{+\alpha} F_\alpha^+ \rangle$

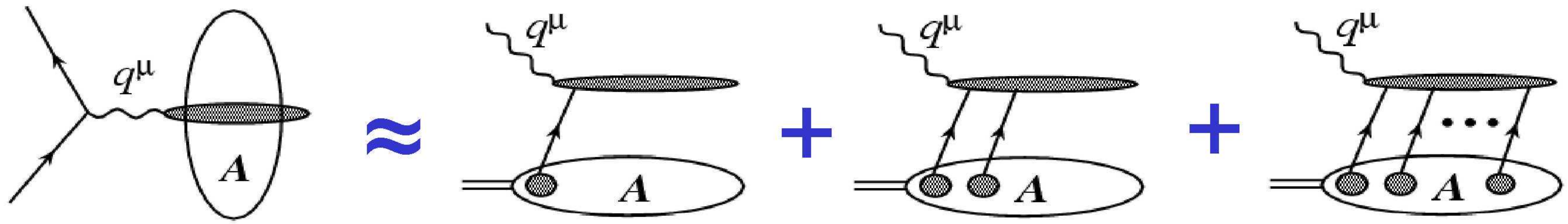
- For a hard probe: $\frac{\alpha_s}{Q^2 R^2} \ll 1$

- Enhanced by nuclear radius: $A^{1/3} \le 6$

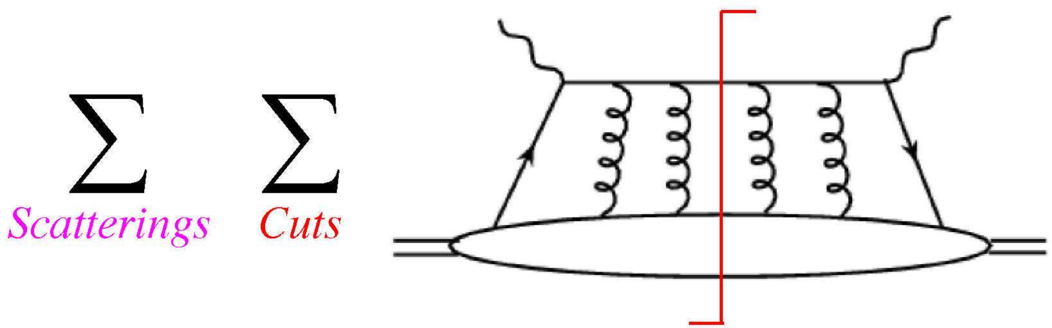
- Enhanced by the slope of small-x distribution: $-\frac{\partial}{\partial x}\varphi(x)$

Calculate multi-parton interactions

At small x, the hard probe covers several nucleons, coherent multiple scattering could be equally important at relatively low Q



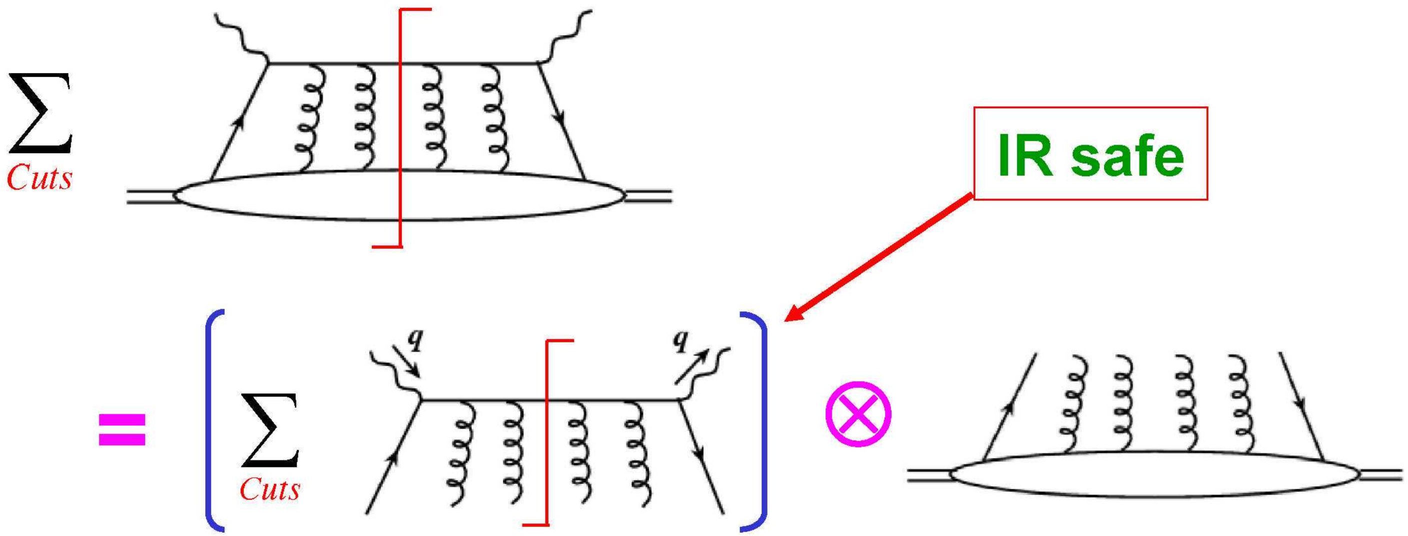
To take care of the coherence, we need to sum over all **cuts** for a given forward scattering amplitude



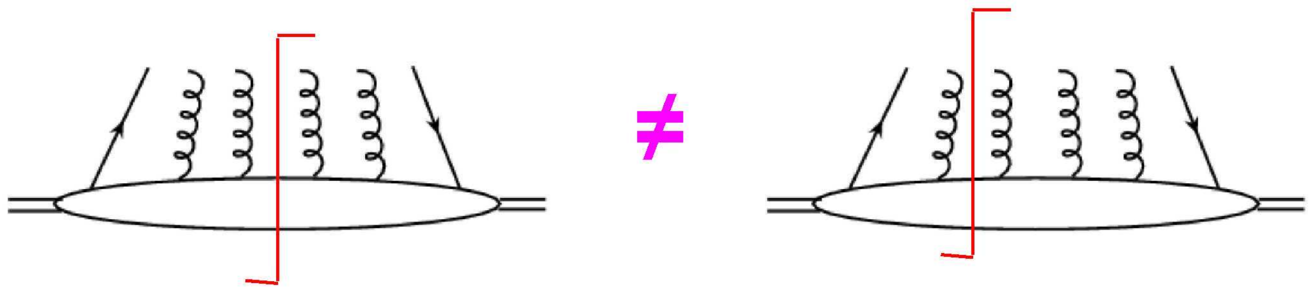
Summing over all **cuts** is also necessary for IR cancellation

Collinear approximation is important

With collinear approximation:



Different cuts for matrix elements of partons with k_T are not equal:



Factorization beyond leading power

- Consequence of OPE:

$$\sigma^h_{phys} = \hat{\sigma}^i_2 \otimes [1 + C^{(1,2)}\alpha_s + C^{(2,2)}\alpha_s^2 + \ldots] \otimes T_2^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1 + C^{(1,4)}\alpha_s + C^{(2,4)}\alpha_s^2 + \ldots] \otimes T_4^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1 + C^{(1,6)}\alpha_s + C^{(2,6)}\alpha_s^2 + \ldots] \otimes T_6^{i/h}(x)$$
$$+ \ldots$$

