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**Fractional momentum correlations in multiple production  
of  $w$  bosons and of  $bb$  pairs in high energy  $pp$  collisions**

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These are preliminary lecture notes, intended only for distribution to participants

# Fractional momentum correlations in multiple production of $W$ bosons and of $b\bar{b}$ pairs in high energy pp collisions

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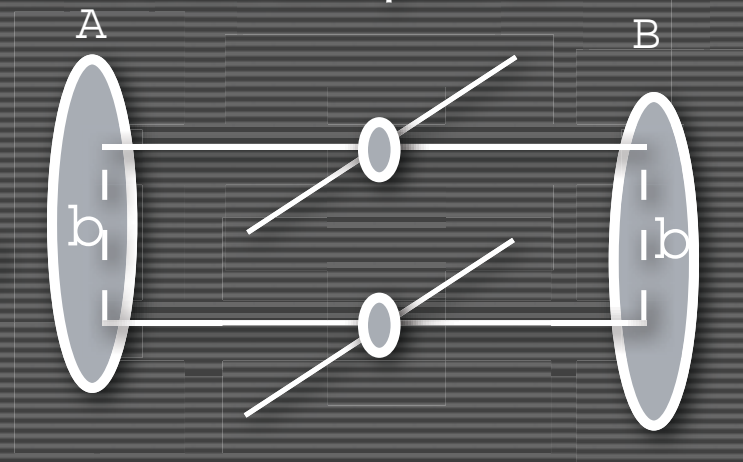
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- Importance of multiparton scattering at new collider energies
- Disconnected collisions: double parton scattering and double parton distribution;
- Double parton distribution in the LLA of pQCD: numerical results for the violation to the factorization ansatz of double parton distributions;
- Multiple production of  $b\bar{b}$  pairs and  $W$  bosons in pp-collisions: numerical results with and without correlation corrections;
- Conclusions

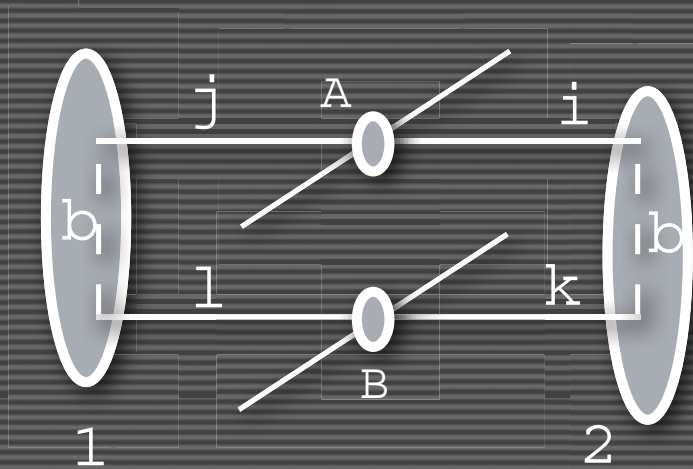
# Introduction

- RHIC and LHC high energy colliders:
  - Regime of very short distances and high momentum transfer:  
asymptotic freedom and validity of perturbative methods
  - small Bjorken  $x \Rightarrow$  growing flux of partons
  - being hadrons extended objects, more than one pair of partons is expected to interact in different points in transverse space, with large transverse momentum exchanged  $\Rightarrow$  pQCD



# Disconnected collisions: double parton scattering

- Double-parton scattering: simplest case of multiparton interaction



- Presence of two different scales:
  - 1) hard perturbative scale
  - 2) inverse of distance in transverse space of the interacting partons
- factorization of perturbative and non-perturbative component of the process

$$D_{(A,B)}^D = \frac{m}{2} \sum_{i,j,k,l} j_l(x_1, x_2, b) \quad i_k(x_1, x_2; b) \quad \hat{A}_{ij}^A(x_1, x_1) \quad \hat{B}_{kl}^B(x_2, x_2)$$

- Double parton distribution: non-perturbative ingredient providing correct dimensionality, related to two body correlations

# Disconnected collisions: double parton scattering

$$i k (x_1, x_2; b) = G_i(x_1) G_k(x_2) F_{ik}(b)$$

- correlation in transverse space and fractional momenta are neglected
- the dimensionality of the two body distribution leads to the introduction of a non-perturbative scale factor, effective cross section

$$\sigma_{eff} = \frac{d^2 b F(b)^2}{-1}$$

$$\sigma_{(A,B)}^D = \frac{m}{2} \frac{\sigma_S^A \sigma_S^B}{\sigma_{eff}}$$

- Inclusive cross-section factorized into the product of inclusive single scattering cross-section of pQCD parton model
- CDF collaboration:  $\sigma_{eff} = (14.5 \pm 1.7) \text{mb}$

