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Effect of the Neutrino Electromagnetic Form Factors on Neutrino Interaction in Dense Matter

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Particles-Nucleus and Nucleus-NucleusScattering at Relativistic Energies
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OUTLINE

- The Aim of Research
- Formalism (Model of matter, Neutrino interaction with matter)
- Results and Discussion
- Conclusion

MOTIVATION

- In Standard Model neutrino have zero charge radius (R), zero magnetic moment (μ_ν) and massless.
- However, there are experimental evidences found that $\mu_\nu < 10^{-10}\ \mu_B$ at the 90% confidence level **[Z. Daraktchieva et al (MUNU Collaboration) PLB 564, 190 (2003)]**
- LAMF experiment also found that neutrino have $R^2 = (22.5 \pm 67.5) \times 10^{-12}\ \mathrm{MeV}^{-2}$ **[P.Vilain et al, PLB 345, 115 (1995)]**
- **According to this results**

OUR AIM

1. We observe the effect of neutrino and anti-neutrino electromagnetic form factors on neutrino mean free path (λ).

2. We also observe effect the weak magnetism of nucleons on neutrino mean free path

RELATIVISTIC NUCLEAR MODEL

- **The Effective Lagrangian of E-RMF Model**

$$\mathcal{L}^{nuc} = \mathcal{L}_N + \mathcal{L}_M,$$

where the nucleon part, up to order $\nu = 3$, has the form

$$\mathcal{L}_N = \bar{\psi}[i\gamma^\mu(\partial_\mu + i\bar{v}_\mu + ig_\rho\bar{b}_\mu + ig_\omega V_\mu)$$

$$+ g_A\gamma^\mu\gamma^5\bar{a}_\mu - M + g_\sigma\sigma]\psi - \frac{f_\rho g_\rho}{4M}\bar{\psi}\bar{b}_{\mu\nu}\sigma^{\mu\nu}\psi,$$

with

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}, \quad \bar{v}_\mu = -\frac{i}{2}(\bar{\xi}^\dagger\partial_\mu\bar{\xi} + \bar{\xi}\partial_\mu\bar{\xi}^\dagger) = \bar{v}_\mu^\dagger,$$

RELATIVISTIC NUCLEAR MODEL

$$\bar{a}_\mu = -\frac{i}{2}(\bar{\xi}^\dagger \partial_\mu \bar{\xi} - \bar{\xi}\partial_\mu \bar{\xi}^\dagger) = \bar{a}_\mu^\dagger,$$

$$\bar{\xi} = \exp(i\bar{\pi}(x)/f_\pi), \quad \bar{\pi}(x) = \frac{1}{2}\vec{\tau}\cdot\vec{\pi}(x),$$

$$\bar{\pi}(x) = \frac{1}{2}\vec{\tau}\cdot\vec{\pi}(x),$$

$$\bar{b}_{\mu\nu} = D_\mu \bar{b}_\nu - D_\nu \bar{b}_\mu + i g_\rho [\bar{b}_\mu, \bar{b}_\nu], \quad D_\mu = \partial_\mu + i\bar{v}_\mu$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu,$$

$$\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu].$$

R. J. Furnstahl, et. Al, Nucl. Phys. A. 598, 539(1996); Nucl. Phys. A. 615, 441 (1997)

RELATIVISTIC NUCLEAR MODEL

$$
\begin{aligned}
\mathcal{L}_{\mathcal{M}} = & \frac{1}{4} f_\pi^2 \mathrm{Tr}(\partial_\mu \bar{U} \partial^\mu \bar{U}^\dagger) + \frac{1}{4} f_\pi^2 \mathrm{Tr}(\bar{U}\bar{U}^\dagger - 2) \\
& + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} \mathrm{Tr}(\bar{b}_{\mu\nu} \bar{b}^{\mu\nu}) - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} \\
& - g_{\rho\pi\pi} \frac{2 f_\pi^2}{m_\rho^2} \mathrm{Tr}(\bar{b}_{\mu\nu} \bar{v}^{\mu\nu}) + \frac{1}{2} \Big(1 + \eta_1 \frac{g_\sigma \sigma}{M} \\
& + \frac{\eta_2}{2} \frac{g_\sigma^2 \sigma^2}{M^2} \Big) m_\omega^2 V_\mu V^\mu + \frac{1}{4!} \zeta_0 g_\omega^2 (V_\mu V^\mu)^2 \\
& + \Big(1 + \eta_\rho \frac{g_\sigma \sigma}{M} \Big) m_\rho^2 \mathrm{Tr}(\bar{b}_\mu \bar{b}^\mu) \\
& - m_\sigma^2 \sigma^2 \Big(1 + \frac{\kappa_3}{3!} \frac{g_\sigma \sigma}{M} + \frac{\kappa_4}{4!} \frac{g_\sigma^2 \sigma^2}{M^2} \Big),
\end{aligned}
$$

where

$$
\bar{U} = \bar{\xi}^2, \quad \bar{v}_{\mu\nu} = \partial_\mu \bar{v}_\nu - \partial_\nu \bar{v}_\mu + i[\bar{v}_\mu, \bar{v}_\nu] = -i[\bar{a}_\mu, \bar{a}_\nu].
$$