Leading twist

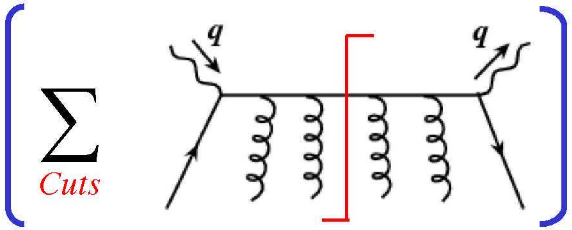
Power corrections

- Predictive power:
  - Coefficient functions are IR safe
  - Distributions/correlations/matrix elements are universal

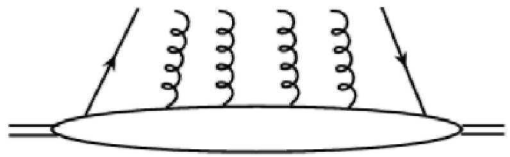
- Distributions are defined to remove all collinear divergences of the partonic scattering

Multi-parton correlation functions

❑ Parton momentum convolution:



$\otimes$



$$\propto \int \prod_i dy_i^- \; e^{i x_i p^+ y_i^-} \left\langle P_A \right| \prod_i F^{+\perp}\left(y_i^-\right) \left| P_A \right\rangle$$

All coordinate space integrals are localized if x is large

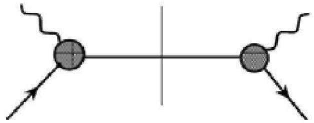
❑ Leading-pole approximation for dx_i integrals :

- ❑ dx_i integrals are fixed by the poles (no pinched poles)
- ❑ $x_i=0$ removes the exponentials
- ❑ dy integrals can be extended to the size of nuclear matter

Leading-pole leads to highest powers in medium length, a much smaller number of diagrams to worry about

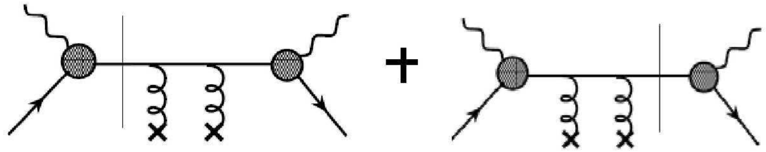
Multiple soft scattering to inclusive DIS

- **LO contribution to DIS cross section:**



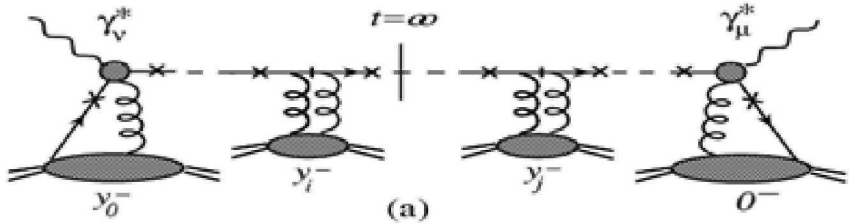
$$\Longrightarrow \quad \delta(x - x_B)$$

- **NLO contribution:**



$$\Longrightarrow \quad \underbrace{\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]}_{\int \frac{dy_2^- dy_1^-}{(2\pi)^2}\left[F^{+\alpha}(y_2^-)F_\alpha{}^{+}(y_1^-)\right]\theta(y_2^-)} x_B \underbrace{\lim_{x_1\to x}\left[\frac{1}{x-x_1}\delta(x_1-x_B)+\frac{1}{x_1-x}\delta(x-x_B)\right]}_{x_B\left[-\frac{d}{dx}\delta(x-x_B)\right]}$$

- **Nth order contribution:**



$$\Longrightarrow$$

$$\left[\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]\right]^N \underbrace{x_B^N \lim_{x_i\to x}\sum_{m=0}^{N}\delta(x_m-x_B)\left[\prod_{i=1}^{m}\left(\frac{1}{x_{i-1}-x_m}\right)\right]\left[\prod_{j=1}^{N-m}\left(\frac{1}{x_{m+j}-x_m}\right)\right]}_{x_B^N\left[(-1)^N\frac{1}{N!}\frac{d^N}{dx^N}\delta(x-x_B)\right]}$$

Infrared safe!

Corrections to DIS structure functions

- **Transverse structure function:** Qiu and Vitev, PRL (2004)

$$F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[\frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)$$

$$\approx F_T^{(0)}(x_B(1+\Delta), Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right)$$

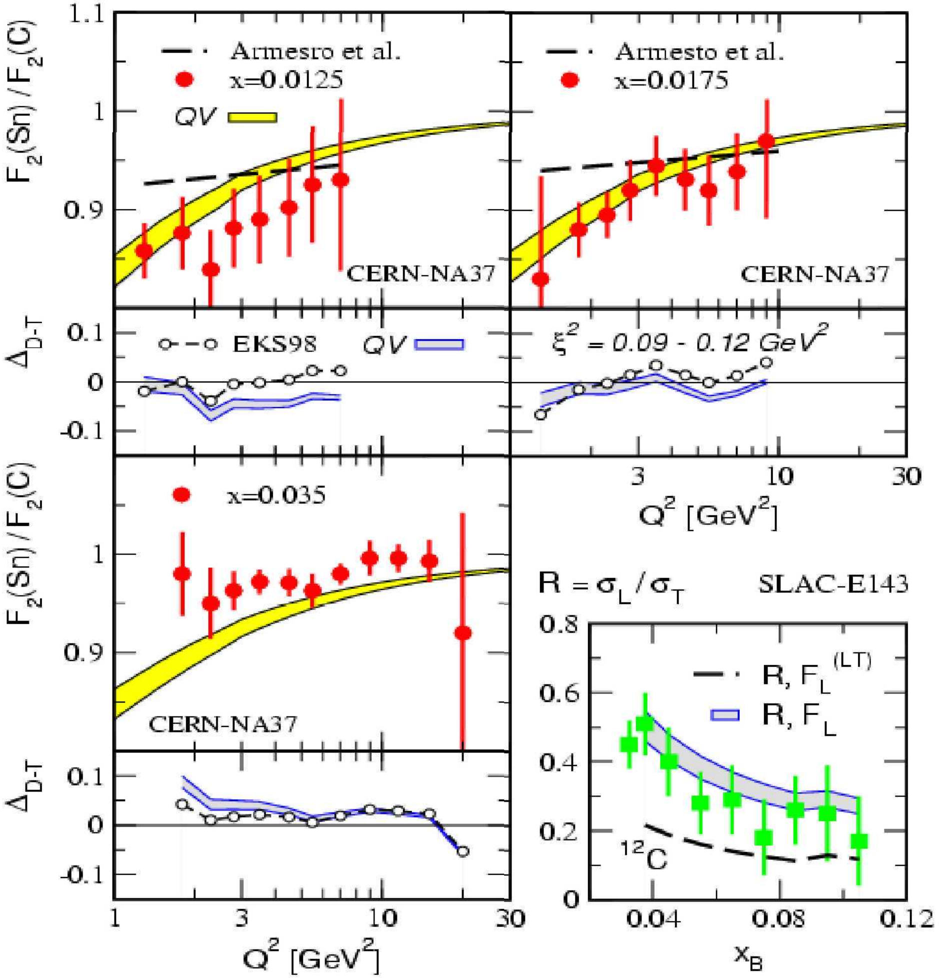
$$\xi^2 = \frac{3\pi \alpha_s}{8R^2} \langle F^{+\alpha} F_\alpha{}^+ \rangle$$