# Double Parton Distribution in LLA of pQCD

- This symplified hypothesis of factorization in contrast with leading logarithm approximation of pQCD<sup>[1]</sup>
- If two parton distribution is factorized at reference scale, it becomes dinamically correlated at any different scale of a hard process
- Degree of violation of factorization ansatz can be estimated by solution of the generalized LAPD equation for two parton distribution

$$\frac{d}{dt} \frac{j_1 j_2}{h} = \frac{j_1 j_2}{h} P_{j_1 j_1} + \frac{j_1 j_2}{h} P_{j_2 j_2} + \frac{j}{h} P_j j_1 j_2$$

where:  $t = \frac{1}{2-b} \ln \left( 1 + \frac{g^2(\mu^2)}{4} b \ln \frac{\mu}{\mu_0} \right)$  and  $b = \frac{33 - 2n_f}{12}$

- $\frac{j}{h}$  satisfies the evolution equation:  $\frac{d}{dt} \frac{j}{h} = D_h^j P_j j$

[1] A. M. Snigirev, Phys. Rev. D 68, 114012 (2003) (hep-ph/0304172);  
V.L. Korotkikh, A.M. Snigirev, Phys. Lett. B 594, 171-176 (2004) (hep-ph/0404155).

# Double Parton Distribution in LLA of pQCD

- If the following initial condition is taken:

$$j_{h,j_1,j_2}^{j_1,j_2}(x_1,x_2;t=0) = j_h^{j_1}(x_1;0) j_h^{j_2}(x_2;0) (1-x_1-x_2)$$

$$j_{h,j_1,j_2}^{j_1,j_2}(x_1,x_2;t) = j_h^{j_1}(x_1;t) j_h^{j_2}(x_2;t) (1-x_1-x_2) + j_{h,j_1,j_2}^{j_1,j_2}(x_1,x_2;t)$$

- the non-factorized contribution is expressed in terms of initial single parton densities and the distribution functions  $j_i^j(x;t)$ :

$$j_{h,corr,1}^{j_1,j_2}(x_1,x_2;t) = (1-x_1-x_2) j_h^{j_1}(t=0) j_{j_1}^{j_1}(t) j_h^{j_2}(t=0) j_{j_2}^{j_2}(t)$$

$$j_{h,corr,2}^{j_1,j_2}(x_1,x_2;t) = \int_0^t dt j_h^j(t) P_j j_{j_1,j_2}^{j_1,j_2} j_{j_1}^{j_1}(t-t) j_{j_2}^{j_2}(t-t)$$



# Double Parton Distribution in LLA of pQCD

- $D_{ij}^j(x;t)$  represents the probability to find a parton  $j$  within a parton  $i$ ; it satisfies the evolution equation:

$$\frac{d}{dt} D_{ij}^j(x;t) = \frac{1}{t} P_{jj} D_{ij}^j(x;t)$$

with the initial condition  $D_{ij}^j(x;t=0) = \delta_{ij} (1-x)$ .

- The equation is solved by introducing the Mellin transforms

$$D_{ij}^j(n;t) = \int_0^1 dx x^n D_{ij}^j(x;t)$$

which lead to a system of ordinary differential equations at first order.

$$D_{ij}^j(x;t) = \int \frac{dn}{2\pi i} x^{-n} D_{ij}^j(n;t) = L^{-1} (D_{ij}^j(n;t) - \ln(x))$$

# Double Parton Distribution in LLA of pQCD

- Once  $\bar{d}_{h,j_1}^{j_1}(x;t)$  has been determined, the integral of the LAPD equation has performed using MRS99 as parametrization for parton distribution and Vegas algorithm for numerical integration.
- In the kinematical regime of interest ( $x$  never exceeds 0.1)  $\bar{d}_{h,corr,1}^{j_1 j_2}$  is negligible;