R. J. Furnstahl, et. Al, Nucl. Phys. A. 598, 539(1996); Nucl. Phys. A. 615, 441 (1997)

PARAMETER SET

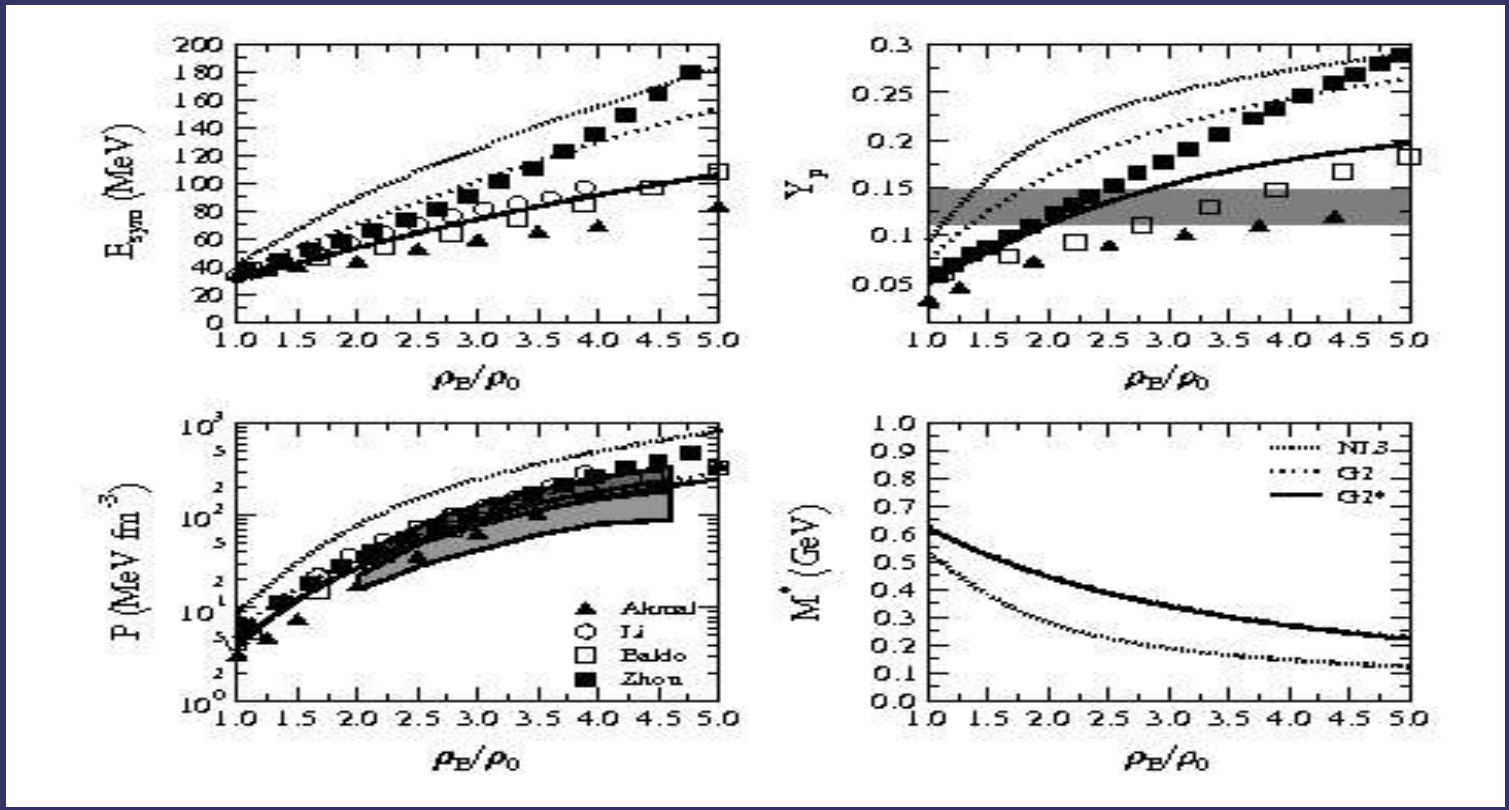
Parameter	G2	NL-3	G2*
m_S/M	0.554	0.541	0.554
$g_S/(4\pi)$	0.835	0.813	0.835
$g_V/(4\pi)$	1.016	1.024	1.016
$g_\rho/(4\pi)$	0.755	0.712	0.938
κ_3	3.247	1.465	3.247
κ_4	0.632	−5.668	0.632
ζ_0	2.642	0	2.642
η_1	0.650	0	0.650
η_2	0.110	0	0.110
η_ρ	0.390	0	4.490

Why we choose this parameter sets ?

A. Sulaksono, C. K. Williams, P.T.P. Hutauruk and T. Mart
Phys Rev C73, 025803(2006)

PARAMETER SET

Because G_2^* parameter set predicts a soft EOS at high density and has coincides with the direct URCA prosess threshold.



Shaded region data experiment from P. Danielewicz

LAGRANGIAN DENSITY

- The Lagrangian density of neutrino matter interactions for each constituent

$$\mathcal{L}_{\rm int}^{j} = \frac{G_F}{\sqrt{2}}(\bar{\nu}\Gamma_W^{\mu}\nu)(\bar{\psi}J_{\mu}^{Wj}\psi) + \frac{4\pi\alpha}{q^2}(\bar{\nu}\Gamma_{\rm EM}^{\mu}\nu)(\bar{\psi}J_{\mu}^{{\rm EM}j}\psi),$$

$$\Gamma_W^{\mu} = \gamma^{\mu}(1-\gamma^5),$$

$$\Gamma_{\rm EM}^{\mu} = f_{mv}\gamma^{\mu} + g_{1v}\gamma^{\mu}\gamma^5 - (f_{2v} + ig_{2v}\gamma^5)\frac{p^{\mu}}{2m_e},$$

$$J_{\mu}^{Wj} = F_1^{Wj}\gamma_{\mu} - G_A^{j}\gamma_{\mu}\gamma^5 + iF_2^{Wj}\frac{\sigma_{\mu\nu}q^{\nu}}{2M},$$

$$J_{\mu}^{{\rm EM}j} = F_1^{{\rm EM}j}\gamma_{\mu} + iF_2^{{\rm EM}j}\frac{\sigma_{\mu\nu}q^{\nu}}{2M}.$$

FORM FACTORS OF NUCLEON

Target	F_1^N	G_A	F_2^N
n	-0.5	$-g_A/2$	$-1/2(\mu_p-\mu_n)-2\sin^2\theta_w\mu_n$
p	$0.5-2\sin^2\theta_w$	$g_A/2$	$1/2(\mu_p-\mu_n)-2\sin^2\theta_w\mu_p$
e	$0.5+2\sin^2\theta_w$	$1/2$	0
μ	$-0.5+2\sin^2\theta_w$	$-1/2$	0

Target	F_1^{EM}	F_2^{EM}
n	0	μ_n
p	1	μ_p
e	1	0
μ	1	0

For neutrinos, while for anti-neutrinos the sign are changed, $g_A \longrightarrow -g_A$

$\mu_p = 1.793$ $\quad$ $g_A = 1.260$

$\mu_n = -1.913$ $\quad$ $\sin^2\Theta_W = 0.231$

P. Vogel and J. Engel, PRD 39, 11 (1989)

FORM FACTORS OF NEUTRINO

$$\Gamma^{\mu}_{W} = \gamma^{\mu}(1 - \gamma^{5}), \qquad (6)$$

while the electromagnetic properties of Dirac neutrinos are described in terms of four form factors $f_{1\nu}$, $g_{1\nu}$, $f_{2\nu}$, and $g_{2\nu}$, which stand for the Dirac, anapole, magnetic, and electric form factors, respectively. The electromagnetic vertex Γ^{μ}_{EM} contains electromagnetic form factors [29,30]. Explicitly, it reads

$$\Gamma^{\mu}_{EM} = f_{m\nu}\gamma^{\mu} + g_{1\nu}\gamma^{\mu}\gamma^{5} - (f_{2\nu} + ig_{2\nu}\gamma^{5})\frac{P^{\mu}}{2m_{e}}, \qquad (7)$$

where $f_{m\nu} = f_{1\nu} + (m_{\nu}/m_{e})f_{2\nu}$, $P^{\mu} = k^{\mu} + k^{\mu\prime}$, and m_{ν} and m_{e} are the neutrino and electron masses, respectively. In the static limit, the reduced Dirac form factor $f_{1\nu}$ and the neutrino anapole form factor $g_{1\nu}$ are related to the vector and axial vector charge radii $\langle R^{2}_{V}\rangle$ and $\langle R^{2}_{A}\rangle$ through [29]

$$f_{1\nu}(q^{2}) = \frac{1}{6}\langle R^{2}_{V}\rangle q^{2} \quad \text{and} \quad g_{1\nu}(q^{2}) = \frac{1}{6}\langle R^{2}_{A}\rangle q^{2}, \qquad (8)$$

where the neutrino charge radius is defined by $R^{2} = \langle R^{2}_{V}\rangle + \langle R^{2}_{A}\rangle$. In the limit of $q^{2} \to 0$, $f_{2\nu}$ and $g_{2\nu}$, respectively, define the neutrino magnetic moment and the Charge Parity (CP) violating electric dipole moment [29,31], i.e.,

$$\mu^{m}_{\nu} = f_{2\nu}(0)\mu_{B} \quad \text{and} \quad \mu^{e}_{\nu} = g_{2\nu}(0)\mu_{B},$$