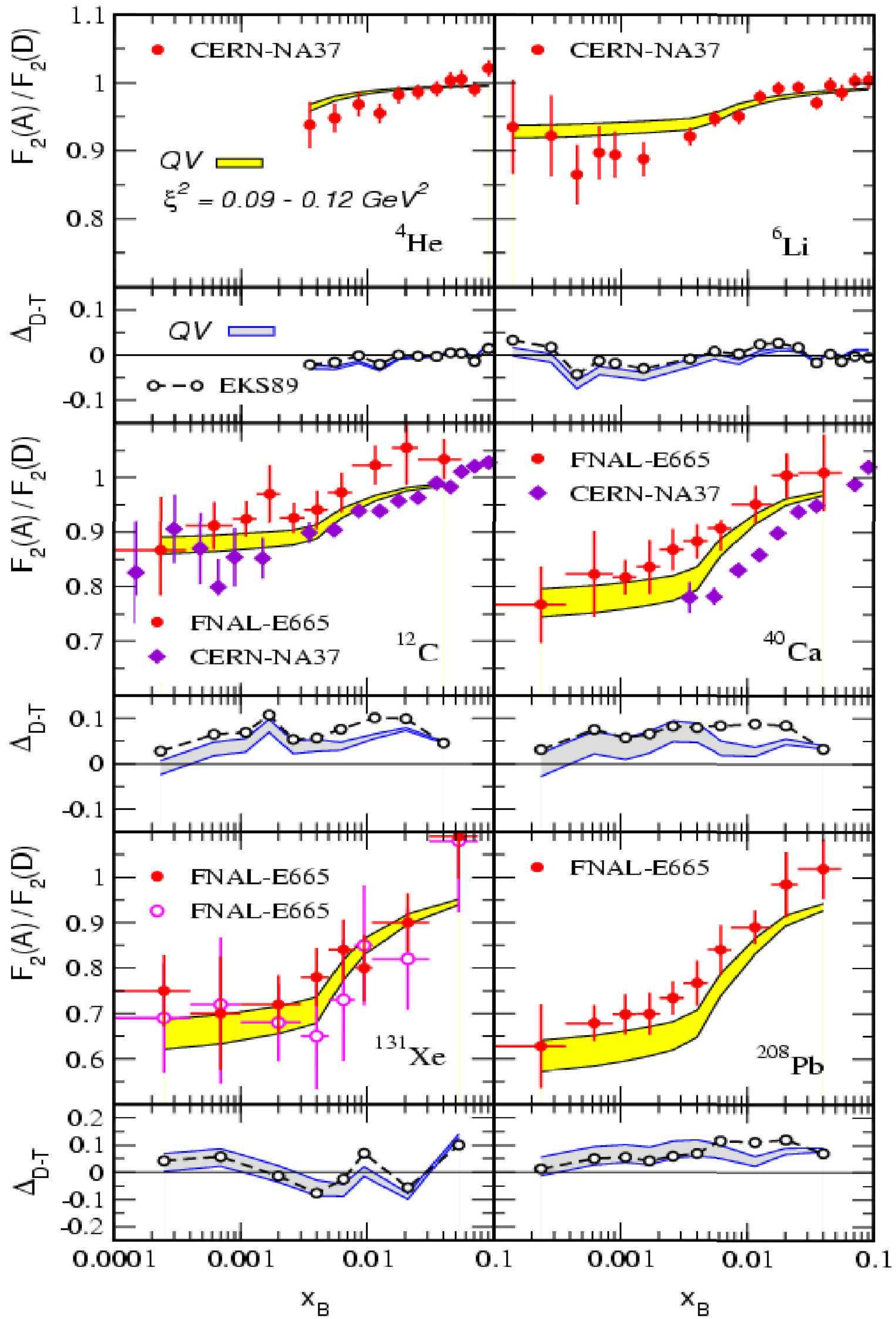
- **Similar result for longitudinal structure function**

Single universal parameter lead to the x-, Q-, and A-dependent suppression to all DIS structure functions at small x

Neglect LT shadowing upper limit of ξ^2

$\xi^2 \sim 0.09 - 0.12\ \mathrm{GeV}^2$





Leading twist shadowing

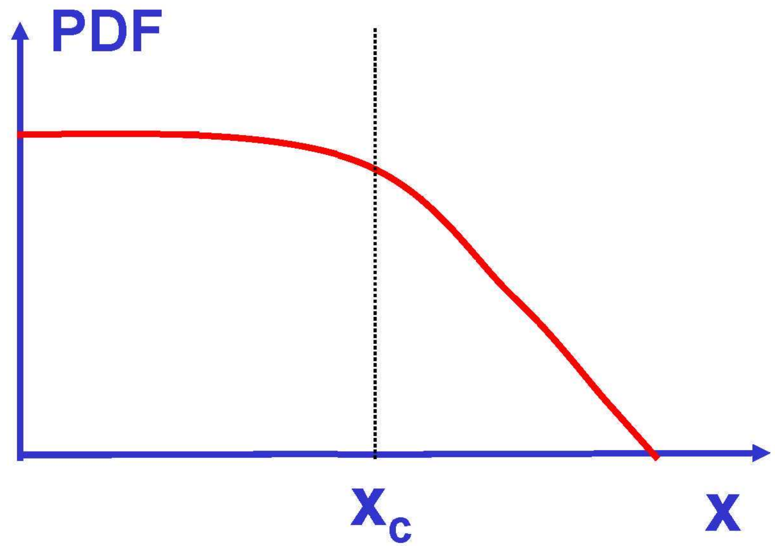
❑ Power corrections complement to the nuclear dependence in nPDFs:

- ❖ Leading twist shadowing changes the x- and Q-dependence of the parton distributions
- ❖ Power corrections to the DIS structure functions (or cross sections) are effectively equivalent to a shift in x
- ❖ Power corrections vanish quickly as hard scale Q increases while the leading twist shadowing goes away much slower

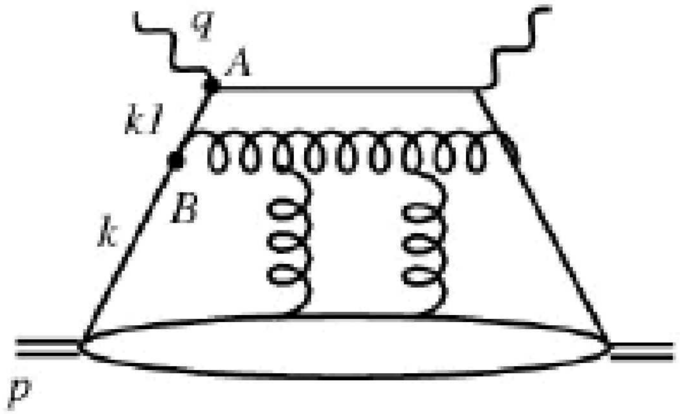
❑ If leading twist shadowing is so strong that x-dependence of parton distributions saturates

for $x < x_c$,

additional power corrections, the shift in x, should have no effect to the cross section!