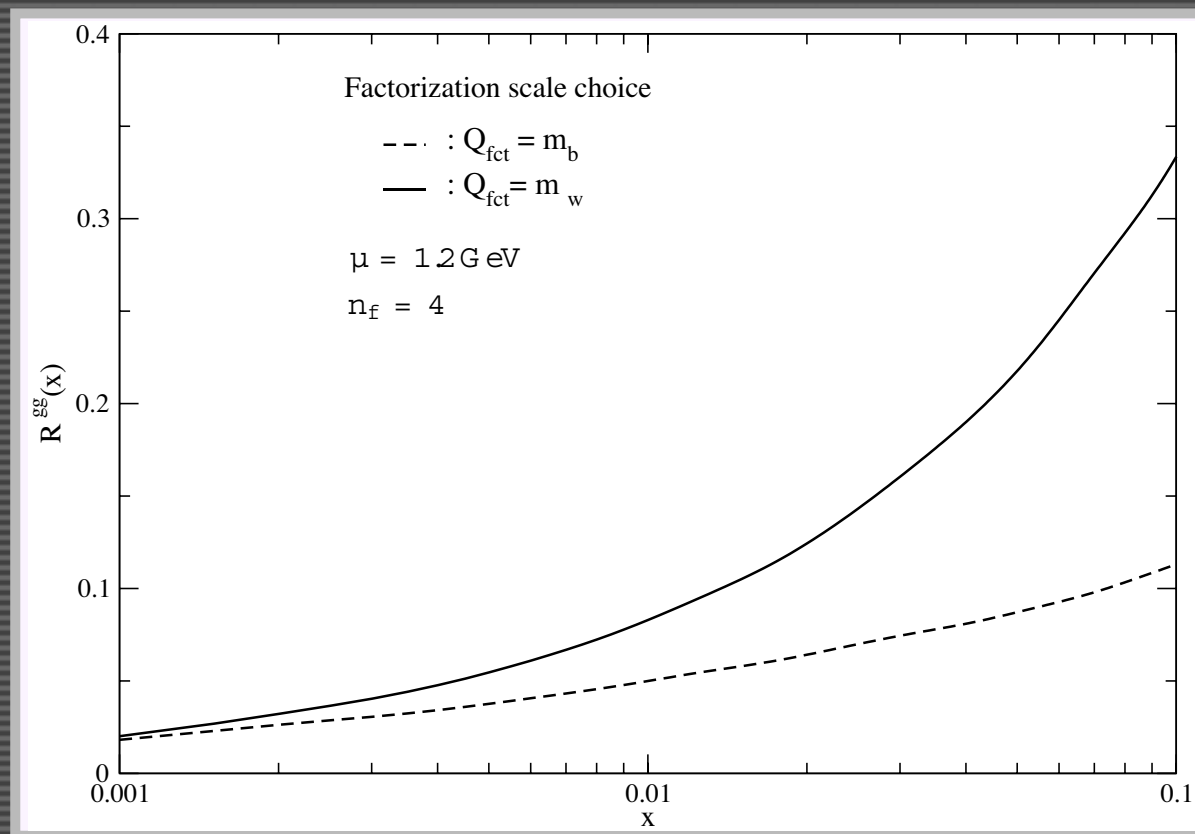
$$\bar{d}_{h,j_1}^{j_1}(x;t) \bar{d}_{h,j_2}^{j_2}(x;t) \left[ \frac{d}{dt} \bar{d}_{h,j_1 j_2}^{j_1 j_2} = \bar{d}_{h,j_1}^{j_1} P_{j_1 \rightarrow j_1} + \bar{d}_{h,j_2}^{j_2} P_{j_2 \rightarrow j_2} \right]$$

- Effect of correlation induced by evolution is estimated by the ratio

$$R^{j_1 j_2}(x_1, x_2; t) = \frac{\bar{d}_{h,corr,1}^{j_1 j_2}(x_1, x_2; t) + \bar{d}_{h,corr,2}^{j_1 j_2}(x_1, x_2; t)}{\bar{d}_{h,j_1}^{j_1}(x_1; t) \bar{d}_{h,j_2}^{j_2}(x_2; t)},$$

plotted as a function of  $x = x_1 = x_2$

# Numerical Results for the Correlation Ratio



## ○ Ratio $R^{\text{gg}}(x;t)$ :

- $Q_{\text{fct}} = M_W = 80.4 \text{ GeV}$ :

  - $R^{\text{gg}}$  35% for  $x = 0.1$
  - $R^{\text{gg}}$  8 – 10% for  $x = 0.01$
  - $R^{\text{gg}}$  2% for  $x = 0.001$
- $Q_{\text{fct}} = M_b = 4.6 \text{ GeV}$ :

  - $R^{\text{gg}}$  10 – 12% for  $x = 0.1$
  - $R^{\text{gg}}$  5% for  $x = 0.01$
  - $R^{\text{gg}}$  2% for  $x = 0.001$