FORMALISM

- Neutrino and Anti-Neutrino Differential Cross Section

$$\left(\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}\right) = -\frac{1}{16\pi^2}\frac{E'_\nu}{E_\nu}\left[\left(\frac{G_F}{\sqrt{2}}\right)^2 \left(L^{\mu\nu}_\nu \Pi^{\mathrm{Lep}}_{\mu\nu}\right)^{(W)} + \left(\frac{4\pi\alpha}{q^2}\right)^2 \left(L^{\mu\nu}_\nu \Pi^{\mathrm{Lep}}_{\mu\nu}\right)^{(EM)} + \frac{8G_F\pi\alpha}{q^2\sqrt{2}}\left(L^{\mu\nu}_\nu \Pi^{\mathrm{Lep}}_{\mu\nu}\right)^{(INT)}\right].$$

A. Sulaksono, C. K. Williams, P. T. P. Hutauruk and T. Mart , PRC 73, 025803 (2006)

- Neutrino and Anti-Neutrino Mean Free Path

LEPTON TENSOR OF NEUTRINO

$$L_\nu^{\mu\nu(\mathrm{W})} = 8\left[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta\right],$$

$$\begin{aligned} L_\nu^{\mu\nu(\mathrm{EM})} &= 4(f_{m\nu}^2 + g_{1\nu}^2)[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q)] \\ &- 8i f_{m\nu} g_{1\nu} \epsilon^{\alpha\mu\beta\nu}(k_\alpha k'_\beta) \\ &- \frac{f_{2\nu}^2 + g_{2\nu}^2}{m_e^2}(k\cdot q)[4k^\mu k^\nu - 2(k^\mu q^\nu + q^\mu k^\nu) + q^\mu q^\nu], \end{aligned}$$

$$f_{m\nu} = f_{1\nu} + (m_\nu/m_e) f_{2\nu},$$

$$L_\nu^{\mu\nu(\mathrm{INT})} = 4(f_{m\nu} + g_{1\nu})[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta],$$

$f_{1\nu}$, $g_{1\nu}$, $f_{2\nu}$, $g_{2\nu}$ are Dirac, anapole, magnetic, electric form factors respectively

POLARISATION TENSOR

$$\Pi^{\mathrm{Lo}(W)j}_{\mu\nu} = (F_1^{Wj\,2} + G_A^{j\,2})\Pi^{Vj}_{\mu\nu} + \left(G_A^{j2} + \frac{q^2}{2mM} F_1^{Wj} F_2^{Wj}\right)\Pi^{Aj} g_{\mu\nu} - 2\left(F_1^{Wj} G_A^j + \frac{m}{M} F_2^{Wj} G_A^j\right)\Pi^{V-Aj}_{\mu\nu} + \frac{F_2^{Wj\,2}}{M^2} \times \left[\left(m^2 + \frac{q^2}{4}\right)(q^2 g_{\mu\nu} - q_\mu q_\nu) - \frac{q^2}{8}\Pi^{Vj}_{\mu\nu}\right],$$

$$\Pi^{\mathrm{Lo}(\mathrm{EM})j}_{\mu\nu} = F_1^{\mathrm{EM}j\,2}\Pi^{Vj}_{\mu\nu} + \frac{q^2}{2mM} F_1^{\mathrm{EM}j} F_2^{\mathrm{EM}j}\Pi^{Aj} g_{\mu\nu} + \frac{F_2^{\mathrm{EM}j\,2}}{M^2}\left[\left(m^2 + \frac{q^2}{4}\right) \times (q^2 g_{\mu\nu} - q_\mu q_\nu) - \frac{q^2}{8}\Pi^{Vj}_{\mu\nu}\right],$$

$$\Pi^{\mathrm{Lo}(\mathrm{INT})j}_{\mu\nu} = \left(F_1^{Wj} F_1^{\mathrm{EM}j} + \frac{q^2}{4M^2} F_2^{Wj} F_2^{\mathrm{EM}j}\right)\Pi^{Vj}_{\mu\nu} + \left[\frac{F_2^{Wj} F_2^{\mathrm{EM}j}}{4M^2}\left(1 + \frac{q^2}{4m^2}\right) - \frac{\left(F_1^{Wj} F_2^{\mathrm{EM}j} + F_2^{Wj} F_1^{\mathrm{EM}j}\right)}{4mM}\right] \times (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi^{Aj} + \left(\frac{m}{M} F_2^{\mathrm{EM}j} G_A^j - F_1^{\mathrm{EM}j} G_A^j\right)\Pi^{V-Aj}_{\mu\nu},$$

CONTRACTION RESULTS

$$\left(L^{\mu\nu}_{\nu}\Pi^{\mathrm{Lo}}_{\mu\nu}\right)^{(W)} = -8q^2 \sum_{j=n,p,e^-,\mu^-} \left[A^j_W\left(\Pi^j_L + \Pi^j_T\right) + B^j_{1W}\Pi^j_T + B^j_{2W}\Pi^j_A + C^j_W\Pi^j_{VA}\right],$$

$$\left(L^{\mu\nu}_{\nu}\Pi^{\mathrm{Lo}}_{\mu\nu}\right)^{(\mathrm{EM})} = q^2 \sum_{j=n,p,e^-,\mu^-} \left[A^j_{\mathrm{EM}}\left(\Pi^j_L + \Pi^j_T\right) + B^j_{1\mathrm{EM}}\Pi^j_T + B^j_{2\mathrm{EM}}\Pi^j_A\right],$$

$$\left(L^{\mu\nu}_{\nu}\Pi^{\mathrm{Lo}}_{\mu\nu}\right)^{(\mathrm{INT})} = -4q^2 \sum_{j=n,p,e^-,\mu^-} \left[A^j_{\mathrm{INT}}\left(\Pi^j_L + \Pi^j_T\right) + B^j_{1\mathrm{INT}}\Pi^j_T + B^j_{2\mathrm{INT}}\Pi^j_A + C^j_{\mathrm{INT}}\Pi^j_{VA}\right]$$