Beyond the tree-level

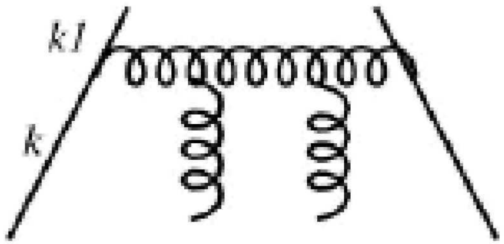


$$\int_0^{-Q^2} \frac{dk_1^2}{(k_1^2)^2}$$

$k_1^2 \rightarrow 0$

$t_{AB} \rightarrow \infty$

Collinear divergence

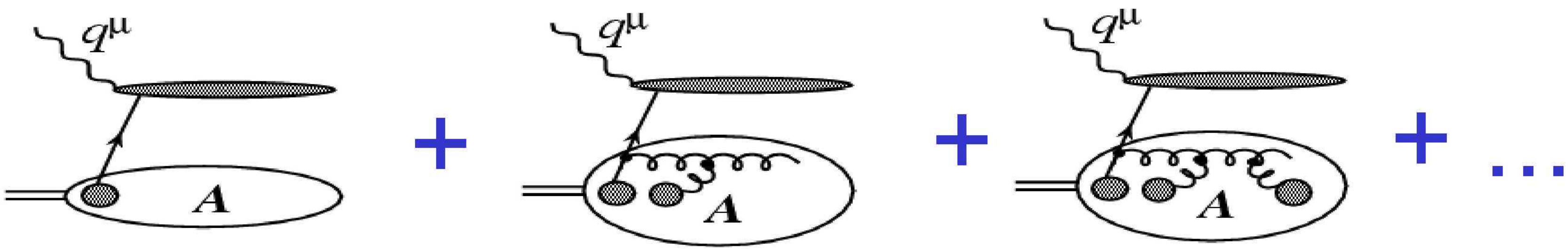


Should be a part of nuclear PDFs

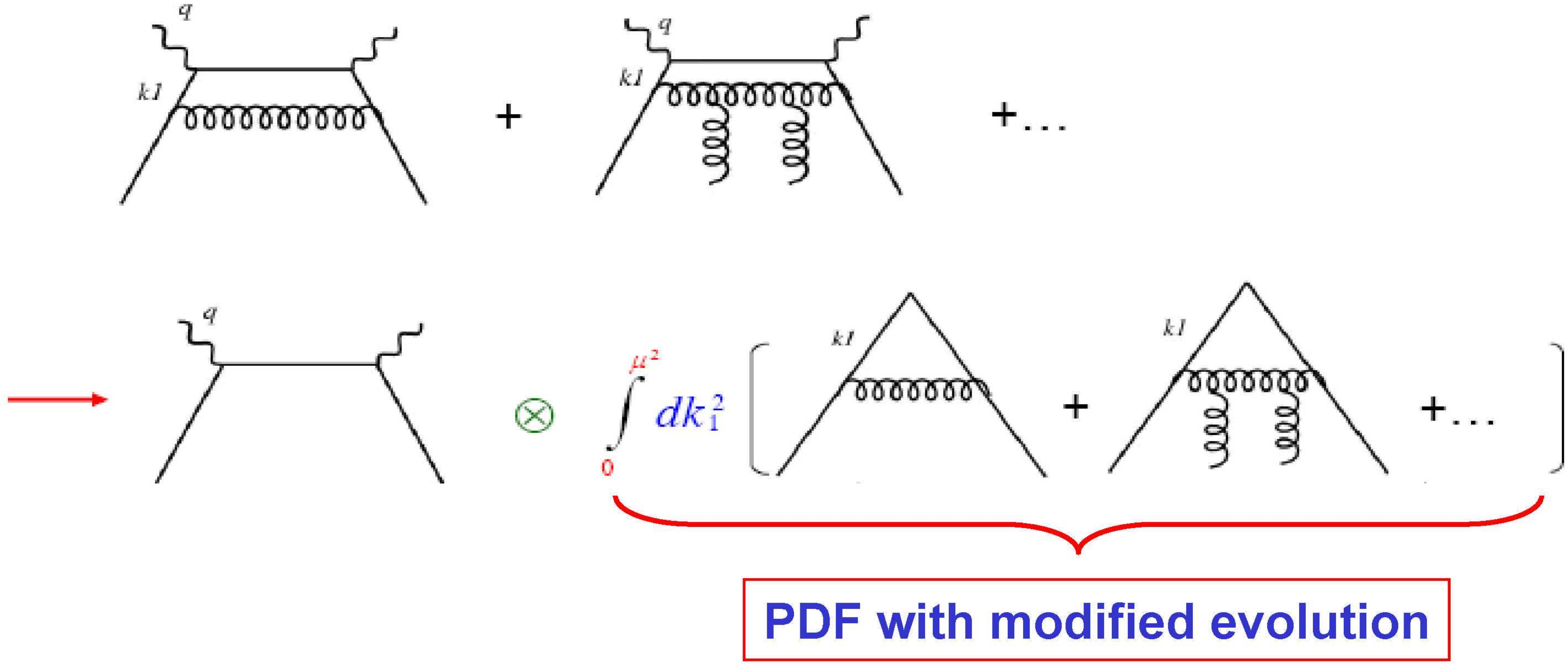
- But, DGLAP evolved nPDFs do not remove this singularity, nor any collinear divergences beyond single scattering
- Redefine nPDFs to include all collinear divergences of partonic subprocesses

Power corrections to PDFs

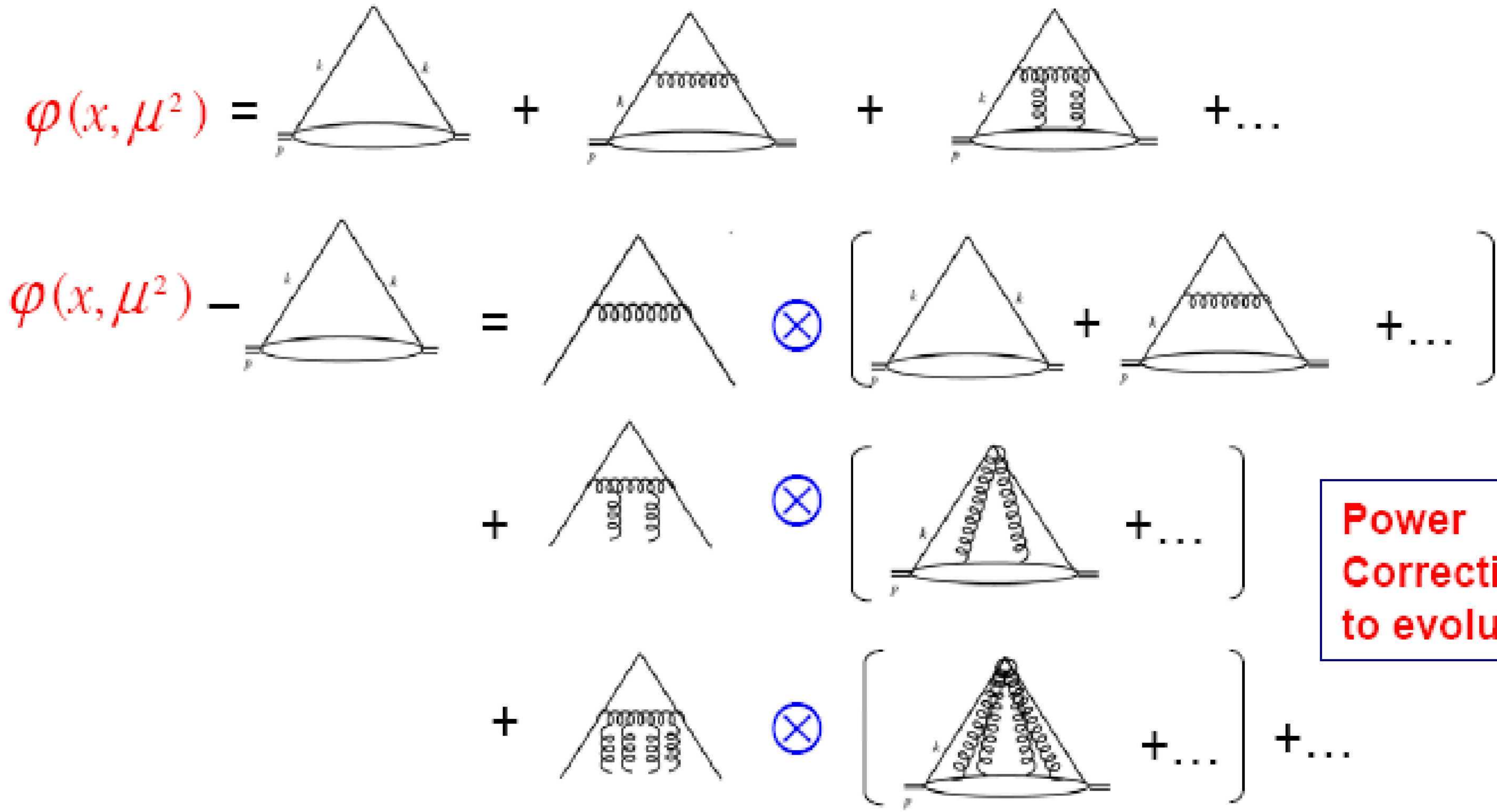
❑ Hard probe sees only one effective parton:



❑ Pinched poles in the ladder diagrams – corrections to evolution



Corrections to DGLAP evolution



Power Correction to evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} \varphi(x,\mu^2) = \gamma\left(\frac{x}{x'},\alpha_s\right) \otimes \varphi(x',\mu^2) - \frac{1}{\mu^2}\,\overline{\gamma} \otimes \varphi^{(4)}(x',\mu^2) + \ldots$$

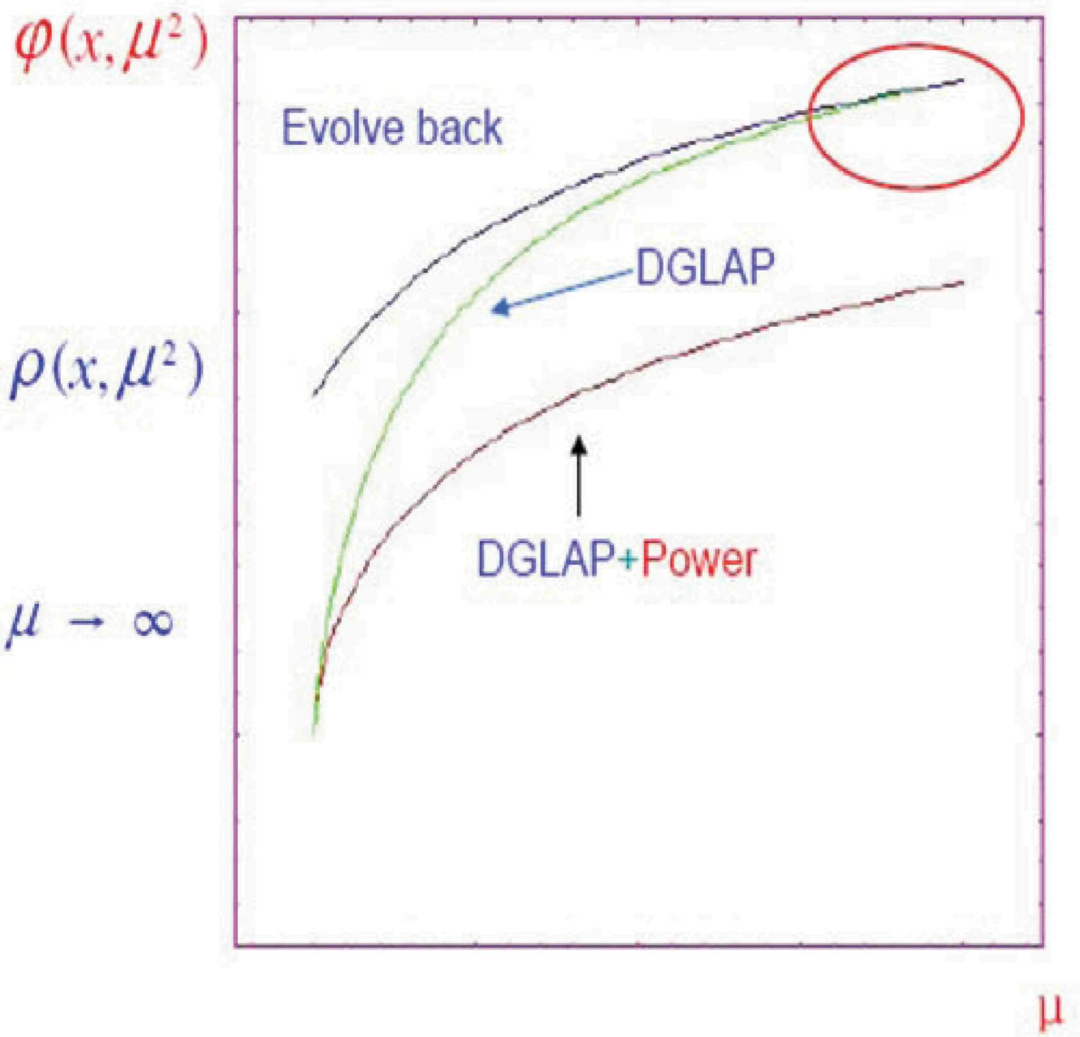
Corrections to PDFs not down by $1/Q^2$

Leading order power correction

$$\frac{\partial \varphi(x,\mu^2)}{\partial \ln \mu^2} = P(x) \otimes \varphi(x,\mu^2) - \frac{1}{\mu^2}\rho(x,\mu^2)$$

$$\int_{\mu_0^2}^{\mu^2} \frac{d\ln\bar{\mu}^2}{\bar{\mu}^2} = \frac{1}{\mu_0^2} - \frac{1}{\mu^2} \rightarrow \frac{1}{\mu_0^2} \quad as\ \mu \rightarrow \infty$$