# Numerical Results for the Correlation Ratio

## Ratio $R^{qq}(x, t)$

$Q_{fct} = M_W = 80.4 \text{ GeV} :$

$R^{qq} \quad 35\% \text{ for } x = 0.1$

$R^{qq} \quad 20\% \text{ for } x = 0.01$

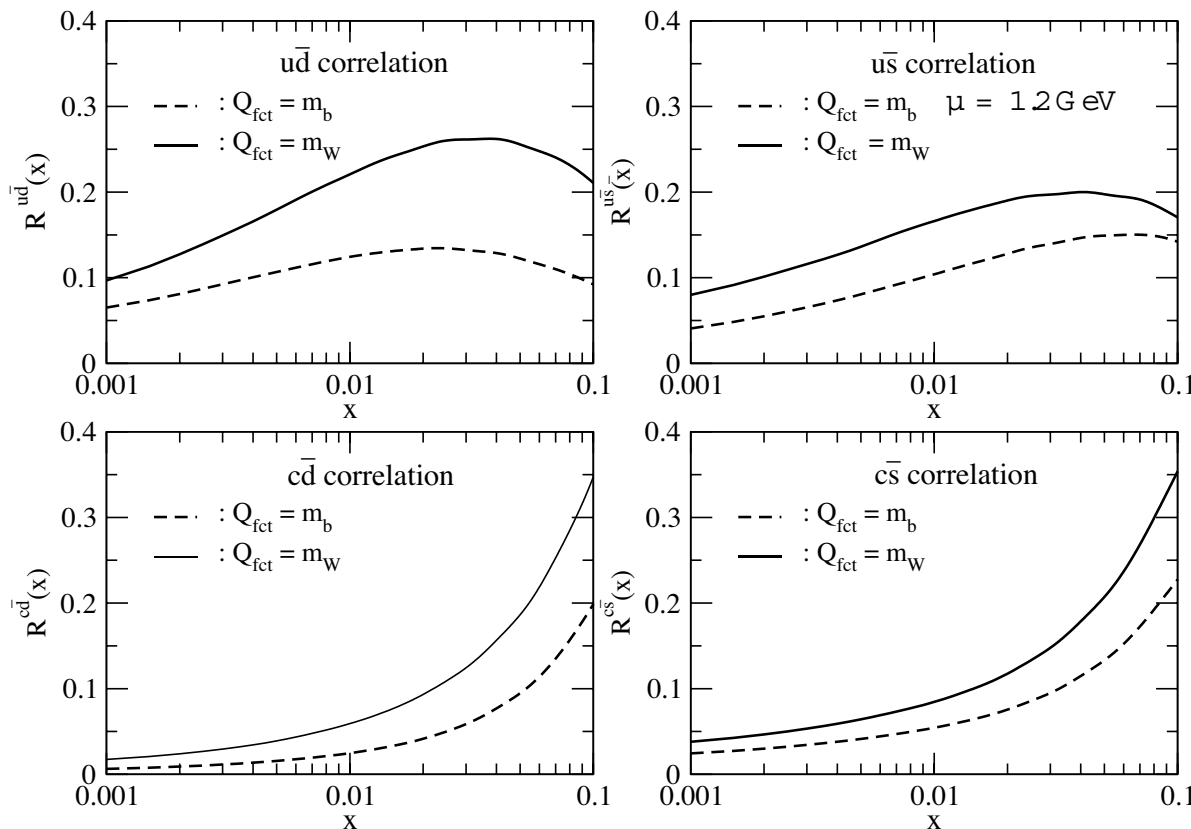
$R^{qq} \quad 10\% \text{ for } x = 0.001$

$Q_{fct} = M_b = 4.6 \text{ GeV} :$

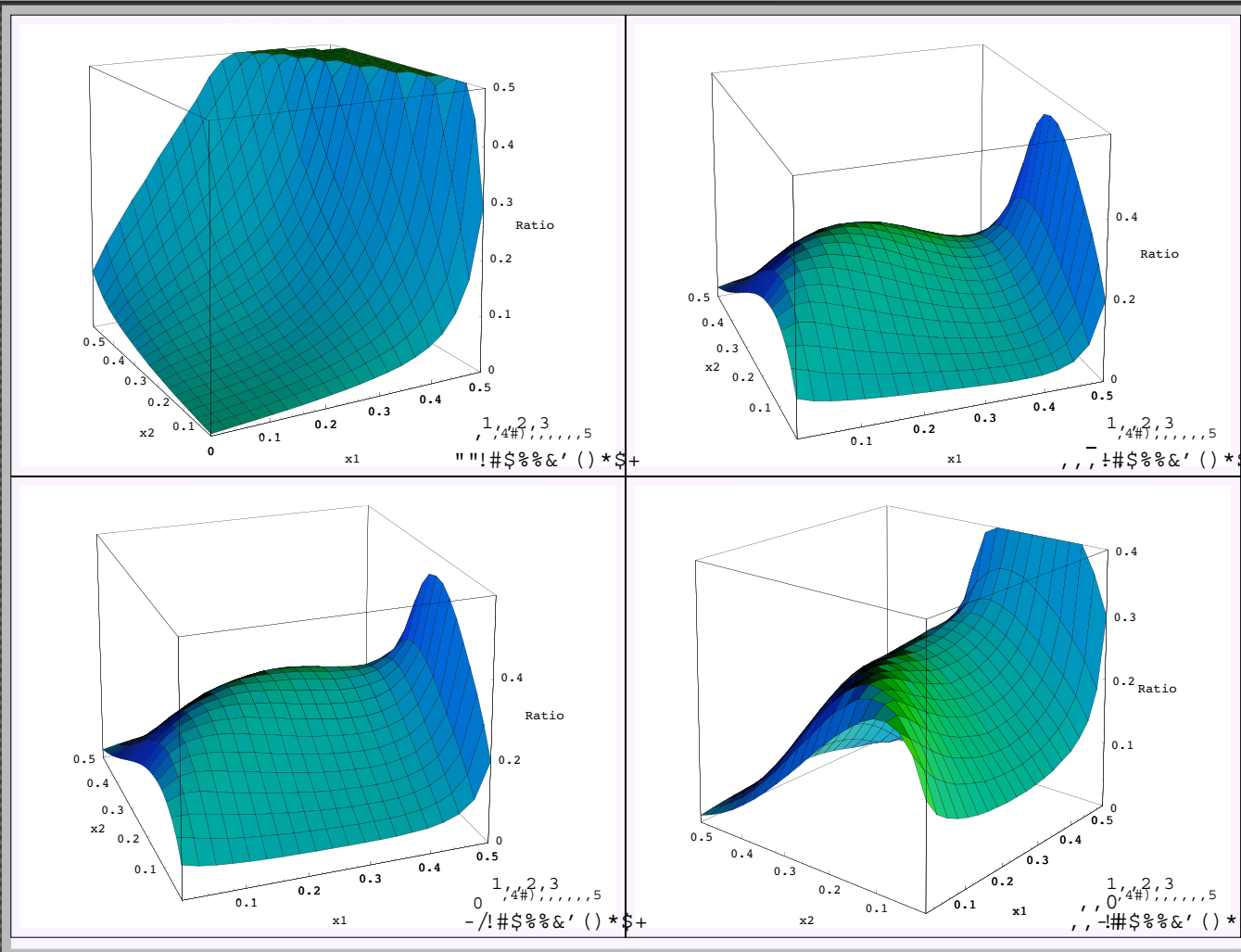
$R^{qq} \quad 23\% \text{ for } x = 0.1$