CONTRACTION RESULTS

$$\Pi_L = \frac{q^2}{2\pi|\vec{q}|^3}\left[\frac{1}{4}(E_F - E^*) + \frac{q_0}{2}\left(E_F^2 - E^{*2}\right) + \frac{1}{3}\left(E_F^3 - E^{*3}\right)\right],$$

$$\Pi_{VA} = \frac{iq^2}{8\pi|\vec{q}|^3}\left[\left(E_F^2 - E^{*2}\right) + q_0(E_F - E^*)\right].$$

$$\Pi_A = \frac{i}{2\pi|\vec{q}|}M^{*2}(E_F - E^*).$$

CONTRACTION RESULTS

$$A^j_W = \left(\frac{2E(E-q_0)+\frac{1}{2}q^2}{|\vec{q}|^2}\right)\left[F_1^{Wj\,2} + G_A^{j\,2} - \frac{F_2^{Wj\,2}q^2}{4M^2}\right],$$

$$B^j_{1W} = \left[F_1^{Wj\,2} + G_A^{j\,2} - \frac{F_2^{Wj\,2}q^2}{4M^2}\right],$$

$$B^j_{2W} = -\left[G_A^{j\,2} + \frac{q^2}{2mM}F_1^{Wj}F_2^{Wj} - \frac{F_2^{Wj\,2}q^2}{4M^2}\left(1+\frac{q^2}{4m^2}\right)\right], \tag{29}$$

$$C^j_W = -2(2E-q_0)\left[F_1^{Wj}G_A^j + \frac{m}{M}F_2^{Wj}G_A^j\right],$$

$$A^j_{BM} = \left[\left(\frac{2E(E-q_0)+\frac{1}{2}q^2}{|\vec{q}|^2}\right)(bq^2-a) + \frac{1}{2}bq^2\right] \times \left[F_1^{BMj\,2} - \frac{F_2^{BMj\,2}q^2}{4M^2}\right],$$

$$B^j_{1BM} = -\frac{1}{2}(bq^2+a)\left[F_1^{BMj\,2} - \frac{F_2^{BMj\,2}q^2}{4M^2}\right],$$

$$B^j_{2BM} = \frac{1}{2}(bq^2+a)\left[\frac{q^2}{2mM}F_1^{BMj}F_2^{BMj} - \frac{F_2^{BMj\,2}q^2}{4M^2}\left(1+\frac{q^2}{4m^2}\right)\right],$$

CONTRACTION RESULTS

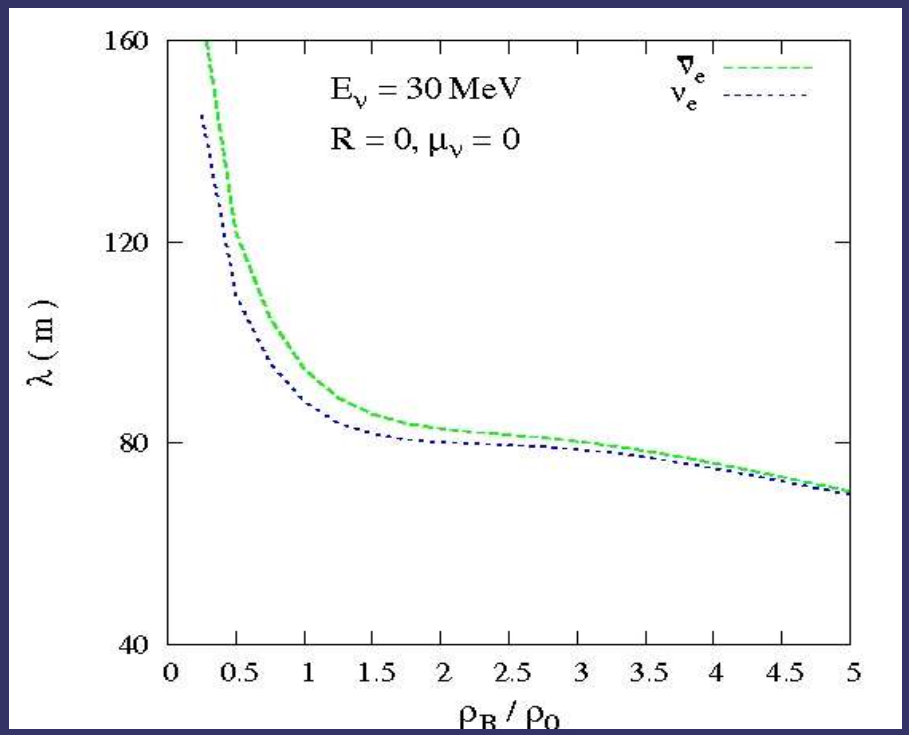
$$A^j_{\rm EM} = \left[\left(\frac{2E(E - q_0) + \frac{1}{2}q^2}{|\vec{q}|^2} \right) (bq^2 - a) + \frac{1}{2} b q^2 \right] \times \left[F_1^{{\rm EM}j\,2} - \frac{F_2^{{\rm EM}j\,2} q^2}{4M^2} \right],$$

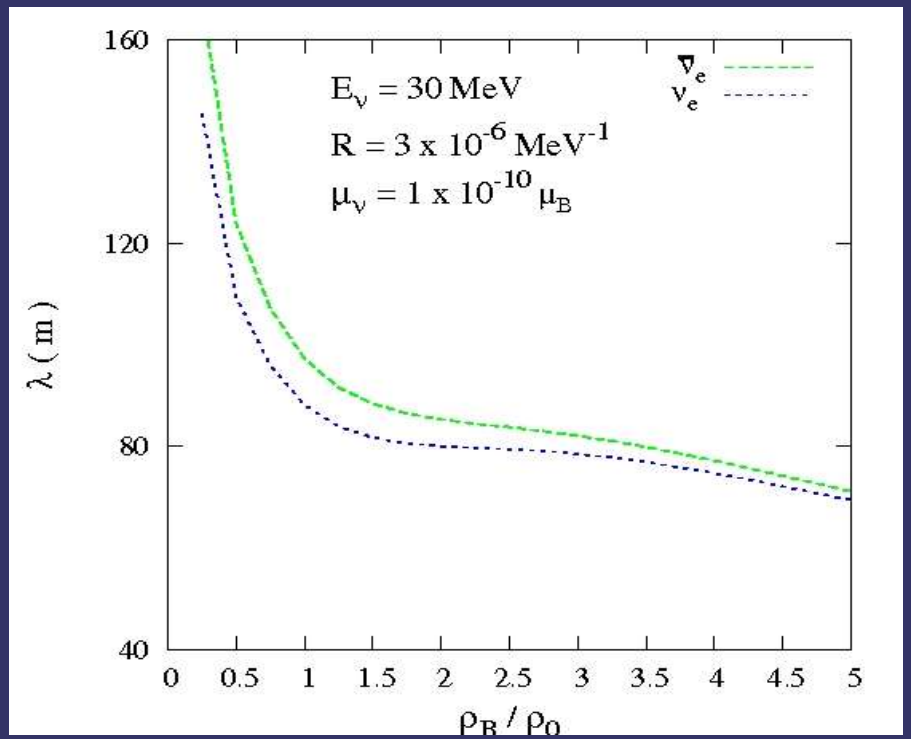
$$B^j_{1\,{\rm EM}} = -\frac{1}{2}(bq^2 + a) \left[F_1^{{\rm EM}j\,2} - \frac{F_2^{{\rm EM}j\,2} q^2}{4M^2} \right],$$

$$B^j_{2\,{\rm EM}} = \frac{1}{2}(bq^2 + a) \left[\frac{q^2}{2mM} F_1^{{\rm EM}j} F_2^{{\rm EM}j} - \frac{F_2^{{\rm EM}j\,2} q^2}{4M^2} \left(1 + \frac{q^2}{4m^2} \right) \right],$$