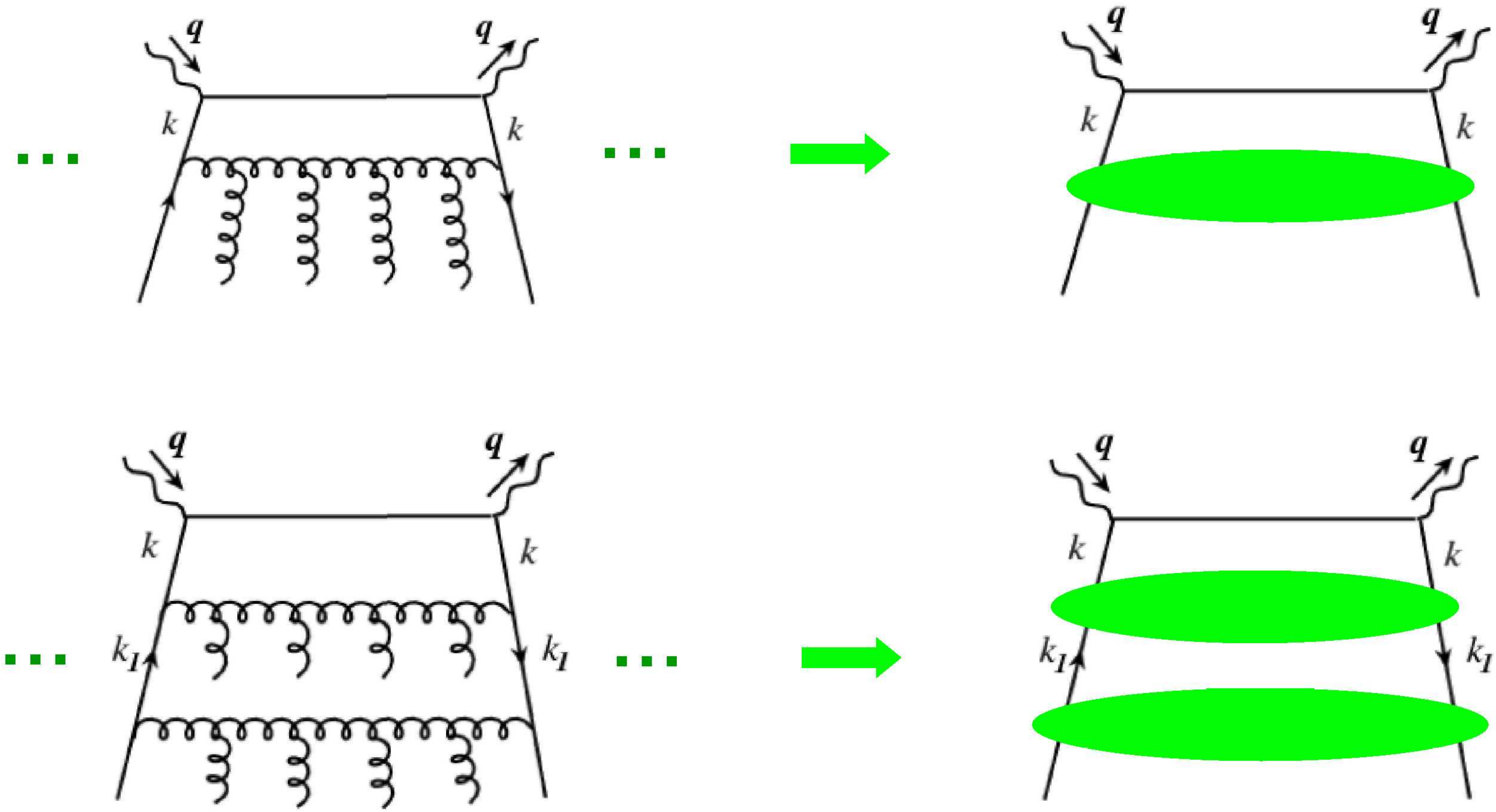
power correction can build up a big effect to low Q^2 distribution



What about high power corrections?

Modified ladder diagrams

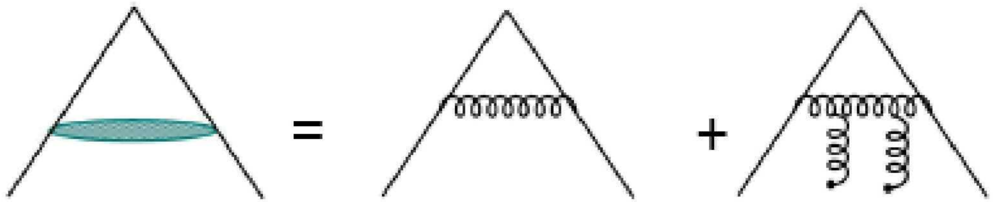
❑ Leading-pole – Leading $A^{1/3}$ term – less diagrams



→ **Modified DGLAP evolution equations at all powers**

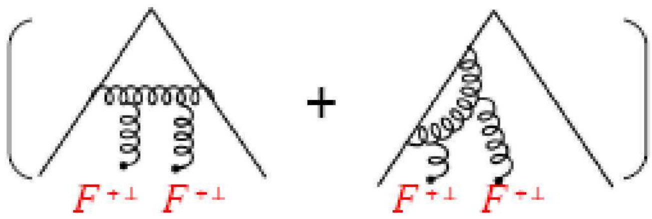
Evolution kennels for modified DGLAP

- Modified $q \to q$ evolution kennel:



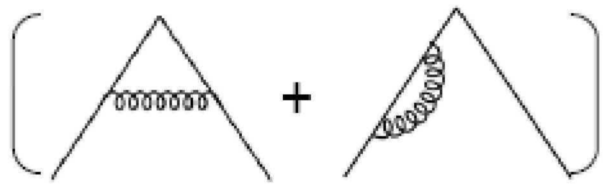
$+\ldots$ + Virtual diagrams

- First non-trivial term:

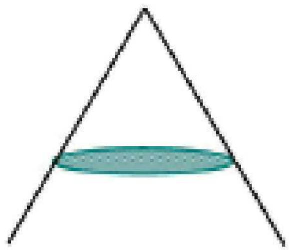


$$\sim [\langle F^{+\perp} F_{\perp}^{+} \rangle \left(\frac{N_c}{N_c^2-1}\right)\cdot\left(\frac{1}{2}\right)(-1)]\, \frac{d}{dy}\delta(y-y_B)$$

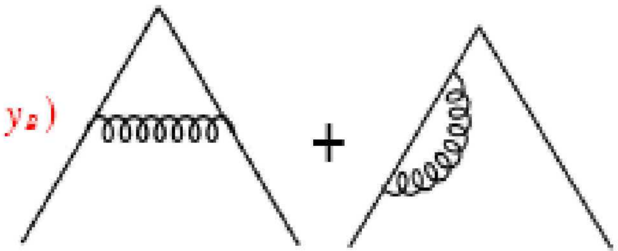
$*$



- Sum of all power corrections:



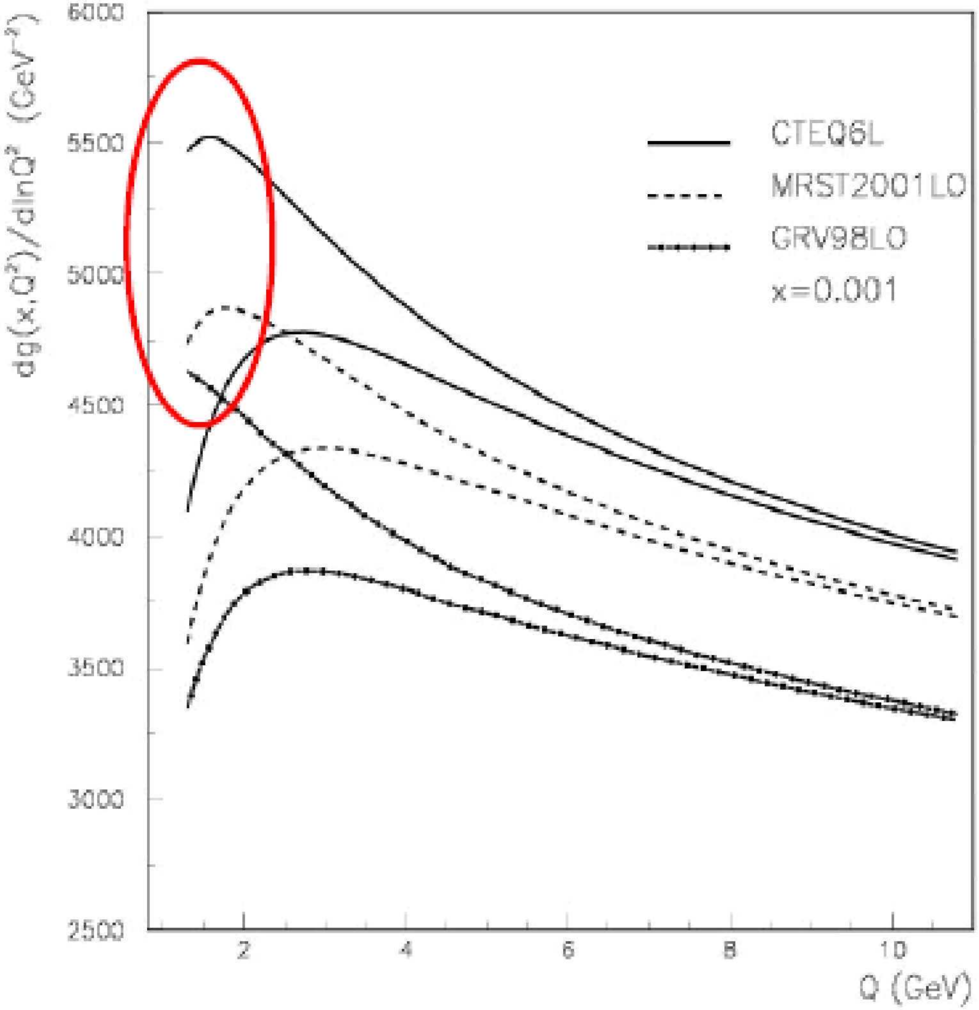
$$\sim \underbrace{\sum_{N=0}[\langle F^{+\perp} F_{\perp}^{+} \rangle \left(\frac{N_c}{N_c^2-1}\right)\cdot\left(\frac{1}{2}\right)(-1)]^{N}}_{\textit{Translation Operator}\quad e^{-c\frac{d}{dy}}} \frac{d^N}{dy^N}\delta(y-y_B)$$

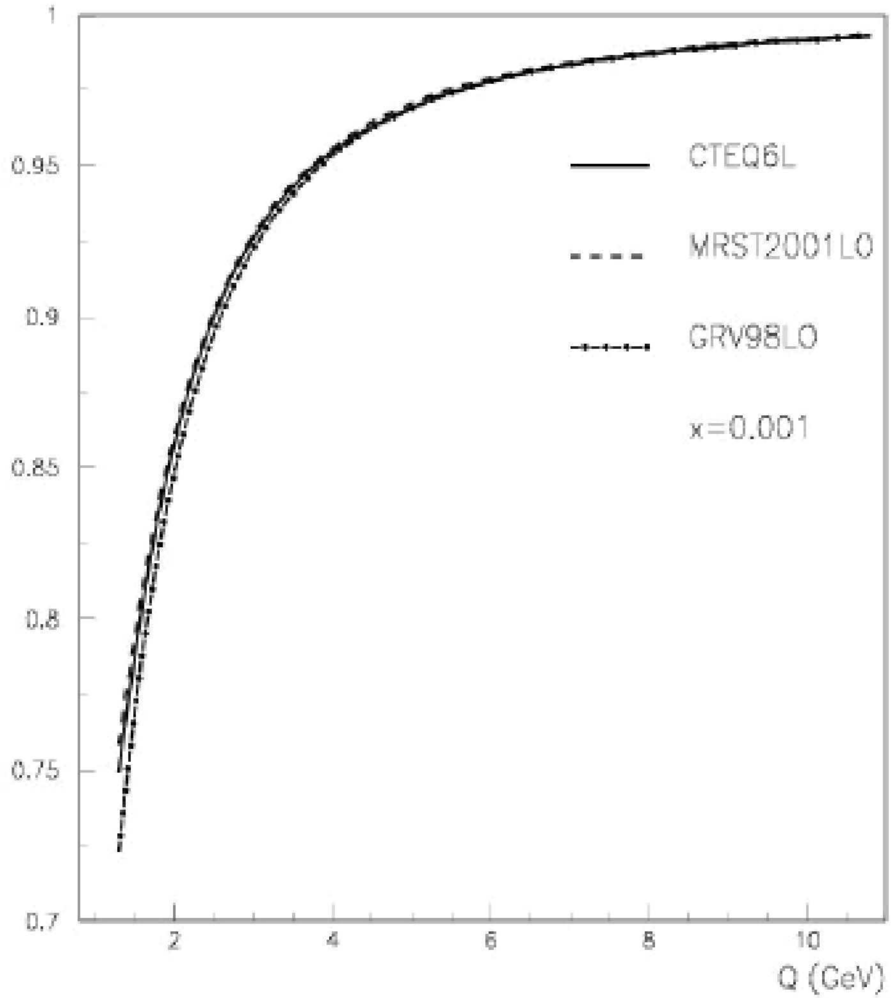


- Similar results for the other kennels

Numerical results

Gluon evolution slope





Important at small x and low Q^2

Negative gluon distribution at low Q?

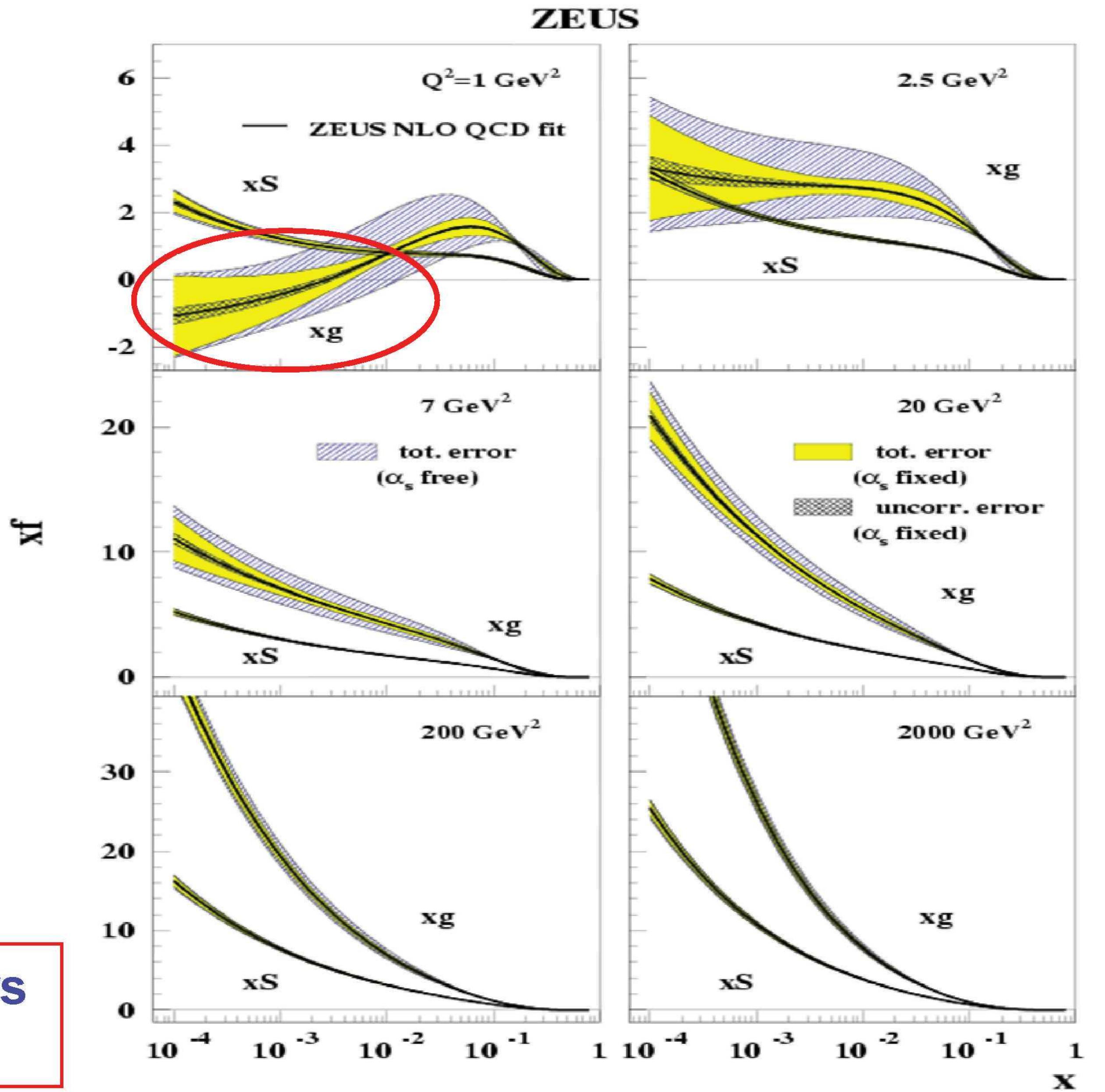
- ❑ NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution

- ❑ MRST, CTEQ PDF's have the same features

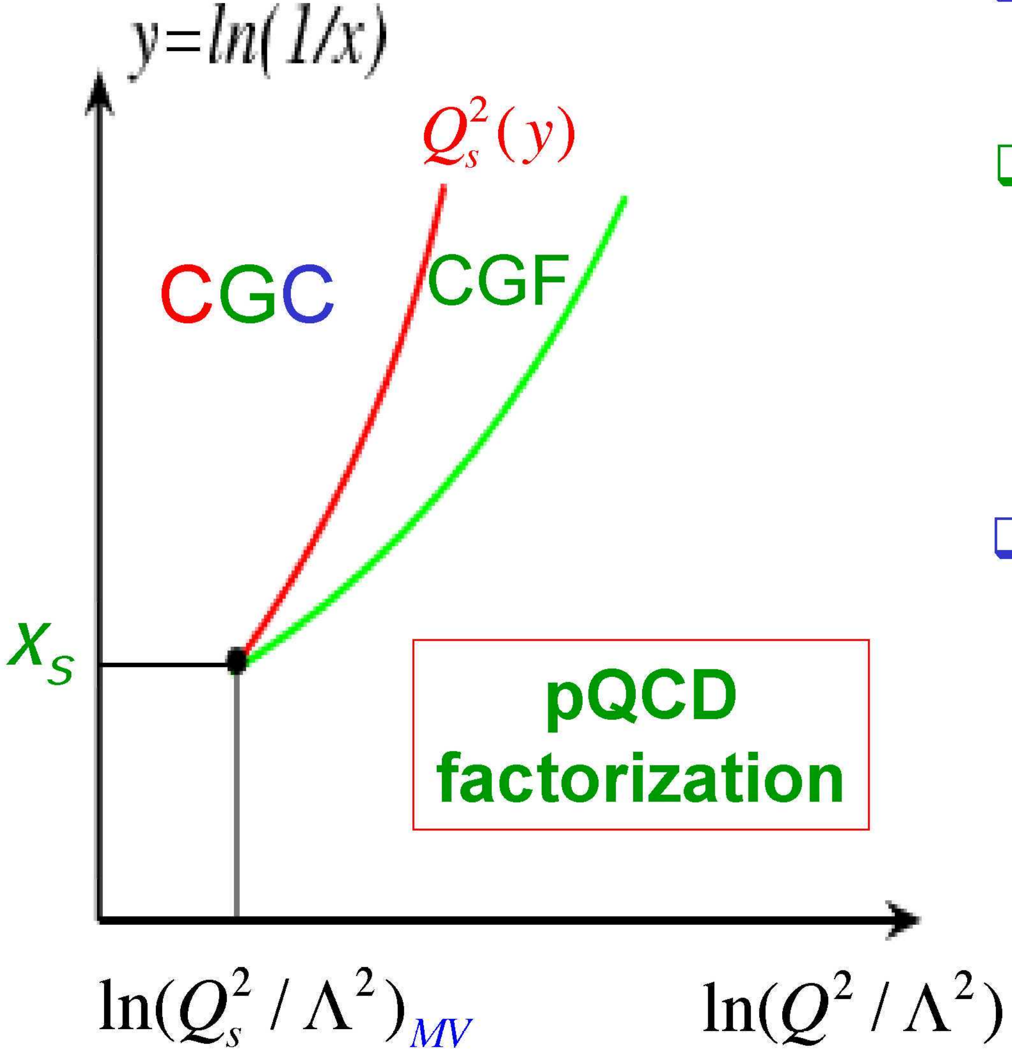
Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

No!

Power corrections slows down small-x evolution



Phase diagram of parton densities



- Experiments measure **cross sections, not PDFs**
- **PDFs are extracted based on**
  - factorization
  - truncation of perturbative expansion
- How to probe the boundary between different regions?

- Look for where pQCD factorization fails
- Power corrections
  – improve predictive power of factorization approach

Summary and outlook

- ❑ Hard probe with an active small x is not "local"
- ❑ Coherent QCD soft scattering – power corrections
- ❑ Leading-pole power corrections could be enhanced at small-x *(steep slope of PDFs)*
- ❑ Leading-pole power corrections are expressed in terms of only ONE universal matrix element

$$\left\langle F^{+\alpha} F_{\alpha}^{+} \right\rangle \sim \frac{1}{p^{+}} \int \frac{dy^{-}}{2\pi} \left\langle N \middle| F^{+\alpha}(0) F_{\alpha}^{+}\left(y^{-}\right) \middle| N \right\rangle$$

- ❑ Power corrections to DGLAP evolution are important
- ❑ Leading-pole power corrections vanish for saturated nPDFs

Global fitting of nPDFs needs to include both power corrections to hard parts as well as the evolution kennels

Backup transparencies

Model for the correlation functions

- **Matrix elements:**

$$\left\langle P_A \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(y^-) \left[\prod_{i=1}^{N} \int \tilde{F}^2(0) \right] \right| P_A \right\rangle$$

- **Approximation:**

Nucleus is made of a group of loosely bound nucleons

$$|P_A\rangle \propto \prod_{i=1}^{A} |p\rangle \quad \text{with } p = \frac{P_A}{A}$$

$$\left\langle P_A \left| \hat{O}_0 \prod_{i=1}^{N} \hat{O}_i \right| P_A \right\rangle \propto A \left\langle p \left| \hat{O}_0 \right| p \right\rangle \prod_{i=1}^{N} \left\langle p \left| \hat{O}_i \right| p \right\rangle$$

- **Reduce the correlation functions to one unknown – a universal matrix element**

$$\left\langle p \left| F^{+\alpha} F_\alpha{}^{+} \right| p \right\rangle$$