$R^{qq} \quad 10\% \text{ for } x = 0.01$

$R^{qq} \quad 5\% \text{ for } x = 0.001$



# Numerical Results for the Correlation Ratio



# Double Parton Distribution in LLA of pQCD

## ○ Remark:

- The non-perturbative input of double-parton scattering cross section is not represented by the distribution functions  $\hat{\sigma}_h^{j_1 j_2}(\mathbf{x}; t)$ , where the transverse variables have been integrated;
- Double parton scattering cross section depends on relative separation of partons in transverse space  $\Rightarrow$  outside control of pQCD;
- Given the different origin of the terms in  $\hat{\sigma}_h^{j_1 j_2}(\mathbf{x}; t)$ , it is not unnatural to have different non-perturbative scales for the transverse separation of the factorized and of the correlated terms;

## ○ Assumption:

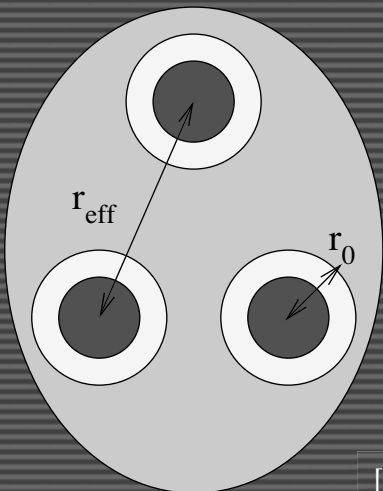
- 1) in  $\hat{\sigma}_{h, \text{fact}}^{j_1 j_2}$  and  $\hat{\sigma}_{h, \text{corr}, 1}^{j_1 j_2} \Rightarrow$  CDF low resolution scale process
- 2) in  $\hat{\sigma}_{h, \text{corr}, 2}^{j_1 j_2} \Rightarrow$  scale related to size of gluon cloud of a valence quark

# Double Parton Distribution in LLA of pQCD

$$j_1 j_2(x_1, x_2, b; t) = j_1 j_2_{h, \text{fact}}(x_1, x_2; t) + j_1 j_2_{h, \text{corr}, 1}(x_1, x_2; t) F_{\text{eff}}(b) + j_1 j_2_{h, \text{corr}, 2}(x_1, x_2; t) F_r(b)$$

where the parton pair densities satisfy:

$$d^2 b F_i(b) = 1 \quad d^2 b F_i(b)^2 = \frac{1}{i} \quad \text{with } i = \text{eff}, r.$$



- $F_{\text{eff}}$ : transverse density of partons at resolution scale leading to scale factor  $\text{eff} = 14.5 m b$ ;
  - $F_r$ : transverse density for partons correlated in fractional momenta, important at higher resolution scale;
- $r = r_0 + \text{eff}$ , with  $r_0 = 2.8 m b$  related to the size of the gluon cloud of a valence quark in the hadron.<sup>[3]</sup>

[3] B. Povh, Nucl. Phys. A 699, 226 (2002)



# Multiple Production of $b\bar{b}$ pairs

## ○ Multiple production of $b\bar{b}$ pairs and $W$ bosons in pp-collisions:

1)  $pp \rightarrow k\bar{k}k\bar{k}$  :