RESULTS AND DISCUSSION

- ν_e and $\bar{\nu}_e$ Mean Free Path Results

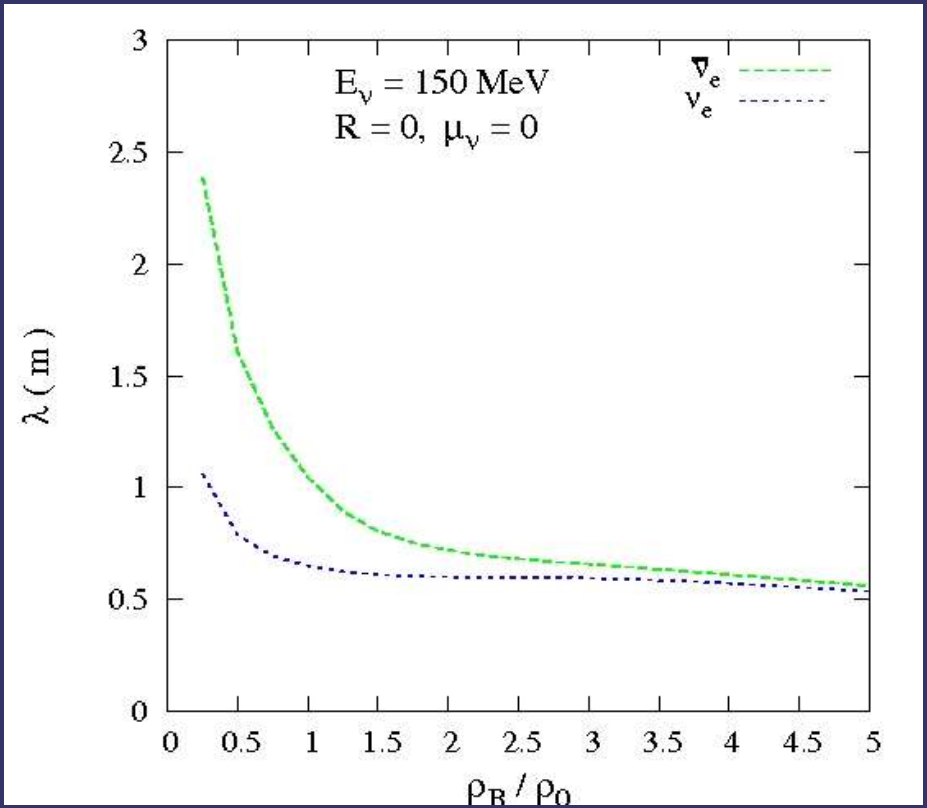


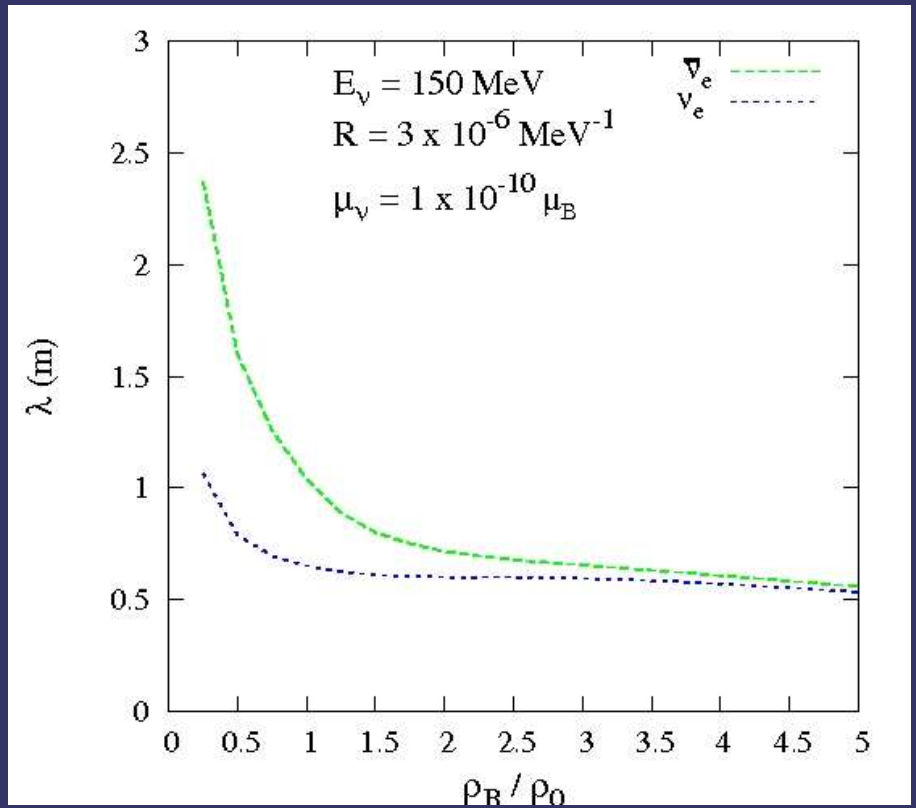


$\bar{\nu}_e$ mean free path is bigger than ν_e ⟷ the effects of weak magnetism of nucleon.

RESULTS AND DISCUSSION

- ν_e and $\bar{\nu}_e$ Mean Free Path Results

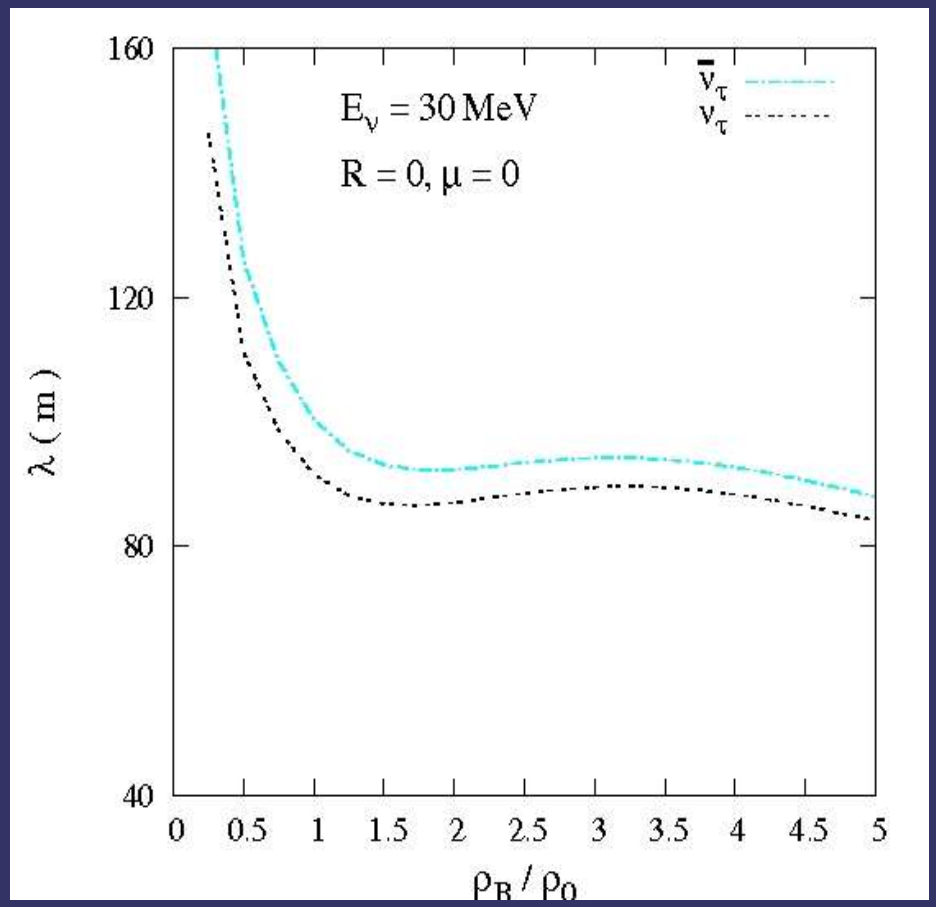


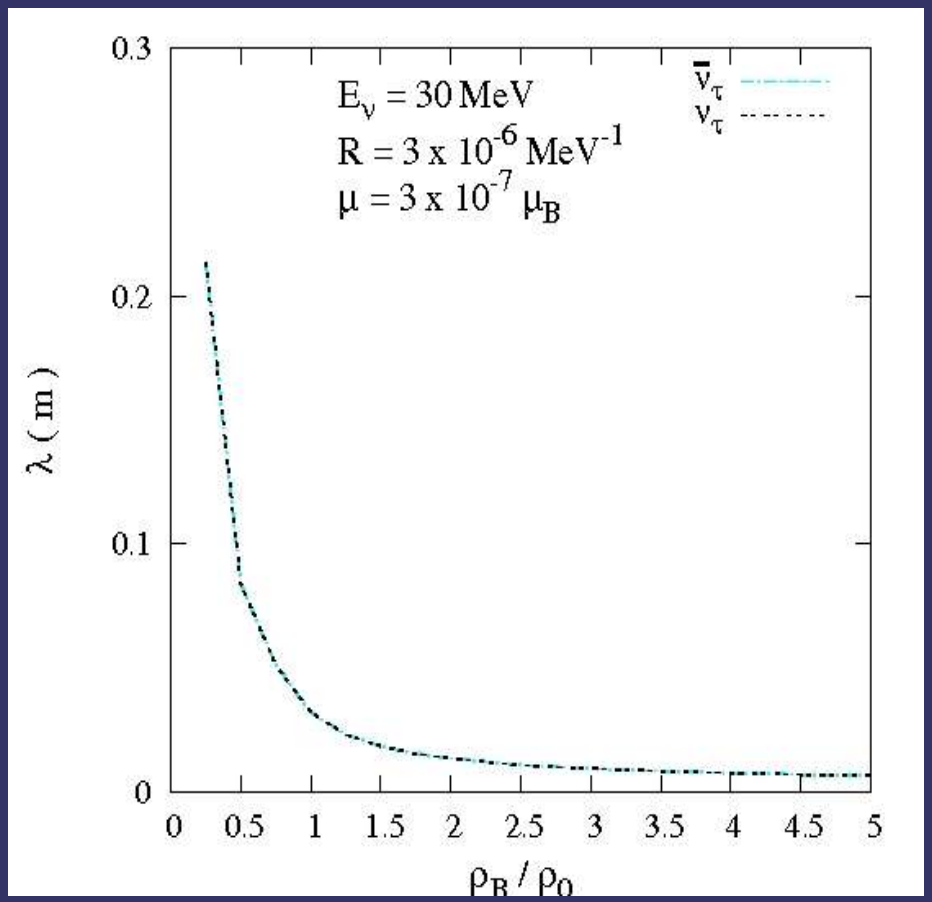


If the E_ν increase, neutrino and anti neutrino decrease. The effect moment magnetic and charge radius is too small

RESULTS AND DISCUSSION

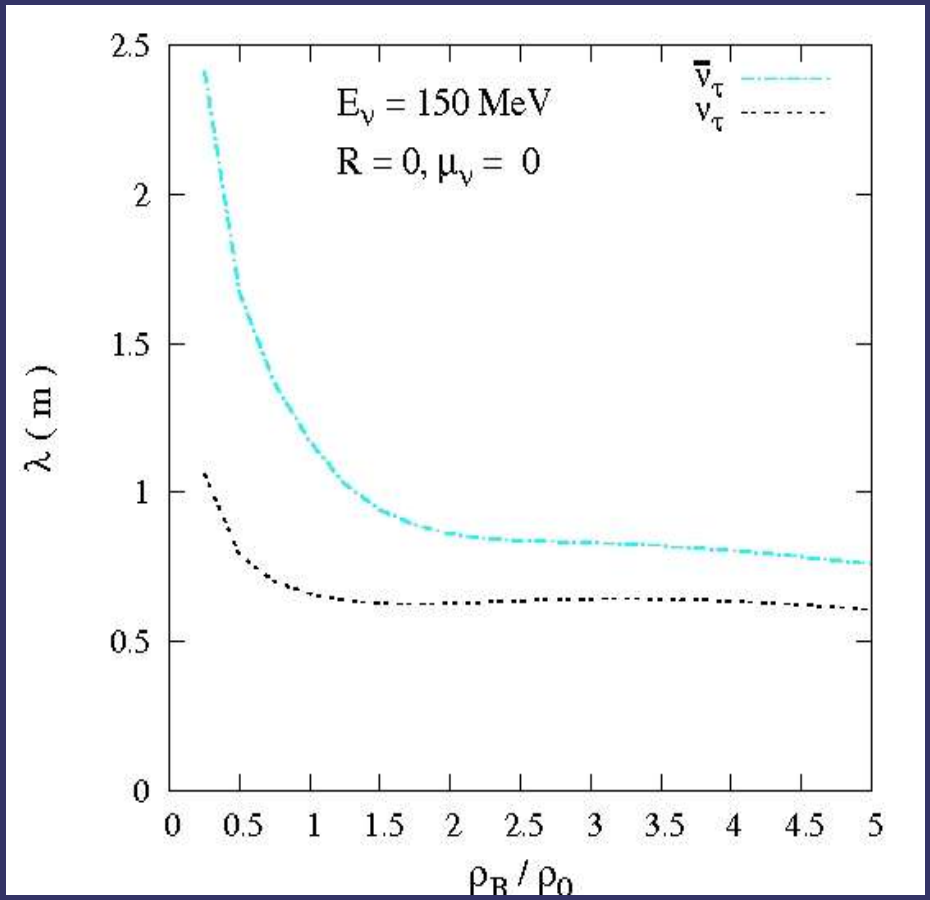
- ν_τ and $\bar{\nu}_\tau$ Mean Free Path Results