- higher order corrections in  $\sigma_s$  are very important ; the whole effect of higher order corrections is reduced to a single numerical value, K factor

$$K = \frac{\langle \sigma \rangle}{\sigma_{LO}}$$

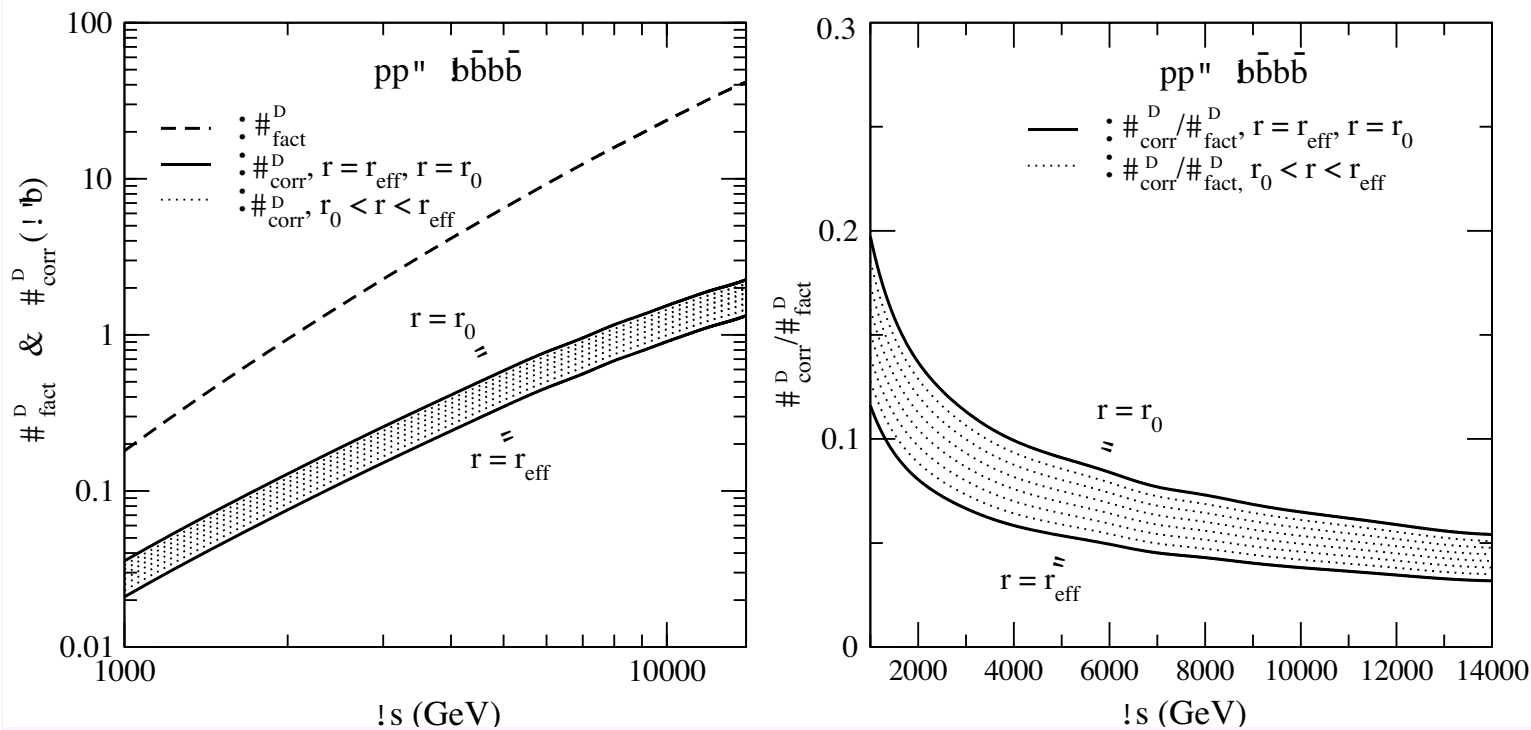
- Our assumption is that higher order corrections in double  $b\bar{b}b\bar{b}$  production can be taken into account by multiplying the cross section of each connected process by the same K factor

- In addition to usual factorized contribution, scaling with  $\sigma_{eff}^{-1}$ , double cross section includes not factorized contributions related to the couplings of  $h_{corr,2}^{ik}$  both with  $h_{fact}^{jl}$  and  $h_{corr,2}^{jl}$



# Numerical Results at LHC: $b\bar{b}$ production cross-section and correlation effect

[4] E. Cattaruzza, A. Del Fabbro and D. Treleani, Phys. Rev. D 70, 034022 [arXiv:hep-ph/0404177].



1)  $K_{\text{factor}} = 5.3$  [4]

2)  $M_{\text{RS99}}$  for PDF

3) Scale choices:

$$Q_{\text{fact}} = Q_{\text{ren}} = m_b$$

4) Gaussian distrib. for  $F_{\text{eff}}$  and  $F_r$

$$\frac{1}{\sigma_{\text{eff}}} \frac{1}{\sigma_{\text{eff}} + \sigma_r}$$

scale factors

for  $r = r_{\text{eff}}, r_0$ : (12 – 20 % at  $\sqrt{s} = 1 \text{ TeV}$  & 3.5 – 6 % at  $\sqrt{s} = 14 \text{ TeV}$ )

- Effect of correlations decreases increasing c.m; the decrease is faster as  $\sqrt{s} \rightarrow 5 \text{ TeV}$ : for larger c.m  $\sigma$  becomes smaller than 0.01 where  $R^{\text{gg}} \approx 0.03$