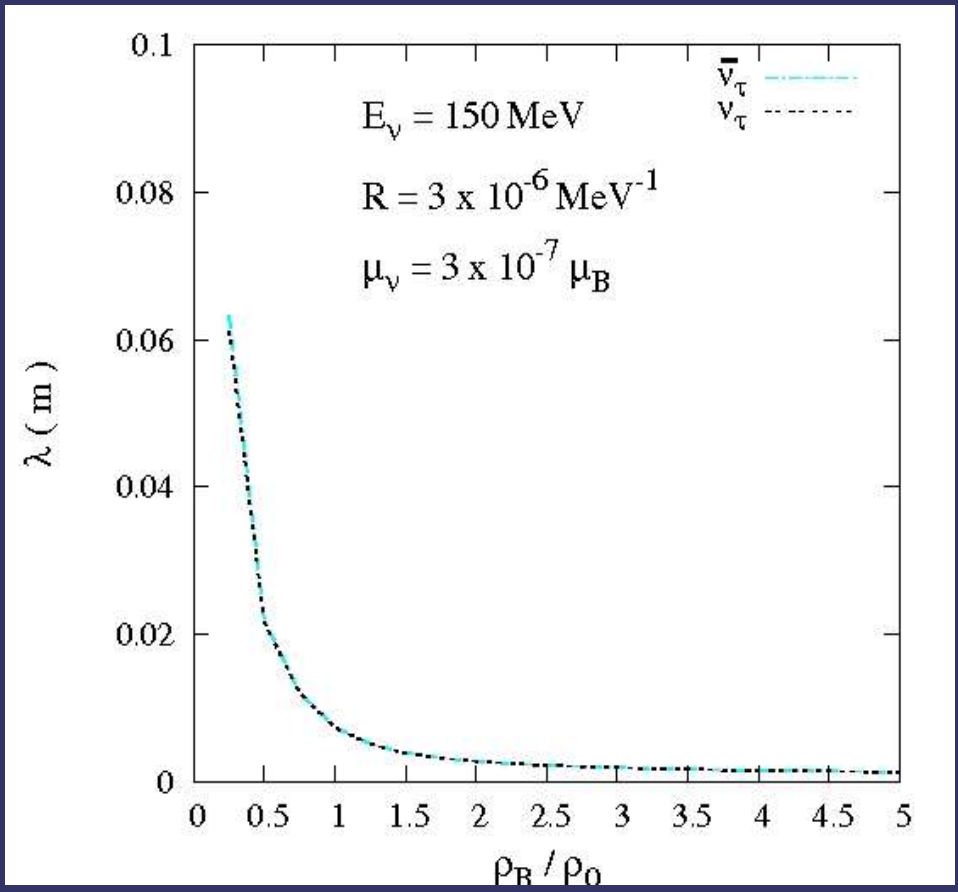




RESULTS AND DISCUSSION

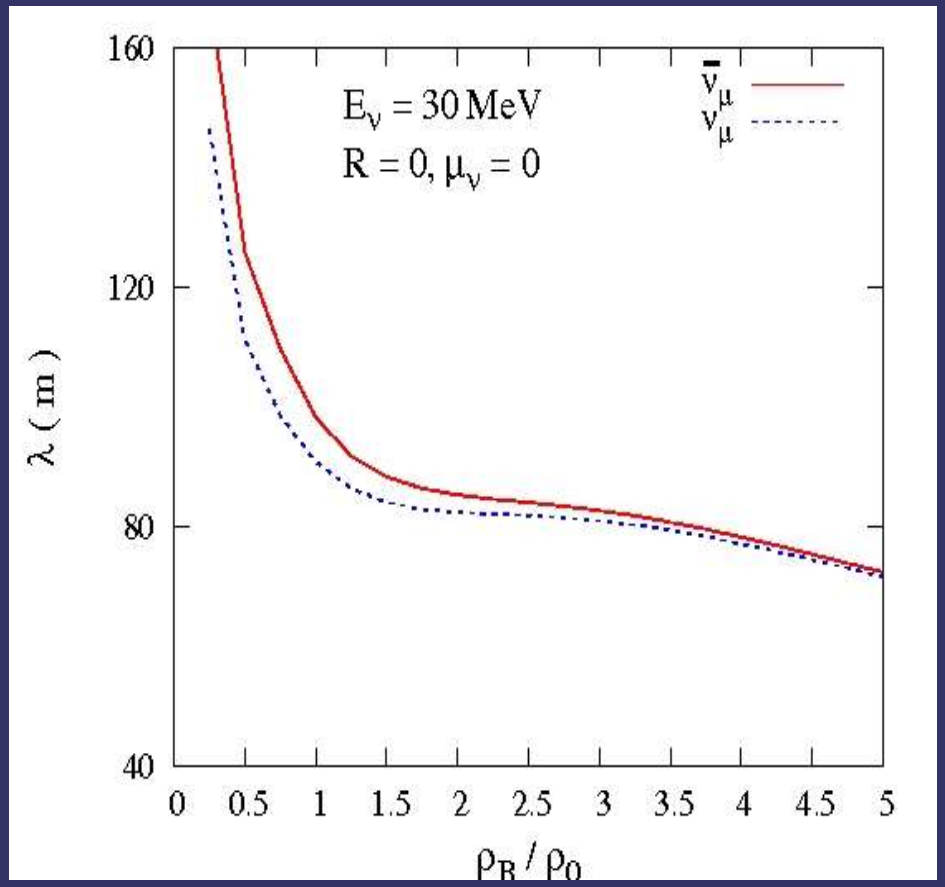
- ν_τ and $\bar{\nu}_\tau$ Mean Free Path Results