# Multiple Production of W pairs

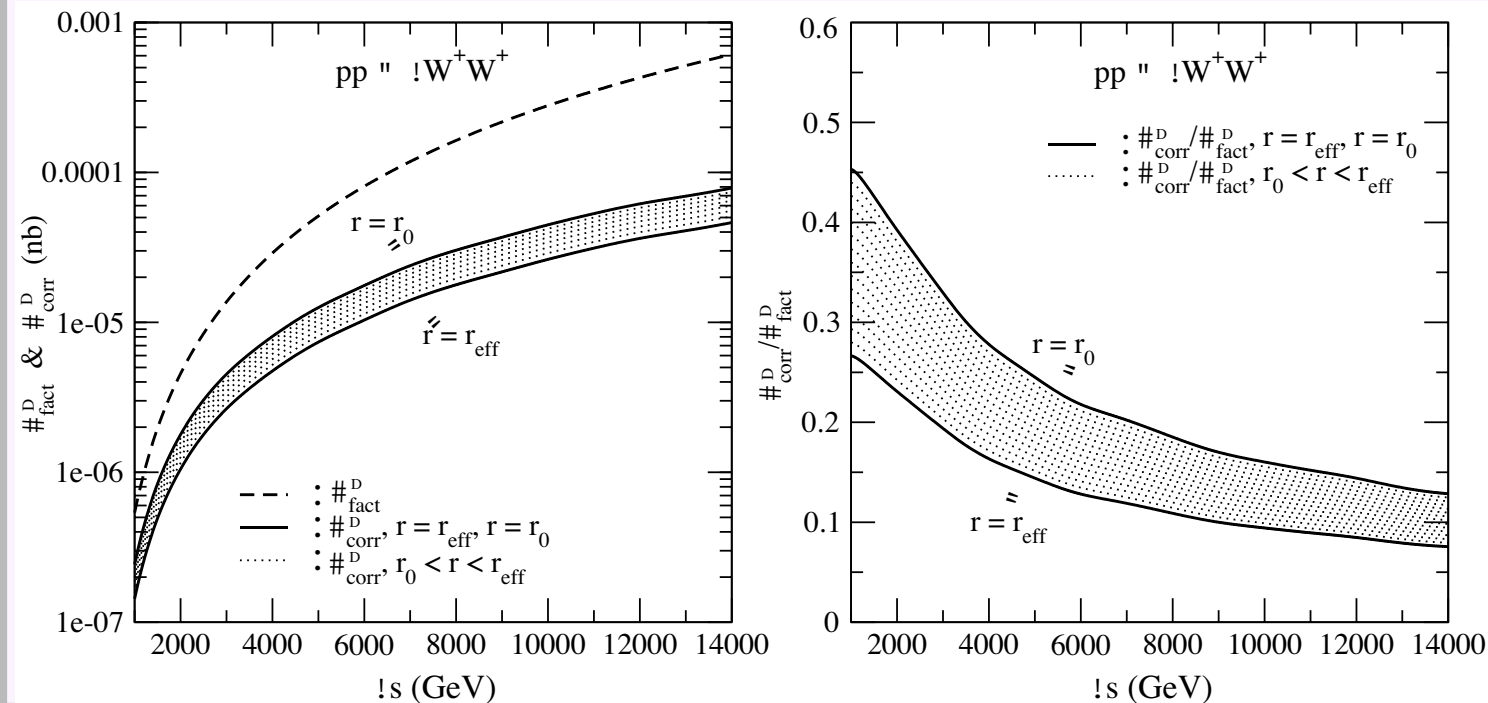
2)  $pp \rightarrow W^+ W^+, W^- W^-$  :

- the cross-sections of like-sign W pair have been evaluated at the leading order, including only quark initiated processes in the elementary interaction ( $q\bar{q} \rightarrow W$ )
- higher order corrections are taken into account multiplying the lowest order correction by the K factor<sup>[5]</sup>

$$K = 1 + \frac{8}{9} s(M_W^2)$$

[5] V. D. Barger, R. J. N. Phillips, Collider Physics (updated edition), Addison- Wesley Publishing Company, Inc. (1996), p.247.

# Numerical Results at LHC: production cross-section and correlation effect



1) MRS99 for PDF

2) Scale choices:

$$Q_{\text{fact}} = Q_{\text{ren}} = M_W$$

3) Gaussian distrib.  
for  $F_{\text{eff}}$  and  $F_r$

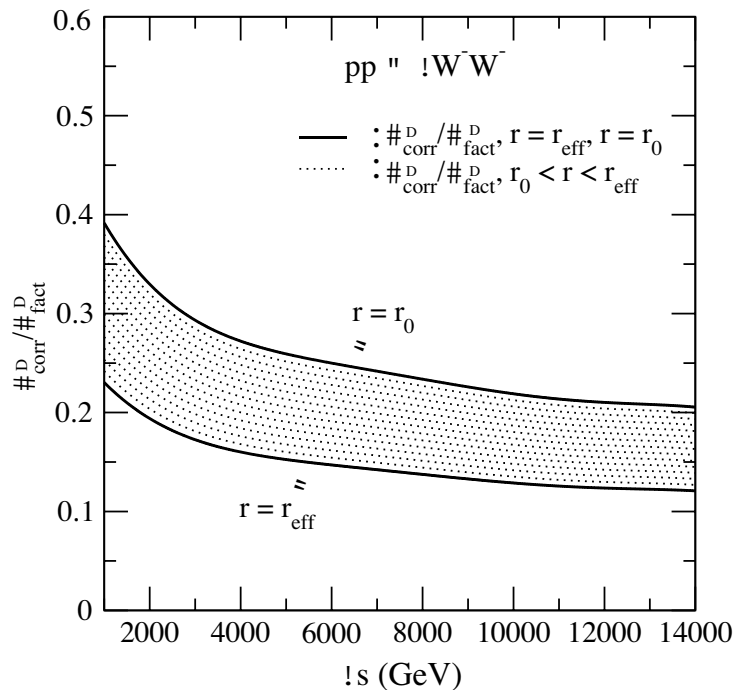
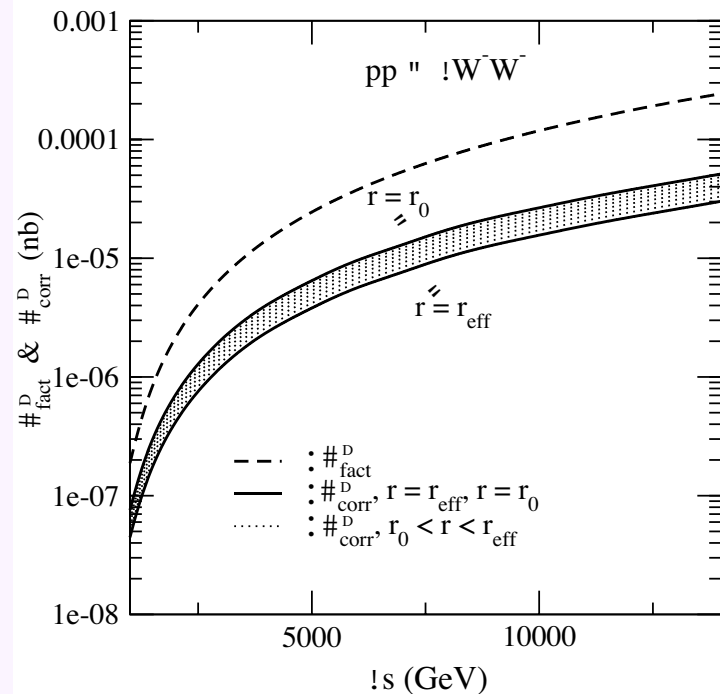
$$\frac{2}{\sigma_{\text{eff}}} \quad \frac{2}{(\sigma_{\text{eff}} + \sigma_r)}$$

scale factors

$$\sigma = 0.26 \times 10^{-2} \text{ as } \sqrt{s} = 1.4 \text{ TeV}$$

$\frac{\sigma_{\text{corr}}}{\sigma_{\text{fact}}}$	27 – 45%	7.5 – 13%	for $W^+$
$\frac{\sigma_{\text{corr}}}{\sigma_{\text{fact}}}$	23 – 40%	12 – 20%	for $W^-$

# Numerical Results at LHC: production cross-section and correlation effect



1) MRS99 for PDF

2) Scale choices:

$$Q_{\text{fact}} = Q_{\text{ren}} = M_W$$

3) Gaussian distrib.  
for  $F_{\text{eff}}$  and  $F_r$

$$\frac{2}{\sigma_{\text{eff}}} \sqrt{\frac{2}{(\sigma_{\text{eff}}^2 + \sigma_r^2)}}$$

scale factors

$$\sigma = 0.26 \times 10^{-2} \text{ as } \sqrt{s} = 1.4 \text{ TeV}$$

$\frac{\#_{\text{corr}}^D}{\#_{\text{fact}}^D}$	27 – 45%	7.5 – 13%	for $W^+$
$\frac{\#_{\text{corr}}^D}{\#_{\text{fact}}^D}$	23 – 40%	12 – 20%	for $W^-$

# Conclusions

- As an effect of evolution, the multiparton distributions are expected to become strongly correlated in momentum fraction at large  $Q^2$  and finite  $x$ ;
- High resolution scale multiparton process (equal sign W pair production) and a smaller resolution scale process ( $b\bar{b}b\bar{b}$  production) in pp collisions in the energy range  $\sqrt{s} \in [1, 14] \text{ TeV}$  have been considered;
- Disconnected contributions to the cross-section, after evolving multiparton distribution at desired resolution scale, have been evaluated;
- Terms with correlation, in equal sign W pair production, are at most 40% of the cross-section at 1 TeV and a 20% effect at LHC
- The effect is much smaller in  $b\bar{b}b\bar{b}$  production, where corrections are of the order of 5%.