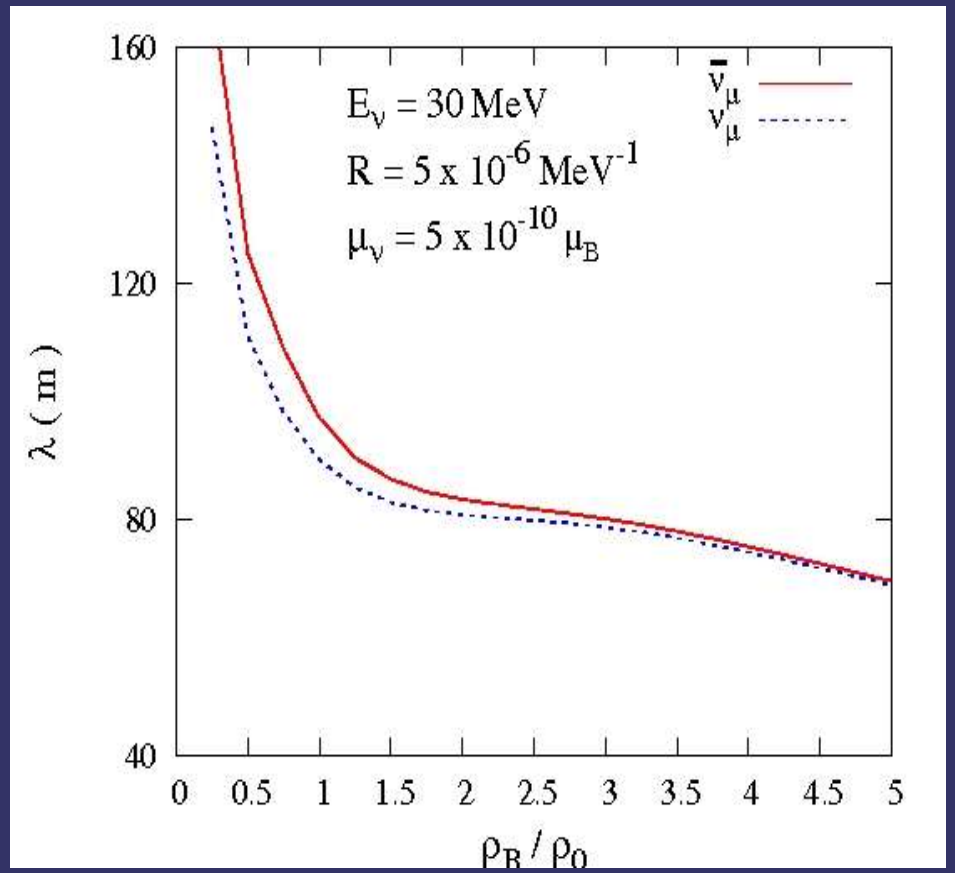




RESULTS AND DISCUSSION

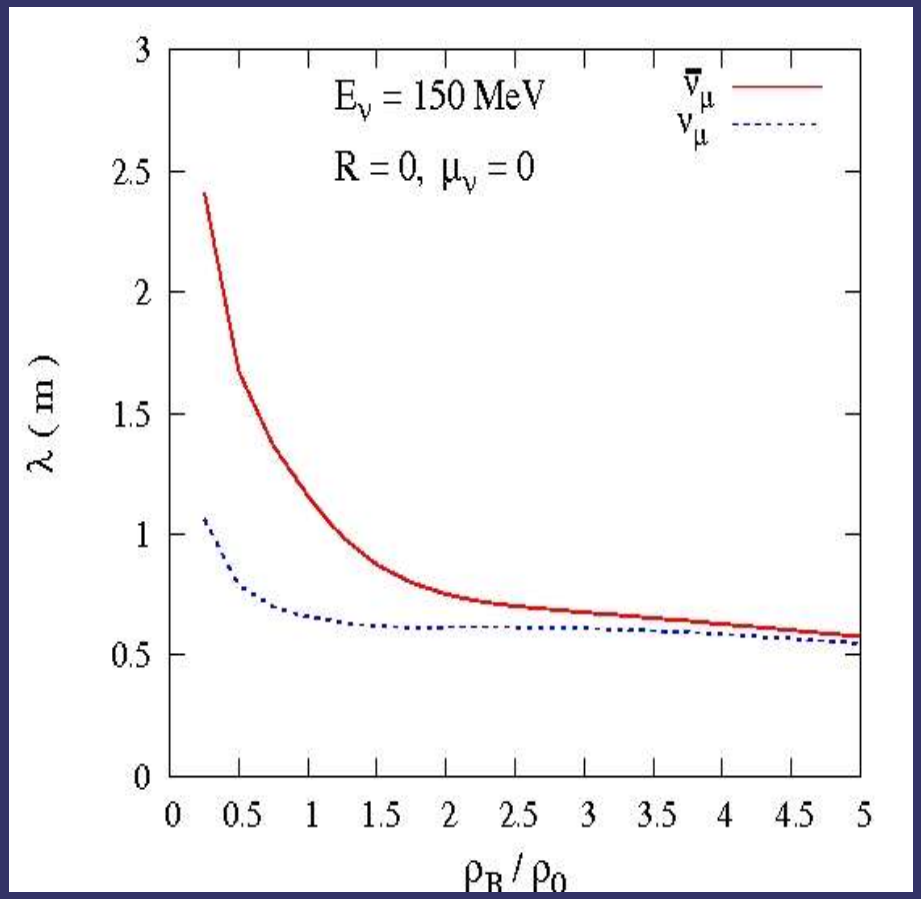
- ν_μ and $\bar{\nu}_\mu$ Mean free Path Results

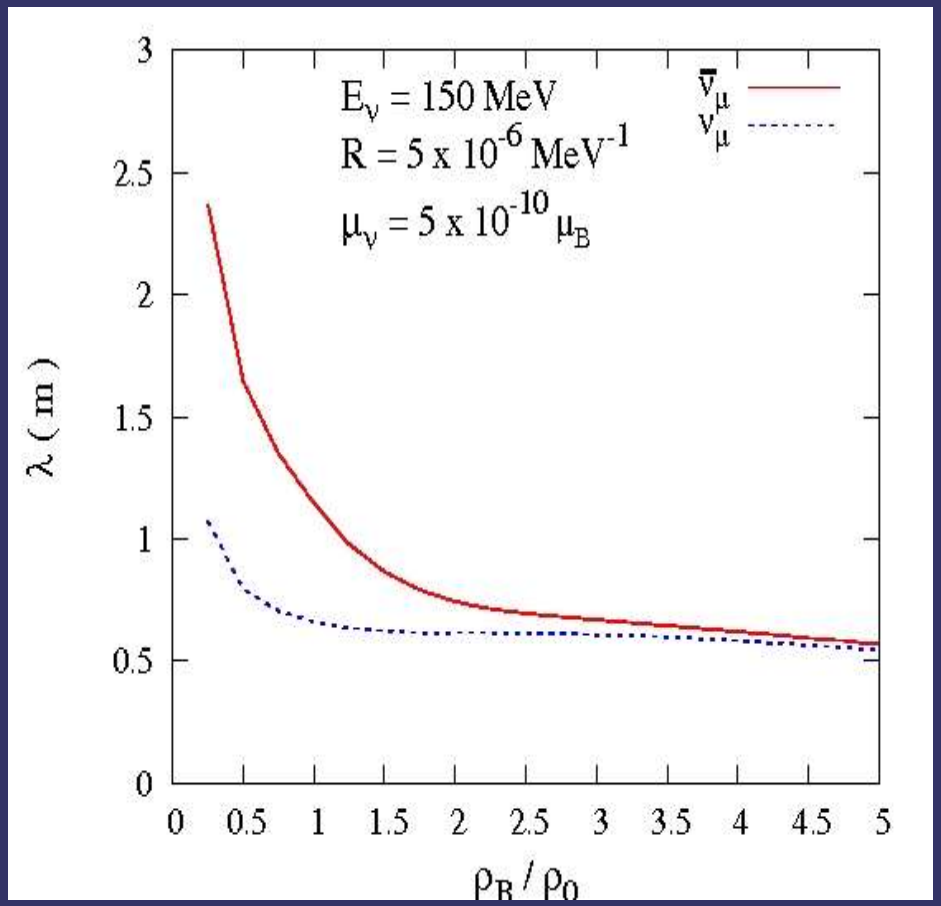




RESULTS AND DISCUSSION

- ν_μ and $\bar{\nu}_\mu$ Mean free Path Results





CONCLUSION

- The effect of electromagnetic form factor depend on the moment magnetic and charge radius of neutrino

$$\nu_\tau, \bar{\nu}_\tau < \nu_\mu, \bar{\nu}_\mu < \nu_e, \bar{\nu}_e$$

- The effect of weak magnetism make a differentiable for neutrino and anti neutrino mean free path. [similar results of C.J. Horowitz. et.al.PRC68,025803 (2003)]
- Neutrino and antineutrino mean free path decrease if energy neutrino increase.
- In general, the neutrino and anti neutrino have influence on neutrino and antineutrino mean free path but too small.


THE END
THANK YOU FOR
YOUR ATTENTION